

1 *A comparison study of mixed convection heat transfer of turbulent nanofluid*  
2 *flow in a three-dimensional lid-driven enclosure with a clockwise versus an*  
3 *anticlockwise rotating cylinder*  
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8 **Abstract**

9 *A turbulent 3D mixed convective flow of pure water, H<sub>2</sub>O, and nanofluid, SiO<sub>2</sub>-H<sub>2</sub>O, inside a*  
10 *differentially heated moving wall enclosure containing an insulated rotating cylinder over a*  
11 *range of rotational speeds,  $-5 \leq \Omega \leq 5$ , Reynolds numbers, 5000 and 10000, and constant*  
12 *Grashof number, is numerically investigated. A cooled lid-driven top wall and a heated bottom*  
13 *wall are the only thermally uninsulated walls in this domain. A standard  $k$ - $\epsilon$  for the Unsteady*  
14 *Reynolds-Averaged Navier-Stokes (URANS) approach is applied to the turbulence*  
15 *calculation. Nusselt number, mean velocity profile, streamline, isothermal and isosurface*  
16 *temperatures are derived and presented in this paper to gain a better understanding of the*  
17 *effects of clockwise and anti-clockwise rotating cylinder directions on the heat transfer and*  
18 *flow patterns. Interesting changes in flow structure and heat transfer have been analysed for*  
19 *all rotational speeds and fluid types at both Reynolds number values. Nonlinear increases in*  
20 *Nusselt number have been observed by using nanofluid instead of pure water. The wall shear*  
21 *stress and turbulent kinetic energy profiles are found to be influenced by changing the*  
22 *Reynolds number and rotational speed and direction. Furthermore, incremental heat transfer*  
23 *rates at the walls can be achieved by increasing the cylinder rotation speeds, but these*  
24 *increases have weaker influences on the top wall than on the bottom wall.*

25  
26 **Keywords:** *Mixed convection, Rotating cylinder, Lid-driven cavity, Turbulent flow, URANS.*

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32 **1. Introduction**

33 The mixed convective flow of moving wall cavity is produced as a result of both natural and  
 34 forced convection. Since heat convection can be represented in many cavity configurations it  
 35 has many industrial and engineering applications such as electronic cooling, lubrication  
 36 technologies, oil extraction, solar collectors, food processing [1-8], many researchers have  
 37 investigated various free or mixed convection problems. Involving some additional passive  
 38 objects within the enclosure to enhance the heat transfer ratio has become popular over the  
 39 years. Sun, et al. [9] added a triangular fin to control the heat transfer of the mixed convection  
 40 case.

<b>Nomenclature</b>			
CFL	Courant–Friedrichs–Lewy number	Y	distance along the non-dimensional y-coordinate (y/H)
D	width of the cavity on z-axis (m)	Z	distance along the non-dimensional z-coordinate (z/D)
d	cylinder diameter (m)		
FVM	finite volume method		
Gr	Grashof number ( $g\beta_m\Delta TW^3/\nu_m^2$ )	<i>Greek symbols</i>	
h	convective heat transfer coefficient ( $W/m^2K$ )	$\alpha$	thermal diffusivity of the fluid ( $m^2/s$ )
k	turbulent kinetic energy ( $m^2/s^2$ )	$\beta$	volumetric coefficient of thermal expansion ( $1/K$ )
L	width of the cavity on x-axis (m)	$\mu$	dynamic viscosity of the fluid (Pa/s)
Nu	Nusselt number	$\nu$	kinematic viscosity of the fluid ( $m^2/s$ )
Pr	Prandtl number ( $\nu_m/\alpha_m$ )	$\rho$	density of the fluid ( $kg/m^3$ )
Re	Reynolds number ( $U_{0,m}W/\nu_m$ )	$\varepsilon$	dissipation rate of turbulent kinetic energy ( $m^2/s^3$ )
Ri	Richardson number ( $Gr/Re^2$ )	$\omega$	rotational speed (rad/s)
$\bar{S}_{ij}$	large-scale strain rate tensor for grid-filter	$\Omega$	dimensionless rotational speed
T	temperature of the fluid (K)		
t	time		
u	velocity component at x-direction (m/s)		
U	dimensionless velocity component at x-direction	<i>Subscripts</i>	
$U_0$	lid velocity (m/s)	av	average value
v	velocity component at y-direction (m/s)	b	buoyancy
V	dimensionless velocity component at y-direction	C	value of cold temperature
W	dimensionless velocity component at z-direction	H	value of hot temperature
x	distance along the x-coordinate	rms	root mean square
X	distance along the non-dimensional x-coordinate (x/L)	sgs	sub-grid scale
		t	turbulent

41

42 Inserting a circular body to enhance the heat transfer of a lid-driven cavity was investigated  
 43 by Oztop, et al. [10]. Appending a heated triangular block at the centre the top moving wall  
 44 square enclosure has an impact on the heat transfer coefficient [11]. Studying the effects of

45 both inserting an isothermal square object and using a nanofluid inside a lid-driven square  
46 enclosure on the heat transfer ratio was undertaken by [Mehmood, et al. \[12\]](#). Nonetheless, it  
47 may be noticed that neither the 3D domain nor turbulent flow condition has been used in the  
48 above references.

49 Moreover, some studies of mixed convection have been completed by introducing one cylinder  
50 or more within the enclosures to control and increase the heat transfer ratio. A 2D laminar  
51 mixed convection heat transfer of nanofluid within a moving wall square enclosure containing  
52 a rotating cylinder was considered by [Mirzakhani, et al. \[13\]](#) to study the impacts of the  
53 Richardson number and rotational speed. It was demonstrated that increasing either the  
54 Richardson number or the nanofluid speed causes an enhancement in heat transfer ratio,  
55 while increasing the rotational speed of the cylinder has a negative effect on the heat transfer  
56 ratio. A 2D free convection study of a laminar power-law fluid within a square cavity containing  
57 a heated cylinder was completed by [Shyam, et al. \[14\]](#), which concentrated on the effects of  
58 changing the cylinder location along the vertical central line for different dimensionless  
59 parameters. It was realised that the heat transfer ratio and streamline and isotherm contours  
60 can be affected by changing the Grashof and Prandtl number or cylinder location. A study of  
61 2D laminar natural convection of air within a cold-walled square enclosure containing a  
62 stationary sinusoidal cylinder was undertaken by [Nabavizadeh, et al. \[15\]](#) to evaluate the  
63 influences of different angles, amplitudes and number of cylinder undulations. It was observed  
64 that changing cylinder parameters can affect the heat transfer and fluid patterns. Three-  
65 dimensional free convection of laminar flow within an enclosure containing a cylinder was  
66 simulated by [Souayeh, et al. \[16\]](#) in order to understand the impact of inclination angles of the  
67 cylinder at different Rayleigh numbers on the fluid patterns and heat transfer ratio. It was noted  
68 briefly that a significant effect was discovered regarding the heat transfer ratio, especially  
69 when the Rayleigh number is  $10^6$  and inclination angle is  $90^\circ$ . Limited research has been  
70 completed into mixed convection turbulent flow within lid-driven enclosures by utilizing an  
71 unsteady approach such as URANS (Unsteady Reynolds-Averaged Navier-Stokes) and LES  
72 (Large Eddy Simulation). Combined convection heat and mass transfer of water within a lid-  
73 driven cavity was studied by [Kareem, et al. \[1\]](#) to analyse the 3D flow structure and heat  
74 transfer by involving unsteady RANS and LES methods at various Reynolds numbers. The  
75 results have shown the ability of both methods to deal with the vortices of the turbulent flow.  
76 Nevertheless, secondary eddies were dealt with more comprehensively by utilizing the LES  
77 method. In addition, it was concluded that a remarkable effect on the heat transfer ratio and  
78 fluid patterns can be observed when the Reynolds number increases. Two different turbulent  
79 methods are used by [Kareem and Gao \[17\]](#) to study a combined mixed convection of different  
80 nanofluid types within a 3D moving sidewalls enclosure. It was demonstrated that heat transfer

81 and flow pattern can be influenced by adding nanoparticles to the pure fluid, and that the  
82 influence of the ratio could be changed by using different types, diameters and concentrations  
83 of nanoparticles. A clear difference in heat transfer rate has also been noticed by utilizing  
84 different turbulence methods.

