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Flux front dynamics and energy losses of magnetically anisotropic 2G-HTS pancake coils under prospective winding deformations

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In this paper, a comprehensive analysis on the local electrodynamics at micron level for the second generation of magnetically anisotropic high temperature superconducting (2G-HTS) pancake coils is presented. Special attention has been paid to the influence of prospective winding misalignment factors onto macroscopical quantities such as the hysteresis losses and critical current density inside each one of the turns of a generic 2G-HTS coil, and their relation with the magneto-angular dependence of the in-field critical current density $J_c(B, \theta)$ of the 2G-HTS tape. It has been shown that for low amplitudes of the applied transport current, $I_a \leq 0.2 I_{c0}$, the flux-front profile for perfectly aligned pancake coils develops a well-shaped flux-free core with a complete absence of magnetization currents in $\sim 30\%$ the middle turns, that are enclosed by transport current profiles in similar fashion to what would be observed in superconducting bulks, but with the outer turns showing a clear dominance of magnetization currents. However, the misaligned pancake coils show a notorious deflection of the flux-front semi-elliptical vertices which induces a repositioning of the co-vertices, shaping then a “worm-like” flux-front profile that is caused by the breakdown in the ordinal symmetry of the mutual inductances. This phenomenon leads to an increment in the Lorentz force between the transport and magnetization currents, what causes an additional source of hysteretic losses which cannot be accounted by classical changes in the size of the flux-front profile. Thus, we have obtained that for relatively large deformations of the pancake coil, up to a 19% increment in the AC losses can be achieved for moderate to low intensities of I_a , whilst for currents greater than $0.7 I_{c0}$, a striking although low reduction on the AC losses can be achieved, whose main physical signature has been connected to the actual disappearance of the flux-free core inside the superconducting coil.

I. INTRODUCTION

The use of the second generation of high temperature superconducting (2G-HTS) tapes onto the manufacturing of pancake shaped coils, that are commonly used in the prototyping and manufacturing of lighter high-power density machines such as, superconducting fault current limiters [1], power generator motors [2], transformers [3], and superconducting magnetic energy storage systems (SMES) [4, 5], is nowadays a subject of intensive research which from the fundamental physical aspects, it mainly focuses on a further understanding and better utilization of the highly non-linear local electromagnetic properties of the 2G-HTS tapes into the high field induced and hysterical conditions of AC coils. Previously, different electromagnetic properties of 2G-HTS pancake coils have been studied either from an experimental point of view for the development of a specific application [6–9], or by the numerical modelling of their physical properties for the understanding of the underlying phenomena and the optimization of these devices [10, 11]. Within these kind of studies, one of the most determining factors which needs to be considered when fostering a superconducting application is the actual estimation of its AC losses, as

it utterly defines the optimal design, performance, and cost assessment of a superconducting machine [12]. However, it is well-known that the type-II superconducting materials used for the fabrication of 2G-HTS tapes are extremely prone to be affected by external and induced changes in the electromagnetic excitations, what results in very complex local distributions of current density and flux front profiles, both being the physical source of the hysteresis losses [13, 14]. In fact, in the case of a superconducting coil, each one of its turns which carries a certain amount of current, then affects the electromagnetic performance of the other turns by magnetic induction where not only profiles of transport current must appear, but also local profiles with magnetization currents. Nonetheless, these local profiles are really difficult to see at the level of individual superconducting tapes, it due to its large aspect ratio (e.g., 1 micron thick x 4 mm width), and consequently to the computational burden than this implies, regardless if the systems of derived inductance matrices is solved by differential [12, 15–17] or integral [18, 19] approaches. Thus, the running of extensive numerical studies in applied superconductivity is somehow limited to the computational resources and time that might be available for the case of study, even if the symmetry can be reduced to a 2D axisymmetric geometry as the one for pancake coils, but still these are of remarkable importance due to the actual costs concerning the experimental testing and manufacturing of these devices.

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Figure 1. Pictorial representation of (a) the overall geometry of the pancake coil and its coordinate system, (b) the cross section of the perfectly aligned pancake coil (mid-section) made of 20 turns of 2G-HTS tapes (not to scale), and (c) its equivalent but with a considerably large turn-to-turn displacement (winding misalignment). The cross section of a single 2G-HTS tape as a periodic array of unit material cells is illustrated in (d), and the temporal process of the applied current $I_{tr}(t)$ is illustrated in the subplot (e) for the time steps with the profiles of current density shown in Figs. 2 & 3.

When studying a superconducting pancake coil either from the experimental or the computational point of view, the most common practice is the assumption that all the turns of the coil are perfectly aligned, and furthermore, that the perfect shape of this coil can be maintained throughout any manufacturing or assembling process. This is certainly a very difficult statement to sustain, as in the reality, winding misalignments can be easily created either by human or machine winding errors, or by the simple fact that the positioning of the tapes can be distorted by the force exerted when installing these coils. However, due to the enormous computational power that is required for this kind of situation, where the turn-to-turn misalignments are in the order of microns, therefore imposing the need to consider a high level accuracy and a substantially fine meshing approach, the influence of the winding misalignment factor on the energy losses of superconducting coils under AC conditions, remains as a fundamental physical and engineering problem to be solved. Thus, in this paper we present a comprehensive study on the AC Losses for diverse misalignment factors in the winding precision of 2G-HTS pancake coils, taking as a benchmark a 20 turns SCS4050 Superpower[®] coil.

In Sec. II, we outline the simulation method and the

main electromagnetic features of the modelled pancake coils, which are captured by the numerical solution of the Maxwell equations under the theoretical framework of the 2D axisymmetric H-formulation [6, 7], and the generalized Kim's like $E - J$ power law model introduced in 20, where the latter allow us to include experimentally measured magneto-angular dependence of the infield critical current density $J_c(B, \theta)$ for commercially available 2G HTS tapes [20, 21]. Then, in Sec. III our results for the time dynamics of the flux front profiles of current density for perfectly wound pancake coils, and its counterpart, coils with a clear misalignment in its winding pattern are both presented. Striking patterns of transport and magnetization currents are disclosed at the micron level, i.e., across the thickness of the 2G-HTS tape and over each one of the turns of the pancake coil, what builds a generalized framework for our discussion of the curves of AC losses for misaligned pancake coils which are disclosed in Sec. III C. Finally, Sec. IV is devoted to summarize the main conclusions from this study.

