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**NONLINEAR IMPACTS OF INTERNATIONAL
BUSINESS CYCLES ON THE UK — A BAYESIAN
SMOOTH TRANSITION VAR APPROACH**

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Approach

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Abstract

Employing a Bayesian approach, we investigate the impact of international business cycles on the UK economy in the context of a smooth transition VAR. We find that British business cycle is asymmetrically influenced by the US, France and Germany. Overall, positive and negative shocks generating in the US or France affect the UK in the same directions of the shock. Yet, a shock emanating from Germany always exerts negative accumulative effects on the UK. More strikingly, a positive shock arising from Germany negatively affects UK output growth more than a negative shock from Germany of the same size. These results suggest that the appropriate UK economic policy depends upon the origin, size and direction of the external shocks.

JEL: C11, C32, C52, E32, F42.

Keywords: International business cycle, Bayesian, smooth transition vector autoregression model.

1 Introduction

The study of international business cycle linkages is of special importance to macroeconomic policy research. Numerous studies have sought to identify a common business cycle across countries (see for instance, Artis and Zhang, 1997, Wynne and Koo, 2000, Inklaar and Haan, 2001). In recent years, nonlinear multivariate models have become more popular among researchers for such models can effectively capture the cross-country asymmetric interdependencies (Smith and Summers, 2005, Artis, Galvao and Marcellino, 2007, Chen and Shen, 2007, to mention a few).

The present paper examines the impacts of international business cycles on the UK economy within the framework of a logistic smooth transition vector autoregression (LSTVAR) model. In particular, we attempt to characterize the behaviour of the UK output growth under the influence of the booms and busts in the US, France, and Germany, respectively.

Business cycle linkages between the UK and the three afore mentioned countries have been examined previously by, for example, Artis and Zhang (1997), Inklaar and Haan (2001), and Perez, Osborn and Artis (2006). However, most of the literature focuses on exploring the business cycles synchronization rather than investigating the propagation of different types of shocks (such as positive and negative or large and small shock) across countries. Although the US' effects on the UK economy are investigated in several studies (for example, Artis, Krolzig and Toro, 2004, Osborn, Perez and Sensier, 2005, Artis *et al.* 2007), to our best knowledge, no evidence on how France and Germany, the two largest continental European economies, influence the UK business cycles has been documented, except for Artis *et al.* (2007), which look into Germany's impact on the UK business cycles at one point.

Our approach for the LSTVAR estimation is Bayesian. In particular, we extend the Bayesian technique in estimating the univariate smooth transition models introduced in Lubrano (1999a, 1999b) into a multivariate form. Compared with the available classical estimation techniques which often require multiple steps and Taylor expansions, our Bayesian method can jointly

estimate the autoregressive coefficients and the nuisance parameters in the transition function in one stage. Therefore, our approach is less susceptible to the sequential testing and inaccurate approximations problems. Furthermore, considering that nonlinear models are generally subject to the criticism of being too parameter rich, we resort to Bayes Factors for model selection and model averaging to reward more parsimonious models.¹

Our results provide strong evidence of asymmetry in the bivariate relationship across the three country pairs. For all cases, LSTVAR models receive overwhelming support over the linear models. Additionally, we find that business cycles in the US, the UK and Germany play important roles in leading regimes changes, while the changes in France output would not cause salient nonlinear effect.

Impulse response analysis implies that features of the impact from the three countries are quite different. Among the three countries, the US' impact is the most persistent. Observe that the effects from France or Germany die out in relatively five years, while with a much clearer cyclical pattern, the impacts of the US growth shocks are still evident after nine years. It is not surprising to observe that the shocks from the US and France would affect the UK in the same direction. However, different from Artis *et al.* (2007), we find that both the expansion and recession of Germany would thwart the UK output growth. Most strikingly, we find that a boom in Germany brings more negative effects to the UK's economy than a bust in Germany.

Overall, we find that the UK's economy is sensitive to the fluctuations of international business cycles in an asymmetric form. Our research nonetheless suggests that relying on linear models would result in systematic mistakes in analysis and policy making due to the presence of substantial nonlinear effect. Furthermore, it goes without saying that pernicious effects on the UK growth rate exerted by Germany is of intrinsic importance to policy makers.

The rest of the paper is structured as follows. Section 2 introduces the LSTVAR model and Bayesian inferences. Section 3 presents empirical results. Section 4 concludes.

¹As discussed by Koop and Potter (1999a, 1999b), Bayes Factors include an automatic penalty for more complex models.

2 Logistic Smooth Transition VAR Model

The vector autoregressive model (VAR) has proven very successful in modeling endogenous relationships among macroeconomic variables without imposing restrictions that may be ‘incredible’ in the sense of Sims (1972, 1980). We therefore model the pairwise business cycle linkages in a reduced form VAR based on two considerations. First, VAR is ideally suited to the analysis of endogenously determined processes where dynamics are important but where we have little or no clear economic structure. Second, VAR provides an atheoretical framework for analysis and allows very rich dynamics.² Considering the possible presence of nonlinearities in the cross-country business cycle linkages, we model the annual growth rates of the two countries of concern in a bivariate LSTVAR system introduced by Weise (1999).

Let $y_t = (y_{1,t}, y_{2,t})$, where $y_{1,t}$ is the annual real GDP growth rate of the country other than the UK (the US, France or Germany), $y_{2,t}$ is the British annual real GDP growth rate. For time $t=1, \dots, T$, the cyclical linkages between the UK and another country can be expressed in the nonlinear autoregressive process of order p as follows.

$$y_t = \Phi + \sum_{h=1}^p \Gamma_h y_{t-h} + F(z_t) [\Phi^z + \sum_{h=1}^p \Gamma_h^z y_{t-h}] + \varepsilon_t, \quad (1)$$

where ε_t is a white noise process, that is $E(\varepsilon_t) = 0$, $E(\varepsilon'_s \varepsilon_t) = \Sigma$ for $s = t$, and $E(\varepsilon'_s \varepsilon_t) = 0$ for $s \neq t$.

The regime changes are assumed to be captured by the first order logistic smooth transition function defined by the transition variable z_t

$$F(z_t) = [1 + \exp\{-\gamma(z_t - c)/\sigma\}]^{-1} \quad (2)$$

In function (2), the parameter γ (which is non-negative) determines the speed of the smooth transition. We can see that when $\gamma \rightarrow \infty$, the transition function becomes a Dirac function, then model (1) becomes a two-regime threshold VAR model along the lines of Tong (1983). When $\gamma = 0$, the

²Many studies of co-movements of business cycles among the main industrial countries (see for example, Norrbin and Schlagenhauf, 1996, Helbling and Bayoumi, 2003) use VAR for modeling the interrelationships.

logistic function becomes a constant (equal to 0.5), and the nonlinear model (1) collapses into a linear VAR(p). The parameter c is the threshold around which the dynamics of the model change. The value for the parameter σ is chosen by the researcher and could reasonably be set to one. However, if we set σ equal to the standard deviation of the process z_t , this effectively normalizes γ such that we can give γ an interpretation in terms of the inverse of the number of standard deviations of z_t . The transition from one extreme regime to the other is smooth for reasonable values of γ .

The principle underlying the LSTVAR is that as z_t increases, moving from well below some threshold c to well above this threshold, the dynamics of the vector process y_t changes from one regime to another. That is, if z_t is very low - i.e., well into what we will call the *lower* regime for nominal purposes - then the process y_t may be generated by the VAR model as follows.

$$y_t = \Phi + \sum_{h=1}^p \Gamma_h y_{t-h} + \varepsilon_t \quad (3)$$

However, when z_t is very high - i.e., well into what we will call the *upper* regime - then the process y_t may be generated by the VAR given by

$$y_t = \Phi^1 + \sum_{h=1}^p \Gamma_h^1 y_{t-h} + \varepsilon_t \quad (4)$$

The transition between these two regimes is smooth and governed by a smooth function of z_t denoted by $F(z_t)$. The value of $F(z_t)$ is bounded by 0 and 1. $F(z_t) = 0$ when z_t is very low, and $F(z_t) = 1$ when z_t is very high.

Thus we may express the full process as

$$y_t = (1 - F(z_t)) [\Phi + \sum_{h=1}^p \Gamma_h y_{t-h}] + F(z_t) [\Phi^1 + \sum_{h=1}^p \Gamma_h^1 y_{t-h}] + \varepsilon_t \quad (5)$$

which can equivalently be written as model (1).

Observe that model (1) encompasses a set of models distinguished by the choice of the transition variable, the order of the autoregressive process, and whether there exist nonlinear effects.

