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Abstract

We build an overlapping generations model with endogenous fertility choices as well as public and private expenditures on health. We find that the complementary effect of public health services on private health expenditures can provide an additional explanation behind a salient feature of demographic transition; that is, the fertility decline along the process of economic growth.

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1 Introduction

In a seminal paper, Bhattacharya and Qiao (2007) analyse a growth model in which public health spending is complementary to the expenditures that individuals incur for the improvement of their health status. They find that an increase in the public provision of health services induces individuals to reduce their saving and, correspondingly,

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increase the resources they devote towards health improvements. As a result, the dynamics of capital intensity become non-monotonic and may admit periodic (endogenous limit cycles) or even aperiodic (i.e., chaotic) equilibria.

In this paper, we build an overlapping generations model in which individuals live for three periods – childhood, young adulthood and old adulthood. Our model utilises the main idea of Bhattacharya and Qiao (2007) – that is, public health spending being complementary to private health spending – but modifies their set-up in the following manner. Firstly, we allow individuals to consume during both periods of their adulthood. Secondly, individuals incur their health expenditure during the final period of their lifetime. Finally, we assume that individuals are reproductive during their young adulthood and they choose the number of their offspring in an optimal fashion.

Our results can be summarised as follows. We find that, rather than generating a trade-off between saving and private health spending, the provision of public health services induces an increase in *both* private health spending *and* saving; therefore, the dynamics of capital accumulation are monotonic and endogenous fluctuations cannot emerge. Furthermore, by motivating individuals to save more, the complementary nature of public and private health expenditures also induces them to bear and raise fewer children during their young adulthood. Hence, this complementarity emerges as an additional explanatory factor behind a well-known stylised fact of demographic transition (e.g., Dyson and Murphy, 1985).[1]

2 The Economy

Time is discrete and indexed by $t = 0,1,...\infty$. We consider an economy which produces a homogeneous commodity and is inhabited by reproductive individuals who live for three periods and belong to overlapping generations. Agents make decisions only after they reach their adulthood. These decisions are dictated by the desire to maximise their lifetime utility function

$$u^t = \ln(c_t) + \gamma \ln(n_t) + \beta[\ln(c_{t+1}) + \ln(h_{t+1})], \quad \gamma > 0,\ \beta \in (0,1), \tag{1}$$

subject to the constraints

$$c_t = (1-\tau)(1-qn_t)\omega_t - s_t, \quad q > 0,\ \tau \in (0,1), \tag{2}$$

$$c_{t+1} = r_{t+1}s_t - x_{t+1}, \tag{3}$$

[1] For theoretical models of economic growth that explain various feautures of demographic transition, see Becker *et al.* (1990), Galor and Weil (2000) and Zhang and Zhang (2005) among others.

$$h_{t+1} = Hx_{t+1}^{\delta\varepsilon_{t+1}}, \quad H > 0, \ \delta \in (0,1). \tag{4}$$

In the previous expressions, c_t denotes consumption during young adulthood, c_{t+1} denotes consumption during old adulthood, n_t is the number of children raised by a young adult, ω_t is the market wage per unit of labour, s_t denotes saving, r_{t+1} is the gross interest on saving, h_{t+1} is the old adult's health status and x_{t+1} is the old adult's spending towards health improvements. When young, each person is endowed with a unit of time which she allocates between raising children and providing labour services. Raising each child requires q units of time. Therefore, the young adult will use her remaining time to earn labour income – an income that is subject to a flat tax rate τ. She divides her disposable income between consumption and saving. The latter is deposited to a financial intermediary with the purpose of providing the agent with retirement income when she becomes an old adult. When old, the agent can potentially face some health problems which she can tackle by using part of her retirement income for the improvement of her health status. The remaining part of retirement income is used so as to satisfy her consumption needs.

With respect to the link between public and private health expenditures, we follow Bhattacharya and Qiao (2007) in using the expression in (4) for which it is assumed that

$$\varepsilon_{t+1} = Z(p_{t+1}), \tag{5}$$

where

$$p_{t+1} = \frac{g_{t+1}}{N_t}. \tag{6}$$

The function $Z(p_{t+1})$ in (5) satisfies $Z(0) = 1$, $Z(\infty) = \bar{\varepsilon} > 1$, $Z' > 0$ and $Z'' < 0$.[2] In equation (6), the variable g_{t+1} is the stock of public capital devoted to health services and N_t is the population of those agents who are young in period t and therefore will be old during $t+1$. The presence of the variable N_t is meant to capture a congestion-type effect. In particular, those who are young in period t will access public health services when they become old, i.e., in period $t+1$. We assume that, for given g_{t+1}, a larger population of agents mitigates the benefit accrued to each agent.

