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A self-gravitating accretion disk in Sgr A* a few million years ago: Is Sgr A* a failed quasar?

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Abstract. Sgr A* is extra-ordinarily dim in all wavelengths requiring a very low accretion rate at the present time. However, at a radial distance of a fraction of a parsec from Sgr A*, two rings populated by young massive stars suggest a recent burst of star formation in a rather hostile environment. Here we explore two ways of creating such young stellar rings with a gaseous accretion disk: by self-gravity in a massive disk, and by capturing "old" low mass stars and growing them via gas accretion in a disk. The minimum disk mass is above 10^4 $M_\odot$ for the first mechanism and is few tens of times larger for the second one. The observed relatively small velocity dispersion of the stars rules out disks more massive than around 10^5 $M_\odot$: heavier stellar or gas disks would warp each other too strongly by orbital precession in an axisymmetric potential. The capture of "old" stars by a disk is thus unlikely as the origin of the young stellar disks. The absence of a massive nuclear gas disk in Sgr A* now implies that the disk was either accreted by the SMBH, which would then imply almost a quasar-like luminosity for Sgr A*, or was consumed in the star formation episode. The latter possibility appears to be more likely on theoretical grounds. We also consider whether accretion disk plane changes, expected to occur due to fluctuations in the angular momentum of gas infalling into the central parsec of a galaxy, would dislodge the embedded stars from the disk midplane. We find that the stars leave the disk midplane only if the disk orientation changes on time scales much shorter than the disk viscous time.

Key words. accretion, accretion disks – black hole physics – Galaxy: center – stars: formation

1. Introduction

The complex chain of events leading to the growth of super-massive black holes (SMBHs) in galactic centers is not yet fully understood (e.g., Rees 2002). Nevertheless, gas accretion is probably the dominant physical process delivering the matter to the giant black holes (e.g., Yu & Tremaine 2002). At small radial distances from the SMBH, the standard thin accretion disk (Shakura & Sunyaev 1973) appears to be an appropriate way to provide the SMBH with gas at rates approaching the Eddington limit. However, at distances larger than $\sim 10^{-2}$ parsec from the SMBH, standard thin disks face several problems.

First, the time necessary for the gas to inflow into the black hole – the disk viscous time scale – becomes too long (e.g. as large as 10^8–10^{10} years for larger radii). In parallel with this, the (standard accretion) disk mass becomes very large. When the latter exceeds about 1% of the SMBH mass, local gravitational instabilities develop (e.g., Paczyński 1978; Kolykhalov & Sunyaev 1980; Shlosman & Begelman 1989; Goodman 2003; Collin & Zahn 1999). The structure of such disks is unclear (see Sect. 9) due to uncertainties in theory and a dearth of relevant observations.

Sgr A* is the closest SMBH (with a mass $M_{\rm BH} \approx 3 \times 10^6$ $M_\odot$; e.g., Schödel et al. 2002). Although Sgr A* has remained extremely dim during the entire history of X-ray observations, there are hints that it was much more active in the past.

The X-ray and γ-ray spectrum of the giant molecular cloud Sgr B2 is most naturally explained as a time-delayed reflection of a source with a flat AGN-like spectrum (e.g., Sunyaev et al. 1993; Koyama et al. 1996; Revnivtsev et al. 2004). The required luminosity is in the range of $\sim$few $\times 10^{39}$ erg s^{-1}, too high by Galactic standards. Sgr A* is then strongly suspected of having been brighter in X-rays by some 6 orders of magnitude 300–400 years ago.

Deeper in the past, a few million years ago, few dozen massive stars were formed and are currently at a distance of the order of 0.1–0.3 pc from Sgr A* (Krabbe et al. 1995; Genzel et al. 2003; Ghez et al. 2003). This is surprising given that the tidal force of the SMBH would easily shear even gas clouds with densities orders of magnitude higher than the highest density cores of GMCs observed in the Galaxy.

In situ star formation scenarios for the Sgr A* young massive stars have been numerically studied by Sanders (1998) with a sticky-particle code and also qualitatively described by Morris et al. (1999). In particular, one of the simulations done by Sanders (1998) assumed that a cold cloud of gas with radial dimensions of 0.4 pc and with a small angular momentum falls into Sgr A* gravitational potential starting from a distance of 2.4 parsec. The cloud gets tidally sheared into a thin band of gas which then forms a precessing eccentric ring. Frequent shocks are assumed to lead to strong compression of the gas and star formation.

The existence of such low angular momentum clouds seems to be questionable. In addition, the initial conditions of the simulations are rather extreme: to be stable against tidal shear (e.g., Eq. (1) of Sanders 1998), the cloud mass should be $M_{cl} \gtrsim 7 \times 10^4 \ M_\odot$, a very large mass for a cloud of 0.4 pc in size. Recent observations (Liszt 2003) seem to contradict the Sanders (1998) model for the ionized gas streamers. Genzel et al. (2003) discount current star formation in the mini-spiral, which is believed to be an ionized streamer. Genzel et al. (2003) also note more generally that "if massive stars are forming frequently in dense gas streamers when outside the central parsec and then rapidly move through the central region, one would expect ~100 times as many massive stars outside the central region as in the central parsec", which is not the case observationally.

Alternatively to the in situ star formation, Gerhard (2001) proposed that the young massive stars could have been formed outside the central parsec in a massive star cluster. Then, due to dynamical friction with the older population of background stars, the cluster would have been dragged into the central parsec and then dissolved there by the SMBH tidal shear. However, this appears to be only possible (Kim & Morris 2003; McMillan & Portegies Zwart 2003) if the cluster is very massive ($M \gtrsim 10^6 \ M_\odot$), or if it is formed very near the central parsec already. In both cases a very dense core for the star cluster is required and appears to be unrealistic. An intermediate mass black hole in the center of the cluster does allow the star cluster to survive longer against tidal disruption and hence transport the young stars in the central parsec more efficiently (as suggested by Hansen & Milosavljević 2003). However the numerical simulations of Kim et al. (2004) show that the mass of the black hole has to be unusually large (~10%) compared to the cluster mass for this idea to work in practice.