II. MODELLING ENVIRONMENT

In this section, we summarize the theoretical framework and modelling strategy, invoked for the numerical simulation of the 2G-HTS pancake coils either with or without winding misalignment features, considering an array of 20 turns of SCS4050 superconducting tapes as shown in Figure 1. Our discussion starts with a brief presentation of the general physical conditions that define the pancake model, and then describing in a more detailed way the coil set-up and computational conditions for an accurate reproducibility of our results.

A. Model Conditions

We built our model based on a two dimensional (2D) axial symmetric H-formulation with the components of the magnetic field $\mathbf{H} = [H_r, H_z]$, perpendicular to the wider and thicker surfaces of the wound tapes. Thus, within the 2D description of the axisymmetric model, the field variables are computed by the so-called H-formulation of Faraday's law:

$$\left| \frac{-\frac{\partial E_\phi}{\partial z}}{r \frac{\partial(r E_\phi)}{\partial r}} \right| = -\mu_0 \left| \frac{\mu_r \frac{\partial H_r}{\partial t}}{\mu_r \frac{\partial H_z}{\partial t}} \right|, \quad (1)$$

which allow to obtain a univocal solution of the local profiles of current density through Ampere's law.

Thus, the profiles of current density inside each one of the turns of the 2G-HTS tapes flows in the ϕ direction, $\mathbf{J}_\phi$, as well as the electric field $\mathbf{E}_\phi$, which has to be defined in accordance with the material properties

of the actual 2G-HTS tape used during the manufacturing of the coils. In this context, a realistic $E - J$ power law function describing the magnetic anisotropy of the critical current density in the Lorentz configuration (perpendicular to the length of the HTS tape), has to be included accordingly with the current-voltage characteristics of commercial 2G-HTS tapes, measured at the aimed operation temperature of the overall coil. In this sense, at 77K, a simplified function $\mathbf{J}_c(f(\mathbf{B}), \theta)$ [20], is capable to describe the magneto-angular characteristics of the infield critical current density of a vast number of 2G-HTS tapes [20, 21] is included into our model:

$$\mathbf{J}_c(f(\mathbf{B}), \theta) = \frac{\mathbf{J}_{c0}}{\left(1 + \epsilon_\theta \left(\frac{\mathbf{B}}{\mathbf{B}_0}\right)^\alpha\right)^\beta}, \quad (2)$$

with

$$\epsilon_\theta = \sqrt{\gamma^{-1} \sin^2(\theta) + \cos^2(\theta)}, \quad (3)$$

and

$$\mathbf{E}_\phi = E_0 \left(\frac{\mathbf{J}}{J_c \left(\frac{\mathbf{B}}{\mathbf{B}_0}, \theta, \gamma \right)} \right)^n, \quad (4)$$

such that the E-J power law field dependence is dominated by the Kim's factor, $\mathbf{B}/\mathbf{B}_0$, whilst the angular dependence is moderated by the electron mass anisotropy ratio, γ , of the YBCO layer inside the 2G-HTS tape.

The results presented in this paper correspond to pancake coils made of the SCS4050 tape, with their main physical properties defined in Table 1, where a Dirichlet boundary condition ($H_r = 0$) is imposed at the center axis of a coil former of 5 cm radius, therefore taking advantage of the axial symmetry conditions of the problem for achieving minimal computing time. Also, it is to be noticed that for the sake of completeness, the profiles of current density in Figs. 2 & 3 are shown in normalized units, J/J_c , such that the model and results presented in this paper are valid for any other 2G-HTS of equivalent dimensions and, obeying Eq. 2 such as, the SCS2050, SCS4050, SCS6050, and SF12100 tapes, all produced by Superpower Inc.; the AMSC8700 and AMSC8612 tapes produced by American superconductor; the ST-05L and ST-06-L fabricated by Shanghai Superconductor Technology Co. Ltd.; or the SQ-12 2G-HTS tape manufactured by SuperOx [20, 21].

B. Coil Setups and Computational Conditions

In order to model diverse misalignment factors in the winding precision of Rare-Earth Barium-Copper-Oxide (ReBCO) pancake coils for practical applications, it is assumed that each one of the tapes is deliberately displaced from each other in the order of μm , replicating

Table 1. Physical and computational characteristics for modelling the SCS4050 [20, 22] superconducting tape^a

	h [mm]	h_l	w [mm]	w_l	I_{c0} [A]	n	B_0 [mT]	β	γ
[20, 22]	0.055	11	4	200	114	30.5	240	1.5	5.02
Cu ₁	0.02	2	4	200	-	-	-	-	-
Ag ₁	0.002	1	4	200	-	-	-	-	-
YBCO	0.001	3	4	200	-	-	-	-	-
Subs	0.05	2	4	200	-	-	-	-	-
Ag ₂	0.0018	1	4	200	-	-	-	-	-
Cu ₂	0.02	2	4	200	-	-	-	-	-

^a h refers to the material thickness which has been divided into h_l layers for attaining a local description of the physical variables. Likewise, the width characteristics are defined by w and w_l respectively, with $h_l \times w_l$ defining the total number of elements composing a material domain. The $E - J$ power law parameters in Eq. 2 for the SCS4050 tape [20, 22] and their overall dimensions are presented in the first row. Material layers are presented in the order shown in Fig 1 (d).

