Mon. Not. R. Astron. Soc. **372,** 143–150 (2006) doi:10.1111/j.1365-2966.2006.10772.x

Massive stars in subparsec rings around galactic centres

Sergei Nayakshin★
Department of Physics & Astronomy, University of Leicester, Leicester LE1 7RH

Accepted 2006 July 4. Received 2006 June 23; in original form 2005 December 16

ABSTRACT

We consider the structure of self-gravitating marginally stable accretion discs in galactic centres in which a small fraction of the disc mass has been converted into protostars. We find that protostars accrete gaseous disc matter at prodigious rates. Mainly due to the stellar accretion luminosity, the disc heats up and thickens geometrically, shutting off further disc fragmentation. The existing protostars, however, continue to gain mass by gas accretion. As a result, the initial mass function for disc-born stars at distances $R \sim 0.03$–3 pc from the supermassive black hole should be top-heavy. The effect is most pronounced at around $R \sim 0.1$ pc. We suggest that this result explains observations of rings of young massive stars in our Galaxy and in M31, and we predict that more such rings will be discovered.

Key words: accretion, accretion discs – stars: formation – Galaxy: centre – galaxies: active.

1 INTRODUCTION

Accretion discs around supermassive black holes (SMBHs) have been predicted to be gravitationally unstable at large radii where they become too cool to resist self-gravity and can collapse to form stars or planets (Paczyński 1978; Kolykhalov & Sunyaev 1980; Lin & Pringle 1987; Collin & Zahn 1999; Gammie 2001; Goodman 2003). There is now observational evidence that the two rings of young massive stars of size ~ 0.1 pc in the centre of our Galaxy were formed in situ (Nayakshin & Sunyaev 2005; Paumard et al. 2006), confirming theoretical predictions. In our neighbouring Andromeda galaxy (M31), Bender et al. (2005) recently discovered a population of hot blue stars in a disc or ring of similar size, i.e. with a radius of $\sim$0.15 pc. The significance of this discovery is that the SMBH in M31 is determined (e.g. Bender et al. 2005) to be as massive as $M_{\rm BH} \approx (1$–$2) \times 10^8\,{\rm M}_{\odot}$, or about 40 times more massive than the SMBH in the Milky Way. This fact alone rules out (Quataert, private communication) the other plausible mechanism of forming stellar discs around SMBHs, e.g. the massive cluster migration scenario (e.g. Gerhard 2001), because the shear presented by the M31 black hole is much stronger than it is at the same distance from Sgr A*, and it is hard to see how a realistic star cluster would be able to survive that (Gürkan & Rasio 2005).

In this paper, we shall attempt to understand what happens with the gaseous accretion disc around an SMBH when the disc crosses the boundary of marginal stability to self-gravitation (Toomre 1964) and forms the first stars. We find that, in a range of distances from the SMBH, interestingly centred at $R \sim 0.1$ pc, the creation of the first low-mass protostars should lead to very rapid accretion on these stars. The respective accretion luminosity greatly exceeds the disc radiative cooling, thus heating and puffing the disc up. The new thermal equilibrium reached is that of a disc stable to self-gravity where further disc *fragmentation* is shut off. Star formation is however continued via accretion on to the existing protostars, which then grow to large masses. We therefore predict that stellar discs around SMBHs should generically possess top-heavy IMF, as seems to be observed in Sgr A* (Nayakshin & Sunyaev 2005; Nayakshin et al. 2006). In the discussion section, we note three main differences between the star formation process in a 'normal' galactic environment and that in an accretion disc near an SMBH.

2 PRE-COLLAPSE ACCRETION DISC

In this section, we determine the structure of the marginally stable accretion disc, $Q \approx 1$ (see equation 3 below), i.e. the disc structure just before the first gravitationally bound objects form. We envisage a situation in which the disc of a finite radius has been created by a 'mass deposition event' on a time-scale much shorter than the disc viscous time, but much longer than the local dynamical time, $1/\Omega$ (see below). Such an event could be a collision of two large gas clouds at larger distances from the SMBH, which cancelled most of the angular momentum of the gas, or cooling of a large quantity of hot gas that already had a specific angular momentum much smaller than that of the galaxy (hot gas can be supported by its pressure in addition to rotation). In these conditions, it is reasonable to expect that the disc will settle into a local thermal equilibrium, in which the gas is heated via turbulence generated by self-gravitation (Gammie 2001) and is cooled by radiation.

As we shall see below, for the parameters of interest, the evolution of the disc after star formation is turned on proceeds on a time-scale again shorter than the local viscous time. Therefore, below we assume that the disc is in the hydrostatic and thermal equilibrium, but not in a steady accretion state, when the accretion rate $\dot{M}(R)$ = const. We now estimate the conditions in the disc (as a

★E-mail: Sergei.Nayakshin@astro.le.ac.uk

function of radius R) when it reaches a surface density large enough to suffer local gravitational collapse. Star formation is a local process in this approach, and different rings in the disc could become gravitationally unstable at different times.

The appropriate accretion disc equations for $Q \sim 1$ have been discussed by many authors (see references in the Introduction and, for the protoplanetary disc case, Rafikov 2005). In particular, we follow most closely the logic of Levin (2006). In our picture, as the mass is fed into the disc from outside, from e.g. a cooling gas cloud, the disc starts off with too little mass (or equivalently surface density) for its temperature to be gravitationally unstable, so that Q is somewhat larger than unity. The viscosity parameter $\alpha \ll 1$ is then controlled by the magnetorotational instability (Balbus & Hawley 1991). As mass is added to the disc, the disc Q parameter approaches unity. Following the literature, we then assume that spiral density waves generate turbulence and angular momentum transport (Lin & Pringle 1987; Gammie 2001) that increase the value of α. In such discs, the radiative cooling time, $t_{\rm cool}$, satisfies (Gammie 2001)

$$t_{\rm cool} = \frac{4}{9}\,\frac{1}{\gamma(\gamma-1)\alpha\Omega}, \tag{1}$$

where γ is the adiabatic index of gas. For gravitational collapse, the cooling time should be shorter than $\simeq 3$ local dynamical times (Gammie 2001). With more and more mass available to drive self-gravitational turbulence, α increases, and $t_{\rm cool}$ eventually approaches the value at which the disc just satisfies the Gammie (2001) cooling criterion. Levin (2006) suggests $\alpha \simeq 0.3$, whereas numerical simulations suggests a rather smaller value of $\alpha \simeq 0.1$ (Rice, Lodato & Armitage 2005).

