A simplified model of a fueldraulic actuation system with application to load estimation

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Abstract

In this work, a simplified model of the compressor variable stator vane fueldraulic actuation system of a jet engine is presented. The actuation system is a sub-assembly of the engine's Hydro-Mechanical Unit (HMU). A unique characteristic of the actuator is an internal cooling flow which prevents the overheating of fuel. It is shown that the effect of the cooling flow is well represented by a static input nonlinearity. The resulting model is of the Hammerstein structure. It is then shown that the model can be used for the estimation of the actuator's external load. The results are validated using an accurate real system simulator.

Keywords

Hydraulic actuators, Fueldraulic systems, Position control, Hammerstein model, Aerospace simulation

Introduction

Hydraulic servo-systems are characterised by high power-to-weight ratios, and the ability to generate forces in locations which are remote and difficult to reach. For these reasons hydraulic servo-systems are widely used in aviation for the actuation of control surfaces and other moving parts. An introduction to hydraulic systems, hydraulic components, physical models, and linearisation techniques, can be found in Merritt¹ and Jalali and Kroll². Modelling and control applications for hydraulic systems have been reported under different approaches. These include: control-oriented models,^{3–6} and PID,^{7–9} robust,^{10–17} adaptive and nonlinear^{18–26} control designs. In some of these contributions, full position and pressure feedback is adopted. However, in aerospace applications, requiring low complexity and high reliability, the use of pressure sensors within the actuators is often avoided.

In this work, the modelling aspects of the Variable Stator Vanes (VSV) fueldraulic actuation system of a Rolls-Royce Trent 1000 jet engine are developed. Here, the term fueldraulic indicates that, in order to reduce overall weight, the actuator uses pressurised fuel in place of an hydraulic fluid. The purpose of the VSV actuator is to position the jet engine stator vanes in such a way that they attain an optimal angle of attack in all engine operating conditions. The vanes are mounted in a casing, so that rotation along their shroud-tip axis is enabled, and are joined in a unison ring which enables their uniform actuation. The actuator is located on the intermediate pressure compressor case, which, during engine operation, is a relatively high temperature zone. As a consequence, a unique feature of this actuator is that its piston has an orifice whose purpose is to establish a cooling flow inside the actuator in order to prevent fuel inside the actuator from overheating. The cooling flow has

a substantial effect on the behaviour of the actuator, similar to that of a sizeable internal leakage. The actuator is also characterised by asymmetric profiles of its servo valve's orifices, resulting in separate low- and high-flow zones, which induce a safe slow retraction of the actuator in the case of electric power loss.

The internal leakage is a feature in many hydraulic actuators' models. In the context of QFT and LPV robust control design,^{10–13,17} previous contributions have taken into account the effect of the internal leakage on the linearised dynamics of the hydraulic actuator for the design of the local controllers. In more complex adaptive and nonlinear design methods based on the use of global nonlinear models^{18–22,24,25} the internal leak is simply included in the full set of nonlinear equations of the model. In contrast with previous works, in this contribution, a simplified model of the global dynamics of the actuation system is developed, in which the effect of the cooling flow is well represented by a static input nonlinearity. The resulting model is of the Hammerstein structure, see e.g. Giri and Bai²⁷. The input nonlinearity turns out to be either a 'deadzone' or a 'preload' nonlinearity depending on the actuator's external load. To the extent of the authors knowledge, such representation of an internal flow or leakage on the global input-output dynamics of the actuator is a new insight into the modelling of hydraulic actuators. In the literature concerned with the modelling for control of hydraulic actuators, deadzone nonlinearities have been considered before, but only to take into account

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Figure 1. The actuation system: two-stage servo valve and VSV actuator depicted in extending mode.

manufacturing imperfection such as valve overlaps (see e.g. Liu and Daley⁷, Lee et al.⁸, Ye et al.⁹, Mohanty and Yao²⁰). It will be shown that the simplified model can be used to estimate the actuator's external load. This task turns out to be a special case of identification of a Hammerstein system with an input nonlinearity of known structure²⁸. In Bai²⁸ the identification method is developed for a one-dimensional parametrisation encompassing different types of input nonlinearities. Here, the algorithms are tailored to the estimation of the two-parameter asymmetric input nonlinearity of the VSV actuation system model.

The actuation system considered in this work is manufactured by Rolls-Royce Control Systems (RRCS) in Birmingham, UK. The simplified model and load estimation algorithm developed in this work are obtained and validated using the Trent 1000 Real System Simulator (RSS) implemented in Simulink. The simulations generated by the Trent 1000 RSS are considered to be very accurate and are employed by RRCS as surrogate experimental data for development and certification purposes. It will be shown that the simplified model achieves excellent performance with a low computational load.