Given the above, the assumptions illustrated through (4), (5) and (6) provide a mechanism through which the provision of public health services is complementary to

[2] We assume that $\delta\bar{\varepsilon} < 1$ in order to ensure the concavity of h_{t+1} with respect to x_{t+1}.

private health expenditures in that they promote their effectiveness in improving the agent's health status during old adulthood.[3] This can be formally expressed through

$$\frac{dh_{t+1}}{dx_{t+1}}\frac{x_{t+1}}{h_{t+1}} = \delta\varepsilon_{t+1} = \delta Z\left(\frac{g_{t+1}}{N_t}\right), \tag{7}$$

i.e, the elasticity of health status with respect to private health spending is increasing to the stock of public capital that the government devotes towards health services.

In any period t, there is a large number of competitive firms who combine labour from young adults, L_t, and capital from financial intermediaries, K_t, so as to produce Y_t units of output according to

$$Y_t = K_t^a (A_t L_t)^{1-a}, \quad 0 < a < 1 \tag{8}$$

The variable A_t indicates some type of labour-augmenting technological progress. Following Frankel (1962) and Romer (1986), we assume that is related to the average capital-labour ratio according to a learning-by-doing externality. That is

$$A_t = \Psi \frac{\bar{K}_t}{L_t}, \quad \Psi > 0. \tag{9}$$

Firms who maximise profits will equate the marginal product of each input with the respective marginal cost. That is,

$$\omega_t = (1-a)K_t^a L_t^{-a} A_t^{1-a}, \tag{10}$$

and

$$r_t = aK_t^{a-1}(A_t L_t)^{1-a}. \tag{11}$$

3 Equilibrium

After some straightforward algebra, the first order conditions associated with an agent's optimal problem allow us to derive the following conditions for saving, fertility and private health expenditures:

$$s_t = \frac{\beta(1+\delta\varepsilon_{t+1})}{1+\beta(1+\delta\varepsilon_{t+1})+\gamma}(1-\tau)\omega_t, \tag{12}$$

[3] We can think of many examples that justify this assumption. The presence of qualified professionals – in the national health system – that offer support and advice on various difficulties that may emerge while people trying to quit smoking (e.g., cravings etc.) may provide an incentive for smokers to seek and buy treatments that support Nicotine Replacement Therapy (patches, gums etc.). Clinical depression can be combated more effectively if sufferers combine antidepressant medication with appropriate counselling by qualified psychiatrists – counselling that is sometimes offered by professionals employed in the national health system.

$$n_t = \frac{\gamma / q}{1 + \beta(1 + \delta\varepsilon_{t+1}) + \gamma}, \tag{13}$$

$$x_{t+1} = \frac{\beta\delta\varepsilon_{t+1}}{1 + \beta(1 + \delta\varepsilon_{t+1}) + \gamma} r_{t+1}(1-\tau)\omega_t. \tag{14}$$

Define $k_t \equiv K_t / N_t$ as capital per worker. Given $L_t \equiv (1 - qn_t)N_t$, we can combine (9), (10) and $\overline{K}_t = K_t$ to write the equilibrium wage as

$$\omega_t = \frac{(1-a)Ak_t}{1 - qn_t}, \tag{15}$$

where $A = \Psi^{1-a}$. Furthermore, assume that the government uses its collected revenues in period t so as to finance the formation of public capital that will be available next period, i.e., during $t+1$, according to a balanced budget rule. That is,

$$g_{t+1} = \tau L_t \omega_t = \tau(1 - qn_t)N_t\omega_t. \tag{16}$$

Combining (16) with (5) and (6) leads to

$$\varepsilon_{t+1} = Z(\tau(1-a)Ak_t). \tag{17}$$

Substituting (17) in (12) and (13) yields

$$s_t = \frac{\beta[1 + \delta Z(\tau(1-a)Ak_t)]}{1 + \beta[1 + \delta Z(\tau(1-a)Ak_t)] + \gamma}(1-\tau)\omega_t = s(k_t)\omega_t, \tag{18}$$

$$n_t = \frac{\gamma / q}{1 + \beta[1 + \delta Z(\tau(1-a)Ak_t)] + \gamma} = n(k_t). \tag{19}$$

These solutions allow us to derive

Proposition 1. *The saving rate $s(k_t)$ is increasing in the stock of capital per worker while the fertility rate $n(k_t)$ is decreasing in the stock of capital per worker.*

The intuition behind the result of Proposition 1 is the following. A higher capital stock increases the government's revenues and allows a greater provision of public capital towards health services. As a result, the effectiveness of private health expenditures increases and old adults will find optimal to devote more resources towards them. Naturally, this implies that old adults would find desirable to have more resources available at the beginning of their old adulthood. Indeed, agents can achieve this by saving a larger fraction of the disposable income they earn when young. Therefore, agents decide to limit the resources they keep during their reproductive period – an outcome to which they respond by reducing the number of children they rear.