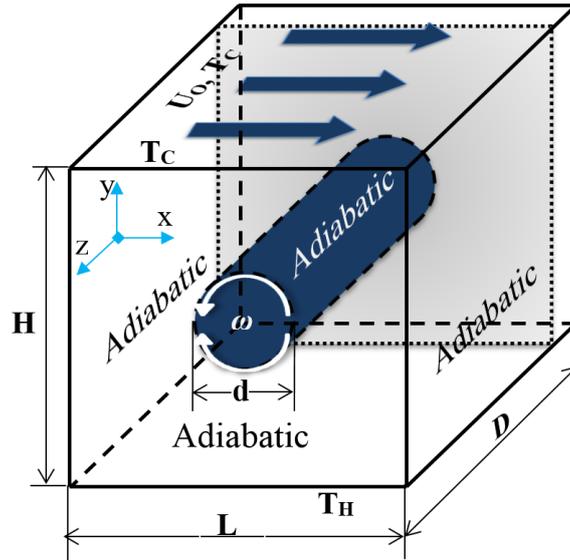
85 Studies into various nanofluid types within cavities have been described by a number of  
86 researchers because of its considerable effect on heat transfer enhancement. Investigation of  
87 free and combined convective heat transfer of a differentially heated circular cylinders within  
88 an adiabatic cavity containing nanofluid was studied by [Garooi and Hoseininejad \[18\]](#). The  
89 influences of nanofluid thermophysical properties and the number of cold cylinders that the  
90 cavity contained and their location and rotational directions are considered at different  
91 Rayleigh and Richardson numbers. It has been found that increasing or decreasing heat  
92 transfer rate strongly depends on these parameters. [Kareem, et al. \[19\]](#) studied a laminar  
93 mixed convection of heat and mass transfer in a 2D trapezoidal moving wall enclosure that is  
94 filled with different types of nanofluid in a numerical manner. The authors aimed at an  
95 understanding of the effects of nanofluid type, nanoparticle diameter, inclined sidewall angle,  
96 aspect ratio, flow direction and Richardson number on the heat transfer ratio and flow  
97 distribution. It was concluded that SiO<sub>2</sub>-H<sub>2</sub>O showed the highest Nusselt number and aiding  
98 flow direction provides for a higher heat transfer ratio. In addition, it has been found that  
99 increasing the nanoparticle concentration and aspect ratio leads to an increase in the heat  
100 transfer coefficient, unlike when the nanoparticle diameter and inclination angle are increased.

101 It can be summarized from the current literature review that a study considering the effects of  
102 three-dimensional rotating cylinders in terms of their speeds and directions (clockwise and  
103 anticlockwise) within a top lid-driven closed cavity on turbulent nanofluid flow, and involving  
104 the unsteady RANS method, is unprecedented, and this consequently forms the main  
105 objectives of this paper.

## 106 **2. Numerical model**

### 107 *2.1. Physical model*

108 A sketch of the three-dimensional flow within a lid-driven cavity configuration containing a  
109 rotating cylinder is shown in [Fig. 1](#). This study deals with an adiabatic central cylinder of  
110 diameter,  $d = 0.2 L$ , that rotates in the anticlockwise and clockwise directions within the lid-  
111 driven cavity with a heated bottom wall and moving cooled wall. The other remaining cavity  
112 walls are assumed to be adiabatic and stationary.



113

114

Fig. 1. Schematic of the analysed configuration.

115 **2.2. Governing equations**

116 An incompressible Newtonian fluid of unsteady flow has been used in this paper. The three-  
 117 dimensional equations governing the continuity, momentum and energy are presented as  
 118 follows [1, 20, 21]:

Continuity equation:

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{1}$$

Momentum equation:

$$\frac{\partial u_i}{\partial t} + \frac{\partial(u_j u_i)}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{1}{Re} \left( \frac{\partial^2 u_i}{\partial x_i \partial x_i} \right) + \frac{Gr}{Re^2} \theta \tag{2}$$

Energy equation:

$$\frac{\partial(\theta)}{\partial t} + \frac{\partial(u_j \theta)}{\partial x_j} = \frac{1}{RePr} \left( \frac{\partial^2 \theta}{\partial x_i \partial x_i} \right) \tag{3}$$

Equations (4) and (5) below represent the turbulent kinetic energy (k) and the dissipation rate (ε) [22], Respectively.

The standard k-ε turbulence model is given by:

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(\rho k u_i) = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + P_k + P_b - \rho \varepsilon + S_k \quad (4)$$

$$\frac{\partial}{\partial t}(\rho \varepsilon) + \frac{\partial}{\partial x_i}(\rho \varepsilon u_i) = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + C_{1\varepsilon} \frac{\varepsilon}{k} (P_k + C_{3\varepsilon} P_b) - C_{2\varepsilon} \rho \frac{\varepsilon^2}{k} + S_\varepsilon \quad (5)$$

where  $C_{1\varepsilon}$ ,  $C_{2\varepsilon}$  and  $C_{3\varepsilon}$  are model constants and  $S_k$  and  $S_\varepsilon$  refer to the user-defined source terms. The remaining terms are written in equations (6-8).

Turbulent viscosity: 
$$\mu_t = \rho C_\mu \frac{k^2}{\varepsilon} \quad (6)$$

Production of k: 
$$P_k = -\rho \overline{u'_i u'_j} \frac{\partial u_j}{\partial x_i} \quad (7)$$

Effect of buoyancy: 
$$P_b = \beta g_i \frac{\mu_t}{Pr_t} \frac{\partial T}{\partial x_i} \quad (8)$$

### 119 2.3. Boundary conditions

120 The boundary conditions for the current study are defined as:

121 Top wall:

$$122 \quad \theta = 0, U = U_{\text{lid}}, V = 0, W = 0$$

123 Bottom wall:

$$124 \quad \theta = 1, U = 0, V = 0, W = 0$$

125 Other walls:

$$126 \quad \partial\theta / \partial Y = 0, U = 0, V = 0, W = 0$$

127 Adiabatic cylinder:

$$128 \quad \omega = \frac{\Omega \times 2U_0}{d}, d = 0.2L$$

### 129 2.4. Thermophysical properties of working fluids

130

131 **Table 1** shows the thermophysical properties of pure water and SiO<sub>2</sub> nanoparticles. This paper  
 132 uses a SiO<sub>2</sub> nanoparticle diameter of 25 nm and a volume fraction of 5%. The effective  
 133 thermophysical properties of the nanofluid formed are calculated by utilizing equations (9-17).

134

135 **Table 1**136 Thermophysical properties of pure water and SiO<sub>2</sub>.

