Towards a Comprehensive Model for Characterising and Assessing Thermoelectric Modules by Impedance Spectroscopy

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Abstract. Thermoelectric devices have potential energy conversion applications ranging from space exploration through to mass-market products. Standardised, accurate and repeatable high-throughput measurement of their properties is a key enabling technology. Impedance spectroscopy has shown promise as a tool to parametrically characterise thermoelectric modules with one simple measurement. However, previously published models which attempt to characterise fundamental properties of a thermoelectric module have been found to rely on heavily simplified assumptions, leaving its validity in question. In this paper a new comprehensive impedance model is mathematically developed. The new model integrates all relevant transport phenomena: thermal convection, radiation, and spreading-constriction at junction interfaces. Additionally, non-adiabatic internal surface boundary conditions are introduced for the first time. These phenomena were found to significantly alter the low and high frequency response of Nyquist spectra, showing their necessity for accurate characterisation. To validate the model, impedance spectra of a commercial thermoelectric module was experimentally measured using a new and parametrically fitted. Technique precision is investigated using a Monte-Carlo residual resampling approach. A complete characterisation of all key thermoelectric properties as a function of temperature is validated with material property data provided by the module manufacturer. Additionally, by firstly characterising the module in vacuum, the ability to characterise a heat transfer coefficient for free and forced convection is demonstrated. The model developed in this study is therefore a critical enabler to potentially allow impedance spectroscopy to characterise and monitor manufacturing and operational defects in practical thermoelectric modules across multiple sectors, as well as promote new sensor technologies.

Keywords: Thermoelectric module, Impedance spectroscopy, Thermoelectric characterization, Thermoelectric generator, Thermoelectric cooling

1. INTRODUCTION

Thermoelectric (TE) materials can directly convert heat into electricity. The TE efficiency in materials is evaluated using a dimensionless figure-of-merit $ZT = S^2T/\rho\lambda$, which comprises of absolute temperature T and three fundamental TE properties: Seebeck coefficient S , electrical resistivity ρ (or electrical conductivity σ) and thermal conductivity λ . While literature shows maturity in measuring TE properties on a material level [1], there is not an international test standard for measuring these properties at a device level. Solving this widely recognized issue [2] is crucial, as device level characterisation is needed to validate the implementation of an ever-growing number of novel TE materials [3], guide system design [4], direct quality control and monitor device longevity. It is therefore critical for the development and application of TE technology to develop standardised, accurate and repeatable high-throughput characterisation methods.

Measurement methods currently used mainly rely on either a heat flow meter method or some variation of the Harman technique [5]. While these two methods have gained popularity [6,7] they each have clear

disadvantages limiting their use. These include, (i) the need for intricate and expensive experimental setups for accurately measuring heat flows with minimal thermal losses, (ii) requires multiple measuring systems to be able to characterise all fundamental TE properties [5], (iii) are known to potentially feature large uncertainties which can reach up to 50 % in some cases [8]. Recently, impedance spectroscopy (IS) analysis has been explored to not only lower uncertainties in the calculation of TE module zT [9,10] but also gain additional information on the three fundamental TE properties without the need of multiple measuring systems [11,12]. In fact, commonplace laboratory equipment such as a lock-in amplifier or a galvanopotentiostat equipped with a frequency-response analyser, is sufficient for room temperature characterisation. IS achieves this by measuring the voltage-current ratio (impedance) across the system as a function of frequency (impedance spectrum) when a small amplitude ac signal is applied under initial steady state conditions. To evaluate device characteristics, a theoretically derived impedance function, representing a collection of physical processes occurring in the system (equivalent circuit), is typically fitted to the experimental impedance spectrum for property extraction. Currently, literature reports various theoretical models which attempt to characterise TE devices under suspended conditions. Initially, Downey *et al.* proposed the use of a simple RC one-port circuit analogy and a transmission line model [13]. On the other hand, De Marchi *et al.* [14,15] and García-Cañadas *et al.* [16,17] later analysed the one-dimensional heat balance equation and performed a more detailed impedance analysis in the complex plane (Nyquist plot). The latter group were the first to explore the potential of IS for providing a complete characterisation of TE modules [11] although neglecting several key effects which take place in TE devices.

Other models, however, were also developed to more specifically analyse the influence of these important effects on the impedance response of a TE module. First Casalegno *et al.* [18] introduced the effect of spreading-constriction on heat fluxes [19] due to area differences between the TE legs and the external ceramic layers. Later, De Marchi *et al.* [14] added convection at the outer surfaces of the ceramics in conjunction with the spreading-constriction effect but did not analyse the effect of convection in the impedance response. This has been recently performed by Beltran-Pitárch *et al.* [20], although only considering free convection. Radiative losses are also important and were first considered by Downey *et al.* on a single TE element, correcting previously underestimated ZT values by up to 40 % [13]. Consequently, large uncertainties as similarly experienced with the Harman method [21] could be present under certain characterisation conditions. While all these models improved the analysis of the impedance response and in some cases the ability to estimate ZT , there is yet a single comprehensive model which includes all these relevant effects with the capability to provide a complete characterisation of TE modules. Consequently, the underlying effect these additional phenomena have on characterising all the fundamental TE properties of a module have yet to be outlined.

The aim of this work is to develop a new single impedance model that can characterise all fundamental TE properties of a module as a function of temperature with accuracy and confidence suitable for practical use. This is made possible, for the first time, by integrating all relevant phenomena: thermal spreading-constriction at layer interfaces and internal-external surface heat losses via convection and radiation, under one easily implementable analytical model. The variations produced in the impedance response due to the introduction of each phenomena are analysed and more importantly their underlying effect on characterising TE properties is reported. In particular, the sensitivity of the impedance response to convective effects is focused upon and the possible use of TE modules as convective heat transfer coefficient sensors, for free and forced fluid flows, is demonstrated using the new model. Such a sensor would be highly desirable and beneficial to a variety of thermofluidic systems which require the characterisation or monitoring of convective heat transfer [22, 23]. Validation of the model is performed by experimentally measuring the impedance spectra of a commercial Bi_2Te_3 -based TE module over various isothermal temperatures using a new contactless infrared heating arrangement. A comparison of fitted TE properties with material property data provided by the module manufacturer is reported and fitting errors are investigated using a Monte-Carlo residual resampling approach (bootstrapping). Good agreement and relatively low errors indicates that IS characterisation is a viable technique for the assessment of TE devices in energy applications and more.

