FRACTIONS: A PIECE OF CAKE?

AN EXPLORATION OF STUDENT TEACHERS' UNDERSTANDING, ATTITUDES AND BELIEFS IN RELATION TO FRACTIONS.

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Helen Fielding

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Abstract

Fractions: A Piece of Cake? An exploration of student teachers' understanding, attitudes and beliefs in relation to fractions.

The title of this study shows the aims on which the research questions were based. These included the areas in which the student teachers felt confident as well as those they perceived to be more difficult. This study adopted a phenomenographic approach in order to provide further insight into each student teacher's subject knowledge. The purpose of this study was to discover the individual and distinct ways in which student teachers understood fractions.

The study was undertaken in two universities with a group of thirteen undergraduate and postgraduate Initial Teacher Education students. The creation of a comfortable, supportive working atmosphere and the use of self-selected small groups, enabled a range of rich and honestly reflective data to be collected. Observations were made of groups working on two collaborative tasks involving the sequencing of fractions by magnitude and finding, followed by diagnostic interviews. Each interview was structured by the student's individual selections from a range of questions where they indicated which they felt were most and least accessible. A constructivist perspective was adopted where the students had the opportunity to reconstruct their own understanding of fractions through the explanation and discussion of their existing ideas.

A range of successful strategies was demonstrated, especially the use of mathematical anchors as a means of comparison and the use of residual or gap thinking to consider differences in magnitude. Improper fractions and reunitising were difficulties cited by many in the group. A certain level of anxiety and lack of flexibility in their chosen approaches was evident with the common assumption that there was a particular method which should be adopted; this was usually based on their "secondary school" experiences.

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Key words

Primary Mathematics
Fractions
Student teacher subject knowledge
Phenomenography

Glossary

SATs - Standardised Assessment Tests (National Tests)

National Curriculum assessments are a series of educational assessments, colloquially known as Sats[1] or SATs,[2] used to assess the attainment of children attending maintained schools in England. They comprise a mixture of teacher-led and test-based assessment depending on the age of the pupils.

National Curriculum levels

At Key Stages 1, 2, and 3, the National Curriculum is accompanied by a series of eight levels. These are used to measure each child's progress compared to pupils of the same age across the country.
By the end of Key Stage 1 (aged 5-7), most children will have reached level 2, and by the end of Key Stage 2 (aged 7-11) most will be at level 4.

GCSE- General Certificate of Secondary Education

An academic qualification awarded in a specified subject, generally taken in a number of subjects by students aged 14–16 in secondary education in England, Wales and Northern Ireland

TDA -Training and Development Agency for Schools

The national agency and recognised sector body responsible for the training and development of the school workforce.

OfSTED - Office for Standards in Education, Children's Services and Skills.

OfSTED reports directly to Parliament. They inspect and regulate services which care for children and young people, and those providing education and skills for learners of all ages in England.

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Chapter 1 Introduction

1.1 Introduction

A professional and personal interest in the learning and teaching of mathematics, and a desire to make it more effective, is the basis of this study. Through a twenty year career in the primary classroom and eleven years in Initial Teacher Education focusing on primary mathematics, the difficulties in understanding and reluctance to learn fractions in both children and adults has been a recurring theme. It is acknowledged that there has already been considerable research (Rees & Barr, 1984, Ball, 1993; Carpenter et al., 1993; English & Halford 1995; Hunting 1984, Ryan & McCrae, 2005, Oppenheimer & Hunting, 1999, Wong & Evans, 2007) focusing on the difficulties encountered by pupils in primary and secondary schools in many countries, especially the UK, USA and Australia. This study focuses on thirteen student teachers' understanding of fractions and the related areas of mathematics. It considers the aspects in which they feel confident as well as those which they perceive as problematic.

It is widely accepted that it is essential for teachers (and student teachers) to have a confident level of subject knowledge and this has been the basis of many studies (Aubrey, 1997, Goulding & Suggate, 2001; French, 2005, Brown et al., 1999; McNamara et al., 2002; Murphy, 2006; Huckstep et al., 2002; Askew et al. 1999, Williams, 2008). There have been relatively few studies which focused specifically on student teachers' understanding of fractions, for example, Ball (1990) Miller (2004), Anderson & Wong (2002), Domoney (2002) Toluk-Ucar (2009). If student teachers, once qualified, are going to be able to teach the mathematics curriculum confidently, it is important that problematic areas such as fractions and the related aspects of mathematics are addressed so that teachers do not perpetuate misconceptions or impart any of their own anxieties to their pupils.

The majority of the studies relating to student teachers, considered in preparation for this study, was quantitative in approach and provided some useful generalisable data based on large samples. The findings of these studies indicated useful trends and overviews of areas of difficulty. I decided to adopt a more interpretative approach, in order to provide further insight into the student teachers' individual subject knowledge. The purpose of this study was to discover the "qualitatively distinct ways" (Steffe 1996:321) in which students understood fractions.

The objectives of the study are :-

a) To explore the nature of student teachers' understanding of fractions.
b) To discover the attitudes and beliefs which student teachers hold regarding fractions.

These were kept intentionally broad and the associated research questions can be found in 3.1.1.

In order to achieve this, a phenomenographic approach was adopted. Phenomenography "takes human experience as its subject matter" (Marton & Neuman, 1996). The study is based on the underlying premise that although participants are all undertaking the same task, there will be a number of qualitatively different ways of experiencing or understanding the question or problem which can be observed and identified. Each participant brings his or her prior experience and learning to the task and this affects the way in which it will be undertaken. The intention of this study was to discover the nature of these differences.

This study was undertaken with a sample of thirteen students from the primary PGCE and the BA in Primary Education courses in University B. It was intended that the student teachers would explore, explain and possibly reconstruct their own understanding of fractions. These methods involved the explanation and discussion of their existing ideas and a consideration of any elements which possibly caused confusion. Two collaborative tasks were observed, which involved the sequencing according to magnitude of a series of fractions, percentages and decimals and matching their equivalents. This was followed by reflective group discussions in groups of two or three.

These tasks focused on the part-whole and the measurement context of fractions (Kieren 1976).

A series of diagnostic interviews were conducted with individuals and pairs. A range of questions were included, based on the findings of the observed tasks and the research literature considered. The participants were asked to indicate the questions about which they felt most and least confident, these perceptions were used to structure their interview. This ensured that each interview followed an individual path depending on the student's choices and enabled them to begin with the questions with which they felt more confident. A questionnaire was also used to provide back up information about each participant's qualifications and feelings about their own learning in mathematics.

One of the challenges in the recruitment of the sample was the student teacher's knowledge that the mathematics content of the study was pitched at a level expected of primary school teachers, student teachers and children. This meant the planning and execution of the observations and interviews needed to be handled sensitively. A further challenge was my dual role of researcher and university tutor and the impact this may have had on the student teachers' willingness to participate. A responsible approach was adopted towards the recruitment and involvement of the participants; this was open and honest in order to demonstrate personal and professional integrity at every stage of the study. Great care was taken to set up a comfortable and confidential working atmosphere where students felt able to discuss and share their understanding and feelings.

The study is presented in the following chapters:-

Chapter 2 - Literature Review. A wide range of literature is discussed in the light of the research questions. The essential nature of teacher mathematical subject knowledge is considered. Research studies which consider the learning of fractions and difficulties encountered by primary and secondary aged children as well as adults are discussed. The attitudes and beliefs which influence the learning of mathematics are considered including the impact of mathematical anxiety.

Chapter 3, Research Design and Methodology

This shows how the research questions were to be addressed through the use of questionnaires, observed tasks and diagnostic interviews. It considers how the findings from pilot studies have influenced the final study. The ethical issues and trustworthiness of the study are also considered.

Chapter 4 - Research Findings.

In this chapter the way each method contributed to the data is considered. Main themes which emerge are categorised and explored. These are presented as a response to each of the research questions. The overarching themes are considered whilst still maintaining the focus of the individual responses.

Chapter 5 – Discussion

In this chapter, each of the broad themes are discussed under the appropriate research question and are contrasted and compared with the associated literature. A critique of the research methods is also included.

Chapter 6 - Conclusions.

This chapter considers the main issues arising from the study and shows how these have addressed the research questions. The findings are reviewed and further possible areas of research considered. The original aspects of the study and particular insights are discussed in the light of the findings and within the review of the critical reflection on research methods.

Chapter 2 The Literature Review

In this chapter a range of appropriate literature and research studies will be considered, these will indicate the existing knowledge relating to the learning of fractions in children and adults. The necessity of confident subject knowledge in mathematics to enable student teachers to deliver their lessons effectively and support individual learners will be a key theme. The beliefs and attitudes held by student teachers towards mathematics will also considered, with a particular focus on mathematical anxiety.

2.1 Introduction

In order for student teachers to share their understanding effectively with their pupils it is essential that they hold a secure level of subject knowledge in all areas of mathematics. This is one of the underlying assumptions of this study and research concerning the knowledge which successful teachers require will be a particular focus of the literature review. It is widely acknowledged that fractions is a difficult area to teach and learn. This is evident in research from many countries. The learning in specific areas of fractions where student teachers found particular difficulty will be explored and discussed. The impact of the attitudes and beliefs held by students on their learning of mathematics can not be underestimated; these will be considered within this constructivist approach.

2.2 Subject Knowledge for Teaching

2.2.1 Subject Knowledge Requirements

There is a widely accepted underlying assumption that secure subject knowledge is required if any subject, in this case mathematics, is to be taught with understanding and clarity (Goulding & Suggate,2001; French,2005; Feiman-Nemser & Remillard, 1995; Brown et al., 1999; McNamara et al., 2002; Murphy, 2006; Huckstep et al., 2002; Askew et al. 1999). A basic premise of my study is that the mathematical understanding of the students and their ability to share that understanding with their pupils is integral to their perceptions of themselves as mathematicians and teachers, as reflected by Aubrey (1997: 3) "If teaching involves helping others to learn then understanding the subject content to be taught is a fundamental requirement of teaching". The need for an effective teacher to have a secure knowledge of the subject is supported by inspection evidence (OfSTED 1994; TTA, 1998; TTA, 2002) which identifies teachers' lack of subject knowledge in mathematics as a contributory factor in low standards of mathematics attainment of pupils (OfSTED 1994).

In order to guarantee that all student teachers have a reasonable level of mathematical knowledge, there is a national minimum requirement of at least a GCSE grade C or equivalent in mathematics for all students beginning an Initial Teacher Education course in England (Teacher Training Agency, 2002). Although this is intended to guarantee a basic level of mathematical understanding and competence from which to develop, the acquisition of a successful GCSE does not guarantee that the mathematical knowledge gained at the time will be retained or can be successfully applied to different situations (Goulding et al., 2007).

In recent years the mathematical subject knowledge of teachers and student teachers has been particularly recognised as an issue for government policy makers in the UK (Alexander, Rose & Woodhead 1992; QCA 2002, Williams 2008, OfSTED 2008). The William's Review (2008) focused on mathematics in primary schools and Early Years settings. The recommendations from this report focused largely on the subject knowledge of teachers and student teachers. One particular recommendation, intended to support the mathematical subject knowledge of student teachers, was the introduction of two complementary GCSEs in mathematics, the successful completion of both would then become a requirement for Initial Teacher Training. It remains to be seen whether a significant proportion of the cohort will take *both* GCSEs, but 'deep subject knowledge' may in future become synonymous with passing both mathematics GCSE I and II with at least a grade C. (Williams et al. 2008:10)

Understanding the Score (OfSTED 2008) focused on the inspection findings in England and considered the main elements of effective mathematics teaching in the primary school. This report reiterated that teachers' subject knowledge is considered to have a limiting effect on pupil progress and considered that the best teachers demonstrate a combination of "deep knowledge and understanding of the subject with a well informed appreciation of how pupils learn mathematics" (2008:38) combined with experience of classroom practice. The following Venn diagram from the report shows the three components, with the implication that ideally all teachers of mathematics would lie within the middle section having a balance of all three attributes.

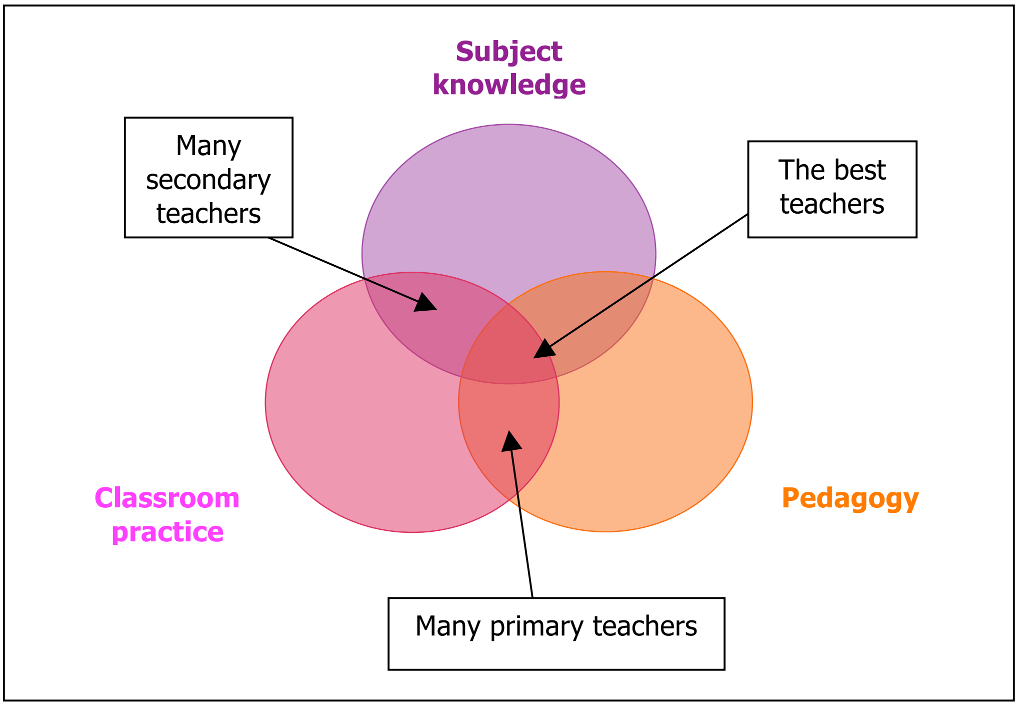

Figure 2.1 Teachers' subject knowledge, pedagogic skills and classroom practice from Understanding the Score (OfSTED 2008:38).

The report also suggested that many primary teachers lack the essential "deep knowledge and understanding" to be an effective teacher of mathematics. The need for this balanced range of mathematical knowledge is supported by Ma (1999) who concluded that no amount of general pedagogical knowledge can substitute for a lack of understanding of particular mathematical concepts. Whilst it is widely considered that secure subject knowledge is an essential element in the successful teaching of primary mathematics, it cannot be assumed that those students with good subject knowledge will necessarily make effective teachers (Tennant, 2006; French, 2005; Askew, 1999). Even students with a sophisticated understanding of mathematics need a perceptive appreciation of the learner in order to be able to share their knowledge effectively.

2.2.2 Types of Teacher Knowledge

The complexities of teaching and learning require that a range of types of knowledge is needed. These were categorised by Shulman (1986) who described seven types of teacher knowledge, all of which are essential for effective teaching in any subject in the curriculum, (see appendix 7.1). Content knowledge can be divided into three areas; subject, pedagogical and curricular knowledge. For the purposes of this discussion, whilst specifically considering mathematics, only two will be considered. First, subject knowledge, this being the substantive knowledge of mathematics and secondly pedagogical content knowledge which in this case would include understanding the effective representation of concepts, useful analogies, how to create suitable examples and the ability to demonstrate and explain mathematical thinking.

McNamara (2002:14) questioned the distinction between subject content knowledge and pedagogical content knowledge, based on the premise that all mathematical subject knowledge is in fact itself "a form of representation" and there is inevitably a large overlap between these two types of knowledge. Dixon (2003) felt that Shulman's categories are limited to a specific representation of knowledge supporting Shulman's own assertion that the "framework is provisional, tentative and probably incomplete" (McNamara, 2002:9). The categories suggested by Shulman did not specifically relate to mathematics but provided a valuable initial starting point from which to consider mathematical subject knowledge.

The evidence of secure subject knowledge and understanding is "*most likely to be found in trainee teachers' teaching*, particularly in how they present complex ideas, communicate subject knowledge, correct pupils' errors and in how confidently they answer their subject-based questions" (TTA, 2002:19). All student teachers are assessed through lesson observation during their training and the complexity of identifying subject knowledge within the process of planning and delivering a lesson is widely acknowledged. This aspect was considered specifically by Rowland et al (2009) who explored further two of the categories (subject matter knowledge and pedagogical content knowledge) identified by Shulman (1986). Their intention was to identify ways of evidencing these aspects within student teacher's lessons. Eighteen categories were identified which were grouped in four areas labeled as the "knowledge quartet" (2003:97). These four areas were foundation, transformation, connection and contingency. *Foundation* includes the knowledge, beliefs and understanding which underpin and inform the teaching and learning choices that teachers made in terms of pedagogy and strategy. *Transformation* is evidenced through putting their knowledge into practice, both during planning and the delivery of the lesson. Key aspects of this are the presentation of ideas and the judicious use of examples. *Connection* refers to "the coherence of the planning or teaching displayed across an episode, lesson or series of lessons" (2003:98). This particular aspect reflects the development of relational understanding (Skemp, 1989) and the connected nature of mathematics (Askew 1999). Connection also includes the structuring of lessons and appropriate differentiation. *Contingency* involves the ability to respond appropriately to the children as the lesson unfolds, including the unexpected, and the incorporation of pupil's ideas to support learning. This is one of a range of studies which considered the link between the students' subject knowledge and their performance in the classroom (Goulding et al, 2003; Huckstep et al, 2002; Rowland et al, 2009; Brown et al, 2001). These studies acknowledged the huge number of factors coming into play when trying to make these causal links. There were student teachers within the sample, who demonstrated a lower level of understanding in their mathematical subject knowledge audit scores than their peers, which was then reflected in their ability to explain and indeed plan effectively, but given the nature of these relatively small studies, only tentative conclusions were drawn.

2.2.3 The Use of Subject Knowledge Audits

A range of studies have considered student teacher subject knowledge through the use of testing and subject knowledge audits,(Goulding & Suggate, 2001; Huntley, 2005). The difficulties in assessing the wide range of mathematics needed to deliver the primary school curriculum are acknowledged. Inevitably the questions posed in audits were mostly of a closed nature and tested an instrumental level of mathematical understanding (Ainley & Briggs, 1999; Murphy, 2006) rather than a more complex relational understanding suggested by Skemp (1989). Some studies considered the areas of mathematics in which the students appear less strong and questioned the effectiveness of using an audit (Goulding & Suggate, 2001; Huntley, 2005; Murphy, 2006). The areas which were found to be less well answered related to algebra, rational numbers and statistics (Ainley & Briggs, 1999).

One of the main roles of the teacher educators, suggested by Dickinson et al, (2004), was that personal subject knowledge should be developed and challenged and to ensure that these changes become reflected in the students' practice within the classroom. This knowledge will continue to develop once the students have completed their teacher education and are teaching. Feiman-Nemser et al (1995) suggested that some aspects of teaching can only develop in the classroom. More specifically Meredith (1993) suggested that students she interviewed felt that the pedagogical knowledge exhibited by teachers may be more closely related to prior knowledge, values and epistemological beliefs, rather than a result of their training. It is possible that only a small proportion of the wide range of pedagogical subject knowledge needed for the effective teaching of mathematics could be considered within some initial teacher training and without the consideration and discussion of attitudes and beliefs, student teachers retain their long held views and are not able to expand their own pedagogical knowledge sufficiently once they begin teaching. It is very difficult to consider knowledge in isolation, especially in the case of mathematics, without considering it in conjunction with understanding (Duffin & Simpson, 2000).

2.3 Mathematical Understanding

2.3.1 Types of Mathematical Understanding

The learning and understanding of mathematics is strongly based on an appreciation of pattern and the relative magnitude of numbers. An interconnected understanding helps to provide an overall structure to mathematics which encourages learners to appreciate the logic of the subject. Skemp (1989) stressed the importance of these mathematical connections. A compartmentalising of concepts tends towards developing *instrumental understanding* "the ability to use algorithms, rules and definitions correctly" rather than *relational understanding*, described by Skemp as "the rich understanding of conceptual relationships and their logical connections" (1989:11). Relational understanding is considered a desirable attribute to be achieved by all learners of mathematics; teachers and students should have developed this level of understanding if they are going to be able to promote this amongst their pupils. This desired depth of relational understanding is not always easy to achieve and involves the ability to transfer knowledge across and within different areas of mathematics (Mitchell 2008). However it was also pointed out by Skemp (1989) and Reason (2003) that there are times when instrumental understanding can have an important place in skill development when the relational understanding is beginning to form.

Sfard (1991) agreed and suggested there is a duality between the two aspects rather than specific and distinct ways of understanding. A very similar distinction was made by Hiebert & Lefevre (1987) when considering conceptual and procedural understanding of mathematics. Hatano (1988) suggested a further level of understanding, known as adaptive reasoning, this was defined as knowing why procedures work. This creates links between having a rich conceptual knowledge and the flexibility to use this understanding appropriately to respond to different types of problem.

These components have been considered further by Kilpatrick et al. (2005) to create a definition of mathematical proficiency as having five intertwining strands, showing the inherent complexity of mathematics. These strands (Conceptual understanding, Procedural Fluency, Strategic Competence, Adaptive Reasoning and Productive Disposition) link strongly to some of the areas already discussed. *Conceptual understanding* and *procedural fluency* would constitute relational understanding as described by Skemp (1989). *Strategic competence,* (Kilpatrick et al., 2005:116) "an ability to formulate, represent and solve mathematical problems" could also be considered as an integral part of conceptual understanding. A certain element of instrumental understanding may come into play whilst procedural fluency develops. *Adaptive reasoning* in this case is described in a similar way to Hatano's definition (1988) this could develop as a result of a linked understanding of the first three strands. The two aspects of mathematical understanding identified by Skemp (1989) permeate these other studies and most of these categories suggested are reliant on the distinction between relational and instrumental understanding.

The fifth strand, *Productive disposition,* is defined as "an habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efforts" (Kilpatrick et al., 2005:116). This introduced an attitudinal quality which could be considered as more of an appreciation of the importance and value of mathematics, rather than an aspect of understanding. Whilst this is an important quality which will no doubt contribute to a greater proficiency in mathematics, it does not seem to be an essential quality in the understanding of mathematics. Ma's (1999) study of Chinese teachers who had become mathematics specialists found that one of the distinguishing features was their attitude towards mathematics supported by their profound understanding of fundamental mathematics rather than a higher level of qualification. She suggested it was possible that this could be attributed to their own experiences in school and their attitude to further professional development.

The value of this more complex interlinked understanding and the categories suggested by Shulman (1986) provide the underlying themes in studies relating to the knowledge required by teachers of mathematics. For example when considering what constituted effective teaching of numeracy, Askew et al. (1999) suggested that areas of mathematical knowledge cannot be separated and that what will ensure greater pupil learning is the "connectedness… in terms of their (the teachers') appreciation of the multifaceted nature of mathematical meanings" (1999:93). This was also reflected by Fischbein who asserts that ideally mathematics should be viewed as a form of human activity (1987). Any mathematical activity requires the use of three closely overlapping dimensions of mathematical knowledge, which are algorithmic, formal and intuitive (which includes common mental models, ideas and beliefs about mathematical entities). This concept of connectedness/relational understanding is a recurring theme in the views of other mathematics educators; Ma referred to ideal teacher knowledge as "A profound understanding of fundamental mathematics" which incorporates four interrelated properties; these being connectedness, multiple perspectives, basic ideas and longitudinal coherence (Ma, 1999:122). Longitudinal coherence is the wider understanding of the pre-requisites and next steps involved in a mathematical concept. This aspect links closely to the area of curriculum knowledge suggested by Shulman (1986).

2.3.2 Constructivism

From a constructivist perspective, learning is a result of an individual's construction of knowledge. This occurs through active cognitive and social engagement in their own experiential world (Von Glasersfeld, 1989). Student teachers bring not only their subject knowledge to their training but also all their own experiences of learning mathematics. Many studies (Ball, 1988; Feiman-Nemser et al, 1995) considered the importance of the role of an "apprenticeship of observation" (Lortie, 1975:61) in students' development towards becoming a teacher. This is the many hours the student will have spent in classrooms observing teachers and building a mental picture of how, in this case mathematics, would be taught. "Learning is a product of the interaction between what the learner is taught and what the learner brings to the learning situation" (Ball 1988: 1). The constructivist approach to learning suggests that the impact of each student's experiences of learning mathematics on their confidence, beliefs and attitudes should not be underestimated when considering their development towards becoming a teacher of mathematics.

Many of the individual student teachers will have developed their own well established methods of working and strongly held beliefs in relation to mathematics; "Constructivism highlights the fact that old ways of thinking are not given up without resistance and emphasises that their replacement or extension to new ways of thinking is guided by already existing conceptions" Booker (1996:382). The opportunity to review and discuss the learning which has taken place was considered as a valuable part of learning by Carpenter and Lehrer (1999). This is a process which involved teacher and learner as the knowledge was constructed collaboratively and was described as a five stage process through which mathematical understanding is promoted, "constructing relationships, extending and applying mathematical knowledge, reflecting on the experience, articulating what one knows and making mathematical knowledge one's own"

(1999:20). This co-constructed knowledge is not only the result of teacher/learner collaboration but is also developed through peer discussion and review.

2.4 The Learning of Fractions

2.4.1 Introduction

Fractions are an integral part of many aspects of mathematics and a secure understanding is necessary in order to develop proportional reasoning. Fractions are also of particular importance in supporting an understanding of algebra, probability and geometry. It is often through fractions that the themes of equivalence and comparison are explored.

There are five interrelated, yet distinct interpretations of fractions, which all contribute to a confident grasp of this area of mathematics (Kieren, 1976). These sub-constructs are defined in the following table.

Sub-construct	**Definition**
Region (Part-whole meaning)	This is a way of representing part of a whole set of objects or complete objects. It involves the partitioning of a shape/number of discrete objects into equal parts, (unitising) or determining how many objects would be in a

	whole set based on a part of the set (re-unitising).
Quotient model	This involves understanding fractions as a result of division. The fraction 2/3 can be interpreted as 2 divided by 3 or the result of sharing 2 cakes among three people.
Measure	This involves using number line representations to demonstrate the relations between fractions and whole numbers as existing along the same continuum at various magnitudes (Hallinen 2009:6).
Operator	This is where fractions are used to transform numbers, it mainly involves the multiplicative aspect of fractions. For example the fraction 2/3 may be perceived as finding two thirds of a given quantity.
Ratio	This involves making “a comparison between two quantities; therefore it is considered a comparative index, rather than a number” (Charalambous & Pitta-Pantazi, 2007).

Table 2.1 The five sub-constructs of fractions and their definitions adapted from Kieren (1976)

Although these sub-constructs reflect very distinct and differing aspects of fractions, Carpenter et al. (1993) suggested that there are three unifying elements to these interpretations of fractions, which are identification of the unit, partitioning and the notion of quantity. These aspects are explored later in the chapter. Fractions are presented to learners in a variety of ways using a combination of written symbols, number lines, diagram/ pictures, concrete objects and real life contexts. Familiarity and confidence in the use of an appropriate representation is necessary in creating an effective personal mental model (Martin 2004).

The five sub-constructs above are interlinked and the learning in one area will support the understanding of the related aspects. Whilst the learning of the sub-constructs above can not be described as consecutive and each learner's route will be different, Kieren (1993:66) suggested that there are four connected types of mathematical knowledge which can be considered in terms of a personal understanding of rational numbers. These were *ethnomathematical,* which is gained by living in a particular environment and the associated experiences of early fractions, *intuitive* which involves the use of thinking tools, imagery and an informal use of fraction language, *technical-symbolic* which develops as the result of working with symbolic expressions of fractions and *axiomatic-deductive* which is demonstrated by logically applying the ethnomathematical, intuitive and technical-symbolic knowledge and organising these preceding forms of knowledge appropriately within an axiomatic structure. This *axiomatic-deductive* level of knowledge develops through a confident application of the other three aspects and a connected understanding of the five sub-constructs of fractions.

2.4.2 Difficulties in Learning Fractions.

There is a wide range of research literature which provides evidence of the ways in which primary and secondary aged school children understand fractions and the types of difficulties they encounter. It is widely accepted that children find fractions difficult to learn as typified by Deheane's statement (1997:87) "All fractions except ½ and ¼ have caused extraordinary conceptual difficulties in centuries past and still impose great hardship on today's pupils". This view is supported by a variety of studies from a range of countries. For example, children in America (Watanabe, 2001, Fantano, 2003 and Mack, 1993), South Africa (Lukhele et al., 1999), Australia (Hunting, 1984) and Holland (Streefland, 1993) all seem to share a similar range of difficulties. Similar problems in learning fractions are regularly reported in the UK, for example, "Children working at all levels had difficulties with prime numbers, equivalent fractions, simple ratios and proportional reasoning" (QCA, 2002:31). Similar difficulties were also reported in Scotland, "Fractions, percentages and ratios is the area of the 5-14 Programme that causes the greatest difficulties for pupils in Scotland… producing the worst performance in any section" (Assessment of

Achievement Survey, 2000 in Howe et al, 2002). The impact of a lack of understanding of fractions in the primary school was considered to have an impact on the study of other areas of mathematics. Knowledge of fractions is necessary to support the understanding of proportional reasoning, algebra, probability and geometry (Kieran 1993). A certain level of understanding is needed in every day life for example, to compare the value of discounts, calculate dosages or estimate quantities when cooking or evaluate the opportunities of winning in various games or in the lottery (Meert et al., 2009). Several studies suggested that fractions and proportions are still difficult to understand for many adults (Bonato et al., 2007, Meert et al., 2010, Morris, 2001, French, 2005).

As indicated above there is a wide range of research which has explored various aspects of fractions and considered some of the reasons why they might prove problematic to the learner (Ball, 1993; Carpenter et al., 1993, English & Halford, 1995, Hunting 1984, Ryan & McCrae, 2005, Oppenheimer & Hunting, 1999, Wong & Evans, 2007, Rees & Barr, 1984). Given the wide range of studies available it is not feasible to consider all aspects within this review. Specific areas which became evident in the pilot studies and earlier literature reviews have therefore guided the focus. In relation to each aspect a range of research studies will be considered which focused on primary and secondary aged pupils as well as adults where appropriate. It is acknowledged that there are fewer studies available to consider adults' understanding in this area (Bonato et al., 2007, Meert et al., 2010).

2.4.2 (i) Whole Number Bias

Most children's early mathematical experiences have put great emphasis on the ability to count and on one to one correspondence. Generally in children's early experience of natural numbers each number has its own unique value, which can be counted as a discrete quantity and is represented in a systematic way. Much of the early mathematics curriculum inevitably focuses on gaining a sound initial understanding of the whole number system. Whilst this is an essential basis from which to build mathematical understanding, it also creates a fundamental difficulty. In contrast to that initial understanding, when looking at a fraction there are two numbers now representing a single value (Westwell, 2002, Liebeck, 1984, Mack in Carpenter et al., 1993, Niekerk, 1999).

Newstead & Murray's study (1998) considered the mathematical understanding of fractions in children between eight and eleven and identified that one of their most common errors was an inability to see the fraction as a single quantity. They tended to respond to the numerator and denominator separately, taking each at face value, especially when considering the addition of fractions or making a comparison of size. This is referred to as whole number bias (Ni & Zhou, 2005). This inclination to consider each part of the fraction as a separate quantity is strongest in those who do not have a clear appreciation of the relative size of the rational numbers they are working with. This lack of distinction between the fraction and a pair of natural numbers is described as N-distractors by Streefland (Carpenter et al, 1993) who identifies differing levels of resistance to this tendency as the understanding develops. This fundamental difficulty of interpretation and representation seems to underpin the different aspects of fractions and is inevitably going to hinder further steps in the development of understanding. Newstead & Murray suggested that the more formal writing of symbols and symbolic algorithms should be "delayed until the children have had the opportunity to conceptualise fractions as single quantities" (1998: 5).

Linked to the issue of *whole number bias* (Ni & Zhou, 2005), is the understanding of equivalent fractions as an area of difficulty identified in a range of studies (Ball, 1993, Graham, 2003, Westwell, 2002, Nunes & Bryant, 1996). Natural numbers have an individual value; however there can be an infinite number of comparable fractions for each rational number which can be shown in different yet similar representation, this level of abstraction can be the source of difficulty (Booker, 1996). Newstead & Murray's (1998) study suggested that children can often generate equivalent fractions using a practised algorithm and by building a pattern but are generally unable to apply them in problem or real life situations. An understanding of equivalent fractions is needed, if effective comparisons are to be made, in order to appreciate the magnitude of less familiar fractions. Using one part of the fraction to make comparisons seems to be a common approach, for instance, treating the denominator as a whole number. Behr et al. (1986) observed an inability to identify the larger fraction if comparing unlike denominators.

This difficulty has been found to be an issue for secondary aged children (14 year olds) as well as for younger ones. Pearn & Stephens (2004) used probing interviews to explore their pupil's understanding of fractions and categorised their responses into three groups (2004:432). The *Proficient multiplicative thinkers* were correct and efficient in working with fractions, the *Residual whole number thinkers* were generally confident but applied inappropriate whole number approaches with less familiar questions or fractions and the *Default whole number thinkers* mostly treated the numerator and denominator separately and discarded the ratio between them. Their study showed that some students within the secondary school still hold this misconception and that it was most apparent when comparing the size of two fractions. The understanding of fractions by adults was considered by Bonato et al. (2007) and initially it was assumed that the whole number bias displayed by children would not be apparent in "well educated adults" (2007:1411). However as a result of their study, where adults were required to compare fractions with the same and differing denominators, they conclude that the whole number bias demonstrated by many children is often carried over into adulthood.

A range of studies have considered the comparison in magnitude of fractions (Bonato et al., 2007, Meert et al., 2009, Meert et al., 2010) which involved considering the reaction times of secondary pupils and adults when establishing which is the larger fraction or comparing to a suggested fraction. This was undertaken initially with fractions with the same, and then differing, denominators. The *distance effect* was considered within these studies, this is the increasing amount of time taken to decide which number is larger/smaller as the numerical distance between them decreases. It was found that learners were able to decide that 7/8 was larger than 1/4, as there was a greater distance between them and they generally took longer to decide when the fractions appeared closer in size, for example 7/8 and 9/10**.** Some studies found that some secondary aged pupils had difficulties in understanding that there are numbers between the natural numbers (Smith 2005) and that they still needed to grasp the concept of *density* where each number does not have "one and only one successor" (Meert et al., 2010:245). In these studies some pupils were unable to appreciate the continuous nature of the number system; that between every pair of natural numbers, a range of fractions can occur.

Bonato et al. (2007:1411) suggested that an adult can process a fraction when they understand its numerical magnitude and can represent it in the appropriate place in a continuum of real numbers. This understanding of the magnitude of fractions was considered by requesting the participants to compare a range of fractions initially with the same denominators, and then with different denominators, so a direct comparison of the numerators was not possible. They also suggested that *whole number bias* is consistent with the use of componential strategies to understand a fraction's magnitude. The participants, who were university students, found difficulty in representing the meaning of a fraction in terms of the numerosities of the numerator and of the denominator, which suggested that the real value of the fraction was still not readily accessible to them. These studies (Bonato at al., 2007, Meert et al. 2010) also suggested that adults, in this case university students, might be able to circumvent this particular problem through the use of a range of strategies that rely on the processing of the integer components when considering the relative size of numerator and denominator. Schneider & Siegler (2010) replicated and extended the study of Bonato et al. (2007) with University and Community College students in the USA. Their findings echoed those of Bonato et al. (2007) they also noted the similarities of strategies used when working with whole numbers and fractions.

2.4.2(ii) Part/whole Understanding

The part/whole construct involves the ability to partition a quantity or shape into equal parts. This can involve the partitioning of a continuous quantity, (for example, length or area), or of a set of discrete objects, into equal sized subsets or subparts. There are many pre-requisite skills required in order to understand this concept and the representation of wholes and parts in a pictorial form.

The seven criteria for the operational understanding of the spatial part/whole of a fraction as identified by Piaget, Inhelder and Szeminska (1984) give a useful indication of the complexity of a seemingly simple concept. These criteria are:-

- A whole region is seen as divisible
- The whole can be split into any number of parts
- The parts must exhaust the whole
- The number of parts do not match the number of cuts
- The parts must be of equal size
- The parts can be seen as wholes in their own right.
- The whole is conserved even when cut up into pieces.

