Table 1. Examples of thixotropic materials with the mechanisms of recovery

Materials	Structural build-up at rest	Structural breakdown under shear
Collodial dispersions and suspensions of solids: 1) Paints 2) Coatings 3) Inks 4) Clay slurries 5) Cosmetics 6) Agricultural chemicals	Flocculation under inter-particle forces	Break-up of flocs (deflocculation)
Emulsions	Flocculation of droplets	Deflocculation
Foamed systems: 1) Mousses	Flocculation of bubbles	Deflocculation Coalescence
Crystalline systems: 1) Waxy crude/fuel oils 2) Waxes 3) Butter/Margarine 4) Chocolate	Interlocking of growing crystals	Break up of long needles
Polymeric systems: 1) Solutions/melts 2) Starch/gums 3) Sauces	Agglomeration of macromolecules Entanglement	De-agglomeration Disentanglement
Fibrous suspensions: 1) Tomato ketchup 2) Fruit pulps 3) Fermentation broths 4) Sewage sludges	Agglomeration of fibrous particles Entanglement	De-agglomeration Disentanglement
Semi-solid metallic systems	Agglomeration of particles	De-agglomeration

Table 2: Tabulation of parameters obtained from shear rate jump experiments on Sn15%Pb alloys (at Fs= 0.36) under different rest times [86].

Shear rate Jumps (s^{-1})	0-100			
Rest times (hrs)	0	1	2	5
η_p (Pas)*	2.1	5.4	8.0	23.0
η_{ss} (Pas)**	0.8	0.8	1.2	2.0
τ_b (s)***	0.18	0.16	0.15	0.12

* *The errors are within 95% confidence limits ($\pm$0.5)*
** *The errors are within 95% confidence limits ($\pm$0.2)*
****The errors are within 95% confidence limits ($\pm$0.03)*

Table 3 Classification of Models of Semi-Solid Die Filling (Commercial codes employed in work are indicated in brackets. If a code is not given then either the authors have written the code themselves or it is not identified in the text of the paper).

	Finite Difference	FEM	Micro-Modelling
One-Phase	Ilegbusi & Brown 1995 (PHOENICS) [94] Barkhudarov et al. 1996 (FLOW3D) [95] Barkhudarov & Hirt 1996 (FLOW3D) [96] Modigell & Koke 1999 (FLOW3D) [84] Kim & Kang 2000 (MAGMAsoft) [97] Modigell & Koke 2001 (FLOW3D) [85] Ward et al. 2002 (FLOW3D) [98] Messmer 2002 (FLOW3D) [99] Seo & Kang 2002 (MAGMAsoft) [100] Itamura et al 2002 (Adstefan) [101]	Zavaliangos & Lawley 1995 (ABAQUS) [103] Backer 1998 (WRAFTS) [104] Alexandrou et al. 1999 (PAMCASTSIMULOR) [105] Burgos & Alexandrou 1999 [106] Alexandrou et al 2001 (PAMCASTSIMULOR) [107] Burgos et al 2001 [108] Alexandrou et al. 2002 [109] Ding et al. 2002 (DEFORM3-D) [110] Jahajeeah et al. 2002 (Procast) [111] Rassili et al. 2002 (FORGE3) [112] Wahlen 2002 (Thixoform) [113] Alexandrou et al. 2003 [114] Orgeas et al. 2003 (Procast) [115]	
Two-Phase	Ilegbusi et al 1999 [102]	Zavaliangos & Lawley 1995 (ABAQUS) [103] Zavaliangos 1998 [116] Koke et al. 1999 [117] Kang & Jung 1999 [118] Binet & Pineau 2000 [119] Choi et al. 2000 [120] Kang & Jung 2001 [121] Yoon et al 2001 (CAMPform2D) [122] Kopp & Horst 2002 (ABAQUS) [123] Modigell et al. 2002 [124]	Rouff et al. 2002 [125]

Table 4. Summary of one-phase and two-phase finite difference simulation papers.