2. THEORETICAL MODEL

2.1. MATHEMATICAL FORMULATION

A typical TE module formed of $2N$ legs (N couples) of lengths L with cross-sectional area A , connected electrically in series and thermally in parallel, and sandwiched by two electrically isolating ceramic layers is considered (see Fig. 1a). It is assumed that the device is suspended vertically (TE legs rest horizontally) in air, the ambient temperature T_∞ does not vary with time and the initial homogenous temperature of the device T_i is in thermal equilibrium with its environment ($T_i = T_\infty$). The total voltage difference across the device is therefore given by the contribution of the total ohmic resistance R_Ω (comprising of the intrinsic electrical resistance of the TE legs R_i , and parasitic resistances R_p , which includes contributions from the copper strips, wires and contact resistance R_c) and the total Seebeck voltage from $2N$ TE legs when subjected to a temperature difference $\Delta T = T_L - T_0$, with T_L and T_0 being the temperatures at positions $x = L$ and $x = 0$, respectively. Therefore, the measured electrical impedance $Z = V/I$ in the time domain t can be summarised as $Z(t) = R + 2NS\Delta T/I$ [11].

Fig. 1. (a) A high-resolution image of a single integrated thermoelectric leg contacted by alumina layers. The image was captured using a Zeta-200 Optical Profiler. (b) Axisymmetric model of half a thermoelectric leg (TE) capped by an external ceramic layer (e). The direction of the conductive heat fluxes produced by the injected Peltier heat (ϕ) flux at the junction, the absolute temperature (T), internal-external convective/radiative losses (h_1, h_2 and h_3), and geometric dimensions, are indicated. The red line depicts qualitatively a possible thermal profile for a positive Seebeck coefficient and current.

Assuming the module is excited in the small signal regime (≤ 10 mA), the contribution of residual Joule heat can be ignored in comparison to the generated Peltier heat [10]. Therefore, under suspended conditions, the temperature at the half length L_h of the TE legs does not change (see Fig. 1b), consequently $\Delta T = -2(T_0 - T_i)$. Considering this and transforming the impedance function to the frequency domain using the Laplace transform $Z(j\omega) = \mathcal{L}\{Z(t)\}$, a more useful expression is obtained as a function of the angular frequency ω and the imaginary unit j ,

$$Z(j\omega) = R_\Omega - \frac{4NS}{i_0} \theta_0 \quad (1)$$

where $\theta = \mathcal{L}\{T - T_i\}$ and $i_0 = \mathcal{L}\{I(x = 0)\}$.

Dividing $\theta_0 = \theta(x = 0)$ by the injected Peltier heat flux density $\phi = ST_i i_0 / A$ (assuming $T_0 \approx T_i$ due to the small sinusoidal current amplitude) a total junction thermal admittance Y_0 can be defined, and using a lumped-

parameter analogy, the latter can be expressed as the series summation of the thermal admittances due to the TE leg Y_{TE} and the external ceramic layer Y_e .

$$\frac{\theta_0}{\phi} = \frac{1}{Y_0} = \frac{1}{Y_{TE} + Y_e} \quad (2)$$

To obtain an expression for the total junction thermal impedance of a complete TE module we consider the thermal admittances produced by the existence of copper contacts, diffusion barriers and solder as negligible, due to their relatively high thermal conductivity ($\sim 300 \text{ Wm}^{-1}\text{K}^{-1}$) and small thickness ($\leq 100 \mu\text{m}$) [11]. Thus, only the external ceramic layer with thickness L_e and longitudinal surface area per TE leg $A_e = A_o/(2N + v)$, where A_o is the total ceramic surface area and v is the number of legs removed in place of wire terminals (typically 2), have been included in the axisymmetric model (see Fig. 1b). To simplify the model further, it is considered that lateral heat diffuses radially, the contacts are homogenous at all interfaces and the TE properties are an absolute average of p- and n-type semiconductors.

Mathematically, this reduces the unknown spatial relative temperature distribution to two transient two-dimensional (cylindrical coordinates: radius r and position x) partial differential equations. After phasor transform the ambient relative thermal diffusion equations can be expressed as,

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial \theta}{\partial r} (x, r, j\omega) \right) + \frac{\partial^2 \theta}{\partial x^2} = \frac{j\omega}{\alpha_\tau} \theta, \quad \tau = \begin{cases} TE & 0 \leq x \leq L \\ & 0 \leq r \leq r_0 \\ e & -L_e \leq x \leq 0 \\ & 0 \leq r \leq r_e \end{cases} \quad (3.1)$$

$$(3.2)$$

which describes the influence of Fourier's conduction law for heat transfer within the single TE leg and the external ceramic separately. It should be noted that the non-linear Thomson effect has been neglected, like the Joule effect its presence is assumed to diminish in the small signal regime [9].

α_{TE} and α_e are the thermal diffusivities of the TE leg and external ceramic layer, respectively. Radial boundary conditions (see Fig. 1b) for Eq. 3 can be expressed as,

$$\frac{\partial \theta}{\partial r} = \begin{cases} -\frac{h_1}{\lambda_{TE}} \theta, & 0 \leq x \leq L \\ & r = r_0 \\ 0, & 0 \leq x \leq L \\ & r = 0 \\ 0, & -L_e \leq x \leq 0 \\ & r = r_e \end{cases} \quad (4.1)$$

$$(4.2)$$

$$(4.3)$$

while the boundary conditions which lie axially are,

$$\lambda_e \frac{\partial \theta}{\partial x} = \begin{cases} \lambda_{TE} \frac{\partial \theta}{\partial x} - \frac{ST_i i_0}{A}, & x = 0 \\ & 0 \leq r \leq r_0 \\ -h_2 \theta, & x = 0 \\ & r_0 \leq r \leq r_e \\ h_3 \theta, & x = -L_e \\ & 0 \leq r \leq r_e \end{cases} \quad (5.1)$$

$$(5.2)$$

$$(5.3)$$

$$\theta = 0, \quad \begin{matrix} x = L_h \\ 0 \leq r \leq r_0 \end{matrix} \quad (6)$$

where λ_{TE} and λ_e are the thermal conductivities of each constituent, h_1 represents a total heat transfer coefficient for lateral surface heat losses of the TE legs and h_2 and h_3 are for longitudinal losses of the external ceramic (see Fig.1b). Assuming that linearised thermal radiation ($T - T_\infty \ll T_\infty$) takes place at all three surface positions and

convection occurs only on the external surface of the ceramic, the total surface heat transfer coefficient for each surface can be approximated as,

$$h_1 = 4\sigma_B \varepsilon_{TE} T_\infty^3, \quad \begin{matrix} 0 \leq x \leq L \\ r = r_0 \end{matrix} \quad (7.1)$$

$$h_2 = 4\sigma_B \varepsilon_e T_\infty^3, \quad \begin{matrix} x = 0 \\ r_0 \leq r \leq r_e \end{matrix} \quad (7.2)$$

$$h_3 = h_c + 4\sigma_B \varepsilon_e T_\infty^3, \quad \begin{matrix} x = -L_e \\ 0 \leq r \leq r_e \end{matrix} \quad (7.3)$$

where h_c is a convective heat transfer coefficient for the external surface of the ceramic layer, σ_B is the Stefan-Boltzmann constant and ε is the emissivity of each material.