2.1 Likelihood Function

For notation convenience, we set $x_t = (1, y_{t-1}, \dots, y_{t-p})$, and $x_t^\theta = [x_t \ F(z_t)x_t]$. Next we stack the vectors over t as $Y = (y_1', y_2', \dots, y_T')'$, $X^\theta = (X_1^{\theta'}, X_2^{\theta'}, \dots, X_T^{\theta'})'$, $B = (\Phi, \Gamma_1, \dots, \Gamma_p, \Phi^z, \Gamma_1^z, \dots, \Gamma_p^z)'$, and $E = (\varepsilon_1', \varepsilon_2', \dots, \varepsilon_T')'$.

Now we can write model (1) in a more compact form as

$$Y = X^\theta B + E \quad (6)$$

where the dimensions of Y and E are $(T \times 2)$, the dimension of X^θ is $(T \times k)$, and the dimension of B is $2k$, with $k = 2(1 + 2p)$.

Given the assumptions on the error terms, the likelihood function of the model can be expressed as

$$L(B, \Sigma, \gamma, c) \propto |\Sigma|^{-T/2} \exp \left\{ -\frac{1}{2} \text{tr} \Sigma^{-1} E' E \right\} \quad (7)$$

Using standard algebraic results, it is possible to show that

$$E' E = S + (B - \widehat{B})' X^{\theta'} X^\theta (B - \widehat{B})$$

where $\widehat{B} = (X^{\theta'} X^\theta)^{-1} X^{\theta'} Y$, and $S = (Y - X^\theta \widehat{B})' (Y - X^\theta \widehat{B})$. Thus, the likelihood function can then be rewritten as

$$L(B, \Sigma, \gamma, c) \propto |\Sigma|^{-T/2} \exp \left\{ -\frac{1}{2} \text{tr} S \Sigma^{-1} - \frac{1}{2} \text{tr} (B - \widehat{B})' X^{\theta'} X^\theta (B - \widehat{B}) \Sigma^{-1} \right\} \quad (8)$$

Vectorizing model (6), we can transform model (1) into

$$y = x^\theta b + e, \quad (9)$$

where $y = \text{vec}(Y)$, $b = \text{vec}(B)$, $x^\theta = I_n \otimes X^\theta$, and $e = \text{vec}(E)$.

Now, using the relationship between the trace function and the vectorising operation, we can write the term in the exponent of (7) as

$$\text{tr} \Sigma^{-1} E' E = e' (\Sigma^{-1} \otimes I_T) e = s^2 + (b - \widehat{b})' V^{-1} (b - \widehat{b}) \quad (10)$$

where $s^2 = y' M_V y$, $M_V = \Sigma^{-1} \otimes (I_T - X^\theta (X^{\theta'} X^\theta)^{-1} X^{\theta'})$, $\widehat{b} = \text{vec}(\widehat{B})$ and $V = \Sigma \otimes (X^{\theta'} X^\theta)^{-1}$.

Hence, the likelihood function in (7) can also be written as

$$L(b, \Sigma, \gamma, c) \propto |\Sigma|^{-T/2} \exp \left\{ -\frac{1}{2} \left[s^2 + (b - \widehat{b})' V^{-1} (b - \widehat{b}) \right] \right\} \quad (11)$$

which has a more familiar Normal form for vector b .

2.2 Priors

In setting the values for the priors we take into account a number of considerations. It is apparent that the LSTVAR model is highly parameterized and the degree of parameterizations influences the quality of inference in finite samples. Priors that are tight around zero (i.e., very informative) tend to improve estimation in VARs (Ni and Sun, 2003). Also, we use Bayes Factors for inference on models. As discussed in Strachan and van Dijk (2004), the Bayes factors are functions of the prior normalizing constants and so the prior settings can have a strong influence on the posterior model weights. Generally, less informative priors will tend to penalize more highly parameterized models. A final consideration is that we have little understanding of the behaviour of economic growth beyond anecdotal evidence and how it can be reasonably modeled. Thus, we face a potential conflict between our desire to specify uninformative priors for a large number of parameters, and priors that are informative which would improve the efficiency of estimation. Furthermore, we do not want to completely avoid or prefer the use of large models *a priori*. Taking into account these considerations, we elicit the priors as follows.

To start with, we assume all models to be *a priori* equally likely. Next, following Zellner (1971), we specify a standard Jeffreys prior for Σ as

$$p(\Sigma) \propto |\Sigma|^{-(n+1)/2}$$

We plan to compute posterior probabilities for model inference. For these probabilities to be well defined, the priors for any parameters that change dimensions, i.e. b , must be proper (see Bartlett, 1957 and Strachan and Van Dijk, 2004 for further discussion). Hence, we assume the prior for b is Normal with zero mean and covariance matrix $\underline{V} = \eta^{-1}I_{nk}$, where η is a shrinkage prior distributed as Gamma with mean $\underline{\mu}_\eta$, and degrees of freedom $\underline{\nu}_\eta$. Note that the prior variance for b depends on η . Large values of η imply greater shrinkage towards zero which will tend to reduce the expected frequentist risk of the estimator. However, smaller values of η will imply a less informative prior. To allow prior for b that is relatively uninformative, but still allow for a degree of shrinkage, we specify the prior

of η distributed as $G(10, 0.001)$, where 10 is the mean, and 0.001 is the degree of freedom.

As explained in Bauwens, Lubrano and Richard (1999), at the point where $\gamma = 0$, the smooth transition function in (2) becomes a constant and, as a consequence, elements of b become unidentified. While when $\gamma \rightarrow \infty$, under a flat prior for γ , the posterior is not integrable. Hence, following the suggestion of Lubrano (1999a, 1999b), we exclude *a priori* the point $\gamma = 0$ from the support of γ . Specifically, we assume the prior of γ is a Gamma distribution with mean $\underline{\mu}_\gamma$ and degree of freedom $\underline{\nu}_\gamma$. Note that although the prior for γ excludes zero, as the prior for b is centered on zero, this restriction does not bias in favor of asymmetry. We define the prior mean of γ as 1, in line with the starting values of grid search in most of the classical works (see, for example, Öcal and Osborn, 2000 and Sensier, Osborn and Öcal, 2002), while our assumption that the degree of freedom of the prior Gamma distribution is 0.001 is for minimizing the prior's influence on posterior computations.

In the end, we assume the prior of the location parameter c as uniformly distributed between the upper and lower limits of the middle 80% of the observed transition variables.

2.3 Posteriors Computations

We use Gibbs Sampling to compute the outputs from the posteriors. Conditional upon γ , c , and η , the model is linear. Thus the conditional posterior distributions of Σ and b and are of standard forms. Combining the likelihood function (7) and the priors, we obtain the conditional posterior distribution for Σ as an inverted Wishart with scale matrix $E'E$ and degrees of freedom T , and the conditional posterior distribution for the vector b as Normal with mean \bar{b} and variance \bar{V} , where $\bar{V} = (V^{-1} + \eta I_{nk})^{-1}$, and $\bar{b} = \bar{V}V^{-1}\hat{b}$.

To obtain the conditional posterior for η , we combine the prior and the likelihood to obtain the expression

$$p(\eta|b, \Sigma, \gamma, c, y, x) \propto \eta^{\frac{\nu_\eta + nk - 2}{2}} \exp\left(-\frac{\eta \nu_\eta}{2\underline{\mu}_\eta} - \frac{1}{2}b'b\eta\right) \quad (12)$$

Thus with a Gamma prior, the conditional posterior distribution of η is

Gamma with degrees of freedom $\bar{\nu}_\eta = nk + \nu_\eta$, and mean $\bar{\mu}_\eta = \frac{\bar{\nu}_\eta \mu_\eta}{\bar{\nu}_\eta + \mu_\eta b' b}$.

The posterior distributions for the remaining parameters, γ and c , have nonstandard forms. However, we can use Metropolis-Hastings algorithms (Chib and Greenberg, 1995) within Gibbs to estimate γ , and the Griddy Gibbs sampler (Ritter and Tanner, 1992) to estimate c .

The Gibbs sampling scheme for our posterior computation, therefore, takes the following form.

1. Initialize $(b, \Sigma, \gamma, c, \eta) = (b^0, \Sigma^0, \gamma^0, c^0, \eta^0)$;
2. Draw $\Sigma|b, \gamma, c, \eta$ from $IW(E'E, T)$;
3. Draw $b|\Sigma, \gamma, c, \eta$ from $N(\bar{b}, \bar{V})$;
4. Draw $\gamma|b, \Sigma, c, \eta$ through Metropolis-Hastings method;
5. Draw $c|b, \Sigma, \gamma, \eta$ numerically by Griddy Gibbs;
6. Draw $\eta|b, \Sigma, \gamma, c$ from $G(\bar{\mu}_\eta, \bar{\nu}_\eta)$;
7. Repeat step 2 to 6 for a suitable number of replications.

To avoid the draws from Metropolis-Hastings simulator getting stuck in a local mode, we try different starting values for the sampler.

