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The effect of stellar-mass black holes on the structural evolution of massive star clusters

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ABSTRACT

We present the results of realistic *N*-body modelling of massive star clusters in the Magellanic Clouds, aimed at investigating a dynamical origin for the radius–age trend observed in these systems. We find that stellar-mass black holes, formed in the supernova explosions of the most massive cluster stars, can constitute a dynamically important population. If a significant ensemble is retained (here we assume complete retention), these objects rapidly form a dense core where interactions are common, resulting in the scattering of black holes into the cluster halo, and the ejection of black holes from the cluster. These two processes heat the stellar component, resulting in prolonged core expansion of a magnitude matching the observations. Significant core evolution is also observed in Magellanic Cloud clusters at early times. We find that this does not result from the action of black holes, but can be reproduced by the effects of mass-loss due to rapid stellar evolution in a primordially mass-segregated cluster.

Key words: stellar dynamics – methods: *N*-body simulations – globular clusters: general – Magellanic Clouds.

1 INTRODUCTION

Globular clusters are central to a wide variety of astrophysical research, ranging from star formation, stellar and binary star evolution, and stellar dynamics, through to galaxy formation and evolution, and cosmology. These objects therefore constitute an integral part of our understanding of the Universe, and it is clearly vital that their internal evolutionary processes are well understood. The Galactic globular clusters, while close, are exclusively ancient objects ($\tau \gtrsim 10^{10}$ yr). We can therefore accurately determine the end-points of their evolution, but must infer the complete long-term development which brought them to these observed states. To directly observe cluster evolution, we must switch our attention to the Large/Small Magellanic Clouds (LMC/SMC, respectively), which both possess extensive systems of star clusters with masses comparable to the Galactic globulars, but crucially *of all ages:* $10^6 \lesssim \tau \lesssim 10^{10}$ yr. These systems are of fundamental importance because they are the nearest places we can observe snapshots of all phases of cluster development.

Elson, Freeman & Lauer (1989) discovered a striking relationship between core radius (r_c) and age for LMC clusters – namely that the observed spread in r_c increases dramatically with increasing age. Here, r_c is the observational core radius, defined as the projected radius at which the surface brightness has decreased to half its central value. Recently, Mackey & Gilmore (2003a,b) used *Hubble Space Telescope* (*HST*) WFPC2 imaging of 63 massive Magellanic Cloud clusters to more clearly demonstrate the radius–age trend in the LMC and show, for the first time, that a radius–age trend also exists in the SMC. An additional 44 objects have since been observed with *HST*/ACS (Program #9891) to improve sampling of the radius–age plane. Structural measurements for all 107 clusters may be seen in Fig. 1.

The observed radius–age relationship provides strong evidence that our understanding of globular cluster evolution is incomplete, as standard quasi-equilibrium models do not predict large-scale core expansion spanning a full cluster life-time (see e.g. Meylan & Heggie 1997). Discerning the origin of the radius–age trend is therefore of considerable importance. A number of groups have investigated possible explanations – these include a size-of-sample bias (Hunter et al. 2003), heating due to binary stars or tidal shocks (Wilkinson et al. 2003), and the formation of cores in primordially cusped clusters due to the sinking of massive stellar remnants (Merritt et al. 2004). However, a model which fully accounts for the observed trend has yet to be elucidated.

The radius–age trend is indistinguishable in the LMC and SMC, and the oldest LMC/SMC clusters have r_c distributions consistent with those of globular clusters in our Galaxy and in the Fornax and Sagittarius dSph galaxies (Mackey & Gilmore 2004). Because these galaxies have very different tidal fields and possible external