Levin & Beloborodov (2003) and Nayakshin et al. (2004) suggested that the origin of the young stars is a massive self-gravitating accretion disk existing in Sgr A* in the past. Here we intend to investigate this idea quantitatively and to also look into some related theoretical questions.

We first estimate the minimum mass of such accretion disks to be around $10^4 \ M_\odot$, for each of the two stellar rings. In addition, we rule out the possibility that a less massive accretion disk could capture enough of low mass stars from the pre-existing "relaxed" Sgr A* cusp and then grow them by accretion into massive stars (Sect. 3).

We then attempt to understand the spatial distribution of the young stars. In particular, we find that the rate of N-body scattering between the stars (Sect. 4) of the same ring can explain the observed stellar velocity dispersions in the inner stellar ring if the time-averaged total stellar mass in the ring was $10^4 \ M_\odot$ or higher. We also find that stellar orbits in both rings should remain close to the circular Keplerian orbits up to this day (if stars were indeed born in a disk). The outer ring is however observed to be geometrically thicker and with a higher velocity dispersion than the inner one. The velocity dispersion of the outer ring may result from the stellar disk warping in the gravitational potential of the inner ring. Such warping sets the upper limit on the disk mass of about $10^5 \ M_\odot$ (Sect. 5).

We also question in Sect. 6 whether it is possible for the disk to leave the newly born stars behind (due to their high inertia) when the disk plane rotates. Dislodging the newly born stars, or proto-stars, from the disk midplane would have significantly reduced the problems faced by accretion disks at large radii since these stars would then stop devouring the disk and instead heat it and speed up the accretion of gas onto the SMBH via star-disk collisions (Ostriker 1983). However, we find that the disk maintains a firm grip on these stars unless the plane change occurs on a time scale much shorter than the disk viscous time (Sect. 6).

It is found that young massive stars would not migrate much radially (Sect. 7) in the disk, meaning that they are probably located at the radius where they were originally formed. Small scale proto-stellar disks around the embedded stars may be gravitationally unstable as well and may create further generations of stars. Hierarchical growth and merging of such objects may result in the creation of "mini star clusters" with the central object collapsing to an intermediate mass black hole (Sect. 8). This could potentially be relevant to the observations of such objects as IRS13 (Maillard et al. 2004).

Since the combined mass of the stellar material in the observed stellar rings presently is $\lesssim 3 \times 10^3 \ M_\odot$ (Genzel et al. 2003), there is the interesting question of whether most of the gaseous disk mass has been used to activate the presently dormant Sgr A* or whether it was reprocessed through star formation and expelled to larger radii via winds and supernova explosions. We believe the latter outcome is more likely since the accretion of gas onto embedded stars is very efficient. We briefly discuss observations that could distinguish between the quasar and the nuclear starburst possibilities (Sect. 9).

2. The minimum mass of a self-gravitating disk in Sgr A* is $10^4 \ M_\odot$

The standard accretion disk solution (Shakura & Sunyaev 1973) neglects self-gravity of the disk. Clearly, this solution becomes invalid when the disk becomes strongly self-gravitating, but here we only want to estimate the minimum disk mass at which the self-gravity becomes important. For numerical values of the standard disk parameters, we follow Svensson & Zdziarski (1994) with their parameters $\xi = 1$ and $f = 0$ (i.e. no X-ray emitting disk corona is assumed). The dimensionless accretion rate $\dot{m}$ is defined as $\dot{m} = \dot{M}/\dot{M}_{Edd}$, where $\dot{M}_{Edd} = L_{Edd}/\varepsilon c^2$ is the accretion rate corresponding to the Eddington luminosity and $\varepsilon \approx 0.06$ is the radiative efficiency of the standard accretion flow[1].
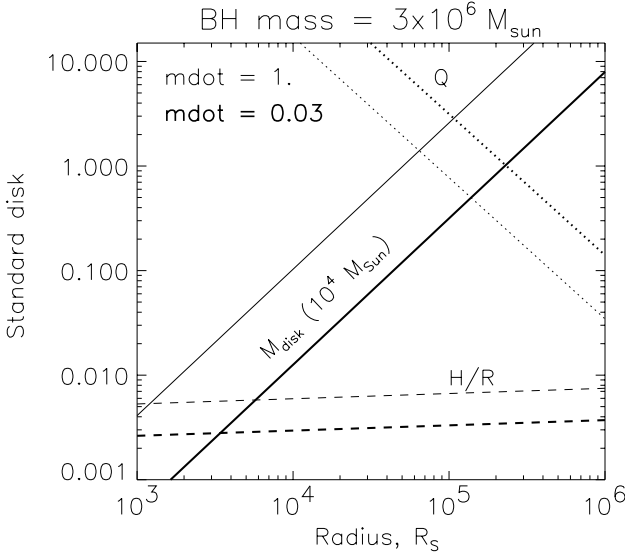
For large radii ($r \gg 1$) the gas dominated equations are appropriate:

$$\frac{H}{R} = 2.7 \times 10^{-3} \ (\alpha M_8)^{-1/10} \ r^{1/20} \dot{m}^{1/5}, \tag{1}$$

$$\Sigma = 4.2 \times 10^6 \ \text{g cm}^{-2} \ \alpha^{-4/5} \ M_8^{1/5} r^{-3/5} \dot{m}^{3/5}. \tag{2}$$

$$T = 6.3 \times 10^2 \ \text{K} \ (\alpha M_8)^{-1/5} \dot{m}^{2/5} \left[\frac{R}{10^5 R_S} \right]^{-9/10}. \tag{3}$$

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[1] Note that our definition of $\dot{m}$ is different from the one Svensson & Zdziarski (1994) use: $\dot{m}_{SZ} = 17.5 \ \dot{m}$.

Fig. 1. The disk mass in units of $10^4 \, M_\odot$, the Toomre instability parameter Q, and the ratio of the disk height scale H to radius R as a function of radius for the standard accretion disk model. The thick curves are plotted for $\dot{m} = 0.03$, whereas thin ones are for $\dot{m} = 1$. In both cases the disk is unstable to self-gravity at $R \gtrsim 10^5 R_S$, where its mass is $M_d \gtrsim 10^4 \, M_\odot$.

