University of Leicester

DEPARTMENT OF ECONOMICS

Contagion and risk-sharing on the inter-bank market

Dan Ladley, University of Leicester, UK

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Daniel Ladley*

Abstract

Increasing numbers of inter-bank lending relationships have an ambiguous effect on financial stability. Studies have shown that fewer inter-bank loans limit the spread of bankruptcies whilst other work has argued that greater connectivity aids risk sharing. In this paper we identify the conditions under which each relationship holds. Using numerical techniques we demonstrate that in response to a large economy-wide shock, higher numbers of inter-bank lending relationships worsen the impact of the event, however, for smaller shocks the opposite relationship is observed. As such there is no optimal inter-bank market structure which reduces contagion under all economic conditions.

Keywords: Systemic risk, Inter-bank lending, Regulation, Network

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*Department of Economics, University of Leicester, United Kingdom. d.ladley@leicester.ac.uk +44116 2522880

1 Introduction

The financial regulation of banks has primarily focused on ensuring that individual institutions have sufficient funds to protect themselves from the risk of their own investments. The events of 2007 and 2008 demonstrated the shortcomings of this approach. Problems in a small number of institutions spread throughout the financial system resulting in the collapse of financial institutions which, according to regulatory requirements, had adequate capital. Systems which had previously been thought to encourage stability and permit risk sharing, such as the inter-bank market, became a route by which financial distress spread. Banks defaulted on inter-bank loans' negatively impacting the balance sheets of their creditors and forcing otherwise sound institutions towards insolvency and collapse. Fragility became contagious as financial distress spread and through inter-bank loans and fire sales of assets. Institutions were not able to predict who would fail next and consequently market confidence evaporated. This created a liquidity crisis, preventing viable institutions from obtaining funds and so exacerbating the system's problems. Regulators and governments were forced to intervene to save the system, injecting capital and rescuing institutions which were judged too-big-to-fail, those who's bankruptcy could have led to further damaging cascades of failures. The financial crisis showed that it was not sufficient to regulate banks in isolation, to protect them against themselves, banks also had to be protected against each other and other financial institutions as the integrity of the system was paramount.

Inter-bank linkages were supposed to provide insurance and stability by allowing banks to access liquidity, however, instead they served to exacerbate the financial crisis by allowing problems to spread between institutions. In this paper we examine how the structure of the inter-bank lending market effects the stability of the financial system[1]. We consider a model of the behavior of heterogenous banks within a closed economy. Households approach banks, placing deposits and borrowing money for risky projects. Banks interact

[1]These are not the only inter-bank linkages which can propagate distress. For instance Allen and Carletti (2006) demonstrate how the transfer of credit risk between institutions may lead to contagion whilst Mendoza and Quadrini (2010) show that in a global financial systems a small shock to bank equity may result in a large reduction in asset prices.

with each other through an inter-bank market, obtaining funds but exposing themselves and other banks to counter-party risk and potentially contagion. The effect of the structure of the inter-bank market is considered in determining the conditions under which the risk sharing or contagion inducing effects are dominant.

The model analyzed in this paper captures the dynamic nature of the financial system. Funds' are lent from banks to households who invest them in projects which in turn leads to them being redeposited into banks, hence within this iterated model money circulates and is multiplied. Banks are presented with a variety of investment opportunities split between loans to households and loans to other banks. These investments are funded through household deposits and potentially borrowing from other banks within the system. Each bank's success is dictated by the performance of their investment portfolio. If a bank invests poorly or is unfortunate it may potentially go bankrupt, if it performs well it will grow. Heterogenous bank sizes arise endogenously within the model.

It is found that the structure of the inter-bank market has a significant effect on the ability of the system to resist contagion in response to system-wide shocks. The optimal structure, however, is dependent on the magnitude of the shock faced. Markets exhibiting a high degree of connectivity share the effects of bankruptcy between more counter-parties reducing the probability of a contagious failure. In contrast, for larger systemic shocks, rather than spreading risk inter-bank connections act to propagate the effects of failures: markets with more inter-bank connections become the most vulnerable. Regardless of the size of shock the cost to a government acting as a deposit insurer is minimized for the most connected markets as more of the cost of failures is borne by surviving banks. The effect of higher equity and reserve ratio's are investigated. Both are found to decrease the market's susceptibility to contagion by reducing the number of banks who cause a second bank to fail. Increasing the equity ratio is found to have a larger effect but at the cost of reducing the ability of banks to offer credit to households. An alternative regulatory mechanism, constraining the size of inter-bank linkages, is also examined. This is found to reduce the number of bankruptcies whilst increasing the quantity of loans given to households. Care must be taken in its use, if it is too loose the regulation has no effect, whilst if it is too tight it severely inhibits the ability of the inter-bank market to distribute funds efficiently and so reduces the loans to households.

The model is shown to be robust to perturbations in parameters, producing qualitatively similar results for a wide range of values. If the constraint of a single inter-bank rate is relaxed, such that larger banks are regarded as more credit worthy and are able to borrow more cheaply, the market is found to be more stable. There is more lending to households and fewer contagious bankruptcies. In contrast, if banks condition their beliefs about being repaid on the inter-bank market on the number of recent bankruptcies the model economy is found to be less stable. Reducing the efficiency of the allocation of funds.

The paper is structured as follows: the next section will give an overview of the related literature on inter-bank markets. Section 3 will set out a model of a financial system in which banks are potentially susceptible to systemic risk. Section 4 will consider the behavior of the model including the potential for contagion under different shocks and a range of market structures. Section 5 examines the effect of regulation whilst Section 6 relaxes modeling assumptions. Section 7 concludes.

2 Literature review

The inter-bank lending market allows financial institutions to lend funds or borrow money to meet liquidity or investment requirements. As such it plays an important role in allowing financial institutions to manage their balance sheets, facilitating the sharing of risk and the efficient allocation of funds. Whilst the inter-bank market provides a mechanism for sharing liquidity risk, participating in the market exposes banks to counter-party risk; The danger is that a bank is unable to recover lent funds due to the failure of a borrower to repay. In their influential work, Allen and Gale (2001) model inter-bank interactions, showing that in equilibrium banks will optimally insure themselves against liquidity shocks by holding deposits in other banks. This protection, however, makes them potentially vulnerable to the failures of their counter-parties. If a very large shock strikes a single bank, which exceeds its available funds, the bank may collapse eliminating a portion of the counter-parties' deposits. If the impact of this bankruptcy is sufficiently large it may potentially cause the default of further, otherwise healthy, banks which may in turn affect others. The effect of these contagious events may be very severe (Gai and Kapadia, 2010), resulting in a loss of equity (Eisenberg and Noe, 2001) and may potentially justify government or regulatory intervention (Kahn and Santos, 2010)[2].

The majority of trading in the inter-bank-market happens over-the-counter (OTC), directly between pairs of banks, as opposed to from a central counter-party. Unlike trades for equities which result in the instantaneous transfer of ownership, interactions within the inter-bank market generally last for an extended period. Funds are initially borrowed by one bank and repaid over a length of time which can range from overnight for certain classes of borrowing, up to periods of several years. At any point a particular bank may be involved in multiple lending or borrowing relationships and as such may be connected to multiple counter-parties. Across all banks these linkages form a structure which may be described by a weighted, directed graph in which nodes are financial institutions and edges are lending relationships of a specific value. Iori et al. (2008) use graph theoretic measures to analyze the structure of the Italian inter-bank market. They show that the structure of the market is characterized by the existence of large 'hub' banks with which many of the market participants interact. The market is also found to be relatively efficient, there being few opportunities to borrow from one institution and lend to another profitably. The structure is shown to vary over time. Towards the end of the month the density of connections increases as banks increase their borrowing and lending to meet their monthly capital requirements. Using similar techniques, Cocco et al. (2009) show that banks tend to form relationships with other institutions that have negative correlated liquidity shocks.

The structure of inter-bank markets, the numbers and distribution of linkages together with their size, has a large effect on how shocks spread and the markets potential susceptibility to systemic events (Haldane and May, 2011). Initially, if a single institution fails only those banks to which it owes money suffer directly, the remainder of the system is unaffected[3]. The direct impact may cause one or more of the initial counter-parties to

[2] Also see Giesecke and Weber (2006), Elsinger et al. (2006) and Brusco and Castiglionesi (2007) for alternative views.

[3] For the present we ignore issues regarding market confidence and beliefs. In reality, a bank that is not directly effected may still fear for their investments and alter their portfolio to limit the possibility

fail which can harm other institutions within the system. Muller (2006) and Upper and Worms (2004), by analyzing data for the Swiss and German banking systems respectively, show that in both cases there is significant potential for this to occur. Highly centralized markets, those with a few large hub banks, are shown to be particularly susceptible to this risk. For instance the UK inter-bank market, which exhibits tiering (Becher et al., 2008), may fall into this category.

Angelini et al. (1996), Boss et al. (2004) and Furfine (2003) draw a different conclusion. They find that there is relatively little danger of systemic events. Only a very small number of banks could cause other banks to fail if they themselves defaulted. This difference in conclusions is, at least in part, driven by differences in the inter-bank markets and the time span of lending. Each of the empirical studies provides a snapshot of a particular market at a particular time under particular financial conditions and is not a general assessment of the susceptibility of inter-bank markets to contagion. The markets studied have different structures, for instance as Angelini et al. (1996) note, the volume traded varies to a large extent across countries. In order to make a complete assessment it would be necessary to perform a large number of similar studies on a range of markets and situations. Unfortunately, the information to conduct such empirical investigations is often into available. For each of the empirical investigations it was necessary to know (or estimate) both the financial position of each market participant and crucially each participant's lending relationships. Whilst the financial positions may be estimated from public balance sheet data, information regarding financial relationships is often proprietary and consequently much less available. In most cases inter-bank lending transactions are conducted directly between institutions, frequently by phone call rather than through an automated exchange[4]. So in contrast to many equities markets where a central body collects trading data, no single body has a complete picture of all transactions. This means that empirical studies are restricted to a relatively small number of countries and occasions where this data is available.

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of future losses. See Lagunoff and Schreft (2001) for an example of this mechanism.

[4]The Italian inter-bank system being a notable exception in that quoted interest rates and transactions go through a central computer system.

Theoretical studies have complemented empirical work in understanding the determinants of systemic risk. Work in this area has shown that there is a relationship between market structure and the effect and scope of financial contagion (e.g. Leitner, 2005), however, the nature of this relationship is ambiguous. Vivier-Lirimont (2006), in a model based on the Diamond and Dybvig (1983) paradigm, find that long chains of loan connections between banks, higher reserve levels and higher liquidation values reduce the severity of contagious events. Increasing the number of inter-bank connections increases severity. This result is partially supported by Brusco and Castiglionesi (2007) who show that increasing cross-holdings increase the extent of contagion but reduces the effect on individual institutions. It differs, however, from Giesecke and Weber (2006) who find that more connections reduce contagion. Boss et al. (2004) demonstrate that the betweenness of a bank, a graph theoretic measure of how central a bank is in a network, is correlated with the contagious effect of its default. Using simulation techniques Nier et al. (2007) show that a small increase in connectivity increases systemic risk but beyond a certain point the degree of systemic risk decreases. In contrast, Lorenz and Battiston (2008) and Battiston et al. (2009) find the opposite relationship, the scale of bankruptcies is minimized for intermediate levels of connectivity. The results above highlight the trade off discussed by Allen and Gale (2001) of risk sharing versus contagious vulnerability. Whilst sparser networks limit the ability of shocks to spread, reducing contagion, they also reduce the risk sharing capacity of the market and so increase the risk of individual banks failing. This finding is highlighted by Iori et al. (2006) who show that in the presence of heterogenous banks the inter-bank market permits a crisis in one bank to spread, however, it also provides stabilization meaning the overall effect is ambiguous.