the effect of imperfections in the wound alignment of the coil (Fig 1 (c)). In particular, we present two different pancake coil topologies: (1) We first considered the case of a perfect alignment ($\delta_m = 0$) between the 2G-HTS tapes (Fig 1 (b)), studying the local dynamics of the flux front profiles as shown in Fig 2. Then, in case (2), we considered the topological misalignment of the 2G-HTS tapes, starting from the innermost turn towards the outermost turn in increments of $\delta_m \times (w_t/4) \times (N - 1)$, with w_t being the overall width of the tape, and δ_m the turn misalignment factor. Five distinctive turn-to-turn displacement factors for the coil misalignment, $\delta_m = 1/50, 1/40, 1/30, 1/20$, and $\delta_m = 1/10$ have been considered, such that when $\delta_m = 1/10$ is assumed for a 20 turns SCS4050 pancake coil, the maximum misalignment between the innermost and outermost turns corresponds to half the width of the SCS4050 tape (2 mm), meaning that the positions of the neighbouring turns are displaced 100 μm from each other (Fig 1 (c)). Analogously, the smaller misalignment factors for this case correspond to turn-to-turn displacements of 20 μm , 25 μm , 33 μm , and 50 μm respectively. Thus, it has been assumed that the condition $\delta_m = 0.1$ defines the maximum tolerance for the misalignment between adjacent tapes, without compromising the mechanical and electrical properties of the 2G-HTS tape at each turn. In this sense, greater misalignments are unlikely to happen as it is virtually improbable to evince a scenario where the imbalance between the positioning of the innermost and outermost turns of a laminar coil will be greater than half of its tape-width, unless the number of turns is much greater than 20. Nevertheless, it is worth mentioning that the threshold value for the misalignment factor, depends on the maximum width of the HTS tape that can be situated out of the collinear configuration (perfect winding), as long as it does not exhibit delamination behaviours, due to shear stresses between the different turns of the

HTS coil [23], which could also be mitigated by the use of adequate insulation systems [24]. Hence, for the case of the SCS4050 tape that has been considered in this study, the maximum misalignment factor $\delta_m = 1/10$ is a plausible threshold value as this has been already demonstrated for up to a 5 – MPa applied stress level with a 8×3 mm width anvil, the shear stress at less than 0.25 mm from the tape edges results negligible [25].

Due to the substantial resolution that is needed to account for the local behaviour of the physical variables, with geometry changes of the order of microns, the finite element meshing that composes the 20-turns pancake coils (Fig. 1) is constituted by 221 domains comprising the 6 layers considered for the internal structure of the 2G-HTS tape, and their surrounding environment. It is therefore to be noticed that due to the geometry, dimensions, and the specific nature of this problem, homogenisation techniques such as the ones proposed in [12] cannot be extensively applied into our study, although these can alleviate the computational burden for the modelling of perfectly wound coils. In the light of this, we have found that for a relative tolerance between internal solutions for the field variables superior than $1E - 6$, and a minimum absolute tolerance tolerance of $1E - 8$, a mesh of 31500 quad elements for the tape-domains renders to the same solution than an equivalent mesh with approximate 15000 quad elements, as long as the number of triangular, edge, and vertex elements of the mesh used for the surrounding spaces (commonly called air domains) doubles the number of quad elements. This computational insight is important, because it does not only allow to define a very small element ratio when a really fine mesh is required for certain regions of space, as it is our case, but also because it allows to simplify or reduce the number of sublayers needed for meshing each one of the material layers of the 2G-HTS tape. For instance, the 31500 quad elements mesh implies that each identified ReBCO layer into the cross-section of the pancake coil modelled, could be divided into 7 sublayers, each containing 75 elements across the width of the tape, and similarly, the Cu and substrate layers can be divided into 4 sublayers each, with the Ag modelled as a single layer. Still, due to the aspect ratio of both the tapes and the overall coil, this is a notoriously big mesh which results in a significantly large demand of processing memory and computing time. However, with a reduced mesh of less than half the number of elements, it is still possible to get results within the same tolerance levels, and this has allowed us to reduce the number of sublayers to its minimum (Table 1), i.e., 3 for the YBCO layers, 2 for the substrate, 2 for the Cu layers, and 1 for the Ag layers, without loosing our fundamental focus of being able to account for current density transitions at the local level.

In fact, for being able to display the local profiles of normalised current density, J/J_c , inside each one of the turns of the superconducting coil, it is necessary as well to have some precautions on how these profiles are plotted into the GUI of commercial software such as COMSOL

5.3a[®] Multiphysics. Although this is a purely technical issue, we consider important to mention this aspect due to the increased community of superconductivity researchers that have adopted this software, where for the case of 2D simulations, it has become more or less standard to show not-to-scale profiles for the current density (overcoming the aspect ratio issue), by applying a cut 2D-line profile that calculates the averaged current along the thickness and width of the HTS layer, which cannot be confused with what the local distribution of profiles of current density means. For instance, from the physics perspective, using a cut 2D-line approach for displaying the average profiles of current density, can render to a reader the false impression of having a large flux-free core region inside the HTS layers for low to moderate amplitudes of the applied current ($I_a \lesssim 0.7 I_{c0}$), as the average of the current on the superconducting regions dominated by the occurrence of magnetization currents results to be zero. Thus, for an appropriate representation of the local profiles of current density, we recommend to export the calculated values over the entire mesh of elements defining the HTS domains for post-processing, analysis, and plotting, such that the displayed flux front profiles could give a better account of concomitant occurrence of transport and magnetization currents inside each one of the turns of the HTS coil.

Finally, concerning to the computing time, it is worth to mention that for calculating the actual AC losses of the concerned systems, the study implies to run the same kind of experimental settings, but with a minimum of 10 different intensities of the applied current, and with a sufficiently small time stepping for at least 1.25 cycles of the external excitation (1000 time steps per cycle is usually enough). Therefore, calculating a single AC losses value for a certain intensity of the AC applied current, and under the reduced mesh conditions described above, the computation can take between 2 days for very low intensities of the applied current, $I_a = 0.1 I_{c0}$, up to 6 days for the threshold current $I_a = I_{c0}$, within workstations equipped with an Intel[®] i7-6700k CPU @ 4GHz core processor with 32GB RAM.