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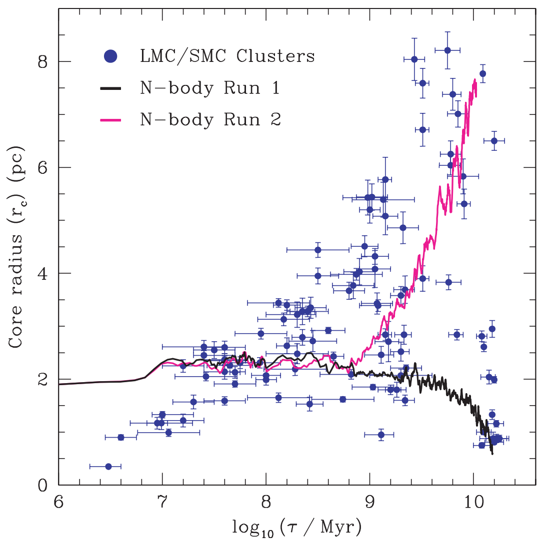

Figure 1. Core radius evolution of Runs 1 and 2, which are initially identical, with no primordial MSeg. They have $f_{\rm BH} = 0$ and 1, respectively. Run 1 evolves exactly as expected, gradually contracting as it moves towards core collapse. In contrast, the BHs in Run 2 induce dramatic core expansion after ≈650 Myr. The plotted LMC/SMC data consists of all clusters from the WFPC2 study of Mackey & Gilmore (2003a,b) as well as preliminary ACS results from Mackey et al. (in preparation) (program #9891).

torques, this suggests that the radius–age relation is driven by internal cluster processes, with any external or tidal effects second order (see also Wilkinson et al. 2003). In this Letter, we report on the results of direct, realistic *N*-body simulations designed to investigate an internal dynamical origin for the radius–age trend. We follow the structural evolution of model clusters with varying degrees of primordial mass segregation (MSeg), possessing populations of stellar-mass black holes (BHs) formed in the supernova explosions of the most massive cluster stars. We demonstrate that a cluster which retains its BHs undergoes dramatic core expansion for most of its lifetime, in contrast to a cluster with no BHs, which proceeds towards core collapse. We also show that primordial MSeg has an important effect on the early evolution of a cluster, when mass-loss due to stellar evolution is severe.

2 NUMERICAL SET-UP

Direct *N*-body modelling is a powerful tool for studying star cluster evolution because it incorporates all relevant physics with a minimum of simplifying assumptions. We have used the NBODY4 code in combination with a 32-chip GRAPE-6 special-purpose computer (Makino et al. 2003) to run simulations of Magellanic Cloud clusters. Full details of NBODY4 are provided by Aarseth (2003). It uses a fourth-order Hermite scheme and evaluation of the force and its first time derivative by the GRAPE-6 to integrate the equations of motion. Close encounters between stars, including stable binary systems, are treated with two-body or chain regularization algorithms. Also incorporated are routines for modelling the stellar evolution of single and binary stars (Hurley, Pols & Tout 2000; Hurley, Tout & Pols 2002). These include a metallicity dependence and a mass-loss prescription such that evolving stars lose gas through winds and supernova explosions.

We generate models with initial properties as close as possible to those observed for young Magellanic Cloud clusters. These objects possess radial surface brightness (SB) profiles best described by Elson, Fall & Freeman (1987, hereafter EFF) models: $\mu(r) = \mu_0(1 + r^2/a^2)^{-\gamma/2}$, where μ_0 is the central SB, a is the scalelength, and γ the power-law fall-off at large r. Typically, their core radii $r_{\rm c} = a(2^{2/\gamma} - 1)^{1/2} \sim 0.2$–2.5 pc and $\gamma \sim 2.0$–3.5 (e.g. Mackey & Gilmore 2003a). Their central densities $\rho_0({\rm M}_\odot\,{\rm pc}^{-3})$ lie in the range $1.5 \lesssim \log \rho_0 \lesssim 2.5$ (except for R136, which is much denser with $\log \rho_0 \sim 4.8$), while their total masses $M_{\rm tot}\,({\rm M}_\odot)$ lie in the range $4.0 \lesssim \log M_{\rm tot} \lesssim 5.6$ (McLaughlin & van der Marel 2005). We generate non-MSeg clusters by selecting stellar positions randomly from the density distribution of an EFF model with $\gamma = 3$. Each star is assigned a velocity drawn from a Maxwellian distribution, where the velocity dispersion σ is calculated using the Jeans equations assuming an isotropic velocity distribution. Expressions for σ are given in a forthcoming paper (Mackey et al., in preparation). We select the initial mass function (IMF) of Kroupa (2001), with a stellar mass range 0.1–100 $M_\odot$. Choosing $N \sim 10^5$ particles results in cluster masses of $\log M_{\rm tot} \sim 4.75$. We adopt [Fe/H] = 0, similar to young LMC clusters. However, both Clouds exhibit strong age–metallicity relationships – this may have important implications for our results.

