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Asymmetric Single-peaked Preferences*

Francisco Martínez-Mora and M. Socorro Puy

Abstract

The asymmetry of single-peaked preferences has scarcely been incorporated as an assumption in economic models. We analyze how to deal with asymmetric single-peaked preferences in a tractable way. We define natural types of asymmetries, provide the tools to compare degrees of asymmetry, and propose concrete utility functions that represent different directions and degrees of asymmetry. As an application, we provide a representative voter theorem which establishes the heterogeneity in degrees of asymmetry across agents that is compatible with the median being the representative voter.

KEYWORDS: single-peakedness, asymmetries, utility representation, single-crossing

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1 Introduction

When preferences are single-peaked with respect to a linear order of a set of alternatives, there exists a most preferred alternative (the bliss point) and alternatives located on one side and closer to this point are preferred to those located further away from it. The domain of single-peaked preferences has been extensively analyzed by social choice theorists as a relaxation of the universal domain of preferences. In this restricted domain, certain social choice functions can be characterized in terms of desirable normative properties (see, e.g., Moulin, 1980; Sprumont 1991). Symmetric single-peaked preferences is an even further restricted domain, as suggested by Massó and Moreno de Barreda (2011): *"Often, an agent preferences over the domain of alternatives is responsive also to the notion of distance, embedding to the preferences its corresponding notion of symmetry"*. Symmetric single-peaked preferences imply that symmetric deviations above and below the peak are identical in terms of utility losses.

In formal political economy as well as in economic policy analysis, the policy space is generally considered to have a meaningful metric: right-wing versus left-wing ideology, expenditure in public good, inflation or marginal taxation among others (Persson and Tabellini, 2000 covers most of these examples). In these fields, preferences of agents or policymakers over the policy space are usually assumed to be symmetric around their ideal policy. The quadratic or the euclidean distance between the peak and the alternatives provides a tractable utility representation of preferences in these settings.

While the quadratic utility representation of preferences over policies is a standard assumption, certain theoretical models could be enriched with a less restrictive utility representation. Thus, in this paper, we provide a detailed analysis on how to represent, in a similar tractable way, asymmetric single-peaked preferences. We are not the first authors raising this point, in fact, Blinder (1997) notes that: *"Academic macroeconomists tend to use quadratic loss functions for reason of mathematical convenience, without thinking much about their substantive implications. The assumption is not innocuous, [...] practical central bankers and academics would benefit from more serious thinking about the functional form of the loss function"*. Following Blinder's suggestion, there are now a number of economic policy studies that incorporate asymmetric preferences as a critical assumption. Thus, Ruge-Murcia (2003) develops a theoretical model of monetary policy-making which endows the central bank with an asymmetric loss function and finds

that some important results (such as the linear relation between unemployment and the average inflation deviation from the target) do not extend to the asymmetric case. Moreover, he provides empirical evidence supporting the existence of a non-symmetric objective function. Along similar lines, Dolado et al. (2004) show that asymmetric preferences are theoretically important, as they imply the existence of a non-linear monetary policy rule, and provide empirical evidence of such nonlinearity in the US. Exploiting the same implication of asymmetric preferences, Surico (2007) offers evidence supporting that the preferences of the Fed were asymmetric with respect to target values before 1979, when output contractions triggered stronger responses than output expansions. Surico stated that " [...] *potential evidence of asymmetries in the central bank objective may be interpreted as evidence of asymmetries in the representative agent's utility.*" Likewise, within the literature on fiscal response to foreign-aid, Heller (1975) and Feeny (2006) highlight the relevance of policymakers' asymmetric single-peaked preferences over deviations with respect to target spending or tax revenues. There are other settings, such as the preferences of citizens over public expenditure which, as suggested by Milyo (2000) do not adjust to the symmetry assumption.¹

In this paper, we explore the difficulties of dealing with *asymmetric* single-peaked preferences. In the same way that the quadratic or euclidean distance functions have facilitated the analysis of symmetric single-peaked preferences, we propose a tractable utility representation of asymmetric single-peaked preferences. Besides, we analyze the degree of asymmetry of preferences across agents for which the preferences of the median agent in the distribution of peaks represents majority preferences.

Within the family of single-peaked preferences, we describe two natural types of asymmetric preferences, those that show *shortfall avoidance* and those displaying *excess avoidance*.² These definitions are based on the comparison of pairs of alternatives symmetrically located about the peak. If every alternative located above the peak is preferred to its symmetric alternative below the peak, we say that preferences show shortfall avoidance. In the opposite case, we say that preferences show excess avoidance.

When preferences over a convex set of alternatives are single-peaked and

¹Waud (1976), in a study of economic policy-making under uncertainty, revealed that the optimal response of the policymaker to changes in uncertainty crucially hinges on the symmetry or asymmetry of preferences.

²This terminology is adapted from Cukierman and Muscatelli (2008). These authors refer to inflation avoidance and recession avoidance preferences.

continuous, every alternative above the peak has an indifferent alternative below the peak. For every alternative below the peak, we propose a preference-bias function that measures the distance between the peak and its indifferent alternative above the peak. We use this function to compare, when possible, the degree of asymmetry across different preferences.

When single-peaked preferences can be represented by a utility function, we provide sufficient conditions to identify whether preferences display one of the proposed types, *shortfall avoidance* or *excess avoidance*. We also deduce conditions on the utility representation that allow the comparison of degrees of asymmetry.³ These conditions are based on the speed at which the slope of the utility function falls. Among other conditions, we find that the coefficient of prudence proposed by Kimball can be used, in some cases, to compare the degree of asymmetry across different utility representations of single-peaked preferences.⁴

We analyze how particular utility specifications can be used to represent asymmetric single-peaked preferences. First, we define a rich family of utility functions, that we call the *generalized single-peaked utility functions*. We show that these utility representations can accommodate every direction and degree of asymmetry. Second, we analyze other utility functions that have been used in the literature, the linear loss function proposed by Varian (1974), and further studied by Zellner (1986), and the piecewise asymmetric function proposed by Waud (1976). We explore the direction and degrees of asymmetry that these functions may accommodate.

Finally, we analyze the heterogeneity in degrees of asymmetry across agents' preferences that is compatible with a representative voter theorem (as defined by Rothstein, 1991). It is common in models of public good provision to consider a set of agents with single-peaked preferences, who have to vote over the level of public good provision. When preferences are symmetric about an ideal point, it is clear that the preferences of the median (that is, the agent with peak in the median of the distribution of peaks) coincide with

³In probability theory, the asymmetry of a probability distribution is classified as negative skew (the left tail is longer) and positive skew (the right tail is longer). Skewness is measured in terms of the third moment about the mean. Observe that in contrast to probability theory, single-peaked preferences cannot be characterized by a mean. Thus, the analysis of asymmetry which is based on the mean, cannot be interpreted in our context of asymmetric single-peaked preferences.