III. LOCAL DISTRIBUTION OF CURRENT DENSITY PROFILES

Although previously published works have given attention to the numerical study of hysteretic losses for perfectly wound pancake coils made of commercially available 2G HTS tapes subjected to AC transport current and self-field conditions [6, 8, 11, 16, 18, 26–30], finding a sufficiently accurate model for the study of the time-dependent physical behaviour of local profiles of current, within the superconducting layers has not been reported yet. This is probably due not only to the computational challenges explained in the previous section, but to the success of applying either reductions on the de-

Figure 2. (color online) Evolution of the normalised current density J/J_c in a perfectly wound pancake coil ($\delta_m = 0$) made of 20 HTS layers under alternating transport current conditions of amplitude I_a . Darker coloured (blue) regions indicate zones with negative elements of current density $J_i = -J_c$, lighter coloured (yellow) regions correspond to $J_i = +J_c$, otherwise (green) corresponds to the flux free core where J_i and B_i are both equal zero. Subplots are labelled in accordance with the monotonic branch of the experimental processes depicted in Fig. 1 (e), starting from the time-step 1 at the leftmost pane to the time-step 10 at the rightmost pane of the figure.

degrees of freedom for the superconducting tape or, common meshing homogenization techniques applied to the cross section of perfectly wound coils, both capable to render an acceptable estimations of the AC losses that are within a 10% average deviation from the experimental measurements, depending upon how refined is the numerical model. However, due to the scale implicit onto the misalignment factor for not homogeneously wound coils, it can be intuitively expected that the induced changes in the AC losses, if any, would be within the same order than the error tolerance aforementioned. Therefore, this implies than simplified meshing minimization techniques would not allow to give a proper account of the physical mechanisms involved for misaligned coils, unless local changes on the distribution of current density inside the superconducting layers of the coil turns could be observed. Thus, this is the first study which presents an exhaustive study on how the local profiles of current density inside individual turns and the overall coil evolve in time within AC transient states of applied current of different amplitudes (Fig 1(e)), covering from very low transport current profiles ($I_{tr} = 0.1 I_{c0}$), up to the critical current threshold value I_{c0} . The presented model not only takes into account the high level meshing approach for the physical and computing time considerations described in previous section, but also the experimentally measured magneto-angular anisotropy of the 2G-HTS tape considered [20]. For a better understanding of the results presented below, we encourage the reader to download the high resolution pictures attached to this paper such that a better visualisation and understanding of the distribution of current profiles in Figs. 2 & 3 could

be achieved.

A. Electromagnetic features of perfectly aligned HTS coils

As previously mentioned, although the numerical computing time for our models could be strongly reduced by simplifying the number of elements inside of each one of the superconducting domains (or HTS coil turns), by allowing a 1D strip symmetry for each one of the HTS layers, therefore eliminating the aspect-ratio problem, this comes under the expense that the physical nature of the local profiles of current density could not be observed within this approach. However, it is well known that the electromagnetic properties of type-II superconductors such as the ReBCO materials used for the manufacturing of 2G-HTS tapes are, strongly affected by the intensity and direction of externally applied magnetic fields [19, 31, 32], which can diminish the intensity of the critical current density, $J_c(B, \theta)$ [20], or lead to rather complicated distributions of current density profiles inside the superconducting material [14, 33–36]. In fact, in order to understand the physical nature and time dependence of the local profiles of current density, which determine the overall electromagnetic properties of the coil, it is necessary to model each one of the HTS layer as a 2D system with a minimum of 2 sub-layers across their thickness, such that opposite magnetization currents can appear within each one of the HTS domains.

For magnetization experiments of stack arrangements

Figure 3. Same as Fig 2 but considering a deformed wound coil with each tape displaced up to the maximum misalignment factor ($\delta = 100\mu m$).

of 2G-HTS tapes in absence of applied transport current, it has been already demonstrated that the strip approximation and the homogenization of multiple HTS tapes as single domains (without further sub-layers across their thickness) can render to approximately the same results, as the stacks of 2G-HTS tapes behave as a system of electrically unconnected layers, with well defined positions for the flux front profiles when the orientation of the applied magnetic field does not change [36]. However, defining the actual symmetry of the flux front profile when a transport current is considered is rather complex, and cannot be determined without the use a refined mesh across the thickness of the HTS tapes. For instance, let's consider the case of having the stack of tapes into the arrangement of the HTS coil illustrated in Fig. 1 (b). In this case, the occurrence of transport current on each one of these 2D HTS layers, which correspond to the wound turns of the HTS coil, implies the manifestation of inductive magnetization currents across the tapes thickness, what therefore implies that if a single sublayer is considered for each one of the HTS domains, the regions with magnetization currents will be computed as flux free-regions, leading to a mathematically valid but physically incorrect result. Consequently, a proper computation of the magnetization losses of the system will be hindered for low intensities of the applied transport current ($I_a < 0.5 I_{c0}$), i.e., when the magnetization currents takes a dominant role, with discrepancies which could be greater than 50% the actual AC losses of the system, as it has been reported in [33], either from the more sophisticated integral numerical methods for the critical state law [19], or purely analytical approaches [37] valid for wires of circular or rectangular cross sections. However, as I_a increases, we have found that the discrepancy between the numerical results within the strip (1D) approximation and the 2D single or homogenized layer

approximations [12, 38] reduces, towards 20% to about 7% between $I_a = 0.8 I_{c0}$ and I_{c0} , which is the normal range of operation of the HTS wire. This is because the area occupied by magnetization currents and the flux-free core for $I_a \geq 0.8 I_{c0}$, is much smaller than the area occupied by the transport current density profiles, a phenomena which is illustrated in the columns 5 and 6 of Fig. 2. In this figure, the distribution of current density profiles for the perfectly wound coil are presented into a matrix order, where depending on the intensity of the applied transport current (rows), the high order meshing within our calculations (see table 1) allows to locally discern between profiles of current density with different physical nature, i.e., inside of each one of the $1\mu m$ thick superconducting layers, as a function of time (columns) correlated with the unit time steps illustrated in Fig. 1 (e).