Fuelhydraulic servo valve and actuator

The fuelhydraulic actuation system treated in this work comprises a two-stage electro-hydraulic servo valve and an hydraulic actuator. A schematic representation of the system is given in Figure 1. The first stage servo valve, together with a mechanical feedback loop between the first and the second stage servo valves, establishes a proportional relation between the control variable TMC (variable stator vanes Torque Motor Current) and the position of the piston of the second stage servo valve. The second stage servo valve, based on the openings of its orifices, establishes supply flows, from a high-pressure manifold at pressure H_p , and return flows, to a low-pressure manifold at pressure L_p , to and from the actuator's chambers. These pressures are determined by the engine speed and can be assumed to be constant in a given flight condition (e.g. 'Ground idle', 'Cruise', etc.). In extension, orifices 1 and 4 are open, and orifices 3 and 6 are closed, while, in retraction, orifices 3 and 6 are open and orifices 1 and 4 are closed. Orifices 2 and 5 are always open. In turn, the supply and return flows alter pressures in the head-end (P_H) and in the rod-end (P_R) chambers causing movement of the piston. The position of the piston is the measurable output of the system and is denoted by X_{VSV} . In the diagram of Figure 1, note the presence of the cooling flow orifice.

In this section, the equations that represent the system in the Trent 1000 RSS are provided. In a preliminary conference contribution,²⁹ a simplified version of the actuation system was considered which omitted the cooling flow. Here, a complete representation of the system is provided, with additional technical explanations, and, crucially, including the cooling flow, which, as it will be shown in the following sections, has a substantial effect on the system's behaviour. It is pointed out that some characteristics of the actuation system are omitted due to confidentiality. However, these characteristics are entirely outside the scope of the analysis and the equations provided constitute a realistic description of the system. In addition, a full set of representative values of the parameters is provided in the section "Simulation results", which would allow the interested reader to simulate the simplified model.

The movement of the piston is described by:

$$\ddot{x}_{VSV} = \frac{1}{M}(F_P + F_L) \quad (1)$$

where M is its mass and F_P and F_L are internal and external loads. The convention adopted is that forces acting on the piston take a positive sign when directed towards extension. The force F_P is developed by the internal pressures acting on the piston. It is given by:

$$F_P = A_H \cdot P_H - A_R \cdot P_R \quad (2)$$

where A_R and A_H are the rod-end and head-end piston's surfaces. Note that $A_H > A_R$ (see Figure 1). The force F_L is an exogenous variable which acts always against the actuator's movement. In addition, according to performance data, F_L is stronger in

retracting mode than in extending mode. This is explained as follows. The external load is interpreted to be the sum of aerodynamic (F_A) and friction (F_F) loads:

$$F_L = F_A + F_F. \quad (3)$$

The aerodynamic load F_A acts on the stator vanes and is always positive (i.e. supporting extension). The friction F_F acts against the actuator movement and is mainly developed in the vanes' pivots and in the transmission from the actuator to the vanes. This is a complex mechanical linkage, containing a large amount of moving joints. In comparison, the friction forces inside the actuator result to be negligible. Hence, the aerodynamic load can be assumed to be always lower than the friction. Finally, in a given flight condition, F_A and the modulus of F_F are assumed to be constant (see more details at the end of the section). Hence, F_L assumes only two possible values: F_L^E (negative, in extension) or F_L^R (positive, in retraction).

The pressures P_H and P_R are described by compressible flow equations:

$$\dot{P}_H = \frac{B}{V_H} (Q_H + Q_{Hd} - Q_C) \quad (4)$$

$$\dot{P}_R = \frac{B}{V_R} (Q_R + Q_{Rd} + Q_C) \quad (5)$$

where Q_R and Q_H are flows through the second-stage servo valve and into or from the rod-end and head-end chambers respectively, Q_{Rd} and Q_{Hd} are displacement flows caused by the piston's movement, and Q_C is the cooling flow. All flows, apart from the cooling flow, take positive sign when incoming (i.e. raising pressures). In extension, Q_H is positive (flow through orifices 1 and 2) and Q_R is negative (flow through orifices 5 and 4). In retraction, Q_H is negative (flow through orifices 2 and 3) and Q_R is positive (flow through orifices 6 and 5). The cooling flow Q_C is considered positive from head-end to rod-end, regardless of the piston's motion. In addition, B is bulk modulus, which characterises the fuel's compressibility, and V_H and V_R are the actuator chambers' volumes. These volumes are function of the piston position, but their variation is assumed to be negligible. This can be done because the servo-valve is located on the fan case, while the actuator is on the compressor case. The manifold between these two elements is relatively long, and contains a high volume of fluid, in comparison to the volume of fluid in the chambers. This assumption, which in some cases has been already adopted in the literature see e.g. Thompson et al.¹¹, Karpenko and Sepehri¹³, introduces a considerable simplification of the equations.