Where H is the disk vertical height scale, R is the distance from the SMBH, T is the midplane gas temperature, α is the dimensionless viscosity parameter, $M_8 = M_{BH}/10^8 \, M_\odot$, $r = R/R_S$, $R_S = 2GM_{BH}/c^2$ is the Schwarzschild radius of the SMBH and Σ is the surface density of the accretion disk. These equations assume Thomson electron scattering opacity for simplicity. Figure 1 shows some of the disk parameters obtained for $M_{BH} = 3 \times 10^6 \, M_\odot$, $\alpha = 1$, and two values of the dimensionless accretion rate, $\dot{m} = 0.03$ (thick lines) and $\dot{m} = 1$ (thin lines). The mass of the disk as a function of radius is approximated as $M_d \approx \pi R^2 \Sigma$.

It is well known that the standard accretion disk becomes self-gravitating at large radii, when the Toomre (1964) parameter Q becomes less than unity,

$$Q = \frac{c_s \Omega}{\pi G \Sigma} \approx \frac{H}{R} \frac{M_{BH}}{M_d} < 1 \tag{4}$$

(c_s is the sound speed inside the disk and Ω its angular velocity). The radius where $Q = 1$ yields the minimum mass of the disk needed for the latter to become *locally* self-gravitating[2].

As can be seen from Fig. 1, the disk should weigh at least $10^4 \, M_\odot$ in order to become self-gravitating. Note that this minimum disk mass estimate is quite robust because H/R depends on α, radius and the accretion rate only weakly. This estimate is also conservative. The basic Shakura-Sunyaev model used here does not include irradiation by the central source, which may increase the disk midplane temperature somewhat, leading to

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[2] Disk self-gravity is local when the disk mass is much smaller than the SMBH mass. Effectively, regions of disk separated by large radial distances do not communicate with each other via gravity in this regime. When the disk mass becomes comparable to that of the SMBH, the instability becomes global, and a very rapid angular momentum transfer via self-gravity occurs.

a slightly larger H/R. In addition, trapping radiation by opacity effects would reduce the efficiency of cooling, adding to the stability of the disk against self-gravity.

3. Capturing low mass stars and growing them by accretion: Too slow

Artymowicz et al. (1993) noted that stars in the nuclear star cluster on orbits relatively close to the local circular rotation of the accretion disk in quasars will be captured by the disk. The stars can then rapidly grow by gas accretion, and then enrich the accretion disk with heavy elements through stellar evolution. For the problem of the observed young massive stars in the Galactic Center (GC), the Artymowicz et al. trapping mechanism may be an alternative route to form the stars. The accretion disk does not have to be self-gravitating for the mechanism to work, provided there is enough stars and the time scale for the star trapping is short enough. One may thus hope to reduce the required disk mass.

To within a factor of the order of unity, Eq. (15) of Artymowicz et al. (1993) yields the number of stars captured by the disk within time Δt as

$$dN_*(R) \sim \frac{\zeta^4}{4} N_*(R), \tag{5}$$

where N_* is the total number of stars in the star cluster within radius R, and the variable ζ is defined by

$$\zeta^4 = 32 C_d \frac{M_* M_d}{M_{BH}^2} \frac{\Delta t}{P}, \tag{6}$$

where P is the orbital period and $M_* = m_* \, M_\odot$ is the typical mass of the stars in the cluster. For an estimate, we take $C_d \simeq 3$ and $q_{-2} \equiv 100 M_d/M_{BH} \lesssim 1$. Note also that we want to start with abundant stellar seeds, so we assume $m_* \sim 1$. At the typical radial position of the young massive stars in Sgr A*, $R = 0.1 \, \text{pc} \sim 3 \times 10^5 R_S$, the circular Keplerian rotation period is

$$P \simeq 10^3 \, \text{year} \left[\frac{R}{3 \times 10^5 R_S} \right]^{3/2}. \tag{7}$$

Thus,

$$\zeta^4 \simeq 3 \times 10^{-4} m_* q_{-2} \frac{\Delta t}{10^3 P}. \tag{8}$$

Now, from results of Genzel et al. (2003) we estimate that

$$N_*(R) \simeq 10^5 \, m_*^{-1} \left[\frac{R}{3 \times 10^5 R_S} \right]^{3/2} \tag{9}$$

(we assumed the cusp power-law index $p = 1.5$ for simplicity; see also Eq. (15) in Nayakshin et al. 2004). Therefore, the number of stars captured by the disk is

$$dN_* \sim 7 \, q_{-2} \frac{\Delta t}{10^6 \, \text{years}}. \tag{10}$$

Note that the radius R and the average stellar mass m_* scale out of this relation.

The number of captured stars is somewhat low if we take into account the fact that the ~12 "helium" stars found in each of the rings are only the brightest end of the stellar distribution, and there are probably many more (less massive) stars in the rings (Sect. 3.7 in Genzel et al. 2003). Therefore one would require $q_{-2} \gtrsim 3{-}10$, that is disk mass $M_d \sim (1{-}3) \times 10^5 \, M_\odot$. With this rather high required disk mass, the disk would have to be self-gravitating, and one expects a large number of stars to be born inside the disk. The disk capture mechanism thus fails to reduce the minimum disk mass. Nevertheless, it should not be forgotten completely because of its ability to bring in some *late* type stars into the plane of the disk, e.g. the plane of the young massive stars. This type of star would not be born inside the accretion disk in a time span of just a few million years.

4. Velocity dispersion of an isolated stellar disk

Stars embedded in accretion disks are often considered in a "test star" regime (e.g., Syer et al. 1991), when each star co-rotates with the accretion disk. The star's velocity is then nearly equal to the local Keplerian circular velocity. When number of embedded stars, $N_* \gg 1$, two-body interactions between stars will increase their local velocity dispersion, σ, potentially leading to some interesting consequences.