	$\rho$ (kg/m <sup>3</sup> )	$C_p$ (J/kg. K)	$k$ (W/m. K)	$\mu$ (Ns/m <sup>2</sup> )	$\beta \times 10^{-5}$ (K <sup>-1</sup> )
Pure Water (H <sub>2</sub> O)	996.5	4181	0.613	0.0001	21
Silicon Dioxide (SiO <sub>2</sub> )	3970	765	36	–	0.63

137

The effective thermal conductivity can be obtained by using the following mean empirical correlations [23]:

$$k_{eff} = k_{Static} + k_{Brownian} \quad (9)$$

$$k_{Static} = k_f \left[ \frac{(k_{np} + 2k_f) - 2\phi(k_f - k_{np})}{(k_{np} + 2k_f) + \phi(k_f - k_{np})} \right] \quad (10)$$

$$k_{Brownian} = 5 \times 10^4 \beta \phi \rho_f C_{p,f} \sqrt{\frac{kT}{2\rho_{np} R_{np}}} f(T, \phi) \quad (11)$$

The effective viscosity can be obtained using the following mean empirical correlations [23]:

$$\mu_{eff} = \mu_{Static} + \mu_{Brownian} \quad (12)$$

$$\mu_{Static} = \mu_f / (1 - \phi)^{2.5} \quad (13)$$

$$\mu_{Brownian} = 5 \times 10^4 \beta \phi \rho_f \sqrt{\frac{kT}{2\rho_{np} R_{np}}} f(T, \phi) \quad (14)$$

where:

Boltzmann constant,  $k$ :  $k = 1.3807 \times 10^{-23}$  J/K

Modelling function,  $\beta$ :  $\beta = 0.0137(100\phi)^{-0.8229}$  for  $\phi < 1\%$   
 $\beta = 0.0011(100\phi)^{-0.7272}$  for  $\phi > 1\%$

Modelling function,  $f(T, \phi)$ :

$$f(T, \phi) = (-6.04\phi + 0.4075)T + (1722.3\phi)$$

The nanofluid density  $\rho_{nf}$  can be obtained from the following equation [23]:

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_{np} \quad (15)$$

where  $\rho_f$  and  $\rho_{np}$  are the mass densities of the base fluid and the solid nanoparticles, respectively.

The effective heat capacity at constant pressure of the nanofluid  $(C_p)_{nf}$  can be calculated from the following equation [23]:

$$(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_{np} \quad (16)$$

where,  $(C_p)_f$  and  $(C_p)_{np}$  are the heat capacities of the base fluid and nanoparticles, respectively.

The effective coefficient of thermal expansion for the nanofluid  $(\beta)_{nf}$  can be obtained from the following equation [24]:

$$(\rho\beta)_{nf} = (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_{np} \quad (17)$$

where  $(\beta)_f$  and  $(\beta)_{np}$  are thermal expansion coefficients of the base fluid and the nanoparticles, respectively.

138

## 139 2.5. Numerical procedure

140 Two different fluids have been used as working fluids inside the enclosure in order to compare  
141 the nanofluid with a conventional fluid. The present study has used the finite volume method  
142 and SIMPLIC algorithm to solve the governing equations of heat and mass transfer, and the  
143 pressure-velocity coupling equations by utilizing the commercial CFD code ANSYS©FLUENT  
144 (version R16.2) [25]. The convection and time evolution terms were dealt with using QUICK  
145 and an implicit second-order scheme. The standard k- $\epsilon$  turbulence model was used for the  
146 unsteady Reynolds-Averaged Navier-Stokes equations. The convergence criterion was  
147 chosen as  $10^{-5}$ .

## 148 2.6. Code validation

149 The present CFD models have been validated against a number of reports in the literature in  
150 order to obtain a trusted numerical solver for the new simulations reported in this paper. The  
151 first comparison was completed for the problem of the 2D steady mixed convection heat  
152 transfer of a lid-driven enclosure containing a clockwise rotating cylinder as originally reported  
153 by Chatterjee, et al. [26]. It can be observed from Fig. 2 and Fig. 3 that the comparisons of the  
154 dimensionless velocity profiles along the vertical line at  $x = 0.25$  and the isotherms and  
155 streamlines show a good agreement. Fig. 4 demonstrates that the second comparison was  
156 achieved successfully against the results originally reported by Alinia, et al. [27] for the  
157 problem of the 2D laminar mixed convection heat transfer of a nanofluid within an inclined  
158 moving wall cavity.

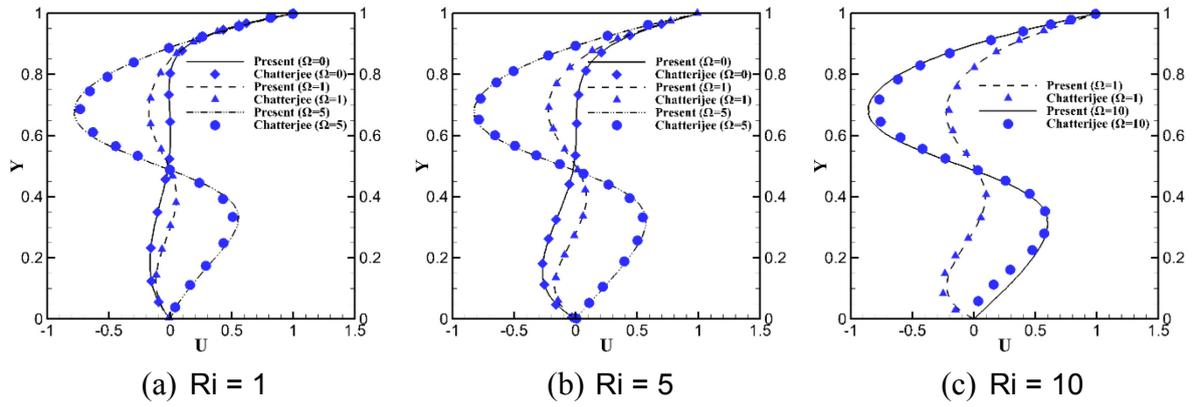


Fig. 2. Comparison of the present work of the dimensionless velocity profiles along the vertical line at  $x = 0.25$  with that of Chatterjee, et al. [26].

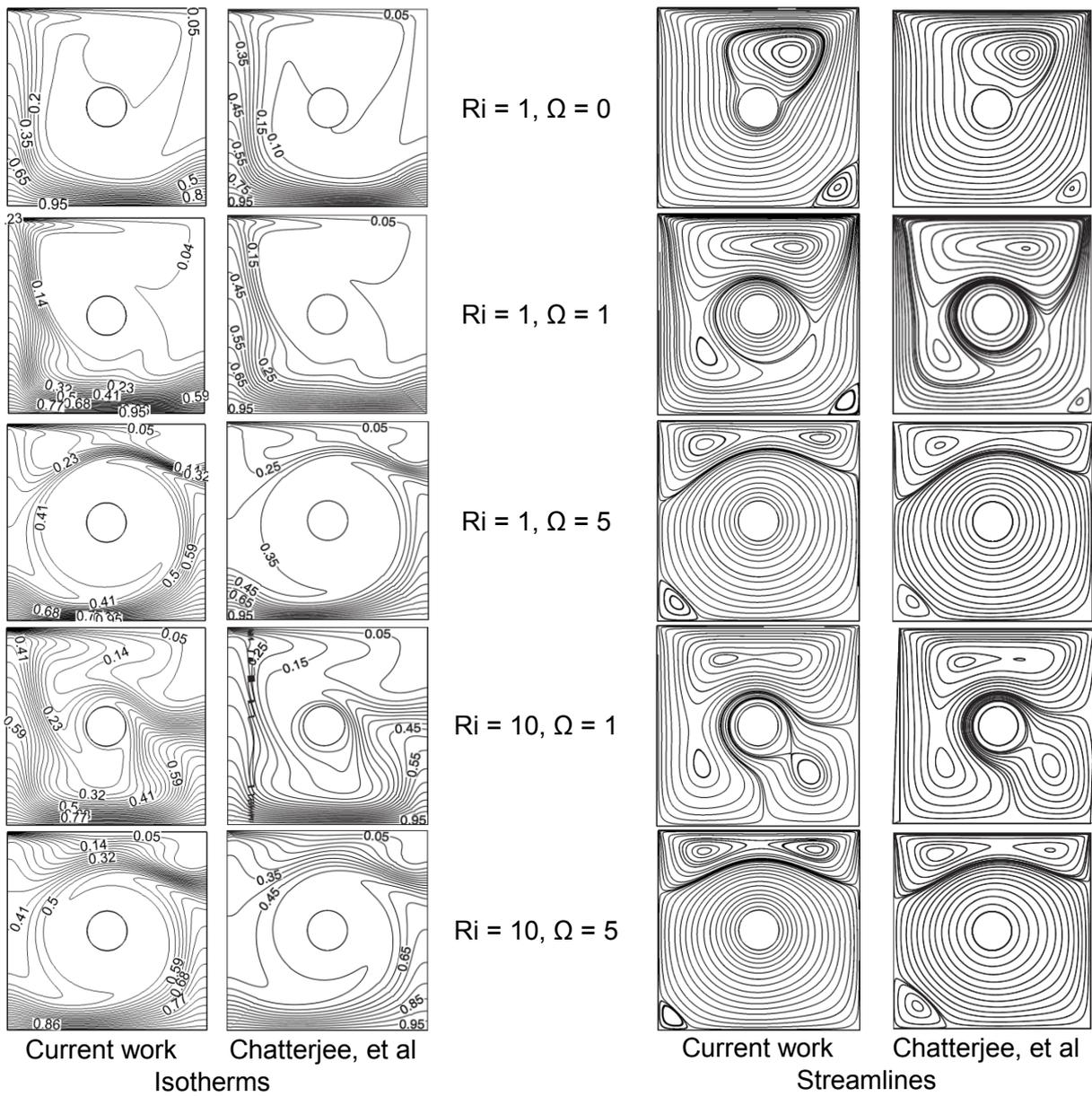
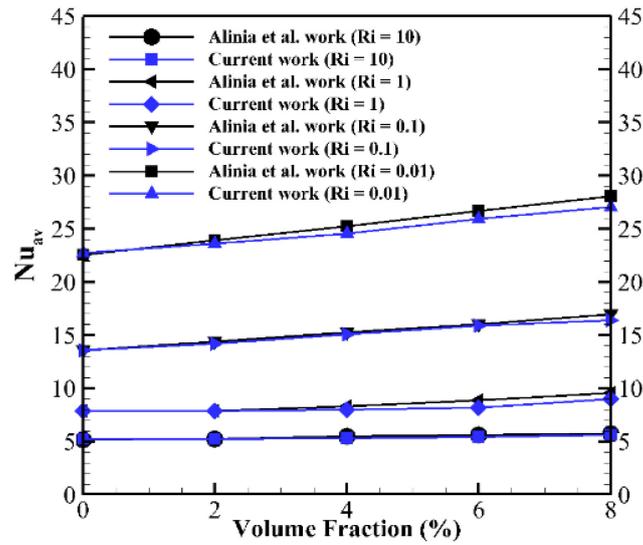