Flows Q_H and Q_R are determined by the flownumbers of the respective servo valve orifices. The flownumber is a parameter integrating the physical characteristics of the orifices with those of the fluid into a single coefficient in the flow equations (see e.g. Jalali and Kroll² where the term 'flow coefficient' is used

Figure 2. Servo valve flownumbers, with highlighted orifices' profiles for the high-flow extension and low-flow retraction modes

instead). The orifices' flownumbers for Q_H and for Q_R in both retraction and extension are approximately equal. Hence, the same variables f^R and f^E are used to denote the flownumbers, in retraction and in extension respectively, for both Q_H and Q_R . The resulting flow equations are:

$$Q_H = \begin{cases} f^E \sqrt{H p - P_H}, & \text{in extension,} \\ -f^R \sqrt{P_H - L p}, & \text{in retraction,} \end{cases} \quad (6)$$

$$Q_R = \begin{cases} -f^E \sqrt{P_R - L p}, & \text{in extension,} \\ f^R \sqrt{H p - P_R}, & \text{in retraction,} \end{cases} \quad (7)$$

where f^R and f^E are determined by the openings of the servo valve orifices as described in the ensuing equations (11), (12), and Figure 2. The remaining flows are described by:

$$Q_{Hd} = -A_H \cdot \dot{x}_{VSV A}, \quad (8)$$

$$Q_{Rd} = A_R \cdot \dot{x}_{VSV A}, \quad (9)$$

$$Q_C = f_C \operatorname{sgn}(P_H - P_R) \sqrt{|P_H - P_R|}. \quad (10)$$

Note that the superscripts E and R distinguish between extending and retracting modes (as in f^E and f^R) while subscripts R and H distinguish between the rod-end and the head-end chambers (as in P_H and P_R).

The steady-state relations between input current TMC and the flownumbers f^E and f^R are described by:

$$f^E = \begin{cases} K^E (TMC - TMC_0), & TMC \geq TMC_0, \\ 0, & TMC < TMC_0, \end{cases} \quad (11)$$

$$f^R = \begin{cases} 0, & TMC \geq TMC_0, \\ K^R (TMC_0 - TMC), & TMC < TMC_0, \end{cases} \quad (12)$$

as displayed in Figure 2. The dynamics of the servo valve includes the drive circuit and motor lags and the motion of the piston. This is represented by a damped third order system with a cutoff frequency at around 10Hz.

The flownumbers profiles depicted in Figure 2 implement the safety requirement that the VSV actuator should slowly retract in case of a power interrupt. This loads the compressor and slows down the engine, thus preventing an engine surge in case of electrical power loss. This behaviour is obtained by means of a positive TMC_0 value, i.e. the current corresponding to all orifices closed, and by means of asymmetric profiles of the orifices. In particular, the reduced gain of the flownumber profile in the retracting mode allows for a slow retraction in the case that $TMC = 0$, i.e. a power interrupt.

In summary, the dynamics of the VSV actuation system in the Trent 1000 RSS is described by a non-linear dynamical system (equations (1)-(12)) with three states (velocity $\dot{x}_{VSV}$, head-end pressure P_H and rod-end pressure P_R), one input (the control input TMC) and three exogenous variables (high-pressure Hp and low-pressure Lp , entering in (6) and (7), and the actuator's loading F_L in (1)). Note that the nonlinearity of the system is due to the flow equations (6-10). The Electronic Engine Controller (EEC) of Rolls-Royce engines maintains accurate estimates of Hp and Lp (which are needed for the control of the fuel distribution). Hence these variables can be assumed to be known with sufficient accuracy. Instead, in the Trent 1000 RSS, the aerodynamic loading F_A and the modulus of the friction force F_F are calculated as functions of the engine speed on the basis of a nominal relation. Hence, they can also be assumed to be constant in a given flight condition. However, in real operating conditions, the relation with engine speed is subject to variations caused by air properties, engine conditions, and other factors. Hence, in a given flight condition, F_A and the modulus of F_F can be considered to be constant but uncertain.

Operating points and linearisation

In this section, in view of the formulation of a simplified model, a linearisation based analysis of the system is developed. In order to find the operating point $\ddot{x}_{VSV}$, $\dot{P}_H$, and $\dot{P}_R$ are set to 0. The derivations are carried out separately for the extending and retracting modes because different flow equations apply. In addition, since the input TMC affects the actuator only through the static relations (11) and (12), in this section, in order to simplify the notation, the flownumber f^E (in extending mode), or f^R (in retracting mode), are considered to be the actuator's input. The subscript 0 denotes input and state variables at the operating point. In addition, v_0 denotes $\dot{x}_{VSV}$ at the operating point. The exogenous variables Hp , Lp and F_L^E (in extending mode) or F_L^R (in retracting mode) are assumed to be constant. Recall that F_L^E is negative

and F_L^R is positive, i.e. always opposing the actuator's movement.

In the following subsections, the operating points for the extending and retracting modes are obtained and then the linearised dynamics is analysed.