In either case, the disc then gradually arrives in a state that perhaps is best described as marginally star-forming, in which the disc mass is just large enough and the cooling time is just short enough to initiate star formation. We emphasize that this is a key assumption of our work. Had the disc instead been created in a violent manner, e.g. by cooling that is more rapid than the local dynamical time, we would expect this disc to cross this marginally star-forming state very quickly and immediately collapse into self-contained objects of mass close to fragmentation masses, which can be as low mass as giant gas planets (see Section 2.1 below and also Shlosman & Begelman 1989).

The hydrostatic balance condition yields

$$c_{\rm s}^2 \equiv \frac{P}{\rho} = H^2\Omega^2, \tag{2}$$

where $c_{\rm s}$ is the isothermal sound speed, P and ρ are the total pressure and gas density, H is the disc scaleheight, and $\Omega^2 = (GM_{\rm BH}/R^3 + \sigma_v^2/R^2)$ is the Keplerian angular frequency at radius R from the black hole. Here, σ_v is the stellar velocity dispersion just outside the SMBH radius of influence, i.e. where the total stellar mass becomes larger than $M_{\rm BH}$. Using equation (2), the disc mid-plane density is determined by inversion of the definition of the Q parameter:

$$\rho = \frac{\Omega^2}{\sqrt{2}\pi G Q}. \tag{3}$$

To solve for the temperature of the disc, we should specify heating and cooling rates per unit area of the disc. The former is coupled with the rate of mass transfer through the disc, $\dot{M}$:

$$Q_{\rm d}^+ = \frac{3\Omega^2\dot{M}}{8\pi}. \tag{4}$$

The accretion rate is given by

$$\dot{M} = 3\pi\nu\Sigma, \tag{5}$$

where $\Sigma = 2H\rho$ is the disc surface density. The kinematic viscosity ν in terms of the Shakura & Sunyaev (1973) prescription is $\nu = \alpha c_{\rm s} H$.

The cooling rate of the disc (per side per unit surface area) is given by

$$F_{\rm rad} = \frac{3}{8}\,\frac{\sigma T^4}{(\tau + 2/3\tau)}, \tag{6}$$

where $\tau = \kappa\Sigma/2$ is the optical depth of the disc. This expression allows one to switch smoothly from the optically thick $\tau \gg 1$ to the optically thin $\tau \ll 1$ radiative cooling limits. We approximate the opacity coefficient κ following table 3 in the appendix of Bell & Lin (1994). For the problem at hand, it is just the first four entries in the table that are important, as disc solutions with $T \gtrsim 2000$ K are thermally unstable (see also appendix B in Thompson, Quataert & Murray 2005) because opacity rises as quickly as $\kappa \propto T^{10}$ in that region. This rather simple approximation to the opacities is justified for the order of magnitude parameter study that we intend to perform here. In addition, we set a minimum temperature of $T = 40$ K for our solutions. Even without any gas accretion, realistic gas discs near galactic centres will be heated by external stellar radiation to effective temperatures of this order or slightly larger. The main conclusions of this paper do not sensitively depend on the exact value of the minimum temperature or exact opacity law.

The radiative cooling time of the disc is given by $t_{\rm rad} = \Sigma c_{\rm s}^2/F_{\rm rad}$,

$$t_{\rm rad} = \frac{8}{3}\,\frac{\Sigma k_{\rm B}\,(\tau + 2/3\tau)}{\mu\sigma T^3}, \tag{7}$$

where $\mu \simeq 2.4\ m_{\rm p}$ is the mean molecular weight, where $k_{\rm B}$ is the Boltzmann constant. In practice, we solve for the radial structure of our pre-collapse models by setting $t_{\rm cool} = t_{\rm rad}$, which is of course the same as solving the heating–cooling energy balance, and we also require $Q = 1$. These two assumptions are completely sufficient to solve this set of equations via a simple iterative method.

2.1 Masses of the first stars in the disc

The upper panel of Fig. 1 shows the resulting disc 'mass' defined as $M_{\rm d} = \pi\,\Sigma R^2$ and the mid-plane temperature (multiplied by 10^3). The lower panel of the figure shows two estimates of the mass of the first fragments in the disc. Different authors estimate the volumes of the first unstable fragments slightly differently, but the reasonable range seems to be from H^3 to $2H \times (2\pi H)^2$. The two curves in the lower panel of Fig. 1 should then encompass the reasonable outcomes, from $M_{\rm frag} = \rho H^3$ to $M_{\rm frag} = \rho 8\pi^2 H^3$. From the figure, the fragment mass is, in the observationally interesting range of radii, i.e. $R \sim 0.1$–1 pc, $M_{\rm frag} \lesssim 1\ {\rm M}_{\odot}$, and hence if the disc were to rapidly and completely collapse into clumps of mass of this order, one would expect low-mass stars or even giant planets to dominate the mass spectrum of collapsed objects.