There are rather interesting physical features to be noticed in Fig. 2, which could not be observed if a reduced mesh for the HTS domains is considered. Starting from a very low amplitude of the applied transport current, $I_a = 0.2 I_{c0}$ (1st row), about 30% of the turns (6 central layers) show a collective 'bulk' behaviour with a well defined flux-free core (green zones) and no magnetization currents [39], i.e., showing that the central turns of the coil are only affected by the extrinsic source of transport current density at the top and bottom of each HTS turn. However, the remaining turns, which correspond to the inner and outer 7 turns of the coil, show a clear dominance of magnetization currents along the width of each tape, these confined by the flux front profile that separates the transport current density (local profiles of current, $\mathbf{J}_i$, flowing in a single direction across the whole thickness of the tape), from the magnetization currents which fully penetrate each one of these turns. Thus, although homogenized or coarser mesh could render to a qualitatively valid macroscopic description of the electro-

magnetic properties of the HTS coil, as power losses are in this range dominated by the transport current [40], it is shown that a simpler model than the one presented in this paper will not be able to give a proper account for the magnetization losses of the system. Then, as the amplitude of the applied transport current increases the number of turns that become fully penetrated by magnetization currents increases too, thence reducing the size of the central flux-free core that disappears when $I_a \gtrsim 0.9 I_{c0}$ (see 2nd column of Fig. 2).

Moreover, there are dominant features on the local distribution of current profiles, which can be directly linked to the material law governing the electrodynamics of realistic HTS materials, and their measured magneto-angular anisotropy [20], which could not be observed by simplified material laws such as the critical state model [32]. For instance, before analysing the peak-to-peak hysteretic behaviour which occurs between the 2nd and 6th columns in Fig. 2, we can analyse the influence of increasing the transport current for a DC application, by just observing the evolution of the profiles of current density along the 2nd column. From this perspective, it is to be noticed that under the critical state condition the local profiles of current must ensure the condition $\mathbf{J}_i = \pm \mathbf{J}_{c0}$, which implies that for $I_a < I_{c0}$, each one of the turns will have to show the occurrence of magnetization currents around the radial axis of the coil, otherwise the system will not be able to satisfy the transport current condition $I_{tr} = \int J_i d\Omega_{tape}$. Therefore, under the critical state condition, the local distribution of profiles of current density for $I_a \lesssim 0.6 I_{c0}$ will show a similar symmetry to the one illustrated in Fig. 2, with exception that the $\mathbf{J}_i$ across and along the width of each turn will equal $\mathbf{J}_{c0}$ for transport current profiles, and $\pm \mathbf{J}_{c0}$ for magnetization currents. However, for $I_a > 0.6 I_{c0}$, the length of the semi-major axis of the quasi-elliptical flux-front profile which delimits the transition from transport current profiles, with the occurrence of magnetization currents will remain constant, whilst the position of the co-vertices linearly approaches the geometrical center of the coil reducing the length of the semi-minor axis of the flux front profile, until the full transport current condition $I_{tr} = I_{c0}$ is reached. Therefore, this implies that for moderate to high currents, $0.6 I_{c0} \lesssim I_{tr} \lesssim 0.9 I_{c0}$, the critical state model will render to a slight overestimation of the magnetization losses that decreases as I_{tr} increases, it due to the consumption of magnetization currents [41], what explains why the CS model is still a very successful physical approach despite neglecting the magneto angular anisotropy of the real HTS material [19].

Nevertheless, a more accurate description of the electromagnetic properties of 2G-HTS tapes can be achieved, by introducing a power law dependence between the critical current density and, the direction and intensity of an extrinsic magnetic field which has been experimentally measured [20]. In this case, even if the angular dependence of J_c is neglected, for instance, by assuming the simplified Kim's model [42], the local distribution of cur-

rent density for $I_a > 0.6 I_{c0}$ and in particular the evolution of the flux-front profile, will both show a better qualitative resemblance with the profiles of current density illustrated in Fig. 2 (columns 1 and 2), where the intensity of the J_i profiles diminishes from J_{c0} at the geometrical center of the coil, towards its outermost and innermost turns. In fact, at this current levels and before changing the direction of the applied transport current, we report that meaningful differences between the Kim's model and the generalized magneto-angular model introduced in Eq. 2 have not been observed. However, it is precisely at applied currents $I_a > 0.6 I_{c0}$ where we have noticed a striking concave feature on the shape of the flux-front profile, when the angular anisotropy characteristics of J_c are considered. This can be seen when the sign of the slope of the applied transport current as a function of time changes, but not the direction of the applied current, i.e., between the time steps 2 to 4 and 6 to 8 in Fig. 1 (e), which in terms of the profiles of current density shown in Fig. 2, corresponds to the columns 3 and 4, and 7 and 8, respectively. This subtle aperture of the flux front profile in concave shape at the bottom and top sides of the pancake coil, can be understood as the predominant physical feature on the local electromagnetic response of a HTS coil with magneto-angular anisotropy, where individual HTS grains [19, 42–45] of the HTS tape respond differently not only to changes on the intensity of applied magnetic field across each one the coil turns, but also to the direction of the inductive magnetic field along the width of each one of the HTS tapes.