One-phase			
	Comments	Flow and viscosity equations	Observations
Ilegbusi & Brown 1995 (PHOENICS) [94]	Slurry incompressible, mass conservation, momentum conservation, energy conservation (enthalpy method), solid fraction from [126], input parameters identified, scalar-equation method for free surface, single internal variable (λ) constitutive model [127-129], experimentally determined values of agglomeration function *H* and disagglomeration function *G* given. Chisel-shaped mould.	$\tau = \tau_y + A(\lambda)\dfrac{(c/c_{\max})^{1/3}}{1-(c/c_{\max})^{1/3}}\eta_f\dot{\gamma} + (n+1)C(T)\lambda f_s\eta_f^{n+1}\dot{\gamma}^n$ (12) $\dfrac{d\lambda}{dt} = H(T, f_s)(1-\lambda) - G(T, f_s)\lambda\dot{\gamma}^n$ (13)	Boundary layer at wall (but not clear in velocity vectors diagram). Low temperature region at wall (but using non-heated die) $\rightarrow$ solid shell. Jetting at central region.
Barkhudarov et al. 1996 (FLOW3D) [95]	Transport equation for η includes advection term and relaxation term which accounts for thixotropy. No yield stress, wall slip, or elastic or plastic behaviour at high f_s. Input parameters given.	$\dfrac{\partial\eta}{\partial t} + \underline{u}.\nabla\eta = \omega(\eta_e - \eta)$ (14) $\dfrac{\partial\lambda}{\partial t} + \underline{u}.\nabla\lambda = b_1(1-\lambda) + b_2\lambda\dot{\gamma}$ (15) Note: We have changed $(\underline{u}\nabla)\lambda$ to $\underline{u}.\nabla\lambda$ for clarity of notation. $\eta = \eta_\infty + c\lambda$ (16) $\Rightarrow \eta = \eta_\infty + \dfrac{b_1 c}{b_1 + b_2\dot{\gamma}}$ and $\omega = b_1 + b_2\dot{\gamma}$ (17)	Match to experimental shear stress hysteresis curves (Sn-15%Pb) [130] with reasonable accuracy. Sensitive to exact values of relaxation time. Die swell in thixoextrusion.

Barkhudarov & Hirt 1996 (FLOW3D) [96]	Transport equation for η with α rate constant for thinning and β rate constant for thickening. $\eta \geq \eta_e$ then material is trying to relax towards the lower equilibrium value η_e (i.e. thinning). Thixotropic data from [69]. Heat transfer, viscous heating and solidification effects included. Heat transfer negligible in time period considered	$\frac{\partial \eta}{\partial t} + (\underline{u}\nabla)\eta = \alpha Min(\eta_e - \eta, 0) + \beta Max(\eta_e - \eta, 0)$ (18) i.e. if $\eta_e - \eta < 0$ then the right hand side = $\alpha(\eta_e - \eta)$ and if $\eta_e - \eta > 0$ then the right hand side = $\beta(\eta_e - \eta)$	Small droplets of Sn-Pb impacting on flat plate. Droplet shapes influenced by relaxation times.
Modigell & Koke 1999 FLOW-3D [84]	Shear stress is function of yield stress and structural parameter κ (which differs from λ in that it varies between 0 and ∞ rather than 0 and 1). Shear stress is assumed to grow exponentially with increasing solid fraction. Input parameters given.	(19) $\tau = \tau_y(f_s) + \exp(Bf_s)k^* \kappa \dot{\gamma}^m$ $\frac{\partial \kappa}{\partial t} = a \exp(b\dot{\gamma})(\kappa_e - \kappa)$ (20) In equilibrium: $\kappa_e = \frac{1}{(\alpha\dot{\gamma})^{m-n}}$ The equilibrium flow curve is then: $\tau = \tau_y(f_s) + \exp(Bf_s)k\dot{\gamma}^n$ with (21) $k = k^* \alpha^{n-m}$	Comparison between Newtonian and thixotropic for flow in a cavity with a round obstacle.