In order to confidently apply the part/whole construct all these criteria must be understood. Streefland (cited in Carpenter et al., 1993), extended this list to include repeated halving to make quarters, having identified this as a common response in young children. The part/whole construct is generally the first interpretation of fractions that children meet in school usually presented in a pictorial form for accessibility (Clarke, 2007, Lamon, 1999). Lamon suggests that "mathematically and psychologically, the part-whole interpretation of fractions is not sufficient as a foundation for the system of rational numbers" (Lamon, 2001: 150). This focus on one construct can be limiting and does not necessarily support the wider understanding of fractions as numbers as the measurement construct might do.

An understanding of the whole, and yet, parts being seen as wholes in their own right, is a sophisticated idea for children to appreciate. Steinke (2000:147) described the concept as having an understanding that the whole and parts of a quantity are in existence at the same time and that in some cases a part can then be considered as a whole, for example considering a cake cut in half and then each half being further partitioned. She proposed that this is one of the most important transitional concepts from concrete to abstract thinking. This seems to be the crux of the understanding of the whole/part concept and an aspect which may cause difficulties for many children and some adults. When this is demonstrated diagrammatically rather than practically, it can further reinforce the abstract nature of the concept.

The whole/part construct was also considered by English & Halford (1995) who viewed this as requiring the concept of inclusion, considering the use of rectangular diagrams with shaded and unshaded parts which were combined to form a whole. They suggest the unshaded parts may often tend to be disregarded and become more of a counting or relational mapping exercise. This was further reflected in Ding's (1996) study which considered what constitutes a unit. If more than one unit was included in an example, the total number of parts was counted rather than a whole unit and a fraction of another. This showed an inability in some children to identify the base unit, possibly when the shading is more prominent and may match the whole unit so it appears more of a discrete object problem rather than a whole/ part question. The understanding that the whole remains constant, and that when divided into parts there will be no remainder, is another aspect identified as difficult for children (Neumann, 1997). Frobisher et al. (1999) described children dividing a section of string in half and cutting off pieces in order to make equal parts with little regard for the original length of the string. In contrast Smith (2002) suggested that this is not an alien concept outside of mathematics, considering "the distinction between part and whole is fundamental to the way infants instinctively organise their experience and is reflected in everyday language" (2002:93), for example, the use of full, empty and half full is regularly used in the early stages of non-standard measurement. There are a large number of studies which identify difficulties in children's understanding of the whole/part concept. In these studies, for example Clarke, (2007) and Lamon (1999), this early experience does not seem to have made an impact on many children's understanding when studied at a later stage or there is a lack of connection made between these types of early experience.

A further aspect of the pre-requisite understanding of the whole/part construct is the inverse relationship between the number of the parts and their relative size, (Nunes & Bryant. 1996, Clarke et al.2007). It can be difficult to appreciate that the larger the denominator, the smaller the fraction as this feels counterintuitive, and consequently this causes confusion for many primary school children. The inability to visualise the size of a fraction is considered a common problem by Graham (2003) who stresses the need for a strong mental picture from an early age is important so that the symbols used to record simple fractions have meaning. The use of a circle seems fairly universal, possibly as it can easily be envisaged and if parts are not included or counted, this can easily be recognised as a whole.

The region representation is often the basis from which fractions is initially taught; failure to grasp the concept at this point will affect the levels of understanding of the related issues and possibly other areas of mathematics. Gabb (2002) considered the initial teaching of fractions observing that many children find it easy to "do" halves and quarters without full understanding, having a pictorial representation to work with. However, she suggested that where there is a lack of basic understanding this often only becomes evident when work on the next step of learning fractions is less familiar and more abstract.

Closely related to the whole/part representation is the consideration of a collection of discrete objects as a whole. Dickson, Brown & Gibson (1984) identified the similarities between this and the whole /part representation but suggested that a whole, when presented as an area or region, usually as a shape or object, is more accessible to children than as a set of discrete objects. Similarly, English & Halford (1995) regarded the most difficult representation for learners to appreciate was a set of discrete objects as a whole entity. However Novillis (1976) (cited in Orton & Frobisher, 1996) considered the development of the understanding of fraction in ten and eleven year olds and concluded that these aspects were of approximately the same difficulty.

Dickson, Brown and Gibson suggested that the representation of fractions when considered as a part of a whole is "inconsistent with the very existence of such improper fractions" (1984; 279). They suggested that the use of a number line enables a clearer understanding about the natural place of improper fractions. This was reflected by the National Mathematics Advisory Panel Report in the USA who stated that, "Conceptual and procedural knowledge about fractions with magnitudes less than 1 do not necessarily transfer to fractions with magnitudes greater than 1" (2008 :28). The report suggested that a problem arising in "fair-share" or other part-whole models was the difficulty of applying this type of representation to fractions greater than one including improper fractions or mixed/fractional numbers.

2.4.2 (iii) Unitising and Re-unitising

An essential aspect of the *part/whole* fraction understanding is the concept of *partitioning* a whole, into *equal* parts and also reconstructing those parts back to create the original whole. This unitising (and re-unitising) refers to the "process of constructing chunks" that constitutes a given quantity (Lamon 2005:78). It is considered to be a subjective process, which most people do naturally. It requires the understanding of partitioning and also of the equivalence of fractions. This strategy is generally used to work out other quantities in relation to a specified unit. It is considered to promote more flexible ways of thinking about fractions when they occur in a variety of representations (Clarke, 2007, Baturo, 2004). However, these essential skills of partitioning, unitising and re-unitising are also considered to be the source of a range of conceptual and perceptual difficulties in interpreting rational-number representations. Re-unitising particularly, the ability to think flexibly and to change their perception of the unit, proves problematic for many children and adults (Kieren, 1983; Baturo 2004, Behr et al., 1992; Lamon, 1999). The "draw me a whole" task used by Clarke et al. (2007), where a child is shown a fraction of a shape and asked to draw what the whole shape could be. This was particularly effective in providing an insight into eleven year old childrens' understanding of the whole/part concept, where 77% of children were able to use unitising successfully to answer questions. Approximately 65% of children answered re-unitising questions which involved making two thirds up to a whole one, fewer children (40%) were able to reunitize when the fraction was larger than one. Children's (and adult's) understanding of fractions can be hindered by a lack of realisation about the importance of the unit and the appreciation that it can be different in each situation (Lamon, 1999).

2.4.2 (iv) Duality

The usual mathematical representations of this aspect of fractions can be confusing, for instance 2/3 = 2/3 (two divided by three equals two thirds is an example considered by Smith (2002:95). It is not as perplexing as it initially appears, the left hand side being the division statement and the right hand side being the resulting fraction solution. Gray and Tall (1994) suggested that the way fractions are represented as a numerator/denominator creates a barrier to understanding especially in light of the similarity with the representation of a quotient when shown as dividend/divider. The similarities which exist, as both are ratios, does not necessarily support understanding as fractions, when displayed in this way, can represent both the process of division and the result of that process. Westwell (2002) suggested that capable students are able to recognise both forms and can comfortably use both representations.

When considering the understanding of fractions some representations can be viewed as either *Processes* or *Objects*. Gray & Tall (1992) described these aspects of mathematics as *procepts* which they defined as "a combined mental object consisting of both process and concept in which the same symbolization is used to denote the process and object which is produced by the process" (1992:217). An example of this would be the consideration of a fraction written as a number - fraction bar - number composite symbol e.g. 2/3. This can be addressed as two divided by three or the result of the process. This complexity of "Fractions as processes or fractions in processes" and the duality between these two situations (Sfard, 1991) was considered as the source of a range of difficulties.

2.4.3 Student Teachers' Understanding of Fractions

Much of the research in the UK which has considered the mathematical subject knowledge held by teachers and student teachers is quantitative in nature (Rowland et al, 1998; Goulding et al, 2003; Huntley, 2005; Draper, 1998; Murphy, 2006, Goulding & Suggate, 2001). These studies expressed concern about the levels of confidence held by student teachers and the difficulties student teachers encounter in the application of their mathematical subject knowledge. A further range of studies found a general lack of confidence amongst student teachers in mathematics. These studies provided more insight into student teachers' consideration of fractions and the difficulties encountered. These included McNamara (2002), Morris

(2001) and French (2005) in the UK, Moss (2005) and Post et al, (1991) in the USA, by Clarke (2006) and Hamlett (2007) in Australia and also Tsao (2005) in China. There have been fewer studies which focused specifically on student teachers' understanding of fractions, for example, Ball (1990) Miller (2004), Anderson & Wong (2002), Domoney (2002) Toluk-Ucar (2009).

In a small scale study, four PGCE students were interviewed by Domoney (2002) to explore the extent of their knowledge and understanding relating to fractions. It was ascertained that although they had a strong visual view of fractions, they did not seem to have had much experience of "thinking of fractions as numbers" (2002:64). These findings indicated an inclination towards a *whole number bias* (Ni & Zhou 2005), where there is a tendency to treat the numerator and denominator separately. These findings were echoed by French (2005), although the focus of his study was not specifically considering fractions; they were used as a vehicle to consider aspects of mathematical understanding and communication. Through the students' explanations misconceptions emerged; the three questions which had caused the most difficulties were discussed and all of these related to fractions. Similar difficulties were also reported in the USA. Moss (2005) found that whilst some of the prospective elementary teachers who were participating in her study were confident, the majority were not. One comment (2005:359) in particular reflected their feelings, "Oh fractions! I know there are lots of rules but I can't remember any of them and I never understood them to start with." It was felt that comments such as these often proved as pertinent and revealing as the written answers to mathematics questions. Fractions, in relation, to division were considered by Ball (1990) with both elementary and secondary pre-service teachers. She found that many in her sample had significant difficulties with the *meaning* of division by fractions. It was found that whilst most could perform the calculations, their explanations tended to be rule-bound, with a reliance on memorising rather than conceptual understanding. They found it difficult to justify their answers.

2.4.4 Developing a Conceptual Understanding of Fractions.

The development of conceptual understanding, which enables learners to apply their knowledge flexibly and to use a variety of representations, is the basis of several studies. Hecht et al. (2003) considered primary school aged pupils solving fraction problems. They suggested that those with stronger conceptual knowledge (Hiebert & Lefevre, 1987) were able to select appropriate strategies relating to each question and review the relative success of each procedure. For example, pupils were more likely to be able to add fractions with different denominators (1/4 + 1/3) if they had an appreciation of the unequal size of these fractions. They considered that conceptual understanding may support the development of an effective mental model, for example, part-whole understanding, which could provide an effective mathematical structure in problems relating to fractional quantities.

The value of a combination of conceptual and procedural knowledge was acknowledged by Hallett et al. (2010) who surveyed nine year old children's knowledge of fractions by studying the individual differences in the use of conceptual and procedural knowledge in their responses to a variety of questions. It was found that, in many cases, conceptual and procedural knowledge appeared to develop in parallel and they were used in response to particular types of questions rather than being specifically favoured as an overall approach. The children fell into five clusters where they used differing levels of conceptual and procedural knowledge, with the anticipated outcome that children with a high level of both types of knowledge performed more strongly. This was also reflected in Hecht & Vagi's (2010) longitudinal study which considered how a range of intrinsic and extrinsic factors affect the development of specific types of mathematical skill, including fraction computation and estimation. One specific area of focus was the consideration of the part-whole and measurement aspects of fractions and decimals. They reported findings which were consistent with the view that the development of procedural and conceptual knowledge are influenced by each other and developed concurrently in some children.

Without a sound conceptual understanding and appropriate real life application, many children fall back on the use of half–remembered rules and algorithms. "All the evidence indicates that many children have serious misconceptions of the concept and operate fractions using incorrect rote procedures" (Orton & Frobisher 1996:107). This assertion was supported by Mack (1993) who found that in the cases where children have learnt by rote rather than understanding that the use of this rote learning inhibited the children in developing a meaningful understanding of the fraction symbols. Lukhele et al. (1999) reinforced these findings in their study into secondary aged pupils' understanding of the addition of fractions. They suggested that most learners' errors are based on treating the numerator and denominator separately combined with the "urge to use familiar (even if incorrect) algorithms for whole number arithmetic" (1999:1). They found that the dominant line of reasoning was to trust answers obtained by an established algorithm without considering whether their answers were appropriate or not, they suggested, in this case the children are not used to "making sense of maths".

Adults do not necessarily develop a suitable level of conceptual understanding of fractions, this was evident in "The Rational number project", a large study undertaken in the U.S.A., which investigated pupil learning and teacher enhancement. They found that many of the practising teachers, (Post et al., 1991) and student teachers (Sowder et al., 1993) who participated in the study had only a procedural understanding and had difficulties with conceptual questions relating to rational numbers.

2.5 Types of Representation

The five sub-constructs of fractions can be represented in a variety of different ways, so throughout their schooling learners will encounter a range of models. The inclusion of a range of shapes in textbooks and other teaching materials where a number of equal pieces are presented to be shared among a group of people are particularly common. These are typically shown as circular, (pizzas and pies) and rectangular (bars of chocolate) diagrams, where shading indicates the identified fraction (Hallinen, 2009) Circles in particular are regularly used by teachers to demonstrate the part-whole concept (Kleve 2009). If these are not explained and modelled appropriately there is the possibility that the questions using these diagrams can become shading and counting exercises (Clarke et al. 2008). This was reflected by Sowder (1988) who considered some of the children participating in their study as "model poor" with many tending to focus largely on circular models as the only possibility. Kleve's study (2009) focusing on one teacher's experience of teaching fractions, found an inclination to use circles to explain when children were experiencing difficulties. This, however, did not prove effective in supporting an understanding of fractions larger than one. These models are used to represent continuous sets; Singer-Freeman & Goswami (2001) also considered the models used for discontinuous sets, for example, numbers of dots. They explored pre-school children's understanding of proportional equivalence by considering familiar references such as pizza and boxes of eggs. They compared the children's responses when matching isomorphic (pizza to pizza) models for a quarter, half and three quarters to those made when non-isomorphic models (pizza to egg box) were used. They found most children were able to make effective comparisons between the two models for a half and some were successful for three quarters. They felt the use as half as a boundary was evident at this pre-school stage (Spinillo, 2004).

Keijzer & Terwel (2003), in a study with nine and ten year olds, contrasted the use of the bar and number line representations with the use of the circle representation and fair sharing. The group using the number line showed a greater proficiency in solving a range of fraction related problems, than those using the other models. It was felt that the number line offered a more transferable model which could be used to cater more easily across different contexts. A range of manipulatives was considered in Cramner et al.'s study (2009). The focus was on the choices of models made by ten and eleven year old pupils. The type of support offered by each model in terms of understanding the part-whole concept was considered. It was thought that these choices might reflect the way the pupils were taught initially, especially if they used particular models consistently for different types of questions. The use of number lines used for representing fractions were thought to be important though they were considered to be more abstract than other continuous models. It provided a link between both conceptual and procedural knowledge. The value of this continuous model is that a length represents a unit and has no separation between consecutive units. It was suggested that in order to make sense of this as a model the symbols and the visual cues must be combined in its interpretation.

2.6 The Biological, Cognitive and Psychological Influences on the Learning of Mathematics with a Specific Focus on Fractions.

Much research suggests that "humans seem equipped evolutionarily and developmentally to manage natural frequencies but not proportions" (Gigerenzer & Hoffrage, 1999). Deheane (1997), Butterworth (2000) Meagher(2002) and Jacob & Nieder (2009) all suggested that the brain is wired to handle whole number effectively and the reason that a great many children find fractions difficult to learn is because "their cortical machinery resists such a counterintuitive concept" (Deheane 1997:7). He suggests that our brains have an automatic inclination to recognise and add whole numbers, linked to numerosity, which begins to develop in preschool children. Whilst this is useful for many aspects of mathematics, it is possibly a hindrance when dealing with rational numbers. The associative aspect of memory probably adds to the difficulty here. Whilst this usually supports a developing "relational" understanding (Skemp 1989) it may be making inappropriate links to the whole number system and helping to cause the inability to see the fraction as a single entity. Domoney supports this, suggesting that using rational numbers is completely different from working with whole numbers and is not a "natural thought process" (2001:1). A further example of the counterintuitive nature of fractions is considered by Smith (2002) looking at multiplying and dividing fractions in particular, having learnt earlier in school that multiplication makes numbers larger, students do not retain their work on fractions as it does not fit with their other understanding of the number system as a whole.

Deheane (1997) and Ni & Zhou (2005) considered the rate at which the brain develops. They suggested that the differing areas of the brain which contribute to counting, number manipulation and use of symbols are all linked and controlled by the pre-frontal cortex area. As this is not so developed in children it is possible that they don't have a "large repertoire of refined control strategies to avoid falling into arithmetical traps" (Deheane 1997:138). This suggests that it takes considerably more effort on the learner's part to understand these "counterintuitive" concepts and that learners need to form their own mental image or model to make sense of the conflicting information. This is supported by a study conducted by Jacob & Nieder (2009) who found that there are specific neurons which particularly respond to fractions. These were discovered whilst scanning adult brains. Fractions were shown to the participants numerically and also in words, and the firing patterns of the neurons were then observed. A notable difference was identified between these responses and those in the control group who were shown ratios of whole numbers. This seemed to suggest that we react to fractions without processing them as actual numbers, when presented as words or symbols which may help to account for the difficulties some people encounter and begin to explain the natural inclination to think in terms of discrete numerosities rather than in terms of fractions or proportions.

2.7 The Attitudes and Feelings towards Mathematics, and in particular Fractions, held by Adults and more specifically Student Teachers.

2.7.1 Introduction

When considering the learning of mathematics, it would be unrealistic to consider the matter of gaining subject and pedagogical knowledge in isolation. Each individual, although theoretically experiencing the same opportunity, is building on their own prior learning and experience (Marton & Neuman, 1996). In the process of learning any subject we all bring a complex mixture of beliefs, emotions and attitudes to each situation. Each learner's prior experiences and reflections have an effect upon the way that learning is approached; these were referred to by Crook & Briggs (1991) as "mathematical baggage". A learner's understanding of mathematics will be "shaped by their self confidence, their repertoire of strategies, what they are able to remember about related areas as well as what they believed about the fruitfulness of trying to figure out the problem” (Ball 1990: 461).

2.7.2 Mathematical Anxiety

Due to the nature of mathematics and the teaching of mathematics many adults have been reported to have a significant disinclination and possibly a dislike of mathematics (Smith, 2002, Crooks & Briggs, 1991, Evans, 2000; Dixon, 2003; Benn, 1997 Boaler 2009). Smith suggested that many adults recall the feeling of “tension, panic or impending crisis” (2002:152) when entering a mathematics class, which would inevitably affect their ability to participate and learn. This feeling has been described as maths anxiety, (Buxton, 1981, Ball, 1990, Bibby, 2002, Ashcraft, 2009, Chinn, 2010). Bessant’s definition reflects the resulting characteristics of this anxiety, “debilitating test stress, low self-confidence, fear of failure and negative attitudes towards mathematical learning” (1995:327).

This anxiety has been considered to fall into two types, firstly there are the socio-cultural factors, one consequence of which is the commonly held belief that only “very clever people can do maths” (Chinn 2010:62) and the issue of adults who feel that it is socially acceptable to admit to being unable to do mathematics, a view widely reported as prevalent in the UK (BBC, 2008). The second type of anxiety is thought to cause “mental blocks” in the process of undertaking mathematics (Chinn, 2010). Hannula et al. (2004) also suggested that these affective qualities of beliefs, emotions and attitudes all have a strong effect on our ability to learn mathematics. The most intense of these are emotions, which inevitably affect cognition. Emotion can diminish the capacity to listen (Wragg & Brown, 2001) and can adversely affect the working memory and hence the ability to pay attention and remember (Buxton, 1981, Ashcraft, 2009, Chinn, 2010). These emotions play an important role in forming coping strategies and adapting to new situations but also can impair the ability to learn when they take the form of anxiety. Attitudes to learning, in this case mathematics, may have been the result of many lessons with the resulting successes or perceived failures or may have been formed by one significant incident (Tooke & Lindstrom, 1998).

It has not only been student teachers in the UK who have experienced anxiety over their own learning and the teaching of mathematics. Bowd & Brady (2001) suggested that this is generally reflected within the student teacher population in the USA. It is considered likely that mathematical anxiety affects student teachers delivering their lessons and a range of studies (Bowd & Brady 2001, Tooke & Lindstrom, 1998) question whether these student teachers may have passed their own anxiety on to their classes. Studies have shown that women demonstrated a higher level of mathematical anxiety than men (Henningsen, 2001; Evans, 2000; Benn, 1997, Boaler, 2009). Similarly in a small scale study in the USA focusing on children aged 10 and 11, Beilock et al. (2009) suggested that female teachers' mathematical anxiety had consequences for achievement of girls in their classes. They speculated that the nature of the role model demonstrated by these female teachers may have promoted some commonly held gender stereotypes to their female students through their own maths anxieties. The study considered the students' achievement in mathematics for one academic year but did not include any specific mention of fractions. They concluded that anxiety relating to mathematics can be diminished through further education and that a greater focus needed to be given to developing a positive attitude as well as strong skills in mathematics in order to teach the subject effectively. This is echoed by Kilpatrick et al. (2005) who included a *Productive Disposition* as one of the key features of proficiency in mathematics. An element of this was "a belief in diligence and in one's own efforts" (Kilpatrick et al., 2005:116).

Several studies (Bowd & Brady, 2001, Evans, 2000) considered that mathematical anxiety can reduce students' levels of confidence in their ability to teach mathematics. A small scale study by Gresham (2008), which involved interviewing twenty pre-service teachers in the United States of America, suggested that their attitudes played a crucial role in their beliefs about their own efficacy in the teaching of mathematics. It was unsurprising that the data showed that those who held negative attitudes toward mathematics also had the highest levels of anxiety and that the students with the lowest degree of mathematics anxiety had the highest levels of efficacy when teaching the subject. This was also reflected by Ball (1990) who suggested that much of the knowledge student teachers bring to their initial teacher education is framed negatively. Ideally initial teacher training should help build a level of mathematical resilience, which is the opposite of 'learned helplessness' where students tend to lack strategies to cope with any barriers or difficulties (Dweck, 2000). In order to develop mathematical resilience, a student needs to believe that they can continue to learn and grow in mathematical thinking and understanding (Johnson–Wilder, 2008).

Most studies which consider mathematical anxiety reflect on the wider learning of mathematics and the situations where anxiety was more prevalent. Chinn (2010) focused on learning situations, for example, taking a test or receiving a report. However he also included some reference to specific areas within mathematics, which were long multiplication, long division and fractions. Over 2000 secondary school students were asked to rate their levels of anxiety from 1 (highest) to 20 (lowest). Although there was not a wide range of areas included to form a useful comparison, presumably the inclusion of these areas were because they were deemed stressful for this age of students. Long division without a calculator was considered the most stressful with an anxiety factor of three whereas "doing fraction questions" scored seven as a mean across the age groups considered. As very few areas of mathematics were included in the questionnaire, it can not be considered particularly significant. In my own experience over a ten year period of working in initial teacher training, it has not been uncommon for student teachers to have expressed a level of anxiety about or a dislike of fractions. These attitudes were reflected in studies by Moss (2005) and Brown et al (2007) and Green & Ollerton (1999).

Maths anxiety can manifest itself in different ways and it is possible to still feel positive in those areas of mathematics in which a level of confidence is felt. Seemingly competent student teachers may feel anxious about teaching and learning mathematics even though this conflicts with how they view their overall professional capacity (Bibby 2002). Apart from explicit comments relating to feelings and uncertainty, anxiety may display itself through exclamations, nervous laugher, giggling, sighs, and shaky voices all of which contribute information about the interaction between feelings and understandings (Buxton, 1981, Chinn, 2010, Bibby, 2002). Indicators within speech may also show anxiety for example, the regular use of disclaimers like only and just, used as qualifiers within explanations. The inclusion of *hedges (two categories of words)* within mathematical discussion was considered by Rowland (1995) who suggested that these were indicators of uncertainty. The first category, 'shields', which usually precede a suggestion, for example, 'I think', 'maybe', and secondly 'approximators', such as 'a little bit' or 'fairly,' the effect of which is to give vagueness to an assertion. His study considered how hedges were used by children between the ages of 10 and 12 when discussing their investigative work, especially generalisation; these findings are also applicable across other areas of mathematics. This particular use of language may also be accompanied by a high level of redundant language and/or apologies used regularly to pre-empt any possible criticism (Bibby, 2002, Buxton, 1981).

Anxiety relating specifically to fractions is a recurring feature in many studies which consider the learning of fractions particularly in adults or pupils in secondary school. Wu (2008:3) described this as a "morbid" fear in US secondary aged pupils, suggesting that this could be attributed to fractions often being children's first experience of the abstract nature of mathematics. Rayner et al. (2009) considered the effect of maths anxiety on pre-service teachers' procedural and conceptual understanding of fractions and suggested that for some students their own weaker mathematical content knowledge may also be a source of their mathematics anxiety; it was also possible that there was a lower level of anxiety in those who had a stronger conceptual understanding.

It is also the case that the feeling of an element of anxiety is not entirely negative and it is an inevitable consequence of learning situations and indeed life in general. A certain level of adrenalin caused by anxiety can promote the urge to succeed and perform well. It should not be just considered in the light of those who consider themselves less confident in mathematics; (Askew & Williams, 1995) suggested that a great level of anxiety can often be felt by those with high expectations of their own achievement in mathematics.

One of the underlying themes of this study is the essential requirement for all primary school teachers to have strong subject knowledge in mathematics. Literature and studies relating to this have been considered in this chapter. A further intention has been to explore the existing knowledge relating to children's and adults' understanding of fractions. The feelings and attitudes held by adults and specifically student teachers towards mathematics have also been considered. These themes have influenced the planning of this study and their impact will be apparent in the following chapters.

Chapter 3

Research design and Methodology

3.1 Introduction

One of underlying assumptions of this study was that the mathematical understanding of the students and their ability to share that understanding with their pupils is integral to their perceptions of themselves as mathematicians and teachers. Aubrey (1997: 3) claims that "If teaching involves helping others to learn then understanding the subject content to be taught is a fundamental requirement of teaching". The purpose of this research was to explore individual student teachers' understanding of fractions which is widely acknowledged as an area of mathematics which is difficult to learn and teach (Morris, 2001, Hunting, 1984, Post et al. 1991, Clarke, 2006, Behr et al., 1992).

There have been many studies which adopt a quantitative approach when considering the range and level of mathematical subject knowledge held by teachers and student teachers (Goulding et al, 2003; Huntley, 2005; Draper, 1998; Murphy, 2006, Goulding & Suggate, 2001). These studies generally used multiple-choice audits or individual written responses to a range of questions. Whilst these types of quantitative studies provided a range of valuable data, I decided to adopt a more interpretative approach, in order to provide further insight into the student teachers' individual subject knowledge in mathematics. This was undertaken through the use of shared tasks followed by reflective and diagnostic group interviews. It was intended that the student teachers would explore, explain and possibly reconstruct their own understanding of fractions. These methods involved the explanation and discussion of their existing ideas and a consideration of any elements which possibly caused confusion. Apart from exploring and explaining their present understanding it provided opportunities for them to develop a more effective, relational understanding (Skemp 1989) of fractions and its related areas of mathematics. It was intended that within the group activities the student teachers would have opportunities to discuss and develop their own subject knowledge in this area. Through the use of collaborative tasks and reflective discussion, it was intended to mirror the process described by Carpenter and Lehrer (1999:20) through which mathematical understanding is promoted, "constructing relationships, extending and applying mathematical knowledge, reflecting on the experience, articulating what one knows and making mathematical knowledge one's own".

In this way the study adopted a constructivist perspective. This is the belief that "knowledge is actively constructed by the cognising subject, not passively received from the environment" (Von Glasersfeld (1989: 162) in Ernest (1991). Ernest (1993:63) described a social constructivist theory of learning mathematics which suggests that "both social processes and individual sense making have central and essential parts to play in the learning of mathematics".

A phenomenographic approach was adopted in order to provide rich detailed descriptions of individual students' understanding and experience (Patton, 2002). The use of phenomenography which is "the empirical study which seeks to understand how individuals experience, apprehend, perceive, conceptualise or understand the world", (Marton 1994: 4424) provides a valuable means for understanding learning from a student's point of view. Phenomenography has been used in a range of mathematical studies which considered the learning of children (Neuman 1997) and of adults (Asghari & Tall, 2005). Phenomenography "takes human experience as its subject matter" (Marton & Neuman, 1996). It is based on the premise that although learners are theoretically experiencing the same opportunity, in this case solving a problem, there will be a number of qualitatively different ways of experiencing or understanding the question or problem which can be observed and identified. The intention of this study was to discover the nature of these differences.

The choice of a phenomenographic approach was also intended to provide a greater exploration of individual beliefs and attitudes. Although this approach is not widely used in studying mathematics education, similarly interpretive approaches have frequently been used to study education and the social aspects of the world (Patton, 2002, Matthew et al. 1994, Strauss and Corbin, 1998, O'Leary, 2004). In order to reflect this interpretivist approach a range of methods favoured by qualitative researchers was adopted. The methods chosen for this study were the observation of a range of collaborative mathematical tasks with subsequent group discussions and individual diagnostic interviews. Written evidence was gathered during the activities which included diagrams, jottings and calculations. These provided useful supporting data and gave a greater insight into the participant's thinking. One of the limitations of using observation alone is that the motivation and views of the participants can only be a matter for speculation (Wragg, 1997, Patton, 2002). In order to overcome this difficulty the students were asked to explain their reasoning to the group whilst undertaking the task in order to justify their decisions and then discuss this further in a follow up group interview.

3.1.1 The Research Objectives and Questions

The objectives of the study are :-

a)To explore the nature of student teachers' understanding of fractions.

b) To discover the attitudes and beliefs which student teachers hold regarding fractions.

This will be achieved by addressing the following research questions.

1) Which aspects of fractions and the related areas of mathematics do student teachers show a confident understanding of?

2) Which aspects of fractions and the related areas of mathematics cause the student teachers significant difficulties?

3) Which representations of fractions do student teachers consider to be the most effective in the learning/ relearning of fractions?

4) What attitudes and beliefs do student teachers hold about fractions?

In order to address the aims of the project a series of observed collaborative tasks was undertaken by small groups (three or four) of students. The findings of these tasks were used to select the questions which were then posed to pairs and individuals in the diagnostic interviews. The use of self-selected groups was intended to allow the participants the opportunity to work together initially, to build their confidence and provide a comfortable working environment (Vaughn et al. 1996, Blaxter et al. 2001) by providing time to talk about their mathematical understanding in a supportive environment. An overall consideration of each participant's annotations and written responses combined with choices made within the tasks contributed to the gathering of evidence, especially towards research question 3. The observations were followed by group interviews where the task was reflected upon and discussed to give a deeper insight into each individual's level of understanding and the reasoning which had led to the decisions made during the task. The nature of the task and the group questions prompted a level of reflection which addressed research question 4 across all group/paired interviews. Emerging themes were then identified and explored in individual diagnostic interviews.

3.1.2 The Context of the Research

The research project was conducted in two universities. The initial stages of the research were completed in University A, this involved piloting and administering the questionnaires, piloting the first observation and the diagnostic interview technique and questions. In September 2008 I moved to work in University B where the main part of the study was completed. Although the majority of the study was undertaken in one Initial Teacher Education institution, it was not considered as practitioner research. The focus of the study was on the mathematical understanding and views of the students and not on the practice within the institution. It was intended that the eventual findings should be informative in the consideration of future mathematics provision of the Initial Teacher Education programmes in University B. The initial intention was to continue the study in University A, the main advantage of this was that the TALOS (Teaching And Learning Observed Space) room provided a very effective method of recording in a situation where the students felt comfortable. A further advantage anticipated was that by working in a university where relationships were established and support was guaranteed from colleagues to encourage students' participation. Permissions had been gained to continue the project from the course leaders and the Dean of the School of Education.

3.1.3 Ethical issues

In this study where the researcher had the role of tutor working with volunteer student teachers it was particularly important that the research practice was "honest, open, empathetic, sensitive, respectful and engaging" (Davies & Dodd in Gray 2004: 346) in order to protect the respondents and to ensure the integrity of the research undertaken. In my role as both student and tutor/researcher the ethical guidelines for the University of Leicester and Universities A and B were all adhered to. At the start of the project, the proposal was approved by the Research Ethics Review Committee at the University of Leicester and by the Joint Inter-College Ethics Committee at University B.

Before beginning the data collection it was necessary to request permission from the course leader, year leader and head of mathematics. The student teachers were also asked to give their written permission to confirm that they understood the nature and purpose of the study and that their participation would be anonymous and entirely voluntary and that they would be free to withdraw at any point. This was in adherence with

University B's ethical guidelines for working with students and the making of video recordings .The guidelines issued by the British Educational Research Association and the principles of the Research Ethics Framework (ESRC) were followed carefully. A video recording was made of each activity with the associated discussion to ensure there was an accurate record of each group's progress in the task and their related reflections. The written responses including annotations and diagrams were also kept as supplementary evidence (with the student's permission) and labeled so they could be considered in the light of the discussions. The privacy of the student teachers was maintained throughout the project; they were labelled in the transcript by successive letters of the alphabet and a suitable pseudonym was created at the analysis stage. Their label continued through the range of activities so their responses could be easily tracked.

The tutor/student teacher relationship was an important issue to consider in order to limit its effect on the student teachers' willingness to participate fully. There was an inevitable concern that student teachers may be reluctant to share their true feelings and levels of competence, especially in the light of possible course assessments and school placement evaluation. It was possible that student teachers may feel that an admission of lack of mathematical confidence may affect their tutor's overall view of their competence as a teacher. In order to avoid this difficulty the participants were recruited from other groups of student teachers with whom I had no direct contact. My main role at University B is the Programme Leader for the Primary Postgraduate course so my responsibilities lie primarily with one group of students. It was, therefore, highly unlikely that I would be involved in the assessment of the participants or in the supervision of their school placements.

The possible benefits of participation in the study were explained. It was suggested that this project could offer student teachers an extra opportunity to further explore their mathematical understanding in a supportive environment as well as providing some more support and practice in an area of mathematics in which the student teacher may feel less certain. This should help them address this area more confidently when they undertake the module which includes fractions, decimals and percentages at a later stage of the course and also support them in their approach to teaching mathematics on future placements and as a newly qualified teacher.

It is the researcher's responsibility to ensure the integrity of the research (O'Leary, 2005, Cohen et al., 2003, Gray, 2004 and Burgess 1989). Aspects of professional integrity relate closely to elements of trustworthiness, especially the qualities highlighted earlier by Davies and Dodd in Gray (2004: 346). These qualities, especially honesty and openness, were viewed as essential ingredients of quality of the research and guided the way in which the research was conducted. By aiming to meet the criteria for trustworthiness, in a conscientious and thoughtful way, it was intended to ensure that the research was undertaken with personal and professional integrity.

3.2 Questionnaires

3.2.1. The Introductory Questionnaire

In the planning stages of the research project a pilot study was undertaken in University A to ascertain whether my perceptions of students' understanding and feelings about fractions and the associated aspects of mathematics were accurate. A questionnaire was selected at this point as it is the most appropriate and efficient way of reaching a large number of people to access a range of types of data (Denscombe 2003, Cohen et al. 2003, and O'Leary, 2004). This pilot study initially used a quantitative approach to gathering data from a large cohort of students. This provided valuable background information from which to plan a more specific small-scale study using qualitative methods. A simple and accessible questionnaire was designed to seek students' views about their levels of confidence across the primary curriculum subjects and then, more specifically, students were asked to examine their feelings towards mathematics reflecting both on their own school experience and the prospect of teaching primary mathematics. The questionnaire also gathered some factual information, for example, levels of qualification in mathematics, (See appendix 7.4). In order to consider the feasibility of the study those students who had not yet begun their mathematics modules were surveyed in order to seek their views before the course had made any impact on their mathematical understanding. It was also important to do this before any mathematics sessions had been attended in order to give a realistic and complete picture of their perceptions. It was intended that the questionnaire sample would be generally representative of an undergraduate cohort (three year BA in Primary Education course with QTS) of student teachers, in terms of age, gender and pre-course qualifications in mathematics. A larger return was more likely if the request for completion is made face to face (Blaxter 2001). The purpose of the questionnaire was explained with the intention that the wider general information gathered could be shared with colleagues and it would provide an insight into the students' perspectives and needs, this could then be incorporated into the planning of more appropriate sessions. It also offered the opportunity for further correlations to be investigated at a later stage of the research; if appropriate. The students were guaranteed confidentiality as the questionnaires were all completed anonymously.