Fig. 3. Comparison of the present work of the isotherms and streamlines with Chatterjee, et al. [26].



160

161 **Fig. 4.** Comparison of the current results of different nanoparticle concentrations and different  
 162 Richardson numbers with Alinia, et al. [27].

163 **2.7. Grid independence test**

164 Structured and non-uniform grids that were considerably finer in the vicinity of the cavity walls  
 165 and around the cylinder are used to discretise the domain. Five different numbers of mesh  
 166 (125868, 292440, 496800, 929160 and 1260762) have been carefully tested to obtain the  
 167 most suitable mesh number and quality. The mesh number 929160 was used in this paper  
 168 because it was found to provide high orthogonal quality of between 0.7 – 1, which can help  
 169 obtain high-quality simulation results with suitable iteration time. For all the simulations  
 170 reported here, the Courant-Friedrichs-Lewy number (CFL) is kept below 0.3, the  
 171 dimensionless time step is 0.004, and the dimensionless wall distance of the first mesh near  
 172 the walls is about 1.

173 **3. Results and discussion**

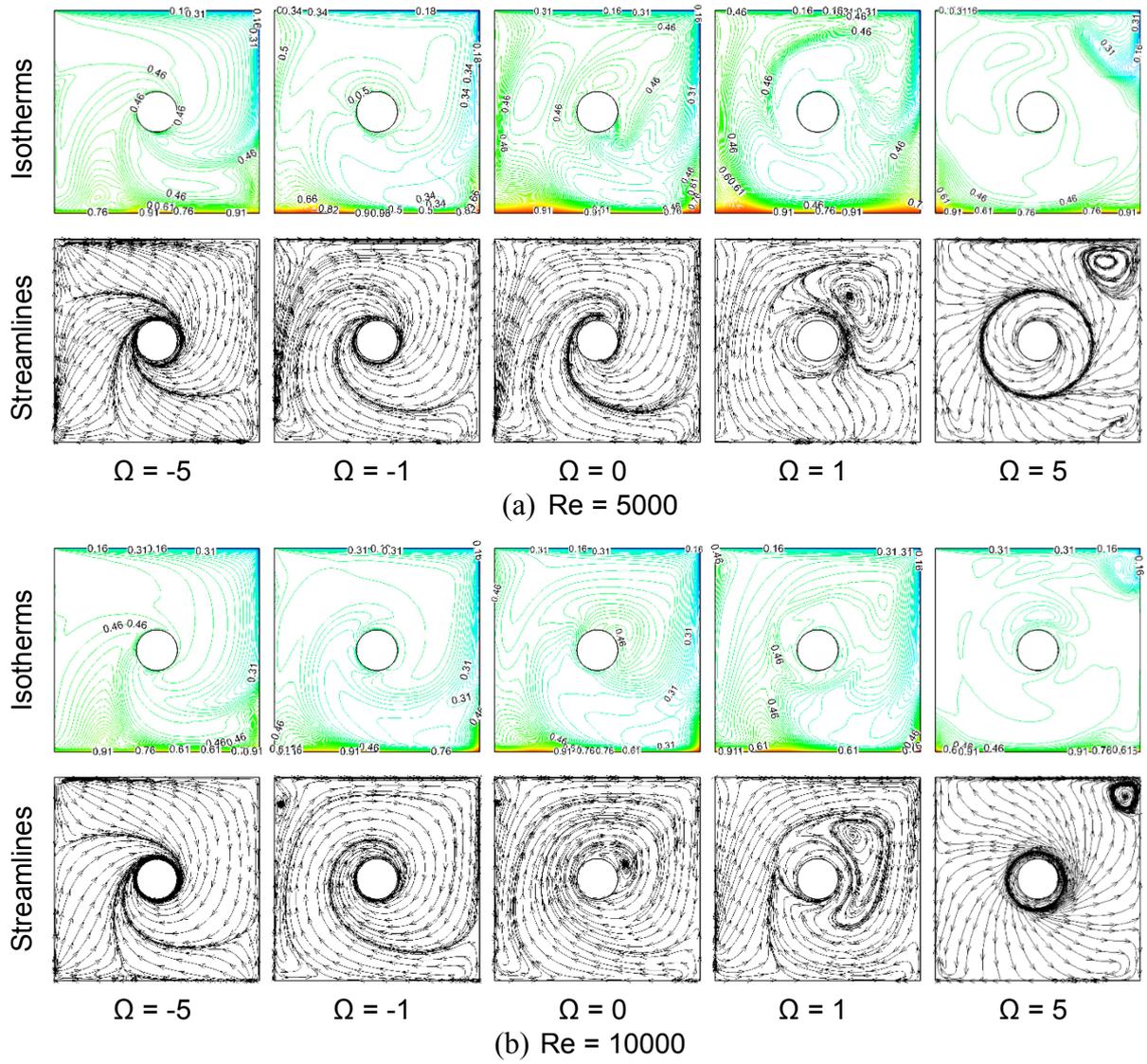
174 Three cases of rotational cylinder conditions have been addressed in this work. The first case  
 175 (Case1) is when the cylinder is stationary ( $\Omega = 0$ ). The second case (Case2) is when the  
 176 rotational direction of the cylinder is clockwise ( $\Omega < 0$ ), whereas the third case (Case3) is when  
 177 the cylinder rotates in the anti-clockwise direction ( $\Omega > 0$ ). For more information, when the  
 178 cylinder rotates in the clockwise direction, it favours the lid-driven motion, unlike when the  
 179 cylinder rotates in the anti-clockwise direction, which opposes the lid-driven wall movement.