Extending mode

From equations (4), (5) and (6) - (10) one obtains

$$f_0^E \sqrt{Hp - P_{H0}} - A_H v_0 - f_C \operatorname{sgn}(P_{H0} - P_{R0}) \sqrt{|P_{H0} - P_{R0}|} = 0 \quad (13)$$

$$-f_0^E \sqrt{P_{R0} - Lp} + A_R v_0 + f_C \operatorname{sgn}(P_{H0} - P_{R0}) \sqrt{|P_{H0} - P_{R0}|} = 0, \quad (14)$$

and from equations (1) - (3) one obtains

$$A_H P_{H0} - A_R P_{R0} + F_L^E = 0. \quad (15)$$

From equations (13) and (14), one obtains that the ratio of the head-end and rod-end flows can be expressed as

$$\frac{\sqrt{P_{R0} - Lp}}{\sqrt{Hp - P_{H0}}} = \frac{A_R v_0 + f_C \operatorname{sgn}(P_{H0} - P_{R0}) \sqrt{|P_{H0} - P_{R0}|}}{A_H v_0 + f_C \operatorname{sgn}(P_{H0} - P_{R0}) \sqrt{|P_{H0} - P_{R0}|}}. \quad (16)$$

It is now assumed that if $v_0 > 0$ then the cooling flow becomes negligible with respect to the displacement flow. The assumption is justified by considering the small flownumber of the cooling orifice. Under this approximation, from equation (16) one obtains that

$$\frac{\sqrt{P_{R0} - Lp}}{\sqrt{Hp - P_{H0}}} \approx \frac{A_R}{A_H}. \quad (17)$$

The approximate operating pressures P_{R0}^E and P_{H0}^E are now defined as the solution obtained from equations (15) and (17) (the latter with an equals sign). The expressions of the approximate operating pressures are

$$P_{R0}^E = \frac{A_R^2 A_H Hp + A_H^3 Lp + A_R^2 F_L^E}{A_H^3 + A_R^3}, \quad (18)$$

$$P_{H0}^E = \frac{A_R^3 Hp + A_R A_H^2 Lp - A_H^2 F_L^E}{A_H^3 + A_R^3}, \quad (19)$$

and the corresponding cooling flow is given by

$$Q_{C0}^E = f_C \operatorname{sgn}(P_{H0}^E - P_{R0}^E) \sqrt{|P_{H0}^E - P_{R0}^E|}. \quad (20)$$

Expressions (18) and (19) are valid provided that $A_H Hp - A_R Lp + F_L^E > 0$ which ensures that $Hp > P_{H0}^E$ and $P_{R0}^E > Lp$. Note that for given Hp and Lp the inequality provides the maximum loading against which the extending mode can be established.

From the equations above, it can be noticed that, under the approximation introduced, the operating

pressures P_{H0}^E , P_{R0}^E depend on Hp , Lp and F_L^E but are independent from the input f_0^E . Then, for a given flownumber f_0^E , one obtains the operating point velocity v_0 from equations (13) and (14). However, note that, since P_{R0}^E and P_{H0}^E were based the approximation (17), they would not fulfil (13) and (14) for the same v_0 . Thus, an approximate expression v_0^E of the operating velocity v_0 is obtained as the arithmetic mean of the operating velocities obtained from (13) and (14). This is given by:

$$v_0^E = \frac{1}{A_H} \sqrt{Hp - P_{H0}^E(f_0^E - f_S^E)} \quad (21)$$

where

$$f_S^E = \frac{1}{2} \frac{A_R + A_H}{A_R} \frac{Q_{C0}^E}{\sqrt{Hp - P_{H0}^E}}. \quad (22)$$

In (21), f_S^E represents explicitly the minimum value of f_0^E corresponding to a positive v_0^E . Note that, since in the derivations a positive velocity is assumed, the expression on the right-hand side of (21) is meaningful only for positive values of v_0^E . Hence, it can be concluded that, for given Hp , Lp and F_L^E , the extending mode is established for $f_0^E > \max(0, f_S^E)$.

In extending mode, the two cases $P_{H0}^E > P_{R0}^E$ and $P_{H0}^E < P_{R0}^E$ can arise. For given Hp and Lp , the actuator loading F_L^E discriminates between the two cases. From (18) and (19) one obtains that $P_{H0}^E = P_{R0}^E$ when the actuator loading is equal to

$$\bar{F}_L^E = \frac{(A_R - A_H)(A_R^2 Hp + A_H^2 Lp)}{A_H^2 + A_R^2}. \quad (23)$$