2.4 Posterior Model Probabilities

There has been a great deal of work on the theories of business cycles and even on the asymmetries observed in business cycles. However, there are relatively fewer formal theories on the nonlinear effects in international business cycle linkages. Thus we have little guidance on how to specify the model prior to introducing the data. Further, notwithstanding the few studies that do exist, we do not wish at this stage of the research to impose any restrictions implied by particular theories. Our interest is on the existence of the linkages and the form of the asymmetries. These concerns were important motivations for considering LSTVAR models. However, we also have reason to expect that the real data generating process might be nonlinear, yet we do not wish to exclude the possibility that the model is linear. A linear model may prove more robust if the asymmetric effect is trivial. Thus, we include the standard linear VAR in our model set. Furthermore, we can not confidently pre-specify the driving force of the asymmetric dynamics (if there is any) nor predetermine the duration of the dynamics, so we allow for

a range of specifications of z_t and lag lengths p .

Bayesian methods provide us a formal method for evaluating the support for alternative models by comparing posterior model probabilities. These posterior probabilities can be used to select the best model for further inference, or to use the information in all or an important subset of the models to obtain an average of the economic object of inference by Bayesian Model Averaging. The posterior odds ratio - the ratio of the posterior model probabilities - is proportional to the Bayes factor. Once we know the Bayes factors and prior probabilities, we can compute the posterior model probabilities.

The Bayes Factor for comparing one model to a second model where each model is parameterized by $\zeta = (\zeta_1, \zeta_2)$ and ψ respectively, is

$$B_{12} = \frac{\int \ell(\zeta)p(\zeta)d(\zeta)}{\int \ell(\psi)p(\psi)d(\psi)},$$

where $\ell(\cdot)$ is the likelihood function and $p(\cdot)$ is the prior density of the parameters for each model.

If the second model nests within the first at the point $\zeta_2 = \zeta^*$, then, subject to further conditions, we can compute the Bayes factor B_{12} via the Savage-Dickey density ratio (see, for example, Koop and Potter, 1999a, Koop, Leon-Gonzales and Strachan, 2006 for further discussion in this class of models). For the simple example discussed here, the Savage-Dickey density ratio is:

$$B_{12} = \frac{p(\zeta_2 = \zeta^*|Y)}{p(\zeta_2 = \zeta^*)},$$

where the numerator is the marginal posterior density of ζ_2 for the unrestricted model evaluated at the point $\zeta_2 = \zeta^*$, and the denominator is the prior density of ζ_2 also evaluated at the point $\zeta_2 = \zeta^*$.

Since the conditional posterior of b is normal, it is easy to incorporate the estimation of the numerator of the Savage-Dickey density ratio in the Gibbs sampler. As to the denominator of the Savage-Dickey density ratio, using the properties of the Gamma distribution and the Normal distribution, we derive the marginal prior for a sub-vector of b evaluated at zeros as

$$\left\{ \left(\frac{\mu_\eta}{\pi\nu_\eta} \right)^{\omega/2} \Gamma \left(\frac{\omega + \nu_\eta}{2} \right) \right\} / \left[\Gamma \left(\frac{\nu_\eta}{2} \right) \right]$$

where $\Gamma(\cdot)$ is the Gamma function, and ω is the number of elements in b being restricted to be zero.

A simple restriction in our application to choose is the point where all lag coefficients are zero, i.e., $\Gamma_h = \Gamma_h^z = 0$, at which point we have the model with $p = 0$. This restricted model is useful as it nests within all models. Once we have the Bayes factor for each model to the zero lag model, via simple algebra we can back out the posterior probabilities for all models.

Taking a Bayesian approach we have a number of options for obtaining inference. If a single model has dominant support, we can model the data generating process via this most preferred model. However, if there is considerable model uncertainty then it would make sense to use Bayesian Model Averaging and weight features of interest across different models using posterior model probabilities (as suggested by Leamer, 1978).

3 Empirical Application

The data we use are quarterly observations of real GDP for the UK, the US, France and Germany over the period of 1970:Q1-2004:Q4. All series are taken from Datastream. For all cases, the first quarter of 1970 is set as the base time for index purposes. We construct the annual growth rates by taking the fourth-difference of log real GDP index.³

The growth rates for the four countries are plotted in figure 1. Note that all the series are stationary and free from seasonal components. The average annual growth rates for the sample period are: 2.34% for the UK, 3.08% for the US, 2.49% for France and 2% for Germany. The correlations between the annual growth rate for the UK and that of the US, France and Germany are 0.5941, 0.3606 and 0.3693, respectively. Note that the dynamics of recessions are quite different from those of expansions, a phenomenon which might imply the presence of asymmetry.

For all countries, we assume the maximum order of the unrestricted bivariate LSTVAR is 4. Although the driving force of the asymmetry can be any exogenous or endogenous variables of concern, following the convention, we simply choose a specific lag of the observed growth rate from our selected

³The jump in German data due to the reunification in 1991 has been corrected.

countries as the transition variable. However, instead of picking a plausible lagged growth rate from a particular country, we allow z_t to be any of the 16 observations of the lagged (from 1-4) annual growth rates for the UK, the US, France or Germany. Note that this specification allows for the driving force of the regimes to be generated within or beyond the two countries being examined under the bivariate VAR. As we allow the order of the VAR to vary from one to four, then for each of the three bilateral relationships we consider a total of 68 models.⁴

3.1 Posterior Evidence on Alternative Models

We calculate Bayesian posterior model probabilities from the Bayes Factors comparing the nested models to the unrestricted LSTVAR(4, z_t) models.⁵ The Gibbs Sampler for each of the unrestricted LSTVAR (4, z_t) model is run for 12,000 passes with the first 2,000 discarded. The convergence of the sequence draws is checked by the Convergence Diagnostic measure introduced by Geweke (1992). We use the MATLAB program from LeSage’s Econometrics Toolbox (LeSage, 1999) for the diagnostic.

The posterior probabilities for the top 10 models evaluated at Bayes Factor are reported in table 1. As we calculate posterior model probabilities with relatively uninformative priors, we would expect this to reward parsimony and, as such, penalize the nonlinear models. However, there is little overall evidence for linear models (which, for a given lag length, is the most parsimonious model). This reinforces the evidence in favor of asymmetry in the bilateral business cycle linkages between the UK and each of the other three countries.

Posterior model probabilities reveal that model uncertainty is not a significant issue in this data. For France and the UK, we find the bivariate relationship can be jointly captured by LSTVAR(4, US_{t-2}) and LSTVAR(4, UK_{t-2}), with posterior probabilities 52.34% and 36.99%, respectively. While

⁴The total number of models is calculated as 4 (maximum order of the nonlinear VAR) \times 4 (choices for z_t) \times 4 (lags of z_t) + 4 (the number of linear VAR models) = 68.

⁵Where the order of the model is 4, and the transition variable z^t equals to $US_{t-1}, US_{t-2}, US_{t-3}, US_{t-4}, FR_{t-1}, FR_{t-2}, FR_{t-3}, FR_{t-4}, UK_{t-1}, UK_{t-2}, UK_{t-3}, UK_{t-4}, GER_{t-1}, GER_{t-2}, GER_{t-3}, GER_{t-4}$, respectively.

model comparison results involving the US and Germany show that a single model receives substantial posterior support in each case. For US-UK, LSTVAR(4, UK_{t-4}) accounts for 90.38% of the posterior probability. For Germany-UK, the posterior model probability of LSTVAR(4, GER_{t-3}) is 92.68%.

We observe four interesting findings from our model comparison results. First, the US growth rates play a leading role in triggering the regime changes for France-UK and a non-negligible role in causing the nonlinear effects for Germany-UK. Second, the regime changes are governed by the UK business cycles in the case of US-UK. Third, Germany's economic performance is important for the regime changes in all cases, in particular, it plays a deterministic role in the case of Germany-UK. Finally, we find that the role of France's growth rate in triggering the regime changes is nearly negligible in all cases. Observe that even though for France-UK, the nonlinear effects are mainly determined by the growth rates of the US and the UK.

It is hard to explain the parameters in such big nonlinear models. Yet, we present the estimated UK equations for the three most preferred models in table 2, for the smooth transition functions and the impulse response analysis we are going to report are based on these results.

To better understand the form of the asymmetric affect, we plot the graphs of the time profile of $F(z_t)$ and the corresponding transition functions over the range of z_t for the three most probable models in figures 2-3. For comparison, we also report the time profiles of $F(z_t)$ derived from Bayesian Model Averaging in figure 4. Observe that for US-UK, the dynamics of the regime changes remains to be between the upper and lower regimes, for France-UK, the model is most often in the upper regimes, while for Germany-UK, more abrupt regime changes can be spotted. From these figures, we can see that the regime changes are rather smooth in all the three cases. Thus, it is improper to model the nonlinear effects using functions that only allow for abrupt changes.