Since the disk velocity field has the radial Keplerian shear, it is the radial velocity dispersion of stars that will grow the fastest. However, when the anisotropy σ_r/σ_z becomes larger than 3, buckling instability will develop and the stellar velocity dispersion will become more isotropic (Kulsrud et al. 1971; Shlosman & Begelman 1989). Thus we assume an isotropic velocity dispersion here for simplicity. The velocity dispersion of stars grows due to N-body interactions at the rate

$$\frac{d\sigma}{dt} \sim \frac{4\pi G^2 M_* \rho_*}{\sigma^2} \ln \Lambda_* \tag{11}$$

where $\Lambda_* \sim H_* \sigma^2 / G M_*$ is the Coulomb logarithm for stellar collisions; H_* is the stellar disk height scale, which in general may be different from the gas disk height scale H.

The growth of velocity dispersions is opposed by the dynamical friction force acting between the stars and the gas. Consider a star moving inside the disk with a relative velocity v_{rel} with respect to the local Keplerian velocity, v_K. Artymowicz (1994) shows that the angular momentum and energy flow between the disk and the star (a small disk perturber in the case of proto-planetary disks), calculated explicitly, coincides within a factor of a few with the hydrodynamical Bondi-Hoyle drag acting on the star during its passage through the disk. The acceleration experienced by the star is thus

$$\boldsymbol{a_d} = -4\pi G^2 M_* \rho C_d \frac{\boldsymbol{v}_{rel}}{g^4 (c_s^2 + v_{rel}^2)^{3/2}}, \tag{12}$$

where $g = \min(1, v_c/v_{rel})$ and $v_c = C_d^{1/4} v_{esc,*}$ is of the order of the escape velocity from the star, $v_{esc,*}$. In perturbative analytical approaches, such as dynamical friction, $C_d \gtrsim 1$ is the Coulomb logarithm, $\ln \Lambda$, where Λ is the ratio of the disk height scale H to Bondi (or accretion) radius (e.g., Ostriker 1983). However, in many circumstances the Bondi-Hoyle formula for

accretion rate onto the star produces super-Eddington values. The drag force (e.g. C_d) should then be reduced to account for the radiation pressure force. One finds that in the disk geometry the largest contribution to the star-gas friction comes from distances ~H from the star. The exact value of C_d depends on disk opacity and the 3-D velocity of the star, but estimates suggest that C_d is not much smaller than unity in this case.

Note that when the relative velocity is high, the drag force is just the hydrodynamical drag, $a_d \propto \pi R_*^2 \rho v_{rel}^2$, where R_* is the stellar radius. For $c_s < v_{rel} < v_{esc,*}$, the classical Chandrasekhar (1943) dynamical friction formula is recovered, with $a_d \propto v_{rel}^{-2}$. Finally, if the relative velocity is smaller than c_s, we have $a_d \sim \dot{M}_B v_{rel}$, which is about equal to the momentum flux accreted by the star ($\dot{M}_B$ is the Bondi accretion rate).

While v_{rel} is not too large, i.e., $g = 1$, the evolution of the stellar velocity dispersion is approximately given by

$$\frac{d\sigma}{dt} \sim 4\pi G^2 M_* \left[\frac{\rho_* \ln \Lambda_*}{\sigma^2} - \frac{\rho C_d \sigma}{\left(c_s^2 + \sigma^2\right)^{3/2}} \right]. \tag{13}$$

As long as $\rho_* \ln \Lambda_* < \rho C_d$, the star-gas drag will be able to keep the stars on local circular Keplerian orbits in the sense that $\sigma \ll c_s$, the gas sound speed, thus the stars indeed behave as test particles. However, when $\rho_* \ln \Lambda_* > \rho C_d$, the stellar velocity dispersion will evolve mainly under the influence of N-body collisions, and it will increase rapidly.

It may appear that the last fact suggests a natural mechanism to stop the very efficient (see Sect. 9) accretion of gas onto the embedded (proto-) stars. As the stellar velocity dispersion grows much larger than the gas sound speed, the stars will be no longer embedded in accretion disks as they would spend most of their orbits outside the main body of the accretion disk. In addition, even when the stars are crossing the disk, the relatively high value of v_{rel} means that the accretion rate onto stars will be strongly reduced. However, the effect is important only when $\rho_* > \rho$ (assuming $\ln \Lambda_* \sim C_d$), that is when the stellar density is already larger than the gas density. Therefore, before this effect may become important, about a half of the initial gas mass should already be consumed by the stars. The accretion onto the stars is curbed by the N-body dispersion effects too late, when the disk is already partially consumed by the stars.

For Sgr A*, we can estimate the expected $H_* \sim R\sigma/v_K$. The relaxation time, defined as the time needed for the stellar disk to thicken to height H_* can be found from Eq. (11):

$$\frac{t_{rel}}{t_{dyn}} \sim \left[\frac{H_*}{R} \right]^4 \frac{M_{BH}^2}{4 M_{d*} M_* \ln \Lambda_*}, \tag{14}$$

where M_{d*} is $\pi R^2 H_* \rho_*$, the mass of the stellar disk. Equation (14) yields

$$\frac{t_{rel}}{t_{dyn}} \sim 2500 \frac{10^4 \, M_\odot}{M_{d*}} \frac{10 \, M_\odot}{M_*} \left[\frac{H_*/R}{0.1} \right]^4 \ln \Lambda_*^{-1}. \tag{15}$$

With $t_{dyn} \sim 300$ years, we have $t_{rel} \sim 10^6$ years. Hence the geometrical thickness of the rings, and the ratio of velocity dispersion to the local Keplerian velocity, σ/v_K, are expected to be of the order of 0.1 for the two young stellar rings in the GC. The individual stellar velocities should thus still be close to the

local *circular* Keplerian values if the origin of the stars is in the gaseous disk.

Levin & Beloborodov (2003) estimate the geometrical thickness of the inner stellar disk in Sgr A* to be of the order of $H_*/R = 0.1$. This ratio is however larger but is not quantified for the outer disk found by Genzel et al. (2003). From their Fig. 15 we estimate that $H_*/R \sim 0.3$ for the outer, counter-rotating, disk.