Resolved observations of very young Magellanic Cloud clusters invariably reveal some degree of MSeg (e.g. de Grijs et al. 2002). Detailed cluster formation models support such observations (e.g. Bonnell & Bate 2006); we would therefore like to include the effects of MSeg in our modelling. We have developed a method to generate clusters with primordial MSeg in a 'self-consistent' fashion; again, full details will be provided by Mackey et al. (in preparation). Briefly, we take a non-MSeg cluster and use NBODY4 to evolve it without stellar evolution, so that the cluster begins to dynamically relax. The degree of primordial MSeg is controlled via the duration of this 'pre-evolution', $T_{\rm MS}$. The positions and velocities of the stars in the pre-evolved cluster are then used as the initial conditions ($\tau = 0$) for a full run with stellar evolution included. Stars slowly escape during pre-evolution, so our MSeg models are marginally less massive than non-MSeg models. We generate MSeg clusters with $T_{\rm MS} = 450$ Myr. These models have structural properties (e.g. density profile, and radial mass-function variation) consistent with those observed for very young Magellanic Cloud clusters.

LMC clusters are observed at galactocentric radii spanning ∼0–14 kpc; our models move on circular orbits of radius 6 kpc about a point-mass LMC with $M_{\rm g} = 9 \times 10^9\,{\rm M}_\odot$. Wilkinson et al. (2003) describe the implementation of an external tidal field within NBODY4. Adopting a point-mass LMC is an over-simplification; however Wilkinson et al. (2003) showed that a weak external field does not result in strong core evolution – hence, here we are only interested in internal processes. Clusters are assumed to initially just fill their tidal radii. The initial tidal radius of a model cluster therefore sets the ratio between the length units used by NBODY4 (see Aarseth 2003) and physical length units (pc). This scaling controls the physical density of the cluster and hence the physical time-scale on which internal dynamical processes occur. Our non-MSeg clusters have central density $\log \rho_0 = 2.31$ and core radius $r_{\rm c} = 1.90$ pc, which matches typical young LMC and SMC objects. The primordially MSeg clusters have $\log \rho_0 = 4.58$ and $r_{\rm c} = 0.25$ pc, which closely resembles the compact, massive LMC cluster R136. Given this correspondence, we are confident in our selection of an appropriate length-scale.

We have modified NBODY4 to control the production of BHs in supernova explosions. We can vary the minimum mass of a BH progenitor star, the masses of the BHs themselves, and the natal velocity kicks they receive. This is implemented in a simple but serviceable manner. All stars initially above 20 $M_\odot$ produce BHs, with masses uniformly distributed in the range $8 \leqslant m_{\rm BH} \leqslant 12\,{\rm M}_\odot$. This range is consistent with dynamical masses obtained from observations of X-ray binaries (e.g. Casares 2006). Each model cluster has the same random seed and so each begins with an identical stellar population: our adopted IMF and total N lead to the formation of 198 BHs in all clusters. In our models, natal BH kicks are either much larger than the cluster escape velocity $v_{\rm esc}$ (i.e. BH retention fraction $f_{\rm BH} = 0$) or zero ($f_{\rm BH} = 1$).

