ESSAYS ON REFERENCE DEPENDENCE IN MACROECONOMICS

Theory and evidence

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by

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Abstract of

<u>"Essays on refernce dependence in macroeconomics"</u>

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There are indications that *psychological factors and biases* are gaining the centre stage of economics and finance once again, as they did during the thirties with the spreading of Keynesian ideas. *Behavioural economics and finance* is a booming field, if judged by the number of contributions in recent years and by its success in explaining phenomena which cannot easily find an explanation according to conventional models.

Against this background, the main objective of this dissertation is to assess whether the psychological factors modelled in the behavioural finance literature, and in particular by *prospect theory* (Kahneman and Tversky, 1979), can have a bearing on *macroeconomic outcomes*, be it prices set in large and competitive financial markets or real economic developments.

In particular, in *Chapter 1* I review the behavioural finance literature to see whether the "anomalies" that this literature has identified could affect market prices. I find many examples in which it could be the case. This is particularly true for anomalies which are widespread among economic agents, for instance because of a social basis. In *Chapter 2*, I look at how a central bank which is characterised by reference dependence and diminishing sensitivity with respect to deviations of inflation from its target, and which weighs probabilities of uncertain events non-linearly. I find that it can behave in a way which is different from what standard models suggest, in particular by violating the certainly equivalence principle. In *Chapter 3*, I see how it is optimal, under quite general conditions, for a prospect theory agent to concentrate, rather than diversify risks. Finally, in *Chapter 4*, I show how loss and disappointment aversion may explain the equity premium puzzle if different assumptions are maintained on the representative agent's investment time horizon and reference point.

Alla mia famiglia, da nonna Lina (97 anni) al piccolo Leonardo (1 anno).

To my family, from granma (97 years old) to Leonardo (1 year old)

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Contents

Preface

"Human decisions affecting the future, whether personal, political or economic, cannot depend on strict mathematical expectation, since the basis for making such calculation does not exist: and that it is our innate urge to activity which makes the wheels go round, our rational selves choosing between the alternatives as best we are able, calculating where we can, but often falling back for our motive on whim or sentiment or chance." (J. M. Keynes, The General Theory, Chapter 12, p. 162.)

Psychological factors in economics once again? Keynes vindicated

In recent decades, the paradigm of rational agents, coldly maximizing their expected utility and possessing a complete knowledge of the model driving the economy has dominated the economics discipline and in particular the finance field. The assumption of rationality and no arbitrage lies at the core of asset pricing models developed since the early seventies, and has made possible an astounding advancement of modelling and insight since then.

There are indications, however, that psychological factors and biases are gaining the centre stage of economics and finance once again, as they did during the thirties with the spreading of Keynesian ideas. Behavioral economics and finance is a booming field, if judged by the number of contributions in recent years and by its success in explaining phenomena which cannot easily find an explanation according to conventional models. While the behavioral approach has not supplanted the mainstream approach based on rationality and optimizing behavior, it has certainly contributed to put human psychology again in a central place in the economics discipline. Behavioral models have been applied especially in finance, due to the richness of data and the fact that the mainstream approach has produced strong and testable implications that have often failed to match the empirical evidence. Reflecting these factors, behavioral finance is the most prominent research agenda applying the behavioral approach and psychological research findings to the economics field.

One important distinctive feature of the behavioral approach, which has grown in importance in the last two decades, is the focus on experimental evidence and on producing simple and analytically tractable models which can be used in rigorous theoretical and empirical analysis. From this perspective, the behavioral approach distinguishes itself from the more "qualitative" emphasis on psychological factors and confidence ("animal spirits") which is in the Keynesian tradition. The focus on analytical tractability and mathematical rigor is especially a feature of *prospect theory*, introduced by Kahneman and Tversky in the seventies, which has gained a prominent status within the behavioral approach as a descriptive theory of agents′ attitudes towards risk and uncertainty. Analytical tractability and parsimony are key features of any good behavioral approach, if the objective is to produce testable conditions in the same way as the mainstream approach based on expected utility maximization and rational expectations.

Overall, the main objective of this dissertation is to assess whether the psychological factors modelled in the behavioral finance literature, and in particular by prospect theory, can have a bearing on macroeconomic outcomes, be it prices set in large and competitive financial markets or real economic developments. This appears to be a key element in evaluating the relevance of psychological factors in explaining economic phenomena.