Kim & Kang 2000 (MAGMAsoft) [97]	Comparison of Newtonian and Ostwald-de-Waele with *n* of –0.48 to +0.45 (depending on T) under shear rate of 3-2500s^{-1}. Input parameters given. Predict defects in product from temperature distribution	Ostwald-de-Waele for viscosity dependence on shear rate.	Good agreement between partial filling experiment and predicted temperature distribution at 80% filling.
Modigell & Koke 2001 (FLOW-3D) [85]	Die filling of steering axle assumed isothermal with wall adhesion.	$\tau = \left(\tau_y(f_s) + k*(f_s)\dot{\gamma}^{m(f_s)}\right)\kappa$ (22)	Models step change of shear rate experiments quite well. Die filling of steering axle. Above critical inlet velocity filling no longer laminar.
Ward, Atkinson, Kirkwood and Chin 2002 (FLOW3D) [98]	As for Barkhudarov and Hirt 1996 [95]	As for Barkhudarov and Hirt 1996 [95]	Modelling of shear rate jumps for Sn-15%Pb. All variable values to fit shear rate jumps consistent with Cross equation and with rate data, except the initial viscosity, which was 2-5 times lower than experimental values. This suggests the initial breakdown of the slurry is very rapid, possibly beyond the detection limits of the data collection system. Modelling of rapid

			compression in a thixoformer suggests aluminium slurries undergo an initial very rapid breakdown and that the subsequent breakdown rate is not strongly shear rate dependent.
Messmer 2002 (FLOW3D) [99]	Thixoforging using approach of Barkhudarov et al. [95].	$\eta_e = A\exp(Bf_s)\dot{\gamma}^m$ (23) $\frac{d\eta}{dt} = \beta(\eta_e - \eta)$ (24)	Forming force measured at end of stroke corresponds well with simulated force. Early part of stroke not well simulated. Attributed to use of only one thinning rate.
Seo & Kang 2002 (MAGMAsoft) [100]	Simple upsetting experiments to obtain rheological data with A356. Input parameters given.	Ostwald-de-Waele compared with Carreau-Yasuda: $\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = \left[1 + (\dot{\gamma}k)^a\right]^{\frac{n-1}{a}}$	No filling results for Carreau-Yasuda presented. Ostwald-de-Waele gives reasonable agreement with partial filling tests.
Itamura et al 2002 (Adstefan) [101]	Compares simulation for die-casting, squeeze-casting and rheocasting for both metal flow and solidification.	No details given.	Die-casting gives air entrapment cf. squeeze casting and rheocasting. Less shrinkage defects in rheocasting.
Two-phase			
Ilegbusi et al. 1999 [102]	Single phase equations solved for whole filling phase. Trajectories of given number		

	of particles computed, assuming they ‘disappear’ when they hit a wall or are trapped in recirculation zone. Measure of segregation obtained by comparing number of particles at given distance from inlet to total number of injected particles.		

Table 5. One phase finite element simulation papers

		Comments	Flow and viscosity equations	Observations
Zavaliangos & Lawley 1995 (ABAQUS) [103]	Medium Volume Fraction Solid $f_s \leq 0.6-0.7$	Single internal variable (λ) constitutive model [127-129]. Same equations as for Ilegbusi and Brown 1995 [94] but without yield stress. Isothermal.	$\tau = A(\lambda)\frac{(c/c_{\max})^{1/3}}{1-(c/c_{\max})^{1/3}}\eta_f\dot{\gamma} + (n+1)C(T)\lambda f_s \eta_f^{n+1}\dot{\gamma}^n$ (25) $\frac{d\lambda}{dt} = H(T, f_s)(1-\lambda) - G(T, f_s)\lambda\dot{\gamma}^n$ (26)	Sn-15%Pb. Free standing billet collapse for $f_s \leq 0.5$. Thixoforming of a simple shape. No validation available.
Backer 1998 (WRAFTS) [104]		1) Herschel-Bulkley	1) $\boldsymbol{\tau} = \tau_y - m\left|\sqrt{D_{II}/2}\right|^{n-1}\boldsymbol{\Delta u}$ (27) Note that this equation is given here exactly as in the Backer paper but it is not clear whether τ_y is being treated as a tensor. In its present form the equation is dimensionally incorrect. For simple shear, the Backer equation reduces to: $\tau = \tau_y + k\dot{\gamma}^n$ (28)	Comparison of Newtonian, Herschel-Bulkley and internal variable results for complex die. Latter tends to fill