Notice that our result concerning the saving rate comes in stark contrast to the result in Bhattacharya and Qiao (2007). In their model, agents consume only during the final period of their lifetime whereas their health spending occurs during their youth because their motive is to increase their life expectancy. Therefore, their set-up generates a trade-off between saving and private health expenditures which, combined with the complementary effect of public health spending, results in the saving rate being decreasing in the stock of capital. This effect is responsible for non-monotonic capital dynamics which converge to limit cycles or, possibly, chaos. Due to the different mechanisms in our paper, such non-monotonicities do not emerge. Moreover, by allowing endogenous fertility choices we add another dimension to the implications from private-public health expenditures complementarities. To see these, we need to analyse the economy's dynamics formally.[4]

4 Economic Growth and Endogenous Fertility

The economy's growth rate can be derived as follows. We begin by using the equilibrium condition in the financial market; that is, $K_{t+1} = s_t N_t$. Combined with $N_{t+1} = n_t N_t$, this condition becomes

$$k_{t+1} = \frac{s_t}{n_t}. \tag{20}$$

Substituting (15), (18) and (19) in (20), we can write the growth rate as $\theta_{t+1} = \frac{k_{t+1}}{k_t} - 1$

$$\theta_{t+1} = \frac{\eta\beta[1+\delta Z(\tau(1-a)Ak_t)]\{1+\beta[1+\delta Z(\tau(1-a)Ak_t)]+\gamma\}}{1+\beta[1+\delta Z(\tau(1-a)Ak_t)]} - 1 = \theta(k_t), \tag{21}$$

where $\eta = qA(1-a)(1-\tau)/\gamma$. It is straightforward to check that $\theta' > 0$. Assuming that $\theta(0) > 0$ holds, we have $k_{t+1} > k_t$ because the growth rate is positive. The long-run rate of growth is, thus, equal to

$$\bar{\theta} = \theta_\infty = \frac{\eta\beta(1+\delta\bar{\varepsilon})[1+\beta(1+\delta\bar{\varepsilon})+\gamma]}{1+\beta(1+\delta\bar{\varepsilon})} - 1. \tag{22}$$

[4] Note that the fact that capital dynamics are monotonic in our framework is not related to the presence of the Frankel/Romer-type externality. This assumption is employed so as to allow the economy to achieve long-run growth. The difference with Bhattacharya and Qiao (2007) is our assumption that individuals try to improve their health status during old adulthood. This is what actually eliminates the possibility of endogenous cycles in our model. For a similar assumption on health costs being incurred during old adulthood, see Gutiérrez (2008).

Consequently, by virtue of (19) and Proposition 1, the fertility rate declines towards its long-run value of $n(\infty) = (\gamma / q) / [1 + \beta(1 + \delta\overline{\varepsilon}) + \gamma]$. More importantly, the decline in fertility is an outcome solely associated with the supportive effect of public health expenditures on the effectiveness of private health expenditures. This argument can be formally established if we compare our existing results with the outcomes that transpire when either $\varepsilon_{t+1} = 1 \ \forall t$ or $\delta = 0$, in which case the term $\ln(h_{t+1})$ from the utility function disappears and $x_{t+1} = 0$ in equilibrium. In both scenarios, variations in the economy's capital stock will not impinge on childrearing decisions and the fertility rate will be constant. We can summarise these results in

Proposition 2. *Given* $k_0 > 0$*, the economy grows at a positive rate whereas the fertility rate declines. The decline in fertility is associated with the complementarity between public and private health expenditures.*

5 Discussion

We have established a new mechanism in explaining a salient feature of demographic transition. In particular, the decline of fertility during the process of growth is attributed to the complementary effect of public health spending on private health expenditures – an effect that has been introduced in the manner of the seminal analysis of Bhattacharya and Qiao (2007) (see equations (4)-(7)). As the economy grows, the public capital available for health services increases and improves the effectiveness of private health expenditures. Old adults will find that it is optimal for them to increase the resources they devote towards the improvement of their health status. In order to ensure the availability of these resources, they reduce their expenditures when young in order to save more. Given that childrearing costs are among these expenditures, reproductive young adults will reduce the number of children they give birth to. Consequently, the fertility rate declines.

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