This ambiguity makes it difficult to design regulations to limit systemic events within the inter-bank market. The Basel III reforms emphasize increasing regulatory capital to provide banks with a larger buffer (and additionally less leverage) in the event of future failures. Rochet and Tirole (1996) highlight the benefits of monitoring to reduce the probability of contagious events whilst Freixas et al. (2000) considers the costs of failures and interventions. The model presented in this paper will consider the susceptibility of different inter-bank market architectures to small and system-wide shocks. It will be used to show how the susceptibility to contagious events varies with market structure along with the effectiveness of different regulatory approaches in limiting the size of failures.

3 Model

We consider a model of a closed economy containing N banks and M households. Each bank, i, has a balance sheet comprising equity (E_i), deposits (D_i), cash reserves (R_i), loans to the non-bank sector (L_i) and loans to the other banks (I_i)[5]. Whilst each household, j, holds depositable funds (d_j), the quantity of which is determined exogenously. Both households and banks occupy locations on the circumference of a unit circle. This circle represents a dimension, not necessarily physical, on which the households and banks differ. Banks are equidistantly spaced with bank 1 being located at the top of the circle and the remaining banks arrayed in index order clockwise around the circumference. The same arrangement is followed by households with household 1 being at the top of the circle. The distance between a particular household and bank affects the banks ability to attract the household as a potential borrower and depositor.

The model operates in discrete time and repeats for an infinite number of time steps. At time zero each bank possesses a single unit of equity and cash reserves. The actions and investments of each bank in each time step effect their financial position in future periods. Successful banks gain more equity and are able to make more investments, potentially allowing further growth. The model is analytically intractable and so is solved numerically by iterating until a steady state is achieved, both in the behavior of banks and their financial positions. We term this fixed steady state the models equilibrium. Once this has occurred the equilibrium is analyzed. The following sub-sections describe the behavior of the banks and households during each period.

[5]Positive values correspond to lending, negative to borrowing.

3.1 Deposits and Lending

At the start of each time period each bank publicly declares its deposit interest rate, $r_i^{deposit}$, and lending interest rate, r_i^{loan}. The description below will show how these values effect household behavior. Banks compete with each other for business, attracting deposit and loan opportunities, through the values of these two interest rates. Each household places all of its depositable funds in the bank which maximizes its expected return, specifically:

$$\arg\max_{i \in N} d_j(r_i^{deposit} - g(i,j)) \tag{1}$$

Where $g(i,j)$ is the distance between i and j[6]. If no i exists such that Equation 1 is positive the household retains its funds and earns no interest. Banks do not refuse any household deposits. All deposits are insured by an agent outside of the system who guarantees that households will be repaid the full value of their deposits in the event of bank failure. Households are, therefore, not concerned with the risk of bank default and so select the bank offering the highest return. We model households as being highly active in their management of deposits, however, in reality deposits tend to be sticky. Individuals are slow to respond to changes in interest rates, frequently maintaining their deposits in institutions paying suboptimal rates, rather than switching[7].

After allocating deposits, each household is presented with a single limited liability investment opportunity, l_j^t. Each opportunity requires an initial investment of a single unit of currency at time t and provides a payoff to the household at time $t+2$ of μ with probability $\theta_{l_j^t}$. With probability $1-\theta_{l_j^t}$ the investment provides zero payoff[8]. Values of μ are fixed across loans whilst $\theta_{l_j^t}$ is drawn from a distribution specified at the start of Section 4. A household with an investment opportunity must fund the investment through bank borrowing. We assume that households wish to retain their deposits for

[6]In line with the majority of the previous literature employing circular city hotelling mechanisms (e.g. Salop, 1979) we assume that transaction costs are linear in the distance between two actors. Alternative functions were tested and had little qualitative effect on the results.

[7]Experiments were performed in which deposits were sticky - depositors only moved their deposits with a fixed probability. Values of this probability greater than 0.02 produced no significant difference in results.

[8]Details of why two period investments are used are provided in the next section.

consumption but will invest in the limited liability opportunity to increase their utility[9]. Each household chooses a single bank to approach. The bank chosen is the one which maximize the household's expected return:

$$\arg\max_{i \in N} \theta_{l_j^t}(\mu - (1 + r_i^{loan})^2) - g(i, j) \quad (2)$$

Investment opportunities are limited liability; in the event of a zero payoff, banks do not have a claim to the households deposits. Consequently if bank i funds an investment opportunity, l_j^t, with probability $\theta_{l_j^t}$ the bank receives $(1 + r_i^{loan})^2$ at time $t + 2$ whilst with probability $1 - \theta_{l_j^t}$ the bank receives nothing. If no i exists such that Equation 2 is positive no funding request is made and the opportunity goes unfunded.

3.2 Investment Behavior

Each time step, banks determine the allocation of assets and liabilities on their balance sheets. Money is distributed from household deposits and inter-bank borrowing to fund loans to households, inter-bank lending and to save as cash reserves. Banks are constrained in this allocation by regulation along with their current holding of two period loans and borrowing from the previous time step.

We consider banks to be victims of a classical principal agent problem. The owners of banks wish to maximize returns in the long term, however, due to imperfect contracting, limited monitoring and limited liability of the managers, the managers they employ are focused on short term returns. This captures a common observation that bank traders and managers receive substantial bonuses for short term performance, encouraging them to take on excess risk and be focused on short term returns. Within this model we do not consider the identity of the shareholders or the managers, we are concerned only with the effect of this relation on bank behavior. Banks operate to maximize short term returns. They do not refuse investment opportunities with positive expected returns in the current

[9] An alternative formulation would additionally include firms. Households would place deposits, whilst firms, without any cash holdings, would approach banks to fund investment opportunities. This formulation is identical in operation to the model presented above, it simply separates the deposit and investment behaviors of the non-bank agents.

period based on the belief that they will receive better opportunities in the next period, therefore, behave as myopic, risk neutral, expected return maximizers.

In allocating their portfolios banks are subject to five key constraints. The first constraint, given by Equation 3, states that each bank's balance sheet must balance; i.e. assets are equal to liabilities.

$$L_i + R_i + I_i = E_i + D_i \tag{3}$$

The second constraint given in Equation 4 fixes the value of the deposit term on the balance sheet. It specifies that the bank's holding of deposits is equal to the sum of deposits placed in that bank by households. The bank may neither refuse deposits nor gain access to additional deposits outside of those contributed by households.

$$D_i = \sum_{j=1}^{M} d_j \text{ where } i = \arg\max_{i \in N} d_j(r_i^{deposit} - g(i,j)) \tag{4}$$

The third constraint, Equation 5, governs the level of liquid cash reserves which the bank holds. The reserve ratio is given by α_i, the bank's preference for cash reserves. Whilst this parameter may be set to any level, regulation imposes a minimum level of liquid cash reserves, forcing the bank to hold at least fraction α_g.

$$R_i \geq max(\alpha_g, \alpha_i)D_i \tag{5}$$

The fourth constrain given by Equation 6 specifies a maximum equity to risky assets ratio. In this equation β_i is the bank's preferred equity ratio and β_g is a minimum value imposed by regulation. The max operator means only positive values, i.e. inter-bank lending and not inter-bank borrowing are considered. Note, whilst reserves are assets, they are not included in the equity ratio. This is because under the Basel accords they are judged to have a risk-weight of zero and so are not included in capital adequacy calculations. In this model inter-bank lending and household lending are equally weighted in the risk calculation.

$$E_i \geq max(\beta_g, \beta_i)(L_i + max(I_i, 0)) \tag{6}$$

The constraint given in Equation 7 states that the amount invested in loans is equal to the total funds invested in individual projects. Here, K_i^t is the set of investments funded by bank i in period t and we define $\|.\|$ to be the sum of the values of loans in the included set. Importantly this constraint includes all projects funded at time t but also those that were funded at time $t-1$. Household lending, along with inter-bank loans, last for two time steps and so are illiquid assets.

$$L_i = \|K_i^t\| + \|K_i^{t-1}\| \tag{7}$$

When calculating its portfolio the level of equity of each bank is determined by the payoffs form its previous investments. The constraints above fix the value of deposits whilst the quantity of reserves are specified by the reserve ratio. Consequently the choice for banks is the distribution of funds between inter-bank lending and borrowing and loans to households. In making this decision bank i determines the composition of K_i^t the set of investment opportunities which it funds. The loans are selected from P_i^t, the set of investment opportunities presented to bank i by households at time t, i.e. $K_i^t \subseteq P_i^t$. The expected return for the bank from each loan, l_j^t, may be expressed as $\theta_j^t(1 + r_i^{loan})^2 - 1$. Bank's invest in zero or more loans in decreasing order of expected return until the expected return falls below the inter-bank lending rate or the bank runs out of funds. If the bank runs out of suitable loan opportunities whilst it still has available funds the bank may lend to other institutions subject to the expected return of the loan being positive. Alternatively if a bank has excess loan opportunities it may borrow money from other banks to fund these investments. Each time step, each bank, i, determines its allocation of funds between investment projects and inter-bank lending and borrowing to maximize its expected return, $E(r_i)$ given by:

$$E(r_i) = (\sum_{k_i^t=1}^{K_i^t} \theta_{k_i^t}(1 + r_i^{loan})^2 - 1) + I_i^t((1 + r^{inter-bank})^2 f(I_i^t) - 1) \tag{8}$$

Where $\theta_{k_i^t}$ is the repayment probability for loan k_i^t (each loan is of unit size) and $f(I_i^t)$ is a function giving an estimate of the probability of inter-bank lending being repaid:

$$f(I_i^t) = \begin{cases} \theta_i^{inter-bank}, & \text{if } I_i^t > 0 \\ 1, & \text{if } I_i^t \leq 0 \end{cases} \tag{9}$$

Here $\theta_i^{inter-bank}$ is bank, i's estimate of the probability of being repaid in the inter-bank market. The failure to repay inter-bank lending results in the bankruptcy of the defaulting bank. In calculating their expected return banks, therefore, assume that they will have to repay inter-bank borrowings so the probability is 1.

3.3 Inter-bank market

Inter-bank lending occurs through an over-the-counter market. In each time period there is a single inter-bank interest rate at which all transactions are conducted. This implies two assumptions, firstly that lenders do not vary their offered rate based on the identity of the borrower and secondly that the market is efficient and so the law of one price holds. The first of these assumptions follows if lenders do not condition their offered rates on the identity, and therefore financial position of their counter-parties or if there is little difference in potential counter-parties. In actual markets, participants form estimates of the risk of default of partners from various information sources including financial statements and the history of past payments. During non-crisis periods the rate at which banks fail is very low and in a steady state there should be little difference in the offered inter-bank rates between the most and least credit-worthy banks. For the initial analysis we assume that this difference is zero, that banks do not condition their lending on their counter-parties financial positions. This assumption simplifies the initial analysis of the model but is relaxed in section 6.