The questionnaire was designed sympathetically so it would not appear too intimidating in the early stages of the course and it was intended to be straightforward to complete (Blaxter, 2001, Cohen et al, 2003). It contained a balance of closed and more open questions which required a written comment. The closed questions generated a range of factual information which was collated and represented quantitatively as nominal data. Although this provided some valuable background data, it did not give the respondents any opportunities to express their views (Denscombe, 2003). The value of using a balance of the questions was the generation of a range of quantitative and qualitative data which gave a much richer and more complex picture of the students at the start of their course. A similar questionnaire issued by Brown et al. (2007) to a large sample of year 11 students gave, through the highlighting of key words used by students to describe feelings towards mathematics, a useful insight into individual feelings as well as a broad overview of the sample.

3.2.2 The Piloting of the Questionnaire

Before surveying the large intended sample, piloting was undertaken to consider the nature of the questions, the design and layout. It also established whether the questions could be analysed effectively and efficiently (Denscombe, 2003, O'Leary, 2004). The piloting was undertaken in two ways; firstly a sample of twelve final year students who were undertaking a research project module completed the questionnaire. They were asked to comment on both its design and content. This feedback proved valuable in terms of accessibility and clarity, with the suggestion of the use of a tabulated list of areas of mathematics to ensure all aspects were considered by each student. This also meant comparisons could be made more easily. The revised questionnaire was then piloted with ten first year students undertaking a BA Hons in Education Studies to ensure it was suitable for students at the start of their university course. No specific issues arose here so it was then given to a cohort of 75 year one students undertaking the BA (Hons) in Primary Education course. In order to guarantee as larger return as possible the request for completion was made face to face (Blaxter 2001).

The findings from the questionnaires provided an interesting range of data. The most conclusive of these resulted from the use of a Likert scale where the students indicated their levels of confidence in the different aspects of mathematics.

Which aspects of mathematics do you feel most/least confident about teaching on your primary school placements?			
	Very confident	**Quite confident**	**Less confident**
Place Value	**28%**	**59%**	**13%**
Addition/ Subtraction	**60%**	**35%**	**5%**
Multiplication/Division	**37%**	**58%**	**5%**
Mental Maths	**23%**	**59%**	**18%**
Fractions	**7%**	**52%**	**40%**

Decimals/ Percentages	**11%**	**51.5%**	**37.5%**
Algebra	**22%**	**36%**	**42%**
Shape/Space	**33%**	**51%**	**16%**
Data Handling	**33%**	**60%**	**7%**
Weight/ Capacity	**23%**	**60%**	**17%**
Time	**46%**	**48%**	**6%**
Length	**35%**	**50%**	**7%**
Investigations	**19%**	**72%**	**9%**

Table 3.1 The results of the question - Which aspects of mathematics do you feel most/least confident about teaching on your primary school placements?

The responding students of this cohort indicated that they felt less confident in teaching fractions (40%), decimals and percentages (37.5%) and algebra (42%) in comparison to other areas of mathematics, this trend was also reflected in the pilot questionnaires.

After an initial attempt to analyse the open ended questions using a cross correlation in order to identify links between responses (Cohen et al., 2003, Blaxter, 2003) it was decided this was not effective or efficient with such a large response rate. Many responses were very pertinent; however as the questions referred more widely to mathematics rather than fractions the information gathered did not sufficiently address the proposed research questions. Although it was valuable to know the nature of the overall cohort which the students participating in the tasks and interviews came from, a more effective method needed to be adopted. The open ended questions were replaced with a series of Likert scale questions considering different levels of agreement for a range of statements relating more closely to fractions and mathematics in general, (see 7.5). An even number of responses were used to engage the students in analysing their views, feelings and confidence and to try to avoid the respondents settling for the middle choice (Edwards et al., 1994). A revised questionnaire was offered to the group participants in the main study using mostly closed questions; it was accepted that such questions can limit the nature of participants' responses (Denscombe, 2003) and these questions would lead to the recording of factual information which could be collated and represented quantitatively as nominal data. Due to the nature and size of the sample it was not assumed that these student teachers were representative of the student teachers in the university, or indeed, of student teachers in general.

3. 3 The Observed Tasks and Group Interviews

3.3.1 The Preparation and Pilot of Observation of Task One

Once the initial research methods had been established and the value of focusing on individuals over a sequence of activities had been established, the opportunity to use a teaching observatory in University A was made available. This provided useful support for my choice of research design. The TALOS (Teaching And Learning Observed Space) enabled the observation and video taping of the mathematical activities in a non-threatening environment. This purpose built room with unobtrusive cameras and microphones and sophisticated recording facilities offered a valuable opportunity to closely observe the students and easily record the activities and discussion. A study by Rees & Barr (1984) used an integrated approach, firstly by analysing student's written responses to questions in a quantitative manner which was combined with the use of language laboratories to enable students to comment on their progress in the relative privacy of a booth. These verbal and written responses both contributed to a useful analysis of a range of areas of mathematics which also includes a useful focus on fractions. A similar combined approach was adopted in order to analyse the students' annotations and the manner in which the task was completed.

In the choice of initial task it was important to ensure that the activity was accessible and sufficiently challenging without being overwhelming to the students. The task consisted of a set of 20 cards to be sequenced in order of magnitude and matched in terms of equality, these included fractions (proper and improper), percentages, decimals and pictorial representations.

0.04 %	**0.4%**	**40%**					**40%**				**400 %**
		$\frac{4}{100}$				$\frac{39}{100}$	$\frac{40}{100}$				
		0.04	**0.043**		**0.38**		**0.4**	**0.43**			
		$\frac{40}{1000}$					$\frac{4}{10}$			$\frac{39}{10}$	
				0.35 **100 square**			**Number line**		**½** **100** **squ-are**		

Table 3.2 The cards used in task one arranged as a solution.

This task was based on a Key Stage 3 teaching pack on Fractions, Percentages, Decimals, Ratio and Proportion (DfEE 2003). This initial task was intentionally chosen to be accessible but with an appropriate level of challenge. It has been used and adapted in university sessions to include a range of simpler fractions, for example, 1, 1/4, 3/4 and 1/3, however for this purpose, these were removed and it was used in its original form. As the intention was to provoke discussion between the students and it was felt it gave a greater level of challenge in this form. A half was still included as a point of comparison as the majority of the fractions were less than a half, so this did not necessarily provide a mid point marker.

This task was selected following a consideration of a wide range of research literature focusing on childrens' and student teachers' understanding of fractions where the main themes which emerged were:

- the inclination to treat the numerator and denominator as natural numbers, rather than a single entity,
- the operational understanding of the spatial part/whole of a fraction
- the wide range of interrelated concepts and representations of fractions which include :- region (part/whole meaning), discrete objects, fractions on a number line, quotient model and the concept of ratio (Kieren, 1976).

It seemed a reasonable assumption that students may still hold similar difficulties as those pupils within secondary school as they may have had little opportunity to extend their understanding since leaving school (French, 2005, Murphy, 2006). The first and third main areas of difficulties, identified above, influenced this choice of activity and both were reflected on the cards by the use of a wide range of equivalent fractions and by using different representations of fractions as well as percentages and decimals.

In order to promote discussion the range of cards stretched past one and included some improper fractions. There were also differing numbers of matching cards; this was intended to prevent decisions about the placing of cards being made for any remaining cards by considering the spaces left in their sequence and removed the opportunity for guesswork. It was intended that through the physical manipulation of the cards and different types of representations that the students would be able to explain their mental manipulation of the concepts under discussion (Moyer, 2001, Drews & Hansen, 2008). The collaborative and practical nature of the task was intended to offer the opportunity for wider discussion. Although the main purpose was to ascertain the students' understanding, it is possible that whilst students were engaged in the task and explaining their approach, their difficulties, thoughts and feelings could be part of their conversation and it may be possible to gather supplementary responses to the second research question. The inclusion of a wide range of representations would provide a range of starting points for discussion and would encourage them to explain their reasoning and possibly make links to other aspects of mathematics.

As this was intended to be the first of a range of activities, it was important that the students felt the activity was manageable, “Impact perception can affect the way the learner sees a task… and can colour the whole route to the solution” (Rees & Barr, 1984:5). A range of equivalent fractions in different forms including percentages, decimals, number lines and shaded grids as well as proper and improper fractions were used in a sequencing activity so that there would be at least one type of representation which seemed accessible to each student. This type of sequencing activity is common in the primary curriculum (Haylock., 2003, Frobisher et al., 1999, Drews & Hanson, 2008), so it would probably be familiar to the students and make the task feel initially manageable.

Observations, where possible, should be undertaken in as natural a setting as possible (Newby, 2010) to ensure a normal experience for the participants. The nature of the group activity was similar to that which might take place in a university mathematics session, however working in the TALOS room meant this was a more unusual situation for the students. The setting was borne in mind, the creation of a comfortable working climate was considered (O’Leary, 2004, Denzin & Lincoln, 2003), to ensure the students would feel able to express their thoughts and share any difficulties. In an effort to make the observed activity feel as comfortable as possible, it was decided that these should be non-participant observations (O’Leary, 2004, Blaxter, 2001). Although these observations were not for a set period of time, the activity had a finite and logical conclusion to be reached. The group agreement of a solution hopefully provided a sense of achievement. After introducing the task and allowing the students to explore the equipment, I moved to an unobtrusive place in the room to allow the participants to respond as naturally as possible (O' Leary, 2004, Silverman, 2001). The cameras in the ceiling were focused on the activity on the table to provide a "bird's eye view" in order to concentrate on the movement of the cards whilst the associated discussion and decision making was recorded. This aspect of the recording seemed to be welcomed by the participants as the cards were the focus rather than themselves. Careful attention was paid in order to avoid the introduction of any bias, the study was introduced in as neutral way as possible so as not to influence the participants or indicate any particular emerging themes.

The students were asked to try and elaborate on their thinking where appropriate during the activity and explain the placement of particular cards. Specific points were noted during the observed task, so these could provide points of guidance for the subsequent group discussion or identify areas requiring further elaboration from the task. It was intended that these should not be apparent to the group so respondents were not aware of which of the points they had raised were noteworthy or alternatively which comments did not prompt a written response (Blaxter, 2001: 173). These notes supplemented the initial list of prompt questions and were used to encourage the flow of conversation and to ensure as complete a coverage of the topic and process as possible.

A pilot study was undertaken which consisted of the initial observed mathematical task described above followed by a group's reflective discussion. The pilot also focused on the nature, appropriateness and level of the task as well as the use of the TALOS room and equipment to discover its effectiveness for this type of observation. The pilot introductory task and follow up group interview was undertaken with a sample of twelve volunteers (three groups of four) from the primary PGCE course. These were self selected and comprised of those who felt they had time to contribute to the study and were interested in developing their own mathematics. This was an opportunity sample (Cohen et al., 2003, Patton, 2002) or an example of volunteer sampling (from a possible sample of 165 post graduate primary students). It is likely that there was also an element of "snowball/chain" sampling (Patton 2002: 82) as these groups were naturally formed by the students, who appeared to be very comfortable working together. There was a potential dichotomy between the way students could be recruited in terms of their suitability for the study and the establishment of a successful working group. There was the possibility that not all members of the group would show as strong a level of commitment or participation as if they had decided to join as an individual. "Friends working together are more likely than non friends to engage in interaction where knowledge is shared. Ideas are challenged, evidence is evaluated and opinions are reasoned about" (Mercer & Howe, 2007 2/1b). Although Mercer was considering children here, it seemed equally applicable to student teachers working together who can prompt and question each other in a supportive manner. This was a particularly important aspect in creating a comfortable setting as these students were not known already to the researcher. The sample was all female (as were 86% of the cohort). The sample was not assumed to be representative of the cohort. In the initial session two groups worked simultaneously at opposite ends of the room placed directly under the cameras. After the activity had been completed, the two groups joined together to discuss the way they had approached the activity. The third group then undertook their activity and follow up discussion.

3.3.2 Reflection of the Effectiveness of the Observation Pilot

The students responded well to the observatory environment with all groups commenting that after a few minutes they had forgotten about the cameras as they were not in their line of vision. Before recording began they had been reassured by seeing where the camera focused and that the cards and their hands were featured in the recording. The students were able to explain successfully as a group how their response to the activity had been achieved. The prompt questions were useful in ensuring all groups covered the same ground in their discussion. In this case there are advantages to the group observation/interview as it gives the students the confidence to speak openly with each other in less intimidating circumstances (Cohen et al, 2003:287). All students were active participants and contributed to the discussion and explained their thinking at some point in the activity. Although relatively clear recordings were created when both groups worked together, there were places when it was difficult to ascertain who was speaking. This was exacerbated by the fact that the students were not known to the researcher, this made following the lines of discussion more complicated and introduced the possibility for error.

The disadvantages were also considered; there was a possibility of one student asserting their views more strongly and affecting the whole group (Cohen et al., 2003, Denzin & Lincoln, 2003). There may be those who feel unable to contribute to the discussion or are unable to make their point at the appropriate point in the discussion. This may prevent a complete picture being available especially if students are in general agreement on a specific point. The self selection of the working groups which should guarantee some level of connection and confidence in the other group members and the use of prompt questions tactfully directed at those who may have not contributed as fully may help counteract some of these disadvantages. The increased clarity of the recording when only one group was working and the opportunity to make specific notes during the observation was a significant advantage. It was decided that the subsequent observations would be recorded separately. The students were asked to comment on the size of the working group and although all groups commented on a supportive nature of working in a group of four, they acknowledged that this group size may be slightly too large to gather individual information about each student's understanding. Two students specifically commented that "someone else said exactly what I was thinking, so I could only agree. That won't be on the tape now." When asked if smaller groups would be suitable, eight of the twelve students thought it would have been an appropriate activity to undertake in pairs.

3.3.3 The Findings from the Pilot Study

The following recurring themes arose from the initial task. At this stage cross tabulation was used to identify emerging themes and establish some preliminary coding. These were issues which were raised by at least one member in each of the three group activities and by both discussion groups. However, at this early stage, it was difficult to ascertain the proportion of students that were in agreement with the difficulties discussed. A common issue occurred, when ordering by comparing denominators then numerators to decide on their relative sizes. This seemed to cause difficulties particularly at the extreme ends of the sequence. A recurring theme which was typified by one student comment when comparing with decimals values, was that fractions "*are misleading, they seem to be bigger when they are actually a smaller number, it just never feels right, you always have to try to* *remember and keep checking*". This type of difficulty is also reflected in studies by Nunes & Bryant (1996) Graham (2003) and Behr et al. (1986).

A further issue arose when considering the shaded grids and a range of equivalent fractions. Students expressed difficulties and deliberated over which fractions would be equal when the denominators were different; one particular example was 40/100, 400/1000, 4/1000, 4/100 and 40/1000. These difficulties showed a lack of understanding of the fraction as a single entity. Similar difficulties are found by Domoney (2002), English & Halford (1995) and Ding (1996). A great deal of the discussion accompanying the task related to this difficulty, constant comparisons of denominators were made throughout, especially by group 2. This aspect was used by groups 2 and 3 as a checking device before completion of the task, resulting in many changes of decision about the placement of the cards.

Another difficulty which was found, to a lesser extent, was the ordering of decimals and percentages for example 0.4%, 0.4 or 4% and matching these to the appropriate fraction, this too was a difficulty discussed by Rees & Barr (1984). This reflects the third main difficulty, identified earlier, which is aptly described by Ball (1993) as "the bewildering array of many related but only partially overlapping ideas that surround fractions" (Ball in Carpenter et al., 1993:168). The review of the findings reinforced that the task was pitched at a suitable level and it had provoked productive discussion between the participants.

There were some difficulties which were apparent in the observations which were then not referred to by the students in the subsequent interview, for example the difficulties/lack of understanding of terms such as 400% as a general term rather than 400% of a specific quantity. A mechanism for raising these aspects was needed so they could form part of the following discussion or be included as an issue for individual interviews.

Throughout all group discussions there were comments from seven of the students, relating to an anxiety about or a dislike of fractions with references to the difficulties they had encountered as children. These types of comments are not uncommon and are also reflected in studies by Moss (2005) and Brown et al. (2007) Green & Allerton (1999). The students were very willing to discuss their perceptions of fractions and several instances of this type of demonstration and explanation occurred, where an individual drew diagrams to illustrate their understanding of a particular fraction or a mixed number/improper fraction. These moments were very illuminating and showed the different visual models the students had adopted.

3.3.4 Group Interviews

Following each observed task there was a group interview. This was planned to gather the perceptions, attitudes and beliefs the student teachers hold regarding fractions, which have been prompted by participation in the task. The flexible nature of the group interview enabled the participants to contribute to those parts of discussion which they felt strongly about and/or comfortable with. However a disadvantage was that it was not always possible to discern the general strength of feeling amongst the group; this was addressed in the diagnostic interviews when individual differences became more apparent.

The range of views and experiences, which showed the differing beliefs held by the student teachers, was considered as they reflected upon how their understanding has been constructed and began to reconstruct some of their established views. This produced richer, fuller and more contextualised data than could be gained in an individual interview (Lederman, 1990, Denzin & Lincoln, 2003). The flexible group discussion based interview also offered the possibility of discovering unanticipated yet relevant issues.

The group interview discussion was prompted by specific initial open-ended questions. This was intended to generate richer data which are "cumulative and elaborating" (Denzin & Lincoln, 2003: 73). It was semi-structured and more conversational in style rather than a series of answers to direct questions (O'Leary, 2004:164). As the research questions focused on attitudes and beliefs as well as understanding, incidences of *unofficial talk* (Houssart & Mason, 2009:59) were also considered. This was where students commented to each other or made asides to the main discussion, as it gave a greater sense of context to the assertions made within the discussion. It is evident within many of the transcripts of discussion. The role of the researcher here was more of "moderator or facilitator" rather than interviewer (O'Leary, 2004: 165). There are aspects of the group interviews which could be construed as focus groups, the specific choice of working groups was designed to "make explicit use of group interaction to generate data" (Roulston, 2010:35) which is a key feature of the focus group. Though the nature of these interviews contrasted in other ways, for example, Cohen et al. (2003) made the distinction where group interviewees are usually familiar with each other whereas focus groups tend to be groups of strangers brought together for consultation. Alternatively Vaughn et al. (1996:4) suggests that a focus group share their "subjective views on a similar concrete experience they all experienced" which would also accurately describe the method employed.

3.3.5 The Preparation and Pilot of Observation of Task Two

The second observed task was a similar sequencing exercise however this time it focused on comparing and sequencing fractions; this contributed to addressing the second research question. The familiarity in the nature of the task was retained whilst the complexity and difficulty increased. The student teachers were asked to record their thinking and notes on paper for later analysis and possibly contribute to the follow up interview. The task was compiled following the consideration of similar sequencing exercises and a range of fraction cards was selected, including some which were equivalent. It was pitched approximately at National Curriculum level 4/5. A greater number of cards (38) were introduced this time, these ranged from 1/14 to 7/2. A range of common fractions were included, but as their less familiar equivalents, e.g. 70/140 with a selection of single fractions to assist with the structuring of the number line. Some more complex fractions were available to be included as an extension activity where required.

This second activity was piloted by a group of four year 1 BA in Primary Education students in University B who also undertook the first observation activity so they would be in the same position approaching the pilot activity as those involved in the main study. In the absence of a teaching observatory, the use of a video camera was piloted. In the previous pilot, one of the main aspects the students had cited that had helped them feel comfortable was the knowledge that they did not appear on the video and that only their hands and voices were recorded. In an effort to recreate this, a camera was balanced on a tall tripod on a table nearby focusing at a steep angle whilst the participants sat facing the activity. After some experimentation, the correct distance was found to ensure all voices were recorded and a wide enough range could be seen for a reasonable sized working area on the table. After the pilot of the first activity, it was discovered that the cards were difficult to read against a similarly pale background and were sometimes placed "off camera". To resolve these issues a black background was used to show working area and to provide a good contrast to make the cards easier to read.

The group who undertook the activity were in agreement that it was more complex than the earlier activity and it took them approximately fifteen minutes as opposed to eight minutes for the first activity. They attributed the longer time partly to the increased level of complexity but also to "we know what we are doing now so we keep talking and explaining which takes longer". The pilot group was able to make valuable suggestions in terms of accessibility, for example, in an effort to make the cards legible they had been made larger than the earlier set and could not be arranged in a single number line /sequence which then could be captured on film. Also they suggested that there were too many cards in this activity and initially it felt "overwhelming and we couldn't get ourselves organised". The cards were then revised in the light of this discussion to use a smaller set of cards (30) of a slightly reduced size with some additional cards for those who felt confident and completed the task quickly.

3.4 The Diagnostic Interviews

3.4.1 Introduction

Based on the two observations and the group interviews, paired and individual diagnostic interviews were planned. The nature of these interviews was similar in some ways to the clinical interviews used by Ginsburg (1981). The intention of Ginsburg's interviews was to explore the nature of children's thinking more deeply and were based on Piaget's earlier work. The main feature of these interviews were a standardised task chosen by the interviewer which was then explored with each child with prompting questions to explore how and why particular decisions had been made. An interesting aspect of the clinical interviews was the contingent nature of the questioning which were based on the child's response. This flexible approach enabled the exploration of a child's "construction of reality" (Ginsburg, 1997:40). The choice of the use of diagnostic interviews for my study reflected a similar purpose to the clinical interview and allowed a flexible approach to explore each person's thinking. It can be seen how the nature of the diagnostic interviews was adapted in section 3.4.2 where the pilot of this part of the study is discussed.

Moyer & Milewicz (2002:294) suggested there has been an increase in the use of "diagnostic interviews which combine questioning and observation" to investigate the mathematical understanding of both children and students. A number of studies carried out in Australia (Clarke, 2006, Mitchell & Clarke,

2004) have adopted this more qualitative approach using a "task based interview" (Mitchell, 2005:545). Through these studies they suggested that there are advantages to using one to one interviews and they can "make more accurate judgments about the extent" of a child's knowledge, though the nature of the questions and need for multiple questions needs consideration in order to validate the responses given. The nature of these semi-structured interviews had further advantages (Roulston, 2010) as it could be tailored to reflect the research questions whilst providing opportunities to clarify any misunderstandings. This type of interview could also be described as "thinking aloud interviews" (Newby, 2010:347) which is valuable for exploring what the respondents consider meaningful. The semi-structured nature put more emphasis on the free response of interviewees and reduced influence of the interviewer. Marton & Neuman (1996) in their phenomenographic study of children's experience of division describe their interviews, which are similar in style, as "open and deep" where children were encouraged to explain, draw and suggest any method they had employed in order to explore their understanding. The disadvantages of this type of interview is that they are time consuming to analyse, (Roulston, 2010, Gray, 2004) consequently a particular need for objectivity and consistency is required to ensure a consistent approach through the analysis.

The use of these final individual/paired interviews was designed to provide a further level of triangulation. Multiple questions which used a variety of forms and representations and provided a development in complexity, whilst retaining an element of repetition, were included, (See 7.6 and 7.9). This was intended to enable direct comparisons to be made within the content of the task as well as between the student teachers.

A difficult aspect to handle within these interviews was when the student had answered incorrectly yet felt that the question has been completed successfully. Considerable care had been taken to encourage working friendship groups and to create a climate where the students felt comfortable, so it was important not to undermine this (Roulston 2010). Prior to the diagnostic interviews it was decided that in these circumstances, the student's attention would be drawn back to the question and they would be asked to review their response. Where this occurred on a couple of occasions within an individual interview, the student then went back through their response seeking reassurance at each step. In each case where this occurred with a working pair, there was disagreement over the correct response and together they were able to reach a satisfactory response. This contrasts in style to some diagnostic interviews undertaken with children in a similar vein, (Mitchell & Clarke, 2004, Clarke et al, 2007) where they responded to a range of questions and explained their thinking but were not told if they were correct. For the purposes of these diagnostic interviews, it was intended that it should also be a useful learning opportunity and a chance for self review. The nature of the questions and the need for students to make choices in order of accessibility required the students to appreciate when they were making errors so these could be discussed in more depth.

3.4.2 The Pilot of the Diagnostic Interviews

Initially it was the intention to provide a series of graduated questions to be answered by all participants in an individual diagnostic interview. This would provide a consistency and give a level of trustworthiness to the results enabling a greater level of comparison to be made between students. A range of questions were piloted with two pairs of students in University A, these were volunteers from the third year who had completed their course and had been part of the group who piloted the questionnaire as part of their research project module. These students were interviewed as they worked through a set of fifteen questions.

This was a valuable opportunity as these students were able to provide an insight into the student's perspective and their comments proved very enlightening. For example "I don't think you should ask that unless you want them to go into complete "meltdown", this might just be too frightening". Their responses to certain questions helped provide appropriate parameters in terms of content and suggested that some questions could be kept in reserve should they be required.

Representative responses which typified all four students were "Do they really have to answer all the questions? What if they can just do it? Or they really can't? " and "I think most students will find it too difficult to sit and do all these and explain to you as they do it". This was encapsulated by "It was ok for us we have finished our course and we know we can do it, but earlier on, I would have got very anxious". After considerable reflection and reviewing of their responses, it was felt that a more accessible strategy was needed. In order to link more closely with the second research question relating to thoughts and feelings, it was decided to ask the students to make choices indicating which questions they perceived they found most and least accessible. This gave a structure to the interviews and ensured that students were able to start with some questions with which they felt more confident and then introduce questions which didn't fit into either category before selecting any which they perceived to be more difficult. This was felt to reflect the phenomenographical nature of the study by allowing each student's prior learning to influence their route through the interview. This aspect of the diagnostic interview differed from the clinical interview (Ginsburg, 1981) in giving the student a greater degree of influence over the nature of the questions they answered as opposed to responding to the interviewer's interpretation of an earlier response.

A possible disadvantage was that all interviews followed a different course but this was balanced with the personal and individualised opportunity for each student to share their thinking and views. A further response to this pilot was the decision to offer the students the choice to work either in pairs or individually for the diagnostic interviews.

The original list of fifteen questions all varied in type and part of the intention of the pilot was to test the order of the questions to ensure they graduated steadily in difficulty. Another productive aspect of this pilot was the discussion between students as they tackled each question. When they perceived a question as more difficult, one pair in particular suggested that another similar question could be included to then build on with a slightly more complex version. In the light of this, some sequences of questions were included, to help the interview to become a more valuable learning experience. A series of nineteen questions was then selected with a final question for all asking each respondent to describe a fraction to someone who was unsure what a fraction was. This question was asked in a very small scale study of student teachers conducted by Domoney (2002:3) which considered their understanding of fractions especially the part-whole concept. The questions included were adopted from a variety of sources, which included mathematics subject knowledge audits, a range of research studies, SATS papers and University Challenge (see appendix 7. 9).

3.5 The Sample for the Main Study

Initially the intention was to work with a sample of twelve volunteers (three groups of four) from the primary PGCE and the BA in Primary Education course in University B. However in the light of the comments made during the pilot, it was decided that smaller groups would be more effective for the purpose of the study. This was intended to ensure that there was time and space for all their contributions to the discussion and a greater opportunity to recognise levels of agreement between students.

The recruitment of the groups for the main study was managed in a similar way to the pilot sample, working with students who had not yet undertaken the fractions input in their course. It was intended to include some reserves so a total of 16 volunteers was required. These again were self selected and comprised of those who felt they had time to contribute to the study and were interested in developing their own mathematics. For this part of the study volunteers to work as either pairs or threes were requested. This opportunity sample (Cohen et al 2003, Patton 2002) was also a type of network sample (Roulston 2010:82) where some individuals, who were particularly interested, recruited their friends to work with them. It was hoped that this would create groups who felt comfortable and confident in sharing their thinking as they would be working together for two observations and subsequent group discussions and a diagnostic interview. The first request brought forward fifteen volunteers; from this original sample, thirteen students continued to complete the project. It was decided that the remaining two who undertook the first observation, would not complete the study, due to illness towards the end of term, so six effective working groups were established (five pairs and one group of three). The participants were nine students who were at the close of their second year (of a four year BA Primary Education degree) and four primary post-graduate students from another tutor group who were yet to undertake their mathematics sessions relating to fractions, decimals and percentages. These students were all aged between 21 and 25 and included one male student. The questionnaire details of these students can seen be seen in appendix 7.8

It was considered that the advantages of using friendship groups would outweigh the possible disadvantages. One important aspect considered was the social nature of working within an established group and how the effect the role they may normally hold within the group, might affect their responses. There was a possibility they may present themselves differently in a group than they would do individually (Roulston, 2010).

Although it was important that the observations were conducted as a group, the diagnostic interviews could also be conducted with individuals. The choice was offered for the final stage of the project and three students selected to be interviewed individually so the groups at the next stage became five pairs and three individuals.

3.6 The Trustworthiness of the Study

At every stage of the study the need for trustworthiness was addressed as it is the researcher's responsibility to ensure the integrity of the study (O'Leary, 2005, Cohen et al, 2003, Gray, 2004, and Burgess, 1989). It was considered essential that the researcher approached the group and individual interviews in a flexible way, being objective, empathetic and a good listener (Denzin & Lincoln 128:2003). The phenomenographical approach to the research contributed to this element of the trustworthiness of the study through the value placed on each individual's contribution to the research as well as the way that each individual's understanding was reviewed separately. Trustworthiness was also considered in terms of "authenticity criteria" as suggested by Guba and Lincoln in Ely, (1999: 95) indicating a need for honesty and believability to be underpinning themes to the processes of the study. The ethical approach, which was considered earlier, contributed to the quality and the trustworthiness of the research. All participants in the main study were student teachers at University B so there were inevitable considerations to be made in the light of my role as tutor/researcher. Although care had been taken to ensure that there was not an academic or professional working relationship, the effect on the volunteers cannot be underestimated. This was carefully managed to ensure the student teachers felt sufficiently comfortable and were able to participate freely. Credibility is also an important measure of trustworthiness; this is the confidence which readers of the study can have in the researcher's ability to be sensitive to the data and to make appropriate decisions (Strauss & Corbin, 1990, Patton, 2002, Gray, 2004). It has been termed "Theoretical sensitivity" and relates to the personal qualities of the researcher (Strauss and Corbin, 1990: 42). The study was well researched prior to data analysis to ensure that the issues and subtleties raised were fully understood and to assist with establishing which data was particularly pertinent. It was suggested that theoretical sensitivity can be developed in a range of ways, including, as in this study, consulting appropriate literature combined with professional and personal experience.

3.7 The Analysis of the Data

The data from the observed tasks and the diagnostic interviews was analysed inductively, aiming to construct a picture that takes shape as the parts are collected and examined (Fraenkel & Wallen, 2006). Although an inductive approach was established, it was not intended to adopt a grounded theory approach where there are no preconceived ideas and all categories emerge from the data set (Strauss & Corbin, 1990). An element of the grounded theory approach was adopted, in terms of constant comparison, to ensure a consistency and flexibility across categories, groups and methods. Inevitably, there was the necessity to also adopt a deductive approach, as some categories were a priori as they were anticipated as the result of the reviewed literature and were integral to the research questions. For example, the application of secondary school methods was an expected category, though what the actual responses of the individual volunteers might be could not be predicted. As in much qualitative research it was appropriate to use both deductive and inductive approaches (Bryman & Burgess, 1994, Patton, 2002).

The approach of Seidel & Kelle (1995: 55) was adopted where the process of coding became an integral part of the analysis. This involved "noticing relevant phenomena, collecting examples of those phenomena and analysing them in order to find commonalities differences, patterns and structures." This careful consideration of individual responses reflected the phenomenographical approach so that all contributions could be considered as common themes emerged. Before any coding could take place it was necessary to become familiar with the data. The first stages involved listening to and watching the videos whilst making some preliminary accompanying notes and recording the timings of key moments of each clip. Each respondent had been given a pseudonym and were labelled alphabetically in the order in which the first observations took place. Part of this initial review involved listing possible codings which could then be considered across individuals, groups, observations and interviews. The identifying of possible conceptual categories was an important early stage where broad themes emerged and descriptive codes began to be established (Richards, 2009). Initial broad codings and notes from the pilot observations were used to affect research design of the main study, for example, reducing larger groups to pairs or, at the most, threes. This assisted with an easier collection of data and aimed to ensure the students felt more comfortable. The phenomenographical approach towards the study influenced the approach to data analysis in terms of response to the pilot studies where student comments were carefully taken into consideration at each stage of the study. This was effective in terms of giving a greater opportunity for each participant to speak giving access to more individual views. It probably enabled the formation of better matched working groups in terms of their social interaction and possibly better balanced in their mathematical competence and confidence. These adaptations enabled individual responses to be included at each stage in the data analysis process.

The initial focus was on the two observations, looking at each activity as a whole and then identifying topics within each. The observation tasks were sufficiently flexible that a form of inductive analysis would be possible to establish whether "discernable patterns, themes and categories would emerge from the data" (Patton, 2002: 390). A convergence of themes was sought by considering recurring themes in the aspects of fractions mentioned by the students and also the methods adopted in undertaking the group task. The second activity was, in most cases, better explained as students began to feel more comfortable in their working groups and in the research setting. This tended to generate the mathematical categories which applied across groups. These could be considered as topic groups (Richards, 2009, Matthew et al., 1994). Through revisiting both sets of activities emerging themes began to be established with overall links between groups and individuals becoming more apparent. Different levels of categories, those which related to attitude or were more explanatory in nature, developed after considering each group's progress through both tasks. At this point coding of categories became "linking rather than labelling" (Richards & Morse, 2007:115). It was considered important that the value of the complexity of the conversation was not underestimated and that the meaning of the students' assertions was not misconstrued by imposing broad codes onto the discussion.

The recording of the categories was a complex issue at this point, involving the compilation of different sectioned documents, where care was taken to record individuals, groups and timings as well as key features so they could be found again easily for further analysis. Diligent cross referencing was maintained so that links could be made where comments or incidents could fall into more than one category. An example of this was the discussion between Jane and Karen when considering the placing of 5/12 in the sequence.

The display and representation of data was an important aspect of the analytical process (Matthew et al., 1994). The organisation of the data in tables required preparation and review so that it was easily accessible for reading, exploring further and reviewing categories to explore the developing picture. This made the consideration of similarities and differences more accessible.

The high occurrence of numbers within the text, either as natural numbers or as part of the fractions referred to, made the analysis of the discussions more complex. Although all the questions had a definite answer, arriving at a correct answer was only part of the interview or observation and the processes and stages undertaken to reach it were equally important. Similarities were found with other qualitative studies involving discussion. For example, in O' Connor's study (2001) 5th grade students were encouraged to share and explain their thinking. The management of this posed similar difficulties in terms of recording and analysing the data and it was suggested that "to make even the most elementary claims about what action has taken place, or what has been accomplished, one must present evidence in the form of actual records of talk" (O'Connor, 2001:144). Charmaz (2000) suggested that "a potential danger of line by line coding is that the original text becomes atomised and much of the meaning lost". Various idiosyncratic and individual methods have been adopted by respondents which could not be clearly explained through a line by line coding. In order to reflect some respondent's explanations accurately, a similar approach to Kvale's (1996) technique of narrative structuring was adopted. Small vignettes which show individual responses have been included to give a more holistic view. These brief inclusions maintain a "narrative story-like structure which preserves the chronological flow" Matthew et al., 1994:81). This type of vignette could be described as a narrative summary constructed by the researcher to demonstrate a particularly rich aspect of the data.

The analysis of the diagnostic interviews initially took a more formalised quantitative approach. An overview of the questions which were selected as accessible or inaccessible was recorded as well as those that did not fall into each category (see appendix 7.10). The numbers of students undertaking each question was recorded and the nature of their responses were noted and cross tabulated. There was a considerable amount of data within these interviews and this made it more complex to analyse. The introduction of the choice element meant every interview followed an Individual path in response to the student's perceived areas of confidence regarding fractions.

A constant system of review across all three elements for all groups was undertaken to establish shared understandings and views as well as identifying individual perceptions. These were again cross referenced back to the research questions. "Coding is a process which enables the researcher to identify meaningful data and set the stage for interpreting and drawing conclusions" (Matthew et al., 1994: 56) who also suggested that coding constitutes the "stuff of analysis".