In principle the temperature varies radially over the cross-section of a TE leg. However, the junction voltage is measured over whole electrode surfaces and not at point locations. Consequently, a complete two-dimensional solution is not required and an average radial temperature distribution over the radius r_0 can be obtained using the following integral,

$$\bar{\theta}(x, j\omega) = \frac{2}{r_0^2} \int_0^{r_0} \theta(x, r, j\omega) r \, dr \quad (8)$$

Applying this integral to Eq. 3.1 and using the boundary relations of Eq. 4.1 and 4.2, in turn allows for the spatial reduction of Eq. 3 (assuming $\theta(r = r_0) \approx \bar{\theta}$) to a simpler one-dimensional homogeneous Helmholtz differential equation,

$$\frac{\partial^2 \bar{\theta}}{\partial x^2}(x, j\omega) - \left(\frac{2h_1}{\lambda_{TE} r_0} + \frac{j\omega}{\alpha_{TE}} \right) \bar{\theta} = 0, \quad 0 \leq x \leq L \quad (9)$$

However, to maintain a mathematical description of the thermal spreading-constriction effect brought about by the area change between the junction and external ceramic, Eq. 8 is only applied to the solution of Eq. 3.2. Consequently, Eq. 3.2 is instead initially solved by the superposition principle $\theta(x, r, j\omega) = \theta_1(x, j\omega) + \theta_2(x, r, j\omega)$ where θ_1 satisfies Eq. 5.1-5.3 although with boundary conditions at the junction replaced by,

$$\lambda_e \frac{\partial \theta}{\partial x} = 0, \quad \begin{matrix} x = 0 \\ 0 \leq r \leq r_e \end{matrix} \quad (10)$$

and θ_2 verifies the same system with boundary conditions at $-L_e$ replaced by,

$$\lambda_e \frac{\partial \theta}{\partial x} = 0, \quad \begin{matrix} x = -L_e \\ 0 \leq r \leq r_e \end{matrix} \quad (11)$$

The first problem follows the same one-dimensional spatial form as Eq. 9 and therefore can be solved in the same manner, while the second can be found using Fourier's method of separation of variables. Hence, the general solution of Eq. 9 and 3.2 respectively are

$$\bar{\theta}_{TE}(x, j\omega) = C_1 \sinh \left[\frac{x}{L_h} \left(\frac{h_1 L^2}{2\lambda_{TE} r_0} + \frac{j\omega}{\omega_{TE}} \right)^{0.5} \right] + C_2 \cosh \left[\frac{x}{L_h} \left(\frac{h_1 L^2}{2\lambda_{TE} r_0} + \frac{j\omega}{\omega_{TE}} \right)^{0.5} \right] \quad (12)$$

$$\begin{aligned} \theta_e(x, r, j\omega) = & h_3 \bar{\theta}_{L_e} \frac{L_e}{\lambda_e} \operatorname{sech} \left(\frac{j\omega}{\omega_e} \right)^{0.5} \cosh \left[\frac{x}{L_e} \left(\frac{j\omega}{\omega_e} \right)^{0.5} \right] \\ & + \sum_{n=1}^{\infty} J_0 \left(\delta_n \frac{r}{r_e} \right) D_n [\cosh(\gamma_n x) - \tanh(\gamma_n L_e) \sinh(\gamma_n x)] \end{aligned} \quad (13)$$

where $\omega_{TE} = \alpha_{TE}/L_h^2$ and $\omega_e = \alpha_e/L_e^2$ are characteristic angular frequencies, J_0 is a zero order Bessel function of the first kind, δ_n is the nth zero root of J_1 and γ_n is an nth value which satisfies the propagation constant of the external ceramic layer such that $\gamma_n^2 = (\delta_n/r_e)^2 + j\omega/\alpha_e$. Constants C_1 , C_2 and D_n are determined from boundary conditions denoted in Eq. 5.1, 5.2 and 6:

$$C_1 = \phi_{0,TE} \frac{L_h}{\lambda_{TE}} \left(\frac{h_1 L^2}{2\lambda_{TE} r_0} + \frac{j\omega}{\omega_{TE}} \right)^{-0.5} ; \quad C_2 = -C_1 \tanh \left(\frac{h_1 L^2}{2\lambda_{TE} r_0} + \frac{j\omega}{\omega_{TE}} \right)^{0.5} \quad (14)$$

$$D_n = [A\phi_{0,e} - (A_e - A)h_2\bar{\theta}_{0,e}] \frac{2r_0 J_1 \left(\delta_n \frac{r_0}{r_e} \right) \coth(\gamma_n L_e)}{A_e \lambda_e \delta_n r_e J_0^2(\delta_n) \gamma_n}$$

where $\phi_{0,TE} = -\lambda_{TE}(\partial\bar{\theta}/\partial x)_{0,TE}$ and $\phi_{0,e} = -\lambda_e(\partial\theta/\partial x)_{0,e}$ and $\bar{\theta}_{L_e}$ and $\bar{\theta}_{0,e}$ are radially averaged Laplace relative temperatures for the r_e -disc at $-L_e$ and annulus of radii r_0 and r_e at the junction respectively.