The second assumption is that the law of one price holds, though in an over-the-counter market it is not immediately obvious that this should be the case. The lack of a central counter-party means that in many inter-bank markets (the Italian market being a notable exception) there is no location at which offered interest rates are made public.

Instead, individuals at banks must spend time directly contacting other banks in order to determine their offered rates. Theoretical work, however, suggests that even limited communication may be sufficient for markets to converge to the equilibrium price (e.g. Axtell, 2005). Here we assume that there is sufficient information exchanged for the market to identify a single price. Empirically this is also supported by Iori et al. (2008) who show that the Italian inter-bank market is efficient in this manner.

The inter-bank rate is dependent on the lending and borrowing preferences of individual banks which are determined by the portfolio allocation set out above. This allocation itself is specific to each individual bank and dependent on the inter-bank rate. There is no closed form solution for the equilibrium, so to identify the market rate and simultaneously solve the bank portfolio problems it is necessary to use a computational approach. Here we use a bi-section method. This operates by taking an interval in which the interest rate is known to lie and calculating the supply and demand at the midpoint. The supply and demand are the total inter-bank loans offered and requested if the interest rate were that at the midpoint of the interval. The interval is then halved to lie between the mid point and either the previous maximum or minimum depending on whether supply or demand are in excess. Iterative application of this algorithm leads to an increasingly small interval in which the equilibrium interest rate lies. Here we calculate the interval such that it is no larger than 10^{-6} with the midpoint taken as the market rate.

In over-the-counter markets, transactions are bilateral, when a bank lends money it lends to one (or more) specific counter-parties who must repay the lender. If those banks go bankrupt the lender may not be repaid. The introduction indicated results showing that the susceptibility of a market to systemic shocks is affected by its structure of inter-bank connections. In the presented model the pattern of inter-bank lending connections is determined exogenously allowing a range of inter-bank market structures to be investigated and compared to different real world examples[10]. We consider the model for different values of λ, the probability that a given inter-bank lender lends money to a particular inter-bank borrower. As λ increases the density of inter-bank connections increases.

[10]A future development of this model would be to make connection decisions endogenous with the desire of finding an optimal inter-bank market structure under a given set of conditions.

The inter-bank connections are constructed as follows. Initially the population of banks is partitioned into three sets by their desired inter-bank positions: lenders, borrowers and those with no position. Each member of the set of lenders is considered in turn in decreasing order of the magnitude of funds offered. Let the set of borrowers to which i lends money be C_i. For each borrower, b, in the population, b is added to C_i with probability λ. If the total amount of funds requested by the members of C_i is less than the amount i wishes to lend, further banks are added to C_i in decreasing order of size of requested funds until this is no longer the case. The lender lends money to each member of C_i in proportion to their requested funds. The loan, I_{ij}, to borrower $j \in C_i$ is of size:

$$L_{ij} = \hat{I}_i^t \frac{\hat{I}_j^t}{\sum_{c=1}^{C_i} \hat{I}_c^t}$$

Where $\hat{I}_i^t$ is the quantity of funds offered or demanded in the inter-bank market by bank i at time t. Once a bank has borrowed its desired amount it is removed from the list.

The parameter λ dictates the structure of the network. If λ is equal to 1 each lender will lend to all borrowers in the market. If λ is close to 0 each lender may potentially only be connected to a single borrower[11]. The above mechanism was chosen as it permits a wide range of market structures whilst the market connectivity responds linearly to changes in λ. Other mechanisms for determining the allocation of connections were considered but were either more complex or resulted in non-linear transitions in connectivity. The results they produced were generally similar to those generated with this mechanism for the same number of connections[12]. In the results section we show that networks generated with this mechanism match many features observed in actual markets[13].

The two period nature of investments is important in capturing the structure of the inter-bank market. In any period each bank may be either an inter-bank lender or a borrower, they may not be both. Consequently if an investment, and therefore the inter-bank

[11] If $\lambda = 0$ (or close to zero) if may be that no banks are initially added to the set C_i, in which case the lender will be connected to the borrowers with the largest demand. Potentially this may result in each lender being connected to a single borrower.

[12] We also considered λ as an endogenous variable set by each bank. It was found that there was no significant difference to the results presented below.

[13] Other classes of network could also be considered for instance Cossin and Schellhorn (2007) examine random graphs but also circular networks and find different effects for firms subject to credit risk.

borrowing funding it, lasted only a single period the network of inter-bank connections would be bipartite. If a borrower failed it could impact on those banks from which it borrowed but there is no potential for the effect spreading any further. Two period loans provide a simple mechanism which allows a bank to be both a lender and borrower (in subsequent periods). In this case the failure of one bank may spread further through the inter-bank market, potentially affecting banks which are not linked to the initial failure. This allows richer and potentially more realistic contagious events than would be possible in the one period model.

3.4 Model Operation

This section details the order of events within each time period in the model. At the start of period t, interest is paid to households on their deposits established during period $t-1$. Banks pay to households the amount of interest defined in Equation 1. After interest is paid, loan success is evaluated for loans established in period $t-2$ and banks repaid by households as appropriate. The inter-bank lending from time $t-2$ which funded these investments is then repaid[14]. If after interest payments and loan success have been evaluated the bank has negative equity it is declared bankrupt. Similarly if a bank has insufficient cash reserves to repay its inter-bank debts it is declared bankrupt. In the event of a bank failure sufficient assets are retained to cover the value of deposits, any remaining liquid assets are used to repay creditors in proportion to the size of their debt. If a bank is not fully repaid it suffers a loss in equity which may, potentially cause it to go bankrupt. If this occurs any inter-bank borrowing on its balance sheet is resolved in the same manner. As such the failure of one bank may spread to its counter-parties and then further within the system. A bankrupt bank is removed from the financial system and takes no further actions.

If a bank fails to which a household or bank owes money, the borrower is still required to repay its loan at the appropriate due date. This is consistent with an administrator ensuring creditors of a bank meet their requirements. Any funds arising from such repayments are considered to either be absorbed by the administrators of the failed bank or to

[14]In periods 0 and 1 of the model their will not be any loans which pay off in that period as no loans had yet been established.

go to the deposit insurer to cover their expenses. This is reflected in Equation 9. After loans and bankruptcies have been resolved the deposits each household possess at time t are set such that:

$$d_j^t = \frac{\sum_{i=1}^{N} L_i^{t-1}}{M} \tag{10}$$

i.e. the total loans from the previous time step are equal to the cash holdings of households available for deposits at the current time step. Money is transferred between households as part of the operation of the real economy. When funds are lent to a household to invest, goods or services are purchased resulting in monetary transfers. In this paper we do not consider the detail and distribution of these interactions and so we assume that funds are distributed uniformly[15].

At this point households place their deposits in banks. Banks then allocate their funds as described above and the inter-bank rate is calculated along with the lending and borrowing relationships. Finally at the end of each period an inflationary process is applied to all values (including cash, loans, reserves etc.) at the following rate:

$$F^t = \frac{\sum_{i=1}^{N} E_i^t}{N} - 1 \tag{11}$$

The effect of the inflationary process is to maintain a fixed value of equity within the system. Doing so simplifies both the analysis and the computational process[16]. An alternative approach would be to model growth of the real economy, increasing the quantity and value of loan request each time step and modeling projects as reallocating and potentially consuming wealth along with creating it. The complexity of this approach together with the many necessary assumptions would complicate the analysis of the model without necessarily adding additional insight.

[15] Alternative mechanisms including having no redistribution and time varying distributions were tested but had little effect on the results.

[16] Without this the model could potentially grow to infinity and prevent a solution being found.

3.5 Parameters and Learning

Banks allocate their funds each time step to achieve the maximum expected return. There are, however, several parameters which affect this allocation along with the behavior and profitability of the bank. These parameters are: reserve ratio (α_i), equity ratio (β_i), lending interest rate (r_i^{loan}), deposits interest rate ($R_i^{deposit}$) and their estimate of being repaid in the inter-bank market ($\theta_i^{interbank}$). There is no closed form solution for assigning optimal values to these parameters within this model with time varying heterogenous banks and under different regulatory frameworks. The values of these parameters are set by a genetic algorithm, an optimization process by which less profitable parameter combination are replaced by those which produce higher returns.

Genetic algorithms (GAs) were first brought to prominence by the work of Holland (1975). They use mechanisms based on the theory of evolution, such as selection and mutation, to find optimal solutions to problems. A genetic algorithm maintains a population of candidate solutions. Each of these solution comprises a vector of values which encodes a particular solution to the problem. In every generation each candidate is evaluated and assigned a score against some criteria. The highest scoring are copied into a new population subject to small perturbations of the parameter values (through mechanisms termed mutation and crossover). This mechanism is repeated over time, resulting in increasingly 'fit' solutions to the problem to be found.

Genetic algorithms have previously been employed in economics and finance model as both a learning and an optimization technique. For example Arifovic (1996) employs a GA to model the learning behavior of traders in an examination of the dynamics of exchange rates. In contrast Noe et al. (2003) and Noe et al. (2006) employ GAs as an optimization technique in investigating corporate security choice along with the optimal design of securities. Within the context of this model we do not claim that a genetic algorithm is a good model of learning. The GA is used as an optimization method and the analysis restricted to the steady state to which the model converges. How the model state changes over time is not analyzed as this will be driven by the specifics of the GA.

Here we optimize the parameters such that they maximize the profitability of banks, i.e. we find those parameters which lead to higher equity. The genetic algorithm functions as follows. Each parameter for each bank is initially randomly drawn from $U(0,1)$. Each time period two banks from the population (including those which are bankrupt) are selected at random with uniform probability. The parameters of the bank with lower equity are replaced by the values of those of the richer bank subject to a small perturbation drawn from $U(-0.0025, 0.0025)$. If the poorer bank is bankrupt it is reintroduced to its previous location on the unit circle with $E = 1$, $R = 1$ and no other assets or liabilities. As such this process also introduces replacements to failed banks. Over large numbers of time periods the random perturbations ensure that the parameter space is explored whilst the copying process results in the population of banks converging to an optimal parameter set for the market.

4 Results

This section considers the robustness of the model economy to financial crisis. The effects of individual bankruptcies and economy-wide shocks are analyzed. The degree to which changes in regulation can mitigate the impact are also considered. In order to quantify these effects and to demonstrate the validity of the conclusions we first consider the steady state behavior of the model. All experiments in this paper use the parameters presented in Table 1 unless otherwise stated. An analysis of robustness to parameters and assumptions is provided in Section 6.

The first two parameter values are chosen based on real world equivalents. Within the model all deposits may be moved in any time-step and so are classed as instantly accessible. We, therefore, use the US reserve requirement of 10%. US banking regulations also define a minimum capital requirement for a bank to be adequately capitalized. This value is calculated as the ratio of Tier 1 and Tier 2 capital to risk adjusted assets. Here we do not differentiate between the two types of capital, instead we simply use equity. We count both inter-bank and household loans as having a risk weighting of 1 whilst reserves are risk-less.

At the start of the simulation $E_i = 1$, $R_i = 1$ for all banks whilst all other assets and liabilities are set to zero. The model was run with 500 different random seeds for each of

11 different values of λ. Each simulation was run for 10000 time steps. To test convergence the average values of market parameters during periods $8000-8999$ and $9000-9999$ were calculated and a T-Test performed to ensure the parameters were stable. At this point market statistics were recorded.