One may try to invert Eq. (15) to constrain the initial stellar mass of the disks in the GC by using the observed velocity dispersions (Genzel et al. 2003) in the rings. Unfortunately the limits are not very stringent due to the strong dependence of t_{rel} on H/R. A disk mass as high as $M_d \sim 3 \times 10^5 \, M_\odot$ could still be consistent with the observations for the inner stellar ring. Interestingly, for the outer stellar ring, the velocity dispersion is too high to be explained by the N-body effects unless the ring mass is unrealistically high.

5. Destruction of stellar rings by orbital precession: The maximum disk mass

Genzel et al. (2003) found that most of the young innermost stars lie in one of *two* stellar rings. There is no noticeable difference in the estimated age of the two groups of stars. The rings are bound to interact gravitationally with one another, and this could lead to observable disk distortions.

In particular, stellar orbits precess around the axis of symmetry in an axisymmetric potential (e.g., Sect. 3.2 in Binney & Tremaine 1987). We represented one of the disks by the Kuzmin potential

$$\Phi_K(R, z) = -\frac{GM_d}{\sqrt{R^2 + (a + |z|)^2}}, \qquad (16)$$

where a is the disk radius, R is the radius in the cylindrical coordinates and z is the perpendicular distance from the disk. We then numerically integrated stellar orbits, starting from nearly circular Keplerian orbits unperturbed by the disk presence. The orbits remain approximately circular, and conserve the inclination angle i between the orbital plane and the disk plane (because the z-component of the angular momentum is rigorously conserved in the axisymmetric potential). The stellar orbital plane precesses with respect to the disk at a rate

$$\dot{\phi} = C_p q P^{-1} \cos i, \qquad (17)$$

where C_p is a constant (for a given orbit and given geometry) of order unity. Angle ϕ here is the azimuthal angle of the lines of the nodes for the orbit in cylindrical coordinates used to define the Kuzmin potential. The precession rate scaling (Eq. (17)) is natural since for small q the effect is linear in q as can be seen for orbits nearly co-planar with the disk (when $i \approx 0°$); for $i = 90°$ there should be no plane precession due to symmetry.

The value of C_p depends on the value of a with respect to the radius of the nearly circular stellar orbit; for a of order the radius, $C_p \sim 1$. Setting $i = 74°$ as appropriate for the two GC stellar rings (Genzel et al. 2003), we obtain

$$\Delta\phi \simeq C_p \frac{q}{0.003} \frac{t}{10^3 P}. \qquad (18)$$

The important point to note is that nearly circular orbits of stars at different radii from the SMBH will precess by different amounts $\Delta\phi$. Therefore such a precession leads to a warping of the stellar disk. After a time long enough to yield $\Delta\phi \gtrsim 1$ somewhere in the disk, the initially flat stellar disk will be disfigured and will not be recognizable as a disk at all by an observer.

An approximate upper limit on the *time-averaged* mass of each of the two stellar rings in Sgr A* can be set. Clearly the exact value of this limit should be obtained numerically with N-body experiments and comparison with the quality (χ^2) of the fits to the two observed planes (Levin & Beloborodov 2003; Genzel et al. 2003). Such a study is underway. Due to an observational uncertainty in the radial dimensions of the rings' inner and outer radii, and theoretical uncertainty in the distribution of gaseous mass (i.e. $\Sigma(R)$) in the accretion disk, it is possible to reduce C_p from its maximum value for some values of parameters. Nevertheless, a rather robust value for the upper mass of the disks appears to be

$$\max M_d \approx 10^5 \, M_\odot. \qquad (19)$$

6. Rotating the accretion disk midplane: Do stars remain embedded?

The accretion disk midplane orientation can in principle change as a result of a new mass deposition coming with a different orientation of the angular momentum vector. In such a rotation, would the newly born stars remain embedded in the disk and follow its rotation or would they stay behind in the "old" accretion disk midplane due to their high inertia? The answer to this question is important for AGN disks in general as embedded stars can significantly influence the accretion process (e.g., Goodman & Tan 2004; Nayakshin 2004).

By orders of magnitude, one can estimate the time needed to turn the accretion disk plane around on a significant angle to be

$$t_{rot} \sim \frac{M_d}{\dot{M}_c} \sim t_{visc} \frac{\dot{M}}{\dot{M}_c}, \qquad (20)$$

where $\dot{M}_c$ is the mass condensation rate onto the accretion disk. If the accretion and condensation processes are in an approximate steady state, $\dot{M}_c = \dot{M}$, $t_{rot} \sim t_{visc}$. The latter is

$$t_{visc} \sim \frac{R}{v_K} \alpha^{-1} \left(\frac{R}{H}\right)^2, \qquad (21)$$

and can be fairly long. Thus in general the disk plane orientation changes rather slowly.

6.1. Forces keeping the stars embedded

Two forces mediating interaction between a star and a gaseous disk are the gravity of the disk as a whole, and the friction acting on a star *moving* inside the disk at a certain velocity with respect to the local circular Keplerian speed. If stars lag behind the rotating disk plane, the characteristic relative velocity at which the star and the gas would be separated is $\sim v_K/t_{rot}$ and is very small compared to the sound speed in the gas (if $t_{rot} \sim t_{visc}$). At small relative velocities the dynamical friction

of a star "leaving" the gas disk is very small too (see Eq. (12)), and a simple estimate shows that the dynamical friction force can be safely neglected in what follows.