Derivation of these constants followed by an r -integration of Eq. 13 using Eq. 8, allows for the calculation of radially averaged Laplace relative temperatures and heat flux densities at the junction and $-L_e$ in the form of simultaneous equations. Assuming $\bar{\theta}_{0,e}$ is equal to the radially averaged Laplace relative temperature of the entire r_e -disc at the junction (valid if $T - T_i \ll T_i$ or $A_e \gg A$), these equations can be represented as one-dimensional quadrupolar matrices [24],

$$\begin{Bmatrix} \bar{\theta}_0 \\ \phi_0 \end{Bmatrix}_{TE} = \begin{bmatrix} \cosh \mu_{TE} & \frac{1}{k_{TE} \mu_{TE}} \sinh \mu_{TE} \\ k_{TE} \mu_{TE} \sinh \mu_{TE} & \cosh \mu_{TE} \end{bmatrix} \begin{Bmatrix} 0 \\ \phi_{L_h} \end{Bmatrix} \quad (15)$$

$$\begin{Bmatrix} \bar{\theta}_0 \\ \phi_0 \end{Bmatrix}_e = \begin{bmatrix} \frac{1}{h_2} \left(\frac{1-\eta}{\eta} \right) & 0 \\ h_2 \left(\frac{1-\eta}{\eta} \right) & 1 \end{bmatrix} \begin{bmatrix} 1 & Y_{sc}^{-1} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cosh \mu_e & \frac{\eta}{k_e \mu_e} \sinh \mu_e \\ \frac{k_e \mu_e}{\eta} \sinh \mu_e & \cosh \mu_e \end{bmatrix} \begin{bmatrix} 1 & 0 \\ h_3 & 1 \end{bmatrix} \begin{Bmatrix} \bar{\theta}_{L_e} \\ 0 \end{Bmatrix} \quad (16)$$

where $k_{TE} = \lambda_{TE}/L_h$, $k_e = \lambda_e/L_e$, $\eta = A/A_e$ (filling factor) and the spreading-constriction thermal impedance across the junction can be defined as

$$\frac{1}{Y_{sc}} = \frac{4}{k_e} \sum_{n=1}^{\infty} \left[\frac{J_1^2(\delta_n \eta^{0.5}) \coth \left(\eta \delta_n^2 \left(\frac{\pi L_e^2}{A} \right) + \frac{j\omega}{\omega_e} \right)^{0.5}}{\delta_n^2 J_0^2(\delta_n) \left(\eta \delta_n^2 \left(\frac{\pi L_e^2}{A} \right) + \frac{j\omega}{\omega_e} \right)^{0.5}} \right] \quad (17)$$

Contrary to the expression obtained by Casalegno *et al.* [17] a hyperbolic cotangent (previously tangent) is instead present in the numerator of Eq. 17. This derives from the addition of a uniform flux density distribution on the external boundary (Eq. 5.3) which was not previously incorporated.

Furthermore, μ_{TE} and μ_e are propagation constants for the one-dimensional wave functions forming Eq. 12 and 13,

$$\mu_{TE} = \left(\frac{h_1 L^2}{2\lambda_{TE} r_0} + \frac{j\omega}{\omega_{TE}} \right)^{0.5} ; \quad \mu_e = \left(\frac{j\omega}{\omega_e} \right)^{0.5} \quad (18)$$

With reference to Eq. 2, thermal impedances generated by the existence of the TE leg and external ceramic layer can therefore be written as,

$$\frac{1}{Y_{TE}} = \left(\frac{\bar{\theta}_0}{\phi_0} \right)_{TE} = \frac{1}{k_{TE} \mu_{TE}} \tanh \mu_{TE} \quad (19)$$

$$\frac{1}{Y_e} = \left(\frac{\bar{\theta}_0}{\phi_0} \right)_e = \left[h_2 \left(\frac{1-\eta}{\eta} \right) + \left(\frac{1}{Y_{sc}} + \frac{1+h_3 \frac{\eta}{k_e \mu_e} \tanh \mu_e}{h_3 + \frac{k_e \mu_e}{\eta} \tanh \mu_e} \right)^{-1} \right]^{-1} \quad (20)$$

Using Eq. 1 and 2, the derived thermal impedances expressed above (Eq. 17–20) can be combined to deduce an equivalent frequency dependent electrical impedance function,

$$Z(j\omega) = R_\Omega + \left(\frac{1}{Z_{TE}^{-1} + Z_e^{-1}} \right) \quad (21)$$

$$Z_{TE} = R_{TE} \left(\frac{h_1 L^2}{2\lambda_{TE} r_0} + \frac{j\omega}{\omega_{TE}} \right)^{-0.5} \tanh \left(\frac{h_1 L^2}{2\lambda_{TE} r_0} + \frac{j\omega}{\omega_{TE}} \right)^{0.5} \quad (21.1)$$

$$Z_e = \left\{ R_{h2}^{-1} + \left[Z_{sc} + (R_{h3}^{-1} + Z_{Weo}^{-1})^{-1} + (Z_{Wec}^{-1} + Z_{Ch3}^{-1})^{-1} \right]^{-1} \right\}^{-1} \quad (21.2)$$

Eq. 21 can be further represented in the form of an equivalent circuit (Fig. 2a), where each element can be defined as,

$$R_{h2} = \frac{4NS^2 T_i}{h_2 A} \left(\frac{\eta}{1-\eta} \right) \quad (22.1)$$

$$Z_{sc} = \frac{4NS^2 T_i}{A Y_{sc}} \quad (22.2)$$

$$R_{h3} = \frac{4NS^2 T_i}{h_3 A} \quad (22.3)$$

$$Z_{Weo} = \eta R_e \left(\frac{j\omega}{\omega_e} \right)^{-0.5} \coth \left(\frac{j\omega}{\omega_e} \right)^{0.5} \quad (22.4)$$

$$Z_{Wec} = \eta R_e \left(\frac{j\omega}{\omega_e} \right)^{-0.5} \tanh \left(\frac{j\omega}{\omega_e} \right)^{0.5} \quad (22.5)$$

$$Z_{Ch3} = \frac{1}{j\omega C_{h3}} \quad (22.6)$$

$$R_{TE} = \frac{2NS^2 T_i L}{A \lambda_{TE}} \quad (22.7)$$

$$R_e = \frac{4NS^2 T_i L_e}{A \lambda_e} \quad (22.8)$$

$$C_{h3} = \frac{A \lambda_e^2 \eta^2}{4NS^2 T_i L_e^2 h_3 \omega_e} \quad (22.9)$$

Z_{TE} is an equivalent circuit element which differs from a constant temperature Warburg element, previously reported [11], by the introduction of $h_1 L^2 / (2\lambda_{TE} r_0)$, which accounts for radiative heat losses at the legs ambient exposed surfaces. R_{h2} is a new resistance due to radiative heat losses taking place at the internal surface of the ceramic layers, which depends on the Seebeck coefficient, h_2 , and the module dimensions. R_{h3} is a resistance due to convective/radiative heat losses taking place at the external surface of the external ceramic layers, which also depends on the Seebeck coefficient, h_3 , and the module dimensions. This resistance, multiplied by η , was identified previously [20]. For this work the introduction of η in this term is neglected since this model now accounts for the spreading-constriction impedance Z_{SC} . Z_{Weo} and Z_{Wec} are open and closed Warburg elements related to the external ceramic layer. The expressions for these last two elements are similar to the $Z_{WCT,C}$ and Z_{CCov} elements, previously reported for convection losses [20]. However, they are modified by the introduction of the filling factor η as a consequence of the addition of the spreading-constriction impedance. Finally, the capacitive term Z_{Ch3} is also present. It is not only affected by convection, but also by radiative heat transfer, as a consequence of Eq. 7.3.