4.1 Steady state analysis

In this section we present statistics describing the state of the converged simulations. The aim of this model is to qualitatively capture the effect of regulation, and the structure of the inter-bank market, on the liklihood of the failure of banks and contagion. For this purpose it is important that key ratios and quantities are of broadly the same magnitude as reality in order for the results to be meaningful. We are not concerned with matching exactly the balance sheets of a particular country. To do so precisely would require a considerably more complex model with many more parameters. A simpler model in this case allows the mechanisms driving the results to be more clearly identified.

Table 2 shows the average asset and liability holdings of all banks within the model economy, together with the balance sheets of all American commercial banks in 2006. Here pre-financial crisis data were chosen as it is compared to pre-shock model data. Balance sheet terms are matched to their closest equivalent, but due to the richness and additional complexity of the real economy this is not possible for all values. In this, and all subsequent tables, the level of inter-bank loans is the total funds lent, the sum of positive positions. The sum of all positions within the market would be 0 as inter-bank lending is equal to inter-bank borrowing within this closed economy.

The ratio of loans to deposits is similar in both the model and empirical data. Relative to equity, however, both of these values are too small in the model. This is a consequence of the inflationary process. In order to maintain a fixed level of equity for computational tractability a relatively high rate of inflation (on average 13%) is necessary. This reduces the value of loans and deposits each time step. This effect is cumulative as loans at time t are used to calculate deposits at time $t+1$. Consequently when inflation along with reserve requirements are taken into account the maximum (post inflation) value of loans possible within the model is:

$$0.87\sum_{t=0}^{\infty}100\times 0.87^{t}\times 0.9^{t}\approx 401$$

This value is very close to the observed value of loans and unused capital. Bank's preferred equity ratio and reserve ratios (Table 3) are both less than the values specified by the regulations i.e. 8% and 10%. This means that the regulated values are used in all cases and the banks are maximally leveraged. If the banks adopted this behavior without the inflationary effect, the value of deposits and loans within the model would be very similar to the empirical data. The banks therefore, behave in a very similar manner to those in reality.

The level of inter-bank lending is high in comparison to the equivalent real word value. There is, however, a key difference between the model and the source of the empirical data. The model represents a closed economy, all borrowing and lending occurs between banks within the model. In contrast American banks were net borrowers during this period, bringing money into the system. A more appropriate measure of the level of inter-bank interaction is therefore the level of borrowing. Here the model and empirical values are much closer and approximately the same magnitude[17].

The deposit and loan rates within the model of 6.9% and 2.8% (Table 3) are empirically plausible. The inter-bank rate of 5.8% is high compared to historical values, however, it is necessary to remember that within this model there is no other source of funds so this rate rises due to demand for funds to lend to households rather than risk. This is highlighted by the bankruptcy statistics which show that bankruptcies are relatively uncommon in the steady state and systemic bankruptcies even less so. The average size of the bankruptcies, as measured by the equity lost, is also very small. The behavior of banks has converged such that in the steady state few go bankrupt.

The model does a good job of matching the magnitudes and key ratios observed in

[17]The level within the model is still slightly higher than seen in the US, however, this difference captures the effect of other inter-bank financial interactions, such as derivative contracts, not considered within this model. In the event of bankruptcy the dissolution of these contracts has a similar effect on the balance sheet to the failure of loans being repaid. Whilst H.8 statements do not provide data on derivatives during 2006 later estimates suggest the value of derivative is at least $400 billion which would place these values very close.

empirical data. We emphasize, however, that the purpose of the model is not to exactly reproduce empirical values and that with the addition of more parameters a closer matching could be achieved at the cost of clarity of results.

4.2 Market Structure

The structure of the inter-bank market is determined by a combination of the endogenous behavior of banks and exogenously specified structure. In particular the number of lenders and borrowers, their size and distribution is determined endogenously by the supply and demand of funds and loan opportunities.

Table 3 shows that in line with the empirical results of Iori et al. (2008), for the Italian inter-bank market, there are more lenders than borrowers and that the majority of banks act as either sources or sink for loanable funds, relatively few both lend and borrow. Examination of the average equity of banks within these groups shows agreement with the findings of Cocco et al. (2009) and Iori et al. (2008) that large banks are net borrowers whilst small banks are net lenders and that large lenders have many small creditors (Muller, 2006). This is because within the model there are only a few large banks, typically around 15%, that are constrained by the amount of funds they are able to raise through deposits. These banks have high equity and so in order to be maximally leveraged they must borrow on the inter-bank market. In contrast small banks are constrained by their level of equity, they would be unable to invest borrowed funds in risky projects. The inter-bank rate is sufficiently high that most small banks lend small amounts to a few large banks. This is supported by the findings of Cocco et al. (2009) who examines the distribution of links between banks, finding that the most common links are between large and small banks whilst the least common are between pairs of small banks. Table 4 shows a similar relationship in the model when the population is partitioned around the median wealth. Banks constrained in the same manner do not tend to lend or borrow from each other as one banks position would worsen. Overall we find that the endogenous structure of the inter-bank market closely matches key structural features observed in reality.

The number of inter-bank connections (lending relationships) is controlled exogenously by λ. As λ is increased Table 4 shows that the number of inter-bank connections increases in direct proportion. For $\lambda = 0$, given the numbers of lenders and borrowers the market is close to being minimally connected[18]. Whilst for $\lambda = 1$ the market is much more densely connected, for any given time step, all borrowers are connected to all lenders. Table 4 also shows the number of components into which the inter-bank network is split. A component is a set of vertices which are all connected through paths but are not connected to any nodes outside of the set. Here we calculate components based on the directed graph, considering i connected to j only if i lent funds to j. Each component therefore represents the maximum extent of contagion from a single bankruptcy. For values of $\lambda > 0.5$ there is on average only one component. This means that there exists at least one bank, who's failure could theoretically affect every other bank within the market. For lower values of λ this is not the case, the maximum impact of any failure is restricted.

4.3 Individual Bankruptcy

Opinion is divided on the effect of the structure of the inter-bank market on the probability and severity of contagion. Two opposing roles have been identified. Allen and Gale (2001) highlight the stabilizing quality, arguing that the more connected a market is the more efficiently risk is shared and the effect of a shock mitigating. In contrast Vivier-Lirimont (2006) and others argue that the more connected an inter-bank market is, the more banks will be involved in failure cascades and the faster these cascades will spread. In order to identify these effects within this model we first consider the bankruptcy of a single bank and its impact on the financial system. A similar analysis has been conducted in a number of studies both analytically and empirically for a range of inter-bank markets[19]. In each case the authors examine how a shock centered on a single bank or region affects the remainder of the financial system, potentially causing the collapse of multiple

[18]The minimally connected market would consist of each lender being connected to a single borrower meaning over two periods the minimum number of inter-bank connections is approximately equal to double the number of lenders. For $\lambda = 1$ each lender is connected to each borrower within a particular time step. The number of connections is close to $lenders \times borrowers$, remembering that the exact number of lenders and borrowers varies each time step.

[19]For example: Boss et al. (2004), Upper and Worms (2004), Nier et al. (2007), Gai and Kapadia (2010) Vivier-Lirimont (2006) and Allen and Gale (2001).

banks in a cascade.

In an analysis using Austrian data, Elsinger et al. (2006) show that systemic failures from the collapse of a single bank only occur in about 1% of cases of bank defaults. Further, only a small proportion of banks are able to cause systemic crisis were they to fail (Boss et al., 2004) and similarly only a small proportion of banks are themselves susceptible to the bankruptcy of a partner institution (Angelini et al., 1996). The effect of contagion when it occurs, however, can be very large (Gai and Kapadia, 2010). Humphrey (1986) shows that the collapse of a large American bank could potentially bankrupt 37% of banks in the market.

The converged economies presented in the previous section serve as a basis for this analysis. The state of the market, the bank positions and inter-bank loans, is frozen and a single bank is made bankrupt by setting its equity and reserves to zero. The effect of this bankruptcy on the rest of the economy is recorded before the state of the market is reset to the frozen state. This is repeated for each bank in turn until the failure of each bank has been considered.

Table 5 shows the impact of a single bankruptcy on the rest of the market. As the market becomes more connected the effect of the bankruptcy decreases. This supports the findings of Allen and Gale (2001), Giesecke and Weber (2006) and Freixas et al. (2000). The mechanism behind this is related to the probability of contagion; i.e. that the collapse of any given bank will induce at least one other bank to collapse (shown in Table 5). The decreasing probability as markets become more connected agrees with the relationship demonstrated by Brusco and Castiglionesi (2007); whilst more banks may be touched by contagion, if the market is more heavily connected the probability that any of them will fail is reduced. The higher level of connectivity spreads the impact of failures, as such being connected to more borrowers reduces, through diversification, the credit risk of the lender. Empirically, Angelini et al. (1996) and Boss et al. (2004) in their analysis of the Italian and Austrian inter-bank markets both find the probability of a bank collapse causing a systemic event to be approximately 4% which corresponds to a market in the upper-middle of the connectivity distribution.

The same table also shows the number of banks which go bankrupt conditional on there being a contagious failure. As the market becomes more connected more banks fail in each contagious event. This appears to suggest a greater vulnerability, however, this is not the case. The table shows that the average equity of the banks which cause contagion increases with connectivity. As the market becomes more connected only the larger banks with more borrowing are able to cause contagious failures. The impact of smaller banks is sufficiently well spread that in many cases they do not cause other banks to fail. The table also shows that the average equity of failing banks is less than the market average of one, indicating that smaller banks are more vulnerable to contagious failure.

An alternative measure of a market's potential susceptibility to contagion is the maximum number of bankruptcies a failure may cause. The sizes of the largest failures in the model are of the same magnitude as those seen in reality. Upper and Worms (2004) find within the German Banking system a single bankruptcy may cause at most 15% of the other banks to fail whilst Humphrey (1986) shows that the collapse of a major US bank could lead to 37% of banks defaulting. The relationship with connectivity differs from that of average contagion. Here the most vulnerable markets are those with an intermediate level of connectivity ($\lambda = 0.4$). Whilst not, on average, the most susceptible to contagion these markets are particularly vulnerable to the failure of crucial banks. Banks within these markets are sufficiently poorly connected that if one fails, the shock is strong enough to drive other banks to failure. At the same time Table 4 shows that for $\lambda = 0.4$ in many cases the market only has a single component, meaning that a single bankruptcy could affect the whole market. The combination of large shocks and wide spread combine to make these markets particularly vulnerable if the wrong large bank fails.

The results in this section have shown that a more connected inter-bank market allows more efficient risk sharing reducing the market's susceptibility to contagion. They also highlighted a vulnerability of intermediately connected markets which, whilst not the most susceptible to contagion, potentially suffer from the largest failures.

4.4 Systemic Shocks

The results presented in the previous section describe how an individual bankruptcy can cause contagion. These results are important in understanding the vulnerability of the financial system to an isolated failure. In reality, however, the failure of a bank is often not a contained spontaneous event. Instead a failure may be caused by a shock which affects the whole financial system. For instance, Gorton (1988) shows that bank panics are most common at the beginning of an economy wide recession. Events such as this can affect multiple institutions simultaneously, weakening balance sheets and potentially causing several unconnected banks to fail at the same time. As a result there may be overlapping cascades of bankruptcies. This section will consider the effect of such a macro-economic shock on the system.