B. Electromagnetic features of uneven winding HTS coils

For the sake of illustration and ease visualization of results for misaligned coils, where the displacement factor between consecutive turns requires the use of a refined 2D meshing approach, as the one introduced in the previous subsection, in Fig. 3 we present the local distribution of profiles of current density inside of each one of the superconducting domains of a 20 turns SCS4050 2G-HTS-tape pancake coil, with a $\delta = 100 \mu m$ turn-to-turn displacement. This frames the maximum misalignment factor that might be allowed during the winding or positioning of much larger coils, where the local displacement between the innermost and outermost turns could reach up to half the width of the wound tape. Consequently, for larger coils, the profiles of current density shown in Fig. 3, can be understood as the homogenization of the local profiles of current density into 20 sub-domains sectioning the thickness of the coil, each one representing the averaged positioning and intensity of the current density into stacks of tapes of relative dimensions. [38, 46]. Thus, as long as the macroscopic electromagnetic quantities are presented under the base of the results obtained for perfectly aligned coils, our results and conclusions are

not limited to the coil dimensions but rather focused on invoked physical principles and material laws.

The most characteristic feature on the local distribution of current density profiles for misaligned coils which can be seen in Fig. 3, is the progressive deflection of the originally semi-elliptical vertices of the flux front profile, when $\delta_m = 0$ (perfectly wound coil) towards the upper and bottom edges of the innermost and outermost turns of the coil, respectively, causing as well a clockwise repositioning of the co-vertices resulting in a “worm-like” flux-front shape. In electromagnetic terms, this repositioning of the semi-elliptical vertices is caused by the breakdown in the ordinal symmetry of the mutual inductance between the coil turns, which causes an unbalance in the local magnetic field at their opposite edges. Thus, as the intensity of the magnetic field at the top edge of the innermost turn must be greater (~ 140 mT at $I_a = I_{c0}$) than the field intensity at its opposite edge (~ 130 mT), the magnitude of the critical current density at the bottom edge increases, what results in a larger accumulation of transport current profiles in this area, forcing the displacement of magnetization current profiles to the opposite edge.

Therefore, although the magnetization and transport current areas separated by the corresponding flux front profile have essentially the same dimensions, it is the added Lorentz-force between the transport and magnetization currents what creates the additional source of AC-losses in the case of misaligned coils. This phenomenon is similar to what has been reported for single AC superconducting wires subjected to synchronous and asynchronous magnetic field excitations [41] although, in the case of misaligned HTS coils, a clearer evolution of the transport current density profiles can be observed in relation with the relative position of the flux front profile and the occurrence of magnetization currents. For instance, it is to be noticed in Fig. 3 that the motion of magnetization currents occurs along the entire length of the wound wire, where certain amount of the profiles of current density associated with the transport current, travels from the bottom edge of the innermost turn to the top edge of the outermost turn in a hysteretic cycle, causing a local imbalance on the heat losses $\mathbf{E}_i \cdot \mathbf{J}_i$ over the surfaces of the innermost and the outermost turns of the coil.

C. AC Losses caused by misalignment factors

The calculation of the AC Losses of high power density superconducting devices is already a classical subject, which is commonly covered through different electromagnetic formulations and material law models for the $\mathbf{E} - \mathbf{J}$ properties of type-II superconductors, where the hysteretic AC losses per unit time and superconducting volume (Φ) for cyclic excitations of frequency ω can be calculated by integration of the local density of power

dissipation $\mathbf{E} \cdot \mathbf{J}$:

$$L = \omega \oint_{f.c.} dt \int_{\Phi} \mathbf{E} \cdot \mathbf{J} d\Phi, \quad (5)$$

where *f.c.* denotes a full cycle of the time-varying electromagnetic sources after the magnetic relaxation period, i.e., after the first half cycle of the applied transport current in absence of an external magnetic field. Otherwise, if the situation considers an applied magnetic field which change of direction along the cycle, the magnetic relaxation period can take several cycles to accommodate the local consumption of magnetization currents, by the concomitant action of the transport current in cross-field and rotating field configurations [14, 33, 34, 41].

However, although the pancake coils studied in this paper are subjected only to an AC transport current, therefore reducing the number of cycles needed for the overall calculation of the AC-losses, the critical aspect behind Eq. 5 is that, an accurate determination of the AC-losses depends not only on the difference between the accumulated power losses at the peak current values, but on the actual history of the local distribution of profiles of current density and electric field inside the superconducting domains. Because of that, and due to the additional computational effort that is demanded for modelling superconducting domains with high aspect ratio, such as the 2G-HTS tapes, it is fairly usual to accept a 10% to 20% relative tolerance between the experimental and numerical evidences, when the later considers a simplified mesh of finite elements as discussed before. In fact, as the most common practice for plotting the AC-losses curve is to display the results in logarithmic scale (Fig. 4 (a)), this aspect is often concealed out of the physical analysis as the visual impact on the difference between the experimental and numerical data is minorized within this approach, particularly for low to medium applied currents $I_a < 0.7 I_c$, i.e., when the magnetization currents inside the superconducting tapes play a dominant role [41, 47]. Thus, although it would be possible to argue that our results for the AC-losses of perfectly aligned pancake coils could be more accurate than the ones predicted by other models with simplified reductions of the local superconducting mesh, it is actually the tolerance window defined above, together with the use of the right material law for the 2G-HTS tape, what allows to define a good estimate of the AC-losses when compared with the experimental evidences. An example of this can be seen in Fig. 4 (a), where besides our simulations for the 20-turns HTS pancake coils with different misalignment factors δ_m , we compare our results (in renormalized units) with the experimentally measured AC losses for a 10-turns HTS pancake coil fabricated by Gömöry et. al. [5], where an excellent resemblance between the experimental and numerical results is achieved. Consequently, from the numerical perspective, when the problem is such that changes on the spatial characteristics of the system are of the same dimensional order, than the size of the finite elements invoked and, the physical

quantities to be determined also depend on their spatial derivatives, it would be normal to expect larger discrepancies between the experimental and numerical results allowing us to explain why, on the one hand, some numerical models including an artificial increase in the thickness of the 2G-HTS tapes with a proper renormalization of the physical quantities [48], have resulted insufficient for the modelling of 2G-HTS coils [10], whilst on the other hand, models with homogenized meshes have shown to be more accurate [12, 46]. Nevertheless, in the case of having a slight displacement of the turn-to-turn alignment δ_m of the 2G-HTS tapes in a superconducting coil, the actual accuracy on the calculation of the AC-losses to distinguish the local changes on the physical variables between the aligned and misaligned coils, utterly demands the use of a fine mesh with finite elements smaller than the size of the spatial displacement between the subsequent turns. Thus, for the coils considered in our study which are made of a 4 mm wide 2G-HTS tape split into 200 finite elements along its width, the minimum displacement that could be conceived without compromising the relative tolerance between the calculation is 20 μm , which corresponds to a turn-turn displacement factor of $\delta_m = 1/50$ in Fig. 4.