	2) Bingham combined with power law dependence. Single internal variable λ [127-129]… apparently the same equations as for Ilegbusi and Brown [94] but without a yield stress. Value of *n=4*.	2) $\boldsymbol{\tau} = \left[A \frac{(c/c_{\max})^{1/3}}{1-(c/c_{\max})^{1/3}} u - C\lambda f_s \left\| \sqrt{D_{II}/2} \right\|^{n-1} \right] \boldsymbol{\Delta u}$ (29) which, for simple shear, reduces to $\tau = A \frac{(c/c_{\max})^{1/3}}{1-(c/c_{\max})^{1/3}} u\dot{\gamma} + C\lambda f_s \dot{\gamma}^n$ (30) (In comparison with Ilegbusi and Brown [94], the liquid viscosity η_f appears to have been included in the function *A*, a velocity *u* is present in the first term and $(n+1)\eta_f{}^{n+1}$ is included in the function C in the second term). The transport equation for λ is: $\frac{\delta\rho\lambda}{\delta t} = \nabla.\rho u\lambda + K(1-\lambda) - G\lambda^2 \sqrt{D_{II}/2}$ (31) which, if ρ is treated as a constant and $\frac{\partial u_x}{\partial x}, \frac{\partial u_y}{\partial y}, \frac{\partial u_z}{\partial z} = 0$, reduces to: $\rho\left(\frac{\partial\lambda}{\partial t}\right) - \rho(\underline{u}.\nabla\lambda) = K(1-\lambda) - G\lambda^2\dot{\gamma}$ (32) (which is very similar to Barkhudarov et al [95] but with a λ^2 in the second term on the right hand side rather than λ).	from side runner rather than bottom because material has flowed further in the runner and disagglomerated in the process. No validation with experiment.
Alexandrou et al. 1999 (PAMCASTSIMULOR) [105]	Herschel-Bulkley. $\tau_y = 9615 \frac{f_s^3}{0.6 - f_s}$ Fluid assumed incompressible. $n = 1$ then Bingham.	$\tau_{ij} = \left\{ \frac{\tau_y \left(1 - \exp\left(-m\sqrt{D_{II}/2}\right)\right)}{\sqrt{D_{II}/2}} \right\} D_{ij}$ (33) $\eta = K(D_{II}/2)^{(n-1)/2}$ (34) For simple shear the equation reduces to:	Bingham set constant and independent of processing conditions…fairly good agreement

	T dependence introduced through making τ_y function of T.	$\tau = \tau_y\left(1 - \exp\left(-m\dot{\gamma}\right)\right)$ (35) $\eta = K\dot{\gamma}^{n-1}$ (36)	between modelling and filling for complex part. Local flow not predicted as well as bulk filling. Comparison of Newtonian and Bingham filling of simple 2-D cavity. Comparison of Newtonian and Bingham filling of 3-D cavity with core. Results show efficacy of 'overflows' on dies beyond 'rewelding' areas.
Burgos & Alexandrou 1999 [106]	Herschel-Bulkley (as for [105])		Predicts time evolution of yielded/unyielded regions for sudden 3-D square expansion.
Alexandrou et al 2001 (PAMCASTSIMULOR)	Bingham fluid. Continuous model due	$\dot{\gamma} = 0 \quad \tau \le \tau_y$	Five different flow patterns

