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Growing Supermassive Black Holes by Chaotic Accretion

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ABSTRACT

We consider the problem of growing the largest supermassive black holes from stellar–mass seeds at high redshift. Rapid growth without violating the Eddington limit requires that most mass is gained while the hole has a low spin and thus a low radiative accretion efficiency. If, as was formerly thought, the black–hole spin aligns very rapidly with the accretion flow, even a randomly–oriented sequence of accretion events would all spin up the hole and prevent rapid mass growth. However, using a recent result that the Bardeen–Petterson effect causes *counter*alignment of hole and disc spins under certain conditions, we show that holes can grow rapidly in mass if they acquire most of it in a sequence of randomly oriented accretion episodes whose angular momenta J_d are no larger than the hole's angular momentum J_h. Ultimately the hole has total angular momentum comparable with the last accretion episode. This points to a picture in which the accretion is chaotic on a lengthscale of order the disc size, that is $\lesssim 0.1$ pc.

Key words: accretion, accretion discs – black holes

1 INTRODUCTION

There is recent observational evidence (Barth et al., 2003, Willott et al, 2003) for supermassive black holes (SMBH) with masses $M \gtrsim 5 \times 10^9 \mathrm{M}_\odot$ at redshift $z \simeq 6$. The existence of such large masses only $\sim 10^9$ yr after the Big Bang is a challenge to theory. For if we accept that SMBH are largely made of matter which underwent luminous accretion (Soltan, 1982; Yu & Tremaine, 2002), and that the accretion luminosity cannot exceed the Eddington value

$$L_{\rm Edd} = \frac{4\pi GMc}{\kappa} \simeq 10^{47} M_9\,{\rm erg\,s^{-1}} \tag{1}$$

(where $M = 10^9 M_9 \mathrm{M}_\odot$ and we have taken κ to be the electron scattering opacity) there is a limit to the rate at which a black hole can accrete. This comes from setting

$$L_{\rm Edd} = \epsilon c^2 \dot M_{\rm acc} \tag{2}$$

where ϵ is the accretion efficiency, specified by the fractional binding energy of the innermost stable circular orbit (ISCO) about the hole. The black hole mass then grows as mass–energy is accreted, at the rate

$$\dot M = (1-\epsilon)\dot M_{\rm acc}. \tag{3}$$

Thus

$$\dot M = \frac{1-\epsilon}{\epsilon}\frac{M}{t_{\rm Edd}} \tag{4}$$

with

$$t_{\rm Edd} = \frac{\kappa c}{4\pi G} = 4.5 \times 10^8 \ {\rm yr}. \tag{5}$$

In general ϵ varies as accretion proceeds (see below), but its minimum value $\epsilon_{\rm min}$ over the hole's accretion history sets a limit to the black hole mass M which can grow from an initial seed value M_0 in a given time. Integrating eqn(4) we have

$$\frac{M}{M_0} < \exp\left[\frac{1}{\epsilon_{min}} - 1\right]\left(\frac{t}{t_{\rm Edd}}\right). \tag{6}$$

At redshift $z \simeq 6$ the last factor in the exponent is $\simeq 2$. We see that large values of $\epsilon_{\rm min}$ severely restrict the growth of SMBH. Thus with $\epsilon_{\rm min} = 0.43$, as appropriate for a maximally rotating hole with dimensionless Kerr parameter $a \simeq 1$ (see below) we find $M/M_0 \lesssim 20$. In this case there is clearly no prospect of growing the inferred SMBH masses $\sim 5 \times 10^9 \mathrm{M}_\odot$ at redshift $z \simeq 6$ from stellar–mass seed holes, and one would have to consider other possibilities (e.g. Volonteri & Rees, 2005; Begelman et al., 2006). Growth from seed holes of mass $10\mathrm{M}_\odot$ requires $\epsilon_{\rm min} \lesssim 0.08$, corresponding to rather lower black–hole spin rates $a \lesssim 0.5$. Still lower values of a are desirable if we wish to avoid the difficulty that accretion must be almost continuous to build up the observed mass.