180 **3.1. Flow and thermal fields**

181 **Fig. 5** and **Fig. 6**, respectively, display the isotherms and streamlines contours, and iso-surface  
 182 temperatures of pure water for Reynolds numbers  $Re = 5000$  and  $10000$ , and rotational

183 speeds,  $-5 \leq \Omega \leq 5$  to show the influences of rotational speed and direction on the turbulent  
184 flow in the cavity. Essentially, it was demonstrated that the effect of increasing the Reynolds  
185 number is more significant when the cylinder is rotating in the anticlockwise direction. By  
186 focusing on the rotational speed condition, it can be seen that for both Reynolds numbers, and  
187 when the rotational move and lid-driven move were in the same direction, the forces of both  
188 movements supported each other, which led to rotating the whole fluid field in the same  
189 direction and producing a reduced number of secondary vortices, unlike when the lid-driven  
190 motion and rotational motion were in oppose directions,  $1 \leq \Omega \leq 5$ , which produces more  
191 secondary eddies within the domain, especially at higher values of Reynolds number and  
192 rotational speed. At  $\Omega = 0$ , it was demonstrated that the force of the top wall movement was  
193 driving the flow structure beyond the minor influence of the buoyancy effect. In the comparison  
194 between pure fluid and nanofluid, it can be clearly observed from [Fig. 7](#) that involving a  
195 nanofluid has its positive influences on the flow patterns and heat distributions across all the  
196 study cases.

197



**Fig. 5.** Isotherms and streamlines contours for  $\phi = 0$  at different Reynolds numbers and rotational speeds.

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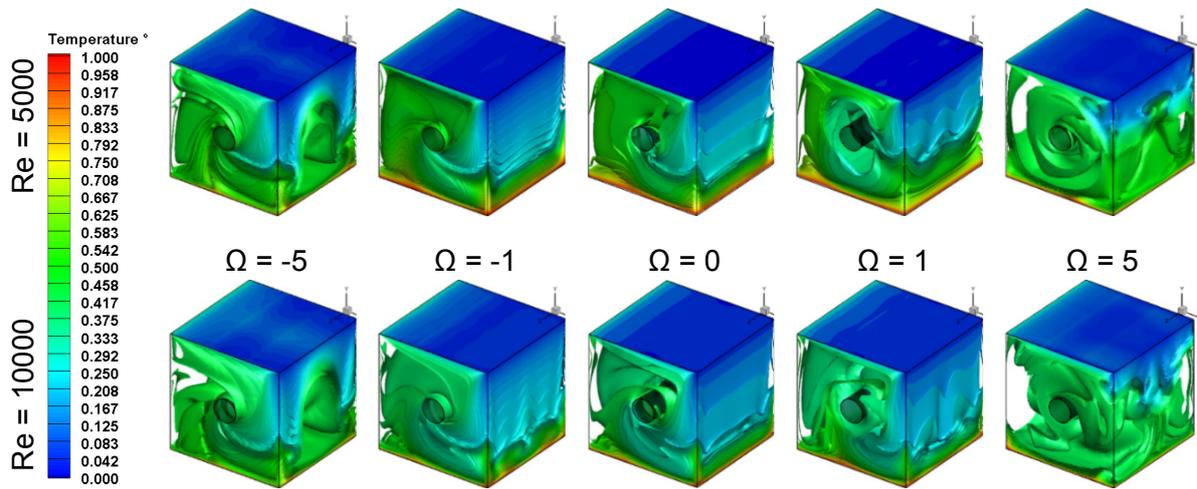


Fig. 6. Three-dimensional iso-surface profiles comparing the clockwise and anti-clockwise rotation of the cylinder for  $\phi = 0$ .

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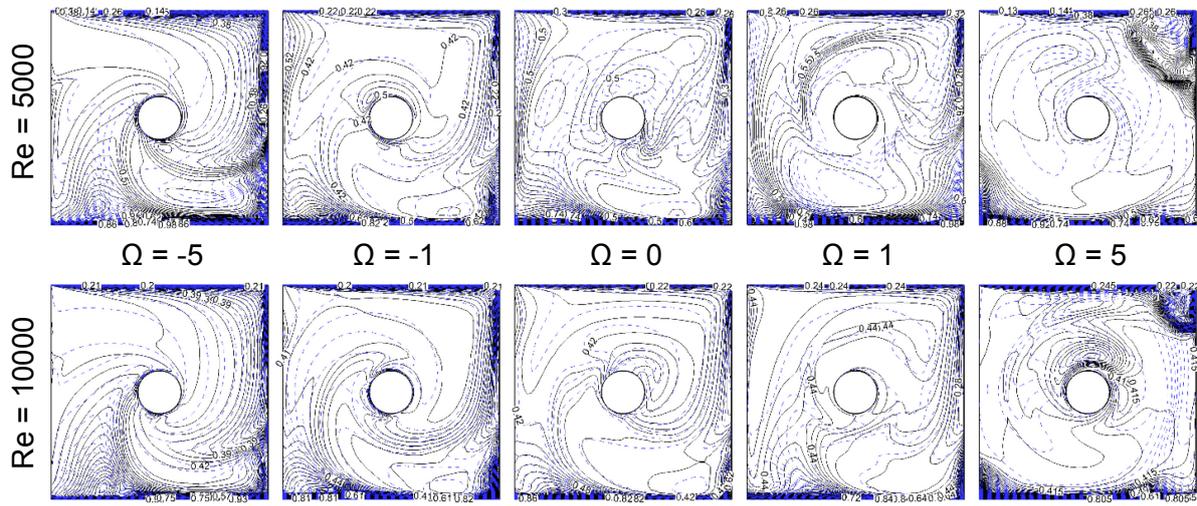


Fig. 7. Isotherm contour comparisons between pure fluid ( $\phi = 0$ , solid lines) and nanofluid ( $\phi = 0.025$ , broken lines) for different Reynolds numbers and rotational speeds.

202

### 203 3.2. Mean velocity profile

204 The variation of the mean velocity profiles of the horizontal lines at 0.25, 0, 0 and 0.25, 1, 0,  
 205 for the pure water and nanofluid at Reynolds numbers of  $Re = 5000 - 10000$  and rotational  
 206 speeds  $-5 \leq \Omega \leq 5$  are shown in Fig. 8. Generally, it can be observed that at all values of  
 207 Reynolds number and rotational speed, the velocity near the moving wall is controlled by the  
 208 lid movement. In addition, it might be noted that the effect of changing the rotating directions  
 209 on the mean velocity profiles, especially at the bottom half of the geometry ( $y$ -axis height is  
 210 0.5 m), is less influenced by the moving top wall. Moreover, at  $\Omega = 5$  and  $Re = 5000$ , it can be  
 211 concluded that the rotating cylinder is controlling the velocity profile over the whole domain,  
 212 unlike when the Reynolds number is increased to 10000, which limited the control of the

213 rotating cylinder to just the bottom part of the cavity. By effectively introducing nanoparticles  
 214 into the pure water, it can be seen that for both Reynolds number values, the rotational speed  
 215 of 5 had a significant influence on the whole flow domain.