[107]	to Papanastasiou [131] to avoid discontinuity at yield surface.	$\tau = \left(\eta + \frac{\tau_y}{\dot{\gamma}}\right)\dot{\gamma} \quad \tau > \tau_y$ (37) $\tau = \left[\eta + \tau_y \frac{1-\exp(-m\lvert\dot{\gamma}\rvert)}{\lvert\dot{\gamma}\rvert}\right]\dot{\gamma}$ [132] (38) Simplifying: $\tau = \eta\dot{\gamma} + \tau_y(1-\exp(-m\dot{\gamma}))$ (39)	'mound', 'disk', 'shell', 'bubble' and 'transition flow' agreeing with observations by Paradies and Rappaz [136]. Map of flow patterns as a function of Reynolds and Bingham numbers.
Burgos et al 2001 [108]	Herschel-Bulkley expanded to include effect of evolution of microstructure. τ_y, K, n are assumed functions of f_s and λ. Single internal variable λ. Assume transient behaviour at constant structure is shear thickening. Material parameters from [133] for Sn-15%Pb with $f_s = 0.45$.	$\frac{\partial\lambda}{\partial t} + \underline{u}.\nabla\lambda = a(1-\lambda) - b\lambda\left[\frac{D_{II}}{2}\right]^{1/2}\exp\left(c\left[\frac{D_{II}}{2}\right]^{1/2}\right)$ (40) which, for simple shear and where λ is not changing spatially, reduces to: $\frac{\partial\lambda}{\partial t} = a(1-\lambda) - b\lambda\dot{\gamma}\exp(c\dot{\gamma}) = -[a + b\dot{\gamma}\exp(c\dot{\gamma})](\lambda - \lambda_e)$ (41) (which for $c = 0$ is equivalent to the Moore equation [72]). $\tau = \left\{K(f_s,\lambda)\left[\frac{D_{II}}{2}\right]^{(n(f_s,\lambda)-1)/2} + \frac{\tau_y(f_s,\lambda)\left[1-\exp\left(-m\left\lvert\sqrt{D_{II}/2}\right\rvert\right)\right]}{\sqrt{D_{II}/2}}\right\}D$ (42) which, in simple shear, reduces to: $\tau = K(f_s,\lambda)\dot{\gamma}^{n(f_s,\lambda)} + \tau_y(f_s,\lambda)(1-\exp(-m\dot{\gamma}))$ (43) (Note that equation (8) in [108] is not correct).	Flow in simple straight channel. Power law index decreases with λ, but consistency index and τ_y decrease. Breakdown is less in the core and in the corners of the square channel than in the higher shear regions.

Alexandrou et al. 2002 [109]	Bingham but using [131] to avoid singularity. Simple compression test. No account taken of evolution of sample's internal structure.	$$\tau = \left[\eta + \tau_y \frac{1 - \exp\left(-m\sqrt{D_{II}/2}\right)}{\sqrt{D_{II}/2}} \right] \dot{\gamma} \quad (44)$$	A356 simple compression. Shape during compression reproduced in simulation using η and τ_y from fitting load versus time curve. Unyielded material at top and bottom in stagnant layers.
Ding et al. 2002 (DEFORM3-D) [110]	Rigid viscoplastic constitutive model. Flow stress for AlSi7Mg obtained from compression on Gleeble machine, ignoring initial transient. Levy-Mises flow rule.	$$\sigma = \exp(a - bT)\varepsilon^m \dot{\varepsilon}^n \quad (45)$$ When written in shear stress terms this is equivalent to: $$\tau = (\exp(a - bT))\gamma^m \dot{\gamma}^n \quad (46)$$	Die with six rectangular orifices heated to 580-586˚C (i.e. isothermal). Good agreement with interrupted flow tests. Metal in biggest orifice flows fastest. Some discrepancy between prediction of load-stroke curve and actual. No