2 KEEPING THE SPIN LOW

In the last Section we showed that growing very large SMBH masses at high redshift from stellar–mass seeds requires *low accretion efficiency*, or equivalently, *modest black–hole spins*. This runs directly counter to the usual expectation (e.g. Volonteri et al., 2005; Madau, 2004) that gas accretion produces systematic spin–up. Volonteri et al. (2005) estimated from semi–analytic cosmological modelling that the fractional change of mass during each accretion episode of a growing black hole is quite large, i.e. $\Delta M/M \sim 1 - 3$. The work of Scheuer & Feiler (1996) and of Natarajan & Pringle (1998) suggested that for accretion via a geometrically thin disc the combination of the Lense–Thirring effect with viscous dissipation (Bardeen & Petterson, 1975) would always align the SMBH spin with the angular momentum of the accreting gas on a timescale typically much shorter than the accretion timescale for mass and angular momentum. Thus every accretion episode would rapidly become a spin–up episode.

Recently however King et al. (2005) (hereafter KLOP) showed on quite general analytic grounds that the Lense–Thirring effect instead produces *counter*alignment on similarly short timescales in particular cases, namely those where the magnitudes of the angular momenta J_h, J_d of the hole and disc obey

$$\theta > \pi/2, \quad J_d < 2J_h. \tag{7}$$

with θ is the angle between the vectors $\mathbf{J}_h$ and $\mathbf{J}_d$. Thus if $J_d < 2J_h$, then for $\mathbf{J}_h, \mathbf{J}_d$ in random directions, counteralignment occurs in a fraction

$$f_c = \frac{1}{2}\left[1 - \frac{J_d}{2J_h}\right] \tag{8}$$

of cases. We note that counteralignment predominantly involves a shift in the angular momentum of the hole to counteralign with that of the disc (see also Lodato & Pringle, 2006).

It follows that if most gas accretion occurs in randomly oriented episodes with $J_d \lesssim J_h$, spinup and spindown episodes tend to alternate in some random way. If instead accretion episodes generally have $J_d \gtrsim J_h$ the hole must consistently spin up, even if hole and disc are initially counteraligned (see Lodato & Pringle 2006 for details). Clearly the first type of accretion, i.e. with $J_d \lesssim J_h$, offers the better chance of keeping the black hole spin and accretion efficiency low, and thus enabling the building large black hole masses in a short time. In fact for similar values of accreted mass, the spindown effect of counteraligned accretion is significantly more effective that the spinup from aligned accretion. This is a straightforward consequence of the fact that the ISCO for retrograde rotation is always larger than that for aligned rotation. Writing the Boyer–Lindquist coordinate radius of the ISCO as $r = xc^2/GM$, we have $x = 9, 6$ and 1 for dimensionless Kerr parameters $a = -1, 0$ and $+1$. The results of Bardeen (1970) show that

$$\epsilon = 1 - \left[1 - \frac{2}{3x}\right]^{1/2} \tag{9}$$

and that the ISCO has specific angular momentum

$$j = \frac{2}{3\sqrt{3}}\frac{GM}{c}[1 + 2(3x-2)^{1/2}]. \tag{10}$$

We thus see that that for $a = -1, 0, 1$, the specific angular momentum j is in the ratio $11 : 9 : 3$, illustrating the point above that spindown is considerably more efficient than spinup.

The general connection between a and x is

$$a = \frac{x^{1/2}}{3}[4 - (3x-2)^{1/2}]. \tag{11}$$

Given a hole specified by initial mass and spin parameters (M_1, x_1), Bardeen (1970) shows that accretion from the ISCO causes these parameters to evolve as

$$\frac{x}{x_1} = \left(\frac{M_1}{M}\right)^2. \tag{12}$$

Thus to discover how the mass and spin of an SMBH evolve over a series of accretion episodes, we have to connect the corresponding sequence of (M, x) values, bearing in mind that the spin parameter a changes sign (hence changing x discontinuously) if accretion switches between prograde and retrograde states.