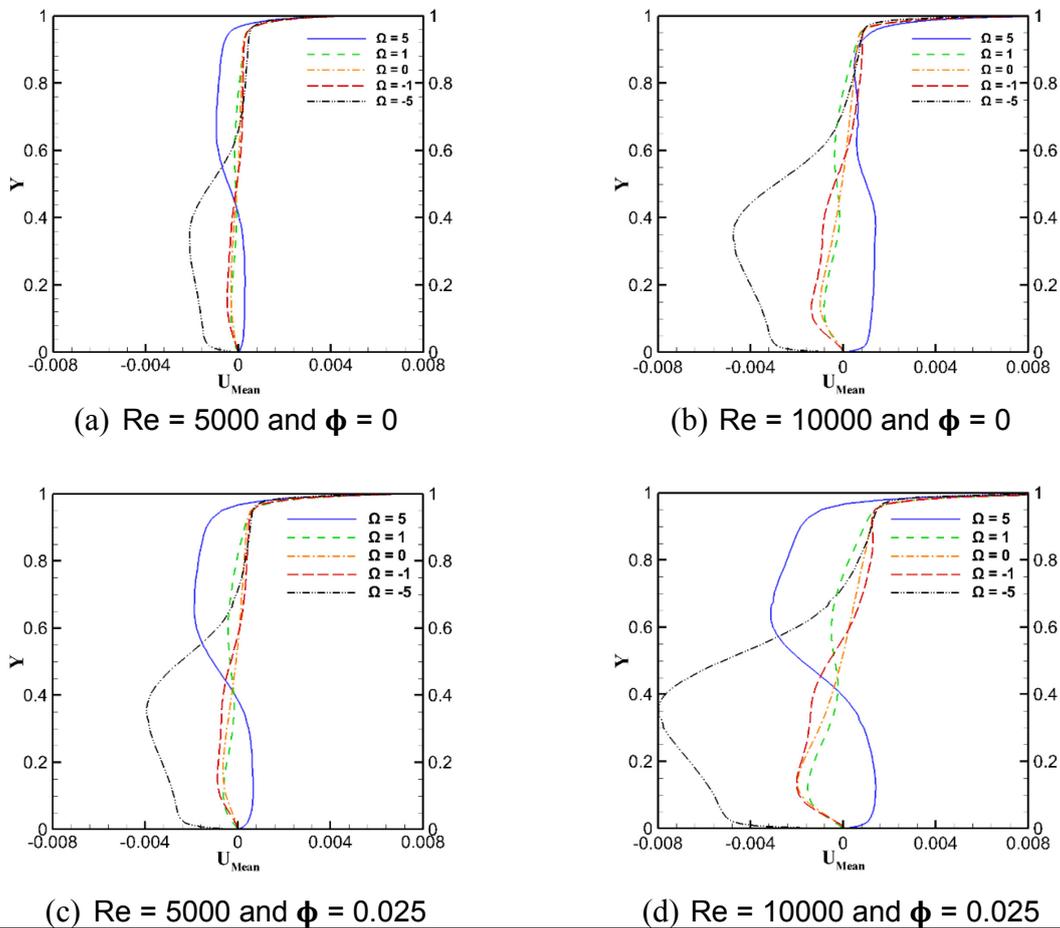


Fig. 8. Mean velocity profiles at vertical lines.

### 216 3.3. Wall shear stress

217 The wall shear stress can be calculated from the dynamic viscosity of the fluid multiplied by  
 218 the fluid velocity gradient at the wall. The variations in the wall shear stress at the moving top  
 219 wall and the heated bottom wall for the selected rotational cylinder speeds and directions for  
 220 the two Reynolds number values are shown in Fig. 9. Clearly, at the top wall for both Reynolds  
 221 numbers and a rotational speed within the range  $-1 \leq \Omega \leq 5$ , the wall shear stress values are  
 222 essentially unchanged. This is due to the fact that, either a speed or direction change of the  
 223 rotational cylinder could not affect the velocity profiles in the boundary layers near the top wall  
 224 significantly, as shown in Fig. 8, though a noticeable effect was observed when the rotational  
 225 speed became -5, suggesting that the boundary layer profile due to the top lid movement is  
 226 now somewhat affected by the fluid motion due to the cylinder rotation. On the other hand, at  
 227 the bottom wall, the wall shear stress is less affected by the moving top wall, and the rotational

228 cylinder plays a leading role in bottom wall shear stress. As a result, the wall shear distributions  
 229 show certain variations for different rotation speeds and directions, reaching their maximum  
 230 effect when the rotational speed reached -5. Clearly, the overall shear stress levels at the  
 231 bottom wall are much lower in comparison to the top wall. Furthermore, the wall shear stresses  
 232 are linked closely to the heat transfer coefficients, which will be discussed in more detail in  
 233 section 3.5 below.

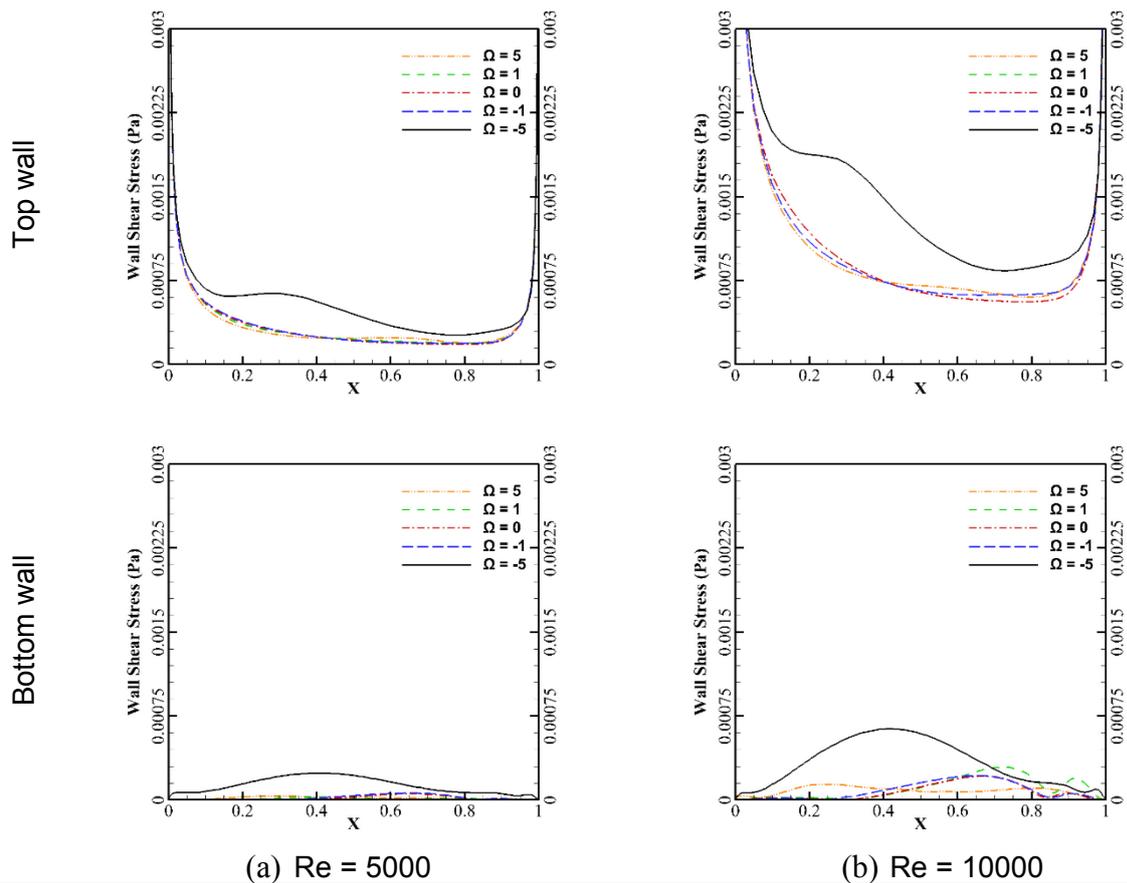


Fig. 9. Wall shear stress profiles for different rotational speeds and Reynolds numbers at the top and bottom walls.

### 234 3.4. Turbulent kinetic energy

235 It is well known that the root mean square (RMS) calculation of the fluid velocity fluctuations  
 236 is defined as the turbulent kinetic energy (TKE), which represents the kinetic energy of the  
 237 fluid motion per unit mass associated with the turbulent eddies. The averaged TKE profiles at  
 238 the two selected positions, one at the mid-height between the centre of the cylinder and the  
 239 bottom wall ( $y = 0.25$ ) and the other at the mid-height between the cylinder centre and the top  
 240 wall ( $y = 0.75$ ) are shown in Fig. 10, which illustrates the effects of changing the cylinder at  
 241 rotational speeds and directions for the two Reynolds number values of  $Re = 5000$ , and  $10000$ .  
 242 The simulated results indicate that for both Reynolds numbers, the TKE trend behaviours are  
 243 more or less the same as for the corresponding rotational speed, though their magnitudes

244 increase significantly as the Reynolds number increases from 5000 (Fig. 10 a) to 10000 (Fig.  
 245 10 b). At either Reynolds number, it can be determined that the rotating cylinder has a  
 246 substantial effect on the TKE profiles, especially with the rotational speed being -5 at  $y = 0.25$   
 247 and with rotational speeds being of 5 and -5 at  $y, x = 0.75$ .