3.2 Impulse Response Analysis

The nonlinear LSTVAR allows for asymmetries in the behaviour of the business cycle linkages. Thus the model provides richer inference on the possible response paths that account for both the nature of the shocks and the current economic environment. In analyzing the response of the UK economy to the foreign shocks we are interested in how the economy responds taking into account the magnitude of the shock, whether the shock is positive or negative and whether UK growth is negative or positive at the time of the shock. For example, it would seem natural to expect that the response to a positive growth shock from the US, say, will have a different effect upon UK's growth if the UK is currently growing quickly than if the UK is in a recession.

As discussed in, *inter alia*, Potter (1995), Koop, Pesaran and Potter (1996), Koop and Potter (2000), impulse response functions of nonlinear models are history- and shock- dependent. This contrasts with the traditional impulse response analysis in a linear VAR in which positive and negative shocks are treated symmetrically and independent of the current state of the business cycle. Thus, the traditional methods of computing impulse responses are unable to inform us on nonlinearities in responses (see Koop *et al.*, 1996 for detailed discussions). We therefore follow these earlier papers and use generalized impulse response functions (GIRF)⁶ to measure the effect of a shock on the asymmetric system.

Following Koop *et al.* (1996), we examine the GIRF where we have a shock v_t and a history ω_{t-1} which is defined as follows

$$GI_y(n, v_t, \omega_{t-1}) = E[y_{t+n}|v_t, \omega_{t-1}] - E[y_{t+n}|\omega_{t-1}] \quad (13)$$

where n is the number of periods into the future after the time t .

The definition in (13) is the expected response path where the expectation is taken with respect to the distribution of all future shocks, the distribution of the parameters and, if model averaging is employed, with respect to the posterior distribution of the models. That is, the impulse

⁶The term impulse response functions, if without any specific description, also refers to general impulse response functions hereafter.

response is the expected deviation of y_{t+n} subject to the shock v_t from the expected value of y_{t+n} without fixed future shocks and conditional only upon the history at time t , ω_{t-1} .

Estimation of the GIRF for a specific model with given parameters is detailed in the literature mentioned above. Here, we only outline how we achieve an estimate that is not conditional upon any parameter values.

We wish to calculate the GIRF for a given shock v_t and history ω_{t-1} . Assume we have the i^{th} draw from the Gibbs sampler of the parameters in the model which we will denote by $\theta^{(i)}$. For each draw we compute $GI_y(n, v_t, \omega_{t-1} | \theta^{(i)})$ which is simply (13) for a given value of the parameters. Next assume we have N draws of $\theta^{(i)}$ where $i = 1, \dots, N$. Then we can compute an estimate of (13) from by

$$\widehat{GI}_y(n, v_t, \omega_{t-1}) = \frac{1}{N} \sum_{i=1}^N GI_y(n, v_t, \omega_{t-1} | \theta^{(i)}).$$

By drawing randomly from histories and averaging across these, we are able to obtain an estimate of $GI_y(n, v_t)$ which is not conditional upon the current state of the economy. Furthermore, we report the estimates of $GI_y(n, v_t, \omega_{t-1})$ conditional upon some special ω_{t-1} since we believe these paths may differ for different histories. To be specific, we are interested in whether the path of $GI_y(n, v_t, \omega_{t-1})$ differs when the UK economy exhibits a positive growth in comparison to a negative growth.

Finally, we report the estimated path of $GI_y(n, v_t, \omega_{t-1})$ when the shock v_t is a negative one/two standard deviations shock to the US, France or German economy, as well as when v_t is a positive one/two standard deviations shock to the US, France or German economy. In the estimation of the posterior distributions of these functions, we found that outliers distorted the posterior means of the GIRFs in some cases. Therefore, we report the median of the GIRFs instead of the mean.⁷

Graphs of the median estimates of the GIRFs for the most preferred model and the BMA results, respectively, are plotted in figures 5-10. In each figure, we use six graphs to examine general impulses from different dimensions. In the upper panel of the figure, we display the impact on the

⁷The mean of the GIRFs with the outliers being dropped share the similar pattern with the median results. Graphs depicting mean values of GIRF are available upon request.

UK growth of positive and negative shocks from the other country but where we have averaged across all the UK histories. The middle panel of the figure shows the same response of UK growth but the path is conditional upon the UK's economy being in expansion at the time of the shock. The lower panel of the figure presents the corresponding effects when UK's economy is in contraction at the time of the shock.

An inspection of all the graphs reveals that the GIRFs plots for the most preferred model and that of the BMA results appear to be similar for all the three country pairs, which is in consistent with the model comparison result earlier reported.

Observing the GIRFs for US-UK plotted in figures 5-6, we see that the impact of a US shock on the UK is in all cases prominent for the first seven to eight quarters, after which there remain much smaller cyclical effects. Finally, the impulse responses die out in about nine years. It is seen that the cumulative effect of a positive US shock will increase the UK's output growth rate, while the cumulative effect of a negative shock from the US will decrease the UK's output growth rate.

With respects to France-UK, from figures 7-8, we can see that while there are strong immediate positive and negative responses to shocks of the same sign, the cyclical effect is much less pronounced than in the case of US-UK. Observe that much of the impact takes place in the first six quarters after the shock. Afterwards, only some smaller cyclical effect remains for another nine quarters. Overall, the impact from France dies out in five years. Similar to that of US-UK, we find a positive shock emanating from France would boost the UK economy, and a negative shock from France would offset the UK's growth.

By visual inspection, we can hardly find any nonlinearities in the GIRFs for US-UK and France-UK. First, the graphs for positive shocks appear to mirror the graphs for negative shocks. Second, the impacts of shocks of differing magnitude seem to have proportionate effects. Third, it looks like that the dynamics of the impulse responses is independent of the status of the UK's economy when the shocks hit.

Noticeable nonlinearities in impulse response functions are observed in the case of Germany-UK. Observing figures 9-10, we find the paths of the

responses will not just differ given the sign and the magnitude of the shock, but also given the current state of the UK economy. Surprisingly, we find that the cumulative effect of any type of innovations in Germany is to slow down the UK economy. More strikingly, we find a positive shock from Germany brings more negative effect to the UK output growth than a negative shock. For a given status of the UK economy when the shock from Germany happens, we can order the shocks by gravity for negatively affecting the UK growth rate. We find, in descending order of severity, that it is the large positive shock, the small positive shock, the large negative shock and the small negative shock. Finally, we observe that when the UK economy is in recession when the shock happens, the overall setting back effect from Germany is less than when the UK's economy is in expansion.

4 Conclusions

In this paper, we investigate bivariate relationships between the UK and three main industrial countries - the US, France, and Germany - within the framework of a LSTVAR model. We employ Bayesian methods to develop an approach to model estimation and evaluation.

The estimation results show that the UK's business cycles are asymmetrically influenced by the other three countries. Overall, it would seem that the UK benefits from positive shocks emanating from the US and France, while suffers from negative shocks from these two countries. However, we also observe that Germany always play a pernicious role in the UK's economy. More strikingly, we find that a boom in Germany would bring more negative impact on the UK than a bust.

As a purely atheoretical study, this paper only describes the behaviour of the linkages between the UK and each of the other three countries. For a better understanding of the forms and sources of these linkages, further investigations (for examples, on the transmission channels) which are beyond our current research are called for.

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Table 1: Top 10 models

No.	US - UK	FR - UK	GER - UK
1	$LSTVAR(4)_{UK,t-4}$	$LSTVAR(4)_{US,t-2}$	$LSTVAR(4)_{GER,t-3}$
2	$LSTVAR(4)_{GER,t-3}$	$LSTVAR(4)_{UK,t-2}$	$LSTVAR(4)_{US,t-1}$
3	$LSTVAR(4)_{UK,t-3}$	$LSTVAR(4)_{FR,t-2}$	$LSTVAR(4)_{GER,t-1}$
4	$LSTVAR(4)_{GER,t-4}$	$LSTVAR(4)_{US,t-1}$	$LSTVAR(4)_{GER,t-2}$
5	$LSTVAR(4)_{US,t-1}$	$LSTVAR(4)_{FR,t-3}$	$LSTVAR(4)_{US,t-2}$
6	$LSTVAR(4)_{UK,t-1}$	$LSTVAR(4)_{FR,t-1}$	$LSTVAR(4)_{UK,t-1}$
7	$LSTVAR(4)_{US,t-4}$	$LSTVAR(4)_{GER,t-4}$	$LSTVAR(4)_{UK,t-4}$
8	$LSTVAR(4)_{UK,t-2}$	$LSTVAR(4)_{UK,t-3}$	$LSTVAR(4)_{US,t-4}$
9	$LSTVAR(4)_{GER,t-23}$	$LSTVAR(4)_{FR,t-4}$	$LSTVAR(4)_{FR,t-1}$
10	$LSTVAR(4)_{US,t-2}$	$LSTVAR(4)_{US,t-3}$	$LSTVAR(4)_{UK,t-3}$

Notes:

Numbers in parentheses are the orders of VAR. The subscript denotes the transition variable.