Therefore the binding force to consider is the direct gravitational attraction between the disk and the star. Near the disk midplane, the infinite plane approximation can be used for the disk gravity. The gravitational attraction force of the gaseous disk for a star that left the disk midplane (i.e., the star-disk midplane separation $|z| \gtrsim H$) is

$$a_{\rm pl} = 2\pi G\Sigma, \qquad (22)$$

where Σ is the local disk surface density. Comparing this acceleration with the centrifugal acceleration of the star moving in a circular Keplerian orbit around the central black hole, $a_{\rm c} = v_{\rm K}^2/R$,

$$\frac{a_{\rm pl}}{a_{\rm c}} = \frac{2M_{\rm d}}{M_{\rm BH}}. \qquad (23)$$

6.2. Critical rotation time

Suppose that the accretion disk midplane turns at a rate given by the time scale $t_{\rm rot}$. Define a critical rotation time scale, $t_{\rm rc}$, such that for disk plane changes occurring on time scales shorter than $t_{\rm rc}$, the stars are dislodged from the gas disk. For $t_{\rm rot} > t_{\rm rc}$, on the contrary, the stars remain bound to the disk. Clearly, we get the critical time scale when $a_{\rm pl} = a_{\rm rot} \equiv v_{\rm K}/t_{\rm rot}$, where $a_{\rm rot}$ is the "rotation acceleration" of the turning disk midplane. We obtain for the critical rotation time

$$t_{\rm rc} = \frac{M_{\rm BH}}{2M_{\rm d}}t_{\rm dyn}. \qquad (24)$$

Figure 2 shows the critical rotation time scales (dotted curves) along with other important time scales for the standard accretion disk model with same parameters as used for Fig. 1 and for a 10 Solar mass star. The thick line curves are for $\dot m = 0.03$, whereas the thin curves are for $\dot m = 1$. The accretion and migration time scales will be discussed in Sect. 7 below.

Note that $t_{\rm rc}$ is longer than $t_{\rm dyn}$ but is much shorter than $t_{\rm visc}$. This implies that if accretion disk plane changes occur on a viscous time scale, the stars would remain bound to the disk. Only very fast plane changes could dislodge the stars from the disk midplane.

6.3. The case of Sgr A*

We have just shown that it is fairly difficult to separate the stars and the accretion disks in slow disk plane rotations or deformations. For the Sgr A* case, this implies that either (i) there were two separate accretion disk creation events that created the two differently oriented rings; or (ii) the accretion disk itself was extremely warped so that its inner part was oriented almost at the right angle with respect to the outer disk part.

7. Accretion onto embedded stars

The Hill's radius $R_{\rm H}$,

$$R_{\rm H} = \left[\frac{M_*}{3M_{\rm BH}}\right]^{1/3} R, \qquad (25)$$
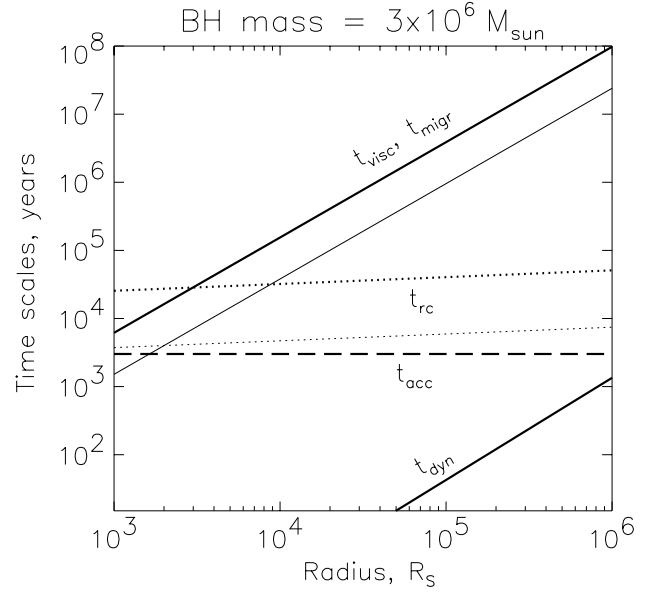
Fig. 2. Time scales for a 10 Solar mass star embedded in the standard accretion disk with parameters as in Fig. 1. As before, thick curves correspond to $\dot m = 0.03$, whereas thin ones are for $\dot m = 1$. The solid lines show the viscous and the dynamical time scales for the disk, as labelled in the figure. The star is massive enough to open up a gap and hence migrates inward on the viscous timescale ($t_{\rm migr} = t_{\rm visc}$). The dashed and dotted lines are the accretion and the critical rotation time scales, respectively. For chosen parameters, the former one is independent of $\dot m$ (see text in Sect. 7 for detail).

defines the sphere around the star where the dynamics of gas is dominated by the star rather than the SMBH. The accretion of gas onto a star is believed to be similar to the growth of terrestrial planets in a planetesimal disk (Lissauer 1987; Bate et al. 2003). For $R_{\rm H} > H$, gas accretion onto the star is quasi two-dimensional. The accretion rate is determined by the rate at which differential rotation brings the matter into the Hill's sphere,

$$\dot M_* = \dot M_{\rm H} \sim 4\pi R_{\rm H} H \rho\, v_{\rm H} \sim 4\pi R_{\rm H}^2 \rho c_{\rm s}, \qquad (26)$$

where $\rho = \Sigma/2H$ is the mean disk density. We used the fact that the characteristic gas velocity (relative to the star) at the Hill's distance from the star, $v_{\rm H}$, is $v_{\rm H} = R_{\rm H}|{\rm d}\Omega/{\rm d}\ln R| \sim c_{\rm s}(R_{\rm H}/H)$ since the angular velocity for Keplerian rotation is $\Omega = c_{\rm s}/H$. Equation 26 is valid as long as $R_{\rm H} > H$ since in the opposite case the gas thermal velocity becomes important and the accretion would proceed at the Bondi accretion rate ($\dot M_{\rm B}$; e.g., Syer et al. 1991). Of course $\dot M_*$ cannot exceed $\dot M_{*,\rm Edd} \simeq 10^{-3}r_* \simeq 10^{-3}\ M_\odot/{\rm year}\ m_*^{1/2}$, the Eddington accretion rate onto the star[3]. We thus estimate

$$\dot M_* = \min\left[\dot M_H, \dot M_{\rm B}, \dot M_{*,\rm Edd}\right]. \qquad (27)$$

One can then define the accretion time scale for a star embedded in a disk:

$$t_{\rm acc} \equiv \frac{M_*}{\dot M_*}. \qquad (28)$$

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[3] We assumed that $r_* = (R_*/R_\odot) \approx (M_*/M_\odot)^{1/2}$. Note that Eq. (10) in Nayakshin (2004) contains a typo.