⁴This coefficient is defined by the third derivative divided by the second derivative of the utility function.

majority preferences. In this case, the location of the peak of the median is sufficient to predict the result of majority voting between every pair of alternatives. When preferences are not symmetric about the ideal point, however, there is an additional difficulty to derive the result of majority voting given that the preferences of the median do not always coincide with majority preferences. The single-crossing property proposed by Gans and Smart (1996) is sufficient for the median agent to be the representative voter according to majority voting.⁵ For the case of asymmetric single-peaked preferences, we translate the single-crossing property into a condition on the heterogeneity of the asymmetry of preferences across agents. This condition, that we call non-decreasing shortfall avoidance, guarantees that majority voting preferences and the preferences of the median voter coincide. Moreover, majority voting is the only aggregation rule for which induced social preferences coincide with the preferences of the median voter.

The remaining of the paper is organized as follows. The next section presents the environment and definitions. Section 3 studies conditions on the utility representation to derive the direction and the degree of asymmetry of preferences. Section 4 proposes and analyzes specific utility functions. Section 5 is devoted to the representative voter theorem in the case of asymmetries. Section 6 concludes.

2 The environment and definitions

An agent has preferences defined over alternatives in $\mathbb{R}$. The space of alternatives we consider has a specific and meaningful metric; thus, it is not possible to alter the spatial location of the alternatives.⁶ The preferences R of the agent on the set of alternatives is a complete preorder. The set of complete preorders on the set of alternatives is $\mathcal{R}$.

The strict and indifference preference relations induced by R are denoted by P and I respectively. Given $R \in \mathcal{R}$, the peak of R , when it exists, is an

⁵Rothstein (1991) proposes an ordering of voters which suffices for the median of this order to be the representative voter. Rothstein named this condition *order restriction*. Gans and Smart (1996) showed that a version of order restriction is equivalent to the single-crossing property and offered easy-checking conditions for a representative voter to exist. The two notions of order restriction, however, are different as explained by Saporiti and Tohmé (2006).

⁶See Eguia (2011) for an analysis that endogeneizes the spatial representation of the set of alternatives.

alternative $e \in \mathbb{R}$ strictly preferred to any other alternative. Let e^* denote the peak of R .

Definition: Preferences $R \in \mathcal{R}$ satisfy *single-peakedness (SP)* if there exists a peak of R , and for all $d_0, d_1 > 0$ such that $d_0 < d_1$, $e^* - d_0 \ P \ e^* - d_1$ and $e^* + d_0 \ P \ e^* + d_1$.

The SP property of preferences requires that, at each side of the peak, alternatives located closer to the peak are preferred to those located further away from it. The set of complete preorders satisfying SP on the set of alternatives is denoted by $\mathcal{R}^{SP}$. We refer to a generic element of $\mathcal{R}^{SP}$ as a SP preferences.

When preferences over a convex set of alternatives are continuous and satisfy single-peakedness, every alternative below the peak can be associated to another alternative above the peak according to the indifference preference relation. In what follows, we interpret $d > 0$ as a deviation off the peak and define a function that links pairs of indifferent alternatives in terms of deviations with respect to the peak.

Definition: The *preference-bias function* δ associated to $R \in \mathcal{R}^{SP}$ assigns to every deviation below the peak d , the corresponding $\delta(d) > 0$ for which $e^* - d \ I \ e^* + \delta(d)$.

In the case that the set of alternatives were bounded, we could also define the preference-bias function. Suppose that $e \in [0, \bar{e}]$, where $\bar{e}$ is sufficiently large as to guarantee that there exists an alternative $\tilde{e} \leq \bar{e}$ such that $0 \ I \ \tilde{e}$. Then, $e^* \in (0, \tilde{e})$ and the preference-bias function is defined so that $\delta : [0, e^*] \rightarrow [0, \tilde{e} - e^*]$.⁷

Observe that by single-peakedness, every preference-bias function is a strictly increasing function. We define three types of SP preferences: symmetric, shortfall avoidance and excess avoidance preferences.

Definition: Let δ be the preference-bias function associated to $R \in \mathcal{R}^{SP}$.

If $\delta(d) = d$ for all d , preferences are *symmetric*,

if $\delta(d) > d$ for all d , we say that preferences show *shortfall avoidance*,

if $\delta(d) < d$ for all d , we say that preferences show *excess avoidance*.

Symmetric SP preferences induce indifference between alternatives located symmetrically around the peak. When SP preferences show shortfall

⁷The assumption that $0 \ I \ \tilde{e}$, $\tilde{e} \in (e^*, \bar{e}]$ is made only to simplify the exposition. Alternatively, if there exists an alternative $\hat{e} \in [0, e^*)$ such that $\bar{e} \ I \ \hat{e}$ then the preference-bias function is defined so that $\delta : [0, e^* - \hat{e}] \rightarrow [0, \bar{e} - e^*]$.

avoidance, the comparison between two alternatives symmetrically located at each side of the peak is such that the alternative located above the peak is higher in the preference ordering than the alternative located below the peak. Of course, it is the opposite when SP preferences show excess avoidance. The proposed types of SP preferences do not fully classify the set $\mathcal{R}^{SP}$. These two types, however, capture two natural and meaningful classes of asymmetries in the set of SP preferences.⁸

Because the preference-bias function is independent of the location of the peak, one can compare degrees of asymmetry between pairs of preferences even when their respective peaks do not coincide. The following definition establishes the binary relation more shortfall avoidance than on $\mathcal{R}^{SP}$.

Definition: Let $R_1, R_2 \in \mathcal{R}^{SP}$ with δ_1, δ_2 denoting their respective preference-bias functions. We say that *preferences R_2 show more shortfall avoidance than preferences R_1* when $\delta_1(d) \leq \delta_2(d)$ for all d (or equivalently, R_1 show more excess avoidance than R_2).

According to this definition, degrees of asymmetry are comparable across pairs of preference relations when their associated preference-bias functions are such that one is always above the other, i.e., $\delta_1(d) \leq \delta_2(d)$ for all d . Observe, therefore, that the proposed binary relations are partial preorders (transitive but not complete) on the sets of SP preferences which show shortfall avoidance or excess avoidance.⁹

3 Conditions on the utility representation

Every preference relation $R \in \mathcal{R}^{SP}$ can be represented by a strictly quasi-concave utility function $V : \mathbb{R} \rightarrow \mathbb{R}$ which has a maximizer. The maximizer (or peak of R) satisfies $e^* = \arg \max V(e)$. We consider differentiable utility representations so that the peak of R satisfies $V'(e^*) = 0$.