Fig. 2. (a) Complete equivalent circuit for a thermoelectric module. (b) Simplified equivalent circuit for standard commercial thermoelectric modules. The equivalent circuit elements framed in the dotted line are related to the ceramic layers. The ones framed by the solid line in grey correspond to the thermoelectric legs (thermoelements).

It should be noted that for the case of TE modules with ceramic plates ($\lambda_e \approx 35 \text{ W m}^{-1} \text{ K}^{-1}$) under conditions where approximately $h_3 < 10^3 \text{ W m}^{-2} \text{ K}^{-1}$, the equivalent circuit from Fig. 2a can be simplified to the circuit in Fig. 2b, since the impedance of the parallel combination of Z_{Wec} and Z_{Ch3} is much lower than the impedance of the parallel combination of Z_{Weo} and R_{h3} .

Rearranging Eq. 22.7 and 22.8, and assuming R_p is negligible (i.e. $R_\Omega = 2NR_i$), expressions for the average TE properties of the thermoelements can be realised; $S = \sqrt{R_e A \lambda_e / (4NT_i L_e)}$, $\lambda_{TE} = \lambda_e R_e L / (2R_{TE} L_e)$ and $\rho_{TE} = R_\Omega A / (2NL)$. Therefore, if the thermophysical properties of the external ceramic layer are known, and R_{TE} and R_e are extracted from experimentally measured impedance spectra by parametrically fitting Eq. 21, one can evaluate all absolute p-n averaged TE properties of the module [12]. Lastly, by combining all three property relations, zT can be expressed as a ratio of measured resistances R_{TE} and R_Ω , such that $zT = R_{TE} / R_\Omega$ [13,16].

2.2. PARAMETRIC ANALYSIS OF ADDED TRANSPORT PHENOMENA

As a first order verification and validation, in the case of $r_0 = r_e$ ($\eta = 1$) and $h_1 = h_2 = h_3 = 0$, the impedance function (Eq. 21) reduces to the model obtained by García-Cañadas *et al.* [11]. Therefore, the effects of the additional phenomena considered in this model can be represented by the filling factor η (thermal spreading-constriction) and the heat transfer coefficient h (convective and radiative heat losses). Fig. 3 shows the results of different simulations performed in order to analyse the effect of the phenomena introduced by the new comprehensive model in the complex plane. These simulations are carried out for a typical Bi_2Te_3 -based module with properties shown in Table 1.

Fig. 3. Simulated impedance responses for a Bi₂Te₃-based thermoelectric module with the room temperature ($T_i=300$ K) properties described in Table. 1. (a) Impedance sensitivity to decreasing the filling factor η . (b) Real impedance values of Z'_{sc} (dashed line) and Z'_{Weo} (solid line) against filling factor η . Points of intersection $Z'_{sc} = Z'_{Weo}$ for different frequencies are displayed as dots (●), while shaded regions indicate feature dominance. (c) Impedance responses of Z_{sc} and Z_{Weo} elements (d) Impedance sensitivity to increasing radiative loss neglecting convection effects ($h_c = 0$, increase of T_{∞} , solid lines), and convective loss discarding radiation effects ($\epsilon = 0$, increase of h_c , dotted lines). The ohmic resistance of the device has been neglected ($R_{\Omega} = 0$) in all cases to gain clarity.

Table. 1. Geometric, thermophysical and thermoelectric properties of the investigated module.

Geometric Properties	
Module Height (mm)	2.0
Outer Area, A_o (mm ²)	6×6
No° of TE Couples, N	17
No° of Terminals in place of TE legs, ν	2
Alumina Surface Area per TE Leg, A_e (mm ²)	1
Alumina Layer Thickness, L_e (mm)	0.4
TE Leg Length, L (mm)	1.1

TE Leg cross sectional Area, A (mm ²)	0.24
Thermophysical Properties	
Thermal Conductivity of Alumina, λ_e (Wm ⁻¹ K ⁻¹) [25]	$85.868 - 0.22972T + (2.607 \times 10^{-4})T^2 - (1.3607 \times 10^{-7})T^3 + (2.7092 \times 10^{-11})T^4$
Thermal Diffusivity of Alumina, α_e (m ² s ⁻¹) [25]	$\lambda_e / [-155496 + 15291T - 19.0182T^2 + 0.0117T^3 + (2.3745 \times 10^{-6})T^4]$
Thermal Emissivity of Alumina, ϵ_e [25]	0.7
Thermal Emissivity of Leg, ϵ_{TE} [26]	0.5
Thermoelectric Properties	
p-type Seebeck Coefficient (μVK ⁻¹) [27]	$-(36.3810 \times 10^{-4})T^2 + (274.3810 \times 10^{-2})T + 28.3381$
n-type Seebeck Coefficient (μVK ⁻¹) [27]	$(15.3073 \times 10^{-4})T^2 - (108.0588 \times 10^{-2})T + 28.3381$
p-type Thermal Conductivity (WmK ⁻¹) [27]	$(36.1558 \times 10^{-6})T^2 - (26.3513 \times 10^{-3})T + 6.2216$
n-type Thermal Conductivity (WmK ⁻¹) [27]	$(33.4545 \times 10^{-6})T^2 - (23.3503 \times 10^{-3})T + 5.6063$
p-type Electrical Conductivity (Scm ⁻¹) [27]	$(15.6017 \times 10^{-3})T^2 - (157.0805 \times 10^{-1})T + 4466.38095$
n-type Electrical Conductivity (Scm ⁻¹) [27]	$(10.5714 \times 10^{-3})T^2 - (101.0588 \times 10^{-1})T + 3113.71429$

Figure 3a shows the influence of the filling factor for the case where heat losses are neglected ($h_1 = h_2 = h_3 = 0$). It is observed that the decrease in η alters both the high (>5 Hz) and mid-low (5 Hz – 5 mHz) frequency responses. For higher frequencies, the effect is more significant, inducing a clockwise phase rotation around the theoretically ideal 45° slope. A similar rotation in numerically simulated thermal impedance spectra of microelectronic structures with spreading-constriction junctions was also observed by Vermeersch *et al.* [19]. Physically, this rotation can be interpreted as a growing lag in the longitudinal thermal response of the system due to an increase in the lateral thermal spreading and constriction as η reduces. At lower frequencies, a decrease in the off-center displacement of the semicircular response is observed.