Little attention has been given to the effect of the inter-bank market during a systemic shock. It is unclear how the risk-bearing and contagion spreading effects interact as equity is eroded. A market in which each bank is connected to more counter-parties may allow system liquidity to be better utilized as the failure of a bank is spread more thinly and so the shock reduced. Alternatively, as the market becomes more connected the weakest banks may be more likely to be effected by bankruptcies causing more of them to fail. One study which looks at this issue is that of Lorenz and Battiston (2008). They find that increasing inter-bank market connectivity at first reduces the incidence of bankruptcy but for more connected markets it increases. Their model, however, does not permit cascades of failures; a key mechanism in the spread of contagion. Whilst not explicitly modeling a systemic shock, Battiston et al. (2009) find a similar pattern when they permit multiple bankruptcies to occur in the same period.

In addition it is not clear whether contagion in the inter-bank market will be significant or if it will be secondary to the financial shock itself (e.g. Giesecke and Weber, 2006). If contagion is secondary within this model it would be expected that the number of failures due to the macro-economic shock would be greater than that caused by contagion.

To investigate these issues we examine the effect of systemic shocks on the model economy. The experiments employ the 500 converged markets as the starting point for these tests. Each converged market suffers a macro-economic shock during the first time step after the converged state. This shock is implemented by changing the probability of project success for projects which finish in the shock time step from θ_i^t to θ^{shock}. All projects ending in other time periods are left unchanged. We perform the experiment for a range of values of θ^{shock} and λ showing how different macroeconomic shock severities effect the stability of the financial system for different market structures[20].

Figure 1 presents results showing the average number of bankruptcies across different market architectures and for different shock severity's. As θ^{shock} decreases fewer projects are completed successfully. This leads to higher losses for banks and consequently more failures. Market connectivity, however, has a non-linear effect on this relationship. For small shocks a more highly connected market reduces bankruptcies, limiting the spread of contagion by spreading the impact of failures. In contrast for larger shocks the pattern is reversed, more sparsely connected markets are less susceptible to contagion. The point at which the effect of the market changes is approximately $\theta^{shock} = 0.775$. For shocks of this size the most fragile market structure is an intermediately connected market. Here both the contagion spreading and risk spreading effects are in evidence and of a similar magnitude. As market connectivity increases the contagion spreading effect leads to an increase in bankruptcies. For $\lambda > 0.5$, however, the ability of the market to spread the effect of failures becomes dominant leading to a reduction in bankruptcies.

The results show that the structure of the inter-bank market influences the number of failures associated with a contagious event. The extent of contagion is highly dependent on the degree to which failures spread. This is governed by two effects both of which vary with market connectivity: the number of banks to which each bank is connected and the probability that the inter-bank loan between two banks is larger than the lender's equity. As connectivity increases each bankruptcy affects more counter-parties. At the same time a lender splits the same amount of funds between more banks meaning the probability that an inter-bank loan is greater than the partner's equity, therefore causing bankruptcy if not repaid, is reduced.

[20]Note θ_i^t is drawn from a distribution for each investment, under a systemic shock the value is always θ^{shock}.

A systemic shock reduces the equity of all banks. For small shocks, in highly connected markets, banks are sufficiently well capitalized and the effect of the shock is sufficiently well spread that the failure of a bank rarely has sufficient impact to cause a counter-party to fail. Inter-bank connectivity acts to reduce risk. As connectivity decreases the average loan size to counter-parties increases and contagious failures becomes more likely. Larger systemic shocks result in reduced bank equities and so smaller counter-party losses may cause failures. Consequently banks in more connected markets start to be at risk from the failure of their counter-parties. For the largest systemic shocks bank equities are damaged to such an extent that regardless of connectivity the size of inter-bank loan losses are sufficient to cause them to fail. Instead of spreading the impact the higher connectivity results in more banks being affected and failing. At the same time the diversification effect from many inter-bank connections is weakened as the failure of banks becomes more correlated. In less well-connected markets banks fail but the scope of contagion is reduced as each bank failure effects a smaller subset of the population.

For $\theta^{shock} = 0.775$ the point at which the liklihood of a bank failing and spreading a shock is maximized at intermediate levels of connectivity. At this level of shock, more connected markets spread impacts sufficiently well that relatively few banks fail whilst less connected markets spread the shock to too few partners, limiting the spread. The intermediately connected markets suffer the most as shocks are sufficient to cause failures and are widely spread.

These results support the findings of Giesecke and Weber (2006) that for small shocks, connections reduce contagion. They also support those of Vivier-Lirimont (2006), that more connected markets result in more banks in the contagion process and the finding of Iori et al. (2006) that larger cascades are observed when the market is more connected. The results for the largest shocks agree with Allen and Gale (2001), the inter-bank market is of little use when there is a system wide shortage of liquidity. In these cases the shocks are so large that the system is unable to spread the effects of failures, instead the inter-bank market acts to worsen the shock by damaging otherwise healthy institutions. The pattern of failures shown in this paper differs from that of Lorenz and Battiston (2008) and

Battiston et al. (2009). Both of these papers find that failures are minimized for intermediate levels of market connectivity. In each case the authors examine different mechanisms to those employed here. The model of Lorenz and Battiston (2008) differs in that it does not permit cascades, a mechanism central to our findings. The results of Battiston et al. (2009), in contrast, are driven by an inter-temporal *financial accelerator*. This mechanism does not have an equivalent within our model as we focus on the short term (within period) effects. If this mechanism is removed, the authors find a similar pattern of results to that seen in this paper for smaller shocks. One area for potential future work would be to add a similar inter-temporal mechanism to this model. This would allow the examination of this effect in the presence of larger shocks when the pattern of bankruptcies is reversed.

In a similar manner to Martinez-Jaramillo et al. (2010) we separate the failures in the banking system into two groups (shown in Figure 1), those which were contagious in nature as opposed to those which were initiated by the systemic shock. In line with the findings of Elsinger et al. (2006), for all but the smallest shock in the most connected markets over half of the bankruptcies are caused by contagion. The systemic shock plays a major role in weakening the banks' equity positions, however, it is the failure of counterparties which induces bankruptcy in the majority of cases. Even for the largest shocks and least connected market nearly 80% of bankruptcies are contagious.

The number and size of banks which fail in the face of a systemic crisis is only one measure of the severity of the impact. An alternative is to consider the cost of bankruptcies to the deposit insurer. During the recent financial crisis many governments around the world were forced to 'bail out' or nationalise banks at huge costs to prevent further losses. If a bank fails the deposit insurer has to pay the cost, the more deposits the bank has the higher the potential cost. The insurer may therefore be interested in the cost of repaying deposits rather than the number of bank failures in judging the optimal inter-bank market structure and whether rescuing banks would be appropriate. Figure 2 shows that as the size of the shock increases, and more banks fail, the cost to the insurer increases. Surprisingly the market architecture has a very different effect from that observed for the number of bankruptcies. In all cases the cost decreases as market connectivity increases.

This relationship is seen because the more connected a market is the more of the cost of failures are born by the surviving banks. When a bank fails in a weakly connected market it has a large impact on a relatively small number of creditors. The impact heavily damages their balance sheets resulting in a large loss in equity and nothing left to pay depositors. In contrast, in a strongly connected market the failure of each bank affects many more counter-parties. This may result in more bankruptcies, but the smaller impacts mean that failed banks may still be able to partially repay depositors. The surviving affected banks bear some of the cost of the failure on their balance sheets reducing the total to be repaid by the deposit insurer. For the insurer increased connectivity is beneficial as it reduces costs, even if it potentially increases the number of bank failures[21]. If insurers are able to influence the connectivity of the market, for instance through regulation or legislation, it would be in their interest to encourage the market to be more connected.

The wider effects of the systemic event on the economy are shown in Table 6 averaged across market connectivities (λ). The results show that the size of the systemic shock is directly related to the damage to the economy, a larger shock results in fewer loans to households. Similarly there is a dramatic reduction in inter-bank lending as banks have little funds available to lend. Table 6 also shows statistics for failures in the next time period. The results show a higher incidence of bankruptcies at this later time compared to data pre-shock with those markets which suffered shocks of intermediate size being the most affected. The banks which go bust at this time are relatively poorly capitalized. Their equity is on average 20% of the average bank equity post-shock. The banks which fail are generally those which were heavily affected by the systemic crash, losing the majority of their equity and reserves. In the next time step they are unable to meet their liquidity requirements and consequently go bankrupt. For more severe shocks these banks are driven to bankruptcy at the time of the initial shock and so do not survive to the following time period.

The effects of market connectivity in the presence of systemic shocks are more complex than for single bankruptcies. We show that, unlike previous studies, there is no optimal

[21]There may be additional social costs due to damage to the payment system if sufficiently many banks fail but we do not consider this here.

level of market connectivity to minimize the impact of a systemic crisis. Connectivity may exacerbate or dampen the effect depending on the shock severity. For deposit insurers, however, there is an optimum structure as more connected markets minimize the cost of repaying deposits.

5 Regulation

The previous section highlighted the effects of the market structure on contagion under both individual and systemic shocks. Here we consider mechanisms for limiting the impact of these events and their wider effect on the market state.

5.1 Equity and Reserve ratio

A key proposal put forward in the Basel III reforms requires banks to hold a higher percentage of capital relative to their risky assets. As a result , banks are more tightly constrained in the degree to which they can leverage their positions and so should be less at risk of failure through poor investment outcomes. An alternative proposal has been made to tighten banks minimum reserve ratios. This change would force banks to hold a higher proportion of liquid reserves which would provide them with increased protection against liquidity shocks. Both of these mechanism are tested within this model. The equity and reserve ratios are varied independently and 500 further experiments conducted for each parameter combination. We consider increases of each requirement by 50%. We focus our analysis on the case of systemic shocks as the effect of these changes on individual failures has already received much attention. Nier et al. (2007), Iori et al. (2006) and Gai and Kapadia (2010) all find that increasing the amount of reserves which banks hold reduces the number of bankruptcies.

Figure 3 shows the effect of the regulatory changes on the probability of contagious bankruptcies. Increasing the equity ratio results in a large reduction in failures in nearly all cases. The reduced level of leverage reduces the level of the macro-economic shock. At the same time there is a reduction in inter-bank lending which limits the impact of failing banks on their counter-parties. Together these two factors combine to reduce the total effect of the shock. Increasing the reserve ratio has a relatively small effect on the markets susceptibility to contagion which is generally only significant for very large shocks. This is because contagion is primarily driven by banks failing through lack of equity. The increased reserve ratio means banks hold more liquid funds which may allow a bank to repay a loan when one of its own loans is not repaid. This effect is more beneficial when inter-bank loans are small so that if they are not repaid the shortfall may be covered by the additional liquid reserves. In the model market, as in real markets, there are relatively few banks which both lend and borrow (Iori et al., 2008) so increasing liquidity has a limited effect. Whilst both of the regulations reduce the number of bankruptcies the mechanism by which they do so, restricted lending to households and banks, has a negative effect on the economy as a whole. The average value of loans to households reduces by 8% to 361.3 for the change in reserve ratio and 12% to 345.1 for the change in equity ratio. The overall effect of these regulatory changes is therefore ambiguous, they reduce bankruptcies but at the same time reduce lending.