Numerical simulations with a constant cooling time show (e.g. Gammie 2001) that if the disc cooling time is at the threshold for the fragmentation to take place, then the first gas clumps will grow very rapidly by inelastic collisions with other clumps, possibly until they reach the isolation mass $M_{\rm iso} \sim (\pi R^2\ \Sigma)^{3/2}/M_{\rm BH}^{1/2}$ (Levin 2006). If this is the case, then the main point of our paper (that stars born in an accretion disc near an SMBH are massive on average) is proven, because the isolation mass can be hundreds of to as much as $10^4\ {\rm M}_{\odot}$ (Goodman & Tan 2004). However, we suspect that the simulations of Gammie (2001) yielded no further gravitational collapse of the gas clumps precisely because the

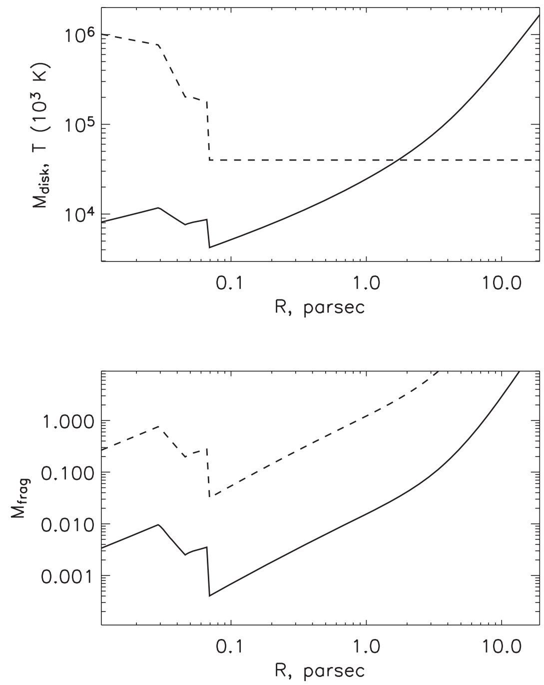

Figure 1. Disc mass (solid curve), $M_{\rm d} = \pi\, \Sigma R^2$, in units of $\mathrm{M}_\odot$, and mid-plane temperature (dashed, in units of 10^3 K) as a function of distance from the SMBH are shown in the upper panel. The SMBH mass is that of Sgr A*. The lower panel shows two estimates of the mass of the first fragments forming in the disc. The realistic value of the mass of the fragments is likely to be in-between these two curves.

cooling times were kept constant. As the clump density increases, the clump free-fall time decreases as $\propto \rho^{-1/2}$, and hence the clumps could not collapse as they could not cool rapidly enough. It is quite likely that, had the cooling time *inside the clumps* been allowed to decrease as the clumps got hotter, the clumps would collapse before they agglomerate into larger ones. Simulations with a more rigorous treatment of radiative cooling, e.g. realistic opacities, were performed by Johnson & Gammie (2003). These show that fragmentation is much more widespread than can be thought based on the analysis of an initial disc state because the radiative cooling time is a very non-linear function of gas column depth and temperature. At the same time, these simulations show that clumps agglomerate into larger ones, as predicted by Levin (2006). We note that these simulations did not include non-radiative cooling, by which we mean hydrogen dissociation and ionization losses. It is well known that these losses are those that allow the 'first' protostellar cores to collapse to much higher densities once the core temperature reaches ∼2000 K (e.g. Larson 1969). Further and even more detailed numerical simulations are needed to investigate these matters further.

3 EFFECTS OF THE FIRST STARS ON THE DISC

We shall now assume that gravitational instabilities in the $Q \approx 1$ disc resulted in the formation of the first protostars. According to the discussion in Section 2.1, we conservatively assume that these protostars are low-mass objects and show that, in certain conditions, even a small admixture of these to the accretion disc may significantly affect its evolution.

3.1 Coupling between stellar and gas discs

As the stars are born out of the gas in a turbulent disc, we assume that the initial stellar velocities are the sum of the bulk circular Keplerian velocity $v_{\rm K}$ in the azimuthal direction and a random component with three-dimensional dispersion magnitude $\sigma_0 \approx c_{\rm s}$. This also implies that, at least initially, the stellar disc scaleheight, H_*, is roughly the same as that of the gas disc, H. Protostars would interact by direct collisions and N-body scatterings between themselves and also via dynamical friction with the gas. The rate of protostellar collisions, $1/t_{\rm coll}$, is the sum of two terms, the geometric cross-section of the colliding stars and the gravitational focusing term (e.g. see Binney & Tremaine 1987). One can show that $1/t_{\rm coll}\, \Omega \simeq \max\,[\Sigma_*\, R_{\rm coll}^2/M_*, (\Sigma_*/\Sigma)\, R_{\rm coll}/H]$, from which it is obvious that collisions are unimportant as long as the collision radius, $R_{\rm coll} \sim 2\, R_{\rm proto}$ (the protostar radius), is much smaller than the disc scaleheight. In all of the cases considered below, this will be satisfied by few orders of magnitude, therefore we shall neglect direct collisions.

The N-body evolution of the system of stars immersed into a gas disc is described by (Nayakshin & Cuadra 2005)

$$\frac{{\rm d}\sigma}{{\rm d}t} \sim 4\pi G^2 M_* \left[\frac{\rho_* \ln \Lambda_*}{\sigma^2} - \frac{\rho C_{\rm d} \sigma}{\left(c_{\rm s}^2 + \sigma^2\right)^{3/2}}\right], \tag{8}$$

where $\ln \Lambda * \sim$ a few is the Coulomb logarithm for stellar collisions, $C_{\rm d} \sim$ a few is the drag coefficient for star–gas interactions (Artymowicz 1994), σ is one-dimensional velocity dispersion, and $\rho_* = \Sigma_*/2H_*$ is the stellar surface density. Therefore, as long as the gas density $\rho \gtrsim \rho_*$, stellar velocity dispersion cannot grow as it is damped by interactions with the gas too efficiently. Recalling that $\rho_* = \Sigma_*/2H_*$, we find that in this situation

$$\frac{\sigma}{c_{\rm s}} \approx \frac{H_*}{H} \sim \left(\frac{\Sigma_*}{\Sigma}\right)^{1/4} < 1. \tag{9}$$

Thus, initially, when $\Sigma_* \ll \Sigma$, stars are embedded in the gaseous disc and form a disc geometrically thinner than that of the gas. However, if stellar surface density grows and approaches that of the gaseous component, then the stellar velocity dispersion will run away. The stars then form a geometrically thicker disc (numerical simulations, to be reported in a future paper, confirm these predictions). Galaxy discs apparently operate in this regime, with molecular gas having a much smaller scaleheight than stars.