The next stage of the analysis corresponded to the third of Bryman's (1994) stages of qualitative analysis where the data was interrogated systematically and codes were reviewed in order to eliminate repetition and to combine any which had a greater level of similarity. The wording of categories were reconsidered to ensure they accurately reflected the data and some previously anticipated aspects which may have provided contrast or balance to common themes were discarded. The names of the codes were generally self explanatory, however, in order to be consistent illustrative sentences, phrases or explanations were considered to give greater clarity.

The data steadily became more meaningful as themes and continuities as well as contrasts, paradoxes and irregularities become more apparent. There was an inevitable need to make judicious selections from the wide range of data collected, as some responses were already discussed in other studies. Some questions proved to be less productive in generating valuable data and so were not included as part of the main study. For example, the inclusion of one question which had more of an algebraic focus, provoked much interesting discussion, however this was beyond the remit of this study.

As this is a small scale personal study with a large quantity of data, it was not possible to share all the codings and data with another researcher to validate the choices and decisions made. In order to consider the trustworthiness of the study, regular discussions were undertaken with my supervisor and with colleagues in both universities and those in the wider mathematics community. Early findings were also presented to a group of mathematics colleagues at the new researchers' day at the British Society for Research into Learning Mathematics. These opportunities have been invaluable in ensuring that the content stayed focused and was not repetitious of other studies. Also views were sought on the preliminary data at Ed.D. sessions and from my study group, this further feedback and questions from these doctorate students, who were not necessarily from a mathematical background, provided valuable opportunities for reflection and review.

Initially it was intended to use Nvivo which is a qualitative analysis software package, to assist in the coding of the dialogue which each observed activity generated. Analysis of this number of observations and diagnostic interviews would inevitably be very time consuming. Following a training session this software was piloted with two transcripts of key moments in the discussion which occurred during the first observations. Nvivo was found to have some value in matching words and short phrases identifying the aspects which related to the student's feelings and attitudes relating to fractions. However the mathematical nature of the discussions, especially the frequent inclusion of numbers used in different contexts, and sometimes less formally within explanations, made analysis undertaken in this way unnecessarily complex.

The data gathered from the observations and diagnostic interviews was typically qualitative as it was "textual, non numerical and unstructured" (Basit 2003:153). When investigating ways of using computer analysis it was important that the students' comments were considered in terms of the developing flow of conversation and their responses to each other as they may have lost their meaning if they were not considered in context. Johnson & Branley (2006) stressed the usefulness of qualitative computer analysis but warn it may encourage researchers to "focus attention on what is said rather than what is not said". This was a valuable aspect to consider in the analysis of the discussions where individuals were undertaking the same tasks and an element of comparison was needed.

During the process of analysis the phenomenographic approach was maintained in order to find the "qualitatively distinct ways" in which students understood fractions through reviewing the findings of the interviews and observations "against the backdrop of the whole and reviewing the whole in the light of the individual pieces of evidence" (Steffe 1996:321). The way the codings were developed and adapted is discussed in the findings chapter.

4. Findings

4.1 Introduction

The main study adopted a phenomenographic approach where thirteen students working in six self-selected groups undertook two group tasks (See table 3.2 and Appendix 7.7) which were followed by reflective discussions. Diagnostic interviews were also conducted where students indicated their perceived levels of confidence by considering the accessibility of a range of questions (See table 4.4), which were then answered and explained. These were undertaken either individually or as a pair. A questionnaire was also used to provide comparable data and to provide a context for the participants. The profile of these students can be seen in table 4.1. The shading indicates the initial working groups. The levels of confidence were identified from their responses to the question, "Which subjects do you feel most/least confident about teaching on your primary school placements?" Students were asked to prioritise the primary curriculum subjects in which they felt most and least confident. The students were given alphabetical pseudonyms which coincided with the order of participation in the first task.

BA in Primary Education with QTS (year 2)	Age	Highest qualification in maths.	Level of confidence in teaching maths (in comparison to other Primary curriculum subjects).	
		At GCSE		Key Stage
Anne	18-20	Grade B	Middle	KS2
Betty	18-20	Grade C	Least	KS1
Carol	18-20	Grade A	Most	KS2

Donna	18-20	Grade C	Least	KS2
Ellen	18-20	Grade B	Most	FS
Fran	18-20	Grade B	Middle	KS1
Gill	18-20	Grade B	Middle	KS1
Holly	18-20	Grade C	Least	FS
Iris	18-20	Grade B	Most	KS1

PGCE Primary				
Jane	21-25	Grade B	Least	KS1
Karen	21-25	Grade B	Least	KS1
Lynn	21-25	Grade A	Middle	KS1
Megan	21-25	AS level	Middle	KS2

Table 4.1 The Profiles of the Participating Students.

The findings will be considered firstly in broad terms within a general overview of the two observations and then diagnostic interviews. The main themes that emerged will then be considered in the light of the research questions. All tasks were videotaped in the manner, described in the methodology, that focused on the task being undertaken and captured the discussion without the students' faces appearing on the tape. This seemed to give the students more confidence when participating and explaining their thinking. Each observed task was followed by the opportunity to reflect in the groups in which they had worked. At this point they were joined by the researcher and these discussions were video taped in the same fashion as the activities. The main focus of the discussions was the consideration of which aspects of the task they had found most and least accessible and the explanation of the strategies involved. The responses to these questions can be found under the appropriate research question.

4.2 The first Observation and discussion

4.2.1 Overview and Completion of Task One

The first activity involved the sequencing of 20 cards with a mixture of fractions, decimals and percentages (DfEE, 2003), (see table 3.2) for cards shown as the solution. These cards ranged from 0.04% to 400% and included a range of representations. This activity was generally undertaken in pairs, with one group of three. The times taken to complete the activity are shown below.

Group	Completion	Time taken to complete task 1
1.Anne / Betty	Successful	21 mins
2. Carol /Donna	Successful	6 mins 20 sec
3. Ellen/ Fran/Gill	Some confusion with equivalent fractions and different representations	22 mins (including 6 minutes to agree final solution with support).
4. Holly/ Iris	Successful	10mins 20 sec
5.Jane/ Karen	Some confusion with very small fractions and improper fractions.	27 mins (including 8 minutes spent checking and revising until they completed the task successfully)
6. Lynn / Megan	Successful	4mins 10secs

Table 4.2 The Timings and Levels of Completion from Task One

A range of approaches was adopted towards the sequencing of the cards. Most groups followed the same procedure of creating one horizontal row based mainly on the decimals, with the equivalents creating vertical lines matched to the appropriate card within the row. Jane and Karen organised their cards in a more tabular fashion so that similar representations were aligned, using the equivalent fractions/decimals as an initial structure. This initially was a slower method but enabled them to cross check effectively. Anne and Betty initially adopted a similar method but then reorganised them into a single line with vertical matches. There was considerable variation in the amount of time by the groups to complete the tasks, from around four minutes to almost half an hour. Four groups completed the task successfully at the first attempt. Ellen, Fran and Gill had some difficulties in matching and ordering and were able to complete with some support (see Vignette 4.5.3 (vi)). Jane and Karen had errors at each end of the sequence but were more certain about those in the middle. However, after a careful review, before deciding they had completed, they were then able to self-correct. Only Lynn and Megan (who took 4mins 30 sec.) didn't seek reassurance that they were correct and seemed certain that it would be right. All other groups after checking through suggested that they had finished but were uncertain whether it was correct. This was typified by the following responses:-

Betty: We think we have finished but it could be **really** wrong.

Jane: Have we got it right ? Really ?

Ellen: We **think** we have finished.

4.2.2 Group Discussions Regarding Task One

The discovery that they had been correct was met with satisfaction and enthusiasm. All groups were pleased with their efforts; this seemed to be roughly proportional to the level of difficulty they experienced in completing the activity. Typical responses, reflecting satisfaction and pleasure at the successful completion of the task, were generally accompanied by applause, laughter and whooping, some representative comments were:-

Anne: Wahay… is it really right ? that has made me so happy.

Betty: I am so chuffed we got it all right…and first time too.

Fran : It's right ! hurray… wooooo… When I first saw the numbers, I went "oh my god" but then I realised I knew much more than I thought I did.

Karen: When you have done it you really do feel dead proud of yourself.

In all cases, some impromptu discussion followed as they reflected on the way they had undertaken the activity. Working as a team featured in many discussions and the support afforded by working together was valued in all groups. Two examples of the types of comments from groups three and five give an indication of these discussions.

Fran: We were a good team and learnt things from each other.

Gill: If I had to do it by myself, well… I could not have done it.

Jane: We should teach together, you can do those …. (pointing out 400%)
We are good at different things.

Karen: You did the squares… I can't see those at all. We got them all between us… what a team!

In the discussions which followed the first observations, the initial focus was the students' reaction to the task rather than specific examples of the mathematics involved. The perceived difficulties in completing the activity were also a feature of the initial discussion. For example, some groups wondered how long such a task might reasonably be expected to take. For example Anne commented "When we were half way through, I suddenly thought, 'Oh dear, we might be here much longer than we thought'. Comments relating to their initial feelings on seeing the cards were an integral part of the discussion. Some typical examples are included below.

Ellen: I just thought "Oh my God, I am so glad I had you guys here to help".

Donna: When I saw the first cards, I thought ok, ok, but then, how many more has she got in that envelope? ... Were there really only twenty?

Gill: I thought what have I let myself in for?

4.3 The second Observation and Discussion

4.3.1. The Completion of Task Two

The second activity involved the sequencing of 32 cards showing a range of fractions. (See appendix 7.7 for cards shown as the solution).

This activity was undertaken in the same working groups as activity one, in pairs and with one group of three. The time taken for completion is shown in the following table. As with activity one this gives a general indication of the level of ease or difficulty with which it was undertaken.

Group	Completion	Time taken to complete task 1
1.Anne / Betty	Successful	24 mins
2. Carol /Donna	Successful Some confusion with equivalent fractions	19 mins 30 sec (including 2 mins checking time)
3. Ellen/ Fran/Gill	Successful	16mins
4. Holly/ Iris	Successful	15 mins 20sec
5.Jane/ Karen	Some confusion with very small fractions and improper fractions.	28 mins (including 5 minutes spent checking and repositioning some cards with support).
6. Lynn / Megan	Successful	7mins 30secs

Table 4.3 The Timings and Levels of Completion from Task Two

All groups continued the task until they were certain it was completed. Three groups took a similar amount of time to the first task, with Carol and Donna who completed the first task relatively quickly taking almost three times as long and Lynn and Megan about twice as long. Four groups were accurate at their first attempt. Ellen, Fran and Gill were more confident and quicker in this task and were correct first time in contrast to the misplacing of several cards in the previous activity. Carol and Donna found this activity much more difficult and this was reflected in the time it took them to complete the task. Their completed task was almost correct with two cards interchanged which were 75/100 and 8/10. They were quick to spot their error when prompted that some reconsideration was needed. Donna responded with "I don't know how that happened… I know that eight tenths is bigger than three quarters". Carol "I think we put down three quarters as a bit of marker and then ignored It….I think it is all correct now". They suggested that the use of percentages as a basis was not as straightforward as in task 1 so they found making comparisons more arduous. Jane and Karen had made a few errors; they were initially happy with their choices but not able to self correct on this activity. Early in the activity 2/5 had been matched with 1/3. In order to make a direct comparison Jane had drawn two circular diagrams and shaded in an approximate 1/3 and 2/5, from this they concluded they must be equal. This, in turn, affected the placement of other equivalent fractions. This will be further discussed later in the chapter when considering circular representations, (see 4.5.4 (i)).

4.3.2 The Respondents' View of the Second Task

As this was the second task, the students seemed better prepared and rather less anxious now that they had a clearer idea about what might expected of them. Generally they were able to explain their thinking in more detail. Most students perceived this task as more difficult as there were less obvious possibilities to change the fractions easily to percentages or decimals and this did not seem such an appropriate strategy given the nature of the fractions included. There were mixed views about the relative complexity of this task. The general view, which was shared by eleven students, was that the second task was more difficult than the first one. This was qualified by observations like, "With this one it is much harder to decide where to start" (Carol)", "I know bits of it, but trying to put them all together...is like trying to do a jigsaw. Some bits fit and you don't know what you know. Some bits of my memories of fractions have completely disappeared." (Jane) and "This is like working in a different language to me." (Fran). It was considered more complex by those who seemed more confident too. "I thought this was one was much more challenging but really good, I enjoyed it" (Iris). Three students felt the second task was more accessible, they suggested that, "It was easier than the first one. All you had to think about was the fractions, rather than having to compare them with the decimals" (Ellen) and "This one was better than the other one, when I saw the cards I thought, 'Oh good…. this one is just fractions… but I'm still not *that* comfortable with them' " (Betty).

4.3.3 Group Discussions Regarding Task Two

During this discussion there were more comments relating to past study than previously, for example "I can't believe we have done it ….. it all begins to come back to you after a while"(Betty). Yeah, we did it….. whehey….Do you know it is 9 years since I did my GCSE?" (Karen). Even the groups who completed successfully sought reassurance that they had placed all the cards correctly. In all cases there was a tentative nature to the announcement that they had finished, for example "Hmm, that's it? Hmm we think… we are finished, yeah? That was hard that one" (Donna). There seemed to be genuine surprise as well as pleasure when they discovered they had completed the task correctly. "Are we really right? That's pretty impressive" (Anne) and "I can't believe we have actually done it …" (Holly). They seemed more pleased with themselves on the completion of this task. This was typified by comments like "Wow, now I feel really clever!"(Anne) and "Can we make it into a photo or screen shot so we have got the evidence that we really completed it!"(Jane).

4.4 Diagnostic Individual and Paired Interviews

4.4.1 Introduction

Before beginning the interviews the students were asked to read through the questions and decide which three questions seemed the most and the least accessible to them. The number of students who selected each question can be seen in the following table to show which questions were perceived as most accessible. A table showing the individual choices made can be seen in appendix 7.10. The selection of questions as accessible did not guarantee that the student would always find them solvable but their initial perceptions were a valuable place to start and gave a structure to each interview enabling the initial focus to be on where each student felt more confident.

The questions used in the diagnostic interviews in the order of which they were perceived to be most accessible.			
Most Accessible **√** Neither ~ Least Accessible **x**			
	√	**~**	**x**
Percentages			
Question 1 **20% of £65=**	**13**	**0**	**0**
Sharing and comparing fractions			
Question 4 **If 3 pizzas are shared between 7 boys and 1 pizza is shared by 3 girls. Who would get the most pizza? How much more?**	**7**	**6**	**0**
Comparison of fractions of quantities.			
Question 2 **Would you rather have :-** **5/6 of £30 or 1/2 of £48 or 1/4 of £104 ?**	**5**	**7**	**1**

Question 8 **Would you rather have :- 6/10 of £520, 2/3 of £600 or 5/7 of £350**	**3**	**10**	**0**
Using a Measurement Context			
Question 18 **How many pieces of ribbon, each 0.08 m long, can be cut from a length of 4 m long?**	**1**	**10**	**2**
Percentage Increase			
Question 13 **In January, fares went up by 20%. In August, they went down by 20%. Sue claims that: "The fares are now back to what they were before the January increase." Do you agree?**	**1**	**9**	**3**
Approximation of magnitude of fractions			
Question 5 **Which is the best estimate for 12/13 + 7/8 =** **a) 1 b) 2 c) 19 d) 21 (multiple choice)**	**1**	**11**	**1**
Addition and Subtraction of Fractions			
Question 7 **1 /4 + 2/3 =**	**2**	**11**	**0**

Question 9. **At the ferry port, 1/4 of the passengers are travelling to France, 1/3 are going to Germany, what fraction are travelling to Holland ?**	**1**	**11**	**1**
Question 10 1 /2 + 1/3 + 1 /4 - 1 =	**1**	**11**	**1**
Question 19 9/16 + 5/64 =	**0**	**12**	**1**
Question 15 5 and 7/ 6 minus 3 and 5/8 =	**0**	**10**	**3**
Flexibility of thinking about unitising.			
Question 3 **X X X X X X = 3/2 of the unit.** **How many is there in a unit?**	**1**	**8**	**4**
Question 11 **These circles represent 3/7 of a unit. How many is the whole unit ? ● ● ● ● ● ● ● ● ●**	**1**	**10**	**2**
Question 12 **These circles represent ¾ of a unit.** **How many is 2/3 of the unit ? ● ● ● ● ● ● ● ● ●**	**0**	**10**	**3**

Multiplication and division of decimals			
Question 17 0.3 divided by 0.3 =	**1**	**9**	**3**
Question 16, 0.2 X 0.1=	**2**	**6**	**5**
An algebraic expression of fractions			
Question 14 **2 x a/b =** **a) 2a/2b b) 2a/b c) a/2b** **d)4a/2b (multiple choice)**	**1**	**6**	**6**
Ordering and magnitude of fractions			
Question 6 **Which fractions come between 2/5 and 3/5 ?**	**0**	**7**	**6**

Table 4.4 The Perceived Accessibility of the Questions in the Diagnostic Interviews

The perceived levels of accessibility were taken into account when considering the students' responses to the questions. This is discussed later under the research questions. The following symbols were used to show the perceived levels of accessibility, Most Accessible **√**, Neither (neutral)**~** Least Accessible **X.** The students were asked to explain and justify their choices of question. For these interviews the students worked either individually or in pairs, those working as a pair were continuing with a student colleague who they had already worked with in the earlier sessions. Fran, Holly and Iris all selected to be interviewed individually. Before the diagnostic interviews began there was the opportunity for some reading time in order to consider the questions. The students generally tended to read the questions in silence, though this was accompanied by a few anxious sounding comments, for example, "What happens if we can't do any of them?" (Karen) and, "are we going to be answering **all** these questions?" (Betty).

4.4.2 Initial Responses

At the start of each interview the initial questions "Which questions do you feel are the most accessible?" and "Can you explain your choices?" were asked. The responses to these questions were regularly accompanied by a great deal of nervous laughter. Some students began by explaining the method they would employ to answer their chosen questions, rather than justifying their choices. A representative example of this came from Megan, "When I looked at question one, I could immediately see that I could do 10% and..." Some students attempted to give an overview and find connections between the categories of questions they had selected. Ellen, for example, said, "I chose ones with a story, like the pizza or the ferry, I like the *wordy ones,* they **tell** you what you need to be looking for." Gill agreed and added, "I was looking for ones with whole numbers too". This contrasted with Anne who chose the purely numerical questions, numbers one (20% of £65=), seven (¼ + 2/3=) and seventeen (0.3 divided by 0.3=) and explained, "You can see exactly what to do straight away".

Each interview took between 45 minutes and an hour and ten minutes depending on the level of explanations given. Paired interviews inevitably took longer to ensure that each student had a full opportunity to share their thinking. The responses to these questions can be found under each research question in order to reflect the commonality of responses across the whole group.

4.5 The Research Questions

4.5.1 Introduction

When considering the students' understanding of the various aspects of fractions and the related areas, it was possible to identify only a few areas where a confident understanding was felt universally across the sample. Inevitably all the participants had a different range of experience and knowledge, so when responding to the research questions, it has been necessary to link confidence and difficulties encountered in order to give a full picture of the groups' understanding and yet reflect individual differences. Due to the small scale and qualitative nature of the study it was not intended to survey all aspects of knowledge and understanding relating to fractions, but rather to explore areas which had become apparent from working with student teachers in mathematics sessions and from the research literature. When considering the areas in which the students felt confident or found difficulties, their perceptions of accessibility, their comments and their explanations during the observations and the diagnostic interviews were all taken into account. It is intended to give an overview of key features arising from the tasks and highlight specific individual responses which gave a particular insight into their understanding.

4.5.2 Which Aspects of Fractions and the Related Areas do Student Teachers Show a Confident Understanding Of?

In tasks one and two, the first question for each group discussion considered which aspects of the activity they found most accessible. It is difficult to consider each suggestion in isolation, as, in some cases, what was considered accessible by some students was also cited as difficult by others. This question was generally greeted with some amusement or a long pause, "It would be much easier to say what was difficult", was Anne's initial response.

The following areas were identified by some students and reflected across tasks one and two as well as the diagnostic interviews.

4.5.2 (i) The Use of Percentages

The most accessible part of the first activity was perceived to be that it offered the possibility of working in decimals/percentages. Seven students suggested that this provided them with a good working basis from which to make their comparisons. Even though the other students did not suggest this as their own initial choice, there was generally agreement that this had been a good approach. This approach was evidently used in most cases and had been reinforced by the inclusion of fractions with ten or a hundred as a denominator. This acknowledged confidence in working with percentages was reflected in Question 1 of the diagnostic interview (20% of £65=). This question was identified as accessible and was answered confidently and quickly by all students, with the majority (12/13) proposing finding 10% (£6.50) and doubling this to give £13. Only one alternative method was offered by Lynn, this was that 20% is equivalent to a fifth. This was explained, using partitioning of £65, in terms of a fifth of £50 to give £10 and a fifth of £15 is £3 so £13 all together.

4.5.2 (ii) Finding Fractions of Quantities

Although the comparisons between fractions of quantities were only selected by three students as two of their more accessible questions, they were identified as accessible by five other students when reflecting on the questions as a whole. "Actually those would be pretty straight forward; they just looked a bit long" (Megan). Students were generally very confident in their approach to these, though they found making a prediction/estimation of which might be the greatest amount of money more difficult.

Question 2 Would you rather have:- 5/6 of £30 or 1/2 of £48 or 1/4 of £104 ?

Only those who had indicated that this was accesible or inaccessible were asked to complete this question. This was only chosen by one student as inaccessible. Holly, who explained that it was initially the size of the numbers which had seemed difficult, "But now I look again, it looks quite doable," she was then able to explain her method and complete the question easily.

Holly: I would probably choose £104 but it probably isn't.
So I would half that £52 then… £26.
Half of that (48) would be £24,
Then five sixths of £30, so a sixth is…hmm
So 5x6 =30, so a sixth is £5… so that's £25 so I would rather have that one.
(circling) so my original choice- a quarter of £104

This was indicative of the methods employed by most students. There seemed to be a general preference for questions of a more concrete nature where a fraction of a specific number was sought. This preference was reinforced during the first task where six students referred to the unknown quantity of the larger percentages, e.g. Donna returned several times to the placing of 400%, " But what is it 400% of? How can we know what to do with that?" This consideration of the necessity to know an actual quantity was only related to percentages whereas fractions seemed to be accepted as a quantity in their own right and regarded as numbers.

4.5.2 (iii) Successful Strategies

One particularly successful strategy which was employed was the inclusion of some marker points, or anchors, defined by the fractions they were most familiar with e.g. 1/2, 1/4 etc. This helped provide useful comparison points for those fractions of which they felt less certain. A common successful strategy based on this which was used regularly by all groups was the selection of two "boundary fractions". For example Iris suggested that two fifths feels bigger than a quarter and it must be less than a half, because that would be two and a half fifths. So it goes in the middle.

The basis of the first two tasks had involved the comparison of the magnitude of a range of fractions and finding those which were equivalent. It was surprising, therefore, that question 6 (Which fractions come between 2/5 and 3/5?) was initially considered difficult by a relatively high number of students (**√**0,**~**7,**X**6). This question was answered by all the participants and a variety of successful methods was employed. Most students interpreted the question as the requirement of finding one fraction between 2/5 and 3/5. These were generally pre-empted by uncertainty about the approach which they might employ, a typical response from Iris, "I don't know where to start with this". She then chose to "turn them into decimals" and confidently selected a half. Jane's initial response was to list fractions, "When I saw that, I was going in my head… "Two sixths, two sevenths, two eighths, two ninths… hmm what comes next? No, I can't do this one". This strategy was abandoned in favour of a circular diagram. Circular drawings were used by Donna, Fran, Jane and Karen (X) who drew circles with shaded sections of 2/5 and 3/5. Donna quickly identified a half and said, "I could have done this easily with percentages, if I had thought about 40% and 60%".

Fran, Jane and Karen surveyed and considered the possibility of two and a half fifths. "You can't really have that … it could be a mixture of decimals and fractions like two point five fifths…" questioned Karen who quickly realised this was a half and Jane confirmed this with annotating her diagram, "Oh it is staring us in the face!" Fran also followed this route and suggested that other equivalent fractions could also be included, "like four eighths and three sixths". Ellen (X) was unable to reach a conclusion from her diagram deciding that there must be quite a few, "If I broke it into tenths".

Two students interpreted question 6 as requiring them to find a range of fractions. Carol (X) and Lynn (X) consequently both identified this as one of the questions they perceived as more difficult. Both converted these to decimals in order to decide what the, "in between fractions" would be. Carol initially decided to use tenths and quickly revised this approach listing all the fractions between 41/100 and 59/100. "Oh Look, I have found a half as well" was her concluding comment.

Lynn gave a very full answer despite marking this as a less accessible question.

Lynn: I chose number 6 as a difficult one, simply because the list would be as long as your arm… I mean that is nought point four and nought point six, so any fraction which gives you a result above nought point four. I could see that one, where as some of these, (pointing to the start of the list) I struggled with the concept of them.

But the thought of listing them all ! So initially I would just go… a half.

Then I would go 41/100, 21/50, 43/100 ….. 44/100 which is 11/25,

Then I would go into the next sub category… thousandths, then ten thousandths,

How many are you looking for?

Only Karen commented on the links between this question and the earlier activities. "I know when we had them in front of us before, well, we struggled then didn't we? So to actually have to do this one with nothing in front of us to move… hmm…" Despite the initial reticence in answering this question, a range of successful strategies was employed. This question is probably the most similar to the tasks in the earlier observations and it is hoped that the discussion and sharing of thinking may have supported the answering of this question successfully.

The use of decimal conversion to compare fractions was a strategy mainly used by Lynn, Iris and Carol. They seemed to prefer considering each fraction as a decimal value in order to make a direct comparison. Lynn demonstrated a strong understanding of place value and used the following strategy for checking the placement of several of the cards in task 2. In this case, decimal conversion was used when considering 7/8 in relation to 4/5.

Lynn: I would do twelve and a half times by seven.

Megan: why?

Lynn: well 8's into a 100 so that's …12, you know 96 and a half. So if… it is 87.5. I am just making sure we are right. Bigger than 0.8… that one is 0.8 (4/5). You can't abstractly compare those two, you need to have some concept of a whole. You would need an idea of the real value.

Megan: Hmm… I would do think of 70/80 would be… well it is nearer a hundred, and you can still see that eighty over a hundred … four fifths is smaller.

Megan seemed to have an intuitive feel for the size of fractions, which was based on a comparison of numerator and denominator in order to establish how close the fraction was to one or a half.

4.5.2 (iv) The Use of Equivalent Fractions

In the second task there seemed to be an increased use of equivalent fractions to make comparisons and to support the decisions made. The larger number of equivalent fractions in the first task may have supported this to a certain extent. The extension of a sequence of fractions was adopted by some students and this seemed to be preferred to cancelling down to a simpler fraction. An example of this was towards the end of task two when Jane and Karen still had 5/12 to place; this had been considered and discarded several times and was now one of the few cards remaining.

Jane: Now two fifths, what about that? We can't reduce it down but could we make it bigger, you know see what else it would be equivalent to.

Karen: Do you mean… five twelfths is...like… ten over twenty four or… if we looked at two fifths. I think it is too difficult

Jane: No I think we are onto something, two fifths, four tenths, six fifteenths (starts to write down sequence) eight twentieths, I don't think this is helping, are these really all the same?

Karen: Yes... the next one is… ten over twenty five so is… two fifths. It is ten over twenty five that's right. I am not sure why we set off on this now.

Jane: Two fifths are ten twenty fifths and five twelfths are ten twenty fourths. Ok… so which is bigger… ten twenty fourths 'cos one twenty fourth is bigger than one twenty fifth. So five twelfths is bigger. Let's swap them!

(Realising it was not correctly placed). Now we must be right!

The same reasoning was then used to reposition 2/9 which was initially placed as larger than a half. It was placed as smaller than 1/4 by comparing 2/8 and 2/9.

The equivalent fractions strategy was used by Lynn and Anne in answering question four (If 3 pizzas are shared between 7 boys and 1 pizza is shared between 3 girls. Who would get the most pizza?). Lynn (~) and Anne (~) both considered the boys share of the pizza as three sevenths without reference to any supporting diagrams or illustrations. They both considered how this might be compared to 1/3 and used the equivalent fraction 3/9 to decide this. The comparison of these denominators enabled them both to give quick and accurate answers. Both these students worked as part of a pair and in each case their student colleagues were not able initially to follow the suggested reasoning behind their answer.

4.5.3 Which Aspects of Fractions and the Related Areas caused the Student Teachers Difficulties?

4.5.3 (i) Introduction

As part of the first observation and discussion, the students were asked to consider which numbers/cards they had found least accessible. Five students suggested that the very small numbers, the decimal percentages had been problematic for them. Lynn began to explain the way she thought of these percentages, (0.4%) "I converted to 0.04 and then you think.. whoah… hang on, hang on, you need to think of it as a percentage again". This view was echoed by Carol, "When they were decimals *and* percentages – now that *was so* confusing".

The consideration of equivalence was discussed in all groups often linked back to areas in which they felt more confident. Though Ellen was confident with the decimals, "If you had given me a bunch of decimals, I could have ordered them straight away…. ", it was tempered with "but given all their equivalents, I can't". This view was supported by Fran, who agreed that, "Switching between them all was *so* difficult."

Fractions which looked similar were considered difficult by three students. Examples given by Jane were 40/100 and 40/1000. Megan pointed out similar cards saying, "These threw me at first.. then they were ok, but I think children might get confused by things over 10 or a 100." Similarly

39/10 and 39/100 which were the most discussed cards within the activity, the inclusion of 39/100 had not supported their understanding. The similar appearance of some groups of fractions was perceived to have caused difficulty and there had been the tendency to place these close to each other initially.

4.5.3. (ii) Improper Fractions and Mixed Numbers

It was evident in both tasks and the interviews that some students found working with mixed numbers and improper fractions difficult. An example of this in the first observed activity the larger numbers, especially the improper fraction (39/10), were identified as problematic by 4 students. As suggested by Carol, "These two (400% and 39/10) because they are not at all clear cut, you had to work out what their equivalent would be to be able to crack it and even then we weren't sure." The 400% was cited as particularly problematic by three students and was typified by Gill, l "I thought …what is it 400% of?, I didn't think we could do this one ?" Five out of the six groups were less certain with fractions larger than one. Karen and Jane found the placing of 39/10 in the sequence particularly problematic. Their discussions can be found in appendix 7.15, this is also included as an example of how the initial codings were applied. A further example of the difficulties posed was shown when Ellen, Fran and Gill worked together to place 39/10 in the number line.

Ellen: 39 over 10 is 3.9 so that's a bit less than 40%.

Fran: Yeah less than a half.

Gill: Just below all these (indicating the equivalents which had begun to be collated in the row).

This was not questioned further until 39/100 was found.

Fran: They can't be the same, 39 over 10 …39 over a 100. hmm, this one is bigger.. I think (picking up 39/10).

They went on to agree that 39/10 was larger than one without trying to establish the exact value.

Ellen: It must be less than this (pointing to 400%) that is huge.

The inclusion of an improper fraction in Question 3 (X X X X X X = 3/2 of the unit. How many is there in a unit?) was perceived by several students to have added another level of complexity, see 4.5.2(iv). Also the use of mixed numbers in question 15 (5 7/ 6 minus 3 5/8 =) caused considerable discussion, (see 4.5.3 (v)).

4.5.3 (iii) Uncertainty About the Relative Size of Fractions.

Fractions which were perceived to be close in size proved problematic for many students, this seemed to be a specific issue with fractions close to one. They were invariable considered more difficult when their denominators were different and with less accessible common factors. This was a common issue across the groups. One example came from Carol and Donna during the second observed task when considering 99/100 and 7/8.

Donna: 99 over100 … 99 out of a hundred and seven eighths is 7 out of 8, both go near the top, they are both one away from a whole.

After further discussion they were still unable to decide. These two cards were returned to later, the rest of the cards have been placed.

Donna: Just these two left and we don't know how to work that one out. Shall we slot them in here before 1? We think we have finished. It was hard that one.

In follow up discussion

HF: which parts did you find difficult?

Carol: (pointing to 99/100 and 7/8) We are still not sure about these, we don't know if there is any difference they are both one away from being a whole. We think these might be the same but we can't work it out. HF: Could you estimate which is larger?

Carol: I think they are about the same.

Donna: This one is about….. 0.9 (99/100) but I can't do the other one.

Carol: We just don't know …I will have to guess here (placing the cards in the correct order).

This type of uncertainty was apparent in other groups, particularly demonstrated by Betty, Holly, Megan and Gill. In these groups, another member was able to explain and demonstrate their own thinking to support their partner's understanding. The following strategy was employed by Anne and Lynn when considering how close 55/100 and 45/80 would be to a half, in order to place them in the number line. Anne was able to explain and demonstrate to Betty, who found the fractions containing two digit numbers less accessible This discussion followed after Betty had decided that 45/80 was equivalent to a whole one and a bit left over.

Betty: Look, I am really struggling here, these don't really look like fractions now.

Anne: They are both five away from a half, but five hundredths is equal to a twentieth, and five eightieths is the same as a sixteenth. One twentieth is smaller so it will be nearer to the half.

Betty: So which is bigger then?

Anne: That (pointing to 45/80) is…(bigger) because it is a half and a sixteenth and that is bigger than a half and a twentieth.

This enabled them to place these two cards in the number line appropriately in relation to a half.

Ellen adopted a more practical approach when explaining to Gill.

Gill: I am not sure that seven eighths is bigger than four fifths (pointing to circular sketch).

Ellen: Seven eighths goes very near the end, if you think of the bit that is missing…. It is an eighth and that is smaller than one fifth.

Gill: So it goes …(beginning to place 4/5 as larger)

Ellen: No, no, think about cutting a piece out, (miming circular shape and a chopping movement) so I cut out an eighth and there is more cake left than when I cut out a fifth so it is nearer to a whole cake …or a pizza.

Here Ellen demonstrated one of the more successful uses of circular representation which was used to effectively explain her reasoning used in the task.

This type of difficulty was also reflected in the diagnostic interviews where the questions which involved the comparison of fractions and required an appreciation of their approximate size. Eight out of eleven students were not able to answer question 5 easily even though only one of them had identified it as a less accessible question.

Question five - Which is the best estimate for 12/13 + 7/8 =

a) 1 b) 2 c) 19 d) 21 (Multiple choice)

Three students, Anne, Carol and Lynn were able to immediately answer that the nearest estimate would be two as both fractions were close to one. An initial answer of 19 was suggested by both Gill and Iris, accompanied by the suggestion that "isn't it nineteen over twenty one ?". Both realised that their suggestion could not be correct as 19/21 would be smaller than one which prompted them to reflect on the size of the fractions in the question and then decide it was two.

Holly suggested these were "both quite big" so it could be 19. The use of a circular diagram helped Holly decide, after dividing a circle roughly into 13 parts and beginning a second drawing, that each fraction was close to one. Similar initial difficulties in considering the size were encountered by Betty who decided that 12/13 would be one point something and following discussion with her working partner Anne, she decided that it would be less than one but "only a little bit". Effective peer support was provided here to agree a suitable conclusion.

Question 4, related to division and the comparison of fractions (If 3 pizzas are shared between 7 boys and 1 pizza is shared by 3 girls. Who would get the most pizza?) This proved problematic for many students. The use of circular representations for this question is considered in 4.5.4 Carol (~) initially established how many people would share each pizza. She made notes to show her thinking to Donna, it was initially written formally as

7/3 = 2.3 and 3/1 =3 followed by this response.

Carol: So the girls would get most, I just divided it. So if seven boys shared three pizzas, they would get two and a third slices each and the girls would get three slices each. I am not sure if that is quite right?

Her working partner Donna (~), used circular diagrams to divide all four pizzas in thirds and established that in fact the boys would get more pizza, as there would be some remaining after they had all had a third. After Donna had demonstrated this, Carol agreed that this must be the case, but despite discussion around the problem, they were unable to decide what had happened in Carol's initial response. She seemed not to appreciate that she had divided the boys by pizza rather than the pizza by boys and had then misinterpreted her findings.