Figure 2 shows the accretion time scale (dashed line) for a 10 $M_\odot$ embedded star. Although we considered two values for the accretion rate onto the SMBH, $\dot{m} = 0.03$ and $\dot{m} = 1$, as in Fig. 1, t_{acc} turns out to be the same for both of these because the accretion rate is close to the Eddington value.

An important point from Fig. 2 is that accretion onto embedded stars is able to double the stellar mass in a few thousand years. Therefore, growing stars as massive as 100 $M_\odot$ in a million years in a disk with gas mass $M_d \gtrsim 10^4\ M_\odot$ appears possible. (One potential uncertainty here is the reduction in the accretion rate onto the embedded stars once these stars are massive enough to clear out a radial gap in the accretion disk. Results of Bate et al. (2003), Fig. 9, show that this reduction can be very large. However, in the case of an AGN disk with *many* embedded stars, the dynamics of the star-gas interaction is likely going to be different from the case of a "test" star or planet. The accretion disk will then be divided into many rings between stars on nearly circular radial orbits. If the orbits are close enough (number of stars $N_* \gg 1$), then the gas in a ring will experience alternating inward and outward pushes from the two stars closest to it and hence the radial gap can in fact be closed, enabling unhindered accretion. The issue deserves future study.)

We also estimated the radial migration time scale, t_{migr}, using the prescription for the radial migration velocity based on the numerical calculations of Bate et al. (2003). For the parameters chosen, the 10 $M_\odot$ stars, and any stars more massive than that, open up a gap in the accretion disk and their radial migration is identical to the viscous flow of matter in the gas disk. Thus $t_{migr} = t_{visc}$ (two solid curves in Fig. 2). The migration time scale is very long, indicating that stars will remain close to where they were born in accretion disks with parameters close to that of the standard disk for Sgr A*. A more realistic self-gravitating disk would not change this conclusion significantly since the migration time scale only gets longer when the midplane disk density decreases as a result of disk swelling due to gravitationally induced turbulence.

8. Growth of "mini star clusters" and intermediate mass black holes in accretion disks

Goodman & Tan (2004) have recently suggested that it is possible to grow supermassive stars in AGN accretion disks. The maximum mass of a star in this case is the gas disk mass in a ring with width of the order of the Hill radius of the star, $R_H = R(M_*/3M_{BH})^{1/3}$. This is the "isolation" mass, $M_i \simeq M_d^{3/2} M_{BH}^{-1/2}$,

$$M_i \approx 550\ M_\odot \left[\frac{M_d}{10^4\ M_\odot} \right]^{3/2} \left[\frac{3 \times 10^6\ M_\odot}{M_{BH}} \right]^{1/2}. \qquad (29)$$

The stability of super-massive stars is briefly summarized in Sect. 2 of (Goodman & Tan 2004). The supermassive star could collapse directly into a black hole if the star is more massive than 300 $M_\odot$ (Fryer et al. 2001).

However, the Hill accretion rate estimate assumes that all of the disk mass delivered by the differential rotation into the Hill radius around the star is accreted onto the star. Even without the gap, this is not obvious because the gas still has to lose most of its angular momentum before it can reach the stellar surface (e.g., Milosavljević & Loeb 2004). Furthermore, quite frequently the accretion rate onto the star estimated in this way exceeds the Eddington accretion rate onto the star (as is the case for Fig. 2). Milosavljević & Loeb (2004) have shown that the fringes of the small-scale disk around the embedded stars themselves become self-gravitating and may therefore also form stars or planets. It is thus possible to grow in situ star clusters. The maximum total mass of such a cluster should be close to the isolation mass.

A qualitative confirmation of these ideas can be found in numerical simulations of a related physical problem by Tanga et al. (2004). These authors simulate the growth and clustering of planetesimals in a proto-stellar disk. They find a hierarchical growth of clusters of particles and find that these "clusters" are intrinsically stable structures. This is likely because of the abundant supply of gas into the Hill sphere: there is always plenty of gas to interact with the particles (gravitationally bound objects in the AGN case) that continue to get more and more bound to the central object in the mini-cluster.

This mechanism of growth of intermediate mass black hole (IMBH) and star cluster bound to it may be relevant to the observations of the IRS13 cluster near Sgr A* (Maillard et al. 2004). The "dark" mass in the IRS13 is estimated to be $\gtrsim 10^3\ M_\odot$. Equation (29) shows that an initial mass of the disk is of the order of several times $10^4\ M_\odot$ would have been sufficient to "in situ" grow an object massive enough to become the IRS13 cluster.

9. Discussion

We have considered here the formation of massive stars in a self-gravitating accretion disk for conditions appropriate for the central ~0.2 pc of our Galaxy. Formation of an accretion disk (instead of a narrow ring) would be a likely outcome of a cooling instability for a hot gas since the gas would realistically have a broad range of the angular momentum values. Additionally, a cloud with an initial size of a parsec or larger, tidally disrupted and shocked, should settle in a disk of a size comparable to its initial radius. There is of course a direct observational test which would distinguish between the accretion disk versus the compact infalling cloud idea of Sanders (1998) – one simply has to establish whether the orbits of the young stars in the two stellar rings are nearly circular or they are strongly eccentric. As we showed in Sect. 4, stars born in an accretion disk in Sgr A* would still retain their nearly circular orbits.

The standard theory of gravitational instability for a thin disk (Toomre 1964; Paczyński 1978) predicts that the minimum mass of the gas in the disk that would make it gravitationally unstable for the parameters appropriate to our Galactic Center is ~$10^4\ M_\odot$ (Sect. 2). It would be interesting to compare the predicted stellar mass resulting from star formation in such an accretion disk with the current stellar content of the rings. Unfortunately theoretical uncertainties for the efficiency of star formation in self-gravitating disks are too large.