Mathematically both the phase rotation and the off-center shift can be understood by examining the evolution of Z'_{sc} and Z'_{weo} real impedances with respect to η and frequency, as analysed in Fig. 3b. From this figure it is shown that for high frequencies, the real impedance due to spreading-constriction Z'_{sc} dominates over Z'_{weo} for $0.4 < \eta < 0.5$, initiating significant phase rotation. For values of $\eta < 0.4$ (typical for most commercial modules), Z'_{sc} dominates over Z'_{weo} at all frequencies, significantly shifting the high frequency vertical asymptote positively and off-center displacement negatively. By quantifying this shift, one can begin to understand the effect to which spreading-constriction has on the characterisation of TE properties. To do so, an expression for the magnitude of the impedance at the turnover frequency for the ideal (Z_{weo} , $\eta = 1$) and non-ideal ($Z_{weo} + Z_{sc}$, $\eta < 1$) cases must be derived, see Fig. 3c. For both cases to match in magnitude it can clearly be seen from Fig. 3c that R_e would have to be reduced in the non-ideal case with expression R_{sc} . In turn, the reduction would correct for an overestimation in both S and λ_{TE} , due to a shared proportional with R_e (see Eq. 22.7 and 22.8).

The difference arises due to the presence of an additional constant (second term in R_{sc}) which is a function of the geometric attributes of the device. It should be noted that since the constant is related to physical properties of the device and not the environment, it will always be present during characterisation. But more importantly as the physical characteristics of a TE device is sensitive to manufacturing and operation defects, the impedance response measured at high frequencies could offer novel damage detection capabilities.

Figure 3d analyses the influence of convective and radiative heat losses on the impedance response. Figure 3d shows separately both the influence of convection losses when radiation does not take place ($\epsilon = 0$,

dotted lines), and radiative losses when convection does not occur ($h_c = 0$, solid lines). It can be observed that both cases only influence the response at mid-low frequencies. The incremental increase in convective/radiative loss can be seen to proportionally reduce the radius of the semicircular response. This reduction can easily be quantified by subtracting the steady state limit ($\omega \rightarrow 0$) of Eq. 21, given by $R_\Omega + (\mathcal{R}_{h1}^{-1}R_{TE}^{-1} + R_{h2}^{-1} + R_{h3}^{-1})^{-1}$, where $\mathcal{R}_{h1} = 1 + h_1L^2/(6\lambda_{TE}r_0)$, from the adiabatic ($h = 0$) case, $R_\Omega + R_{TE}$. Therefore for both cases to fit identical semicircular responses, the additional presence of a loss factor ($\mathcal{R}_{h1}$) and resistances in parallel (R_{h2} and R_{h3}) would have to be compensated by an increase in R_{TE} . This equates to a correction for an underestimation in zT and overestimation in λ_{TE} , as they are proportional and inversely proportional to R_{TE} respectively (see Eq. 22.7).

Intriguingly, both high temperature thermal radiation and forced convection ($h_c > 25 \text{ Wm}^{-2}\text{K}^{-1}$) can cause comparable reductions to the radius of the semicircular response (see Fig. 3c). This sensitivity theoretically suggests a potential use of TE modules as impedance-based thermal sensors for both convective and radiative systems. Although, it should be noticed that at room conditions under free convection at its maximum ($h_c = 25 \text{ Wm}^{-2}\text{K}^{-1}$ [28]), a significant reduction in the semicircle is observed (see Fig. 3c), which it is not the case for radiative losses ($T_\infty = 300 \text{ K}$), which would overlap with the spectrum for $\varepsilon = 0$ in Fig. 3c. Consequently, radiative losses can be neglected for measurements at room temperature conditions.

2.3. FITTING ERROR METHODOLOGY

Fitting experimental impedance results to Eq. 21 can be performed using a least square fitting routine. The fitting is able to provide the values of different parameters of interest (resistances and characteristic frequencies).

To evaluate the parametric variance of the fitted variables and therefore assess the technique's fitting precision for characterising TE properties, Monte-Carlo simulations were carried out. The procedure for these Monte-Carlo simulations is as follows. Firstly, model output and experimental observations are fitted by varying initial parameter input between an infinite range. A single solution is obtained for each parameter by minimization of a least square function. After which, a surrogate distribution of measured impedance values (surrogate spectrum) is generated by adding a normal distribution of randomized solution residuals to the modelled impedance spectrum. For simplicity, solution residuals are resampled using phase randomization. Lastly, each surrogate spectrum is fitted to the experimental observations using the same least square routine.

Using a resample size of 10^3 (i.e., the number of surrogate realisations) zero mean covariance distributions for fitted parameters were generated. Confidence intervals of 2-sigma order (two standard deviations) for each calculated TE property was derived from the variance-covariance matrix of the simulated distribution.

3. EXPERIMENTAL VALIDATION

3.1. MEASUREMENT SETUP

To experimentally validate the developed model, electrical IS was carried out on a commercial (Adaptive®, European Thermodynamics Ltd.) Bi_2Te_3 -based TE module, using a Galvano-Potentiostat PGSTAT302N equipped with a FRA2 impedance module (Metrohm Autolab B. V.). The module's properties can be found in Table. 1. The experimental configuration is shown schematically in Fig. 4. The measurement bench was configured using 4 ultra-low impedance wires in a two-terminal configuration. Two low impedance PFA coated wires were used to connect the module to the measurement bench as well as suspend the module vertically between the centerline of two 500 W infrared heat lamps. The module and heating system was mounted to a

custom rig and placed in vacuum ($<10^{-5}$ mbar). The vacuum chamber's walls were 3 mm thick steel, therefore also functioning as a Faraday cage to minimize electromagnetic interference from external sources.

Fig. 4. Schematic of experimental configuration.