5.2 Borrowing Constraints

An alternative to constraining the total lending or borrowing is instead to constrain the maximum funds a bank may lend to a single counter-party. This approach forces banks to diversify their inter-bank lending, making them less susceptible to the failure of a single debtor. Here we implement this regulation by limiting the maximum a particular lender may lend to a particular borrower to be no more than a multiple η of the borrowers equity. As a consequence larger banks with more equity may borrow more from any given lender.

Table 7 presents the results of 500 simulation for three different borrowing constraints. For $\eta = 10$ it can be seen that the constraint does not effect the results, there is no significant change in any of the market statistics. As η is decreased the constraint becomes binding. For $\eta = 5$ the effect of the regulation is beneficial, the number of systemic bankruptcies is significantly reduced in all but one case. The regulatory change limits the size of the inter-bank connections reducing the probability of a bank failing due to the collapse of one of its creditors. The regulatory change also has a broader beneficial effect. There is a reduction in the demand for inter-bank loans which, reduces the total volume of loans and the interest rate in this market. As a result the volume of loans to households increases and there is more competition between banks forcing down the household borrowing rate.

Care, however, must be taken with the implementation of this regulation. If the borrowing constraints are too tight there can be substantial negative effects. For $\eta = 2$ there is still a significant reduction in bankruptcies. The function of the inter-bank market, however, is severely impaired, meaning funds are no longer efficiently allocated and the total value of loans to households is heavily reduced. By regulating too heavily the economy is severely restricted.

6 Model Sensitivity

This section presents results detailing the robustness of the model to changes in parameters and specification. The initial model presented above provides a relatively simple framework which captures the key behaviors of banks and households. Assumptions were made in forming the model, which whilst making it more transparent, over simplified important aspects of real world behavior. Here we relax several of these assumptions which move the model closer to reality whilst also permitting a greater degree of heterogeneity within the system. By comparing the modified model behavior to the base case we are able to determine the effect of the changes in a clear manner, which would not have been possible if they had been included in the initial model formation.

6.1 Parameter sensitivity

The results presented above are based on one parameter combination. In order fully to understand the model it is important to determine the robustness of the results and how behavior changes if parameters are varied. Table 1 details the models six key parameters. Of these six, changes to α_g and β_g have already been considered as regulatory actions. Here we will consider the remaining four. Further simulations were run in which the parameter values were changed and the affects reported[22].

Varying the payoff from investments, μ, affects the loan, deposit and inter-bank interest rates. Greater returns from investments allow banks to charge households higher interest rates which in turn allows banks to pay higher rates for funds from both depositors and on the inter-bank market. The model is robust to a wide range of values. $\mu = 1.15$ was chosen as it produced deposit and loans rates comparable to reality.

The parameters controlling the probability of a successful investment, θ, and the number of households, M, are closely linked. Together they control the supply of potentially fundable loan requests. A decrease in households results in fewer loan requests per time-period, whilst a decrease in θ reduces the expected return of projects making fewer profitably fundable[23]. The results of the model are robust across a wide range of parameter values ($0.9 < \theta < 0.999$, $M > 20N$), if either or both values are too low there may be insufficient profitable investment proposals resulting in unallocated funds and potentially no inter-bank lending. $M = 10000$ and $\theta = 0.99$ was chosen for computational reasons whilst providing sufficient supply of funding request. Increasing M beyond this point leads to significantly slower program execution without changing the results.

While θ and M describe the supply of investment projects, N, the number of banks, controls the demand. The model produces qualitatively similar results for a wide range of values ($N > 40$). $N = 100$ was chosen as it is of the same magnitude as the number of banks in many of the worlds inter-bank markets, though some are much larger or smaller.

6.2 Inter-bank confidence

One of the key features of the recent financial crisis was the loss of liquidity within inter-bank markets. Banks observed the failures of other financial institutions and became reluctant to lend due to the fear of not regaining their funds. The loss of confidence resulted in a shortage of liquidity and an exacerbation of the crisis. In the model presented above the failure of a bank may cause other banks to fail both in the current and

[22] Tables of results demonstrating the relations are available from the author upon request.

[23] Note this parameter also interacts with μ. The larger the value of μ the lower θ may be whilst maintaining a profitable project.

future time periods (Table 6). Banks, however, do not take this into account, they do not become more reluctant to lend even though the probability of funds not being returned is increased. A parallel may be drawn here with the work of Lagunoff and Schreft (2001) who show that banks may change their portfolio of investments to reduce their exposure to potential losses even if they have not directly suffered.

To capture this effect the model is modified. Equation 9 is changed such that:

$$f(I_i^t) = \begin{cases} \theta_i^{inter-bank} - \kappa_i f^t, & \text{if } I_i^t > 0 \\ 1, & \text{if } I_i^t \leq 0 \end{cases} \tag{12}$$

Where f^t is the number of banks which have failed in the current time step t and κ_i is a parameter controlling the size of bank i's reaction to bankruptcies. A larger value of κ_i means that bank i reacts more strongly to a bankruptcy with a greater loss of confidence in the inter-bank market. The value of κ_i is assigned randomly at the start of the simulation and is optimized in the same way as deposit and loan interest rates. f is set each time period based on the number of bank failures.

Allowing banks to react to failures negatively affects the stability of the market. Table 8 shows that there are fewer loans to households and fewer inter-bank loans, both quantities also have a higher standard deviation. The inter-bank interest rate in particular is very volatile. During some periods it is similar to the base case but in others, particularly after the failure of one or more banks, it can be very high, essentially preventing inter-bank lending. The average size of contagion in response to a single bankruptcy is similar to that of the base model (Table 9), however, there is less variation with connectivity. Less connected markets are less vulnerable whilst more connected markers are more so. This is because there is less inter-bank lending between fewer banks. Consequently the magnitude of both the risk spreading and contagion inducing effects are reduced making the effect of connectivity smaller. The effect of the more volatile market may be seen in the size of the largest failures, these are in most case much larger than the base model and increasing with connectivity. The sudden fluctuations in market conditions can damage the positions of banks, amplifying the effect of an individual failure by making counter-parties more likely to fail. The consequences of the reduction in lending may also be seen in the reduction in bankruptcies due to systemic shocks (Table 10). Less inter-bank lending means fewer banks fail due to contagion, but this is accompanied by a much larger reduction in loans and inter-bank lending than in the base case. Banks react to the failure of counter-parties by stopping lending on the inter-bank market. As a consequence funds are less efficiently allocated and the economy as a whole suffers. Overall if banks react to the failure of other banks by becoming less willing to lend on the inter-bank market the system is destabilized as was seen in the 2007-2008 financial crisis.

6.3 Credit Worthiness

In the base model it was assumed that there existed a single inter-bank interest rate. It was argued that this was a reasonable assumption if banks have limited information about each others states, the probability of systemic events is low, and the market is efficient. In reality, however, banks vary their inter-bank rates depending on the counter-party. More credit worthy banks, those thought less likely to fail, pay lower rates. At the same time banks tend repeatedly to interact with the same counter-parties (Cocco et al., 2009) potentially allowing more attractive interest rates due to improved information. A banks state and history affect the rate at which it can lend and borrow. Here we integrate this observation.

Each time period each bank has associated with it a risk premia, ζ_i drawn from $|N(0, 1/E_i)|$ which is the market estimation of the necessary compensation to lenders for the risk of it failing. This is to some extent a simplification of a potentially very complex effect. In reality a banks risk premia is dependent on its own situation and the risk attitudes of all other market participants. This mechanism, however, uses the observation that larger banks are less likely to fail (e.g. Section 4.3) and so should receive more favorable terms. This rate is added to the inter-bank rate bank i pays when it borrows. If a bank lends money it lends at the base inter-bank rate. The recipients premia is not included when determining lending preferences as any additional value received over the base inter-bank rate is considered to be fair compensation for the additional risk borne.

The addition of a risk premia reduces inter-bank lending, however, unlike allowing bank to vary their confidence in the inter-bank market, it does so in a relatively stable manner. As a result the market is less volatile and more funds are allocated to households, there are fewer bankruptcies and interest rates are lower (Table 8). This is reinforced in the results for single bankruptcies, Table 9 shows the size of the contagious event is in nearly all cases reduced (along with the size of the maximum bankruptcy). The system as a whole is also more resilient, even in large crisis the extent of lending to households is less heavily reduced (Table 10). These results are in-line with the findings of Park (1991), who shows that historically the availability of solvency information regarding individual banks reduces the severity of panics. Here the risk premia is conditional on bank equity and so is equivalent to giving banks this information. The introduction of the risk premia makes it relatively more expensive for smaller and potentially more vulnerable banks to borrow. As a consequence inter-bank lending along with the potential for systemic risk are both reduced making the market more stable and the allocation of funds to households more efficient.

7 Conclusion

The structure of the inter-bank lending market has a major effect on the stability of the financial system. In response to individual shocks inter-bank connections spread the impact of failures. Consequently the expected number of failures decreases as the number of inter-bank connections increases. Despite this relationship it is found that intermediately connected markets potentially suffer the largest contagious failures. These markets share risk less well than those better connected yet are potentially susceptible to the failure of a single bank spreading and affecting the whole market making them particularly susceptible to the failure of the largest banks. For systemic shocks the relationship is more complex. The optimal inter-bank market connectivity varies with shock size. Previous work has shown two contradictory relationships, both an increasing and decreasing liklihood of failures with increasing market connectivity. The model presented here demonstrates the conditions under which each effect is dominant. For small shocks higher connectivity helps to resist contagion but for larger shocks it has the opposite effect. As a consequence there is no single best market architecture able to limit contagion from systemic shocks. There is, however, an optimal market structure for reducing the costs of these shocks. The more connected a market is, the more the costs of failures are internalized reducing the cost to an insurer.

In order to limit the effects of contagion several regulatory actions were examined. Changes to both the reserve and equity ratios were considered but were found to have ambiguous results. In both cases increasing the ratios resulted in a decreased size of contagion but also decreased lending, though both effects are more marked for changes in the equity ratio. Loan constraints that limit the amount a lender may lend to a particular borrower, were also considered. If the constrains were too lax they had no effect, whilst if they were too tight they reduced bankruptcy but heavily damaged the efficiency of the economy, reducing the amount of funds allocated to household loans. For intermediate levels of regulation bankruptcies were reduced and more loans given to households, suggesting this could be a promising mechanism for limiting systemic risk. It was also shown that if banks react to the bankruptcies of their peers the economy is destabilized and funds are allocated less efficiently. In contrast if banks condition their lending rates on the size of their counter-parties this reduces risk and makes the market less susceptible to contagion.

The model is sufficiently general that it invites further extension. The architecture of the market considered in this paper was imposed exogenously, banks had no choice about their counter-parties. A richer model would relax this constraint, allowing lenders to select and decline potential borrowers and to offer different interest rates based on the counter-parties financial position. This would allow issues such as the characterization of the optimal market structure to be addressed. Even without making this endogenous there are other market structures which could be investigated, for instance hierarchical networks as seen in the UK inter-bank market.