			examination of liquid segregation in the samples.
Jahajeeah et al. 2002 (Procast) [111]	Power Law Cut-Off (PLCO) model of Procast [134] i.e. isotropic, purely viscoplastic, independent of pressure, deformation homogeneous.	$\eta(\dot{\gamma},T)=\eta_0(T)\dot{\gamma}_0^{\,n(T)}$ for $\dot{\gamma}\le\dot{\gamma}_0$ $\eta(\dot{\gamma},T)=\eta_0(T)\dot{\gamma}^{\,n(T)}$ for $\dot{\gamma}>\dot{\gamma}_0$ (47)	Brake calliper divided into different regions each with different cut-off values $\dot{\gamma}_0$. Reasonable agreement with interrupted filling tests. Defect prediction with less than optimum runner design.
Rassili et al. 2002 (FORGE3) [112]	Visco-plastic constitutive model from force recordings of extrusion tests. Friction assumed very low. No time dependence.	$K(\bar{\varepsilon},T)=K_0\exp\left(\frac{\beta}{T}\right)\bar{\varepsilon}^{\,n}$ (48) K is equivalent to the current yield stress, K_0 to a yield stress and $\bar{\varepsilon}$ to an effective strain. In shear terms, this is analogous to: $\tau=\tau_y\exp\left(\frac{\beta}{T}\right)\bar{\gamma}^{\,n}$ (49)	Several combinations for tool displacement. Ejector goes up, punch starts to go down when ejector stops….buckling leading to lap. Punch and ejector move simultaneously or punch goes down

			first then ejector goes up both avoid buckling but lap is formed on each ‘ear’ of part. Estimation of forging force.
Wahlen 2002 (Thixoform) [113]	Model based on viscoelasticity and thixotropy.	$\sigma = \sigma_0 \left[1 - \exp\left(-\frac{\varepsilon}{\dot{\varepsilon}\tau_M} \right) \right] \frac{f_s \lambda}{2} \left(\frac{R_b}{R} \right)^3 \left[\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \exp\left(\frac{Q}{RT} \right) \right]^m \quad (50)$ This could be written in shear stress terms as: $\tau = \tau_y \left[1 - \exp\left(-\frac{\gamma}{\dot{\gamma}\tau_M} \right) \right] \frac{f_s \lambda}{2} \left(\frac{R_b}{R} \right)^3 \left[\frac{\dot{\gamma}}{\dot{\gamma}_0} \exp\left(\frac{Q}{RT} \right) \right]^m \quad (51)$ Note: R means both the particle size in the R_b/R term and the gas constant in the exponential term.	Good agreement between model and curve of flow stress versus true strain. Cylindrical specimens produced in backward extrusion. Results allow prediction of temperature of transition from plastic deformation of interconnected particles to viscous flow of a suspension of solid particles. Discrepancies between prediction and experiment

			though.
Alexandrou et al. 2003 [114]	2-D jets of Bingham and Herschel-Bulkley fluids impacting on vertical surface at distance from die exit in order to account for flow instabilities (eg. 'toothpaste' effect) in semi-solid processing.	Papanastasiou model [131]: $$\boldsymbol{\tau} = \left[\eta + \tau_y \frac{1-\exp(-m\dot{\gamma})}{\dot{\gamma}}\right]\dot{\boldsymbol{\gamma}} \qquad (52)$$ Generalized to Herschel-Bulkley fluid by specifying $$\eta = \kappa\dot{\gamma}^{n-1} \qquad (53)$$	'Bubble' pattern gives unstable jet, 'shell', 'disk' and 'mound' stable along with most 'transition' cases. Instabilities are result of finite yield stress and the way yielded and unyielded regions interact. Plots of Bingham number versus Reynolds number identify stable and unstable regions.
Orgeas et al. 2003 (Procast) [115]	PLCO model as for Jahajeeah et al [111] (see above) but with only one value of cut-off $\dot{\gamma}_0$ (determined by geometry). n and η_0 dependent on f_s.	$$\eta = \eta_0\left(\frac{\dot{\gamma}_0}{\dot{\gamma}_c}\right)^{n-1} \text{ for } \dot{\gamma} < \dot{\gamma}_0 \qquad (54)$$ $$\eta = \eta_0\left(\frac{\dot{\gamma}}{\dot{\gamma}_c}\right)^{n-1} \text{ for } \dot{\gamma} \geq \dot{\gamma}_0$$	Bifurcation of Poiseuille-type flow near a shaft inserted in a tube. Without shaft, experimentally, thin segregated layer of liquid at wall led to 'plug flow'. Reliable