To obtain structural measurements consistent with those for real clusters, we simulate observations of our N-body models. That is, we mimic the reduction procedures from which the *HST* $r_{\rm c}$ measurements were derived. In those observations, the bright (saturation) and faint (background-limited) stellar detection levels are a weak function of cluster age, reflecting the requirement for longer exposure durations to image main-sequence stars in older clusters. This is not responsible for the radius–age trend, but must be accounted for in our analysis. Further, the WFPC2 and ACS fields-of-view limit radial profiles to a maximum extent of ∼70–100 arcsec. To simulate these observations we first convert the luminosity and effective temperature of each N-body star to magnitude and colour using the model atmospheres of Kurucz (1992) and Bergeron, Wesemael & Beauchamp (1995). Next, we impose appropriate bright and faint detection limits along with the field-of-view limits. We use the remaining stars to construct a SB profile, following Mackey & Gilmore (2003a). Stellar positions are projected on to a plane, and the SB calculated in circular annuli about the cluster centre. A varying annulus width is used to evenly sample both the cluster core and halo. Finally, we fit an EFF model to the resulting profile to derive $r_{\rm c}$ and γ. To reduce noise we average the results for three orthogonal planar projections.

3 SIMULATIONS AND RESULTS

The parameter space of interest is spanned by non-MSeg clusters and those with significant primordial MSeg. For each of these, we consider evolution with BH retention fractions $f_{\rm BH} = 0$ (large natal kicks) and 1 (no natal kicks). These four runs define the extremities of the parameter space, and hence are expected to cover the limits of cluster behaviour. Their properties are listed in Table 1. No special significance should be attached to $\tau_{\rm max}$ – these simply represent the most convenient termination points for each simulation after $\tau =$ 10 Gyr had been reached.

3.1 *N*-body pair 1: no mass segregation

The evolution of our non-MSeg runs is illustrated in Fig. 1. Run 1 constitutes the simplest case – no primordial MSeg and no retained BHs. It behaves exactly as expected for a classical globular cluster. There is an early mass-loss phase ($\tau \lesssim 100$ Myr) due to the evolution of the most massive cluster stars. During this phase, BHs are formed in supernova explosions between 3.5–10 Myr; however, all receive large velocity kicks and escape. The severe early mass-loss is not reflected in the evolution of $r_{\rm c}$, presumably because it is evenly distributed throughout the cluster. Subsequently, the core radius slowly contracts as two-body relaxation proceeds and mass segregation sets in. The median relaxation time at $\tau = 10^8$ yr is $t_{\rm rh} \sim 2$ Gyr. At $\tau_{\rm max} = 12$ Gyr $\approx 6\,t_{\rm rh}$ the cluster has not yet entered the core-collapse phase.

Now consider Run 2, which is identical to Run 1 except that $f_{\rm BH} = 1$. Once early stellar evolution is complete, the BHs are more massive than all other cluster members (of mean mass $m_* \approx 0.5\,{\rm M}_\odot$) and are hence subject to mass stratification on a time-scale of $\sim (m_*/m_{\rm BH})\,t_{\rm rh} \approx 100$ Myr. By 200 Myr, the mass density of the BHs within a radius of 0.5 pc is already roughly equal to that of the stars; by 400 Myr it is about three times larger. Soon after, the central BH subsystem becomes unstable to further contraction (Spitzer 1987, Equation 3-55) and decouples from the stellar core in a runaway gravothermal collapse. At 490 Myr, the central density of the BH subsystem is ∼80 times that of the stars. This is sufficient for the creation of stable BH binaries in three-body interactions – the first is formed at ∼510 Myr, and by 800 Myr there are four. Until this phase, the evolution of Run 2 is observationally identical to that of Run 1. Neither BH retention nor the subsequent formation of a central BH subsystem leads to differential evolution of $r_{\rm c}$. This contrasts with the results of Merritt et al. (2004), who found significant early expansion in their models due to the sinking of BHs. We attribute this difference to the much higher degree of central mass concentration in their initially cusped clusters, which thereby respond more strongly and more rapidly to the perturbations induced by sinking remnants. These authors also noted the possibility of further cluster expansion due to subsequent evolution of the BH subsystem. We indeed observe expansion due to such processes (see below).