⁸In particular, these definitions do not cover SP preferences which show shortfall avoidance in some ranges of the domain and excess avoidance in others. It would of course be possible to generalize them to account for any SP preference in $\mathcal{R}$.

⁹Again, the definition could be generalized to compare degrees of asymmetry in specific intervals of the domain of d .

3.1 Conditions for asymmetric SP preferences

In terms of the utility representation, when SP preferences display shortfall avoidance, $V(e^* - d) < V(e^* + d)$ for all d , and when they display excess avoidance, in turn, $V(e^* - d) > V(e^* + d)$ holds for all d . Figure 1 depicts two examples of utility functions representing shortfall avoidance, excess avoidance and its respective preference-bias functions.

Figure 1: Alternatives to symmetric SP preferences

A characterization of symmetric single-peaked preferences in terms of the properties of the utility representation V is straightforward to establish. Preferences are symmetric if and only if $V'(e^* - d) = -V'(e^* + d)$ for all d . That is to say, if and only if marginal utility at every pair of symmetric deviations above and below the peak coincide. Similarly, certain properties of the slope of V guarantee that preferences show shortfall avoidance or excess avoidance.

Proposition 1 Let V be a differentiable utility representation of $R \in \mathcal{R}^{SP}$. Properties (1a) or (2a) guarantee that R show shortfall avoidance:
 (1a) $V'(e^* - d) > -V'(e^* + d)$ for all d ; (2a) V' strictly convex for all $e \in \mathbb{R}$
 Properties (1b) or (2b) guarantee that R show excess avoidance:
 (1b) $V'(e^* - d) < -V'(e^* + d)$ for all d ; (2b) V' strictly concave for all $e \in \mathbb{R}$.

Proof. The following claims prove our statement.¹⁰

Claim 1: (1a) $\Rightarrow \delta(d) > d$ for all d .

Proof of Claim 1: Utility loss derived from reducing e^* to $e^* - d$ is measured by $\int_{e^* - d}^{e^*} V'(e)de$, whereas utility loss from increasing e^* to $e^* + d$ is measured by $-\int_{e^*}^{e^* + d} V'(e)de$. By (1a), $\int_{e^* - d}^{e^*} V'(e)de > -\int_{e^*}^{e^* + d} V'(e)de$ for all d . Solving for the integral and simplifying $V(e^* - d) < V(e^* + d)$ for all d , i.e., $\delta(d) > d$ for all d .

Claim 2: (2a) $\Rightarrow$ (1a)

Proof of Claim 2: By strict convexity of V' we have $V'(e^*) < \frac{V'(e^* - d) + V'(e^* + d)}{2}$ for all d . By SP, $V'(e^*) = 0$, and substituting in the inequality yields $V'(e^* - d) > -V'(e^* + d)$ for all d . ■

By conditions (1a) and (1b), the comparison of the slopes of the utility representation of SP preferences at every symmetric deviation with respect to the peak reveals the direction of the asymmetry. Conditions (2a) and (2b) also reveal the direction of the asymmetry by checking whether the marginal utility function is strictly concave or strictly convex.¹¹

3.2 Comparison of degrees of asymmetry

Figure 2 depicts two utility representations of preferences $R_1, R_2 \in \mathcal{R}^{SP}$ where preferences R_2 (represented by V_2) show more shortfall avoidance than preferences R_1 (represented by V_1). The peak of both preferences is the same and their associated preference-bias functions δ_1 and δ_2 can be easily compared.

¹⁰The analogous statements on preferences showing excess avoidance can be proved following a similar reasoning (that we omit in the interest of brevity).

¹¹For a concrete application of conditions (2a) and (2b) see Cukierman and Muscatelli (2008) where central banks' preferences show excess avoidance over inflation and shortfall avoidance over output gap.

Figure 2: V_2 shows more shortfall avoidance than V_1

Next, we show that the comparison between the slopes of different utility representations indicates the strength of the asymmetry across different preferences. For each pair of preferences $R_1, R_2 \in \mathcal{R}^{SP}$, we denote by e_1^*, e_2^* their respective peaks.

Proposition 2 *Let V_1, V_2 be two differentiable utility representations of $R_1, R_2 \in \mathcal{R}^{SP}$, respectively. If $V_1'(e_1^* + \gamma) \leq V_2'(e_2^* + \gamma)$ for all $\gamma \in \mathbb{R}$, then R_2 show more shortfall avoidance than R_1 .*

Proof. Condition $V_1'(e_1^* - d) \leq V_2'(e_2^* - d)$ for all d implies that the utility loss derived from reducing e_1^* to $e_1^* - d$, which is measured by $\int_{e_1^* - d}^{e_1^*} V_1'(e) de = B$, in comparison to the utility loss derived from reducing e_2^* to $e_2^* - d$, which is measured by $\int_{e_2^* - d}^{e_2^*} V_2'(e) de = A$, is such that $B \leq A$. By definition of the preference-bias function, there is $\delta_2(d) > 0$ for which $A = \int_{e_2^*}^{e_2^* + \delta_2(d)} V_2'(e) de$, at the same time, and given that $0 > V_2'(e_2^* + d) \geq V_1'(e_1^* + d)$ for all d ,

$$\int_{e_1^*}^{e_1^* + \delta_2(d)} V_1'(e) de \geq A. \quad (1)$$

By definition of the preference-bias function there is $\delta_1(d) > 0$ for which

$$\int_{e_1^*}^{e_1^* + \delta_1(d)} V_1'(e) de = B. \quad (2)$$

Since $B \leq A$, then (1) and (2) imply $\int_{e_1^*}^{e_1^* + \delta_1(d)} V_1'(e) de \leq \int_{e_1^*}^{e_1^* + \delta_2(d)} V_1'(e) de$ from where we derive that $\delta_1(d) \leq \delta_2(d)$ for all d . ■

In other terms, the proposed sufficient condition for R_2 to show more shortfall avoidance than R_1 implies that $V_1'(e_1^* - d) \leq V_2'(e_2^* - d)$ and $V_1'(e_1^* + d) \leq V_2'(e_2^* + d)$, for all d ; i.e., below the peak, equal-distance deviations generate more disutility with V_2 than with V_1 , and above the peak, equal-distance deviations generate more disutility with V_1 than with V_2 .

The degree of asymmetry can also be compared using the degree of concavity or convexity of the marginal utility function. For this, one of the marginal utility specifications must be obtained as an increasing transformation of the other.