All the other groups, which included the seven students who had indicated this question as a confident choice, followed a similar route supported by circular pizza diagrams. The initial decision to share the girls' pizza into thirds, then prompted them to adopt a similar approach for the boys'. This was followed by the consideration that the remaining 2/3 could be shared between the 7 boys, this was usually referred to as 'a third and a little bit more each'. They were all uncertain about what that "little bit" might be or how they might find out. In several cases there was initial confusion between numbers of slices and specific fractions. Some students had developed quite successful strategies using circular representations, however in this case, the model supported the strategy adopted but did not always enable them to decide on the actual size of the slices. A more successful example occurred when viewing all of the pizzas. Fran suggested, "This won't work unless you cut all the pizzas into smaller fractions so like if you did this (drawing three new circles) into sevenths. Then you would all get a slice of that one, and that one and that one (pointing to each pizza in turn,) then there would be nothing left over."

4.5.3. (iv) Unitising and Re-unitising

Unitising (and re-unitising) refers to the "process of constructing chunks" which constitutes a given quantity. (Lamon 2005:78). It is an essential aspect of the *part/whole* fraction understanding is the concept of *partitioning* a whole, into *equal* parts and also reconstructing those the parts back to create the original whole. Questions 3, 11 and 12, which involved the identification of the basic unit from both proper or improper fractions proved problematic initially. A series of linked question were included within the diagnostic interview and all students undertook these questions. This was mainly because at least one of this series of 3 questions was perceived as less accessible by the three out of the five students in the first two groups and in each case it had provoked an interesting discussion. When selecting neutral questions for the later groups, this set was also included in order to further explore the findings from earlier groups. Generally the complete set was undertaken rather than taking one question in isolation. It was intended that question three might act as an opportunity to build confidence for the later questions.

Question 3 (X X X X X X = 3/2 of the unit. How many is there in a unit?) was undertaken by all students. It was selected by one student (Megan) as an accessible question. This question was answered confidently by students Anne (X), Carol(~) Gill(~) Iris((~) and Lynn(~) who were able to provide a quick and accurate response. To Anne, Iris and Lynn this was obvious and immediate; the others explained that they had divided by three to find the size of each half before deciding what the whole number might be. Gill reached a correct answer though her explanation did not seem to match the method she employed. "Three halves is a whole and a half, so I divided it into 3 to find out what one is and then there is a half left." This explanation did not reflect the process she was undertaking, when she appeared to be dividing one and a half by three by sectioning the six crosses into three parts. Betty (√), Donna (√), Holly (~) Jane (~) Karen (~) and Megan(x) struggled with the nature of the question initially and the improper fraction appeared to add another level of complexity. It seemed to be met by a degree of uncertainty as shown in the discussion between Jane & Karen.

Jane: Which one have you ticked ?

Karen: Question 3 ? Yeah because…yeah hmm…well actually maybe not….

Jane : What made you go " well, actually maybe not ?" ?

Karen: I just looked again and it's one of the top heavy ones :Now I am thinking about how to work that out and I can't do that. I *ticked* it , but now I can't do it .

Jane: Yeah now I come to look at it, it seems much harder. So….., not sure how to get started on that one.

Karen: Well, it's a top heavy one again That's one and a half… on its own right?

Jane: Why is one and a half ?

Karen: Hmm, well it is top heavy; doesn't that mean… one and a half? Three over two is one and a half, isn't it ?

Jane: so, two over two, would be a whole, so… And that is a half and there's (counting) six… (writing X=3/2).

After discussion of possible sized units and the rows of crosses were divided into two by a line which was then dismissed by Jane "If I do that I will only have two halves instead of three".

The exploration of the other possibilities then led to the following.

K: Six is one and half … is that right ?

J: so you need to split six to make one and a half so a whole would be 4 and a half would be 2. and 4 and 2 make 6 .yeah ?

K: Yeah, I see what you mean …

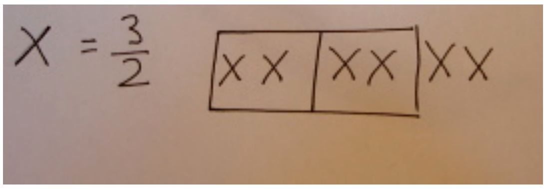

Figure 4.1 Jane's response to question 3

The problematic nature of starting this question was also reflected by Anne, Betty and Holly. The improper fraction prompting responses like, "I am really not sure what this question is asking" (Betty) and "How can it be more than a whole one?" (Holly). These conversations gave a real insight into the attitudes and beliefs held by the students and will be discussed further below. This question was considered to be more difficult by Donna, Jane and Karen as "this fraction, you know, three halves could not be simply scaled up to find the whole number" (Donna). All these students began to draw or annotate the question sheet. Most students recorded 3/2 as 1 and 1/2, in the process of recording their responses; this seemed a more accessible form to consider. Most groups having solved this question successfully were keen to try questions 11 & 12 which were similar. The inclusion of an improper fraction added to the perceived difficulty of this question which reflected the findings from the first two activities.

Question11 (These circles represent 3/7 of a unit. How many is the whole unit ? ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐) was undertaken by all the students. It was attempted a little more confidently by all groups of students; once more those who had answered the question easily did so again with the exception of Lynn. Question 3 had given the opportunity to develop a way of thinking about the magnitude of the unit. Betty built effectively on the previous learning, "So three sevenths is 9 and we need to divide it by three to find one seventh. This is much clearer now we have done the other question".

Megan and Lynn were also able to answer the question after a rather more hesitant start. The inclusion of sevenths, a less familiar fraction, also seemed to have posed an initial obstacle.

Megan: So I would really have to think about this one a lot more, as it isn't seven or a multiple of seven dots.

Lynn: I saw that nine is… you can cancel the nine to three. But again, I did find that one hard, I needed to think about that one a lot more… tough

Megan: I had to think about that one a lot more, it would have been easier if there was a multiple of seven. So I just broke it up into threes….. and continued it. Hmm… tricky… sevenths.

There was some uncertainty in two groups, amongst Gill, Holly Jane and Karen who used a step by step approach based on the earlier question. Various techniques were used, in several cases, dividing the nine circles into three and then drawing a continuation to reflect all twenty one dots was considered helpful.

This was typified by Holly, who after a less confident start tackled it logically.

Oh no, not another unit one!

So… to make it a whole you would need seven sevenths, so nine is three sevenths.

Then divide that into that, so each little bit… (circling three dots at a time) hmmm… so I need some more sevenths, so add three and… (counting up in threes) twenty one. Is it twenty one?

Most students checked their thinking by annotating the diagram or redrawing the problem in a more accessible fashion. Betty drew her diagrams for each question and completed the "whole" as a checking mechanism each time.

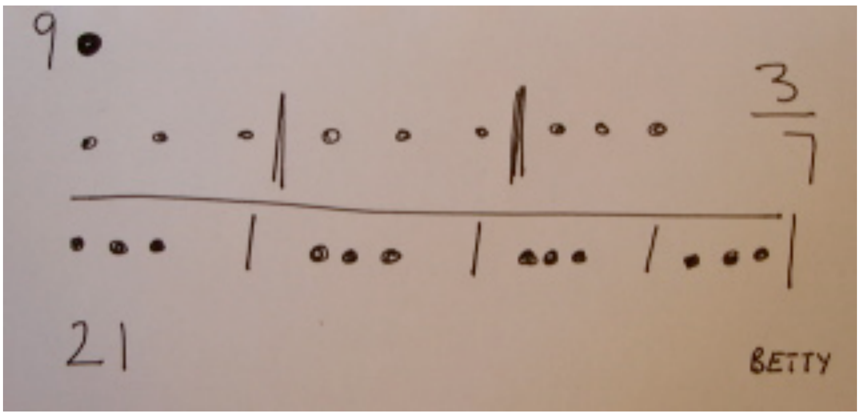

Figure 4.2 Betty's response to question 11

The third in the series of questions relating to unitising was question 12 (These circles represent ¾ of a unit. How many is 2/3 of the unit ? □ □ □ □ □ □ □ □ □)

This again was answered by all the students. There was general agreement that the earlier questions 3 and 11 had helped with the understanding of this question. The inclusion of a second fraction was considered to provide another level of complexity in question 12, which several students found more perplexing. Responses which were typified by Ellen, "It looks much more scary." and Gill, "You feel it will be even more confusing when you see the two thirds." All the students used the initial number to find one quarter, this was demonstrated by Gill who explained that, "I am using the same routine as before, (drawing three more dots to continue the line to show twelve dots). I have got a system going now…. Now we need to find two thirds of this... Is that right? I realised I need to work out a whole number first."

Iris, who had managed the other two questions easily, tried to complete the question in one step by finding a common denominator. She was unable to proceed with this saying

Iris: What have I done? I have confused myself now. I am going to have to start again on that one. I was trying to get the bottom number the same so I would know what I am doing with three quarters and two thirds.

She eventually adopted the same approach as the other students, of finding the whole number twelve before finding two thirds.

4.5.3.(v) The Use of "Methods Remembered From Secondary School"

An interesting range of methods was adopted in response to question 15, several of which were hindered by recall of what they perceived *you were supposed to do* with these types of questions rather than matching their method to the numbers within the question. The subtraction of fractions, particularly in those questions involving mixed numbers and improper fractions, were perceived as difficult by the majority of students. Question 15 (5 and 7/ 6 minus 3 and 5/8 =) was undertaken by 10 students and initially identified as less accessible by three. Megan completed this question quickly using the techniques discussed with Lynn whilst undertaking question 10. She used 24 as the lowest common denominator and kept the numbers in the form they appeared in the question.

Anne (X) and Carol both immediately converted 5 and 7/6 to 6 and 1/6, "This is what you did at school, you always simplified it," explained Carol. They successfully identified 24 as the lowest common denominator but were then unable to proceed with the numbers in the revised forms. Anne quickly appreciated that she needed to return to 5 and 7/6, Carol, however, was, "stuck now." Following a suggestion that this conversion may not have helped, she was quickly able to proceed but, "I would not be able to decide that by myself, I would still be struggling." She was very excited to have completed what she perceived as, "a particularly difficult question".

Betty followed a similar route after converting to twenty-fourths,

28/24= 1 & 4/24= 1& 2/12= 1 & 1/6. After some deliberation she suggested

Do you think I should have left it as it was?

So could I do 28/24 minus 15/24ths? and then 5 minus 3? .

Lynn, Fran (X) and Iris all converted the mixed numbers completely into improper fractions, Lynn and Iris made arithmetical errors in the working through of the question, once the question was answered, Lynn explained the choice of method to Megan.

Lynn: I have put it into numbers I can access, I can't access those (pointing at the question containing mixed numbers, those are completely abstract to me.. I mean ….what does 6 and a sixth look like ? nothing !

Megan: 6 pizza and a sixth of a pizza… ?

Lynn : These are like…abstract numbers and I like to work in decimals so my first attempt was 6.1666 minus 3 … and 5/8 , What would 1/8 be ?... and times by 5 would be 6.25 and now I am in all sorts of trouble I could do it like that, but I am going to be like grrr (groan) and end up with a number that's 6 decimals long and that isn't a fraction at all. And because I have got a recurring number, you can't possibly start because fractions don't give whole numbers. So you have got *that* problem so I converted them all into vulgar fractions so when you are subtracting.

That's the easier thing to do, to get one thing they are both divisible by. So I thought 8.16. 24, both divisible by 24

Lynn continued to explain in order to justify her method.

Lynn: Did you see what I did there then, I just converted them into vulgar fractions and I just got that bit there wrong , so I know if I can if I have got them both .. even if they are vulgar expressed over 24.

Then it just eases doing it so 148 minus 87 which I can do like that (clicked fingers) is 61. I can find the whole numbers when I have got my results at the end. Just makes more sense to me. I would prefer to do it all as a whole, do you see what I mean which is why I have got these huge vulgar fractions.

Megan's response, "Hmm that was very impressive …… do people who are good at maths like to make it more complicated?" This comment was particularly interesting in the light of her relatively fast completion of the task by a straight forward and accurate method. She had responded to the size of the numbers and adjusted her method accordingly yet did not perceive this as, "good at maths".

4.5.3 (vi) Vignette – An Individual Case – The Little World of Fractions.

Ellen, Fran & Gill worked together on the first sequencing task. After five minutes discussion, it became clear that Ellen and Gill held conflicting views about the placing of the fraction and decimals and appeared to be placing the cards in opposing directions. Gill was uncertain about the place of fractions in the number system, particularly in relation to zero and negative numbers. Fran was initially unable to decide who was correct.

Ellen: I think that (pointing to 0.43) is bigger, if that is 0.4 then only because it has more on the end of it , it is 0.4**3** , 0.4**4** **0.45**

Gill: What a sec… shouldn't that go there (rearranging the order of cards) that would go 0.43, 0.42 ..

Ellen: But that's **going down** in size…

Gill: so it goes 0.38, 0.37 …. And then you get to zero and then it goes up…..

Fran: I don't know I think Gill is right ?

Ellen: That's confused me…

Gill: Well I don't know…but

(They move onto matching some equivalent cards successfully. 4/10= 40%, 4/10= 0.4)

Gill: this is 0.1,(counting squares) 2 3 so … It goes

Ellen :I don't understand, I am really sorry but I don't understand why is 0.3 is in front of 0.38.

Fran: I am not sure what you mean

Gill: Right if you have got a number line with zero on.

Fran: Can you draw it ?

Ellen: It should be going that way to get to 0.4

Gill: but 0.4 should be down there. If you have a line, there's zero. you go one, two, 3, 4 (moving right) and coming this way, it is 1,2,3,4 So then if have zero 0.1 (moving to negative side) it goes,

Ellen: but it is not a minus , it **goes,** (reaching for pen- goes right) right up to 0.9.

Gill: Oooh… (head in hands) argghhh

Ellen: These are all back to front, they don't go that side… when it gets to 0.49 it will turn to 0.5.

Gill: arghhhhh …I get it now, I see what you are doing…

During the feedback, there were still some cards to review.

Ellen: Am I right in thinking that goes? 0.4, 0.43

Just that before, you were saying it was minus…

Gill: I got confused, I thought that the small ones were going down like a minus… , I know they are not minus numbers but they get smaller like in their own little world different to proper numbers…….

Fran: Yes you really confused us, I couldn't see what you were doing.

Gill: Well I just thought that… I was counting up to zero… hmmm before you… I can see it now.

When reflecting on the difficulties each had encountered.

Ellen: When we were doing these, we got really confused…. You kept moving them.

Gill: I don't know why, I just thought that's' where they went, I know they are not minus…..

Fran: They tell you that at school….

Gill: yeah… but they are really small and go before the numbers, like in their own little world… but I don't know why I did that. I really confused you all. You get taught that anything less than the whole numbers is a minus.

This working group of Ellen, Fran and Gill spent 22 minutes on this initial task. The selection of some equivalent fractions gave the basis to their structuring of the task. It gradually became apparent that there were contrasting views held by Gill and Ellen, Fran was uncertain about who was correct. The exchanging of placed cards between Ellen and Gill revealed that they were working in opposite directions. Eventually Ellen began to realise that Gill was placing fractions below zero, as if they were negative numbers. The following discussion revealed that Gill believed that fractions existed "in their own little world" and was uncertain about their place within the number system. Gill was placing the cards in the reverse direction with the larger numbers closer to minus one and smaller number closer to zero.

4.5.4 Which Representations of Fractions do Student Teachers Consider to be the Most Effective in the Learning/ Relearning of Fractions?

4.5.4 (i) Circular Representation

Circular drawings were the most commonly used representations amongst the thirteen students. They were applied in a range of questions with some students using this as their main source of recording and as a means of exploring the comparative size of fractions. In some cases, they were used as a means of checking or for demonstrating to another group member. As can be seen by the following examples, the use of circular representations was not always effective in supporting the students' thinking. They were not a universally popular choice as Carol generally favoured rectangular drawings and Lynn's strongest inclination was to covert the fractions to decimals wherever feasible.

The use of circular diagrams is evident in many of the examples already discussed. Some questions prompted the use of these by the wording of the question, (See 4.5.2 (iii)), in the solving of question 4 (If 3 pizzas are shared between 7 boys and 1 pizza is shared between 3 girls. Who would get the most pizza? How much more?). As can be seen earlier, the accurate use of the diagram was supportive in some cases, but there was also confusion in the vocabulary used when referring to slices rather than actually naming the fractions they had created in the dividing of the circle.

Circular diagrams were used by several students when answering question 9 (At the ferry port, 1/4 of the passengers are travelling to France, 1/3 are going to Germany, what fraction are travelling to Holland?). Ten students attempted this question which was not perceived as particularly accessible or inaccessible, with most students leaving this as a neutral choice. Both Holly and Karen suggested this was more complicated than it initially appeared as you were not told how many people were travelling.

Circular diagrams were generally considered helpful here. Jane initially decided that it must be a third, based on making an estimation from her diagram, though after discussion with Karen, it was decided that the fraction they were looking for, "must be smaller than a half and larger than a third so that it can actually make a whole one". The use of finding a common denominator was then decided on and used effectively to find the solution.

Gill (X) had drawn a circular diagram but was uncertain about how to establish what the remaining piece might be.

Ellen: Well, a quarter looks like this…. Like a clock and if you
drew a third, then this bit is Holland…

Gill: So that bit is quarter past… and….

Ellen: I couldn't work it out straight away like this…. (new drawing)

Can you see the 5 minutes…… round the clock. 1/12 and 2/12 (pointing to each 5 minute section) Does that makes sense? 1/4 =3/12 and 1/3=4/12 so that is 5/12 , it looks like 25 to…..

Gill: hmmm. Yes

Ellen: I only know 'cos my dad used to teach me fractions on the clock.

Similarly Holly used circular diagrams and decided by drawing sections on her diagram that the remaining parts in each half would be1/4 and 1/6 but was unable to work out what this might be in total.

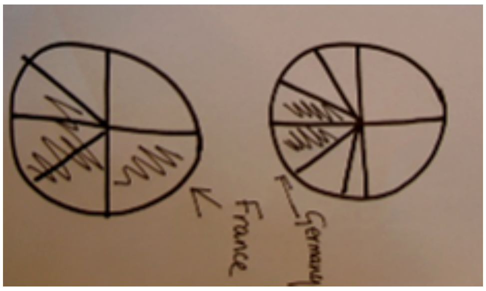

Figure 4.3.

Holly's sketch for question 9

The inaccurate sketching of circular diagrams was evident during the second task, Jane created two circular drawings and then shaded in an approximate 1/3 and 2/5, from this they concluded they must be equal. This false premise persisted until the end of the activity and was not noticed despite the close checking of the order of the sequence. On completion they were initially unable to decide where they needed to reconsider so were prompted to re-examine 1/3 and 2/5, to which Jane responded, "I wonder if it was the way I drew it, they must be very close though." After some discussion they decided that 2/5 was correctly matched with 40/100 and used percentages to consider as this as 40% and 1/3 as 33%. The 2/3 and 4/5 were then correctly placed. The conversion to decimals helped identify the error. "I don't think my diagram can have been accurate enough!" was the final response.

4.5.4 (ii) Use of Rectangular Diagrams

In the first task some fractions were represented as shaded hundred squares. When asked which aspects were considered most accessible, these shaded diagrams were identified by Donna, Gill and Jane who all decided these were the "easiest". Jane typified their responses "The shaded shapes make the most sense as you can see exactly what they are". However this view was not shared by Donna or Karen "I can't work these out at all. When I look at them, they just… well, I find it very difficult, I just can't see it". Carol tended to use rectangular diagrams to check her answers. For example, the following discussion took place when comparing 5/12 and 2/5 (here the use of a circular and rectangular diagrams were considered, as part of the discussion relating to identified difficulties, following task two).

Donna: It was quite confusing (pointing to 5/12 to 2/5) it was hard, because half is two point five fifths so that's only nought point five less and that's 1 less because half would be six twelfths.

Carol: Five twelfths was difficult because you couldn't' make it match or break it down. We put it there before six twelfths but don't know if it is right.

Donna: (sketching on a circular diagram) I don't know that's just how I imagine it. You can cut it up into slices, I simplified it and did two and a half out of six.

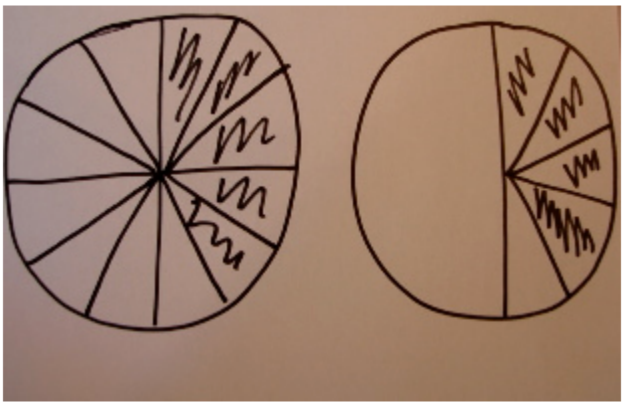

Figure 4.4 Donna's sketch to compare 5/12 and 2/5

Carol: I drew two fifths as well, (rectangle divided into five vertical sections and drawing a horizontal line across all to make ten parts). I was tempted to put it there (pointing to part before 2/5) just because you have to colour a whole other square in get to a half but only half a square on the other one. I don't know if that is right though.

Before Carol drew her comparison diagram for 5/12, she re-examined Donna's sketch and decided that was more helpful. "I think two fifths is probably smaller" was her conclusion.

4.5.5 What Attitudes and Beliefs do Student Teachers Hold About Fractions?

4.5.5. (i) Introduction

Each participating student was asked to complete a questionnaire before undertaking the first task. The results which show their responses to a variety of statements relating to their personal attitudes and beliefs about mathematics can be found in appendix 7.11. The students' responses were included under the initial letter of their pseudonym so that links could be made between their responses to the statements and their contributions to the subsequent discussions and interviews. An initial coding system was adopted when reviewing the video evidence to try to ascertain the trends in attitude and belief about fractions held by the students. Large amounts of dialogue were coded and themes began to emerge to create these initial categories, (see appendix 7.13). It gradually became apparent that these coded groups fell into five main attitudinal categories, which form the sub headings below. Three categories seem closely linked; the lack of confidence, anxiety and acknowledgement of confusion are inevitably linked but several students exhibited signs of one of these without necessarily showing all these aspects.

The five main categories were:-

4.5.5(ii) A Belief That There is a Single/Correct Way of Answering a Question (B1)

Eight students referred to their secondary school experience. Comments relating to whether they would be able to remember suitable methods were made by four students; this was typified by Donna, "I just hope I can remember what you are **supposed to** do with some of these questions". Six students referred to the length of time since they had studied mathematics prior to university. "It is a very long time since I did my GCSEs…." (Carol) "I do remember doing fractions like these but it was so long ago…. "(Megan). This became more evident in the diagnostic interviews. For most students undertaking the tasks, comparing fractions which were relatively close in size and with differing denominators proved difficult. There was regular reference to methods used in secondary school which generally had been partly forgotten. See 4.5.3 (v). This view was typified by Fran's comment, "when we get stuck, shouldn't we be doing that thing..? you know… (miming the drawing of a horizontal line) where you draw a line and find the LCD (Lowest Common Denominator) and then I think… you check to see what goes into it ? Did you do that at school?" As a group they were uncertain what this had entailed so did not proceed with it as a method for making comparisons. Throughout the tasks and particularly when answering questions in the diagnostic interviews there was frequent reference to the way a question "ought" to be answered and discussion about whether they were answering a question "the right way".

4.5.5(iii) Lack of Confidence (B2)

The following are sections from tables showing participants' responses to the questionnaire when asked to judge their levels of confidence in all areas of mathematics. The complete table can found in appendix 7.12.

	Very confident	Quite confident	Less confident	I am not sure
Division	**C**	**E J L**	**B D F G H I K M**	
Fractions	**L**	**A D K M**	**B C E F G H I**	**J**
Percentages	**L**	**B D J**	**A C E F G H I K M**	
Decimals	**G L**	**A B D F H K**	**C E I J M**	

Table 4.5 Participant's Individual Perceptions of their Confidence in the aspects of Mathematics relating to Fractions.

The participants were aware of the nature of the study before they volunteered. It can be seen from in appendix 7.12 that the group included students demonstrating a wide range of levels of confidence in their understanding of mathematics. For example, Lynn had indicated a *very confident understanding* in the area of fractions and decimals, her responses to the tasks tended to favour the use of decimals with a predominant approach throughout to turn most fractions to decimals, "To make them more accessible" (comment from task 2). Whereas Jane indicated that she was unsure, when asked about this in the diagnostic interview, she perceived that, *I am not sure,* on the scale, was a lower level of confidence than *less confident*. Her response being "Fractions are something I am *really* not sure about." This was the only area she included in the *I am not sure* category.

Interestingly a greater number of students indicated a lower level of confidence in percentages than in fractions. This initial perception contrasted with the discussion and question choices made in the diagnostic interviews where percentages seemed to be an area of success for most students as discussed in 4.5.2 (i). Although five students indicated that they were quite/very confident in fractions, virtually all the students, showed a tentative approach especially when approaching the tasks. This was evidenced by regularly seeking reassurance that they were doing it correctly. Betty's comment was a typical example. "You would tell us if we were doing this completely wrong wouldn't you?" The lack of certainty that the tasks had been completed successfully (See 4.2.2) was evident in five out of the six groups. Iris, in contrast, seemed the most at ease with the tasks despite, initially indicating a lower level of confidence. The participants were generally very willing to share their thoughts and feelings; this was probably helped by working in self-selected groups and a reflection of the fact that they had volunteered for the study in the first place. Carol provides an interesting contrast indicating she agreed very strongly with *I feel confident in my understanding of maths* and that it is one of the subjects she feels most confident to teach. However also she felt *less confident* in her understanding of fractions, percentages and decimals. She regularly made assertions about her thinking followed by "But I really don't know, I am just not sure".

4.5.5 (iv) Signs of Anxiety Relating to Fractions (B4)

	1	2	3	4	5
Maths often makes me feel anxious.	**J**	**A B G I**	**C D F K M**	**E L**	**H**
I feel comfortable using maths.	**L**	**A C D E F H I M**	**G J**	**B**	**K**

Table 4.6 Participant's Individual Perceptions of their feelings towards Mathematics.

The table above, which is a section of the appendix 7.12, shows the students' responses to the statement *maths often makes me feel anxious,* where one indicated a strong agreement. In response to this statement, Jane was the only one whose response indicated that she strongly agreed with this. The other participants who suggested that mathematics often made them anxious were Anne, Betty, Gill and Iris. This is quite often reflected in their comments within the discussions, for example, "Who wouldn't be nervous when faced with all these fractions?" This was Jane's comment at the start of the second task. Betty regularly referred to her feelings about mathematics and the following typifies this. "I panic because I am not sure of all the rules, you know, when you are dividing you need to flip them over. I am never sure if that is the right way around, I get the rules all mixed up".

Holly indicated that she was not sure in this category and used, *I am not sure*, as a response to four of the categories, only Karen included more "*I am not sure*" answers but annotated her questionnaire with some qualifying comments for example, she included "sometimes" as a response to "I feel comfortable using maths". This statement providing a link to the anxiety statement and several of the same students Jane, Gill and Betty, responded in a similar way with Betty giving the least positive response to this statement. Although this was not specifically pitched at fractions, it gave a useful indicator of the participants' attitudes and feelings about mathematics and the coding of discussions indicated a greater level of anxiety when working with fractions. Generally the discussion focused on the student's own subject knowledge; however Jane and Karen considered their feelings on facing the first task and considered how children may feel in the same situation.

Karen: When you look at these all together (indicating the cards spread on the table) then you panic, well I did definitely.

Jane: It makes you realise how children must feel though.

Karen: Yes when you are faced with all these numbers.

Jane: And you, as the teacher say "Come on, come on, I am sure you can do this"

Karen: Hmmm, makes you think though…

4.5.5(v) Acknowledgment of Confusion (B5)

In the discussion following the first task, the admission of confusion was a common theme mentioned by six students from three groups. These were usually general comments as shown by Jane, "Some of these were so confusing, we did get **very** confused didn't we." Also, relating to specific aspects of the activity, Gill apologised for her own perception of the number line. "I really confused us all, I am **so** sorry, I really caused havoc with the fraction line". There were some questions which seemed to promote a greater level of acknowledged confusion, for example, 5 and 7/ 6 minus 3 and 5/8 =?

4.5.5(vi) Satisfaction on Successful Completion of a Task (B3)

An unanticipated attitude, which was a very pleasant surprise, was the evident pride and pleasure that the participants showed on the successful completion of a task or a question, especially if they perceived it to be more taxing, see 4.2.3 for examples. Completion of the tasks by most groups was met by relief and in some cases astonishment that they were correct. Ellen, Gill and Fran on finishing task 2, "Is it really correct? are you sure? Wow, how did we do that?" followed by much mutually congratulatory laughter, clapping and high-fives. The level of satisfaction often seemed to be proportionate to the level of difficulty and the time taken to complete it. Some students reflected on their success compared to their initial views. These types of comments tended to come from those who had been more anxious initially. For example during the diagnostic interviews, Betty commented, “Maths is not my strong point anyway and I realised that I have forgotten a lot. But some things I thought I didn’t know I found I knew really after all.” And Jane, “Sometimes I did surprise myself; I knew more than I thought I did. I haven’t done maths since GCSE which was ages ago.”

Chapter 5 Analysis, Synthesis and Discussion

5.1 Introduction

The research questions will provide the structure of this chapter. The methodology, research literature and findings in the earlier chapters will be drawn together to consider the main issues. There will be some inevitable overlap between questions as connections between them are explored.

It is acknowledged that there is a wide range of research in the field relating to the teaching and learning of fractions. The focus of this study was to explore a small group of student teacher's understanding of fractions. In an effort to explore this specific group of adults' understanding, only areas which have been explored less in the previous research have been included in the findings and the main issues arising will be considered in this chapter. The inclusion of the tasks one and two was intended to explore further some of my initial observations from working with students in university primary mathematics sessions. Both the tasks were focused on the part-whole and the measurement sub-constructs of fractions. One of the key aspects considered within the methodology, which was consistent with the phenomenographic approach, was the provision of the opportunity for the students to engage in mathematical discussion. This focus on the prior learning, as well as creating situations which enabled students to explore and extend their understanding, gave a real insight into individual's perceptions of fractions. It was intended that the tasks would encourage the participants to give explanations and justify their thinking to their peers. This was considered an important part of developing mathematical confidence in student teachers as they develop the ability to validate their answers (Ball,1990).

The successful completion of these tasks involved making comparisons between a range of fractions and whole numbers to establish their magnitude by placing them along the same continuum. Task one was intended to be more accessible as an introduction so this included decimals and percentages as well as proper and improper fractions. These tasks and the diagnostic interviews were also intended to explore further the areas of research which indicated that some secondary school pupils and student teachers had a limited appreciation of fractions as numbers (Streefland, 1993, Domoney, 2002).

5.2 Which Aspects of Fractions and the Related Areas do Student Teachers Show a Confident Understanding of?

5.2.1 Introduction

The participants displayed a greater level of confidence in their use of percentages up to 100%, finding fractions of quantities and the finding of straightforward equivalent fractions. The strategies employed by the students are discussed in the following section.

5.2.2. Successful Strategies

A successful strategy adopted by many groups, as could be anticipated, was the initial placing of the more common fractions and their equivalents to provide a structure. This reflected the use of *mathematical anchors* (Singer-Freeman & Goswami 2001, Spinillo, 2004). These were often used as a guide and referred to as "boundaries" (Jane, placing a half in task 2) or "markers" (Megan referring to 0.4 in task 1 and Carol used 3/4 in task 2). This comparison to the more accessible numbers, for example 1/2 or 1 is also referred to as *Benchmarking* by Clarke at al. (2008). The use of mathematical anchors for comparison reflected the use of a half, as an anchor, made by eight and nine year old children when adding fractions (Spinillo 2004) where it was considered to further facilitate their understanding.

A particularly effective use of a half as a mathematical anchor was made by Anne in her explanation, when Betty was uncertain about which might be larger, 55/100 or 45/80. The size of the denominator and numerator in this case were proving to be very inaccessible. Betty had observed that they were both larger than a half but was unable to proceed further. Anne suggested that,

"They are both five away from a half, but five hundredths is equal to a twentieth, and five eightieths is the same as a sixteenth. One twentieth is smaller so it will be nearer to the half."

When further explanation was needed she elaborated with, "That (pointing to 45/80) is…(bigger) because it is a half and a sixteenth and that is bigger than a half and a twentieth." This was an interesting and natural use of a mathematical anchor, where her prior knowledge of a half and its equivalents was used effectively to enable her to compare smaller simple fractions. This was one of the more complex explanations offered by a student to support the learning of another. Anne demonstrated an element of axiomatic deductive knowledge here (Kieren, 1993) by drawing on her intuitive understanding of the relative size of the fractions under comparison (See 7.2). She then used her technical symbolic knowledge to find the equivalent fractions and removed a half from the consideration of each in order to make the comparison of a sixteenth and a twentieth more accessible to Betty. This enabled her to deduce that 45/80 was larger than 55/100. Anne's strategy is described by Clarke et al. (2008) as *Residual* thinking where a learner refers to the amount needed to make a fraction up to a more accessible number, usually one or, in this case, a half.

A similar strategy was described as *Gap Thinking* by Pearn & Stephen (2004) in their study of secondary aged pupils. An example of where this was used effectively was demonstrated by Ellen when comparing 4/5 and 7/8, she explained to Gill and Fran "so I cut out one eighth and there is more cake left than when I cut out a fifth, so it is nearer to a whole cake" (mimed circles and the resulting sized slices in the air). This was a practical explanation which demonstrated her ethnomathematical and intuitive knowledge (Kieren, 1993) enabling her student colleagues to envisage and compare the cakes and the remaining fractions. Here Ellen was able to "transform" her knowledge (Rowland et al, 2009) and provide a suitable model to explain to her student colleagues.

A range of personal strategies was employed throughout which reflected the findings of Bonato et al. (2007) in their study where adults were asked to compare fractions in terms of size. They suggested that the main difference between children and adults, when making comparisons between the relative size of numerator and denominator, was that many adults had developed their own methods to overcome these difficulties. One such example was the discussion of contrasting methods used during task 2 where Lynn and Megan were considering 2/9 and 1/4. Both were initially uncertain, where Lynn focused on known facts "1/9 is 0.11111" so 2/9 was 2.222 and consequently less than a quarter. Whereas Megan converted 1/4 to 2/8 and then compared it to 2/9 with the comment "so if the tops are the same…". These approaches typified the respective approaches where Megan was generally one of the quickest to notice possible equivalents and make comparisons based on either the numerators or denominators. In contrast Lynn's use of *known facts* to support the finding of equivalent fractions was evident in several discussions, especially the inclination to convert these to decimals, if they could not easily be compared. In many cases learning was evident from the discussions as students compared methods, however Lynn's focus on decimals throughout, even when this involved quite complex arithmetic, showed a consistent and close adherence to previously firmly held beliefs. In Lynn's case the views that the conversion to decimals would offer a simpler means of comparison and also that some fractions were difficult to visualise, were referred to several times. This reflected an aspect of Constructivism highlighted by Booker (1996) that, in some cases, learners are reluctant to reconsider their established ways of thinking and that a development in understanding may be guided by their existing views of the concept.

5.3 Research Question 2: Which Elements of Fractions and the Related Areas Cause Student Teachers Difficulties?

5.3.1 Introduction

The difficulties encountered in the tasks and within the diagnostic interviews will be considered in the following sections. The aspects chosen for discussion are those most commonly agreed by the participants, though it will be evident in the following sections that the students had very differing levels of understanding and in all cases there were some students who showed a greater level confidence in these areas. As this was a small scale qualitative study it will not be possible to make generalisations based on these findings.