To examine the variation of the impedance spectra as a function of Isothermal temperature, infrared power was raised stepwise until thermal equilibrium was reached (steady state >1 hour) at five incremental locations between room ambient (297 ± 1 K) and module maximum service temperature (400 K). To monitor the surface temperature of the module a calibrated Pt100 temperature sensor (Innovative Sensor Technology IST AG) was placed in close proximity. IS characterisation was performed in galvanostatic mode in a frequency range of 5 mHz to 50 kHz (50 logarithmic increments) with a peak amplitude of 3 mA in the ac regime without dc bias (0 A dc). In all tests, a parasitic inductive behavior was initially measured for frequencies >500 Hz, which has been omitted from this study. Experimental observations were logged using Nova (Metrohm Autolab B. V.) and fitted with the presented analytical model (Eq. 21) using a complex non-linear least square fitting routine [29], written in a Matlab (The Mathworks Inc., MA, USA) programming environment. A convergence criterion of 10^{-4} and a maximum number of fitting iterations of 10^4 was prescribed. The goodness of fit was evaluated by the root mean square error (RMSE) of prediction and the coefficient of determination (R^2).

3.2. MEASUREMENT AND MODEL RESULTS

Firstly, the effects of convection are examined. Figure 5a experimentally shows a comparison of room temperature impedance measurements taken of the vertically suspended TE module in passively stabilised air (Fig. 5b), forced airflow (Fig. 5c) and vacuum (Fig. 4). For both convective responses, a systematic reduction in the radius of the mid-low frequency semicircular response is clearly identified, as seen theoretically in Fig. 3c. The 42 m Ω reduction in radius as a result of free convection represents a ~ 3.5 % underestimation in module zT and overestimation in λ_{TE} . This can be seen as significant and should be taken into consideration if an open-air setup is to be used. Furthermore, an external convective heat transfer coefficient of 21.6 Wm $^{-2}$ K $^{-1}$ was measured by initially fitting all elements of Eq. 21 to the high vacuum measured spectrum ($h_c \approx 0$), and then separately fitting h_c to the open-air measured spectrum. The obtained value can be found to be in agreement with the typical values of h for natural air convection (see Table. 2). Surprisingly, it is considerably lower than the one obtained by Beltrán-Pitarch *et al.* [20]. This is believed to be partly due to the consideration of spreading-constriction impedance which was not accounted for in the mentioned study, but also due to the relatively large difference in

square width of the modules used (40^2 compared to 6^2 mm²). The larger size would increase the onset of free circulation as described by the Grashof number Gr which is proportional to the third power of the modules square width when vertically suspended. In fact, since $Gr < 10^2$ for the small module in this study, it is likely that the value of h_c characterised is representative of natural convection from its environment and not due to its own temperature perturbations. This suggests that smaller TE modules are better suited for the application of measuring environmental convection.

To further examine the possibility of using TE modules as h sensors, the impedance response measured under a forced fluid flow (see Fig. 5c) was also characterised and found to be $56.3 \text{ Wm}^{-2}\text{K}^{-1}$. This value can be compared to a theoretical prediction (see Table. 2) based on the well know expression for the Nusselt number $Nu = h_c l / \lambda_{air} = 0.332 Re_l^{1/2} Pr^{1/3}$ of a flat plate under laminar flow [28], where l is the distance from the leading edge (8 mm), λ_{air} is the thermal conductivity of the air at room temperature ($\sim 25.7 \text{ mWm}^{-1}\text{K}^{-1}$), Re_l is the Reynolds number at location l ($\sim 3 \times 10^3$) and Pr is the room temperature Prandtl number of the air (~ 0.71). While the value measured shows reasonable agreement with the prediction it is also higher. This is likely due to the overall temperature of the module dropping due to the relatively high convective heat loss. Such a temperature reduction would cause a further reduction of the semicircular response due to a proportional drop in S which would wrongly be perceived as an increase in h_c . This limitation will have to be overcome if TE modules are to be adopted as h sensors for systems with practical heat transfer. Although, it should be noted that if convective effects are eliminated using high vacuum and radiative effects are made more significant by increasing the measurement temperature, alternatively TE modules could also potentially be used as ϵ sensors utilising the same characterisation approach.

Fig. 5. (a) Nyquist plot of the investigated thermoelectric module measured at room temperature in passively stabilised air, forced airflow and at high vacuum ($< 10^{-5}$ mbar). (b) and (c) illustrate experimental configurations used for free and forced convection respectively.

Table. 2. Characterised convective heat transfer coefficients.

	Open-air Model Fitted	Natural Convection Ref [28]	Forced Airflow Model Fitted	Forced Convection Theoretical
h_c ($\text{Wm}^{-2}\text{K}^{-1}$)	21.6	5 – 25	56.3	45 – 52

Secondly, modelled impedance spectra and extracted TE property curves yielded from model fittings, are here analysed and compared to experimental observations and material property data provided by the module

manufacturer. For this part of the study experimental measurements were taken under vacuum. Convective losses can therefore be neglected ($h_c = 0$) while radiative effects can be isolated.

Figure 6 shows the experimental impedance results and associated model fittings for the TE module tested using the measurement setup described in Fig. 4. The fitting was performed by setting all elements of Eq. 21 in terms of R_{TE} , ω_{TE} , R_e and ω_e (free variables), and regarding all other terms as constant. It should be noted that since the error in fitting R_Ω is small ($<0.1\%$) and can easily be identified as the real axis (Z') intercept [30], it has been treated as a constant. A good fit ($R^2 > 0.995$, $RMSE < 0.028$) between experimental and theoretical spectra was achieved for all temperatures. At high frequencies, spreading-constriction impedance, previously investigated theoretically in Fig. 3a, can be seen to be a significant feature of the experimental results. This demonstrates that the incorporation of Z_{sc} is necessary and has not been considered in previous applications of this method [20,28].

Fig. 6. Nyquist plot of the investigated thermoelectric module at six steady state isothermal temperatures. Solid squares depict experimental results while solid lines depict model fittings. The high frequency response is shown below with a zero-mean normalised Z' . The yellow solid line represents an impedance response with no spreading-constriction ($\eta = 1$).

For increasing module temperature, two distinct impedance features become identifiable. Firstly, a notable positive shift in Z' at an approximate rate of $9.5 \text{ m}\Omega\text{K}^{-1}$ can be seen. This is due to the incremental increase in the internal ohmic resistance R_Ω of the module. Secondly, an amplification to the magnitude of both the high frequency snout and mid-low frequency semicircular responses were also observed. This magnification corresponds to a proportional increase in R_e and R_{TE} respectively, as it can be seen in Fig. 7a.