Once formed, binary BHs undergo superelastic collisions with other BHs in the core. The binaries become 'harder', and the released binding energy is carried off by the interacting BHs. This leads to BHs being *scattered* outside $r_{\rm c}$, often into the cluster halo, as well as to BHs being *ejected* from the cluster (we retain this terminology henceforth). Eventually a BH binary is sufficiently hard that the recoil velocity imparted to it during a collision is larger than the cluster escape velocity, and the binary is ejected. A BH scattered outside the cluster core gradually sinks back into the centre via dynamical friction, thus transferring its newly-gained energy to the stellar component of the cluster. Most is deposited within $r_{\rm c}$, where the stellar density is greatest. The ejection of BHs also transfers energy to the cluster, as a mass m escaping from a cluster potential well of depth $|\Phi|$ does work $m|\Phi|$ on the cluster. This mechanism is particularly effective in heating the stellar core, as BHs are ejected from the very centre of the cluster, and the energy contributed to each part of the cluster is proportional to the contribution which that part makes to the central potential.

Together, these two processes (scattering and ejection) result in significant core expansion, starting between $\tau \approx 600$–700 Myr. Expansion continues for the remainder of the simulation, which

Table 1. Details of N-body runs and initial conditions. Each cluster begins with N_0 stars with masses summing to $M_{\rm tot}$, and initial central density ρ_0. Initial cluster structure is 'observed' to obtain $r_{\rm c}$ and γ. Each model is evolved until $\tau_{\rm max}$.

Name	N_0	$\log M_{\rm tot}$ (M$_\odot$)	$\log \rho_0$ (M$_\odot$ pc^{-3})	$r_{\rm c}$ (pc)	γ	Initial MSeg $T_{\rm MS}$	BH Retention $f_{\rm BH}$	$\tau_{\rm max}$ (Myr)
Run 1	100 881	4.746	2.31	1.90 ± 0.09	2.96 ± 0.17	None	0.0	12 000
Run 2	100 881	4.746	2.31	1.90 ± 0.09	2.96 ± 0.17	None	1.0	10 668
Run 3	95 315	4.728	4.58	0.25 ± 0.04	2.33 ± 0.10	450 Myr	0.0	11 274
Run 4	95 315	4.728	4.58	0.25 ± 0.04	2.33 ± 0.10	450 Myr	1.0	10 000

terminates at $\tau_{max} \approx 10.6$ Gyr. The size of r_c is roughly proportional to $\log \tau$, consistent with the upper envelope of the observed cluster distribution. However, in this model the expansion begins too late for the evolution to trace the upper envelope exactly; rather, it runs parallel.

The number of stable BH binaries in the system peaks at 5, at $\tau \approx 890$ Myr. After this point, there are 0–5 BH binaries at any given time. Single and binary BHs are continually ejected; however, empirically, both escape rates decrease with time such that $d^2N_e/d\tau^2 \propto -1/\tau$. This arises due to the decreasing density of the central BH subsystem – the number of BHs is falling because of ejections; these ejections also heat the BH core. The BH–BH encounter rate therefore decreases with time. Hence, the BH binary hardening rate decreases, as do the BH ejection rates. Furthermore, the stellar core is also less efficiently heated with time – this is reflected in the roughly logarithmic dependence of r_c on τ. By $\tau_{max} \approx 10.6$ Gyr, 96 single BHs, 15 binary BHs and one triple BH have escaped; 65 single BHs and two binary BHs remain in the cluster. This is at odds with early studies (e.g. Kulkarni, Hut & McMillan 1993; Sigurdsson & Hernquist 1993) which predicted depletion of BH populations on time-scales much less than cluster lifetimes. The decreasing BH encounter rate seen in our models prolongs the life of the BH subsystem for much longer than previously appreciated.