Table 2: Estimated parameters for the most preferred models

	US-UK	FR-UK	GER-UK
η	8.9416 (3.4356)	17.7540 (6.0606)	15.6150 (5.1681)
c	0.0353 (0.0103)	0.0097 (0.0056)	0.0378 (0.0050)
γ	0.9886 (0.3812)	2.7632 (1.4184)	6.5763 (3.7615)
lower regime			
Φ	0.0058 (0.0244)	0.0183 (0.0080)	0.0138 (0.0035)
$\Gamma_{1,1}$	1.0682 (0.1833)	0.5870 (0.2198)	0.8401 (0.1487)
$\Gamma_{2,1}$	0.7063 (0.1913)	0.5491 (0.1743)	0.5352 (0.1419)
$\Gamma_{1,2}$	0.0867 (0.2200)	0.0757 (0.2246)	0.1233 (0.1686)
$\Gamma_{2,2}$	0.0250 (0.1954)	-0.0275 (0.1684)	0.0343 (0.1551)
$\Gamma_{1,3}$	-0.2249 (0.2223)	-0.4639 (0.2379)	-0.1322 (0.3041)
$\Gamma_{2,3}$	0.3403 (0.1975)	0.3824 (0.1697)	0.3026 (0.1480)
$\Gamma_{1,4}$	-0.1364 (0.1948)	-0.3274 (0.2209)	-0.3537 (0.1477)
$\Gamma_{2,4}$	0.0748 (0.3669)	-0.1476 (0.1539)	-0.0572 (0.1305)
upper regime			
Φ^1	0.0280 (0.0724)	-0.0010 (0.0105)	0.0499 (0.0404)
$\Gamma_{1,1}^1$	0.1948 (0.3747)	0.4036 (0.2478)	-0.2431 (0.2850)
$\Gamma_{2,1}^1$	-0.1739 (0.3531)	0.2292 (0.2082)	0.0808 (0.2565)
$\Gamma_{1,2}^1$	0.0603 (0.4005)	0.1724 (0.2599)	0.0359 (0.2918)
$\Gamma_{2,2}^1$	-0.3365 (0.3568)	-0.0735 (0.2237)	0.2225 (0.2631)
$\Gamma_{1,3}^1$	-0.0867 (0.4138)	0.0456 (0.2570)	-0.2639 (0.3687)
$\Gamma_{2,3}^1$	0.0886 (0.3877)	-0.2466 (0.2300)	-0.0758 (0.2437)
$\Gamma_{1,4}^1$	-0.1776 (0.4054)	0.3180 (0.2502)	-0.5306 (0.2856)
$\Gamma_{2,4}^1$	-0.9271 (0.4002)	-0.0724 (0.2038)	-0.6705 (0.2598)

Notes:

*Standard errors are in parenthesis.

** The first subscript indicates the country, where 1 denotes the country other than the UK, 2 denotes UK. The second subscript denotes the lag length of the variable.