Proposition 3 *Let V_1, V_2 be strictly concave utility representations of $R_1, R_2 \in \mathcal{R}^{SP}$. If $-\frac{V_1'''(e)}{V_1''(e)} \leq -\frac{V_2'''(e+\Lambda)}{V_2''(e+\Lambda)}$ for all $e \in \mathbb{R}$ where $\Lambda = e_2^* - e_1^*$, then R_2 show more shortfall avoidance than R_1 .*

Proof. By strict concavity of V_1 and V_2 , the functions V_1' and V_2' are strictly decreasing functions and they can be related by a strictly increasing transformation g such that $V_1'(e) = g(V_2'(e + \Lambda))$ where $\Lambda = e_2^* - e_1^*$. This implies that when $e = e_1^*$, $g(0) = 0$.¹² Differentiating the expression,

$$\begin{aligned} V_1''(e) &= g'(V_2'(e + \Lambda))V_2''(e + \Lambda) \\ V_1'''(e) &= g''(V_2'(e + \Lambda)) [V_2''(e + \Lambda)]^2 + g'(V_2'(e + \Lambda))V_2'''(e + \Lambda). \end{aligned}$$

Dividing the second expression by $V_1''(e)$ and rearranging terms:

$$-\frac{V_1'''(e)}{V_1''(e)} = -\frac{V_2'''(e + \Lambda)}{V_2''(e + \Lambda)} - \frac{g''(V_2'(e + \Lambda))V_2''(e + \Lambda)}{g'(V_2'(e + \Lambda))}.$$

Given that $-\frac{V_1'''(e)}{V_1''(e)} \leq -\frac{V_2'''(e+\Lambda)}{V_2''(e+\Lambda)}$, $V_2'' < 0$ and $g' > 0$, we deduce that $g''(V_2'(e + \Lambda)) \leq 0$ (i.e., g must be concave).

¹²If $e_2^* = e_1^*$, then $\Lambda = e_2^* - e_1^* = 0$, and it is possible to compare degrees of asymmetry just by comparing $\frac{V_1'''(e)}{V_1''(e)}$ to $\frac{V_2'''(e)}{V_2''(e)}$.

By definition of δ_1 , it follows that $V_1(e_1^* - d) = V_1(e_1^* + \delta_1(d))$, or equivalently, $\int_{e_1^* - d}^{e_1^* + \delta_1(d)} V_1'(e) de = 0$. Substituting function g ,

$$0 = \int_{e_1^* - d}^{e_1^* + \delta_1(d)} V_1'(e) de = \int_{e_1^* - d}^{e_1^* + \delta_1(d)} g(V_2'(e + \Lambda)) de. \quad (3)$$

By concavity of g , $\int_{e_1^* - d}^{e_1^* + \delta_1(d)} g(V_2'(e + \Lambda)) de \leq g(\int_{e_1^* - d}^{e_1^* + \delta_1(d)} V_2'(e + \Lambda) de)$, where $\int_{e_1^* - d}^{e_1^* + \delta_1(d)} V_2'(e + \Lambda) de = V_2(e_2^* + \delta_1(d)) - V_2(e_2^* - d)$. Thus, by Expression (3), $0 \leq g(V_2(e_2^* + \delta_1(d)) - V_2(e_2^* - d))$ and given that g is strictly increasing with $g(0) = 0$, then $V_2(e_2^* + \delta_1(d)) - V_2(e_2^* - d) \geq 0$. By definition of $\delta_2(d)$, $V_2(e_2^* - d) = V_2(e_2^* + \delta_2(d))$, we deduce that $V_2(e_2^* + \delta_1(d)) \geq V_2(e_2^* + \delta_2(d))$ and by single-peakedness, it must be the case that $\delta_1(d) \leq \delta_2(d)$ for all d , i.e., R_2 show more shortfall avoidance than R_1 . ■

The proof of Proposition 3 is based on the theories developed by Arrow (1971), Pratt (1964) and Kimball (1990). In Arrow-Pratt's theory of risk aversion, concavity of the utility function over consumption indicates the presence of risk aversion, while according to Kimball's theory of precautionary savings, concavity of the marginal utility function entails precautionary saving behavior. In each case, the degree of concavity of the utility function or the degree of concavity of the marginal utility function measures risk aversion or precautionary savings respectively.¹³ These behavioral traits become thus comparable across pairs of concave functions such that one is a concave transformation of the other.

In our context, as long as V is strictly concave, the curvature of the marginal utility function determines the degree of asymmetry. Interestingly, we can apply the coefficient of prudence (proposed by Kimball, 1990) to measure whether preferences show more shortfall avoidance for the *more convex* marginal utility representation (or, equivalently, more excess avoidance for the *more concave* marginal utility representation). We use this idea to define a coefficient that measures the asymmetry of SP preferences.

Definition: Let V be strictly concave utility representation of $R \in R^{SP}$. We refer to the ratio $p(e) = -\frac{V'''(e)}{V''(e)}$ as the *coefficient of shortfall avoidance*.

According to Proposition 3, the higher $p(\cdot)$, the greater the degree of shortfall avoidance. When $p(\cdot)$ is constant and $p(\cdot) < 0$, we say that pref-

¹³ Arrow-Pratt's coefficient of risk aversion is defined by $-\frac{u''}{u'}$, whereas Kimball's coefficient of prudence is defined by $-\frac{u'''}{u''}$ where u measures utility over private consumption.

erences show constant shortfall avoidance. Likewise, when $p(\cdot)$ is constant and $p(\cdot) > 0$, we say that preferences show constant excess avoidance. We can also define increasing or decreasing shortfall avoidance or increasing or decreasing excess avoidance.¹⁴

In the case of a constant coefficient of asymmetry, shortfall avoidance can be directly compared among different utility specifications.

Corollary 1 *Let V_1 and V_2 be strictly concave utility representations of $R_1, R_2 \in \mathcal{R}^{SP}$. If the coefficient of asymmetry is constant and such that $p_1 \leq p_2$, then R_2 show more shortfall avoidance than R_1 .*

Proof. When $p_2(\cdot)$ is constant, $-\frac{V_2'''(e)}{V_2''(e)} = -\frac{V_2'''(e+\Lambda)}{V_2''(e+\Lambda)}$ for all Λ . Thus, $-\frac{V_1'''(e)}{V_1''(e)} \leq -\frac{V_2'''(e)}{V_2''(e)}$ implies that $-\frac{V_1'''(e)}{V_1''(e)} \leq -\frac{V_2'''(e+\Lambda)}{V_2''(e+\Lambda)}$ for all $e \in \mathbb{R}$, which according to Proposition 3, this is equivalent to asserting that R_2 show more shortfall avoidance than R_1 . ■

Corollary 1 can be useful to compare degrees of asymmetry across different utility functions (see, for example, the linex loss function that is analyzed in the following section).

4 Asymmetric SP utility representation

In this section, we analyze utility functions representing asymmetric SP preferences. First, we propose a new utility function based on the preference-bias function. Second, we analyze two other utility functions that have been proposed in different contexts to represent asymmetric single-peaked preferences.