3.2 Accretion on to protostars

Because the stars remain embedded in the disc, they will continue to gain mass via gas accretion. Such accretion has been previously considered by many authors (e.g. Lissauer 1987; Bate et al. 2003; Goodman & Tan 2004). We assume that the accretion rate is

$$\dot{M}_* = \min[\dot{M}_{\rm Bondi}, \dot{M}_{\rm Hill}, \dot{M}_{\rm Edd}], \tag{10}$$

where the accretion rates in the brackets are the Bondi, the Hill and the Eddington limit, respectively (e.g. Nayakshin 2005). The latter is calculated based on the Thomson opacity of free electrons instead of dust opacity because we assume that the cooler regions of accretion flow on to the star are shielded from the stellar radiation by the inner, hotter accretion flow (Krumholz, McKee & Klein 2005): $\dot{M}_{\rm Edd} = 10^{-3} r_*\, \mathrm{M}_\odot\, \mathrm{yr}^{-1}$, where $r_* = R_*/\mathrm{R}_\odot$, and R_* is the main-sequence radius of the star.[1]

[1]This note is made in response to a referee comment. A more rigorous treatment would attempt to model the evolution of the stellar radius as the

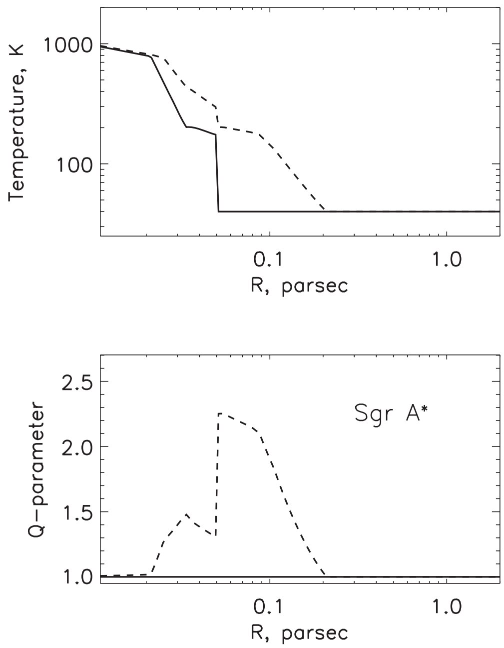

Figure 2. Temperature (upper panel) and Q parameter (lower panel) versus radius in parsec for the marginally self-gravitating disc with (dashed) and without (solid) protostars embedded in the discs. The protostars have masses $M_* = 0.1\,\mathrm{M}_\odot$ and surface density $\Sigma_* = 0.001\ \Sigma$.

3.3 Heating of the disc by protostars

The presence of stars will lead to additional disc heating via radiation and outflows, and N-body scattering. The energy liberation rate per surface area due to N-body interactions is given by

$$Q^+_{*N} \sim \Sigma_* \sigma \left(\frac{\mathrm{d}\sigma}{\mathrm{d}t}\right)_* \sim 4\pi G^2 M_* \Sigma_* \frac{\rho_* \ln \Lambda_*}{\sigma}, \tag{11}$$

where $(\mathrm{d}\sigma/\mathrm{d}t)_*$ stands for the first term only in equation (8). Internal disc heating with the α parameter equal to unity is

$$Q^+_{\mathrm{d}} = \frac{9}{4}\Sigma c_{\mathrm{s}}^2 \Omega. \tag{12}$$

Comparing the stellar N-body heating with that of the internal disc heating, we have

$$\frac{Q^+_{*N}}{Q^+_{\mathrm{d}}} \sim \left(\frac{\Sigma_*}{\Sigma}\right)^{3/2} \frac{M_*}{M_{\mathrm{BH}}} \left(\frac{R}{H}\right)^3. \tag{13}$$

We have assumed above that the parameter $Q \sim 1$ when the stars just appear in the disc and that $\Omega^2 = GM_{\mathrm{BH}}/R^3$, i.e. that we are within the SMBH sphere of influence. Considering this expression for typical numbers, one notices that N-body heating is never important for large black holes and discs with finite disc thickness, i.e. at distances of tens of parsec and further away, but it may become important for smaller SMBHs, such as Sgr A*, and subparsec distances.

The radiative internal output of stars is another source of disc heating. We shall use very simple parametrization for the internal stellar luminosity as a function of mass, $L_* \propto M_*^3$. To this radiative output, we should also add the accretion luminosity,

$$L_{\mathrm{acc}} = \frac{GM\dot{M}_*}{R_*}. \tag{14}$$

The sum should not exceed the Eddington limit, $L_{\mathrm{Edd}} = 4\pi GM_* m_{\mathrm{p}}c/\sigma_{\mathrm{T}} \approx 10^{38} m_*\ \mathrm{erg\ s^{-1}}$, and hence our prescription is

$$L_* = \min\left(\mathrm{L}_\odot \frac{M_*^3}{\mathrm{M}_\odot^3} + L_{\mathrm{acc}}, L_{\mathrm{Edd}}\right), \tag{15}$$

where $L_{\mathrm{Edd}} = 4\pi GM_*\, m_{\mathrm{p}}\, c/\sigma_{\mathrm{T}}$ is the Eddington limit with Thomson opacity σ_{T}. The radiative disc heating per unit surface area is then

$$Q^+_{*\mathrm{rad}} = \frac{\Sigma_*}{M_*} L_*. \tag{16}$$

4 ANALYTICAL ESTIMATES

As equation (3) suggests, accretion discs on parsec scales are as dense as 10^{12} particle cm^{-3}, which is multiple orders of magnitude denser than the densest gas in molecular clouds far from galactic centres. Therefore, it is not surprising that the first protostars will be accreting at super-Eddington rates, typically. The corresponding surface Eddington accretion luminosity heating (equation 16 with $L_* = L_{\mathrm{Edd}}$) is

$$Q^+_{*\mathrm{Edd}} \sim 10^5\ \Sigma_*\ \mathrm{erg\,s^{-1}\,cm^{-2}}, \tag{17}$$

which is a function of the stellar surface density only. At the same time, the disc intrinsic heating at $Q \approx 1$ is, from equation (12),