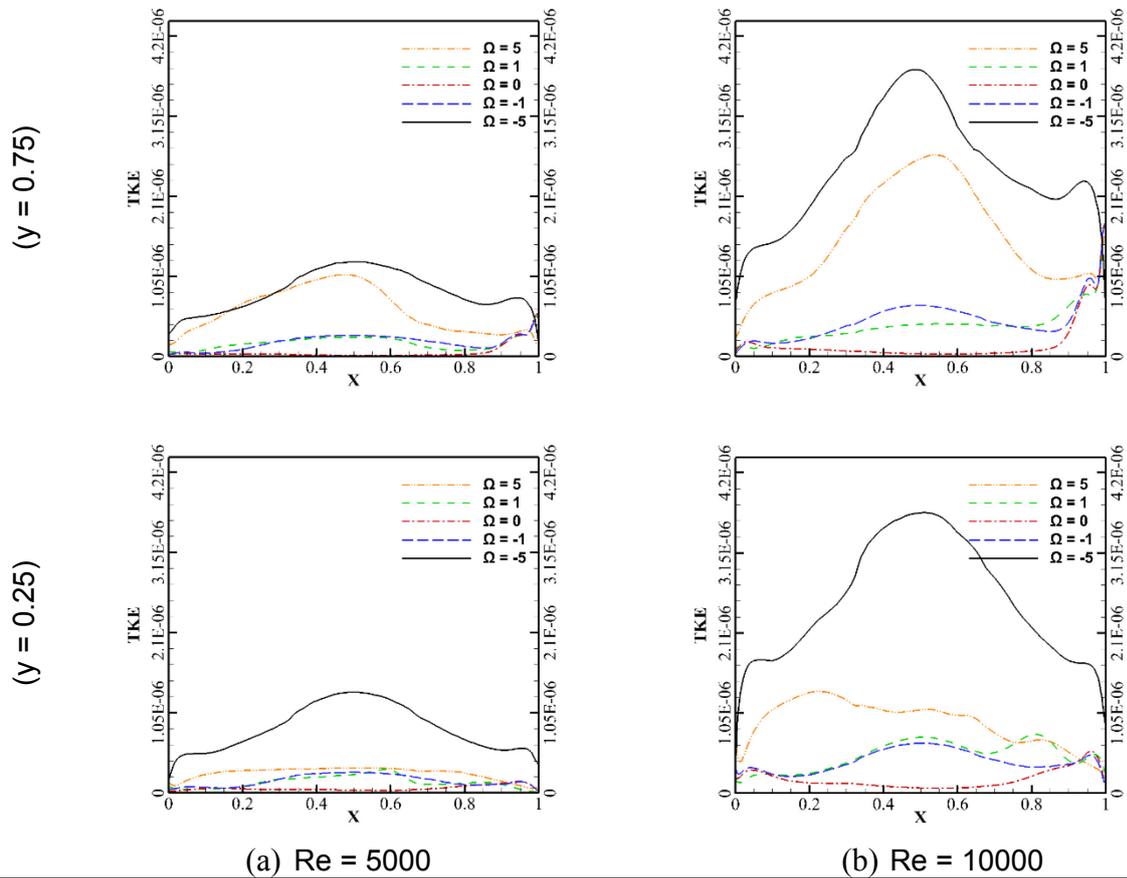


Fig. 10. Turbulence kinetic energy profiles for different rotational speeds and Reynolds numbers.

### 248 3.5. Nusselt number

249 It is widely recognised that heat transfer can be rapidly enhanced by involving turbulent flow,  
 250 unlike when the flow is laminar. Which can develop an insulating blanket near the solid walls  
 251 [28]. Any intermixing of the fluid would not happen when the flow motion is slow because the  
 252 boundary layer velocity reduces smoothly due to the viscous drag, which can lead the heat  
 253 transfer relying only on molecular convection and conduction. By contrast, the heat transfer  
 254 rate can be enhanced significantly by incrementing the fluid velocity, which generates turbulent  
 255 vortices where the boundary layers break away from the cavity walls and mix with the bulk of  
 256 the fluid further from the obstructed enclosure walls [29]. Fig. 11 and Fig. 12 illustrate the local  
 257 Nusselt number distributions for pure water and the nanofluid for the selected Reynolds  
 258 numbers,  $Re = 5000$  and  $10000$ , and rotational speeds,  $-5 \leq \Omega \leq 5$ , for turbulent flow condition

259 at the midlines of the bottom and top walls, respectively. For both Reynolds numbers, the  
 260 bottom wall Nusselt numbers are affected by changing either the rotational speed or the  
 261 rotational direction. The combined motion of the moving top wall and rotating cylinder provide  
 262 for higher local Nusselt numbers at the bottom wall, particularly when the rotational speed is -  
 263 5. On the other hand, the influences of the rotational speed on the moving top wall for both  
 264 Reynolds numbers are less remarkable unless the rotational speed is -5, as the heat transfer  
 265 characteristics in this region are mainly controlled by the fluid motion due to the wall  
 266 movement. It is worth noting that the heat transfer distributions at the top and bottom walls  
 267 bear a close resemblance to the wall shear profiles, as discussed in section 3.3, which can be  
 268 explained by the well-known Reynolds analogy between heat and momentum transfer. Both  
 269 the top and bottom wall Nusselt numbers for pure water and the nanofluid at different Reynolds  
 270 numbers and rotational speeds are compared in Table 2 to quantify the nanofluid's effect on  
 271 the heat transfer coefficient. It can be observed that for all different Reynolds numbers and  
 272 rotational speed cases, the effects of the nanofluid on the heat transfer coefficients were  
 273 always positive, and were more visible at the bottom wall than at the top wall.