Note that $\bar{F}_L^E$ is negative, since $A_H > A_R$. Since F_L^E is also negative, the two cases can arise: $F_L^E < \bar{F}_L^E$ and $F_L^E > \bar{F}_L^E$. If $F_L^E < \bar{F}_L^E$, then $P_{H0}^E > P_{R0}^E$, Q_{C0}^E is positive (i.e. from head-end to rod-end) and $f_S^E > 0$. If $F_L^E > \bar{F}_L^E$, Q_{C0}^E is negative (i.e. from rod-end to head-end) and $f_S^E < 0$. The difference between the two cases is now illustrated. If $F_L^E < \bar{F}_L^E$ and $0 < f_0^E < f_S^E$ then the valve opening is not enough to establish an incoming flow which is bigger than the cooling flow and so $v_0^E = 0$. Hence, if $F_L^E < \bar{F}_L^E$, the extending mode is established only for $f_0^E > f_S^E$. If $F_L^E > \bar{F}_L^E$ then the flow coming into the actuator has direction opposite to that of the cooling flow. In this case any $f_0^E > 0$ is sufficient to establish the extending mode. In particular, a strictly positive v_0^E is established for $f_0^E = 0^+$.

Illustrative examples of the relation between f_0^E and v_0^E are shown in Figure 3 (b) for selected flight conditions: 'Ground idle', 'Take-off', 'Cruise' and 'Descent idle'. In each case, the corresponding values of Hp , Lp , and F_L^E were obtained from the Trent 1000 RSS. In the case denoted 'Cruise outlier' the actuator loading is 20% lower than the nominal actuator loading. In the other cases the loading takes nominal values. It can be seen from the figure that $f_S^E > 0$, in the four cases corresponding to nominal

conditions, while $f_S^E < 0$ in the case 'Cruise outlier'. In addition, note that the slope of the linear part is different for each flight condition.

Retracting mode

Following a derivation similar to that presented for the extending mode, with the obvious modification in the notation, one obtains the following expressions which define the operating point in the retracting mode:

$$P_{R0}^R = \frac{A_H^3 Hp + A_R^2 A_H Lp + A_R^2 F_L^R}{A_R^3 + A_H^3}, \quad (24)$$

$$P_{H0}^R = \frac{A_R A_H^2 Hp + A_R^3 Lp - A_H^2 F_L^R}{A_R^3 + A_H^3}, \quad (25)$$

$$Q_{C0}^R = -f_C \sqrt{P_{R0}^R - P_{H0}^R}, \quad (26)$$

$$v_0^R = -\frac{1}{A_R} \sqrt{Hp - P_{R0}^R(f_0^R - f_S^R)}, \quad (27)$$

$$f_S^R = \frac{1}{2} \frac{A_H + A_R}{A_H} \frac{-Q_{C0}^R}{\sqrt{Hp - P_{R0}^R}}. \quad (28)$$

In this case the expressions are valid provided that $A_R Hp - A_H Lp - F_L^R > 0$ which ensures that $Hp > P_{R0}^R$ and $P_{H0}^R > Lp$.

In the retracting mode, in contrast to the extending mode, the actuator's movement can be established only if $P_{H0}^R < P_{R0}^R$. This is because the rod-end area of the piston is smaller than its head-end area (see Figure 1). Therefore, since F_L^R is positive, the rod-end pressure must be greater than the head-end pressure in order to enable retraction. This fact can be recovered from the equations. Using (24)-(28) it can be shown that $P_{H0}^R = P_{R0}^R$ is obtained when the actuator's load is equal to

$$\bar{F}_L^R = \frac{(A_R - A_H)(A_H^2 Hp + A_R^2 Lp)}{A_H^2 + A_R^2}. \quad (29)$$

Note that $\bar{F}_L^R$ is negative, since $A_H > A_R$. However F_L^R is positive. Hence only the case $F_L^R > \bar{F}_L^R$ is possible which, in turn, corresponds to having $f_S^R > 0$. Hence, $v_0^R = 0$ for $0 < f_0^R < f_S^R$, and the retracting mode is established for $f_0^R > f_S^R$. The expression of $\bar{F}_L^R$ in (29), which appears to be of theoretical interest only, nevertheless will be used to shorten the expressions used to define the simplified model in the next section. The relation between f_0^R and v_0^R is illustrated in Figure 3 (a) for the same flight conditions considered in Figure 3 (b).

Linearised models

Given Hp , Lp and $F_L^{E/R}$, linearised models around operating points $[P_{H0}^{E/R}, P_{R0}^{E/R}, f_0^{E/R}]$ are obtained by linearising the flow equations (6), (7) and (10). The linearised system has a pair of high-frequency lightly-damped poles ($\approx 10^3$ Hz). In addition, it can be seen that the choice of the operating points has very little effect on the linearised dynamics since it affects only the amplitude of the high-frequency resonance

Figure 3. Relation between flownumber and velocity in extending and retracting modes for different flight conditions and actuator loadings. (The profiles shown are for two actuators in parallel in accordance with the simulation cases presented in the last section.)