$$Q^+_{\mathrm{d}} \approx \Sigma T_2 \frac{M_{\mathrm{BH}}}{3\times 10^6\,\mathrm{M}_\odot} \left(\frac{0.1\,\mathrm{pc}}{R}\right)^{3/2}\ \mathrm{erg\,s^{-1}\,cm^{-2}}, \tag{18}$$

where Σ and Σ_* are in units of $\mathrm{g\,cm^{-2}}$, and T_2 is the disc temperature in units of 100 K. We see that even a very small admixture of protostars ($\Sigma_* \ll \Sigma$) accreting at Eddington accretion rates will result in stellar heating much exceeding the intrinsic one. Because the disc thermal equilibrium is established on time-scales comparable to the disc dynamical time, the disc will heat up at a constant Σ until its radiative losses can balance the accretion luminosity. This will increase the disc sound speed and the Q parameter *above unity*. Therefore, the accretion feedback will stop further fragmentation from happening. The stars embedded into the disc will however continue to gain mass at very high rates. This should lead to a top-heavy initial mass function for the stars.

Note that a similar conclusion has been already reached by Levin (2006) who considered rather later stages in the evolution of a more massive active galactic nucleus (AGN) disc, when the high-mass stars were turned into stellar mass black holes. He pointed out that accretion on to these embedded black holes will likely heat the disc, driving the Q parameter above unity for radii somewhat smaller than 1 pc.

protostar accretes mass, as initially R_* may be considerably larger than the main-sequence value, but we neglect it here as this modelling would be rather cumbersome and poorly constrained: protostellar rotation, metalicity, radiative cooling in the collapsing envelope, etc., would all play a role. Most importantly, the change in the results will be small as the Hill accretion rates would still remain super-Eddington (e.g. Nayakshin 2004 noted in his equation 14 that Hill accretion rates on a single star embedded in the accretion disc may be larger than that through the whole disc to the central SMBH). For example, we increased the protostellar radii by a factor of 10 compared with the main-sequence stellar radii and observed that results were affected only for the largest disc radii, R. In particular, for the same parameters as those used in Fig. 2, the region where Q is larger than unity was limited to $R < 0.1$ pc rather than $R < 0.2$ pc.

5 NUMERICAL RESULTS

We shall first consider the case of Sgr A* for which the mass of the SMBH is estimated to be $M_{\rm BH} \simeq 3.5 \times 10^6\,{\rm M}_{\odot}$ (Schödel et al. 2002; Ghez et al. 2003). For this particular example, we shall accept that $\Sigma_* = 0.001\,\Sigma$ and that the initial masses of protostars are $M_* = 0.1\,{\rm M}_{\odot}$. The upper panel of Fig. 2 shows the disc mid-plane temperature before the stars are introduced (solid) and after (dashed) versus radius R. The temperature of the gas increases for radii $0.03 \lesssim R \lesssim 0.2$ pc. The Q parameter after the stars are introduced is plotted in the bottom panel of Fig. 1. Q indeed becomes greater than unity in the same radial range, thus shutting off further gravitational collapse.

The radial range where the protostars shut off further fragmentation is rather insensitive to the assumptions of our model. Fig. 3 shows the Q parameter after stars are introduced into the $Q = 1$ gaseous disc for the Sgr A* case, but with varying assumptions. In particular, the solid curve is the same as that in Fig. 2 (lower panel); the dotted one is calculated for the opacity coefficient κ multiplied arbitrarily by 3, whereas for the dashed one κ was divided by $\sqrt{T}$. These arbitrary changes were introduced to estimate the degree to which the results are dependent on the (uncertain) opacity detail. Finally, the dot-dashed curve is calculated assuming the standard opacity but increasing the protostellar mass to $1\,{\rm M}_{\odot}$ and stellar surface density Σ_* to 0.01, respectively. The stellar heating is then more pronounced and a larger area of the marginally stable disc can be affected.

We also consider the case of a more massive black hole: in particular, we set $M_{\rm BH} = 1.4 \times 10^8\,{\rm M}_{\odot}$, as is thought to be the case for M31. Fig. 4 shows the disc temperature structure (upper panel) before the collapse (dashed) and after the collapse. Note that these curves are almost identical to those for the Sgr A* case except for a general shift to larger radii. This shift is about a factor of 3 only, which should not be surprising given that, in the standard accretion disc theory (Shakura & Sunyaev 1973), the mid-plane disc temperature is a very weak function of the central object mass.

6 MAXIMUM GROWTH RATE OF STARS BY ACCRETION

The second referee of this paper, Jeremy Goodman, raised a very interesting and potentially important point on the maximum possible accretion rate on *low-mass* protostars that we discuss here. As is well known, for stellar models in which the ratio of gas pressure to total (gas plus radiation) pressure, β, is constant throughout the star, the Lane–Emden equation can be solved to form the ‘Eddington quartic relation’

$$\frac{1-\beta}{\mu^4\beta^4} = 0.003\left(\frac{M}{{\rm M}_{\odot}}\right)^2, \qquad (19)$$

where μ is the average molecular weight in units of hydrogen mass. The derivation of this equation states nothing about internal energy sources, so it should apply not only during the main-sequence life of the star but also during contraction of the star and (perhaps) even when the power is generated via accretion.

Now, in the star-forming disc models that we explored in this paper, the low-mass protostars were assumed to be able to accrete gas at the Eddington limit for their main-sequence mass. By definition, then, radiation pressure should be significant, and hence β is small. The Eddington quartic relation then predicts that the mass of the star should be much larger than $1\,{\rm M}_{\odot}$, i.e. $M \sim 100\,{\rm M}_{\odot}$ for $\beta = 0.5$ and $\mu = 0.5$, in stark contrast to our assumption $M \ll 1\,{\rm M}_{\odot}$.