The mean mass of stellar escapers is identical in both Runs 1 and 2, at 0.33 $M_\odot$. This is less than m_* at all times. The distributions of velocities with which stars escape are also indistinguishable. These results imply that both models lose stars solely due to relaxation processes. There is only a tiny group of ~20 high velocity escapers in Run 2, indicating that stars interact closely with BH binaries only very rarely. Heating of the stellar component via close interactions is negligible – the hardening of BH binaries is driven solely through interactions with other BHs. At $\tau = 10$ Gyr, the masses of Runs 1 and 2, respectively, are $0.36\,M_{tot}$ and $0.29\,M_{tot}$, reflecting the fact that Run 2 is more loosely bound than Run 1 for the majority of its evolution.

3.2 *N*-body pair 2: strong mass segregation

Runs 3 and 4 are primordially MSeg versions of Runs 1 and 2, respectively. Early mass-loss due to stellar evolution is highly centrally concentrated – hence the amount of heating per unit mass lost is maximized, leading to dramatic early core expansion (Fig. 2). Run 3 traces the observed upper envelope of clusters until several hundred Myr. Run 4 retains its BHs and hence loses less mass than Run 3 – this is reflected in its smaller r_c. After the early mass-loss phase is complete, core expansion stalls in both runs. Two-body relaxation gradually takes over in Run 3, leading to a slow contraction in r_c. At $\tau = 1$ Gyr, $t_{rh} \approx 4$ Gyr; hence this cluster is not near core collapse by $\tau_{max} \approx 11.2$ Gyr. At $\tau = 10$ Gyr, the remaining mass in Run 3 is $0.30\,M_{tot}$.

In Run 4, the BH population evolves similarly to that in Run 2. One might naively expect the earlier development of a compact BH subsystem in Run 4, because the BHs are already located in the core due to the primordial MSeg. However, the centrally concentrated mass-loss hampers the accumulation of a dense BH core, and the first binary BH does not form until 570 Myr, a similar time to the non-MSeg model. The BH subsystem evolves more slowly than that in Run 2 – by $\tau_{max} = 10$ Gyr, there are still 95 single BHs and two binary BHs remaining in the cluster. As in Run 2, the evolution of the BH subsystem leads to expansion of r_c. This begins at $\tau \approx 800$ Myr and continues until τ_{max}. As previously, r_c behaves roughly as $\log \tau$ during this phase. By τ_{max}, Run 4 has $r_c \sim 11$ pc, comparable to that observed for the most extended old Magellanic Cloud clusters (e.g. Reticulum). However, it is only weakly bound, retaining $\sim 0.13\,M_{tot}$. This mass loss is not the driver for the core expansion, so a more massive cluster could have comparable expansion while retaining more of its total mass. Indeed, extended old LMC clusters typically have masses $\approx 10^5$ $M_\odot$ (Mackey & Gilmore 2003a), which may easily be $\sim 0.1\,M_{tot}$, for these clusters.

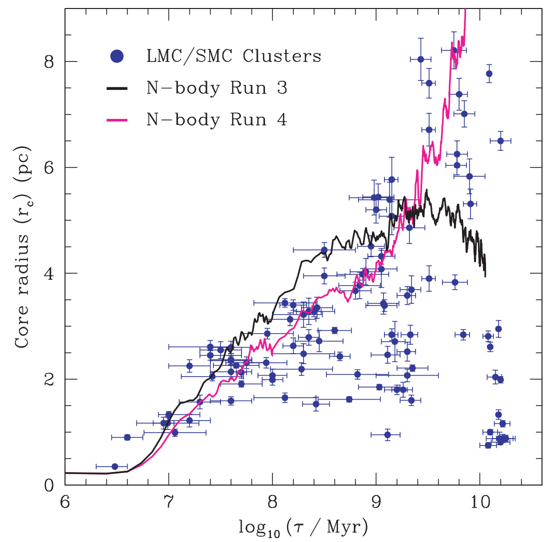

Figure 2. Core radius evolution of Runs 3 and 4, which are initially identical, with significant primordial MSeg. They have $f_{BH} = 0$ and 1, respectively. Both expand dramatically at early times due to mass-loss from stellar evolution. Subsequently, Run 3 begins to contract as two-body relaxation proceeds. In contrast, Run 4 continues expanding due to its BH population.