Figure 4. Bode plots of the linearised dynamics between flownumber and velocity in extending mode for the same flight conditions considered in Figure 3(b). The vertical lines indicate the bandwidth of the VSV actuation control system and the sampling frequency.

peak. This is illustrated in Figure 4 where the Bode plots of the linearised dynamics from f^E to $\dot{x}_{VSVA}$ corresponding to the flight conditions considered in Figure 3(b). In each flight condition, changing f_0^E does not cause noticeable changes in the Bode plots. In each Bode plot, the low-frequency gain corresponds to the slope of the linear part of the corresponding graph in Figure 3(b).

High-frequency lightly-damped poles are a common occurrence in hydraulic systems, as shown in e.g. Fales⁴, Thompson et al.¹¹, Karpenko and Sepehri¹⁴ and Rubagotti et al.²¹. Here, the bandwidth of the VSVA control system is $10\text{rad s}^{-1}/1.6\text{Hz}$ and the available sampling frequency is 40Hz . Hence, the high frequency resonance peak is well beyond the control bandwidth (see Figure 4).

Simplified model

In this section, a simplified control-oriented model is presented, which approximates the dynamics between the control input $\Delta TMC = TMC - TMC_0$ and the actuator's position x_{VSVA} . The rationale behind the simplified model is that, following the linearisation-based analysis at different operating points performed in the previous section, the high-frequency dynamics can be neglected in order to represent the behaviour of the actuator in the frequency range of interest. In particular, over the frequency range up to the sampling frequency 40Hz , it can be assumed that ΔTMC is linked to $\dot{x}_{VSVA}$ by a delay plus a static nonlinearity.

In order to simplify the notation, the 'signed flownumber' f is introduced to represent both extending and retracting flownumbers with one single variable. In addition, the dynamics of the servo valve inside the bandwidth is well represented by a pure delay $T_d = 0.025\text{s}$. Hence, from (11) and (12) one obtains the static relations

$$f = \begin{cases} K^E \Delta TMC_d, & \Delta TMC_d \geq 0, \\ K^R \Delta TMC_d, & \Delta TMC_d < 0. \end{cases} \quad (30)$$

where ΔTMC_d is the delayed ΔTMC . Then, one can link f to $\dot{x}_{VSVA}$ through the non-linear static relation

$$\dot{x}_{VSVA} = \begin{cases} N^E(f - f_S^E), & f \geq \max(0, f_S^E), \\ 0, & -f_S^R < f < \max(0, f_S^E), \\ N^R(f + f_S^R), & f \leq -f_S^R, \end{cases} \quad (31)$$

where N^E and f_S^E are functions of F_L^E, H_p, L_p and N^R and f_S^R are functions of F_L^R, H_p, L_p as follows.

Figure 5. Input nonlinearity for selected flight conditions. Note the different scalings for positive and negative values of ΔTMC_d (compare with Figures 2 and 3).

Figure 6. Simplified model of the actuation system

Substituting (18) in (21) and (24) in (27), one obtains

$$N^E = \sqrt{\frac{A_H H p - A_R L p + F_L^E}{A_H^3 + A_R^3}}, \quad (32)$$

$$N^R = \sqrt{\frac{A_R H p - A_H L p - F_L^R}{A_H^3 + A_R^3}}. \quad (33)$$

Similarly, using (22) and (28) and taking into account that in extension the cooling flow can be positive or negative, one obtains

$$f_S^E = \text{sgn}(\bar{F}_L^E - F_L^E) \times \frac{f_C}{2} \frac{A_R + A_H}{A_R A_H} \sqrt{\frac{(A_H^2 + A_R^2) |\bar{F}_L^E - F_L^E|}{A_H H p - A_R L p + F_L^E}}, \quad (34)$$

$$f_S^R = \frac{f_C}{2} \frac{A_R + A_H}{A_R A_H} \sqrt{\frac{(A_H^2 + A_R^2) (F_L^E - \bar{F}_L^E)}{A_R H p - A_H L p - F_L^R}}. \quad (35)$$

Instances of the static nonlinearity obtained by combining (30) and (31) are shown in Figure 5.

In conclusion, the simplified model of the VSV actuator linking ΔTMC to $x_{VSV A}$ is composed of a pure delay, an input static nonlinearity, and a linear integrator plant (see Figure 6). The coefficients of the static nonlinearity include some physical parameters of the actuator and the exogenous variables $H p$, $L p$ and $F_L^{E/R}$. Some other parameters of the full nonlinear

model become irrelevant in the simplified model, these are: the head-end and rod-end volumes of the actuator ($V_{H/R}$) and the bulk modulus (B). However, it is remarked that $V_{H/R}$ and B were needed to perform the linearisation analysis of the previous section which led us to the simplified model.

Estimation of the actuator's loading

In a given flight condition, as discussed in Section 2, F_L^E and F_L^R can be considered to be uncertain constants, while accurate values of $H p$ and $L p$ are available in the EEC. Here, it is shown that F_L^E and F_L^R can be estimated by a simple extension of the method presented in Bai²⁸. In this section, in order to simplify the notation, the symbols $u = \Delta TMC_d$, $v = \dot{x}_{VSV A}$, and $y = x_{VSV A}$ are used.