One point to make here is that the Eddington quartic mass prediction for radiation-dominated stars should however not be taken *at the face value* for the problem at hand. Equation (19) is derived under the assumptions of constant β and a spherical symmetry. We expect both of these to be very poor approximations here. Accretion on to the star definitely proceeds in non-spherical geometry: through an accretion disc due to a considerable amount of vorticity in the gas gravitationally captured by the protostar from the AGN accretion disc. Close to the stellar surface, where most of the gravitational energy is released, we picture the accretion process to be akin to that in the theory of matter spreading on to the neutron star surface during disc accretion, as developed by Inogamov & Sunyaev (1999). Most of the radiation comes out in that model inside the disc and the boundary/spreading layers, and the non-spherical

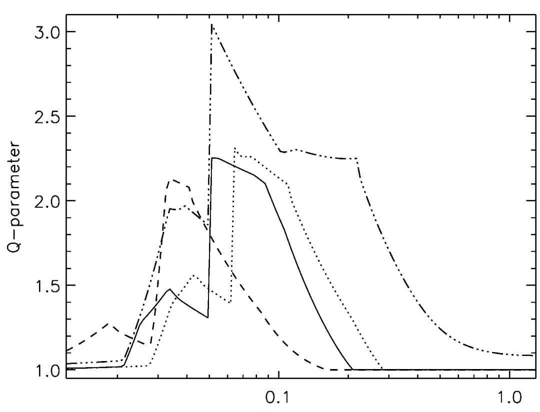

Figure 3. The Q parameter versus radius for the self-gravitating disc in which the first stars were born. The solid line is the same as that in the lower panel of Fig. 2. The other curves are obtained by varying the assumption of the model to test the sensitivity of the results (see text).

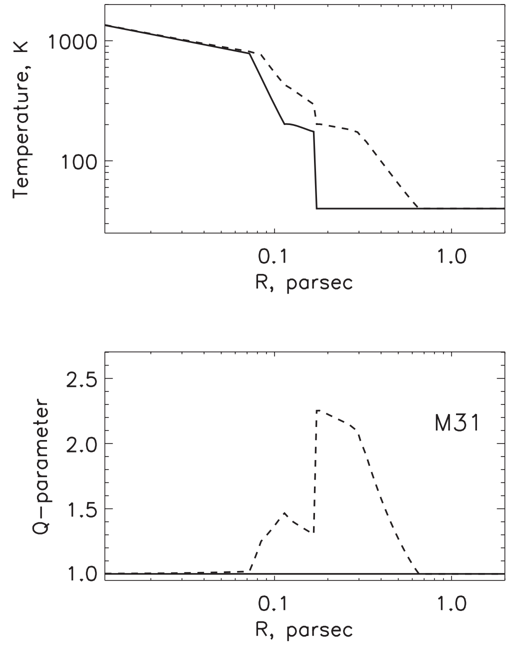

Figure 4. Same model as that used for Fig. 1, except for a higher SMBH mass, $M_{\rm BH} = 1.4 \times 10^8\,{\rm M}_{\odot}$. Note that, compared with Fig. 1, the radial range where protostars succeed in heating the disc up has shifted to slightly larger radii.

geometry of the problem is absolutely crucial. One reason why the Eddington quartic relation does not apply in this situation is that radiation transfer occurs mainly in the direction perpendicular to the flow of matter, in stark contrast to spherical geometry.

Next, radiation-dominated accretion discs certainly do not satisfy $\beta=$ const throughout their interiors; rather, they have approximately constant gas density (Shakura & Sunyaev 1973). More elaborate distributions of the radiation and gas density with vertical height can be derived in a steady-state approach, and those again do not reduce to $\beta=$ const (Hubeny et al. 2000).

Nevertheless, having said this, it is still quite plausible that the geometrical and other factors will only relax but not remove completely the constraints from the Eddington quartic relations. Only detailed numerical simulations of the type performed by Krumholz et al. (2005, but with an additional treatment of boundary conditions due to the stellar surface) may answer this question reliably, it seems. In the absence of such simulations, we explore the sensitivity of our results (Section 5) on the assumed maximum accretion rate on to the protostars. The corresponding stellar accretion luminosity is written as $\lambda L_{\rm Edd}$, where λ is a dimensionless parameter $\leqslant 1$.

The unit surface heating rate via Eddington-limited stellar accretion depends only on the total stellar surface density and *not* on the mass of individual stars (equation 17). Therefore, the disc heating due to such accretion is simply reduced by the factor λ. The ratio of the disc internal stellar heating to the internally generated heating (equation 18) is

$$\frac{Q_{*\rm Edd}}{Q_{\rm d}^{+}} \approx 10^5\, \frac{\lambda \Sigma_*}{\Sigma}\, \frac{3\times 10^6\,{\rm M}_{\odot}}{M_{\rm BH}} \left(\frac{R}{0.1\ {\rm pc}}\right)^{3/2} T_2^{-1}. \tag{20}$$

In our model, stellar feedback is important from the start, i.e. when stellar surface density is still very low compared with that of the gas, or else the stellar mass spectrum cannot be influenced by gas accretion as there is not enough gas left in the disc. In particular, if $\Sigma_* \sim 10^{-3}\Sigma$, then stellar heating feedback becomes negligible for $\lambda \lesssim 0.01$. On the other hand, if that is the case, then fragmentation in the disc will continue until Σ_* becomes sufficiently large to satisfy the criterion given by equation (20).

To summarize, our results are not strongly influenced by the assumption that the maximum of the protostellar accretion luminosity is the corresponding Eddington limit unless λ is as small as $\lambda \ll 10^{-3}$. Three-dimensional numerical simulations of gas accretion on low-mass protostars are needed to establish whether λ can indeed be that small when the angular momentum of accreting gas is significant.

7 DISCUSSION

In this semi-analytical paper, we studied the 'first minutes' of an accretion disc around an SMBH after the disc became unstable and formed the first stars. We assumed that the disc accumulated its mass over time-scales much longer than the local dynamical time and is thus in thermal equilibrium before gravitational collapse. In this case, irrespective of the typical mass of the first protostars, even a 0.1 per cent admixture (by mass) of these significantly alters the thermal energy balance of the disc. The protostars accrete gas from the surrounding disc at very high (super-Eddington) rates at a range of disc radii. The accretion luminosity of these stars is sufficient to heat the disc up in that range of radii to the point where it becomes stable to self-gravity ($Q > 1$), which then shuts off further fragmentation of the disc. The protostars already present in the disc would however continue to gain mass at very high rates. Quite generally, then, an average star created in such a disc will be a massive one, in stark contrast to the typical galactic star formation event.