3 THE ACCRETION EPISODES

Volonteri et al. (2005; see also Wilson & Colbert, 1995; Hughes & Blandford, 2003) showed that if successive accretion episodes add angular momentum to the growing hole at random angles, then the eventual spin of the hole is small. We have argued that a small spin is beneficial in helping the hole to grow more rapidly. But moreover we argue that if hole growth occurs through a series of accretion episodes with $J_d \lesssim J_h$, then the spindown process is more efficient than envisaged by Volonteri et al. (2005). In that paper the black–hole spin was assumed to align very rapidly with the accretion flow. Then significant spindown would require a matter supply whose angular momentum reversed on timescales short compared with the timescale $t_{\rm al}$ for alignment/counteralignment, which is only a few times 10^4 yr (Natarajan & Pringle, 1998) and thus unacceptably short. However KLOP showed that with $J_d < 2J_h$ the black hole spin can counteralign with the disc, so that accretion then occurs in a fully retrograde fashion. A randomly–oriented sequence of such accretion episodes can thus keep the spin low, rather than all aligning the hole very quickly and causing systematic spinup.

The condition $J_d \lesssim J_h$ requires

$$M_d \lesssim Ma\left(\frac{R_g}{R_d}\right)^{1/2}, \tag{13}$$

where R_d is a typical outer disc radius, and $R_g = GM/c^2$. This agrees with the estimate (eq. 20) in KLOP, which explicitly uses the AGN disc properties computed by Collin–Souffrin & Dumont (1990). One could now follow the procedure outlined at the end of the last Section to follow the hole's evolution under a random series of accretion events obeying eq (13). However a straightforward argument shows the likely outcome. To simplify the analysis we assume that all the episodes have the same mass and angular momentum. We take the disc angular momentum as either precisely aligned or counteraligned with $\mathbf{J}_h$, which follows from the result that the alignment/counteralignment timescale is shorter than the disc accretion timescale. Then M_d, R_d are the same for all episodes. Since spindown is more efficient than spinup, a random sequence of events must tend to decrease $|a|$ for the hole towards zero. However this process stops once we reach equality in eq (13), when further episodes cause a to oscillate between a small positive value

$$a \approx \frac{M_d}{M}\left(\frac{R_d}{R_g}\right)^{1/2} \tag{14}$$

and a negative value which is still smaller (i.e. closer to zero: recall that spindown is more efficient than spinup). Clearly for suitable choices of M_d, J_d one can arrange to keep $|a|$ small enough ($\lesssim 0.5$) to ensure the low accretion efficiency required for rapid mass growth, assuming an adequate supply of mass, i.e. sufficiently frequent small-scale mergers.

For typical parameters if $J_d \sim J_h$ we find that $R_d \sim 8000 R_g a^{10/19}$ (cf. KLOP, eq 20) and then the mass M_d in each event is of order 1% of the hole's mass or less. Thus we require ~ 100 such episodes to occur during the growth time $\sim t_{\rm Edd}$. Using the results of Section 2, we find that if each accretion episode adds as much as $\Delta M = 0.18M$ to the hole, a would oscillate between values ± 0.3, with accretion efficiencies ϵ ranging from 0.049 to 0.069. With the smaller accretion episodes required to satisfy $J_d \lesssim J_h$ we find that with e.g. $\Delta M = 0.016M$ the hole spin oscillates between $a = \pm 0.03$ with ϵ in the range $0.056 - 0.058$. This allows a mass $M \sim 5\times 10^9 {\rm M}_\odot$ to grow in about 5×10^8 yr, about one half of the time available at redshift $z = 6$. Growth of such a mass in a significantly shorter time would require systematically retrograde accretion, which appears implausible.

4 CONCLUSIONS

We have shown that SMBH can grow their mass rapidly if most of it comes from a sequence of randomly oriented accretion episodes whose angular momenta J_d are no larger than the hole's angular momentum J_h. This comes about because the hole then has a low spin and thus a low radiative accretion efficiency, so rapid mass growth is possible without breaching the Eddington limit. The hole ends up with total angular momentum comparable to that accreted during the last accretion episode. Volonteri et al. (2005) suggested that most SMBH mass growth occurs through a series of major galaxy mergers in each of which the black hole accretes $2-4$ times its original mass. However, cosmological simulations, including the ones upon which these estimates were based, are not capable of following the detailed hydrodynamics of the accretion process. To satisfy the conditions discussed here we need the accretion hydrodynamics to be chaotic at the level of the size of the disc, that is on a scale of around $\lesssim 0.1$ pc. Given that any such merger is likely to involve major episodes of star formation, especially in the central regions, this seems quite likely to be the case.

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