Figure 7b shows the temperature dependence of the characteristic angular frequencies extracted from the fitted spectra of Fig. 6. For the external ceramic layer, ω_e shows inverse temperature proportionality, while for the TE legs ω_{TE} remains relatively constant with a minimum around 340 K. This is valid since the ceramic used in the module (alumina) is known to have a thermal diffusivity which decreases with temperature (Table. 1) and Bi_2Te_3 -based alloys typically exhibit relatively constant specific heat capacities [31] accompanied by a thermal conductivity which does not show large variations in this temperature range. Although, one should note that while ω_e is reasonable in magnitude, ω_{TE} is around an order of magnitude less than what would be expected from Bi_2Te_3 -based alloys. This was similarly experienced by García-Cañadas *et al.* [11] who suggested its cause to be due to

the diffusion of heat towards the ceramic layers slowing diffusion along the TE leg. This was later addressed by Apertet *et al.* [32].

Fig. 7. Fitted (a) resistances and (b) characteristic angular frequencies as a function of temperature.

Utilising the extracted resistances in Fig. 7a, all temperature dependent TE properties of the investigated module were calculated and compared with manufacturer data in Fig. 8a, 8b and Fig. 9. Initially it can be identified that the extracted S and λ_{TE} closely follow a similar trend to the averaged absolute p- and n-type properties (see Fig. 8a and 8b). Neglecting spreading-constriction can be seen from Fig. 8c to cause an over estimation in S ($\sim 3\%$) and also in λ_{TE} ($\sim 4\%$) from Fig. 8d, while ignoring thermal radiation only causes an overestimation in λ_{TE} ($\sim 2\%$). However, it should be noted that the effects of thermal radiation are proportional to the third power temperature (Eq. 9) and therefore would become significant for TE devices operated at higher temperatures [13]. Lastly, for adiabatic conditions and not considering the spreading-constriction impedance leads to overestimations in both S ($\sim 3\%$) and λ_{TE} ($\sim 5\%$), as it is shown in Fig. 8c and 8d. This proves the need for a comprehensive model in order to perform high accuracy characterisation of TE module properties.

Fig. 8. Comparison between model fitted and manufacturer provided thermoelectric material properties of the investigated module. (a) Absolute Seebeck coefficient and (b) thermal conductivity as a function of temperature. The influence of neglected thermal effects is also shown for (c) the Seebeck coefficient and (d) the thermal conductivity. Vertical error bars represent associated fitting error (two standard deviations).

Furthermore, there is a difference between manufacturer provided materials ZT , σ and the equivalent fitted values (see Fig. 9). This difference is due to parasitic resistance, mainly governed by the electrical contact resistance at the thermoelement-solder interfaces. This difference can therefore be corrected by a simple dimensionless factor, $(1 - R_c/R_\Omega)^{-1}$, that is a function of the ratio between the contact resistance R_c and the ohmic resistance R_Ω . Physically, this parameter quantifies the losses in both ZT and σ when moving from material to module-level. By measuring the magnitude of the dimensionless factor from Fig. 9a and or 9b, one can estimate R_c for each temperature by simple algebraic rearrangement. Based on the data collected, an average specific electrical contact resistance of $2.5 \times 10^{-9} \Omega\text{m}^2$ per junction was estimated, which after summation, accounts for approximately 28 % of the total internal electrical resistance of the module investigated. This is in good agreement with typical values for Bi_2Te_3 -based modules previously published in literature (10^{-9} to $10^{-8} \Omega\text{m}^2$ [33]). Figures 9a and 9b visually illustrate the importance of module-level characterisation, as poor joint consolidation often limits final module performance.

Fig. 9. (a) Electrical conductivity and (b) figure-of-merit zT as a function of temperature. Vertical lines indicate the measurable difference between material and module-level characterisation due to the existence of parasitic resistances.

Lastly, Fig. 10 shows zero-mean covariance distributions for fitted model parameters using Monte-Carlo residual resampling, indicating how the fitting error of one parameter influences the other. Based on simulated distributions for each isothermal temperature, predicted 2-sigma confidence intervals are shown as error bars on Fig. 8a, 8b and Fig 9. These show relatively larger intervals for S (± 2 %) and λ_{TE} (± 4 %) in comparison to zT ($\pm < 1$ %) and σ ($\pm < 0.1$ %) where the error bars are obscured by the data points. This is the case since S and λ_{TE} are functions of R_e (Eq. 22.8), which according to Fig. 10a has an order of magnitude more fitting variance than R_{TE} . Furthermore, Fig. 10b shows R_e has relatively strong correlation ($r_c = -0.7$) to the fitting variance in ω_e . Overall indicating that adequate modelling of high frequency dependent phenomena (R_e fitting) and an accurate initial estimation of the external ceramic layer's thermophysical properties (predicting ω_e) are both necessary for improving the techniques repeatability. Moreover, Fig. 10c and 10d shows ω_{TE} to have relatively large variance, corroborating the idea of a probable gap in the presented model as previously indicated by Fig. 7b.

Fig. 10. Zero-mean covariance distributions of fitted parameters for residual resampled surrogate data accumulated across all temperature dependent datasets. The color map represents probability density, with lighter shades (yellow) indicating higher density and darker shades (blue) indicating lower density. Distribution correlation coefficients r_c are displayed in the top right corner.

4. CONCLUSIONS

A new impedance model was mathematically derived (including a suitable equivalent circuit) which can now determine key TE properties from a non-destructive module-level test with greater accuracy than previously possible. By integrating all relevant phenomena for the first time, the new model identifies three important Nyquist features, (i) spreading-constriction induces a clockwise phase rotation at high frequencies; (ii) spreading-constriction decreases the off-center displacement of the semicircular response at high-mid frequencies; (iii) both convective and radiative losses reduced the effective radius of the semicircular response.

The mentioned features corrected for an underestimation in module zT and overestimation in the absolute Seebeck coefficient and thermal conductivity. Using Monte-Carlo simulations it was also found that the new model's ability to conform to spreading-constriction phase rotation in the high frequency region is also highly necessary for enhancing characterisation precision.

Experimentally this study revealed that module zT , electrical conductivity, the average absolute Seebeck coefficient and the thermal conductivity of the module thermoelements can be characterised accurately over a temperature range. More so, the observed reduction in radius of the impedance response due to convection/radiation can be exploited for the potential use of TE modules as convective heat transfer coefficient sensors. Similarly, emissivity sensing could potentially be realised using higher measurement temperatures although future work is needed to test this.

Lastly, since the model is directly related to physical features of typical modules, it offers an exciting potential for the characterisation and monitoring of manufacturing and operational defects in practical TE modules.

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