Shlosman et al. (1990) have shown that if the cooling time of a self-gravitating disk is shorter than $t_{\rm dyn} = \Omega^{-1}$, then the disk will fragment and form stars and/or planets. For longer cooling times, it was argued that the disk does not fragment (Shlosman et al. 1990). Numerical simulations with a constant cooling time by Gammie (2001) confirmed this, and have shown that the disk settles into a stable state where the cooling is offset by the energy input generated by gravitational instabilities (see also Paczyński 1978). Yet for disk temperatures of the order of 10^3 Kelvin, the opacity is strongly dependent on the temperature. Johnson & Gammie (2003) showed that in the non-linear stage of the instabilities, the local cooling time may be orders of magnitude smaller than that found in the unperturbed disk model. However AGN disks are usually hotter than this and hence the non-linear effect should be weaker.

Nevertheless, we believe that the Sgr A* accretion disk was likely consumed almost entirely in the star formation episode rather than accreted by the SMBH. There is no doubt about star formation here: there are dozens of the brightest and quite massive stars in each of the stellar rings with 3-D velocity measurements. There are additional numbers of dimmer stars that have only 2-D velocities measured but are strongly suspected of belonging to these same rings (Genzel et al. 2003, Sect. 3.7). The accretion time scale of the embedded stars is very short (Sect. 7 and Fig. 2) compared to the disk viscous time scale. We have also shown in this paper that neither disk plane rotations, warps, or the stellar *N*-body scattering (unless there is already more mass in the stars than in the gas, see also Cuadra & Nayakshin 2004) can "shake" the stars off the disk midplane. In addition, each massive star opens up a radial gap in the accretion disk around it. These stars would not let *the standard* accretion disk transfer the gas into the SMBH simply because they are in the way of the gas flow.

While the standard accretion disk equations are not applicable to the region where the disk becomes self-gravitating, the stellar accretion time scale is shorter than the viscous time by 3–4 orders of magnitude. We experimented with a prescription for the accretion disk equations that introduces turbulent energy and pressure in addition to the thermal ones to keep the disk marginally stable (i.e., $Q \gtrsim 1$), and found that the turbulent energy content must be unrealistically high to reduce the accretion rate onto the stars sufficiently.

We have also found (Sect. 4) that the geometrical thickness of the inner 2–4″ stellar ring, and the stellar velocity dispersion, could be explained by *N*-body scatterings between the members of the same ring if the initial stellar mass was as high as $10^4 M_\odot$. However, the outer 4–7″ projected distance ring (Genzel et al. 2003) is too thick to result from the internal scatterings. We believe that a better explanation is stellar ring warping due to a non-spherical gravitational potential (Sect. 6), e.g. due to the presence of the inner ring. Estimating the rate at which the rings become distorted, we tentatively set an upper limit on the *time-averaged* total mass of each of the gas-star disks (rings) at around $10^5 M_\odot$. Future numerical *N*-body modeling and direct comparison to stellar orbits may tighten this limit.

We hope that future observations of the stellar mass content in the two rings in Sgr A*, and also observations of the inner Galaxy ISM budget, could be used to constrain the initial mass of the gaseous accretion disk and its further fate. As we have shown, the gaseous disk mass should have been in the range of $(1-10) \times 10^4 M_\odot$, 10 to 100 times higher than the present day mass in the observed stellar rings (Genzel et al. 2003). If most of the disk gas was used to make stars, then more of these stars and/or their remnants should be found in the future in the inner ~0.2 pc of the Galaxy. In addition, one may look for evidence of a hot, high metallicity bubble in the inner 1 kpc of the Galaxy produced by stellar winds and supernova explosions.

If instead the gas was mostly accreted by Sgr A*, then there should be evidence of a past quasar phase. The required accretion rate, $\sim (10^4-10^5) \, M_\odot/10^6$ year $= 10^{-2}$ to $10^{-1} \, M_\odot$ year^{-1} is comparable with the Eddington accretion rate for Sgr A*, $\dot{M}_{\rm Edd} \sim 0.03 \, M_\odot$ year^{-1}, corresponding to perhaps $L_{\rm X} \lesssim 10^{43}$ erg s^{-1} or even more. This would have to be a very rare event in Sgr A*'s recent life since the recent star formation episode appears to be an isolated event in Sgr A*'s past (Krabbe et al. 1995). In addition, such star formation and accretion events should also be rare in nearby galaxies, since these either have no detectable AGN or have very weak ones with an X-ray luminosity usually less than $L_{\rm X} \lesssim 10^{40}$ erg s^{-1} (e.g., Zang & Meurs 2001; Pellegrini 2005). A past bright AGN phase should also leave a hot buoyant radio bubble in the Milky Way halo, as accretion onto the SMBH is widely believed to go hand in hand with superluminous jet outflows. Future observations will hopefully constrain Sgr A* accretion activity a few million years ago.

10. Conclusions

Our main results are as follows:

1. The minimum mass of each of the disks needed to form the observed young stars by self-gravity is around $10^4 M_\odot$.
2. The observed stellar velocity dispersions in the outer ring are too large to result from *N*-body interactions between stars belonging to the same ring. The orbital precession of stars caused by the potential of the other disk can explain the observed disk thickness and velocity dispersion if the time-averaged stellar and gaseous mass in the inner disk is in the range $(3-10) \times 10^4 M_\odot$.
3. A few million years ago, Sgr A* had a good chance to become a very bright AGN with a bolometric luminosity $L \sim 10^{44}-10^{45}$ erg s^{-1}, but was robbed of most of its gaseous fuel by nuclear star formation in a self-gravitating accretion disk. Nevertheless, even if only a few percent of the available disk fuel was captured by Sgr A*, the SMBH in our Galactic Center was as bright as $L \sim 10^{42}-10^{43}$ erg s^{-1}.

We have also shown that capture of stars from the "old" relaxed isotropic Sgr A* star cluster (the cusp; see Genzel et al. 2003) by the accretion disk is inefficient unless the gaseous disk mass was as high as $10^5 M_\odot$. In addition, the role of possible accretion disk midplane changes was estimated. It was found that the embedded stars inertia would have been efficient in taking the stars out of the body of the disks only if the disk plane change its orientation on time scales much shorter than the disk viscous time.

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