4.1 The generalized single-peaked utility function

The utility function representing SP preferences and based on the distance function is such that $V(e) = -f(e - e^*)$ where f is a continuous and strictly increasing distance function between the peak e^* and the alternative e . Particular examples of f are the quadratic function, in which $f(e - e^*) = [e - e^*]^2$, the distance function induced by a norm, in which $f(e - e^*) = \|e - e^*\|$, or any function $f(e - e^*) = |e - e^*|^\lambda$ where $\lambda > 0$.

¹⁴When $p(\cdot) > 0$ for all $e \in \mathbb{R}$ and $p(\cdot)$ is increasing (decreasing), preferences show increasing (decreasing) shortfall avoidance. When $p(\cdot) < 0$ for all $e \in \mathbb{R}$ and $p(\cdot)$ is decreasing (increasing), preferences show increasing (decreasing) excess avoidance.

Given preference $R \in \mathcal{R}^{SP}$, the preference-bias function δ associated to R assigns to each deviation below the peak, the deviation above the peak for which the agent is indifferent. Let $\delta^{-1} : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ be the inverse of the preference-bias function.¹⁵ The *generalized single-peaked utility function* is defined by

$$V(e) = \begin{cases} -f(e - e^*) & \text{when } e \leq e^* \\ -f(\delta^{-1}(e - e^*)) & \text{when } e > e^* \end{cases} . \quad (4)$$

Below the peak, the proposed utility function coincides with the distance function. In order to capture the preference-bias, the level of utility derived from any alternative above the peak is equal to the corresponding utility value at its indifferent alternative below the peak. If the SP preference relation is symmetric, i.e., $\delta^{-1}(d) = d$, the generalized single-peaked utility function collapses to $V(e) = -f(e - e^*)$ for all e .

The generalized single-peaked utility function can also be defined in the case in which the set of alternatives is bounded. If $e \in [0, \bar{e}]$ and $0 I \tilde{e}$, the inverse of the preference-bias function is bounded above by $\delta^{-1}(\tilde{e} - e^*) = e^*$, which corresponds to the indifference relation $0 I \tilde{e}$. We can extend the domain of δ^{-1} to every $d \in (\tilde{e} - e^*, \bar{e} - e^*]$ in a strictly increasing way. Thus, the generalized single-peaked utility function is defined by

$$V(e) = \begin{cases} -f(e - e^*) & \text{when } e \in [0, e^*] \\ -f(\delta^{-1}(e - e^*)) & \text{when } e \in (e^*, \bar{e}] \end{cases} .$$

Observe that the utility derived from alternatives $e \in (\tilde{e}, \bar{e}]$ is below $V(\tilde{e})$ and that the utility function is strictly decreasing in $e \in (\tilde{e}, \bar{e}]$.¹⁶

Arguably, the generalized single-peaked utility function is derived from the set of indifferent alternatives of the primitive preference relation. Another is that it maintains the tractability of the distance functions (including

¹⁵Because the preference satisfies the SP condition, the preference-bias function δ is bijective and it has an inverse.

¹⁶If, for example, we take a linear preference bias-function $\delta(d) = kd$ with $k > 0$, the corresponding generalized single-peaked utility function would be

$$V(e) = \begin{cases} -f(e - e^*) & \text{when } e \in [0, e^*] \\ -f(\frac{e - e^*}{k}) & \text{when } e \in (e^*, \bar{e}] \end{cases} .$$

Note that this function extends, in a natural way, the domain of δ^{-1} to alternatives in the interval $e \in (\tilde{e}, \bar{e}]$.

smoothness if the distance function is smooth). Next, we show that it can accommodate every SP preferences, including those which satisfy our definition of shortfall and excess avoidance.

Proposition 4 *Every preference $R \in \mathcal{R}^{SP}$ can be represented by the generalized single-peaked utility function.*

Proof. First, we show that every preference relation $R \in \mathcal{R}^{SP}$ is represented by the generalized single-peaked utility function. The preference ordering across alternatives located at the same side of the peak is captured by the distance-metric utility function below the peak, and above the peak, by a function that is strictly decreasing in distance (given that δ^{-1} is a strictly increasing function in all its domain). The preference ordering of pairs of alternatives located at opposite sides of the peak can be deduced by identifying the pairs of alternatives yielding equal utility. Thus, $V(e^* + \delta(d)) = -f(\delta^{-1}(e^* + \delta(d) - e^*))$ and simplifying $V(e^* + \delta(d)) = -f(d)$. Since $V(e^* - d) = -f(d)$, we deduce that $e^* - d I e^* + \delta(d)$ for all d .

■

Observe that the generalized single-peaked utility function is that it can be used to compare pairs of preferences such that one of them shows more shortfall avoidance than the other. Given a distance function f , the utility derived from every alternative below the peak is normalized. Thus, if we account for two different functions V_1, V_2 we have that $V_2'(e_2^* - d) = V_1'(e_1^* - d)$ for every alternative below the peak. Above the peak, however, the derivative does not coincide when we represent two distinct preferences. In the case that $\delta_1(d) \leq \delta_2(d)$, condition $V_1'(e_1^* + d) \leq V_2'(e_2^* + d)$ for all d , is a sufficient condition to show that R_2 show more shortfall avoidance than R_1 .¹⁷

If we take a linear preference bias-function $\delta(d) = kd$ with $k > 0$, the corresponding generalized single-peaked utility function is

$$V(e) = \begin{cases} -f(e - e^*) & \text{when } e \leq e^* \\ -f(\frac{e - e^*}{k}) & \text{when } e > e^* \end{cases} \quad (5)$$

Under this utility specification, $k > 1$ represents a particular class of SP preferences that show shortfall avoidance, while $k < 1$ corresponds to another

¹⁷This is a simplification of Proposition 2 that can be used when we account for the generalized single-peaked utility function and f is the same in both functions V_1, V_2 .

class of SP preferences that exhibit excess avoidance. Because the preference-bias function does not need to be linear; we can think of diverse functional forms yielding different asymmetric SP preferences. Thus, it is possible to represent more complex asymmetries than the basic types previously defined. For instance, we can represent preferences that display excess avoidance for some values of d , and shortfall avoidance for others.¹⁸

Our proposal accommodates every continuous and strictly increasing distance function, in particular, a rich family of utility specifications are given by:

$$V(e) = \begin{cases} -|e - e^*|^\lambda & \text{when } e \leq e^* \\ -|\delta^{-1}(e - e^*)|^\lambda & \text{when } e > e^* \end{cases} \quad (6)$$

where different values of $\lambda > 0$ yield different utility functions. It is straightforward to check that these functions are smooth, i.e., they are continuously differentiable in all the domain, including the peak.