4 DISCUSSION

Our four simulations cover the observed cluster distribution in radius–age space, thereby defining a dynamical origin for the radius–age trend. At ages less than a few hundred Myr, cluster cores expand due to centrally concentrated mass-loss from stellar evolution. At later times, expansion is induced via heating due to a BH population. Although early mass-loss may result in significant core expansion, a cluster cannot reach the upper right-hand corner of the radius–age space by this means alone – the mass-loss phase is too short, and the maximum allowed expansion rate during this phase is defined by the observed upper envelope of clusters. Only with prolonged expansion due to BHs can ~10 Gyr old clusters with $r_c > 6$ pc be explained in this model. Although we have assumed $f_{BH} = 1$, full retention is not necessary for cluster expansion. BH kicks of order $10 \lesssim v_{kick} \lesssim 20$ km s^{-1} would result in $f_{BH} \sim 0.5$ in our models; we expect r_c evolution in such systems to be intermediate between that of Runs 1 and 2, or Runs 3 and 4. We will address this issue further in an upcoming paper (Mackey et al. in preparation).

Galactic globular clusters, with $N \sim 10^6$, are an order of magnitude more massive than our present models. However, we expect the evolution described above to scale to such objects – reflected in the fact they possess an r_c distribution consistent with that observed for the oldest Magellanic Cloud clusters (Mackey & Gilmore 2004). This is because the mass fraction of BHs formed in a cluster is dependent only on the IMF and minimum progenitor mass, neither of which should change with M_{tot}, while a larger M_{tot} implies a larger f_{BH} because it is easier to retain newly-formed BHs. The densities in our models are consistent with those observed for globular clusters; hence we expect the same processes to operate on similar time-scales, although BHs are likely to be more difficult to eject in more massive clusters – increasing the potential of each BH to heat the cluster via additional scattering-sinking cycles. Core expansion due to mass-loss or BH heating has strong implications for the observed properties of Galactic globular clusters (e.g. the fraction which are core-collapsed) as well as their survivability. Extended clusters are significantly more susceptible to tidal disruption, so it is important to account for expansion effects in studies of the evolution of the globular cluster mass function, for example. Core expansion due to BHs may also offer a viable explanation for the origin of the luminous, unusually extended globular clusters found in M31, which are >10 Gyr old metal-poor objects (Mackey et al. 2006).

Our model requires variations in BH population size between otherwise similar clusters. There are a variety of possibilities in this regard. First, the number of BH-forming stars in a cluster is small, so there will be sampling-noise variations between clusters. Further, any dispersion in stellar rotation may introduce mass-loss variations and further dispersion in BH numbers. Natal BH kicks are poorly constrained at present – typical estimates lie in the range $0 \lesssim v_{\rm kick} \lesssim 200$ km s^{-1}, with kicks of a few tens of km s^{-1} possibly favoured (e.g. Willems et al. 2005, and references therein). Stellar binarity may therefore play a significant role in retaining cluster BHs, as will the initial cluster mass and degree of primordial MSeg, especially if $v_{\rm kick} \approx v_{\rm esc}$. Metallicity may also be a key factor, as theory suggests that BH production is more frequent, and $m_{\rm BH}$ is greater for metal poor stars than for metal-rich stars (e.g. Zhang, Woosley & Heger 2007). In this respect, the age–metallicity relationships of the Magellanic Clouds (where [Fe/H] decreases for clusters of increasing age) may play a central role in shaping the radius–age trend. Similarly, the spread in [Fe/H] for Galactic globulars may have been important in determining the structural properties of these objects. Our results imply that clusters possessing significant BH populations are, for most of their lives, low-density objects in which the time-scale for close encounters between stars and BHs is very long. It is therefore unsurprising that no BH X-ray binaries are seen in the ∼150 Galactic globulars (Verbunt & Lewin 2006).

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