The significance of accretion feedback on to embedded *stellar mass black holes* for accretion discs near galactic centres was pointed out by Levin (2006). He noted that the accretion discs can be stabilized by the feedback out to a radius of about 1 pc, in good agreement with our results. Because the Eddington luminosity depends only on the disc opacity and the mass of the central object, it is then not really surprising that the feedback is effective for accretion on to stars as well.

The range of radial distances from the SMBH where this effect operates is a slow function of the SMBH mass and is typically from a few per cent of a parsec to a few parsecs, with the peak of the effect taking place at $R \sim 0.1$ pc. The reason why the feedback is only effective in a range of radii is that, at large radii, i.e. tens of parsec, the gas density in the disc drops significantly so that accretion rates on to the protostars become much smaller than the respective Eddington-limited rates. At radii much smaller than $\sim$0.1 pc, the intrinsic disc heating (equation 18) becomes very large. A related point is that steady-state constant accretion rate disc models show that there is always the minimum radius where star formation becomes impossible as $Q > 1$ there (e.g. Goodman 2003; Nayakshin & Cuadra 2005; Levin 2006). The value of the minimum distance where star formation should be expected is comparable to the minimum radius for which we predict favourable conditions for development of a top-heavy IMF.

We suggest that the uncommonly effective feedback from star formation on low-mass protostars may be relevant for the formation of rings of massive stars observed in the Galactic Centre and in the nucleus of M31. In particular, Nayakshin & Sunyaev (2005) and Nayakshin et al. (2006) have shown from two completely independent lines of evidence that the IMF of stellar discs in the Galactic Centre must have been very top-heavy, with the mass of solar-type stars accounting for no more than $\sim$50 per cent of the total, the remainder being in the $M \gtrsim 30\,{\rm M}_{\odot}$ stars. The insensitivity of our results to details of the model and the SMBH mass suggest that the IMF of stars born inside accretion discs near galactic centres may be generically top-heavy, which would have long-ranging consequences for the accretion theory in AGN.

7.1 Generality and shortcomings of this work

In this paper, we concentrated on the growth of protostellar mass via accretion of gas. As discussed in Section 3, in certain conditions the mass of gas clumps before they collapse to form a star may be much higher than the Jeans mass. This would only increase the expected final mass of a typical star in the disc. The same is true for direct collisions of protostars, the other channel via which protostars may grow (see Section 3.1).

In addition, here we have limited the rate at which the protostar would grow to the Eddington accretion rate on to a star. This is important if the rate at which the gas is captured in the sphere of influence of the protostar, the Hill or the Bondi radius, whichever is smaller, exceeds the Eddington accretion rate. It is possible that, in reality, the excess gas settles into a rotationally supported 'protostellar' disc from which further generations of stars may be born (Milosavljević & Loeb 2004). It is not obviously clear whether this effect would increase the average mass of the stars or would rather decrease it. On the one hand, fragmentation of the protostellar disc may give rise to many low-mass stars. On the other hand, though, these stars may be then driven into the central more massive star by the continuing gas accretion, as suggested by Bonnell & Bate (2005). In the latter case, the central star may in fact grow faster than the Eddington accretion rate.

On balance, we believe that the main conclusion of our work, e.g. the unusually high (perhaps dominant) fraction of the total mass going into the creation of high-mass stars as opposed to low-mass stars, may be rather robust. One clear exception to this will be a very rapid (dynamical) gravitational collapse of a disc. For example, when a large quantity of gas (compared with the minimum needed for the disc to become self-gravitating, see Fig. 1) cools off very rapidly and settles into a disc configuration, and the cooling time is shorter than dynamical time, the disc will break into self-gravitating low-mass objects before it can establish thermal balance (e.g. Shlosman & Begelman 1989).

We deliberately stayed away here from discussing the much more complicated question of the eventual disc evolution. The answer depends not only on the initial radial structure of the disc but also on how the disc is fed with gas after it has crossed the self-gravity instability threshold. We shall investigate these issues in our future work (note that Thompson et al. 2005 recently developed a model for kiloparsec-scale star-forming discs in ultraluminous galaxies).

7.2 Why is star formation is AGN discs different from 'normal' star formation?

It is instructive to emphasize the differences in star formation rates near an SMBH and in a galaxy. Consider the relevant gravitational collapse time-scales t_c. For accretion discs around the SMBH, this is $t_c \simeq 1/\Omega \approx 60\,\mathrm{yr}\,(R/0.04\,\mathrm{pc})^{3/2}\,(M_{\mathrm{BH}}/3\times10^6\,\mathrm{M_{\odot}})^{-1}$, which is shorter than the Eddington limit doubling time of $\sim$1000 yr. Compare this time with the free-fall time for a molecular cloud of mass $10^3\,\mathrm{M_{\odot}}$ and size 1 pc: $t_{\mathrm{ff}} \sim 10^6$ yr. Clearly, then, an average accretion rate in the galactic environment is orders of magnitude below the Eddington limit, and no significant radiation feedback should be expected from *low-mass* protostars. The latter can then form in great numbers with little damage to the rest of the cloud, unlike in the case of a massive disc.

Another significant qualitative difference is that the escape temperatures, $T_{\mathrm{esc}} \sim GM\mu/kR$, are vastly different near an SMBH and inside a molecular cloud. For the former, the escape temperature is typically in the range of 10^6–10^7 K, whereas for the latter it is only $\sim$few $\times\,10^3$ K. Hence, while photoionizing feedback from massive stars may unbind most of the gas in a molecular cloud, stopping not only further fragmentation but also further accretional growth of protostars, in SMBH discs the effect is local (e.g. Milosavljević & Loeb 2004). In particular, it simply increases the disc scaleheight until the gravity of the SMBH (which increases as z/H for thin accretion discs, e.g. Shakura & Sunyaev 1973) is strong enough to hold the gas in place. In other words, accretion or any other star formation feedback in a disc environment may be strong enough to prevent further disc fragmentation but not the growth of the existing protostars.