4.2 The linex loss function

This function was proposed by Varian (1974) and further studied by Zellner (1986). The linex (for linear-exponential) loss function has been used to represent policy makers' preferences over inflation and output gap (e.g. Ruge-Murcia, 2003; Dolado et al., 2004; Surico, 2007) as well as in Bayesian econometrics (Zellner, 1986) and optimal forecasting (Christoffersen and Diebold, 1997). According to our notation, this utility specification is defined by:

$$V(e) = -\exp(\alpha[e^* - e]) + \alpha[e^* - e] + 1 \quad (7)$$

where $\alpha \in \mathbb{R}$. Observe that this specification yields $V(e^*) = 0$.

Proposition 5 *The linex loss function can represent shortfall avoidance when $\alpha > 0$ and excess avoidance when $\alpha < 0$. Besides, its coefficient of shortfall avoidance is constant and equal to α in the case of shortfall avoidance, and equal to $-\alpha$ in the case of excess avoidance.*

¹⁸A preference-bias function that creates such kind of asymmetry is $\delta(d) = [d(1-a)]^\beta$, where $a \in [0, 1)$, $\beta > 0$ and $\beta \neq 1$; β determines whether preferences exhibit excess avoidance near the peak and shortfall avoidance from some distance onwards, or viceversa. In particular, if $\beta < 1$, then $\delta(d) \geq d$, $\forall d \leq d^*$ and $\delta(d) < d$, $\forall d > d^*$ so that preferences show shortfall avoidance below d^* and excess avoidance above d^* . The opposite case occurs when $\beta > 1$, which implies that $\delta(d) \leq d$ $\forall d \leq d^*$, and $\delta(d) > d$ $\forall d > d^*$. The threshold distance is given by $d^* = (1-a)^{\frac{\beta}{1-\beta}}$.

Proof. We can directly solve for the first, second and third derivatives of V , which can be used to measure the direction and the degree of asymmetry:

$$\begin{aligned} V' &= \alpha \exp(\alpha [e^* - e]) - \alpha \\ V'' &= -\alpha^2 \exp(\alpha [e^* - e]) \\ V''' &= \alpha^3 \exp(\alpha [e^* - e]). \end{aligned}$$

The second derivative guarantees concavity of the utility specification. By Proposition 1, the sign of the third derivative implies that the linex utility function represents shortfall avoidance when $\alpha > 0$ and excess avoidance when $\alpha < 0$. By Proposition 3, the coefficient of asymmetry is $-\frac{V'''}{V''}$, which equals α when there is shortfall avoidance and $-\alpha$ when there is excess avoidance. ■

According to our results, the linex function can accommodate the two types of asymmetry (shortfall and excess avoidance). Because its coefficient of asymmetry is constant, by Corollary 1, it is easy to represent and compare different degrees of asymmetry just by changing or comparing the value of α . This utility representation, however, cannot represent every preference $R \in \mathcal{R}^{SP}$. For instance, it cannot represent preferences which show excess avoidance for some values of d , and shortfall avoidance for others.

4.3 The piecewise asymmetric function of Waud

Finally, we analyze another utility representation of asymmetric SP preferences proposed by Waud (1976). Using our notation, this utility specification is defined by:

$$V(e) = \begin{cases} -\lambda f(e^* - e) & \text{when } e \leq e^* \\ -f(e - e^*) & \text{when } e > e^* \end{cases} \quad (8)$$

where f is a strictly increasing and convex function, $f(0) = 0$ and $\lambda > 0$. Observe that $\lambda < 1$ represents preferences showing shortfall avoidance, while $\lambda > 1$ corresponds to preferences showing excess avoidance. Additionally, by Proposition 2, when $\lambda > 1$, the higher the coefficient λ , the more excess avoidance the preferences show. Thus, the function proposed by Waud can represent preferences of the two basic asymmetric types (shortfall and excess avoidance). Furthermore, this allows for comparisons of degrees of asymmetry just by checking the value of λ . Nevertheless, this utility representation cannot represent every preference $R \in \mathcal{R}^{SP}$.

By the definition of the preference-bias function, $V(e^* - d) = V(e^* + \delta(d))$ for all d , which implies $\lambda = \frac{f(\delta(d))}{f(d)}$ for all d . Given a convex function f , that condition imposes a restriction on the preference-bias function and thereby on the preferences it can represent. To illustrate such restriction, let f be the quadratic function; then, the preference-bias function must satisfy $\delta(d) = \lambda^{\frac{1}{2}}d$; i.e., δ has to be linear in d . This example illustrates that this utility specification cannot represent every preference $R \in \mathcal{R}^{SP}$. Additionally, there are preference-bias functions for which no function f exists such that Expression (8) represents these preferences (consider again the case of preferences which show excess avoidance for some values of d , and shortfall avoidance for others).

5 Representative voter theorem

A representative voter theorem establishes conditions under which the preferences of a specific agent (e.g. the agent with median peak if the profile of preferences is single-peaked) are predictors of majority voting rule.¹⁹

In this section, we consider a continuum of agents with SP preferences. We assume that agents with the same peak are of the same type so that their preferences off-the-peak are also equal. Types of agents are ordered according to the peaks, so that if i, j are two different types and $i < j$, then $e_i^* < e_j^*$.

A preference profile R_N describes the preferences of all the agents. There is a continuous distribution function of peaks across agents. The peak of the median type is denoted by e_m^* and her preferences are denoted by R_m .

Let $e, e' \in \mathbb{R}$ be two different alternatives and let $N(e, e')$ be the percentage of voters for whom $e R_i e'$. We say that e is preferred to e' under *majority rule* (denoted by $e R_M e'$) if and only if $N(e, e') \geq N(e', e)$. We refer to R_M as *majority preferences*. The strict and indifference preference relations induced by R_M are denoted by P_M and I_M respectively. The proposed definition of majority rule is equivalent to that of plurality rule given that we only account for pairwise comparisons of alternatives. Observe, however, that this concept

¹⁹See the related literature (Rothstein, 1991; Gans and Smart, 1996; Austen-Smith and Banks, 1999; Saporiti and Tohmé, 2006; Barberà and Moreno, 2011). Conditions such as "order restriction" defined by Rothstein (1991) and "single-crossing" proposed by Milgrom and Shannon (1994) are sufficient to guarantee that preferences of the median coincide with majority preferences.

differs from simple majority rule by which e is preferred to e' if and only if $N(e, e') \geq 1/2$.

The property of single-crossing is a sufficient condition for the existence of an agent whose preferences coincide with majority preferences.

Definition: A preference profile R_N satisfies (*strict*) *single-crossing* if for all $e, e' \in R$ where $e < e' : e' R_i e$, implies $e' P_j e$ for all $j > i$.