Based on all the above, in Fig. 4 (b) we present our numerical results for misaligned 2G-HTS pancake coils as a percentage function of the predicted AC losses for perfectly wound coils, allowing an ease reading and physical interpretation of our results. Therein, it is to be noticed that for low to moderate amplitudes of the applied transport current $0.1 \leq I_a/I_{c0} \leq 0.6$, i.e., when the local profiles of current are dominated by the occurrence of magnetization currents across the turns of the HTS coil, the turn-to-turn misalignment in the wound coil is capable to produce a large increase in the AC-losses from the base value, when the later refers to the hysteretic losses of a perfectly wound pancake coil. This well-known increase becomes larger as the turn-to-turn displacement factor is greater, reaching up to $\sim 19\%$ increment in the extremal condition, when the relative displacement between the innermost and the outermost turn is about half the tape width, a condition that could be reach for much larger coils than the one presented in Fig. 3, although on these cases, the AC-losses ratio between perfectly wound and misaligned coils will be equivalent, as the distribution of profiles of current shown in Figs. 2 & 3 will be governed by the same physical principles onto the local dynamics of flux front profiles described above.

Thus, in conjunction with Figs. 2 & 3, it is to be noticed that any increment or decrement on the AC-losses, is bounded to the displacement or consumption of magnetization currents by the injected transport current, and ultimately to the disappearance of a flux-free core into the coil turns as $I_{tr} > 0.7 I_{c0}$. Then, although an increment in the AC-losses can always be expected for applied currents lower or equal than $0.7 I_{c0}$, this has given the fact that under these conditions, there is still place for the motion of magnetization currents by the Lorentz

force induced by the transport current, this increment is gradually smaller as the size of the flux-free core decreases. Moreover, as it can be observed in Fig. 4 (b), and from our analysis of Figs. 2 & 3, it is possible to conclude that as larger is the deformation of the flux fronts caused by the wound misalignment as larger will be the increment of the AC-Losses. Also, we demonstrated that for overall displacements between the innermost and the outermost turns of the pancake coil, smaller than just a quarter of the 2G-HTS tape width, the increment in the AC-losses ratio is less than 6%. Therefore, although for practical purposes it is always recommended to ensure that the winding alignment of the coil must be straight and not deformed, such that the homogeneity characteristics of the magnetic field are not compromised, we have proven that at least for these transport current conditions, rather conventional and unintended misalignments generally smaller than $w_t/6$ (i.e., $\leq 33 \mu\text{m}$ turn-to-turn displacement in a 20 turns 4 mm tape-width pancake coil) lead to nearly negligible increments in the AC-losses ($< 3\%$). Nevertheless, for applied currents greater than $\sim 0.7 I_{c0}$ we have encountered a striking decay on the AC-losses ratio curve (Fig. 4 (b)) which is connected with the disappearance of the flux free core in Fig. 3. This decrease in the AC-losses becomes more evident as the winding misalignment is exacerbated, and although this improvement could be almost negligible for very small winding displacements, what is remarkable about this unique feature on the curve of AC losses, is that this allows a direct macroscopic method to determine the actual intensity of the applied transport current when the full disappearance of the flux free core is achieved inside a HTS coil, an event that cannot be determined by any other method, including the magnetization measurements, as in the later a magnetization signal will still be observed at $I_a \geq 0.7 I_{c0}$.

IV. CONCLUDING REMARKS

In this paper, we disclosed how systematic misalignments in the winding of 2G-HTS tapes can impact the hysteresis losses of pancake coils subjected to alternating currents. For this purpose, we have presented an exhaustive study on the numerical modelling of pancake coils, at a meshing level that allows to foresee the dynamics of flux front profiles at a local level, i.e., inside each one of the 2G-HTS layers that composed the different turns of the pancake coil, such that for the first time, a physical discerning of magnetically induced and transport current profiles can be evinced (see Figs. 2 & 3). The presented model takes into account the experimentally measured magneto-angular anisotropy of commercial 2G-HTS tapes which has been reported in 20, illustrating here the profiles of current density as function of the applied transport current for an exemplary pancake coil made of 20 turns of SCS4050 SuperPower

Figure 4. (Color online) a) Hysteretic AC losses per cycle for diverse misalignment factors in the winding precision of 2G-HTS pancake coils presented as a function of the amplitude of the applied transport current, compared with the experimental measurements reported in 5. b) The curves of AC losses are also displayed as a function of the innermost to outermost turns misalignment distance (top abscissa) which for general purposes is measured in units of the 2G-HTS tape width, corresponding therefore to the shown turn-to-turn displacement factor δ_m (bottom abscissa) for the 20 turns SCS4050 pancake coil with the profiles of current density illustrated in Fig. 3.

Inc. tape.