			results for flow around the shaft but some instabilities/ discrepancies when flow first encounters shaft. Filling of reservoir (giving ‘disk+shell’) well-simulated.

Table 6. Two-phase finite element simulation papers and micro-modelling

	Comments	Flow and viscosity equations	Observations
Zavaliangos & Lawley 1995 (ABAQUS) [103]	High Volume Fraction Solid $f_s \geq 0.7$. Porous viscoplasticity model; fluid flow in porous medium; continuity equations. Isothermal. Behaviour symmetric under tension and compression.	See paper for details.	Compression of semi-solid billet indicating liquid segregation. At higher strain rates less liquid is lost. No experimental validation.
Zavaliangos 1998 [116]	Deformable porous medium saturated with liquid. Stress partitioned into stress carried by solid phase and purely hydrostatic component for pressure in liquid phase. Solid phase has two limits: fully cohesive porous solid and cohesionless granular material. Degree of cohesion represented by internal variable which does evolve with deformation (cf. single internal variable in [127-129]). Permeability equation implies that solid-liquid segregation decreases as the grain size decreases. Behaviour not symmetric under tension and compression.	See paper	Converging (conical) channel. High strain rates result in near-undrained conditions and minimal phase segregation.

Koke et al. 1999 [117]	Liquid phase assumed Newtonian. Solid phase is pseudo-fluid with Herschel-Bulkley viscosity. Darcy Law, Carmen-Kozeny capillary approach. $f_s \geq 0.5$.	$\eta_s = \left(\frac{\tau_y}{\dot{\gamma}} + k^* \dot{\gamma}^{m-1} \right) \lambda + \eta_f$ (55) At equilibrium, $k^* = k$, the coefficient in the Herschel-Bulkley power law term (see Backer above), and $m = n$. τ_y and k are assumed to increase exponentially with f_s. This equation gives a different expression for shear stress from that due to Brown and co-workers [127-129]. Note: It isn't clear where λ has gone to in equation 16.	Vertical compression of cylindrical billet. Phase segregation. Qualitative agreement with experiment [135].
Kang & Jung 1999 [118]	Compressible viscoplastic model for the solid phase and Darcy's law for the flow of liquid through a porous medium. Separation coefficient introduced $S = S_0 + (1 - S_0)\frac{\bar{\varepsilon}}{\bar{\varepsilon}_{cr}}$ where S_0 is the ratio of the actual separation to the initial separation, $\bar{\varepsilon}$ is the equivalent strain and $\bar{\varepsilon}_{cr}$ a critical strain.	$\bar{\sigma} = K \exp(S) \dot{\varepsilon}^m \exp\left(\frac{Q}{RT}\right)[1 - \beta f_l]^{2/3}$ (56) for $\varepsilon < \bar{\varepsilon}_{cr}, \varepsilon > \bar{\varepsilon}_{cr1}$ $\bar{\sigma} = K \exp(1 - S) \dot{\varepsilon}^m \exp\left(\frac{Q}{RT}\right)[1 - \beta f_l]^{2/3}$ (57) for $\bar{\varepsilon}_{cr} < \varepsilon < \bar{\varepsilon}_{cr1}$. These equations could be written in shear stress terms by replacing $\bar{\sigma}$ with $\bar{\tau}$, $\bar{\varepsilon}$ with $\bar{\gamma}$, $\bar{\dot{\varepsilon}}$ with $\bar{\dot{\gamma}}$ etc.	The higher the strain rate the more homogeneous the distribution of the solid fraction. In compression forming, macroscopic phase segregation occurred with densification of the remaining solid in the central region.
Binet & Pineau 2000 [119]	Mixture approach. Hydrodynamic part same as for most incompressible CFD codes but velocity field represents velocities of the mixture and a source term is added to the momentum equations to take account of the diffusion velocities of the individual phases. Relative velocities	See paper.	Predictions of segregation at corners of entrance and outlet of diverging channel in a simple die.