5.3.2 Improper Fractions and Mixed Numbers

All the participants, except Iris, identified the examples of mixed numbers or improper fractions as problematic either within the tasks and/or the diagnostic interviews. During task 1 the relative size of 39/10 and 39/100 provoked debate in all groups, a lack of certainty was evident in most discussions. For example "I was doubting myself, I know thirty nine over ten is 3.9 it is something I know, but because I was questioning everything else, it just made me question what I thought about that too" (Jane) (See Appendix 7.15). Jane & Karen discussing the ordering of 39/10 and 39/100). This lack of certainty was often indicated by the language used to explain these fractions, in most cases where problems occurred they were referred to as *thirty nine over ten* or *thirty nine over a hundred*. Also in the case of proper fractions with a large numerator and, in some cases, the denominator, seemed to affect the way these examples were described. This was also reflected by Betty's comments in task 2 when comparing 45/80 and 55/100. "Look, I am really struggling here, these don't really look like fractions now." This was in direct contrast to the majority of proper fractions which were described in more commonly used fractions language, e.g. 3/7 as three sevenths. In cases where students acknowledged this as an area of recurring difficulty, (Jane and Karen) this way of describing fractions continued with smaller numerator and more accessible fractions (see 4.5.3 (iv)). Here again 3/2 was described as three over two before deciding it is one and a half (question 3 in diagnostic interview). The difficulty identified above as the inability to recognise the whole within an improper fraction which relates to the seven criteria for the operational understanding of the spatial part/whole of a fraction as identified by Piaget et al. (1984). It is possible that in some cases these criteria were not fully understood. In particular "The parts can be seen as wholes in their own right" (1984:277) seemed to be the criteria that caused difficulties in situations where unitising or re-unitising (Lamon 1999) was needed. The confident understanding of the part-whole sub-construct of fractions was considered essential for learners to extend their knowledge of proper fractions to improper fractions. It is possible that the circular models commonly used through many primary schools (Clarke, 2008, Hallinen, 2009) had contributed to a more limited understanding in some learners as they are not considered as supportive for improper fractions (Dickson et al., 1984). This was reflected by Lynn**,** when considering question 15 (5 7/ 6 minus 3 5/8 =) "I can't access those (pointing at the question containing mixed numbers, those are completely abstract to me.. I mean …what does 6 1/6 look like?... nothing", after converting this from 5 and 7/6. This difficulty was particularly surprising as Lynn appeared to be one of the most confident within the group and had demonstrated a strong grasp of the equivalence between fractions and decimals during both tasks. The focus on the use of circular representation combined with a lack of confidence with improper fractions demonstrated by many participants seems to suggest that they have had a similar primary experience suggested by Clarke (2008) and Hallinen (2009). This may have provided a focus on part/whole examples of proper fractions without making the connection to the measurement construct to enable greater links to be made with the number system. This may have inhibited the development of the understanding of fractions as part of the whole number system especially as it extends past one. This lack of connection with the measurement construct was evident in many explanations. Although there was widespread evidence of ethnomathematical and intuitive knowledge (Kieren, 1993) within the student's explanations when comparing fractions, in many cases this did not extend to the consideration of fractions as numbers especially to those larger than one.

Six students referred to 400% as difficult to place in the sequence during task 1 or within the ensuing discussion. An indicative comment came from Donna during task 1, "But what is it 400% of? How can we know what to do with that?" This problem occurred with 400%, which was the only percentage included that was larger than one. This was surprising as during the reflection of task 1, the percentages were cited by most students as the most accessible part of the task. This did not reflect their questionnaire responses where four students indicated they were very (1) or quite (3) confident in their understanding of percentages with the remaining nine selecting this as one of the areas in which they felt less confident. The indication that they found percentages more accessible in task 1 and that they used these as a structure for selecting equivalents, was possibly more of a reflection of their levels of confidence about the specific examples of fractions and decimals examined.

Percentages, particularly 40% and 4% were often used in this task as a mathematical anchor (Spinillo, 2004), as a structure by which to make comparisons, or to help find equivalent fractions or decimals. The students appeared generally confident with percentages less than one and the perceived necessity to know what the percentage was of, only related to 400%. This contrasted with the improper fraction (39/10), which although cited as difficult, seemed to be accepted as a quantity and regarded as a number. The inclusion of the improper fraction 7/6 in question 15 was also considered problematic (see 4.5.2 (iv)). This will be considered further when secondary school methods are discussed.

5.3.3 Uncertainty About the Relative Size of Fractions

An area of difficulty which was evident in both tasks and in the questions, which related to the part-whole and the measurement sub-constructs, was an uncertainty about the relative size of some fractions. This became particularly apparent during the comparison of fractions which were perceived to be close together in size, (see 4.5.3 (iii)). The *distance effect* was evident in many cases; this is the increased amount of time taken to decide which number is larger/smaller, when the numerical distance between them is smaller (Deheane 1997, Butterworth 2000). Four students (Carol & Donna, Jane & Karen,) had particular difficulty ascertaining which the larger/smaller fraction was. They were particularly uncertain with fractions with larger numerators and denominators which appeared less familiar and were not straightforward equivalents of any common fractions, (See 4.5.3 (iii) Comparison of 99/100 and 7/8, Carol & Donna).

The use of a mathematical anchor or benchmarking (Spinillo, 2004, Clarke, 2008) combined with residual or gap thinking (Clarke, 2008, Pearn & Stephen, 2004) was used effectively in some cases. However this strategy was not universally successful and was partly employed by those participants who were less confident in making comparisons between fractions. The concept of a "mathematical anchor" offered some support in initial sequencing of more familiar or simple fractions. This was described as finding "the space" (Carol during the second task) between each fraction and a known fraction with which they were more confident, often half or one. An example of where the use of residual or gap thinking did not provide an effective strategy occurred during the second observed task when Carol and Donna were comparing 99/100 and 7/8. Both fractions were compared to one but, in this case, their conclusion that each fraction was "one part away" did not support them in deciding which might be the larger fraction. This was an unusual incident within the project where the students were unable to proceed. They did not try to draw on their ethnomathematical or intuitive knowledge (Kieren, 1993) to support them or make connections with prior learning or the methods employed when making other comparisons. Their subject knowledge here was reliant on the finding of a method they felt they had known in the past and lacked several elements of the Knowledge Quartet (Rowland et al, 2009) in particular the making of connections between concepts .

This difficulty in making comparisons was similar to the findings of Pearn & Stephen *(*2004) in their study of secondary aged pupils who also referred to fractions with different denominators as *one bit away* from one, for example 4/5 and 7/8. Their next consideration which was the conversion of the fractions to decimals, did not offer further support to Carol and Donna, although 99/100 was considered to be close to 0.9, the difficulty of dividing by eight prevented any further comparison with 7/8. This particular case was unusual, when neither method worked, the cards were placed correctly but with the comment "We just don't know …I will have to guess here" (Carol). In most similar cases, the students' activity reflected the findings of Bonato et al. (2007) and Schneider & Siegler (2010) who suggested that many adults had found their own methods to overcome these difficulties when considering the relative size of numerator and denominator.

Similar difficulties were also evident in the answering of question 5 (Which is the best estimate for 12/13 + 7/8 = a)1 b) 2 c) 19 d) 21). This question focused on addition but the underlying concept was a consideration of the size of a fraction and the appreciation that both fractions were close to one in size. The inclusion of this question was intended as a triangulation of the focus of the tasks. This question had been used with grade 5 children, in

National assessments in Australia and had been included in studies by Mitchell & Clarke (2004) and Smith et al. (2005). The inclusion of the multiple choice element was intended to generate a greater level of discussion. It was anticipated that this question would be answered confidently as it was based on estimating the size of the two fractions, a skill already used regularly in the tasks. It was surprising therefore, that only three students gave accurate and immediate answers. Four students displayed some evidence of whole number bias (Ni & Zhou, 2005), they responded to the question by considering the numerator and denominator separately taking each as a natural number. Two initially responded with "nineteen over twenty one" (Iris and Gill), having added each pair of numerators and denominators. Whole number bias (Ni & Zhou, 2005) where fractions are not perceived as a number is a common issue for primary school children, but was also found be the case in research studies with some secondary school pupils (Pearn & Stephens, 2004) and remaining a difficulty for some adults (Bonato et al., 2007). This question prompted an unexpected range of answers and responses. "It must be quite big… probably 19, but I can't remember what you are supposed to do" (Betty) was a typical comment. It generated discussion in several groups about the addition and multiplication of fractions and the methods they had employed during secondary school, these mostly resulted in the assertion that they were no longer certain about what these methods had been. This question was interesting in the discussion it produced, as 9 out of the 13 did not respond to the size of the fractions or appreciate the estimation aspect of the question. The prospect of adding the fractions seemed to be problematic and acted as a distraction from any possible simpler approaches (See 4.5.3(v) and 5.3.5). This instrumental approach (Skemp 1989) adopted by the majority of students provided a useful insight into their wider understanding of fractions. The majority seemed unable to make connections with their earlier learning of fractions, or connections with their wider understanding of mathematics, in particular place value or estimation. These participants seemed to be searching for a procedural approach (Hiebert & Lefevre, 1987) to support them as they do not yet seem to have developed an effective conceptual understanding of the relative sizes of fractions.

The inclusion of Question 4 (If 3 pizzas are shared between 7 boys and 1 pizza is shared by 3 girls. Who would get the most pizza? How much more?) was intended to offer the opportunity to explore further the comparisons of the relative sizes of fractions made in tasks 1 and 2. This was based on the question asked by Clarke et al. (2008) with the extension of "how much more?" As anticipated, the wording of the question prompted most students to use circular representations. This question was identified as accessible by seven of the students. Iris and Lynn used a division approach, "well, it is a third and then three sevenths for the boys"(Iris) when questioned further, " three sevenths is nearly half so that must be more, I could work it out in twenty oneths .. is that what you would call them ?" The most common response was that the girls would all have a third and the boys would have "a third and a little bit", see Holly's drawing below.

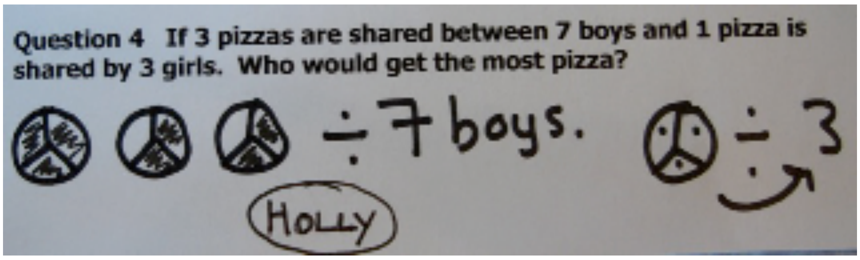

Figure 5. 1 Holly's response to question 4

These types of answers demonstrated both ethnomathematical and intuitive knowledge (Kieren, 1993) using their wider experience to provide a sensible answer and to make a direct comparison between the amounts allocated to the boys and girls. The consideration of the familiar fraction, a third, first and then trying to make a comparison by drawing the pizzas, in most cases left the students considering how they could then divide 2/3 by 7, with only Lynn deciding this was "two twenty oneths". Most participants approached this through drawing the pizzas. This focus on partitioning and the part-whole aspects of fractions reflected similar methods used by children in the middle grades (Clarke, 2006, Lamon, 1999). The participants generally did not consider, the division sub-construct despite use of the word "sharing" in the question. The phrasing of the question did not influence their approaches. The connection between fractions and division did not appear to be strongly developed in the majority of the participants. Where division was applied, it lead to some confusion, as can be seen in Carol's initial jottings of 7/3 = 2.3 and then 3/1 =3 and accompanying explanation. "So the girls would get most, I just divided it. So if seven boys shared three pizzas, they would get two and a third slices each and the girls would get three slices each. I am not sure if that is quite right?" For a moment her working partner Donna was in agreement but then after making a comparison with her own circular diagrams using the "third and a bit" approach was able to explain that Carol's division method had not been successful. They were unable to explain the error or to reflect on how the division statements had been arrived at. The sharing of people by pizzas, (rather than pizzas by people) had been the cause of the initial confusion; this was compounded by the use of vocabulary. The use of slices rather than named fractions meant Carol was unable to question or explain her own logic. Although Carol showed some elements of fundamental mathematics, there were elements of profound subject knowledge (Ma, 1999) in which she appeared less confident, for example, the connected nature of mathematics and the consideration of multiple perspectives. The use of 7/3 as a quotient indicated an understanding of the duality of fractions (Sfard, 1999) where fractions can indicate the process and also show the result of that process. This response showed an interesting mix of fractions and decimals. Decimals seemed to be a representation that Carol felt more comfortable with. This was evident in task 1 and question 6. This was also reflected in the completion times for the tasks, with the second one which included only fractions taking three times as long. The lack of clarity about what was sought by question 4 gave an insight into Carol's understanding and linked to her earlier comment that she found the circular diagrams unhelpful (Task 2). She generally demonstrated a rather instrumental approach (Skemp, 1989) and favoured seeking a more formal arithmetic approach rather than considering the more accessible context of the question and consequently added an extra level of complexity to the question. Carol tended to focus on the technical symbolic (Kieren, 1993) aspect of mathematical knowledge and generally showed little intuitive understanding, see above when considering slices rather than fractions of a pizza.

5. 3.4 Unitising and Re-Unitising

The ability to unitise and re-unitise in a flexible way was a particular problem for eight of the students. Unitising involves *partitioning* a whole, into *equal* parts and also reconstructing (reunitising) the parts back to create the original whole (Lamon 2005). The focus of these three questions (3, 11 and 12) was re-unitising, which involved identifying the original whole, only question 12 involved unitising once the whole had been established. It was thought that re-unitising would be a more appropriate focus based on earlier studies which suggested that this remains a problem in some secondary school pupils and adults (Kieren, 1993; Baturo 2004, Behr et al. 1992; Lamon, 1999). This also seemed to be the case in this study, the students were generally confident with unitising but the concept of re-unitising seemed unfamiliar to the majority of the students (see 4.5.3.(iv)). Re-unitising was considered an important strategy whether undertaken physically or mentally and the inability to identify the base unit was considered as a factor in inhibiting development of a greater level of understanding (Behr et al. 1992, Ding, 1996, Mitchell, 2004).

It was evident that some students found the consideration of *numbers as fractions* problematic and the inclusion of discrete objects seemed to compound this. The two stages of initially identifying the number with the given fraction, and then "scaling the fraction back up to the whole" (Donna), were considered difficult by eight students. The difficulties encountered here contrasted sharply with the confidence displayed by most students when finding *a fraction of a number* as in questions 5 and 8. This could have been partly due to the less familiar presentation of the questions.

Question 3 (X X X X X X = 3/2 of the unit. How many are there in a unit?) was intended as an introductory question offering the possibility of working in halves. However the inclusion of the improper fraction counteracted this possible level of accessibility, (see section 5.3.2). Question 3 was met with a range of confused responses, for example "How can it be more than a whole one?" (Holly). There was a general increase in confidence across all groups when tacking question 11 based on a successful outcome from question 3. However, it was still met with some hesitancy in some cases, for example, the inclusion of sevenths which were considered "unusual" (Megan). Virtually all students divided the dots up on the page with seven students continuing the pattern of three dots for each seventh to make 21. (See figure 4.2) There seemed to be an uncertainty about how large the whole could be. "I had expected it to be much more… with all those dots to start with" (Betty), this was one of the many examples where the students' mathematical understanding of estimation or place value was drawn upon to support them. Several students seemed to consider these questions in isolation, with little connection to their wider mathematical context, in this case, the use of estimation or knowledge of multiples. This was just one example where an instrumental approach (Skemp 1989) was evident and the question seemed to be under consideration as if the concepts were new to the student. Virtually all students used *the continuation of the pattern* strategy to complete question 12. There was apprehension about the two part nature of the question, especially as reunitising was required first, once the first part was completed then most students found the unitising part of the question accessible. Iris attempted a more relational approach to the question trying to solve it in one step through the use of common denominators, however this proved too difficult and a two step approach of re-unitising and then unitising was adopted. The inclusion of three similar questions with a steady progression was productive in building confidence and provoking discussion and within the working pairs there was evidence of collaborative learning where knowledge was co-constructed (Carpenter & Lehrer, 1999). Particularly in the cases of Anne & Betty and Jane & Karen valuable learning took place through the extension and application of their mathematical knowledge. Both pairs were honestly reflective and through the articulation of what they knew they began "making the mathematical knowledge their own" (Carpenter & Lehrer 1999:20). It was intended that through the choice of activities and questions the opportunity for relational understanding (Skemp, 1989) might be promoted. Although not all aspects of fractions were explored, the aspects which students found difficult were reflected more generally in other studies which examined student teachers' subject knowledge (Domoney, 2002, French, 2005).

5.3.5 A Belief That There is a Single/Correct Way of Answering a Question and a Consideration of "Secondary School Methods"

An interesting and unanticipated aspect of the study was the students' assumption that more complex or "secondary school" methods were "clever" and intuitive or straight forward methods were not considered as valuable. This view was apparent in several of the discussions between working groups and was typified by Megan's response "Hmm that was very impressive … do people who are good at maths like to make it more complicated?" (This was following a complicated and unsuccessful attempt by Lynn to answer question 15 (5 7/ 6 minus 3 5/8 =) (see 4.5.3.v) Whereas Megan herself had solved the question simply and accurately keeping the whole numbers intact and using the finding of the common denominator to subtract the fractions. Yet Megan, who had responded to the size of the numbers and adjusted her method accordingly, did not perceive this as "good at maths".

There was a tendency for some participants to return to the more formal methods they had learnt in secondary school. This was particularly evident in question 5. This was reflected in the earlier findings, (see 5.3.2) where fractions with larger numerators proved problematic and were perceived as inaccessible. More effective and straightforward strategies which had been applied with familiar or simple fractions were often not considered. This question also prompted discussion about secondary school methods, the addition sign was a distraction from the suggested estimation in the question. Several students commented on the need to remember "the method or way" of adding fractions, typified by Betty. "I am sure I did this at school, but I can't remember what you are supposed to when you add fractions". This was just one of many incidents throughout the diagnostic interviews where participants tried to remember a method rather than consider the relationship between the numbers and/or fractions. These responses reflected constructivist views that in many cases, learners are reluctant to change and adapt their thinking of long held views or concepts, (Booker, 1996). In most cases, this belief that there was a correct method was counter-productive to developing a relational understanding of fractions. The inclusion of multiple choice answers in question 5 was certainly effective in provoking discussion especially amongst those who considered several of the answers might be possible.

The use of “secondary school methods” was particularly evident in question 15 (5 7/ 6 minus 3 5/8 =) (See 4.5.3. (v)). This was included intentionally in this accessible format, where no simplification would be needed so that it could still be considered as feasibly asked at primary school level. It was based on similar questions used by Lamon (2005). The inclusion of the improper fraction was included as a result of comments relating to the difficulties of working with improper fractions made in the pilot study. This initial finding was reiterated by the participants in the main study during task 1. It was hoped that the relative sizes of the mixed numbers and the inclusion of 7/6 as a mixed number would prove accessible. Only one student (Megan) responded to the question in its original form. The predominant inclination was to convert 5 and 7/6 into 6 and 1/6 straight away. The usual explanation was typified by Betty “That’s what you do first …simplify them”. This initial step caused confusion for most participants, as suggested by Lynn, “Now I am in all sorts of trouble” when the arithmetic became complicated. The size of the numbers and improper fractions did not seem to influence the choices about how to tackle this problem. When she reflected on this question Jane suggested that, “improper fractions always seem so much harder so it made sense to convert it”. This assumption that there was an established method tended to focus on finding a technical symbolic approach (Kieren, 1993) and in many cases did not appear to value the more intuitive knowledge which they had successfully employed.

5.3.6 Vignette – An Individual Case – The Little World of Fractions

One particularly individual misconception relating to Gill's understanding of fractions and their place in the number system became apparent during the first task, see 4.5.3 (vi) for the transcript. Ellen, Gill and Fran seemed unable to agree on the ordering of the fractions in the sequence. After several attempts to clarify the way the cards were being placed, Ellen suggested the use of a number line to ask Gill to explain her ordering of the fractions. It then became clear that Gill was placing fractions below zero, as if they were negative numbers and that she was placing the cards in the reverse direction with the larger numbers closer to minus one and smaller number closer to zero. When asked to explain further, she explained she knew they were not minus numbers and continued "*but* they are *really* small and go before the numbers, like in their own little world… but I don't know why I did that. I really confused you all. You get taught that anything less than the whole numbers is a minus." Mitchell (2005) refers to these types of misconceptions as "blind spots" which occur within a student's conceptual understanding of fractions and persist through secondary school education based on an early misunderstanding. Gill's view of the number system seems to have gone unquestioned through her years since primary school and she has not been able to create an effective personal mental model from which to develop her understanding further (Martin 2004). Gill's understanding of fractions seemed similar, in some respects, to some of the students in Domoney's study (2002) who did not seem to have had much experience of "thinking of fractions as numbers". However they were reported to have a strong visual view of fractions which for Gill did not seem to be the case. Gill's view was unique amongst the participants though, when the debate about ordering took place in the group, Fran was unable to decide which of the members of her group were correct. Her confusion, however, lay in the relative size of fractions between zero and one rather than considering fractions as placed between zero and minus one.

During the diagnostic interviews Gill and Ellen worked together again. In the discussion following Question 15 (5 7/6 minus 3 5/8 =) Ellen asked Gill about how she saw these fractions, " Well I never thought of them as together like that… like with a number"(miming a pushing together with her hands), she then went back to the "confusion" of the first task. "I thought it would be good to join in with your project cos I haven't done fractions since GCSE, actually did we even have to do them then? I haven't had to do it on placement yet and I don't think we do fractions on our course until next year, do we?" It seems likely that her view of the place of fractions within the number system had been long held and had not been questioned. It is difficult to envisage how this perception may have impacted on her wider understanding of mathematics. She had formulated her understanding of fractions around this but had managed to make sense of the equivalence of fractions and had a sound grasp of the conversion between decimals and fractions in the next task and within the diagnostic interview. This seemed to confirm that despite believing that these were numbers "below zero" they were not treated as negative numbers. Her comments in the diagnostic interview implied that she had not understood the measurement sub-construct of fractions (Kieren, 1976, Hallinen, 2009) which is the understanding of the relationship between fractions and whole numbers as part of the same continuum.

Her questionnaire responses indicated that mathematics did cause her some anxiety and it was not an area in which she felt comfortable. There was an unusual balance between her responses regarding different aspects of mathematics where she included fractions, percentages and division as areas in which she felt less confident in clear contrast to decimals which she indicated as a very confident area. Her lower level of confidence in fractions was also reflected in her initial choices for the diagnostic interview where she indicated that she was looking for questions with whole numbers. Many of her responses throughout the tasks and interview indicated an uncertainty and tentativeness in her mathematical subject knowledge and this may be partly responsible for her acknowledged level of anxiety (Rayner et al., 2009).

Gill's understanding of fractions was very particular and unusual and this "blind spot" had remained unquestioned into adulthood. The phenomenographic nature of the study was valuable in uncovering the student's perceptions which may not have become so apparent in a more quantitative study. This was supported by the nature of the diagnostic interviews (Clarke, 2006) where it was possible to act more as a facilitator (O'Leary, 2004) and allow the group interaction to generate the data (Roulston, 2010). Surprisingly Gill was not deterred by her earlier "confusion" and joined in willingly with the remainder of the tasks and interview. The questioning and probing by peers in a self selected group provided a constructive opportunity to explore each other's thinking even when the discussion could be potentially embarrassing. This potential pitfall and embarrassment was dissipated by the good humour and encouragement of her group. Her persistence was partly due to the supportive nature of her group but largely as a result of her own positive approach to learning as demonstrated in her given reasons for joining the project. Her comment in the diagnostic interview typified this "Good job I found that out before I tried to teach it!" The confidential nature of the project and the way it was structured in order to offer the opportunity to discuss and reflect may have also contributed to her willingness to participate so openly with the project. The importance of student teachers holding secure mathematical subject knowledge was an underlying premise of this study (Askew 1999, Goulding & Suggate 2001) . It became evident there were specific areas in which individual students felt less confident and that they were aware of their own difficulties, (See section 4.5.2). Gill's "blind spot" reflected some of the findings of Meredith (1993) who found that the pedagogical knowledge, displayed by some of the teachers in her study, was more closely related to their prior knowledge and beliefs than a result of their training.

5.4 Research Question 3: Which Representations of Fractions do Student Teachers consider to be the Most Effective in the Learning/ Relearning of Fractions?

5.4.1 Introduction

The main focus in the data collection of this study was the dialogue which took place during the observation of the two sequencing tasks and a diagnostic interview. However, this data was supported by annotations, diagrams and occasionally the more formal recording of the mathematics undertaken. This written evidence helped to supplement and reinforce the data gathered. The video taping of the discussions also gave an insight into the types of models the students were envisaging and using to support their explanations. In some cases, the miming of actions, such as chopping into sections or the drawing circles in the air to demonstrate to their colleagues provided a clearer indication of their thinking. The majority of the questions for the diagnostic interviews and the tasks were presented in an intentionally neutral way so as not to suggest particular methods or models to the students. Only question 4 which included pizza as an object for sharing suggested a particular representation.

5.4.2 The Use of Circular Diagrams

Circular diagrams were the most consistently used model by the participants, see 4.5.3(iii) and 4.5.4. Most students identified this as a natural and accessible way to explore questions they were uncertain of or as checking mechanism. The majority of the questions related to the part-whole sub-construct of fractions (Kieren 1976, Kleve, 2009) where this model would be most appropriate. The only question which may have influenced the choice of model was question 4 (If 3 pizzas are shared between 7 boys and 1 pizza is shared by 3 girls. Who would get the most pizza? How much more?) where, as anticipated, circles were the favoured model.

During task 2 the use of circles was favoured more consistently where the fractions were either drawn on one diagram or on separate circles in order to view their comparative sizes. The second task prompted the use of circular diagrams to support the making of comparison between two fractions which were perceived to be similar in size. This occurred usually when the denominators were not immediately comparable. An example of effective use was given by Ellen (See 4.5.3 (iii)), however this was not always the case and their use resulted in confusion, for example Jane and Karen, decided that 1/3 and 2/5 were equivalent based on their rough sketches (See 4.3.1). This approach was used to a lesser degree in the first task as most of the denominators were divisible by ten and the use of percentages was favoured to make comparisons.

The choice of circular diagrams is the most common model included in primary school mathematics texts book, (Clarke, 2008, Hallinen, 2009, Kleve, 2009) and the participants' decision to use them to answer a range of questions reflected Cramner et al.'s study (2008) where it was thought that there is a tendency for students to return to the models they were taught in the initial stages of learning fractions. There was evidence of students using a similar representation regardless of the nature of the question, for example Lynn' use of decimals or Carol's rectangular diagrams. An important aspect of the teaching of mathematics is the effective choice of representation to support learning, (Drews & Hanson, 2008). The careful consideration of which types of representations might best promote understanding is an aspect that some participants may find difficult at this stage .

5.5 What Attitudes and Beliefs do Student Teachers Hold About Fractions?

5.5.1 Introduction

The design of the study was specifically intended to elicit the attitudes and beliefs held by the students. The nature of the diagnostic interview was shaped in part by the participants' responses to the tasks. As the majority of participants had been very explicit and forthcoming in their discussions, it was not considered necessary to ask a direct question relating to attitudes and beliefs. This data which was gathered in situ seemed more "real" and contextual. Some views were explored further by members of the group who, in effect, adopted the role of interviewer, for example, Ellen questioning Gill, which can be found in the vignette, (4.5.3 (vi)). A possible disadvantage with this approach was that it generated a greater quantity of data for analysis, however this was balanced by the deeper and richer nature of the data (Denzin & Lincoln 2003). The way the camera focused only on the activity undertaken, the "disembodied hands" (Betty) and the voices recorded in discussion, seemed to promote a more confident approach and encouraged a greater level of participation. Many of the participants accepted the offer to view the setting of the camera and see the way their colleagues would appear in the recording. As the project progressed there was generally an increased willingness to share their views and feelings as they became more familiar with each other, the researcher, the way of recording and types of activities involved. All the students seemed to view the opportunity as a useful learning experience aiming to clarify areas of uncertainty and misunderstanding. An example of this would be Gill's reasons for participation, "I thought it would be good to join in with your project cos I haven't done fractions since GCSE". Her approach reflected an aspect of the fifth of Kilpatrick's strands of mathematical proficiency, a *Productive disposition (*2005:116) where the students demonstrated diligence and were prepared to make an effort to develop their understanding further.

Careful coding was undertaken to try to ascertain the implicit attitudes and beliefs within their explanations as well as the explicit references to their feelings. The initial codings included three aspects. These were; Mathematical themes (T), Attitudinal (A) and Explanatory (E). These proved useful initially in keeping track of emerging themes and issues but it became apparent that there was repetition and overlap within the initially chosen codes and they were then combined or discarded as the data was reviewed. The range of initial and revised codings can be seen in the appendices, 7.13 and 7.14.

5.5.2 Lack of Confidence and Signs of Anxiety Relating to Fractions

Unsurprisingly one of the main themes, which emerged based on a combination of four of the original codes (See 7.13), was a lack of confidence in their approach to working with fractions. These indicators were often linked to signs of anxiety regarding their understanding of fractions, so these aspects will be considered in conjunction with each other. These findings were not unexpected as the questionnaire responses had shown the majority of the participants felt less confident in fractions, decimals, percentages and division, than in other areas of mathematics, (see Appendix 7.12). The snowball or chain sample (Patton, 2002) was comprised of self selected volunteers who created their own working groups based on friendship. This produced a range of mixed groups who seemed motivated to develop their understanding further. The second year volunteers, identified alphabetically from Anne to Iris, were nine out of a possible 70 students. These participants were not part of the twenty five students who chose a mathematics specialism for their final year core subject module. This gives some indication of the context of a large part of the sample (9/13) within their cohort. A lack of confidence was expressed by all students at some point and these are evident throughout the findings, (see 4.5.5 (iii)).

The discussions, which accompanied the tasks, were very revealing in showing how the students felt during the activities. They seemed to speak naturally to each other without feeling the need to be formal or to particularly adjust their language for the recording. Within the discussions there was a great deal of redundant language indicating a level of tentativeness and possibly anxiety (Bibby, 2002, Buxton, 1981). Much of their discussion was tentative in nature, for example, *I think, I might* or *I hope* before making an assertion, as typified by Anne, "We think we have finished but it is probably really wrong," after completing task 1 (successfully). This use of language was indicative of the findings of Rowland (1995) who categorised these types of comments as *plausibility shields* which can either indicate a personal view but also show doubt in the assertion made, especially when its veracity is under discussion. The frequent use of disclaimers like *only* and *just*, which were used as qualifiers within explanations, had a very similar effect. At points where the participants were uncertain about the accuracy of their answer, there was a regular use of *approximators* (Rowland, 1995). For example, *rounders,* such as around or about, "It is around a third? Well, a bit more really" (Karen when considering question 4) and *adaptors* which usually act as qualifiers, for example "We are *fairly* sure we are right, aren't we?" (Betty on completion of task 1). The tasks were also accompanied by nervous laughter and giggling. As the codes were revised, it became apparent that these aspects of the students' behaviour indicated a certain level of anxiety (Buxton, 1981, Chinn 2010, Bibby 2002). ; this was reviewed and combined to create an overall heading *signs of anxiety* when referring to fractions. This included such aspects as nervous laughter, a greater use of redundant language, regular seeking of reassurance from others within the group as well as more explicit comments relating to anxiety. Most groups described some anxiety when reviewing the tasks (see 4.5.5(iv)) and the representative comments for the revised codings can be found in 7.14. Even students who had indicated a more confident approach in their questionnaire and had proved competent in a range of questions, for example Lynn, had moments where anxiety was evident in their discussion with their colleague, see 4.5.3 where their decision to make the conversion of mixed numbers into fractions (question 15, 5 7/ 6 minus 3 5/8 =) rendered the question much more difficult to answer.

There were a range of more explicit comments that indicated that the tasks were creating some anxiety, for example, "(It) ...is like trying to do a jigsaw. Some bits fit and you don't know what you know. Some bits of my memories of fractions have completely disappeared." (Jane) and "This is like working in a different language to me" (Fran). The students were very open about their feelings and shared their difficulties frankly and willingly. This may be due to the creation of a comfortable collaborative working environment for the tasks and interviews, but it may also reflect a widely held view in the United Kingdom. This is the perception that it is socially acceptable for adults (and children) to admit to a dislike of mathematics and to share the assumption that it is a difficult subject for them to understand (BBC,2008, Chinn, 2010).

Apart from the findings of the questionnaire there was little direct evidence within the tasks that students felt differently about their competence in fractions in comparison to other areas of mathematics. No direct question was asked relating to this in the diagnostic interviews and although students commented on the difficulties they were finding with fractions, it is not possible from this study to say whether these difficulties related to other areas of mathematics as well. However two students (of the three Carol, Ellen and Iris), who had indicated that mathematics was one of the subjects they felt most confident in teaching on school placement, commented on their confidence in mathematics in general. "I am normally O.K. with maths" (Ellen) or "I was one of the ones who was good at it when I was at school" (Carol) were examples which were used as introductions, before going on to explain which aspect of a question was proving problematic to them. Both students used these qualifying types of statements several times, as if the difficulties they were encountering were a surprise to them. This seemed to reflect some of the findings of Bibby (2002) where apparently confident student teachers felt anxious about mathematics and this was in conflict with their personal perception of their own professional capacity.

During the discussions most students did not make reference to using their subject knowledge to inform their teaching. However this was a theme considered by Jane and Karen within their discussions. "We should teach together… we are good at different things" (Jane) reflected their consideration of impact of their subject knowledge on their ability to teach and on the learning of the children. They showed a clear appreciation of the value of good subject knowledge and how, if this was not in place, this may impact negatively on the children they teach. They regularly shared their perceptions of themselves as mathematicians and as potential teachers (Aubrey 1997). This focus may have been because they were closer to the end of their course than the majority of the participants.

The only other comment which reflected the role of mathematics in the process of becoming a teacher came from Donna, "I hated maths at school, but now I have started teaching it, I have begun to understand and enjoy it. I might even do the maths specialism. My mum would be astonished!" This provided some further insight into her reasons for taking part of the project.

5.5.3 The Appreciation of the Value of Working Together and Learning From Each Other.

It became increasingly apparent that the value of working in a group had been recognised and appreciated across the groups. There were unsolicited comments made in the discussions following the group tasks, for example, "We were a good team and learnt things from each other" (Fran). Several students seemed to be aware of the way that understanding had been reviewed and in some cases, had been co-constructed as a group. For example, Jane & Karen discussion when answering question 3 together, (see 4.5.3. (iv)). This articulation of their thinking and reflection enabled several students to "make the learning their own" reflecting the process suggested by Carpenter and Lehrer (1999). This was most evident between pairs when difficulties were considered and addressed. All groups worked in a supportive manner but those who had encountered greater difficulties, included the discussion and shared learning as a part of the development of their successful understanding, (see 4.2.3).

A collaborative approach was adopted by most groups and they worked together in a supportive fashion. This may be because the groups had been self selected. In the early stages, before a negotiated way of working had become established, there was a tendency towards the acceptance of all the assertions offered, (see the Vignette 4.5.3 (vi)). A questioning of each other's decisions took longer to develop in some groups. This was a reflection of a dilemma considered in the early planning stages of the project. This was the consideration of how suitable working groups could be created where students with similar levels of confidence would work effectively together to re-examine their own understanding. The anticipated difficulty was whether participants would be able to discuss their thinking honestly without appearing to patronise or alienate their less confident peers. Fortunately effective working groups were established for the tasks and the smaller group size, based on the findings from the pilot, proved more productive in discovering individual's views and understanding.

One recurring feature of this collegiate approach was the seeking of reassurance and in most groups this was particularly evident in the first task. In most groups there was an evidently less confident member who sought reassurance from their student partner. This reflected a lack of certainty in their assertions which was demonstrated in group responses at the end of tasks and after individual questions in the diagnostic interview. This seeking of an "External authority - a belief in knowledge as validated by experts and as fixed and absolute" (Hodgen & Johnson 2004:225) was apparent through the questioning and tentative nature of their statements when seeking agreement from their peers and the researcher. Boaler and Greeno (2000) considered the idea of authority and different types of mathematical knowledge. They found links between procedural knowing and an acceptance of external authority in mathematics. This was reflected in the belief, which was regularly demonstrated by most of the participants, that there was an established way of answering a particular question (See 4.5.5 (ii)). This reflected Ball's findings (1990) who suggested that those student teachers whose subject knowledge tended to be bound by rules and procedures were reluctant to change their attitudes and views about mathematics. Within this study some questions became more difficult to answer by those seeking of a specific method rather than responding to the nature of the question or the size of the numbers or fractions, for example, responses to question 15 (see 4.5.3. (v)).

5.6 A Critique of the Research Methods.

5.6.1 Trustworthiness and Credibility

In order to ensure the trustworthiness of the study initial ethical concerns about the tutor/student and researcher/interviewee relationship were borne in mind throughout the tasks and interviews (Davies & Dodd 2004). There was an introductory discussion with each group, which occurred before task 1, about the nature of the project and the manner in which it would be conducted. This reinforced the details of the original discussion where the students' agreement to participate had been obtained and permission forms completed. This open and honest approach was adhered to throughout the project. This was perpetuated by the students in the way they supported each other and the need to participate sensitively within discussions was considered throughout by both the researcher and the students. The participants seemed to view the project as a genuine opportunity to explore their own understanding and refresh and develop their own subject knowledge. The value of discussing each other's thinking was noted by several groups and typified by Ellen after task 2. "It has been really interesting seeing how other people approach the same problem, we all had *such* different ways of doing things". Karen commented on the links to working with children during the diagnostic interview, "Jane and I do lots of things differently, I have learnt a lot by doing questions together, … but just imagine how many methods a whole class of children might have… we would have to be ready to cope with all of those".