For the modelling and physical understanding of misalignment factors replicating the effect of possible imperfections in the alignment of 2G-HTS wound pancake coils, it has been assumed that each one of the tapes can be displaced from each other in the order of μm , such that for the sake of generality the set of results is compared with the flux dynamics and AC losses of

perfectly wound coils ($\delta_m = 0$, Fig. 4). In particular, we have considered topological misalignments that start from the innermost turn towards the outermost turn ($\delta_m \times (w_t/4) \times (N-1)$), with collateral turn-to-turn displacements of $20\ \mu\text{m}$, $25\ \mu\text{m}$, $33\ \mu\text{m}$, $50\ \mu\text{m}$, and $100\ \mu\text{m}$ for the case of the $N = 20$ turns SCS4050 pancake coil ($w_t = 4\ \text{mm}$), which correspond to displacement factors of $\delta_m = 1/50, 1/40, 1/30, 1/20$, and $1/10$, respectively, or in a more general perspective, to innermost to outermost turn-misalignments of $1/10, 1/8, 1/6, 1/4$, and $1/2$ the width of the HTS tape.

On the one hand, in terms of the flux fronts dynamics for perfectly wound coils, it has been shown that for very low amplitudes of the applied transport current, $I_a \leq 0.2 I_{c0}$, about 30% of the middle turns into the pancake coil show a collective bulk-like behaviour, with a well shaped flux-free core and complete absence of magnetization currents, it delimited by the occurrence of extrinsic profiles of transport current at the edges of each one of the HTS turns. Then, for the remaining 70% of the coil, the transport current flux front profile is preserved, but a strong dominance of magnetization currents along the width of each tape appears with no evidence of local flux-free cores, i.e., with these HTS turns being fully penetrated by the induced magnetic field, by the neighbouring HTS turns and the injected transport current. Thus, as the amplitude of the applied transport current increases the size of the flux-free core reduces, pushing then the magnetization currents towards the middle turns, until a full disappearance of the flux free core is reached at under critical current conditions $I_a \approx 0.9 I_{c0}$. Moreover, we have demonstrated that although for applied currents $I_a \leq 0.6 I_{c0}$, the distribution of flux-front profiles obtained with the magneto-angular anisotropy features of 2G-HTS tapes shows a negligible difference with the critical-state model predictions, for moderate to high currents, $0.6 I_{c0} \lesssim I_{tr} \lesssim I_{c0}$, the critical state model will render to a slight overestimation of the magnetization losses that decreases as I_{tr} increases, although no significant differences have been encountered between the simplified Kim's model (no angular dependence) and the generalized magneto-angular model introduced in Eq. 2, other than a striking concave feature on the shape of the flux-front profile when the angular anisotropy characteristics ($J_c(\mathbf{B}, \theta)$) are considered.

On the other hand, concerning to the flux front dynamics for misaligned pancake coils ($\delta_m > 0$), a notorious deflection of the semi-elliptical vertices with respect to the front profiles for perfectly wound coils ($\delta_m = 0$) has been observed, which evolves towards the upper and bottom edges of its innermost and outermost turns, inducing then a clockwise repositioning of the co-vertices to form a “worm-like” flux-front shape, this is caused by the breakdown in the ordinal symmetry of the mutual inductance between the coil turns that produces an unbalance in the local magnetic field at the coil opposite edges. Thus, as the intensity of the magnetic field at one of the edges of the innermost turn of a misaligned pancake coil must

be greater than the field intensity at its opposite edge, then the magnitude of the critical current density at this region decreases, due to the magneto-angular anisotropy of the 2G-HTS tape, which ergo forces a larger accumulation of transport current in the side of the 2G-HTS turns with greater critical current density, leading thence to the Lorentz motion of magnetization currents towards the edge with the weakest critical current density. Moreover, we have found that regardless of the magnitude of the considered misalignment factor in the winding of pancake coils, the resulting size of the flux front profiles is essentially unaltered. This fact allow us to conclude beyond the shadow of doubt that the physical mechanism that serve as a source of additional AC-losses in the case of misaligned coils, is the induced Lorentz-force between the transport current and the magnetization currents, which is caused by the collateral displacement of the HTS turns and the magneto-angular dependence of the critical current density.

Finally, we have found that when the magnetization currents play a dominant role in the local distribution of profiles of current density inside of the pancake coil, i.e., for applied currents of relative amplitude $I_a/I_{c0} \leq 0.6$, a turn-to-turn misalignment in the coil winding is able to produce a large increase in the AC-losses with regards to the losses produced by a perfectly wound coil. In fact, the added hysteresis losses are more substantial as the turn-to-turn displacement factor increases, and it can even reach up to an astonishing 19% increment in sufficiently large pancake coils, capable to sustain a relative displacement between the innermost and the outermost turn of about half the width of the employed 2G-HTS tape. This increment in the AC losses is developed by the displacement and consumption of magnetization currents by the injected transport current, within the deformed flux fronts caused by the winding misalignment, diminishing as the size of the flux free core reduces until its full disappearance at about $0.7 I_{c0}$. Nevertheless, although from the engineering perspective it would always be better ensuring a complete absence of deformations in the wound 2G-HTS coil, either during its manufacturing or practical application, such that the expected homogeneity features of the magnetic field will not be

compromised, we have demonstrated that for unintended misalignments that are generally smaller than $w_t/6$ (e.g., equivalent to a $\leq 33 \mu\text{m}$ turn-to-turn displacement in a 20 turns 4 mm tape-width pancake coil), the increment in the AC losses could be neglected if a $\sim 3\%$ relative tolerance is accepted. However, for high intensities of the applied current, $I_a > 0.7 I_{c0}$, and relatively large deformations in the wound coil, $> w_t/4$, a more counter-intuitive phenomena has been observed as unexpectedly, the AC losses of the deformed pancake coil results smaller than the AC losses generated by a perfectly aligned coil with the same overall dimensions and 2G-HTS tape. This phenomena has been found to be directly linked with the time dynamics of the flux front profiles and its physical nature, as it reflects the exact intensity of the applied current that is needed to have all the turns of the pancake coil completely penetrated by profiles of superconducting current density, therefore determining the exact conditions for which a complete absence of a flux free core could be observed by thermal measurement methods [49], which can not be resolved by the conventional use of electromagnetic methods.

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