***The superscript 1 indicates the parameter is of the upper regime.

Figure 1
Annual Growth Rates

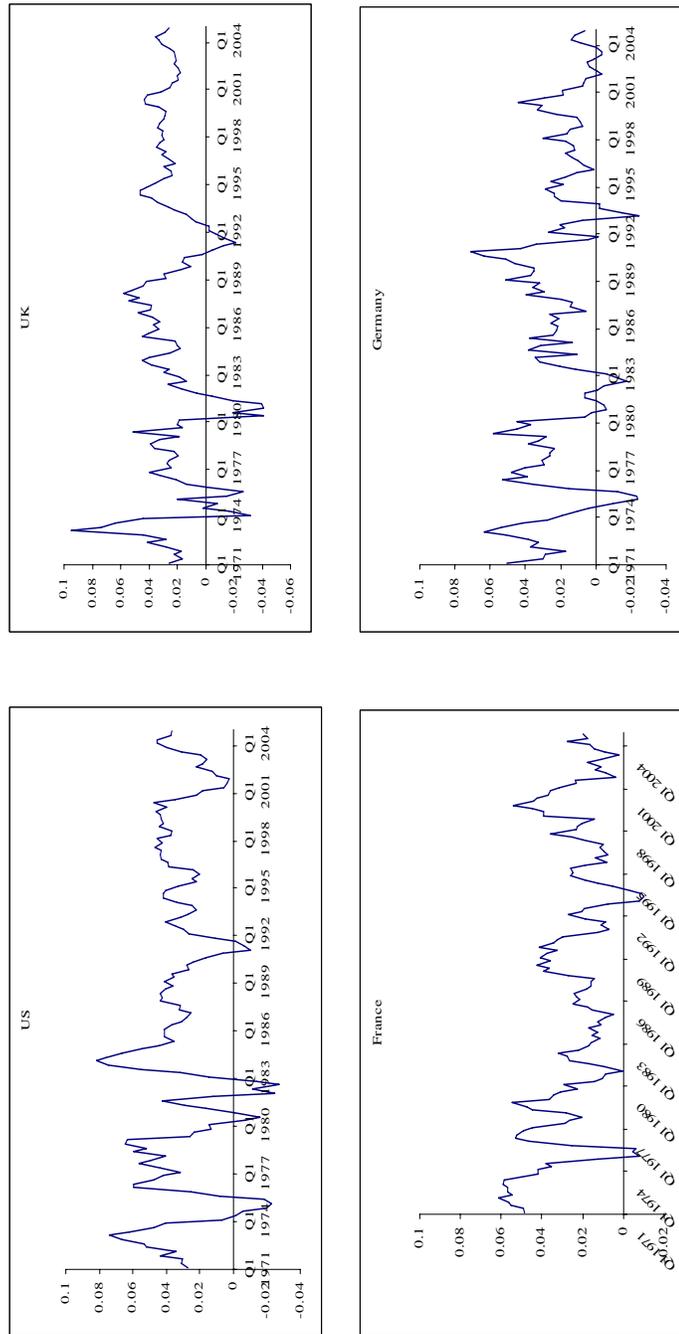


Figure 2
Time Profiles of Smooth Transition Functions — Most Preferred Models

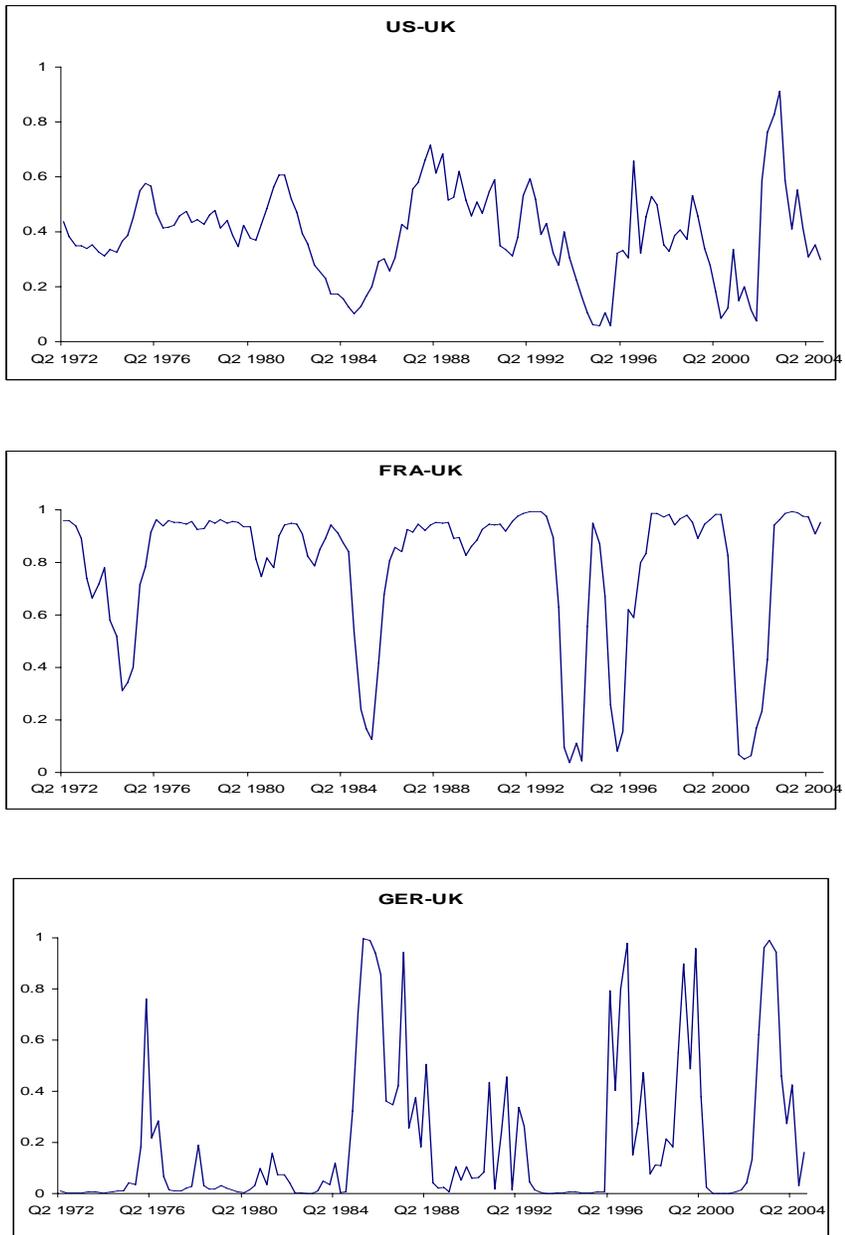


Figure 3
Smooth Transition Functions

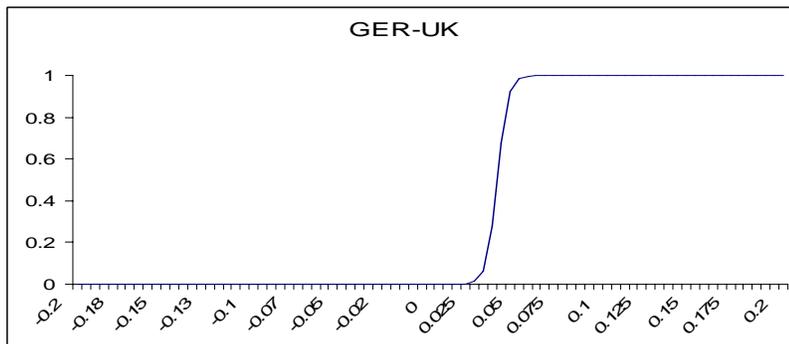
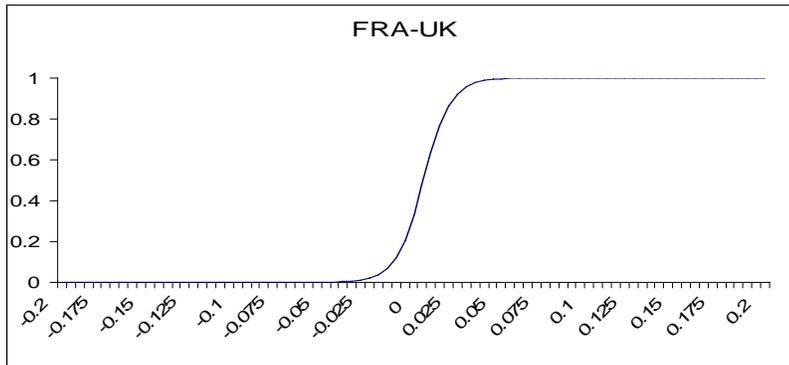
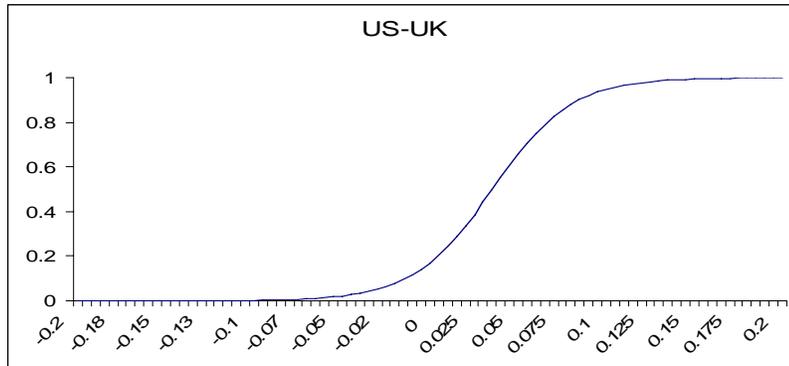


Figure 4
Time Profiles of Smooth Transition Functions __ BMA results

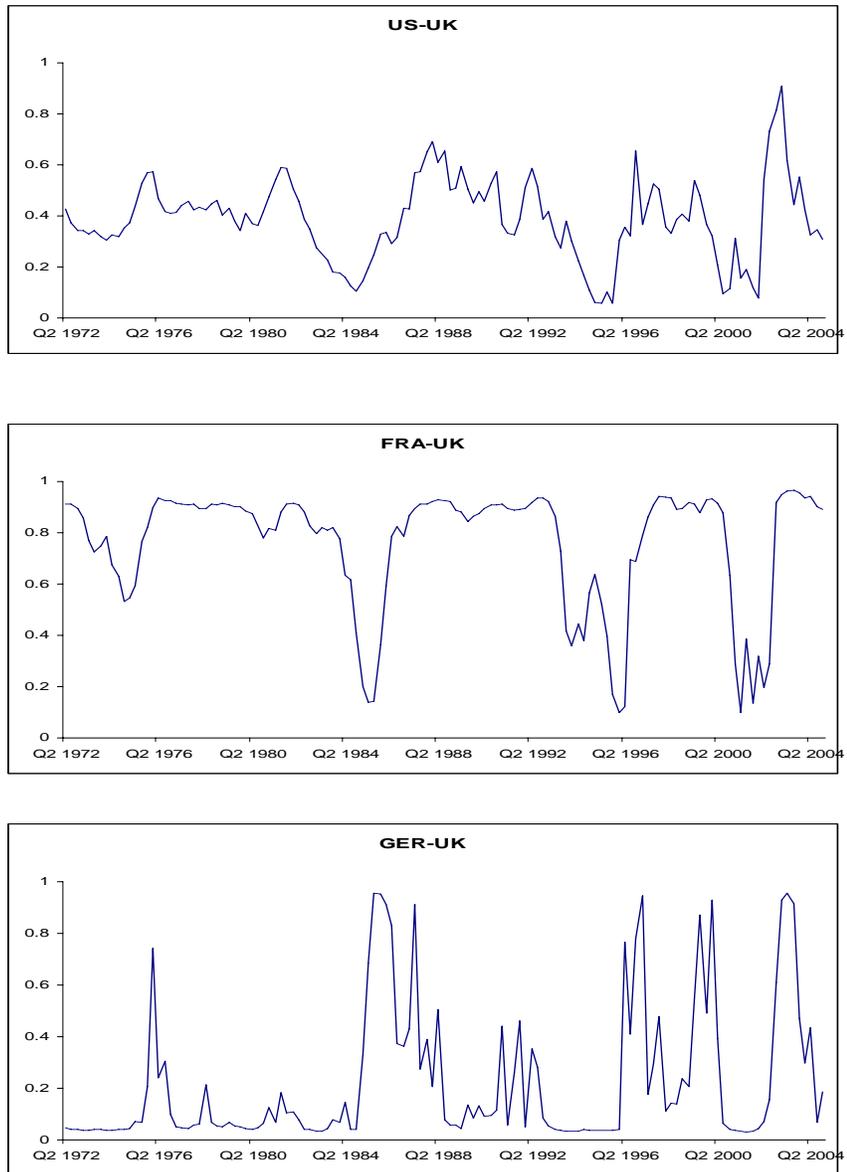
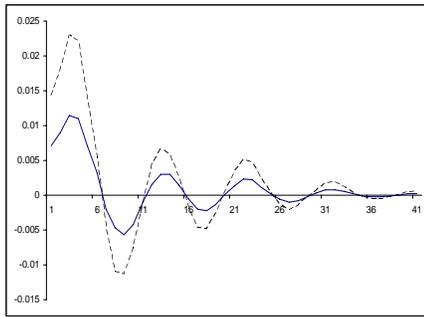
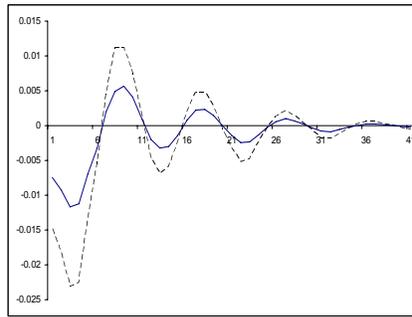


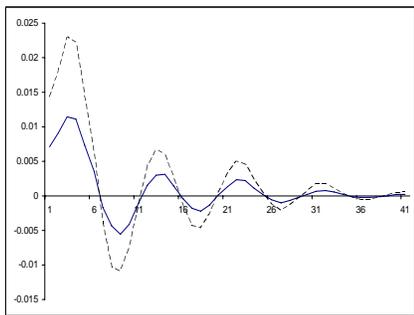
Figure 5
 General Impulse Response Functions ___ Most Preferred Models