The role of the researcher varied within the interviews which followed more the format of a discussion, which often became student lead. This inclusive approach contributed to the integrity of the project (O'Leary, 2005, Cohen et al. 2003,) as the project was planned to try to make the participants feel as comfortable as possible (Roulston, 2010) and to ensure all contributions felt valued.

5.6.2 The Recruitment and Participation of Groups

The reflections of the students taking part in the pilot project had indicated that groups of two or three would be more effective for the purposes of the study. When student teachers were being recruited for the main study, the seeking of comfortable cooperative working groups became a focus. As the nature and purpose of the study was explained to possible participants, the ways of working and types of activities were discussed, the collaborative nature of these methods became apparent to the students who then volunteered as groups of three or pairs, providing a type of network sample (Roulston, 2010:82). This change in size of working group proved effective in meeting the intention of providing more opportunity for all contributions to the discussion to be heard and included. It also allowed for a greater recognition of the levels of agreement between students which was more difficult in groups of 4. The smaller groups helped to support the phenomenographical nature of the study by permitting a greater focus on each individual within the group so that their specific contributions could be fully recognised.

Due to the small scale qualitative nature of the study it was inevitable that an opportunity sample (Cohen et al., 2003, Patton, 2002) would be sought. This initially included those who were particularly interested, who then encouraged their friends to join the project, to form a working group. 13 students (rather than the intended 12) participated in the study, which benefited from their willingness to share their thinking and their generosity with their time. The advantages of using friendship groups were evidently beneficial in the nature of the discussion which, after some initial anxieties, seemed relaxed and forthright. This enabled a greater insight to be gained into the student's thinking and feelings. An anticipated possible disadvantage was the difficulty of working in an established friendship group and how this might affect their contributions to the discussion. There was a concern that the role they may normally hold within the group, might affect their responses, especially if there were differing levels of confidence within the group. Fortunately this did not appear to be an issue and the groups worked in a very collaborative and supportive manner. Some students opted to work individually for the diagnostic interviews. In the case of Iris and Holly, this was beneficial as they were quite contrasting in their levels of confidence and working with them separately helped gain a greater insight into their individual levels of understanding.

5.6.3 Questionnaires

The original questionnaire which was issued to whole cohorts of student teachers in University A, prior to the main study, proved very efficient in providing a range of data (Denscombe, 2003, O'Leary, 2004). The data gathered supported the view that further exploration into student teacher's personal understanding of fractions would be valuable. A revised version of the original questionnaire was used to provide triangulation for the other research methods, as it provided supporting evidence about each participant's views, perceived levels of confidence and qualifications in mathematics. The results of the questionnaire were considered in the light of specific responses of individual students especially relating to their feelings and beliefs. For example see 4.5.5 (iii), where comparisons were made between the indications of a lack of confidence on the questionnaire and student responses during the diagnostic interview.

5.6.4 The Observations of Tasks 1 and 2

The choice of task although based on a key stage 3 sequencing activity proved a valuable introduction to the study. The inclusion of fractions (proper and improper), percentages, decimals and pictorial representations enabled each student to use their favoured representation as a way of accessing the activity. The familiarity with the type of activity which might take place within a primary classroom helped build their confidence and the manipulation of the cards as decisions were made helped to make explicit their thinking (Moyer & Milewicz, 2002, Drews & Hansen, 2008). The similar nature of the sequencing tasks gave the opportunity to contrast and compare responses between groups and individuals which were then reflected in the diagnostic interviews, this provided further triangulation of the data. Although observation is a common research method, this combination of an observed activity with a commentary and a reflective discussion proved very powerful and gave a real insight into their understanding. A disadvantage was the huge amount of data generated as the tasks took some groups more than 20 minutes to complete. This was somewhat unexpected and the time needed to complete the tasks had been underestimated, however the length of time taken was partly due to the diligent nature of the students who were explaining exactly what they were doing and making their decisions explicit to the rest of the group. This thorough approach provided deep and rich data (Lederman, 1990, Denzin & Lincoln, 2003). There was one particular incident in task 1 which warranted a greater level of consideration and discussion, where Gill explained her understanding of the place of fractions within the number system. In order to fully reflect the unexpected nature of this data, a vignette was included, (see 4.5.3 (vi)). The richness of a "narrative story-like structure" (Matthew et al., 1994:81) was used which showed the chronology of events and the discussions based upon it. This was considered in a similar way to Kvale's (1996) technique of narrative structuring which gave an overview of the main incident and the ensuing discussion.

5.6.5 The Diagnostic Interviews

The diagnostic (Moyer & Milewicz, 2002) or task-based (Mitchell, 2005) interview proved highly effective in encouraging the participants to explore their understanding of fractions, whilst giving a commentary which made their actions and thinking more explicit. At the start of each diagnostic interview the students were asked to consider the range of questions and to identify the three which they perceived to be the most and the least accessible. This added an extra dimension to the way that these types of interviews had been used in earlier studies (Clarke 2006, Mitchell & Clarke, 2004) by providing an indication of each student's perceptions of accessibility/inaccessibility of each question.

This preliminary activity proved valuable in several ways and provided the interview with a structure which was shaped by each individual's choices. The purpose of these diagnostic interviews was based on the principles of the clinical interview (Ginsburg, 1981) which was to enable the deeper exploration of an individual's understanding. However it differed in structure in terms of the choices made by each participant at the start of the interview. This structure was in keeping with the phenomenographic approach (Marton, 1994) where each individual's learning and experience was explored. The study was based on the constructivist premise that although the participants are theoretically experiencing the same opportunity, there will be a number of qualitatively different ways of experiencing or understanding the question or problem which will be influenced by their previous experiences of learning, in this case, fractions (Glasersfeld, 1991).

The range of questions remained consistent through all the interviews to ensure that there was the opportunity to make comparisons between students and approaches. This would contribute to the reliability of the design. Each interview took a different route where both the interviewer and interviewee had the opportunity to steer the direction of the discussion. The individual question choices can be seen in appendix 7.10. All interviews followed a similar broad structure, focusing initially on the participant's more confident choices, leading on to some neutral questions and ending with some of those which they perceived to be more difficult. Where the interview was conducted as a pair, a negotiated route was agreed by considering the consistencies in their choices. This was followed then by undertaking some of each individual's choice of question, where students answered both sets of question choices and undertook discussion in the same way as the individual interviews. The paired and individual interviews were equally enlightening and there did not seem to be particular advantage in either approach. However several of those who selected to be interviewed as a pair, indicated that they felt more comfortable with that arrangement. A typical comment, from Karen "At least there is a chance that one of us will be able to answer the question!" The diagnostic interviews generally took about an hour and the students answered between 9 and 13 questions in that time. The nature of the interviews was intended to provide an opportunity to explore and review prior learning so where errors were made, these were identified in order that discussion could take place to address any misconceptions. For example, Holly's response to question 5 (Which is the best estimate for 12/13 + 7/8 = a) 1 b) 2 c) 19 d) 21 (multiple choice)) where there was clear evidence of whole number bias (Ni & Zhou 2005) in her answer '19/21' which was not one of the options. This lead to a discussion about the relative size of 12/13 and 7/8 and then as a result of Holly's response the size of 19/21. This study's approach contrasted the way in which diagnostic interviews (Clarke et al., 2008) had been used with children who were not informed when their responses were incorrect. This gave a clear indication about what could be achieved by each child but did not offer the opportunity for reflection and discussion which formed such a valuable part of the diagnostic interviews with student teachers.

5.6.6 The Effectiveness of the Question Selection for the Diagnostic Interview

One aspect of the research design which required careful planning was the selection of appropriate questions for the diagnostic interviews. It was necessary to choose sufficient questions so the students would have time to consider each one but not begin to answer them. This was intended to give an initial focus on their perceptions of accessibility of each question. On reflection there were probably too many questions for the students to consider, this was reflected by the fact that fewer of the later questions were selected even though they had not been ordered by difficulty. It is possible that there was an expectation that the questions would become more difficult and this may have affected their choices. It was considered necessary to include some repetition in order to give some further triangulation to the tasks for example questions 3, 11 and 12, which focused on unitising and reunitising. It was not feasible therefore to consider all the questions in the findings. For example, question 14,

2 x a/b = a) 2a/2b b) 2a/b c) a/2b d)4a/2b (multiple choice) which included an algebra focus proved rather a distraction from the consideration of fractions. This question, which was pitched at a slightly higher level, was included to probe their ability to generalise about fractions and equivalence. However the algebraic notation added a greater level of perceived complexity than had been anticipated. This prompted much discussion about algebra, rather than the fractions it represented; this generated some interesting data which is, however, beyond the scope of this study. The students in the pilot studies had indicated that the questions were pitched at an appropriate level and generally they seemed suitable for inclusion in the final study. The questions were considered to provide sufficient challenge without being overwhelming or too complex. In the process of choosing and designing questions, care was taken to be clear and accessible. Each question was intended to be pitched at a suitable level for primary school aged children. The context and research behind the interview questions can be found in 7.9. Questions were included from a range of sources including earlier research studies, university mathematics subject knowledge audits and University Challenge.

The inclusion of questions 3, 11 and 12 was intended to provide a progression in difficulty relating to unitising and re-unitising (Lamon 1999).

Students acknowledged a growing confidence as a result of working on all three questions, (see 4.5.3(iv) and 5.3.4). The layout of each question as a horizontal line of discrete objects rather than an array was intentional in order to provide a more open ended approach to the task. Similar questions for children have been presented as rectangular arrays where the number could be more easily divided with vertical lines (Cramer & Lesh, 1988, Clarke, 2007). This rearrangement of the objects could be considered as a visual/perceptual distractor (Behr & Post, 1981). The presentation of the questions in this way was to try to avoid the possibility of the equivalent to shading and counting which is prevalent in many school text books (Clarke, 2007, Ding, 1996). An unanticipated difficulty became apparent in the answering of question 11. This was introduced by the choices of numbers, the use of three as the numerator and also as the result where one seventh equalled three caused some confusion. This difficulty occurred when some students (Megan, Betty and Holly) were trying to explain their thinking and began to doubt their assertions, as typified by Megan, "Are we sure? Let's check again… …Somehow that makes it more difficult… not sure why...are you?" (Megan). If future questions were planned, a closer review of the nature of the answers would be considered as a result of this and question 11 would be adapted to avoid the problem. This confusion contrasted with views held on the repeated use of 4/40/400 as numerators in the first task which was considered helpful by some students, "It helped you make the comparisons as you could see if they were hundredths or tenths" (Lynn whilst reflecting on the first task).

5.6.7 Data Analysis

The data generated by the observations and diagnostic interviews reflected the phenomenographic approach and it was considered important to maintain the essence of each individual's contribution whilst exploring the similarities and differences (Steffe 1996). Possible methods of reviewing and exploring the data were considered in order to show the varying nature of each student's understanding, confidence and attitudes. It was intended to approach the data with as open a mind as possible and although there were inevitably anticipated issues based on prior research, it was hoped to view the data as inductively as possible. However it quickly became apparent, as in much qualitative research, that it would be necessary to use both deductive and inductive approaches (Bryman & Burgess, 1994, Patton, 2002). The use of the phenomenographical approach enabled a range of rich and detailed data to be gathered and also influenced the nature of the coding as part of the analysis. This involved "noticing relevant phenomena, collecting examples of those phenomena and analysing them in order to find commonalities differences, patterns and structures" (Seidel & Kelle, 1995: 55).

As there was considerable video footage to analyse, the noting of key moments and timings became imperative and a rigorous approach to note taking and the capture of dialogue was developed. This involved observing each video several times to become familiar with data and to begin to gain a sense of the individual responses. This was approached systematically by considering the same task for each group so direct comparisons could be made and similarities and differences noted. A paper based timeline was produced for each observed task where main emerging issues could be recorded with names of participants and exact timings so these could easily be found and reviewed. The contributions of individuals began to emerge as the data was reviewed and specific elements of their own understanding became more apparent as they could be tracked throughout an observation.

At this early stage large themes began to emerge so, in conjunction with the timelines, possible incidents of note were recorded under each theme. It became apparent that many of the brief conversations could be classified under several themes, for example, Jane and Karen's discussion about the placement of 39/10 and 39/100, indicated difficulties with improper fractions and mixed numbers as well as a belief that there was a "correct" way of working with these fractions. These incidents were recorded under the appropriate theme and carefully cross referenced to any other emerging themes. Initially the intention was to transcribe all the video footage in order to preserve the totality of the students' experiences and to ensure that the phenomenographical nature of the study was reflected. However it soon became clear that this was not feasible or necessary. The nature of each discussion was segmented and focused on the consideration of specific comparisons with the sequence so it became evident that key moments/snap shots of conversation should be transcribed; these varied in length from 30 seconds up to eleven minutes. Selected incidents were transcribed and linked to the appropriate themes.

Initially this process was repeated for the diagnostic interviews although it soon became apparent that using a cross tabulation for the responses to the questions was more efficient and effective, this in turn enabled the production of transcripts which could then be coded. The focus on incidents and brief conversations maintained the phenomenographical approach as they provided a real insight into an individual's understanding and prior learning and often gave an indication of their attitude towards mathematics or fractions in particular. The use of the student's notes and representations was helpful in conjunction with the accompanying dialogue to give a fuller picture of a student's understanding.

Betty's written response to question 11 can be seen below in figure 5.2 which indicates a confident response to the question and shows an effective approach to checking her answer. Her initial response was less certain and then showed her reflection of the basis of the previous question.

Betty: Hmm, it is another one of those…I really am not sure here. Sevenths sound trickier… (starts to draw dots). That last one was difficult … this one is less than one… so can I just keep drawing ? If three is a seventh? hmm… is that right ?

Anne nods, Betty continues to draw her dots sectioning them into threes, counting under her breath as she does so.

Betty: So it is 21! I will just count again… yes 21, so a whole would be 21. I did it! I think that is right… isn't it? I don't think I could do it without my diagram though.

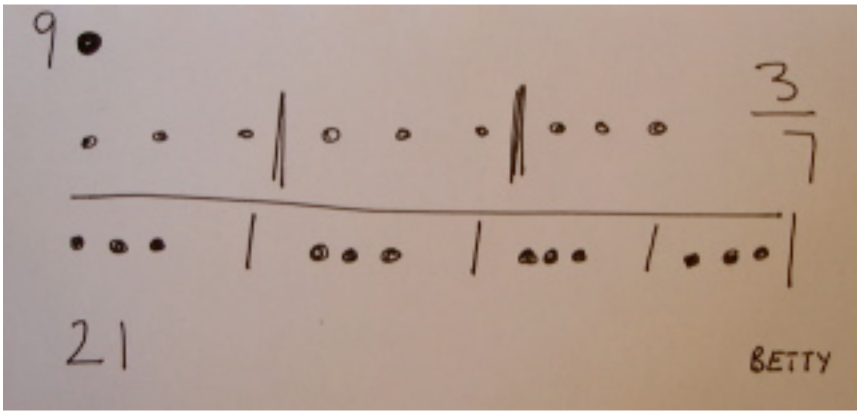

Figure 5.2 Betty's response to question 11

This combination of Betty's explanation and the recording of her thinking provided a full picture of her exploration of the question, including the seeking of reassurance from her partner Anne. This was recognised as a key moment in the study where Betty showed a more confident approach although still couched in quite anxious terms. At the early stages of analysis these critical moments were recorded as above to capture the discussion and reflect each student's role within it. By adopting a phenomenographical approach it was intended to consider the holistic nature of each student's response in order to understand their individual perspective. The collection of many such incidents began to inform the way that codings might be established in order to find some commonalities within mathematical explanations or attitudes or beliefs held by the students. The codes attached to this explanation by Betty can be found in appendix 7.16.

The value of deep and contextualised data cannot be overestimated but the quantity generated in the observations and diagnostic interviews was initially rather overwhelming. The use of a qualitative analysis software package was considered to try to make the data more manageable. After an exploration of Nvivo, it was found to be of limited value due the nature of the data. This was partly due to the reoccurrence of numbers throughout the discussion and the way the same numbers are used in different contexts depending on the nature of the question. This problem was unintentionally exacerbated by the design of the first task where the number 4 was used as the common feature to assist with comparison. This made computer analysis of the first task very difficult, especially the tracking of the students' responses to particular numbers. The student's lack of formality when referring to the mathematics in their discussion made this computer analysis more complicated, as the use of the words denominator and numerator were not used consistently. An example of this would be the way the same fraction might be referred to, as three sevenths, three over seven or within discussion as "this one" so the video needed to be reviewed to ascertain the fraction under discussion. As such detailed and rich data had been gathered, the use of a software package may not have been able to fully reflect the extent of this and may have affected the phenomenographical nature of the study by making it more difficult to retain the individual nature of each student’s responses.

As the data was explored a range of categories and codes began to emerge which reflected the attitudes and aspects of greater/lesser mathematical confidence relating to fractions, (See Appendix 7.13). This process of coding was integral to the analysis (Seidel & Kelle, 1995) as the categories were established and the patterns, similarities and differences were discovered. These initial codings were inevitably quite descriptive in order to label the emerging phenomenon (Richards, 2009). As the data became more familiar, the identified categories seemed to converge for example, *Uncertainty* and *tentativeness* initially were pursued as separate lines of enquiry, though once an overview of the categories was considered they were combined as part of a more overarching theme which was then coded as *lack of confidence*. Care was taken to continue to reflect the complexity of the conversation so although increasingly broad categories were becoming established, the meaning of the students’ assertions would not be lost or misconstrued. The organisation of the data was crucial at this point so cross referencing could take place (Matthew et al., 1994). This occurred where a particular incident may fall into different categories; much of the dialogue reflected an identified attitude as well as a mathematical theme. It was considered important to keep track of these key moments. This proved a time consuming process but by making constant comparisons between the tasks and the diagnostic interviews the final categories emerged (See Appendix 7.13). The larger themes were linked to the key moments and although much dialogue was coded in a line by line fashion, care was taken to avoid the meaning being lost (Charmaz, 2000). This line by line coding was very enlightening and gave a real insight into each individual's understanding. The phenomenographic nature of the study ensured a focus on the whole response from each person and enabled them to be considered individually as well as within the larger emerging themes. For example, Betty's response to question 11, as introduced earlier (appendix 7.16) showed the tentative nature of her response with reference to her difficulties with the earlier related question and the seeking of reassurance which supported her in reaching her conclusion. Even though the question was successfully answered, her final comment of "I don't think I could do it without my diagram though" is revealing, possibly implying that she feels she should be able to answer this type of question mentally. The phenomenographical nature of this study was reflected at each stage of the data analysis as all the aspects of the discussion were considered in the creating of codes and themes to ensure that each student's responses were valued and included.

The value of the collection of qualitative contextual data was appreciated and the methods generated a great deal more than could be accurately reflected within the report. A judicious selection was made based on areas which emerged strongly between the groups and care was taken to consider the contrasting aspects to those in the findings of research studies which had been reviewed as part of the preparation for the study. A diligent and organised approach was adopted towards the analysis of data, though due to the small scale nature of this study it was not possible to triangulate or validate the interpretations made with another researcher or colleague.

When reviewing the data it was important to remember that all the interpretation had been made by one researcher as "when interpreting interaction analysts are inseparable from their analyses which have been filtered through their own experience" (Barwell; 2003:112). In order to be reflexive, an objective approach to the review of data was essential at every stage of the analysis. This involved a critical self awareness of how established views or assumptions might impact on the study and influence the way in which the data was considered (Roulston, 2010). Brown in Barwell (2003) makes a useful distinction between "being aware of judgement and interpretation and acting upon it". An awareness of this reinforced the need for careful and consistent categorisation. It is entirely possible that another researcher would conceptualise the themes differently, for example, following the progress of each individual through the activities, or they may interpret the same incident in a different way. This type of dilemma in reviewing interpretive data is a common feature of qualitative research (Matthew et al., 1994).

In this chapter the research questions have been considered and the themes and issues from the research literature and findings have been discussed. It has been acknowledged that there is a degree of overlap between questions as the individual understanding of the student teachers have been explored. The research methods have been critiqued. The main themes considered are pulled together in the next chapter.

Chapter 6 Conclusions, Further Considerations and Recommendations

6.1 Introduction

This chapter brings together the main aspects of the findings and shows how these have addressed the research questions. The findings, based on this small sample of student teachers, will be reviewed and further possible areas of research considered. The broad nature of the questions and the phenomenographic nature of the study could have generated many different types of responses and I appreciate that the analysis undertaken has generated a very personal interpretation. The original aspects of the study and particular insights are discussed in the light of the findings and within a critical reflection on the research methods used.

The research questions were phrased broadly to avoid prejudging or pre-empting particular responses from the student teachers. It was anticipated that the participants would already have a connected understanding of fractions and their relationship with percentages and decimal fractions. It was inevitable therefore that these elements would become part of the study as many adults use a range of methods when working with fractions (Bonato et al., 2007). The qualitative nature of the methods employed, and the structure of diagnostic interviews based on a phenomenographic approach, resulted in a vast amount of individualised data. The large amount of data generated was very detailed and multi-layered. The small sample of volunteers each brought a different range of experience and knowledge to the project. Consequently, when considering the research questions, it was necessary to link confidence and difficulties encountered in order to give a full picture of the group's understanding and yet still reflect individual differences. It was evident on several occasions that areas where some students felt confident posed specific difficulties for others, so the drawing of generalised conclusions was not feasible nor would have been expected given the phenomenographic nature of the study.

This study was intentionally small scale considering a group of thirteen students. The pilot studies which proved so valuable in guiding the level of challenge and finding ways of making tasks accessible were undertaken in a different Initial Teacher Education Institution. The inclusion of students from two Initial Teacher Education routes (Undergraduate and Post graduate) in the final study added strength to the findings and although the results are not presumed to be generalisable (to the wider student teacher population), this does demonstrate that the findings are not limited to a level of confidence with fractions in student teachers on a particular Initial Teacher Education route or within one Initial Teacher Education institution.

6.2 The Research Objectives and Questions

6.2.1 Which Aspects of Fractions and the Related Areas of Mathematics do Student Teachers Show a Confident Understanding Of?

The main areas where the student teachers felt more confident are shown in sections 4.5.2 and 5.2. These included the use of percentages and equivalent fractions and the ability to find fractions of numbers. There was a range of successful strategies employed throughout the tasks and diagnostic interviews. One of particular interest was the use of a *mathematical anchor* (Singer-Freeman & Goswami 2001, Spinillo, 2004) or *benchmarks* (Clarke et al., 2008). Although the students did not seem aware of this as a technique, it was introduced by various students using their own terminology, as "boundaries" by Jane or as "markers" by Megan and Carol. This use of an anchor/marker/boundary was effective in making successful comparisons between two less familiar fractions and this reflected earlier studies where the technique was used by primary aged children when making a comparison with a familiar fraction such as a half. This use of mathematical anchors in conjunction with *Residual* thinking (Clarke et al., 2008) or *Gap Thinking* (Pearn & Stephen, 2004), where a comparison was made between the numerator and denominator, was effective as their closeness was examined in order to make a comparison. These studies consider the early use of these ways of thinking demonstrated by primary and secondary aged pupils (respectively). The use of these techniques by adults and their application to more sophisticated questions, for example making a comparison between 55/100 or 45/80 by Anne and Betty in task 1, was an interesting and valuable outcome of this aspect of the study. Bonato et al. (2007) suggested that adults find ways to circumvent the difficulties they encounter when working with fractions. This focused approach of considering individual student teacher's understanding enhances the existing knowledge relating to adults' understanding of fractions. Throughout the study there was considerable evidence within the student's individual approaches of the successful use of their own ethnomathematical and intuitive knowledge (Kieren, 1993) which was often explained in a reasoned way to their colleagues. The value of these successful individual methods were not always appreciated by the student teachers and contrasted to the regular seeking of an established method for answering specific types of questions which would constitute technical symbolic knowledge.

A further review of the methods of comparing fractions employed by adults and student teachers through diagnostic interviewing and reflective discussion would be an interesting and productive development of this study.

6.2.2 Which Aspects of Fractions and the Related Areas of Mathematics Cause the Student Teachers Significant Difficulties?

One of the intentions of the study was to further explore the areas of fractions which had proved difficult for student teachers in university primary mathematics sessions. The predominant difficulties arising appeared to be related to the part-whole and measurement sub-constructs of fractions (Kieren 1976). The focus of the tasks and questions in the diagnostic interviews had mainly related to these aspects, with the broader inclusion of the areas specified by the National Curriculum relating fractions at primary school level in the UK, in order to gain as complete a picture as possible. Although there is a growing body of research which considers student teachers' mathematical subject knowledge , (Goulding et al, 2003; Huntley, 2005; Murphy, 2006, Goulding & Suggate, 2001, Rowland et al, 2009) there was a limited amount of research available which looked specifically at fractions: recent studies included Miller (2004), Anderson & Wong (2002), Domoney (2002) and Toluk-Ucar (2009).

The phenomenographic approach taken in this study enabled a focus on the areas which arose from individual contributions to discussions which allowed common themes to be identified. A recurring difficulty was the understanding of improper fractions and mixed numbers (see sections 4.5.3 (ii) and 5.3.2.) This area of difficulty occurred through both tasks and in a range of interview questions. Improper fractions were often referred to in a different way to proper fractions, for example as "thirty nine over ten" or “three over two”. Occasionally proper fractions were expressed in this way but this tended to be if the numerator or denominator was large e.g. fifty five over eighty. Improper fractions were often referred to as being problematic and students commented on their lack of experience in using these. In general they did not appear to apply their understanding of proper fractions when working with improper fractions. In several cases they seemed to adopt a very procedural approach to aspects of the tasks and interview questions which included improper fractions.

The difficulties encountered in considering the relative size of fractions were evident throughout and indicated a lack of experience with unfamiliar fractions where they found it difficult to estimate an approximate size in order to make a comparison or add, for example, ‘Which is the best estimate for 12/13 + 7/8 = a) 1 b) 2 c) 19 d) 21’ (multiple choice). The residual or gap thinking (Clarke et al., 2008, Pearn & Stephen, 2004) where the numerator and denominator are compared was used effectively by four students whereas others were not able to place more unusual fractions easily on the ordered sequence and tended to look for a lower common denominator. The creation of an effective mental model, (Martin 2004) for example a number line or the use of a clock/circular diagram with which they felt confident and would support their learning, was not apparent in most cases.

The ability to re-unitise, which involves the reconstruction of the parts of a fraction to create the original whole (Lamon 2005), was found to be a particular area of difficulty (see 4.5.3 (iv)). The essential skill used in conjunction with unitising is a key aspect of the ability to think flexibly about fractions. This study contributes to the knowledge of this aspect of part-whole sub-construct of fractions by exploring this issue with adults as opposed to primary and secondary school aged children (Behr et al. 1992, Ding, 1996, Mitchell, 2004).

This is considered as an unfamiliar aspect by nine students and seemed initially problematic for most of the participants. The inclusion of the three progressive questions 3, 11 and 12, which involved re-unitising with increasing levels of complexity, gave the opportunity to practise and discuss this skill. A valuable follow up would be to explore whether what appeared to be a learning experience during the diagnostic interview has had a lasting effect and whether this aspect was considered by these student teachers when planning for the teaching and learning of fractions.

When the student teachers were faced with aspects that they found difficult, the majority of the group demonstrated a rather instrumental understanding (Skemp, 1989) which was reinforced by the assumption that there was an established method which they could not entirely remember. It was often considered a matter of memory rather than the use of prior knowledge, mathematical reasoning or relational understanding which would support them when overcoming difficulties.

One particularly valuable outcome of the study was considered in the vignette in 4.5.3 (vi) and 5.3.6, The Little World of Fractions. The phenomenographic nature of the study enabled an individual's understanding to be reviewed; in this case it was the place of fractions within the number system. Gill's "blind spot", Mitchell (2005), became apparent within the first task and revealed an early misunderstanding. Her view of the number system seemed to have gone unquestioned through her school career and strongly showed that she did not perceive fractions as numbers within the natural number system. This aspect also proved a useful learning experience as was indicated in her comment following the reflective discussion of task 1 "Good job I found that out before I tried to teach it". This consideration of individual understanding proved to be very revealing and reinforced the value of this phenomenographic approach in generating rich and valuable data. This study offers further insight into individual understanding of student teacher's understanding especially within this vignette where an unusual view had been held unquestioned until adulthood.

6.2.3 Which Representations of Fractions do Student Teachers Consider to be the Most Effective in the Learning/Relearning of Fractions?

Predominantly the students tended to favour the use of circular diagrams (see 4.5.4 and 5.4.2). This finding was not surprising and reflected the findings of Cramner et al.'s study (2008) where middle school children reverted to using the representations they had been initially introduced to when first learning fractions. Many students adopted the use of such diagrams for a range of questions whether there was a suggestion towards that representation or not within the question. It was relied upon by some students as part of their explanations to their colleagues, e.g. Ellen using a clock analogy in Question 9 or the description of a circular cake is used by Anne task 1 to help make comparisons in her explanations to Betty. Circular representations were not always found to be suitable or helpful for example when used as a rough sketch to help with estimation, see Jane and Karen comparing 1/3 and 2/5 in task 2 (4.5.2).

The use of a number line was rarely considered. A key moment occurred when there was confusion within a group (see the Vignette, the little world of fractions, 4.5.3 (vi) and 5.3.6) and for clarification, Ellen suggested the drawing of a number line which helped the group to appreciate each other's perspective and shed light on the source of the confusion. The lack of the use of a number line may possibly be an indicator of the way these students were taught fractions initially. There were several occasions when it was evident that some students did not always see fractions as numbers and exhibited evidence of whole number bias, (Ni & Zhou, 2005). At these moments the use of a number line would have been valuable for considering the question and clarifying the magnitude of the numbers they were comparing. The tendency to favour one representation reflected the findings of Toluk-Ucar (2009) who suggested that teachers need to be able to move flexibly between different types of representations in order to present concepts clearly depending on the nature of the question. The careful use of representations to ensure that the mathematics is accessible and appropriate to the learner is an important aspect in the sharing of the teacher's own subject knowledge (Rowland et al. 2009). This has wider implications for Initial Teacher Training to ensure that students have a greater familiarity and confidence in a range of representations.

One of the implications of the study has been to review the nature of the representations used in presenting fractions to student teachers in university mathematics sessions and within their directed study. I intend to ensure that, in the future planning of the mathematics curriculum of my current institution, there is a greater emphasis placed on the use of the number line and the place of fractions within the number system. This should reinforce the understanding of the relationship between fractions and whole numbers as part of the same continuum (Hallinen, 2009).

6.2.4 What Attitudes and Beliefs do Student Teachers Hold About Fractions?

The design of the study was intended to elicit the student teachers' beliefs and attitudes throughout the activities and from their discussion together as they answered the questions. The intention was to gather the data in-situ to ensure it was as real and contextualised (Lederman 1990, Denzin & Lincoln 2003) as possible. This approach was largely successful and on some occasions students' individual views were explored further by members of their group who asked probing questions and adopted the role of the interviewer in order to seek clarification.

The main theme which pervaded both tasks and the diagnostic interviews was a lack of confidence in their understanding of fractions (see 4.5.5(iii) and 5.5.2). A lack of confidence and/or signs of anxiety were exhibited in a variety of ways, for example, tentativeness of approach, the use of redundant language, nervous laughter and the need for reassurance. This was not entirely surprising as the occurrence of mathematical anxiety in adults and children has been widely researched (Smith, 2002; Crook & Briggs, 1991; Evans, 2000; Dixon, 2003; Benn, 1997 Boaler 2009). Although the participants were not highly anxious, in terms of exhibiting the signs described in these studies, for example panic or inability to think, there was a consistent occurrence of evidence in the initial coded categories which indicated a lack of certainty in their assertions. The consideration of the language used provided a valuable insight into the underlying levels of uncertainty. The frequent use of disclaimers and qualifiers within their assertion was evident. Uncertainty was also indicated through the regular use of hedges (Rowland, 1995) within many of the discussions. This consideration of the language used within diagnostic interviews enhances the existing knowledge relating to adult's tentativeness or uncertainty relating to fractions and the related areas.

Great care was taken to ensure the working climate was as comfortable and supportive as possible in order to avoid adding any further reasons for anxiety. Bowd & Brady (2001) and Tooke & Lindstrom (1998) considered the possibility of student teachers passing on their own anxiety to the pupils they teach, and this continues to be a key issue in Initial Teacher Education. As a result the planning of future provision for my institution's teaching of mathematics will include a greater focus on addressing student teacher anxiety. Additionally, a further consideration of beliefs and attitudes will be included in conjunction with the review of substantive and pedagogical subject knowledge. This study has reiterated that it is possible that seemingly confident students still have areas about which they feel less certain or have an established way of working which they are reluctant to question and explore, for example Lynn's approach using decimal conversions when comparing fractions. A valuable extension of the study would be the consideration of whether student teachers' attitudes and beliefs about fractions differ from their views of mathematics as a whole.

The appreciation of the value of working together and learning from each other was an unanticipated attitude which was expressed regularly throughout the group discussions (see section 5.5.3). This was particularly valuable where students co-constructed their learning when reviewing and reflecting on areas in which they felt less certain, they were then able to "make the learning their own" (Carpenter and Lehrer, 1999). The comments from students indicated that the study had been valuable in terms of their own learning and had been conducted in a way which made them feel comfortable. This aspect of the data particularly echoed the intended constructivist nature of the study and an area considered for a further study was a consideration of the extent to which student teachers perceived their subject knowledge had developed as part of the study. These aspects of the study combined with the review of the methodology has reinforced the value of the small scale interpretative approach, which will be employed again to further explore the mathematical understanding of student teachers.

6.3 Critical Reflection on Research Methods

6.3.1 Introduction

One of the key elements which underpinned the trustworthiness of the study was the responsible approach which was adopted towards the recruitment and involvement of the participants. It was felt important to demonstrate personal and professional integrity at every stage of the study (O'Leary 2005, Cohen et al 2003, Gray 2004, and Burgess 1989). This honest and open approach included the sharing of the purpose, aims and research questions of the study with the all students who were considering taking part.

The adoption of a phenomenographical approach has contributed to existing knowledge (Neuman, 1997, Asghari & Tall, 2005) by further exploring the way it can be used to consider learning in mathematics. This approach explores the way "individuals experience, apprehend, perceive, conceptualise or understand the world", (Marton 1994: 4424) and in this case proved very effective in understanding the learning of fractions from a student teacher's point of view. The students, although undergoing the same experience, brought their prior learning and attitudes to the task which resulted in a range of qualitatively different responses which were identified and categorised.

The first research question which considered the areas of fractions in which the student teachers felt confident gave a sense of perspective to the study. It also provided a level of encouragement and a sense of objectivity by implying there were no preconceptions about what the outcomes might be. Unexpected and satisfying aspects resulting from this question were the confidence with which the students gave explanations and participated in some aspects of the study and particularly the pleasure expressed by all groups following the successful completion of a task or question. This tended to be directly proportional to the level of difficulty encountered and it was heartening to see students who struggled with a particular aspect exhibiting such pleasure at their own (and partner's) success.

6.3.2 A consideration of the Limitations of the Research Methods

A possible limitation was that one of the underlying premises of the study is the expectation and assumption that primary school teachers (and student teachers) will have a secure knowledge across the curriculum. The participants, therefore, appreciated that the mathematical content of the activities and diagnostic interviews were an expectation of them as prospective teachers and also of primary school children. With this reservation in mind, particular care was taken to ensure sensitive management of the methods used. The tasks and questions could potentially have resulted in the participants feeling under pressure or embarrassed which would have undermined the students' confidence and resulted in a less productive outcome. The creation of a climate which provided a learning opportunity and a chance to reflect and explore prior knowledge in a safe and comfortable environment was essential to the success of the project. It was also intended to benefit the volunteers who gave their valuable time to take part.