To start with, the parametrisation of the 'deadzone' and 'preload' nonlinearities of Bai²⁸ is adopted to express the static nonlinearity in the model. In order to do this, the following coefficients are defined

$$M^E = N^E K^E, \quad M^R = N^R K^R, \quad (36)$$

$$\bar{u}^E = f_s^E / K^E, \quad \bar{u}^R = -f_s^R / K^R. \quad (37)$$

Then, by combining (30) and (31) one obtains

$$v = \begin{cases} \frac{M^E}{2} [1 + \text{sgn}(u - \bar{u}^E)] (u - \bar{u}^E), & u \geq 0 \\ \frac{M^R}{2} [1 - \text{sgn}(u - \bar{u}^R)] (u - \bar{u}^R), & u < 0. \end{cases} \quad (38)$$

In the above expressions M^E and $\bar{u}^E$ are functions of F_L^E and M^R and $\bar{u}^R$ are functions of F_L^R . Note that M^E and M^R are always positive. The coefficient $\bar{u}^R$ is always negative (deadzone in retraction), while $\bar{u}^E$ can be positive (deadzone in extension) or negative (preload in extension).

Given a set of input (u) and output (y) data one can use (38) to compute an estimate of the actuator's velocity $v(u, F_L^E, F_L^R)$ as a function of F_L^E and F_L^R . Then, estimates $\hat{F}_L^E$ and $\hat{F}_L^R$ can be obtained by direct minimisation of a least-squares criterion. In general the problem is not convex but is two dimensional and, thus, is not computationally intensive. In addition, as pointed out in Bai²⁸, the estimate is consistent and has good properties when the data are affected by noise. In particular, if the data are affected by i.i.d. zero-mean Gaussian noise then $\hat{F}_L^E$ and $\hat{F}_L^R$ are a maximum likelihood estimator. Here, this property is particularly well suited since the noise affecting the data is low-intensity electrical noise which is well described by the i.i.d. Gaussian assumption (see illustrative results in the following section).

Simulation results

In this section, the simplified VSV model is assessed in a comparison with the Trent 1000 RSS. The Trent 1000 RSS is a large Simulink model which includes: Pumping unit, Hydro-Mechanical Unit (HMU), Electronic Engine Control (EEC), and Engine.

$K^R = 4.36 \text{ mm}^3\text{s}^{-1}\text{Pa}^{-1/2}\text{mA}^{-1}$	$M = 0.90 \text{ kg}$
$K^E = 32.97\text{mm}^3\text{s}^{-1}\text{Pa}^{-1/2}\text{mA}^{-1}$	$H_p = 2061 \text{ kPa}$
$A_H = 5748 \text{ mm}^2$	$L_p = 358 \text{ kPa}$
$A_R = 4940 \text{ mm}^2$	$F_L^E = -1664 \text{ N}$
$f_C = 0.582 \text{ mm}^3\text{s}^{-1}\text{Pa}^{-1/2}$	$F_L^R = 4430 \text{ N}$

Table 1. Representative values of parameters in 'ground idle'.

In the RSS, the VSVA component includes the servo valve and two actuators in parallel. In the RSS VSVA component, the actuation system is implemented by the full non-linear equations (1)-(12). In Table 1 numerical values of the parameters of the RSS VSVA component, which are representative of a 'ground idle' state, are disclosed. Note that Table 1 includes all parameters which appear in the simplified model. For confidentiality, the parameters' values in the other flight conditions are not disclosed. In order to generate the simulations presented in this section, RSS VSVA component was isolated from the rest of the RSS. Hence H_p , L_p , F_A and $F_L^{E/R}$ were set at constant values corresponding to the given flight condition. In order to generate the closed-loop simulations, the existing PI controller from the EEC module was included. The simulations of the RSS VSVA component were generated using the Simulink Runge-Kutta solver with a 10^{-6} s fixed-step size compiled in rapid accelerator mode. The reason for the choice of such a short step size is that the RSS VSVA component includes stiff differential equations (see the high-frequency resonance peak shown in Figure 3). In order to generate a 10s response, the solver required approximately two minutes of computational time. In contrast, the simplified model was simulated using a variable-step solver requiring a comparatively negligible time.