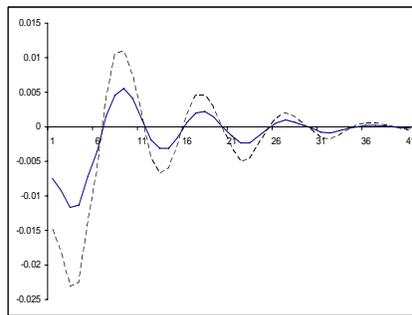
Impacts of US' Positive Shocks



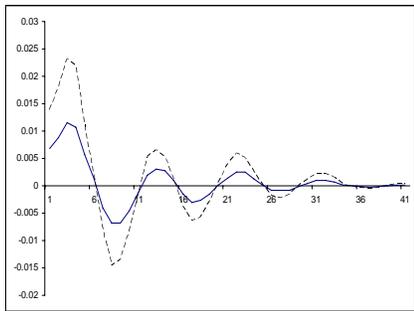
Impacts of US' Negative Shocks



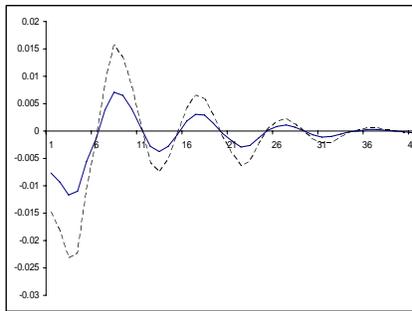
Impacts of US' Positive Shocks When UK is in Expansion



Impacts of US' Negative Shocks When UK is in Expansion



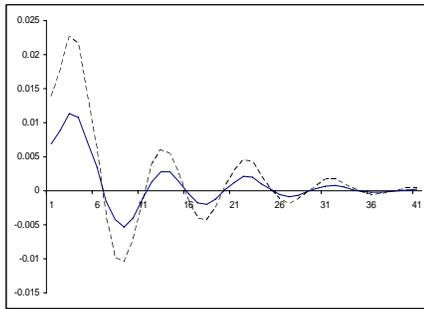
Impacts of US' Positive Shocks When UK is in Recession



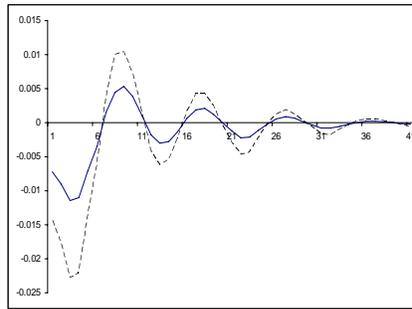
Impacts of US' Negative Shocks When UK is in Recession

Notes:
 Solid line is for the impulse response function when the shock equal to the standard deviation of the US growth rates. Dashed line is for the impulse response function when the shock equal to two times the standard deviation of the US growth rates.

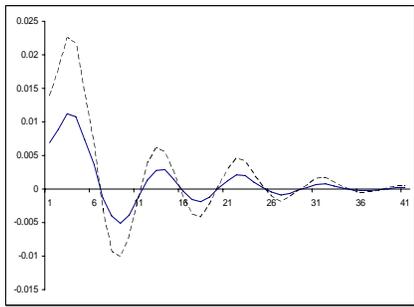
Figure 6
 General Impulse Response Functions ___ BMA



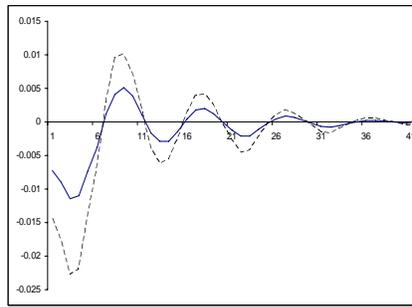
Impacts of US' Positive Shocks



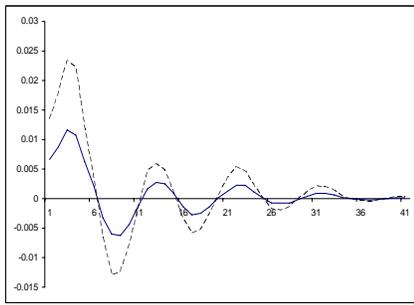
Impacts of US' Negative Shocks



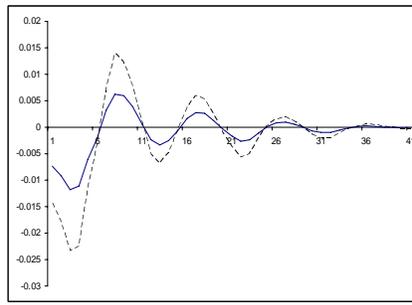
Impacts of US' Positive Shocks When UK is in Expansion



Impacts of US' Negative Shocks When UK is in Expansion



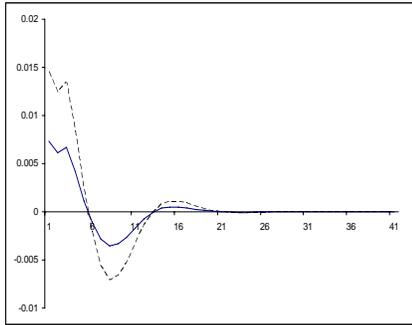
Impacts of US' Positive Shocks When UK is in Recession



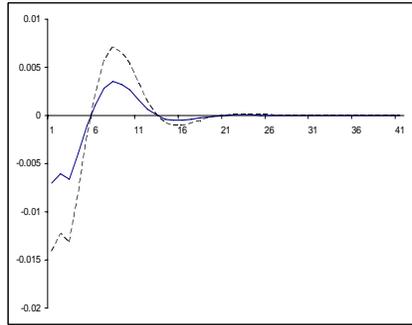
Impacts of US' Negative Shocks When UK is in Recession

Notes:
 See notes in figure 5.

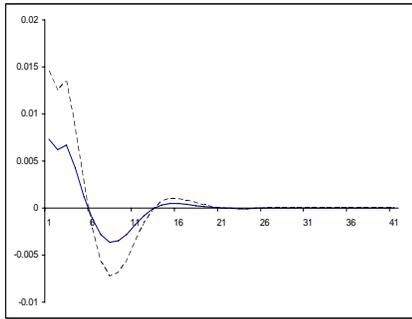
Figure 7
 General Impulse Response Functions ___ Most Preferred Model



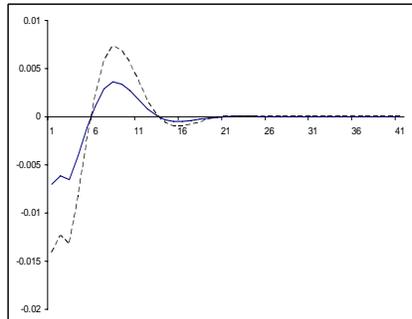
Impacts of France's Positive Shocks



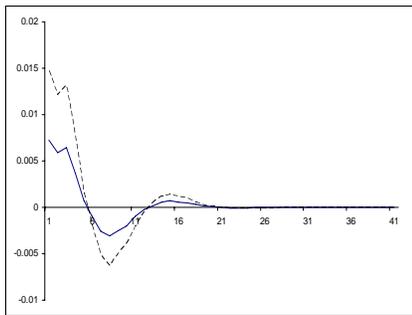
Impacts of France's Negative Shocks



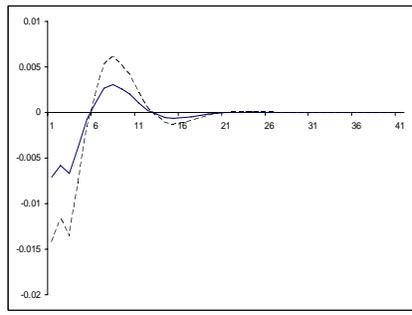
Impacts of France's Positive Shocks When UK is in Expansion



Impacts of France's Negative Shocks When UK is in Expansion



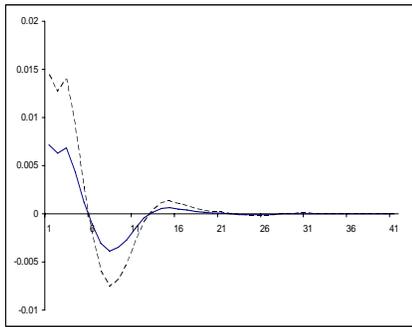
Impacts of France's Positive Shocks When UK is in Recession



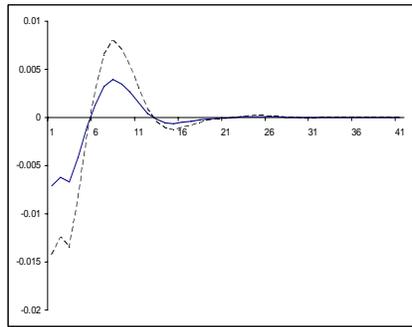
Impacts of France's Negative Shocks When UK is in Recession

Notes:
 Solid line is for the impulse response function when the shock equal to the standard deviation of France's growth rates. Dashed line is for the impulse response function when the shock equal to two times the standard deviation of France's growth rates.