The third important difference is geometry. Most stars in a molecular cloud move on orbits different from those of the gas, as the latter is influenced by both gravity and pressure forces, whereas stars obey only gravity. Hence, the gas and the stars may be separated out in space, terminating accretion on to the stars. In contrast, as is well known from standard accretion theory (Shakura & Sunyaev 1973), gas pressure forces are very small compared with SMBH gravity for thin gas discs in galactic centres, and so both stars and gas follow essentially circular Keplerian orbits around the SMBH (Nayakshin & Cuadra 2005). Therefore, the stars are always not too far away from the gas and hence have a much better chance to gain more mass by accretion.

Finally, turbulent or magnetic field support against gravitational collapse is not likely to be dominant in the disc, in a complete reversal to the case of molecular clouds. Molecular clouds have dynamical time-scales of orders of a million years, and the lifetimes are estimated between 1 and 10 million years (e.g. Tassis & Mouschovias 2004). This implies that molecular clouds are dynamically young systems. Any turbulence caused by settling in or energy/turbulence input from outside (e.g. Bonnell et al. 2006) may plausibly last for the lifetime of these clouds. Accretion discs of subparsec scale, on the other hand, have dynamical times of the order of 1 to 10^3 yr, depending on the SMBH mass, but probably live for 10^6–10^8 yr or even longer (indeed, the Salpeter mass e-folding time-scale is $\sim 4\times 10^7$ yr). Therefore, these discs are dynamically old systems. Any turbulence generated initially as the disc settles in is likely to decay away. It is the commonly accepted view that turbulence there is subsonic, expressed in the standard theory by the requirement $\alpha < 1$ (Shakura & Sunyaev 1973), which has been amply confirmed by numerical magnetohydrodynamic (MHD) simulations (e.g. Fleming, Stone & Hawley 2000).

8 CONCLUSIONS

In this paper, we have shown that the birth of even a small number (by mass fraction) of low-mass protostars inside a marginally unstable star-forming accretion disc near a galactic centre will unleash very strong accretion thermal feedback on to the gaseous disc. In a subparsec range of radii, the disc will be heated and thickened so that it becomes stable to further fragmentation. However, the feedback is not strong enough to unbind the gas from the deep potential well of the SMBH. Therefore, while the feedback stops further disc fragmentation, accretional growth of stars already present in the disc proceeds. Quite generically, this scenario should lead to the average star created in the SMBH accretion disc being 'obese' compared with its galactic cousins.

ACKNOWLEDGMENTS

The author acknowledges very useful comments on the draft by Yuri Levin, and fruitful discussions with Andrew King, Jim Pringle and Jorge Cuadra. The second referee, Jeremy Goodman, is thanked for a useful report that improved this paper.

REFERENCES

Artymowicz P., 1994, ApJ, 423, 581
Balbus S. A., Hawley J. F., 1991, ApJ, 376, 214
Bate M. R., Lubow S. H., Ogilvie G. I., Miller K. A., 2003, MNRAS, 341, 213
Bell K. R., Lin D. N. C., 1994, ApJ, 427, 987
Bender R. et al., 2005, ApJ, 631, 280
Binney J., Tremaine S., 1987, Galactic Dynamics. Princeton Univ. Press, Princeton, NJ
Bonnell I. A., Bate M. R., 2005, MNRAS, 362, 915
Bonnell I. A., Dobbs C. L., Robitaille T. P., Pringle J. E., 2006, MNRAS, 365, 37
Collin S., Zahn J., 1999, A&A, 344, 433
Fleming T. P., Stone J. M., Hawley J. F., 2000, ApJ, 530, 464
Gammie C. F., 2001, ApJ, 553, 174
Gerhard O., 2001, ApJ, 546, L39

Ghez A. M., Becklin E., Duchjne G., Hornstein S., Morris M., Salim S., Tanner A., 2003, Astron. Nachr., 324(S1), 527
Goodman J., 2003, MNRAS, 339, 937
Goodman J., Tan J. C., 2004, ApJ, 608, 108
Gürkan M. A., Rasio F. A., 2005, ApJ, 628, 236
Hubeny I., Agol E., Blaes O., Krolik J. H., 2000, ApJ, 533, 710
Inogamov N. A., Sunyaev R. A., 1999, Astron. Lett., 25, 269
Johnson B. M., Gammie C. F., 2003, ApJ, 597, 131
Kolykhalov P. I., Sunyaev R. A., 1980, Sov. Astron. Lett., 6, 357
Krumholz M. R., McKee C. F., Klein R. I., 2005, ApJ, 618, L33
Larson R. B., 1969, MNRAS, 145, 271
Levin Y., 2006, astro-ph/0603583
Lin D. N. C., Pringle J. E., 1987, MNRAS, 225, 607
Lissauer J. J., 1987, Icarus, 69, 249
Milosavljević M., Loeb A., 2004, ApJ, 604, L45
Nayakshin S., 2004, MNRAS, 352, 1028
Nayakshin S., 2005, MNRAS, 359, 545
Nayakshin S., Cuadra J., 2005, A&A, 437, 437
Nayakshin S., Sunyaev R., 2005, MNRAS, 364, L23
Nayakshin S., Dehnen W., Cuadra J., Genzel R., 2006, MNRAS, 366, 1410
Paczyński B., 1978, Acta Astron., 28, 91
Paumard T. et al. 2006, ApJ, 643, 1011
Rafikov R. R., 2005, ApJ, 621, L69
Rice W. K. M., Lodato G., Armitage P. J., 2005, MNRAS, 364, L56
Schödel R. et al., 2002, Nat, 419, 694
Shakura N. I., Sunyaev R. A., 1973, A&A, 24, 337
Shlosman I., Begelman M. C., 1989, ApJ, 341, 685
Tassis K., Mouschovias T. C., 2004, ApJ, 616, 283
Thompson T. A., Quataert E., Murray N., 2005, ApJ, 630, 167
Toomre A., 1964, ApJ, 139, 1217

This paper has been typeset from a TEX/LATEX file prepared by the author.