In Figures 7 and 8 the position and velocity responses generated by the RSS VSVA component are compared with the responses generated by the simplified model. It can be seen that the responses provided by the simplified model mostly overlap those generated by the RSS. Hence, the simplified model provides computationally light yet accurate simulations. For completeness, the mean squared error (mse) between the two responses, computed with sampling time 0.025s, is reported for each plot. For the position responses of Figure 7 the values are $\text{mse} = 3.66 \times 10^{-3}$ and $\text{mse} = 1.77 \times 10^{-5}$ for the open and closed-loop responses respectively. For the velocity responses of Figure 8 the values are $\text{mse} = 1.388 \times 10^{-3}$ and $\text{mse} = 9.51 \times 10^{-3}$ for the open and closed-loop responses respectively. Note that the mse of the closed-loop position responses is much lower than the mse of the open-loop position responses. This is expected since position is the controlled variable in the closed-loop system. In all cases, the mse values, which are very low in comparison with the excursion of the responses, confirm the accuracy of the simplified model. It can also be seen that the velocity responses generated by the RSS VSVA component are characterised by bursts of high-frequency oscillations at the transition points

(see magnified inset in Figure 8). These oscillations match the frequency of the resonance peak seen in Figure 4. The oscillations are smoothed by integration and do not appear in the position responses. It can be seen, by comparing the velocity responses, that the velocity generated by the simplified model at the transition points follows a smoothed version of the velocity response generated by the RSS.

In Figure 9, the evolution of the TMC inputs is displayed. In the open-loop simulations, the inputs to the RSS and to the simplified model were the same. In the closed-loop simulations, the TMC inputs were generated by two independent closed-loop systems fed by the same reference signal. Note that the TMC inputs are still almost overlapping. In Figure 9 the value of TMC_0 and the limits of the input deadzone of the simplified model are also displayed. Note that the deadzone is smaller on the positive side (compare with Figure 5). Comparing Figure 9 with Figure 8, it can be seen that the time intervals in which TMC is inside the dead-zone of the simplified model coincide with time intervals in which the velocity is null. This is more evident in the open-loop simulations, in which the input TMC is inside the deadzone for longer intervals.

In Figure 10, the evolution of P_H and P_R in the simulation generated by the RSS is displayed. In the figure, the operating pressures $P_{H0}^{E/R}$ and $P_{R0}^{E/R}$ are also displayed and are plotted only over the time intervals in which the actuators are moving. From the figures it can be seen that the pressure P_H and P_R converge very rapidly to the operating values whenever the actuator is moving independently of the attained velocity. In this respect, recall that the independence of operating pressure from the velocity was a key point in the formulation of the simplified model. In addition, note that P_H and P_R are subject to the same bursts of oscillations seen in the velocity plots. However, in the case of pressures, the amplitudes of the oscillations are relatively too small to be seen in the plots.

To conclude, the results of the estimation of the actuator's loads are illustrated. The estimates were obtained using sampled open-loop and closed-loop data generated by the RSS and acquired with sampling time 0.025s. The open-loop and closed-loop inputs were those used to generate the simulations in Figures 7-10. The acquisition of data was simulated by including the transducer component which includes a realistic source of low-intensity electrical noise. The velocity of the actuators was then reconstructed in discrete-time using a backward difference discretisation scheme.³⁰ The optimisation of the least-squares estimation criterion was performed by a simple grid search. The estimates, and relative errors with respect to the true values in Table 1, were $\hat{F}_L^E = -1721 \text{ N}(3.5\%)$ and $\hat{F}_L^R = 4423 \text{ N}(0.2\%)$ obtained from the open-loop data, and $\hat{F}_L^E = -1664 \text{ N}(0\%)$ and $\hat{F}_L^R = 4430 \text{ N}(0\%)$ obtained from the closed-loop data. Note that the relative errors range from negligible to 3.5%. Thus, the estimated loads can be considered to be very accurate. The contour plots of the least-

Figure 7. Position responses in open-loop (left) and in closed-loop (right).

Figure 8. Velocity responses in open-loop (left) and in closed-loop (right).

Figure 9. TMC inputs in open-loop (left) and in closed-loop (right).

Figure 10. Pressure responses in open-loop (left) and in closed-loop (right).

Figure 11. Estimation criterion using open-loop data (left) and using closed-loop data (right).

-squares criteria in both cases are shown in Figure 11. Note how the cost functions are such that the optimum is easily determined. The estimation of the actuator's loads was repeated for all flight conditions considered in Figure 3, which correspond to significantly different shapes of the input nonlinearity in the simplified model, leading to similar results in each case.

Conclusions

In this paper, a simplified model of the compressor variable stator vanes (VSVA) actuation system of the Rolls-Royce Trent 1000 engine was presented. The actuator is characterised by the presence of a cooling flow orifice which significantly affects its behaviour. It has been found that the behaviour of the actuator is accurately represented by a static input nonlinearity followed by a linear system. The accuracy of the simplified model was verified using the Rolls-Royce Trent 1000 Real System Simulator. The VSVA simplified model has enabled us to develop a fast algorithm for computing a snapshot estimate of the

actuator's external load in a given flight condition. The external load is by far the largest uncertainty affecting the actuation system. In our current work we are developing a continuous estimator suitable for changing flight conditions with a focus on fault detection^{31–34}.

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