	calculated from interaction force between phases. Darcy's Law, Carman-Kozeny relation. Rheological data from [137].		
Choi et al. 2000 [120]	Compressible visco-plastic solid, liquid phase following Darcy's law. Kuhn's yield criterion [137] for deformation of solid phase. Friction equation at die/material surface from [138].	See paper.	Head of a trench mortar shell in which forward and backward extrusion are taking place simultaneously. Higher die temperature (400˚C) gives better product. Qualitative agreement with experiment for segregation of liquid.
Kang & Jung 2001 [121]	As for Kang & Jung 1999 [118]	As for Kang & Jung 1999 [118]	Prediction of overflow positions in scroll component. Liquid segregation in the narrow cross-sections. Higher strain rates gave less segregation.
Yoon et al 2001 (CAMPform2D) [122]	Von Mises yield criterion. Semi-solid treated as single phase with incompressibility. Flow stress as function of strain (with f_s and 'breakage ratio'). Mixture theory and D'Arcy's Law to update f_s. Material properties for Sn-15%Pb from [134]. Input parameters for	Equations are summarised in Fig. 1 in the paper. $\bar{\sigma}_f = K\left(\frac{\bar{\varepsilon}}{\bar{\varepsilon}_{cr}}\right)^n (\exp(b))\bar{\dot{\varepsilon}}^m$ for $\bar{\varepsilon} < \bar{\varepsilon}_{cr}$ (58) $\bar{\sigma}_f = K\left(\exp\left(b\frac{\bar{\varepsilon}-\bar{\varepsilon}_{st}}{\bar{\varepsilon}_{cr}-\bar{\varepsilon}_{st}}\right)\right)\bar{\dot{\varepsilon}}^m$ for $\bar{\varepsilon} \geq \bar{\varepsilon}_{cr}$ (59) These equations could be written in shear stress terms	Isothermal predictions of liquid segregation in good agreement with experiment [134]. Non-isothermal simulation for ball joint gives qualitatively useful information.

	non-isothermal Al2024 alloy given.	in an analogous way to Kang and Jung 2001 above.	
Kopp & Horst 2002 (ABAQUS) [123]	Drucker-Prager yield criterion (yield strength different in tensile and in compressive strain). Finite element mesh attached to solid phase.		
Modigell et al. 2002 [124].	Equilibrium flow behaviour modelled with Herschel-Bulkley. Thixotropy modelled with structural parameter following first order differential equation. Pseudo-fluid approach for the solid phase. All non-Newtonian properties shifted to the solid. Liquid Newtonian. Continuity and momentum equations solved for each phase. Interaction between phases modelled with Darcy law.		Simulation and experiment for Sn-15%Pb match well for $f_s > 0.55$. Maps of laminar, transient and full turbulent filling produced.
		Micro-modelling	
Rouff et al. 2002 [125]	Volume solid fraction of the 'active zone', f_A^s ,is the internal variable. The 'active zone' consists of the solid bonds between spheroids and the liquid between spheroids which is not internally entrapped. During deformation the bonds are broken and liquid is released. Spheroids and 'active zone' treated as isotropic and incompressible.	$f_A^s = 0$ for $f_s \leq f^c$ $f_A^s = \dfrac{f_s}{f_s + D(1 - f_s)\dot{\gamma}^n}$ for $f_A^s > f^c$ (60)	Good agreement with experimental data on viscosity versus shear rate.