The regulatory changes considered in this paper were of a static nature, regulations were changed and the model simulated to find the new equilibrium. This does not have to be the case. There is scope to investigate the application of regulations dynamically, for instance changing capital or reserve requirement or providing banks with additional liquidity at particular points in time. The role of the central bank was also not realistically considered. Allen et al. (2009) have shown how a central bank may limit volatility through open market operations. Central bank intervention, in the form of bail outs or quantitative easing could be examined. The model may provide a test bed to investigate these issues.

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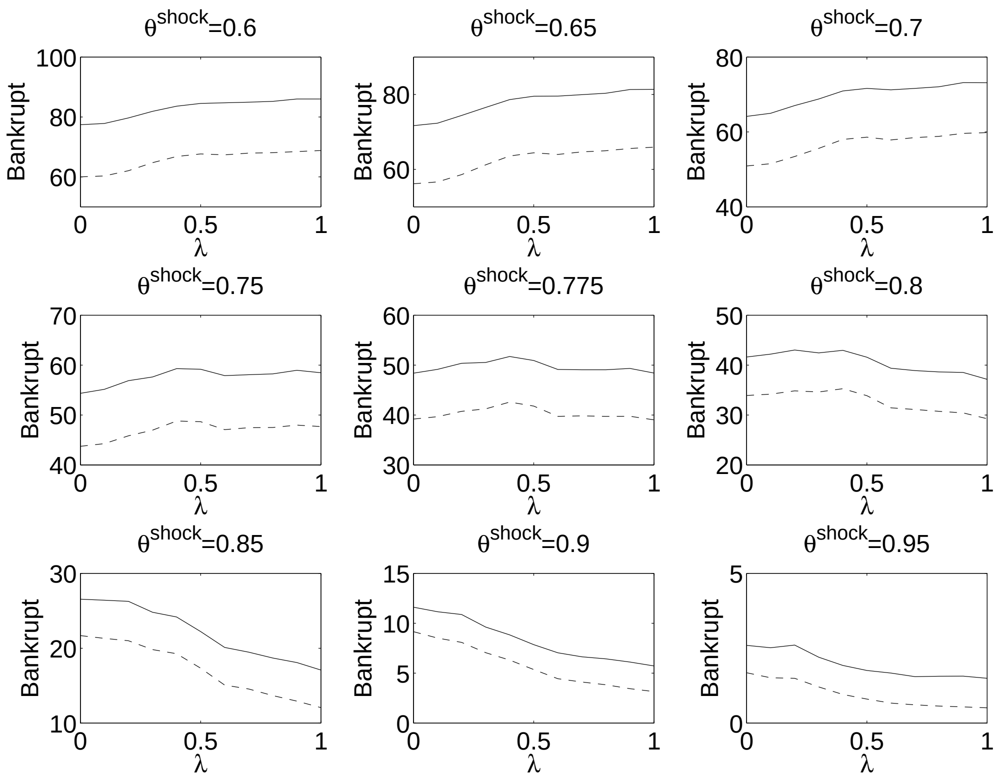

Figure 1: Total number of bankruptcies occurring on shock period (solid line) and the number of bankruptcies which were caused by contagion (dashed line), for different values of θ^{shock} and λ. Note the scale on the Y axis changes to illustrate the effect of λ. All shocks conducted at period 10000 and averaged over 500 repetitions.

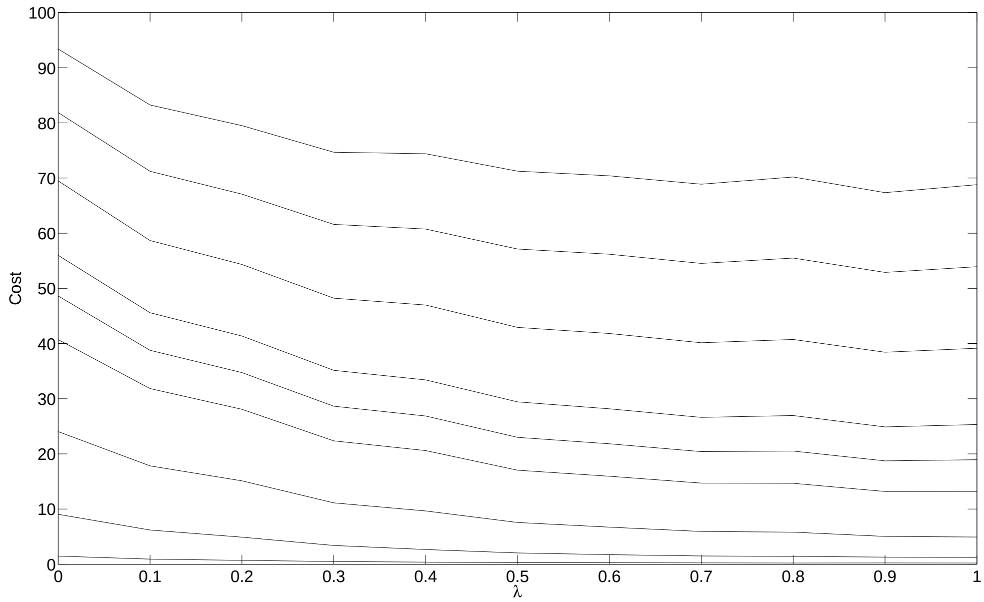

Figure 2: Total cost of repaying depositors of failed banks for different values of θ^{shock} and λ. The top line corresponds to the largest shock ($\theta^{shock} = 0.6$) the lines below are for shocks of decreasing size. All shocks conducted on period 10001 and averaged over 500 repetitions.

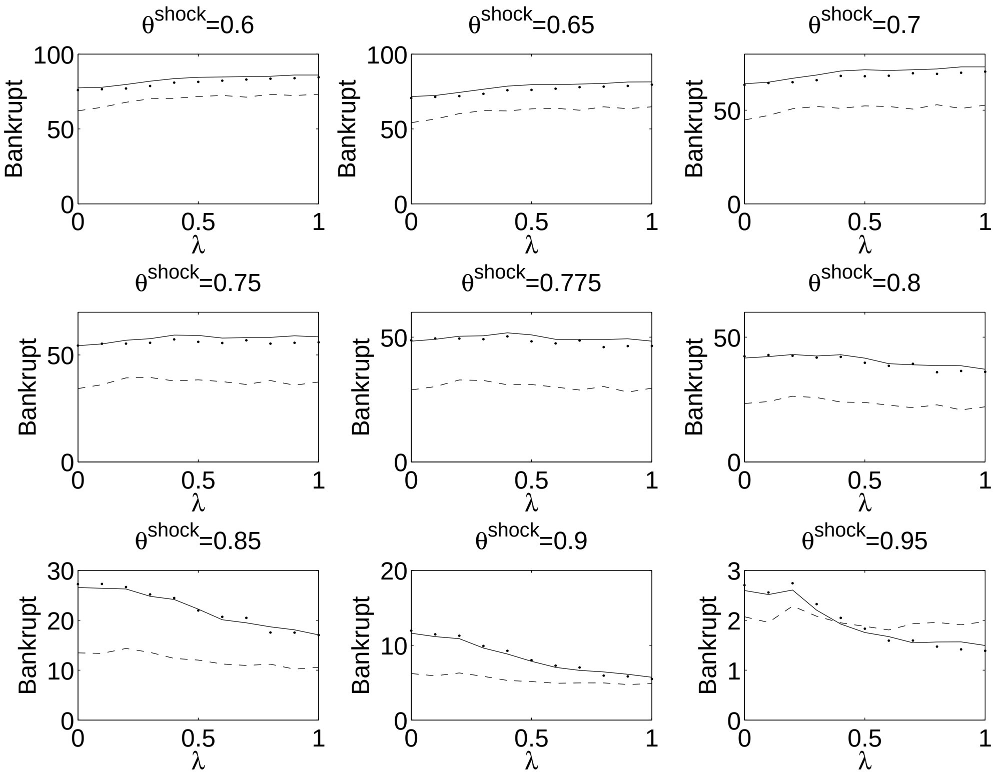

Figure 3: Total number of bankruptcies occurring on shock period for the base model (solid line), increased equity ratio (dashed line) and increased reserve ratio (dotted line), for different values of θ^{shock} and λ. Note the changing scale on the Y axis to illustrate changes with λ. All shocks conducted at period 10000 and averaged over 500 repetitions in each case.

Parameter	Meaning	Value
α_g	Reserve Requirement	0.10
β_g	Capital Requirement	0.08
N	Banks	100
M	Households	10000
θ_j^t	Project success probability	$U(0.99, 1.0)$
μ	Project payoff	1.15

Table 1: Parameters used for all simulations (unless otherwise stated).

Model Type	Value	SD	Empirical Type	Normalized	Real
Loans	391.5	(32.6)	Loans	950.2	8330.1
Inter-bank Loans	283.3	(36.9)	Inter-bank Loans	41.5	364.5
Reserves	34.8	(3.42)	Cash Assets	36.3	317.1
Unused capital	14.3	(6.8)			
			Other Assets	94.55	829.0
Deposits	341.3	(31.1)	Deposits	721.8	6327.3
			Borrowings	221.7	1943.9
			Other Liabilities	71.9	630.1
Equity	99.1	(5.13)	Residual	99.1	868.7

Table 2: Assets and liabilities of model data along with data for commercial banks in the USA (billions of Dollars), December 2006, source: H.8 statement, Board of Governors of the Federal Reserve System. The left hand side of the table presents the model data whilst the right hand side presents empirical data normalized such that the Residual is equal to the model Equity. Unused capital is capital placed in reserves above that which the banks reserve ratio specifies due to the bank being unable to find a profitable way to allocate the funds. The level of inter-bank lending in the model is the sum of all positive positions. By definition the sum of all positions, positive and negative is 0.

Term	Value	SD	Term	Value	SD
Loan Rate	0.069	(0.011)	Inter-bank Rate	0.058	(0.01)
Deposit Rate	0.028	(0.006)	Inflation Rate	0.13	(0.02)
Lenders	77.6	(6.1)	Average Lender Equity	0.83	(0.08)
Borrowers	21.1	(4.9)	Average Borrower Equity	1.67	(0.61)
Both	4.57	(2.79)	Average Both Equity	0.87	(0.29)
Bankrupt	0.18	(0.81)	α_i	0.06	(0.03)
Systemic Bankrupt	0.03	(0.49)	β_i	0.06	(0.04)
Equity value	0.14	(0.66)	$\theta_i^{inter-bank}$	0.99	(0.05)

Table 3: Aggregate model statistics at period 10000 averaged over 500 runs. Standard deviations in parenthesis. Values calculated prior to inflation/consumption effect. 'Both' in the table refers to those banks in the system who were lenders in one period and borrowers in the next (or vise versa).