By single-crossing, if an agent prefers an alternative e' over another e , where e' is greater than e , then every agent indexed above this one also prefers alternative e' over e . Neither single-crossing implies single-peakedness nor single-peakedness implies single-crossing.²⁰ The important implication of this property is the following:

Proposition 6 (*Gans and Smart 1996, Theorem 2*) *If the preference profile R_N satisfies (*strict*) single-crossing, then the majority preferences and the preferences of the median voter coincide.*

Thus, single-crossing is a sufficient condition for a representative voter theorem. When the SP preferences of all the agents are symmetric, the single-crossing property holds. When preferences of the agents are asymmetric, as pointed out by Rothstein (1991), we cannot guarantee a representative voter theorem. The author, however, shows that this is not true for particular cases.

Proposition 7 (*Rothstein 1991, Theorem 4*) *Let $\{\delta_1, \delta_2, \dots\}$ be the preference-bias functions associated to the different types of a single-peaked preference profile R_N . If for all i , $\delta_i = kd$, with $k > 0$ common for every agent, then majority preferences and the preferences of the median voter coincide.*

There are some examples in which an agent, different from the median of the peaks, is such that her preferences coincide with majority preferences.²¹ We, however, focus on those cases in which the representative voter (when it exists) coincides with the agent whose peak is the median of the peaks.

Next, we translate the single-crossing property in terms of degrees of asymmetry of preferences across types of agents. We go beyond Rothstein (1991) showing that certain heterogeneity in the asymmetry of preferences across types of agents is compatible with a representative voter theorem. We

²⁰Sapority and Tohmé (2006) illustrate this point.

²¹See for instance Rothstein (1991), Figure 1.

consider a particular type of heterogeneity of preferences across agents that we call non-decreasing shortfall avoidance.

Definition: Let $\{\delta_1, \delta_2, \dots\}$ be the preference-bias functions associated to the types of single-peaked preference profile R_N . We say that R_N satisfies *non-decreasing shortfall avoidance* when $\delta_i(d) \leq \delta_j(d)$ for all d and all $j > i$. Figure 3 depicts a particular profile of preferences satisfying this property. Observe in the figure that those agents with higher peak show higher degrees of shortfall avoidance.

Figure 3: An example of non-decreasing shortfall avoidance

According to our results in Section 3, we can provide conditions on the differentiable utility representation of preferences which guarantee that non-decreasing shortfall avoidance holds.

Corollary 2 *Given the single-peaked preference profile R_N and its corresponding utility representations, each of the following conditions guarantee that R_N satisfies non-decreasing shortfall avoidance:*

- i) $V'_i(e_i^* + \gamma) \leq V'_j(e_j^* + \gamma)$ for all $\gamma \in \mathbb{R}$ and $j > i$,
- ii) $-\frac{V'''_i(e)}{V''_i(e)} \leq -\frac{V'''_j(e+\Lambda)}{V''_j(e+\Lambda)}$ for all $e \in \mathbb{R}$ and $j > i$ where $\Lambda = e_j^* - e_i^*$,
- iii) the coefficient of shortfall avoidance is constant and such that agents with higher peak have (weakly) higher coefficients, i.e., $p_i \leq p_j$ for all $j > i$.

An aggregation rule, f , assigns to every profile of preferences, a reflexive and complete binary relation. Among the family of all aggregation rules, the voting rules are a particular type.²² We refer to R_f as the social preferences induced by an aggregation rule f where, in particular, R_M refers to the social preferences induced by majority voting rule.

Proposition 8 *If the single-peaked preference profile R_N satisfies non-decreasing shortfall avoidance, then majority rule is the only aggregation rule such that the induced social preferences coincide with the preferences of the median voter*

Proof. By non-decreasing shortfall avoidance, $\delta_i(d) \leq \delta_j(d)$ for all d and all $j > i$. Given that, by definition, the preference-bias function is increasing, $\delta_j(d + e_j^* - e_i^*) > \delta_i(d)$ when $e_j^* - e_i^* > 0$ (i.e., when $j > i$) and

$$\delta_j(d + e_j^* - e_i^*) + e_j^* - e_i^* > \delta_i(d). \quad (9)$$

Let e, e' be two different alternatives, where $e < e'$, such that $e' R_i e$. If either $e' < e_i^*$ and/or $e' < e_j^*$ where $j > i$, by single-peakedness $e' P_j e$. For single-crossing to hold, we have to show that $e' P_j e$ even when $e' > e_j^*$. Given that $e' R_i e$ we deduce that

$$\delta_i(d) > e' - e_i^* \text{ where } d = e_i^* - e \quad (10)$$

Combining inequalities (9) and (10), $\delta_j(e_i^* - e + e_j^* - e_i^*) + e_j^* - e_i^* > e' - e_i^*$, which simplifying yields $\delta_j(e_j^* - e) > e' - e_j^*$. In terms of the preference relation of agent j this implies that $e' P_j e$ when $e' > e_j^*$. Thus, the (strict) single-crossing property holds and by Proposition 6, $R_M = R_m$. Consider an aggregation rule f different from majority rule. Then, there must be a profile of preferences for which two alternatives e_0, e_1 satisfy that $e_0 P_f e_1$ when $N(e_0, e_1) < N(e_1, e_0)$, i.e., $R_f \neq R_M$. As a consequence, $R_f \neq R_m$. ■

According to Proposition 8, heterogeneity in the asymmetry of preferences in the form of non-decreasing shortfall avoidance guarantees that the information about social preferences induced by majority rule is contained in the preferences of the median voter. Besides, the social preferences induced by any other aggregation rule require more information about the preferences of voters than that contained in the preferences of the median voter.

²²Following Austen-Smith and Banks (1999), among the family of all preference aggregation rules, the voting rules are characterized by two properties: neutrality and monotonicity (for example, Borda rule).

6 Conclusion

Both in formal political economy and in economic policy analysis, it is relevant to incorporate political actors endowed with asymmetric single-peaked preferences. For example, the asymmetry of preferences of central banks over inflation, and over output gap is crucial to derive optimal monetary policies²³; the asymmetry of policymakers or the preferences of voters over public expenditure and public debt is also relevant in the analysis of political decisions. Whether the predictions of some models are robust to the asymmetry of preferences is a question that, from our point of view, deserves more careful attention.²⁴

We have explored the way of representing and comparing different degrees of asymmetric single-peaked preferences. We have identified sufficient conditions on standard utility representations that reveal the direction (if any) of the asymmetry of preferences and another that allows the comparison of degrees of asymmetry across different preference relations. We have pay particular attention to the utility representation of these asymmetries. We have come up with a new proposal, the generalized single-peaked utility function, that not only represents the single-peaked preferences with any direction and degree of asymmetry, but also maintains the analytical tractability and smoothness of the distance functions. We have also explored in detail two other utility representations of asymmetric single-peaked preferences: the linex loss function and the utility function proposed by Waud. Interestingly, we find that the linex loss function shows a constant coefficient of asymmetry. In this case, different degrees of asymmetry can be parametrized by a single variable. Finally, we have explored the degree of heterogeneity on asymmetries across agents that is compatible with a representative voter theorem. The well-known single-crossing property implies, in the domain of single-peaked preferences, non-decreasing degrees of shortfall avoidance across agents with higher peak. Thus, if the modeling assumption is such that the preferences of agents show non-decreasing shortfall avoidance, then

²³As shown by Cukierman and Muscatelli (2008), the asymmetry of preferences is not innocuous as it induces a non-linear Taylor rule. See also Blinder (1997), Ruge-Murcia (2003), and Dolado et al. (2004).