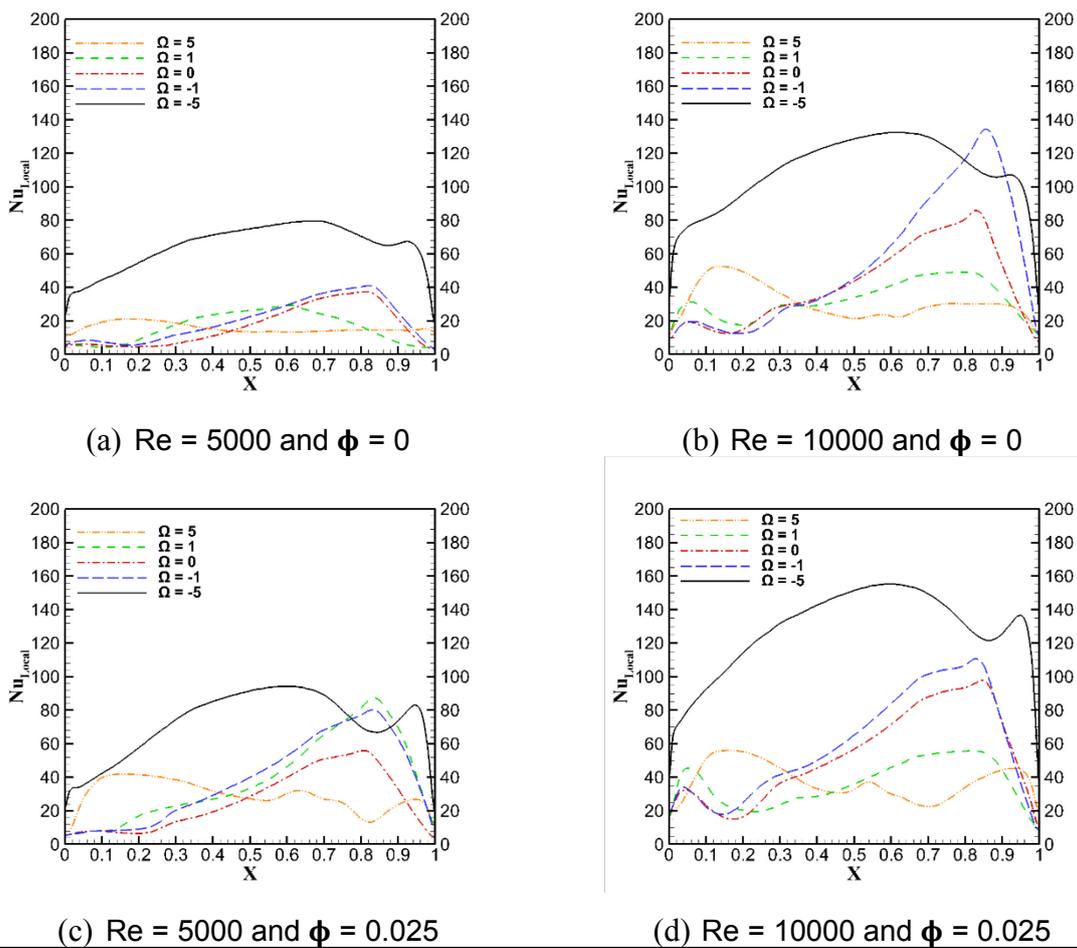


Fig. 11. Local Nusselt numbers at the bottom wall for pure fluid and nanofluid.

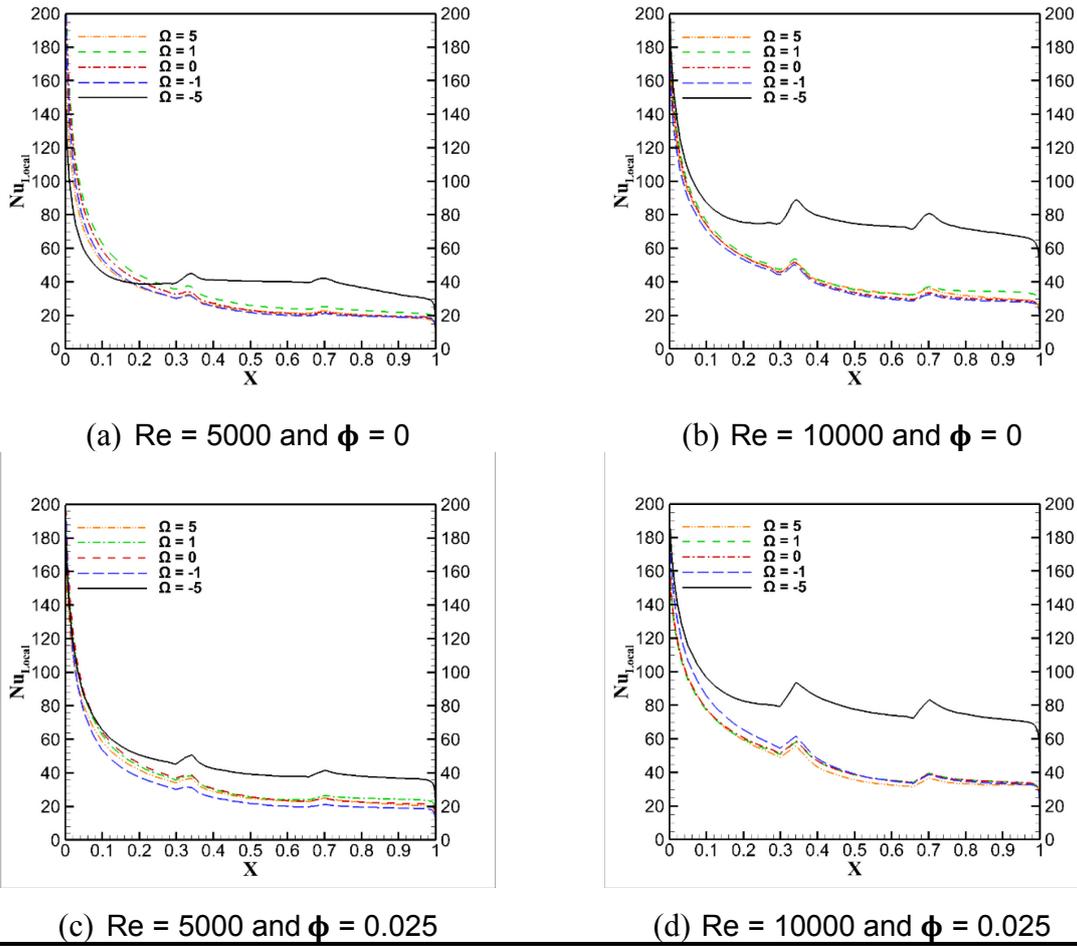


Fig. 12. Local Nusselt numbers at the top moving wall for pure fluid and nanofluid.

274

275 **Table 2**

276 Comparison of Average Nusselt numbers between pure fluid and nanofluid at the top and  
 277 bottom walls.

Rotational speed	$\Omega = -5$	$\Omega = -1$	$\Omega = 0$	$\Omega = 1$	$\Omega = 5$	
Top	$Nu_{nf}/Nu_{H_2O}$	59.97/55.41	50.24/41.76	47.11/44.77	46.99/47.85	42.78/39.74
	(Re = 5000)	= 1.08	= 1.20	= 1.05	= 0.98	= 1.07
Bottom	$Nu_{nf}/Nu_{H_2O}$	87.85/84.20	58.63/50.39	55.65/53.17	55.29/55.89	54.00/53.24
	(Re = 10000)	= 1.04	= 1.16	= 1.04	= 0.98	= 1.01
Bottom	$Nu_{nf}/Nu_{H_2O}$	63.92/56.83	31.83/17.14	21.74/14.26	31.92/14.40	25.14/14.85
	(Re = 5000)	= 1.12	= 1.85	= 1.52	= 2.21	= 1.69
Bottom	$Nu_{nf}/Nu_{H_2O}$	113.48/96.0	32.23/28.96	50.31/34.98	45.76/44.82	33.44/26.80
	(Re = 10000)	= 1.18	= 1.11	= 1.43	= 2.02	= 1.24

278 **4. Conclusion**

279 In this paper, the mixed turbulent convection heat transfer of a 3D lid-driven cavity containing  
 280 a cylinder that could rotate either clockwise or anticlockwise is addressed by involving both  
 281 pure water and the nanofluid. The simulations cover a range of cylinder rotation speeds,  $-5 \leq$

282  $\Omega \leq 5$ , and Reynolds numbers,  $Re = 5000$  and  $10000$ , through the URANS method. The  
283 following points have been concluded from this research:

- 284 • The temperature distributions and flow patterns were found to be influenced by both the  
285 speed values and the direction in which the cylinder was rotating, and nanofluid always  
286 positively effectives the heat transfer enhancement.
- 287 • The bottom wall shear stress can be influenced by both the rotational speeds and the  
288 Reynolds numbers, whereas the top wall shear stress values remain roughly the same for  
289 different rotational speeds and directions, except when  $\Omega = -5$ .
- 290 • When  $\Omega = -5$ , the lowest number of secondary eddies was found and the highest Nusselt  
291 number observed, especially in the case of the latter when the nanofluid was involved,  
292 unlike when  $\Omega = 5$  where the highest number of secondary vortices was produced.
- 293 • The cylinder rotation speed has a weaker influence on the top wall than on the bottom  
294 wall, where the largest difference in bottom wall Nusselt number between the conventional  
295 fluid and the nanofluid was found at a rotational speed of  $\Omega = 1$ .

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299

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