λ	Connections	Component	Largest Component	Large to Large	Large to Small	Small to Small
0.0	180.0	12.0	24.1	65.4	97.5	17.1
	(26.7)	(3.1)	(10.3)	(9.4)	(21.2)	(13.2)
0.1	386.5	6.9	40.7	123.3	210.4	52.7
	(55.0)	(1.5)	(10.5)	(13.8)	(44.6)	(29.1)
0.2	684.2	4.3	58.7	207.2	364.3	112.7
	(109.4)	(0.9)	(10.8)	(26.2)	(89.3)	(57.6)
0.3	1017.7	2.9	70.3	307.8	537.2	172.7
	(154.2)	(0.8)	(8.7)	(39.6)	(124.8)	(81.5)
0.4	1307.4	1.9	77.5	408.9	694.5	204.0
	(204.0)	(0.7)	(6.2)	(56.8)	(165.7)	(104.6)
0.5	1643.0	1.5	79.8	517.4	875.5	250.1
	(253.3)	(0.6)	(5.0)	(69.9)	(205.6)	(130.5)
0.6	1965.0	1.2	80.9	627.4	1054.7	282.9
	(298.9)	(0.4)	(5.0)	(83.1)	(244.0)	(151.3)
0.7	2298.5	1.1	81.4	727.2	1227.1	344.2
	(339.4)	(0.2)	(4.5)	(95.1)	(272.5)	(178.5)
0.8	2598.6	1.0	81.7	829.6	1391.5	377.4
	(394.2)	(0.1)	(5.0)	(111.2)	(314.7)	(209.7)
0.9	2984.0	1.0	80.9	942.2	1597.3	444.6
	(440.6)	(0.0)	(5.0)	(123.0)	(359.6)	(222.9)
1.0	3298.9	1.0	81.6	1049.1	1778.5	471.2
	(494.8)	(0.0)	(5.0)	(137.4)	(403.6)	(251.2)

Table 4: Statistics describing the structure of the inter-bank market network for variation in λ. Statistics collected at day 10000 and averaged over 500 runs. Standard deviations in parenthesis. The last three columns give the number of lending relationships between large banks (above median size) and small banks (below median size).

λ	Contagion	Probability	Size	Equity	Cause Equity	Largest
0	1.62	0.226	7.16	5.45	2.08	19.8
	(0.61)	(0.059)	(3.98)	(1.80)	(3.20)	(10.5)
0.1	1.59	0.213	7.45	5.93	1.84	24.6
	(0.45)	(0.049)	(2.87)	(1.66)	(1.15)	(11.7)
0.2	1.43	0.183	7.82	6.16	1.92	28.9
	(0.47)	(0.036)	(3.30)	(2.07)	(0.83)	(13.1)
0.3	1.17	0.144	8.10	6.23	2.15	28.8
	(0.52)	(0.029)	(3.90)	(2.55)	(0.90)	(14.3)
0.4	0.96	0.105	9.15	6.92	2.52	29.8
	(0.60)	(0.029)	(4.88)	(3.32)	(1.05)	(16.5)
0.5	0.71	0.074	9.58	7.23	2.81	27.5
	(0.75)	(0.030)	(6.06)	(4.32)	(1.06)	(18.1)
0.6	0.57	0.052	10.89	8.13	3.15	27.2
	(0.93)	(0.029)	(8.06)	(5.91)	(1.31)	(20.3)
0.7	0.43	0.036	11.75	8.77	3.28	25.8
	(1.18)	(0.026)	(10.90)	(8.19)	(1.74)	(23.30)
0.8	0.35	0.026	13.46	9.98	3.34	26.0
	(1.42)	(0.024)	(14.56)	(10.88)	(2.29)	(26.5)
0.9	0.26	0.018	13.93	10.19	3.24	23.4
	(1.77)	(0.022)	(18.42)	(13.85)	(2.94)	(28.7)
1	0.22	0.013	16.79	12.24	3.13	23.1
	(2.10)	(0.019)	(25.70)	(19.14)	(3.51)	(32.4)

Table 5: Statistics showing the effects of single bankruptcies on the economy for variation in λ. Contagion is the average number of banks which fail as a consequence of a single bank being made bankrupt (excluding the initial bank). Probability is the chance that contagion will occur. Size is the average number of banks which go bankrupt conditional on contagion occurring whilst equity is the value of these banks. 'Cause Equity' is the average equity of the banks which cause contagion. Largest is the size of the largest contagion. Data collected using market states saved at period 10000 and averaged over 500 runs.

Time	t				t+1	
$\theta^{inter-bank}$	Bankrupt	Systemic Bankrupt	Loans	Inter-bank Loans	Bankrupt	Bankrupt Equity
0.6	82.9	65.6	67.2	16.1	1.33	0.22
	(12.2)**	(13.4)**	(42.8)**	(18.1)**	(1.96)**	(0.54)**
0.65	77.8	62.3	84.3	24.2	1.96	0.31
	(14.7)**	(15.5)**	(48.6)**	(24.3)**	(2.21)**	(0.59)**
0.7	69.9	56.6	107.1	37.6	2.87	0.44
	(18.0)**	(18.5)**	(53.3)**	(32.0)**	(2.48)**	(0.64)**
0.75	57.6	46.9	136.7	59.6	3.80	0.57
	(21.5)**	(21.5)**	(54.5)**	(39.6)**	(2.77)**	(0.68)**
0.8	40.6	32.7	175.6	93.5	3.91	0.58
	(22.3)**	(22.0)**	(51.0)**	(43.3)**	(3.31)**	(0.79)**
0.85	22.2	17.2	228.4	140.6	2.58	0.36
	(17.8)**	(17.3)**	(45.0)**	(40.6)**	(3.50)**	(0.84)**
0.9	8.4	5.8	296.5	197.1	1.09	0.09
	(9.4)**	(8.8)**	(39.5)**	(34.7)**	(3.65)**	(0.86)**
0.95	1.9	1.0	360.7	249.8	0.45	0.03
	(3.1)**	(2.5)**	(34.8)**	(33.7)**	(3.73)**	(0.86)

Table 6: Market statistics post shock during the shock time period and following period, averaged across λ. All shocks conducted at the start of period 10000 and averaged over 500 repetitions.

Shock Size	η ∞	10	5	2
0.6	82.9	82.5	66.1	37.1
	(12.17)	(12.49)	(15.48)**	(13.51)**
0.65	77.8	77.5	59.4	31.7
	(14.67)	(14.88)	(16.59)**	(12.32)**
0.7	69.9	69.7	50.5	26.3
	(18.03)	(18.12)	(17.39)**	(10.57)**
0.75	57.6	57.6	39.0	21.6
	(21.46)	(21.44)	(17.01)**	(8.92)**
0.8	40.6	40.6	26.1	17.4
	(22.43)	(22.40)	(15.95)**	(8.10)**
0.85	22.2	22.3	15.1	13.3
	(17.76)	(17.75)	(9.43)**	(5.89)**
0.9	8.4	8.4	7.8	9.4
	(9.37)	(9.35)	(5.41)**	(4.55)**
0.95	1.9	2.0	3.3	5.4
	(3.07)	(3.22)	(3.08)**	(3.02)**
Loans	391.5	392.9	404.0	303.3
	(32.62)	(35.44)	(75.58)**	(86.72)**
Inter-bank Loans	283.3	282.2	189.3	66.1
	(36.91)	(39.98)	(63.64)**	(27.25)**
Lending Rate	0.069	0.068	0.050	0.025
	(0.008)	(0.006)	(0.008)**	(0.001)**
Inter-bank Rate	0.058	0.058	0.045	0.016
	(0.010)	(0.013)	(0.018)**	(0.011)**

Table 7: Statistics showing the effects of systemic shocks on the economy for different borrowing constraints averaged across λ. All shocks conducted at period 10000 and averaged over 500 repetitions in each case. $\eta = \infty$ corresponds to the base case where there is no constraint. The market statistics at the bottom are pre-crash values.

	Inter-bank Confidence		Credit Worthiness	
Bankrupt	0.258	(1.113)	0.04	(0.26)**
Systemic Bankrupt	0.045	(0.662)	0.002	(0.06)**
Loans	341.04	(89.2)**	410.65	(20.32)**
I-B Loans	246.64	(80.42)**	247.65	(28.77)**
I-B Rate	0.155	(0.379)**	0.054	(0.008)**
Loan Rate	0.065	(0.009)**	0.066	(0.008)**
Deposit Rate	0.026	(0.005)**	0.027	(0.006)**
$\theta^{inter-bank}$	0.97	(0.08)	0.99	(0.004)**
Reaction	0.47	(0.28)	-	-

Table 8: Steady state market statistics for two model variations. Values consistent over λ, calculated in time period 10000 and averaged over 500 repetitions in each case.

	Inter-bank Confidence				Credit Worthiness			
	Shock		Max Size		Shock		Max Size	
0	1.39	(0.51)**	20.52	(10.20)	1.30	(0.71)**	21.85	(10.74)**
0.1	1.34	(0.64)**	28.98	(14.07)**	1.39	(0.50)**	25.43	(11.68)
0.2	1.28	(0.61)**	31.00	(15.42)*	1.19	(0.51)**	23.98	(12.74)**
0.3	1.10	(0.59)	30.59	(14.86)	0.78	(0.63)**	22.30	(14.66)**
0.4	1.00	(0.60)	34.39	(17.41)**	0.58	(0.71)**	22.76	(15.57)**
0.5	0.84	(0.67)*	33.47	(18.61)**	0.44	(0.87)**	22.74	(17.49)**
0.6	0.68	(0.82)*	32.46	(21.53)**	0.32	(1.04)**	20.81	(17.20)**
0.7	0.67	(0.88)**	35.05	(22.64)**	0.26	(1.27)*	20.56	(18.76)**
0.8	0.49	(1.14)*	32.29	(26.31)**	0.20	(1.55)	20.59	(22.06)*
0.9	0.60	(1.15)**	39.55	(30.33)**	0.19	(1.79)	21.65	(25.61)
1	0.57	(1.29)**	41.90	(35.11)**	0.16	(1.82)	23.16	(28.76)

Table 9: Statistics showing the effect of a single bankruptcy for different values of λ for two different model cases. Results collected in time period 10000 and average over 500 repetitions in each case.

	Inter-bank Confidence			Credit Worthiness		
	Bankrupt	Loans	I-B Loans	Bankrupt	Loans	I-B Loans
0.6	66.37	58.4	16.5	79.65	84.8	14.9
	(30.80)**	(87.7)	(55.8)	(8.99)**	(81.9)**	(27.4)
0.65	62.43	69.2	20.0	73.77	109.3	24.0
	(30.60)**	(93.5)*	(57.4)	(11.92)**	(98.3)**	(43.4)
0.7	56.79	83.9	25.9	64.74	141.1	40.7
	(30.10)**	(102.0)**	(61.6)*	(15.86)**	(112.0)**	(64.9)
0.75	48.22	103.0	36.3	51.21	180.1	68.7
	(29.00)**	(109.6)**	(69.0)**	(19.41)**	(112.5)**	(83.3)
0.8	35.46	126.8	53.7	33.71	227.8	109.4
	(26.00)**	(109.5)**	(75.6)**	(19.32)**	(91.9)**	(80.9)*
0.85	20.07	156.4	79.0	17.20	285.6	158.5
	(19.50)	(99.7)**	(75.4)**	(13.78)**	(62.5)**	(56.8)**
0.9	7.57	199.2	113.9	6.56	347.4	205.0
	(10.00)	(93.9)**	(77.6)**	(6.27)**	(37.7)**	(35.6)**
0.95	1.79	267.6	171.3	1.70	392.2	235.0
	(3.30)	(104.1)**	(92.7)**	(2.00)	(24.4)**	(29.7)**

Table 10: Statistics showing the effect of systemic shocks for two different model cases. Values averaged over λ, collected at period 10000 for 500 repetitions in each case.