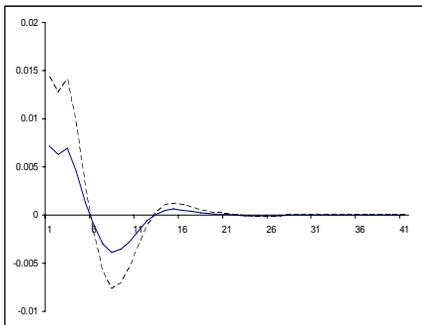
Figure 8
 General Impulse Response Functions ___ BMA



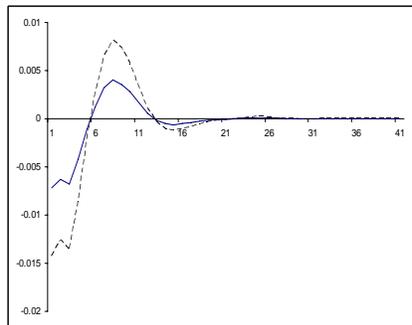
Impacts of France's Positive Shocks



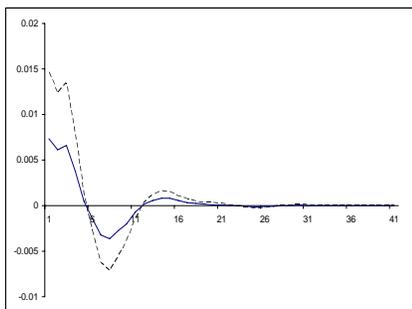
Impacts of France's Negative Shocks



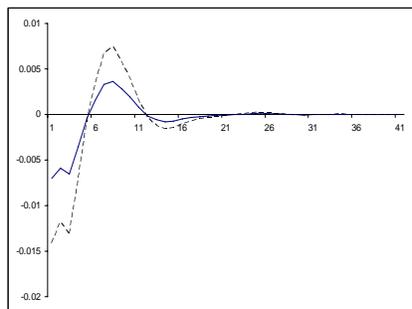
Impacts of France's Positive Shocks When UK is in Expansion



Impacts of France's Negative Shocks When UK is in Expansion



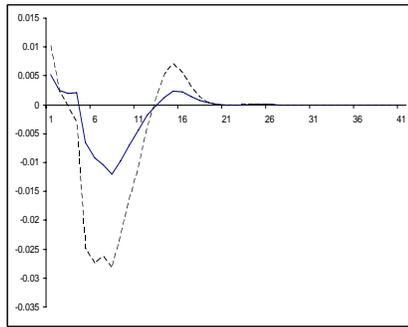
Impacts of France's Positive Shocks When UK is in Recession



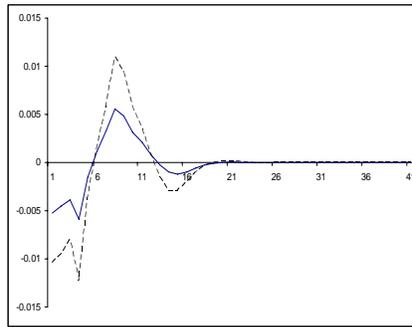
Impacts of France's Negative Shocks When UK is in Recession

Notes:
 See notes in figure 7.

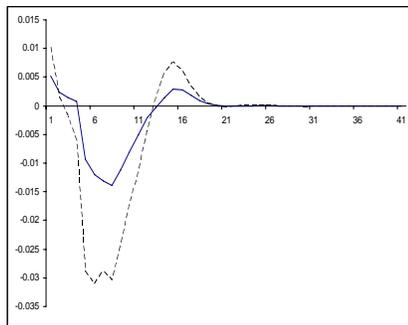
Figure 9
 General Impulse Response Functions ___ Most Preferred Model



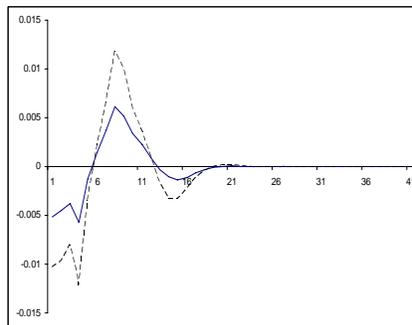
Impacts of Germany's Positive Shocks



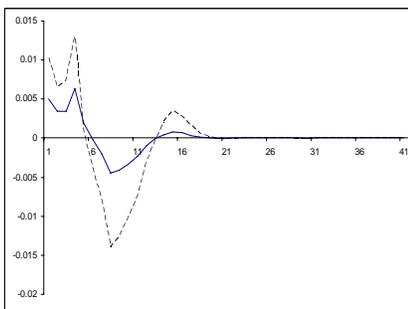
Impacts of Germany's Negative Shocks



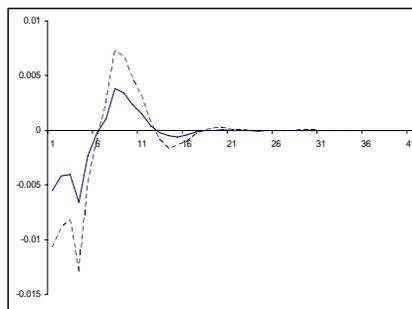
Impacts of Germany's Positive Shocks When UK is in Expansion



Impacts of Germany's Negative Shocks When UK is in Expansion



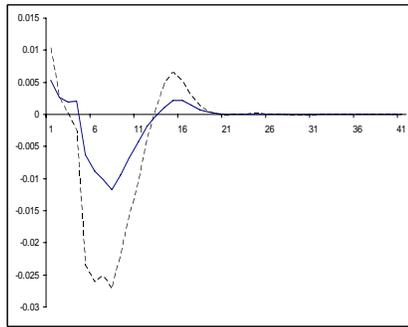
Impacts of Germany's Positive Shocks When UK is in Recession



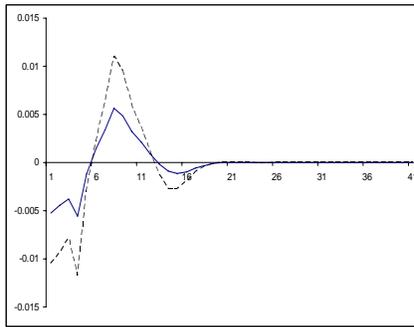
Impacts of Germany's Negative Shocks When UK is in Recession

Notes:
 Solid line is for the impulse response function when the shock equal to the standard deviation of Germany's growth rates.
 Dashed line is for the impulse response function when the shock equal to two times the standard deviation of Germany's growth rates.

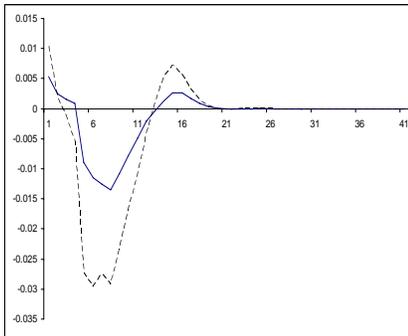
Figure 10
 General Impulse Response Functions ___BMA



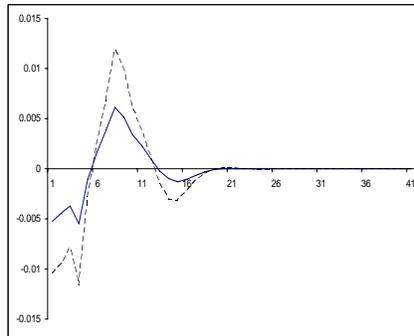
Impacts of Germany's Positive Shocks



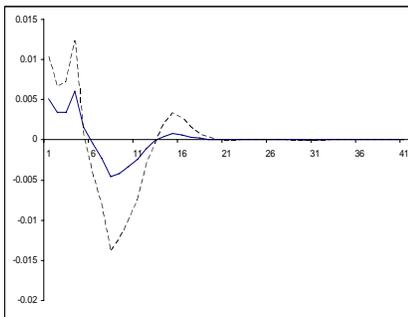
Impacts of Germany's Negative Shocks



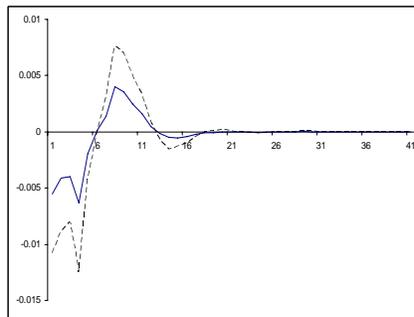
Impacts of Germany's Positive Shocks When UK is in Expansion



Impacts of Germany's Negative Shocks When UK is in Expansion



Impacts of Germany's Positive Shocks When UK is in Recession



Impacts of Germany's Negative Shocks When UK is in Recession

Notes:
 See notes in figure 9.