²⁴In a companion paper, Martínez-Mora and Puy (2012), we show that these asymmetries modify the location of the platforms as well as the winning probability of the political parties that compete in a general election, when competition follows either the citizen-candidate model or a probabilistic voting model.

the preferences of the agent with peak in the median represents the social preferences according to majority rule.²⁵ Besides, the social preferences induced by any other aggregation rule differ from the preferences of the median.

We believe that our results can be very useful to incorporate, in a tractable way, a richer domain of single-peaked preferences.²⁶

References

- [1] Austen-Smith D. and J. S. Banks (1999), *Positive Political Theory I: collective preferences*, Michigan studies in political analysis, University of Michigan Press.
- [2] Arrow K. (1971), *Essays in the Theory of Risk-Bearing*. Chicago, IL: Markham Publishing Company.
- [3] Barberà S. and B. Moreno (2011), Top monotonicity: A common root for single peakedness, single crossing and the median voter result, *Games and Economic Behavior* 73: 345–359.
- [4] Benartzi S. and R. H. Thaler (1995), Myopic Loss Aversion and the Equity Premium Puzzle, *Quarterly Journal of Economics* 110: 73-92.
- [5] Blinder A. (1997), Distinguished lecture on economics and government: what central bankers could learn from academics and vice versa, *Journal of Economic Perspectives* 11: 3–19.
- [6] Christoffersen, P. F. and F. X. Diebold (1997), Optimal Prediction under Asymmetric Loss, *Econometric Theory* 13: 808-817.

²⁵Consider, for example, that preferences of agents over the one-dimensional policy space are represented by the linex loss function. We can incorporate heterogeneous preferences in terms of the peak of the agent and the degree of asymmetry of preferences (measured by parameter α) while maintaining the median voter as the representative voter according to majority rule.

²⁶Prospect theory also accounts for asymmetric preferences, though in a different sense. Following empirical and experimental evidence, this literature assumes that agents are more sensitive to losses than to gains with respect to a reference point (Kahneman and Tversky, 1991; Benartzi and Thaler 1995). In contrast to our analysis, the reference point is not a maximizer. We believe, however, that some of our results can also be adapted to this context.

- [7] Cukierman A. and A. Muscatelli (2008), Nonlinear Taylor Rules and Asymmetric Preferences in Central Banking: Evidence from the United Kingdom and the United States, *The B.E. Journal of Macroeconomics* 8, Article 7.
- [8] Dolado, J. J., R. Maria-Dolores and F. Ruge-Murcia (2004), Nonlinear Monetary Policy Rules: Some New Evidence for the U.S., *Studies in Nonlinear Dynamics and Econometrics* 8: 1-32.
- [9] Eguia J. X. (2011), Foundations of Spatial Preferences, *Journal of Mathematical Economics* 47: 200-205.
- [10] Feeny S. (2006), Policy Preferences in Fiscal Response Studies, *Journal of International Development* 18: 1167-1175.
- [11] Gans J. S. and M. Smart (1996), Majority voting with single-crossing preferences, *Journal of Public Economics* 59: 219-237.
- [12] Heller P. S. (1975), A model of Public Fiscal Behavior in Developing Countries: Aid, Investment, and Taxation, *American Economic Review* 65: 429-445.
- [13] Kahneman D. and A. Tversky (1991), Loss Aversion and Riskless Choice: a Reference Dependent Model, *Quarterly Journal of Economics* 106: 1039-1061.
- [14] Kimball M. S. (1990), Precautionary Saving in the Small and in the Large, *Econometrica* 58: 53-73.
- [15] Martínez-Mora F. and M. S. Puy (2012), The electoral effect of voters with asymmetric preferences, Mimeo Universidad de Málaga.
- [16] Massó J. and I. Moreno de Barreda (2011), On strategy-proofness and symmetric single-peakedness, *Games and Economic Behavior* 72: 467-484
- [17] Milgrom P. and C. Shannon (1994), Monotone Comparative Statics, *Econometrica* 62: 157-180.
- [18] Milyo J. (2000), A Problem with Euclidean Preferences in Spatial Models of Politics, *Economics Letters* 66, 179-182.

- [19] Moulin H. (1980), On strategy-proofness and single peakedness, *Public Choice* 35: 437-455.
- [20] Persson T. and G. Tabellini (2000), *Political Economics: Explaining Economic Policy*, The MIT Press.
- [21] Pratt J. W. (1964), Risk Aversion in the Small and in the Large, *Econometrica* 32: 122-136.
- [22] Rothstein P. (1991), Representative voter theorems, *Public Choice* 72: 193-212.
- [23] Ruge-Murcia F. J. (2003), Inflation Targeting under Asymmetric Preferences, *Journal of Money, Credit and Banking* 35: 763-785.
- [24] Saporiti A. and F. Tohmé (2006), Single-Crossing, Strategic Voting and the Median Choice Rule, *Social Choice and Welfare* 26: 363-383.
- [25] Sprumont Y. (1991), The Division Problem with Single-Peaked Preferences: A Characterization of the Uniform Allocation Rule, *Econometrica* 59: 509-519
- [26] Surico P. (2007), The Fed's Monetary Policy Rule and U.S. Inflation: The Case of Asymmetric Preferences, *Journal of Economic Dynamics and Control* 31: 305-324.
- [27] Varian, H. (1974), A Bayesian Approach to Real State Assessment, in Feinberg, F. E. and Zellner, A. (eds.) *Studies in Bayesian Economics* in honour of L. J. Savage. North-Holland. Amsterdam.
- [28] Waud, R. N. (1976), Asymmetric Policymaker Utility Functions and Optimal Policy Under Uncertainty, *Econometrica* 44: 53-66.
- [29] Zellner, A. (1986), Bayesian Estimation and Prediction using Asymmetric Loss Functions, *Journal of the American Statistical Association* 81: 446-451.