The creation of a comfortable non-judgmental working atmosphere where students worked in self selected friendship groups was effective and this was evidenced by the students' willingness to discuss the areas in which they were less confident. It was also apparent in the way that the relaxed and frank discussions were conducted. As the study progressed and the students became more familiar with the nature of the tasks, a greater level of unofficial talk was evident (Houssart & Mason, 2009:59). This was valuable in providing a further insight into the students' attitudes and small details within their explanations. The unusual angle used for the video recording, which meant that students did not appear on the tape, was also conducive to openness and enabled the discussions to be put into context. They were aware they would appear as "disembodied hands" and voices and felt this was fairly anonymous compared to a normal video angle where their faces would be seen. The focus on the cards in the activities and on their gestures could be followed on the recording; this was valuable in putting the discussions in context. This manner of conducting observed tasks and diagnostic interviews would be generalisable to other interpretative research situations as it was highly effective in promoting open and reflective discussion.

A possible limitation was the way the sample was obtained. The use of an opportunity (Cohen et al, 2003) or snowball/chain sample (Patton 2002: 182) could be very problematic in the nature of the overall group or the balance of the working groups which may be created. It was appreciated that this study is the result of working with a small specific sample of student teachers from the two Initial Teacher Education routes and that different data may have been gained with a different sample. The diligent approach of several participants who encouraged their friends to join in was effective in creating comfortable and effective working groups. The dual role of university tutor and researcher was carefully considered and I attempted to anticipate what possible problems this might create for the participants. The clarity and openness in which the research was approached was intended to address concerns as well as contributing to the effectiveness and the trustworthiness of the study. The data collection was timed for late in the university year when all assessment points had passed so that all students would feel they could confidently share all their perceptions and views. It was essential to have an established professional relationship with the participants where they felt at ease to participate and speak freely in confidence. The anticipated value of developing their own subject knowledge was also a key factor in the students' willingness to continue and to participate fully in the study. The opportunity to reflect and discuss based on a shared activity was also considered valuable by the participants.

6.3.3 The Task Design

The success of this study relied on the careful choice of tasks which were pitched at an appropriate level of challenge whilst remaining accessible. The two observed tasks were both effective in generating rich data. The interactive nature and relative familiarity of the sequencing activity contributed to this. Many similar studies make comparisons between pairs of fractions; this study extends this to allow the students to explore the relationships between a range of fractions. This generated a range of data which gave the broader picture whilst offering a greater insight into individual levels of understanding. Although the tasks had one correct answer, in terms of the order in which the cards should be sequenced, each group took a very individual route towards its completion. The nature of the tasks offered many possible starting points and allowed all participants a way to contribute to the construction of the sequence and the finding of equivalent fractions. This flexible way of approaching each task was intended to ensure that the participants appreciated that there were no pre-conceived or anticipated difficulties inherent in the task. The aim was to contribute to the trustworthiness and integrity of the study. This approach would be generalisable across other areas of mathematics in order to explore misconceptions and difficulties in more depth.

The introductory activity at the start of each diagnostic interview, where students were asked to identify the questions which they perceived to be the most and the least accessible, proved very valuable in creating an effective structure for the forthcoming interviews. This was an original approach to the structure of a diagnostic interview and was introduced as a result of reflection on the pilot diagnostic interviews. The students who contributed to that part of the study all had differing views on how the questions should be ordered. This revised approach was briefly piloted with some year 1 students who found it reassuring and reported that it enabled them to "get started" in the diagnostic interview. This initial selection and opportunity to briefly review the nature of the forthcoming questions enabled each interview to be matched to each student's levels of confidence and areas of possible difficulty and provided an individual structure. This phenomenographical approach (Marton, 1994) enabled each individual's learning and experience to be explored. The student's indications were used mainly to help structure the interview but were recorded as a further means of exploring their subject knowledge (see (Appendix 7.10). Where a participant's perceptions of the accessibility of the questions were particularly pertinent in the light of their response to the question, this was included in the study. However it was beyond the scope of the study to consider all aspects raised by these indicators of accessibility.

At the pilot stage one anticipated limitation was that the interviews would be difficult to sustain in terms of maintaining the students' interest and attention. The selection of questions at the start of each interview gave a sense of structure which was evident to each student and, surprisingly, two students were keen to try all the questions (Iris and Karen). This preliminary exercise of indicating the levels of accessibility of the questions had a profound effect on the diagnostic interviews and enabled the students to structure their own route through the questions and take greater ownership of their own learning. It also seemed to engender an interest in their own progress within the interview. This is an area which would benefit from greater exploration. It would provide a useful basis for a further study where an exploration of student teachers' perceptions of their own capabilities could be considered with their actual understanding through their responses to specific questions . The study has generated some valuable issues for consideration and wider application both in terms of the mathematical subject knowledge of student teachers and in the choice of methodology. The methods adopted, which provided the opportunity for student teachers to work collaboratively and reflect on their own and each other's learning, are already well established on many Initial Teacher Education courses. In this study the activities were carefully pitched and thoroughly piloted to ensure they offered an appropriate level of challenge whilst being sufficiently supportive and, most importantly, providing the opportunity for discussion. These types of activities provide a useful way of undertaking formative assessment of student teachers whilst suggesting a suitable activity which is transferable to the primary school classroom. They also prompted the students to consider their own priorities for further personal study. The design of the diagnostic interviews has wider applications than the learning of mathematics for researchers considering learning. The adaptation of the clinical interview (Ginsburg, 1981) to be more in line with the phenomenographical approach and the review of the structure was intended to encourage adults to reflect more deeply on their own learning. This type of diagnostic interview was particularly successful when working with a pair as at times one participant took on the role of the interviewer, often to seek clarification, and this gave a much greater insight as they developed a strategy together. The possible implications of this approach are quite far reaching and could have a range of useful applications when working with either children or adult learners. Possible examples of this might be exploring and furthering scientific understanding or specific higher order skill development.

Through working with adults it has become apparent that many of the views and types of understanding established during their primary school years have remained unchallenged and the belief that there is one formal way of answering questions relating to fractions is still commonly held. The students displayed many successful strategies, some of which were informal and intuitive (Kieren 1993). Often they did not appreciate the effectiveness of the methods they had employed or consider how they might use these in other situations. The implications of this for Initial Teacher Training when considering the teaching and learning of fractions is quite wide ranging. It became evident that there were aspects of the learning of fractions which were considered problematic by the majority of the participants (see sections 4.5.3 and 5.3). Although unitising is a strong theme throughout most primary school curricula, the re-unitising proved unfamiliar and difficult, this way of considering fractions offered the possibility to think more flexibly about fractions and explore their understanding further. Similarly the consideration of mixed numbers and improper fractions was acknowledged as problematic with a recurring view that fractions were less than one. Linked to this was an aspect of representation, which was the value of using a number line when considering the relative magnitude of fractions, which again was not a representation the students tended to consider, though they found it a helpful model when suggested.

The inclusion of these aspects as part of Initial Teacher Training in mathematics may be a consideration for some courses to broaden the aspects of fractions considered. The findings of this study have been discussed with primary teachers who are undertaking a Masters in Mathematics Education who felt it had implications for working with their less confident colleagues. They also suggested that the nature of the mathematics curriculum in many primary schools does not address the issues highlighted in the study so it is felt that the study has implications for a reconsideration of the way that fractions is taught, especially in Key Stage 2.

This study has revealed that student teachers' knowledge and understanding of fractions is complex, individual and varied. The establishment of a supportive environment in which participants can articulate and explore their individual beliefs, understandings and misconceptions has contributed to a better understanding of the difficulties which they experience with fractions and related areas. It has highlighted several implications which will impact on the primary mathematics initial teacher education courses in my current institution.

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Appendix 7.1 Seven Types of Teacher Knowledge

The Seven Types of Teacher Knowledge

as delineated by Shulman (1986:8)

- content knowledge - both 'substantive' and 'syntactic'
- general pedagogical knowledge - generic principles of classroom management;
- curriculum knowledge - materials and programmes;
- pedagogical content knowledge - which for a given subject area includes forms of representation of concepts, useful analogies, examples, demonstrations;
- knowledge of learners;
- knowledge of educational contexts, communities and cultures;
- knowledge of educational purposes and values.

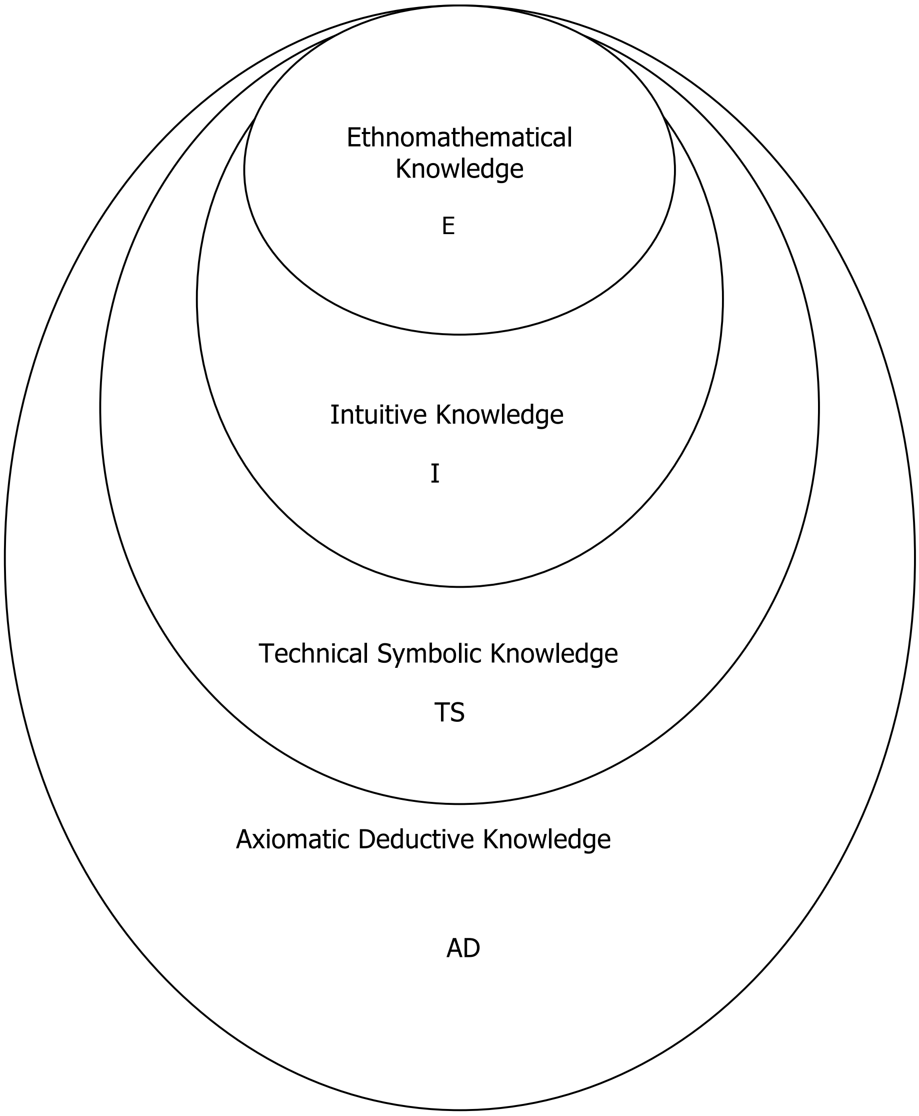
Shulman, L. (1986) *Those Who Understand: Knowledge Growth in Teaching.* Educational Researcher15, 4-14.

Appendix 7.2

Kieren's Model of Mathematical Knowledge Building.

Rational and Fractional Numbers: From Quotient Fields to Recursive Understanding in

Carpenter,T.P., Fennema, E. & Romberg, T.A. (Eds.)(1993), Rational Numbers: An Integration of Research. Hillsdale N.J.: Lawrence Erlbaum



Appendix 7.3

The results of the question "Which aspects of mathematics do you feel most/least confident about teaching on your primary school placements?"

Which aspects of mathematics do you feel most/least confident about teaching on your primary school placements?			
	Very confident	**Quite confident**	**Less confident**
Place Value	**28%**	**59%**	**13%**
Addition/ Subtraction	**60%**	**35%**	**5%**
Multiplication/Division	**37%**	**58%**	**5%**
Mental Maths	**23%**	**59%**	**18%**
Fractions	**7%**	**52%**	**40%**
Decimals/ Percentages	**11%**	**51.5%**	**37.5%**
Algebra	**22%**	**36%**	**42%**
Shape/Space	**33%**	**51%**	**16%**
Data Handling	**33%**	**60%**	**7%**
Weight/ Capacity	**23%**	**60%**	**17%**
Time	**46%**	**48%**	**6%**
Length	**35%**	**50%**	**7%**
Investigations	**19%**	**72%**	**9%**

A questionnaire was administered to the yr1 students at the start of their course, BA in Primary Education with recommendation for QTS.

Eighty five out of ninety one (93%) trainees registered on the course returned the questionnaire.

Appendix 7.4 Student Teacher Questionnaire (First version).

The purpose of this questionnaire is to provide a profile of this cohort of students. It is intended to be completely anonymous. The results will inform my research and hopefully enable us to meet your cohort's needs more effectively.

Please circle as appropriate

Age 18-20 21- 29 30+ **Gender** Female Male

Qualifications in Mathematics

GCSE Grade C GCSE Grade B GCSE Grade A

My GCSE was achieved at my :-

1st attempt 2nd attempt more than 2 attempts

AS level A level Other (Please specify)____________

Degree with some mathematical content (Please specify)____________

Which Key Stage would you prefer to teach? **(**Please circle one)

Foundation Stage Key Stage 1 Key Stage 2

Which subjects do you feel most confident about teaching on your primary school placements?

Which subjects do you feel least confident about teaching on your primary school placements?

What do you consider the most important qualities/skills needed to be an effective teacher of mathematics?

Which aspects of mathematics do you feel most/least confident about teaching on your primary school placements?

Please tick the appropriate boxes in the table below.

	Very confident	Quite confident	Less confident	Don't know
Place Value				
Addition/ Subtraction				
Multiplication				
Division				
Mental Maths				
Fractions				
Percentages				
Decimals				
Algebra				
Shape/Space				
Data Handling				
Weight/ Capacity				
Time				
Length				
Investigations				
Others ? (please specify)				

Did you enjoy mathematics when at primary school? Yes No

Please expand on your answer.

Did you enjoy mathematics when at secondary school? Yes No

Please expand on your answer.

Describe your feelings towards your learning of mathematics.

Thank you for your time and contribution

Appendix 7.5 Student Teacher Questionnaire (Revised)

The purpose of this questionnaire is to provide a profile of trainees on this course. It will be completely anonymous. The results will inform my research and hopefully enable us to meet the trainees' needs more effectively.

Please circle as appropriate

Age 18-20 21- 29 30+

Gender Female Male

Qualifications in Mathematics

GCSE Grade C GCSE Grade B GCSE Grade A

AS level A level Other (Please specify)_____________

Which Key Stage would you prefer to teach? (Please circle one)

Foundation Stage Key Stage 1 Key Stage 2

Which subjects do you feel most /least confident about teaching on your primary school placements?

Please choose up to 3 in each case. Indicate your priorities e.g. 1st, 2nd etc.

	Most confident	**Least confident**
English		
Maths		
Science		
ICT		
History		
Geography		
RE		
PE		
Art		
D&T		
MFL		
Other ?		

What do you consider the most important qualities/skills needed to be an effective teacher of mathematics?

Which aspects of mathematics do you feel most/least confident about teaching on your primary school placements? Please tick the appropriate boxes in the table below.

	Very confident	Quite confident	Less confident	I am not sure
Place Value				
Addition/ Subtraction				
Multiplication				
Division				
Mental Maths				
Fractions				
Percentages				
Decimals				
Algebra				
Shape/Space				
Data Handling				
Weight/ Capacity				
Time				
Length				
Investigations				
Others ? (please specify)				

How do you feel about maths?

Please tick the appropriate column below.

1. I strongly agree 2. I agree 3. I disagree 4. I strongly disagree 5. I am not sure

	1	2	3	4	5
I enjoyed maths when I was at primary school.					
I enjoyed maths when I was at secondary school.					
I look forward to teaching maths on my school placement.					
I feel confident in my understanding of maths.					
I have to work hard to make sure I understand new concepts.					
I often feel frustrated when doing maths.					
I welcome new challenges in maths.					
Maths often makes me feel anxious.					
I usually grasp new concepts quickly.					
I enjoy working on new problems.					
My level of confidence in maths has influenced my choice of key stage for teaching.					
I use my memory to help me learn maths.					
I feel comfortable using maths					
Maths usually makes sense to me.					

Thank you for your time and contribution.

Appendix 7.6

The Diagnostic Interview Questions

Question 1 **20% of £65 =**

Question 2 **Would you rather have :-**

$\frac{5}{6}$ of £30

$\frac{1}{2}$ of £48

Or $\frac{1}{4}$ of £104

Question 3 **X X X X X X = 3/2 of the unit. How many is there in a unit?**

Question 4 **If 3 pizzas are shared between 7 boys and 1 pizza is shared by 3 girls. Who would get the most pizza?**

Question 5 **Which is the best estimate for 12/13 + 7/8 = (multiple choice)**

a) 1 b) 2 c) 19 d) 21

Question 6
Which fractions come between 2/5 and 3/5 ?

Question 7
1 /4 + 2/3 =

Question 8 **Would you rather have :-**

$\frac{6}{10}$ of £520

$\frac{2}{3}$ of £600

Or $\frac{5}{7}$ of £350

Question 9 At the ferry port, ¼ of the passengers are travelling to France, 1/3 are going to Germany , what fraction are travelling to Holland ?

Question 10 1 /2 + 1/3 + 1 /4 - 1 =

Question 11 These circles represent 3/7 of a unit. How many is the whole unit ?

● ● ● ● ● ● ● ● ●

Question 12

These circles represent ¾ of a unit. How many is 2/3 of the unit ?

● ● ● ● ● ● ● ● ●

Question 13

In January, fares went up by 20%. In August, they went down by 20%. Sue claims that: "The fares are now back to what they were before the January increase." Do you agree?

Question 14 2 x a/b = (multiple choice)

a) 2a/2b b) 2a/b c) a/2b d) 4a/2b

Question 15

5 and 7/ 6 minus 3 and 5/8 =

Question 16 0.2 x 0.1 =

Question 17 0.3 divided by 0.3 =

Question 18 How many pieces of ribbon, each 0.08 m long, can be cut from a length of 4 m long ?

Question 19 9/16 + 5/64 =

Appendix 7.7 **Observed Task 2 cards (in correct order).**

$\frac{1}{14}$	$\frac{1}{10}$	$\frac{2}{18}$	$\frac{1}{8}$	$\frac{2}{12}$	$\frac{1}{5}$	$\frac{2}{9}$	$\frac{1}{4}$	$\frac{27}{100}$	$\frac{1}{3}$	$\frac{40}{100}$	$\frac{5}{12}$	$\frac{6}{12}$
							$\frac{8}{32}$					$\frac{70}{140}$

$\frac{2}{5}$	$\frac{55}{100}$	$\frac{45}{80}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{8}{10}$	$\frac{7}{8}$	$\frac{99}{100}$	$\frac{6}{5}$	$\frac{16}{9}$	$\frac{9}{4}$	$\frac{7}{2}$
			$\frac{6}{10}$		$\frac{75}{100}$	$\frac{4}{5}$						

Appendix 7.8 **Profile of Participants** **University B**

B.A.(Hons) Primary Education	Age	Highest qualifications in maths at GCSE.	Level of confidence indicated in maths
A Anne	18-20	Grade B	Middle
B Betty	18-20	Grade C	Least
C Carol	18-20	Grade A	Most
D Donna	18-20	Grade C	Least
E Ellen	18-20	Grade B	Most
F Fran	18-20	Grade B	Middle
G Gill	18-20	Grade B	Middle
H Holly	18-20	Grade C	Least
I Iris	18-20	Grade B	Most

PGCE Primary			Level of confidence indicated in maths
J Jane	21-29	Grade B	Least
K Karen	21-29	Grade B	Least
L Lynn	21-29	Grade A	Middle
M Megan	21-29	AS level	Middle

Appendix 7.9
The Source and Justification of the Inclusion of The Questions used in the Diagnostic Interviews.

The Questions used as a Basis of the Diagnostic Interviews.	
Questions	The Source and Justification of the Inclusion of each question.
Percentages	
Question 1 20% of £65=	A mental calculation question from KS2 SATs (2003) and also in QTS skills test practice, which was in the context of 20% discount on a £65 book order. Selected as an opening question as it was perceived as very accessible by all participants in the pilot study.
Question 13 In January, fares went up by 20%. In August, they went down by 20%. Sue claims that: "The fares are now back to what they were before the January increase." Do you agree?	DfES (2005) Malcolm Swan, Improving Learning in Mathematics: Challenges and Strategies. Page 28 The use of true or false was considered more accessible than a closed question.
Comparison of fractions of quantities.	
Question 2 Would you rather have; 5/6 of £30 or 1/2 of £48 or 1/4 of £104 Question 8 (√3:~10:X0) Would you rather have :-	Based on a selection of examples for Key Stage 2 from Nrich.org.uk. In order to identify the largest or smallest, for example. Would you rather:- Be given 60% of 2 pizzas or 25% of 5 pizzas? Be bitten by 15% of 120 mosquitoes or 8% of 250 mosquitoes?

6/10 of £520 or 2/3 of £600 or 5/7 of £350	
Flexibility of thinking about unitising.	
<u>Question 3</u> X X X X X X = 3/2 of the unit. How many is there in a unit? <u>Question 11</u> These circles represent 3/7 of a unit. How many is the whole unit ? □ □ □ □ □ □ □ □ □ <u>Question 12</u> These circles represent ¾ of a unit. How many is 2/3 of the unit/ □ □ □ □ □ □ □ □ □	<u>Unitising</u> refers to the process of constructing chunks in terms of which to think about a given commodity. It is a subjective process (Lamon 2005:78) It is a natural process and plays an important role in several processes needed to understand fractions especially in partitioning and in equivalence. The questions are based on examples from Cramer, K. & Lesh, R. (1988). Rational Number Knowledge of Preservice Elementary Education Teachers. The progression of questions building on the slightly simpler version (Question 3) this was based on the views of students undertaking the pilot.
Sharing and comparing fractions	
<u>Question 4</u> If 3 pizzas are shared between 7 boys and 1 pizza is shared by 3 girls. Who would get the most pizza? How much more?	Based on questions from Clarke, D. Roche, A. & Mitchell, A. (2008) 10 Practical Tips for Making Fractions Come Alive and Make Sense. Mathematics Teaching in the Middle School. Vol. 13 No7.
Approximation of magnitude of fractions and benchmarking task using near equivalence to 1.	
<u>Question 5</u> Which is the best estimate for 12/13 + 7/8 = a) 1 b) 2 c) 19 d) 21	Mitchell (2004) Question used with grade 5 children, it was chosen by Mitchell as it had also been used in National assessments and earlier research studies. The use of multiple choice provided a greater level of

(multiple choice)	discussion. Fraction density between 0 and 1 (Smith, 2005)
Addition and Subtraction of Fractions	
Question 7 1 /4 + 2/3 = Question 15 5 and 7/ 6 minus 3 and 5/8 = Question 19 9/16 + 5/64 = Question 9 At the ferry port, ¼ of the passengers are travelling to France, 1/3 are going to Germany, what fraction are travelling to Holland ? Question 10 1 /2 + 1/3 + 1 /4 - 1 =	Based on questions in Lamon, S. (2005) More in depth discussion of the Reasoning Activities in "Teaching Fractions and Ratios for Understanding" 2nd Edition A year 2 audit in University A used online to prompt students to review their own subject knowledge. It was considered problematic by the students who undertook the pilot. University Challenge - Monday 14th January 2008 A quarter final between Trinity College, Cambridge and Worcester College, Oxford. Answers given during the programme were 0 and then 1.
Ordering and magnitude of fractions	
Question 6 **Which fractions come between 2/5 and 3/5 ?**	This question developed the idea of continuity and fraction density. It was intended as a an extension of sequencing activities The open nature of the question was intended to promote discussion.

Multiplication and division of decimals	
Question 16 **0.2 X 0.1=** Question 17 (√ 1:~9:X3) **0.3 divided by 0.3 =**	Both questions were from a year 3 audit in University A used online to prompt students to review their own subject knowledge. They also both appear in the QTS skills tests practice for mental calculation.
An algebraic expression of fractions	
Question 14 **2 x a/b =** **a) 2a/2b b) 2a/b c) a/2b** **d) 4a/2b** (multiple choice)	A year 3 audit in University A used online to prompt students to review their own subject knowledge. It was considered problematic by the students who undertook the pilot. The inclusion of the equivalent fractions and the algebraic nature was included as a possible extension if required.
Using a Measurement context	
Question 18 **How many pieces of ribbon, each 0.08 m long, can be cut from a length of 4 m long?**	Lamon, S. (2004) Teaching Fractions and Ratios for Understanding 2^{nd} Edition

Table 7.10 A Table to show the Students' Question Choices in the Diagnostic Interviews

	Question Choices √= considered accessible **~**=unselected X=considered inaccessible															
	BA in Primary Education									PGCE primary				Total	Total	Total
	A	B	C	D	E	F	G	H	I	J	K	L	M	√	~	X
1	√	√	√	√	√	√	√	√	√	√	√	√	√	13	0	0
13		X			√			X		X				1	9	3
2			√	√		√		X		√	√			5	7	1
8			√	√		√								3	10	0
3	X	X		X	X								√	1	8	4
11				X	X		√							1	10	2
12	X		X	X										0	10	3
4		√			√	√	√			√	√		√	7	6	0
5										X		√		1	11	1
6		X	X		X			X			X	X		0	7	6
7	√							√						2	11	0
9							X					√		1	11	1
10									√				X	1	11	1
15	X					X	X							0	10	3
16					√	X		√	X	X	X	X		2	6	5
17	√		X						X	X				1	9	3
14		X				X	X		√		X	X	X	1	6	6
18		√							X				X	1	10	2
19											X			0	12	1

Appendix 7.11

The levels of confidence in teaching the primary mathematics curriculum identified by the participants of the study.

A questionnaire was completed by all the participants before the first task.

These are the results from question 7.

Which aspects of mathematics do you feel most/least confident about teaching on your primary school placements?

	Very confident	Quite confident	Less confident	I am not sure
Place Value	**ACJ**	**BDGHIL**	**EFKM**	
Addition/ Subtraction	**ACDEFGIJL**	**BHKM**		
Multiplication	**CL**	**ABDEFHJKM**	**GI**	
Division	**C**	**EJL**	**BDFGHIKM**	
Mental Maths	**D**	**ACEFHIKLM**	**GJ**	**B**
Fractions	**L**	**ADKM**	**BCEFGHI**	**J**
Percentages	**L**	**BDJ**	**ACEFGHIKM**	
Decimals	**GL**	**ABDFHK**	**CEIJM**	
Algebra	**L**	**G**	**ACEFHIKM**	**BD**
Shape/Space	**EFIL**	**BCDGHJKM**	**A**	
Data Handling	**EDFIL**	**CGHJKM**	**AB**	
Weight/ Capacity	**DF**	**ABCEGHJKLM**	**I**	
Time	**BDLM**	**AEFGHIJK**	**C**	
Length	**ADEL**	**BCFGHIJKM**		
Investigations	**DL**	**ABEFGHIJ**	**CKM**	

	Very confident		Quite confident		Less confident		I am not sure	
	no	%	no	%	no	%	no	%
Place Value	3	23	6	46	4	31		
Addition/ Subtraction	9	69	4	31				
Multiplication	2	16	9	69	2	16		
Division	1	8	3	23	8	62		
Mental Maths	1	8	9	69	2	16	1	8
Fractions	1	8	4	31	7	54	1	8
Percentages	1	8	3	23	9	69		
Decimals	2	16	6	46	5	38		
Algebra	1	8	1	8	8	62	2	16
Shape/Space	4	31	8	62	1	8		
Data Handling	5	38	6	46	2	16		
Weight/ Capacity	2	16	10	77	1	8		
Time	4	31	8	62	1	8		
Length	4	31	9	69				
Investigations	2	16	8	62	3	23		

Appendix 7.12 Questionnaire Results From the Participants in the Main Study.

Please tick the appropriate column below.

1. I strongly agree 2. I agree 3. I disagree 4. I strongly disagree 5. I am not sure

	1	2	3	4	5
I enjoyed maths when I was at primary school.	G L	C E F H J M	B I K	D	A
I enjoyed maths when I was at secondary school.	E	C F G I K M	F J L	B D H	A
I look forward to teaching maths on my school placement.	AD	C E F H I J L M	B G	------	K
I feel confident in my understanding of maths.	CL	A D F I J M	B G	------	E H K
I have to work hard to make sure I understand new concepts.	B G J K	A C D F H I	E M	L	C
I often feel frustrated when doing maths.	EG	A B J K	D F I	L M	CH
I welcome new challenges in maths.	EL	A F G H M	D I J	-----	B K
Maths often makes me feel anxious.	J	A B G I	C D F K M	E L	H
I usually grasp new concepts quickly.	L	A C I M	B D F J K	-----	H
I enjoy working on new problems.	L	A B D E F H J M	G I	-----	C K
My level of confidence in maths has influenced my choice of key stage for teaching.	B	C H I J	A D F K	E G L M	-----
I use my memory to help me learn maths.	D E L	A C F G H I J K	B	-----	M
I feel comfortable using maths	L	A C D E F H I M	G J	B	K
Maths usually makes sense to me.	CL	A D E F H I M	G J	B	K

Appendix 7.13 Initial Codes Generated From the First Level of Data Analysis.

	Attitudinal Codes		Explanatory Codes
A1	Uncertainty, in terms of correctness of answers	**E1**	When I was at school…
A2	Offering agreement and reassurance (social)	**E2**	Observed during school placement
A3	Seeking reassurance from colleagues	**E3**	Qualified certainty
A4	Nervousness –typified by laughter	**E4**	Unqualified certainty
A5	Tentativeness	**E5**	Vocalisation of thinking
A6	Anxious comments	**E6**	Appreciation of an incorrect answer.
A7	Lack of confidence in own ability	**E7**	Seeking confirmation from working partner and/or researcher.
A8	Joy		
A9	Satisfaction on completion of task		
A10	Views of others success/confidence		
A11	Confusion		

	Emerging Themes (T)
T1	Difficulties with improper fractions and mixed numbers.
T2	Uncertainty about relative size of fractions
T3	Suggestion that there is a "proper" way of answering a question
T4	Place Value difficulties
T5	Whole number bias
T6	Use of Circular Representations
T7	Unsuccessful use of "secondary school methods" which are applied regardless of the question.
T8	Uncertainty about unitising/reunitising
T9	Confidence in use of other representations e.g. decimals and percentages
	Mathematical themes (T) Attitudinal (A) and Explanatory (E) were initially used to help identify areas but were then combined or discarded as the data was reconsidered.

<u>Appendix 7.14</u> <u>Revised Attitudinal Codes</u>

Revised Code	Revised Title	Initial codes included	Representative examples of comments demonstrating the code.
B1	Consideration of half remembered methods regardless of the nature of the question. (linked to D5)	T3 – A presumption that there is a single/correct way of answering a question	Donna: I just hope I can remember what you are **supposed to** do with some of these questions (prior to the diagnostic interviews).
		T7 - Use of "secondary school methods"	Carol: This is what you did at school, you always simplified it… but I'm stuck now (Question 15).
B2	Lack of confidence	A9 - Lack of confidence in own abilities	Betty: That was really hard. Are we really meant to know all that? (task1).
		A1 – Uncertainty	Anne: We think we have finished but is probably really wrong (task 2).
		A10 -Views expressed on other student's performance as more successful than their own.	Megan: Hmm that was very impressive … do people who are good at maths like to make it more complicated? (following question 15).
		A5 -Tentativeness	Karen: What happens if we can't do any of them?" (task 1)
B3	Satisfaction on successful completion of a task	A8- Joy / Pleasure on the successful completion of a task	Jane: Is it right? Whay ahey! (high fives) we are cleverer than we thought! (questions 3,11 & 12 completed as a progressive sequence).
		A9 -Satisfaction	Karen: When I first saw the cards I thought… oh my God !... but now I have done it , I realise I know loads more than I thought I did. (task 2)

Revised Code	Revised Title	Initial codes included	Representative examples of comments demonstrating the code.
B4	Signs of anxiety relating to fractions	A4- Nervous laughter	From all groups following the question "Which aspects did you find most accessible?".
		A3 - Seeking reassurance from other group members.	Ellen: I just thought "Oh my god, I am so glad I had you guys here to help". (task 1)
		A6- Anxious comments	Jane: I know bits of it, but trying to put them all together...is like trying to do a jigsaw. Some bits fit and you don't know what you know. Some bits of my memories of fractions have completely disappeared (task 2).
B5	Acknowledgment of confusion	A11- confusion	Iris: What have I done? I have confused myself now. I am going to have to start again on that one (question 12).

Codes were initially generated as a response to the comments made by different groups of students. Although some had been anticipated based on the findings of the literature review, it was intended to generate categories from the three tasks.

Different aspects of coding were used, Mathematical themes (difficulties) (D) and Successful (S), Attitudinal (A) and initially Explanatory themes were also used to help identify areas but were then discarded as the data was reconsidered.

Traits of behaviour displayed were initially coded, to consider the respondent's reaction. This generated a wide range of codes which gave ways of considering each student's views and strategies. On further reflection it became apparent that some codes had overlapping content, for example, a tentativeness of approach and uncertainty in ways of beginning a question.

Similarly joy and satisfaction on the completion of the problem were coded as one aspect.

Table 7.15 An Example of the use of the Initial Codings to Explore Dialogue about Improper Fractions (see 4.5.2(ii)).

This discussion took place during task one and followed the successful placing of 0.39.

Line	Name	Dialogue	Codes
1.	Karen	39% so that will go here so it reads, 38%, 39 % and these are 40 %.	E5
2.	Jane	Ok so now..39 over 10 hmm… 39 divided by 10 … 3.9 then divided by 100 to get a %,	
3.	Karen	If this is decimal, what is it as a decimal?	A1
4.	Jane	3.9	
5.	Karen	Then decimals to percentages you times by 100, don't you?	
6.	Jane	Hmm Not sure … so 39 %?	A1
7.	Karen	If you have a percentage …	A5
8.	Jane	That's what I was getting at, I think that they were the same….	T4
9.	Karen	Hmm (placed with 39/10 with 39%).	A1
They returned to this card at the checking stage.			
10.	Jane	So this one as a decimal would be 0.39	T10
11.	Karen	So to get a % you times by a 100 which would be 39% so they are both the same.	
12.	Jane	Oh, well we had some up here that were the same… (pointing 4% pile)	
13.	Karen	But I think it is because we converted them to different things….. How could 39 over 100 and 39 over 10 both be the same?	E7

14.	Jane	Oh no. (removed both from the line) So 39 over 100 is 39% so that goes there, matching it before but level with 40 %. This is 3.9 so where…?	T1
15.	Karen	It will come just before that? (pointing to 400%) if you had that it would be 3000…	T1
16.	Jane	390	
17.	Karen	Yes 390, 390%.. then we have got 400 % that's just 10 more..	
18.	Jane	I am really not sure if we have got it right or not…have we ?	A3, E7
In the following discussion Jane reflected on areas she found difficult.			
19.	Jane	I was confusing myself, then I was doubting myself, I know 39/10 is 3.9 it is something I know , but because I was questioning everything else, it just made me question what I thought about that too . Then we were trying to organise it into a way so they were connected and to be systematic. Half way through we had got a mixture of % and fractions along the bottom line and it was confusing.	A11,A1, A6. E5, A5

Appendix 7.16 Betty's Response to Question 11

Line	Name	Dialogue	Codes
1	Betty	Hmm, it is another one of those…I really am not sure here.	T8 A7
2	Betty	Sevenths sound trickier… (starts to draw dots).	T2 A7
3	Betty	That last one was difficult … this one is less than one…	E5 Ref to T1 (qu 3)
4	Betty	so can I just keep drawing ? If three is a seventh? hmm… is that right ?	E5 A3 E7
5	Anne	Nods in agreement (A2)	
6	Betty	continues to draw her dots sectioning them into threes,	
7	Betty	counting under her breath . 10,11………. 21	
8	Betty	So it is 21 ! I will just count again… yes 21, so a whole would be 21.	E3
9	Betty	I did it ! I think that is right… isn't it ?	A9 A3 E7
10	Betty	I don't think I could do it without my diagram though.	A5 A7