Against this background, this dissertation aims at providing an answer to the following key questions:

a) is the maximization of individual expected utility by economic agents enough to understand aggregate behavior and macroeconomic outcomes?

b) Are there systematic psychological traits of economic agents which are not consistent with the axioms underlying the expected utility model, but

c) are plausible and systematic enough to affect macro variables and prices?

Chapter 1: Can psychological factors and biases affect aggregate market prices?

This chapter contains a focused review of the behavioral finance literature, especially in the last decade, which provides the background for the remainder of the dissertation. In particular, this literature - which has grown very large in recent years - is reviewed with the objective of understanding whether behavioral biases and factors may affect aggregate prices and competitive financial markets. The chapter sets out to provide a (tentative) answer to the following two key questions:

- What are the most important and systematic behavioral biases which characterize economic agents?

- Are they relevant to understand aggregate market behavior, namely do they affect aggregate prices and competitive markets?

The analytical framework of the chapter is as follows. There is a large competitive market where a certain asset is priced. Moreover, there is a relationship linking the asset price to some exogenous variable, i.e. a "fundamental", derived from the maximization of the representative agent's expected utility. The key question is whether agents' psychological factors and biases can affect *market prices* so as to make them deviate from fundamentals. To the extent that the fundamental itself may be affected by fluctuations in prices due to behavioral influences (for example, the firm of an irrationally overvalued stock can benefit from agents' enthusiasm and increased sales) a *self-fulfilling* mechanism might establish itself.

The review of the behavioral finance literature is then focused on whether "anomalies", i.e. systematic behavior that violates the axioms of expected utility, can affect prices set in a large, competitive market. In particular, I group the most important anomalies into five categories:

1. Anomalies that derive from deliberation and optimization costs with no role played by emotions and "visceral" factors. This field broadly covers the literature on *decision heuristics* and *bounded rationality*.

2. Anomalies which are related to the role played by *emotions and visceral factors*.

3. Anomalies related to *choice bracketing:* in this category, anomalies relate to the fact that agents "edit" problems in narrower frames compared with the standard maximization of lifetime utility popular in economics and finance models.

4. In the fourth category, we survey recent contributions which claim that a set of well-defined and deterministic preferences does not exist. Rather, *stochastic and context-dependent preferences* should be considered.

5. Finally, I review *reference dependence* and in particular *prospect theory*, which is built on the assumption that preferences are built around *reference points* and not in abstract terms (Kahneman and Tversky, 1979).

I identify numerous examples in the literature of anomalies which can affect market prices, although there are also anomalies which are unlikely to do so. This represents a very important distinction between the various anomalies identified. In general, two features seem to have an important bearing on whether a certain anomaly is a good candidate to affect market prices. First, the anomaly should be sufficiently widespread in market participants' individual psychology and relevant in a market context. For example, the "irrational" reluctance to realize losses is a factor which is probably widespread and relevant in a market context, while "caring about others" is a tendency which is unlikely to emerge in a large, competitive market. Second, anomalies which have a social dimension are more likely to become widespread and affect asset prices. For example, irrational enthusiasm or pessimism by agents on a certain asset is likely to have a social basis which makes this kind of behavior sufficiently widespread to significantly affect market prices on aggregate.

A main conclusion of the chapter is that some behavioral factors and biases do have the potential to affect market prices, and this should be taken seriously (although not necessarily acted upon) by policy-makers. So, while there is no hard evidence that the market may be "irrational" in the beat-the-market sense normally maintained by mainstream finance theorists (i.e., it is practically impossible to make extra-money in the market without taking on more risk), there are many indications to suggest that large, competitive financial markets may be irrational in other relevant senses. For example, they might not optimally reflect fundamentals and so create a misallocation of capital.

After this broad review of the behavioral finance literature, I narrow down the focus of the dissertation on *prospect theory* (Kahneman and Tversky, 1979). In particular, I concentrate my attention on three related aspects of the theory: (i) reference dependence, from which (ii) diminishing sensitivity is a natural consequence; and (iii) non-linear weighing of probabilities. These aspects represent key innovations compared with the expected utility theory, and my objective in the continuation of the dissertation is to understand whether these features may have an important bearing on macroeconomic outcomes.

Chapter 2: Reference dependence and non-linear weighing of probabilities by central banks

Due to their size and importance, policy-makers can certainly affect aggregate market outcomes. It is therefore *prima facie* interesting to study whether it is plausible to assume that the psychological features of prospect theory may apply to policy-makers, and whether this might affect macroeconomic outcomes.

In this chapter[1] I apply prospect theory to the modelling of central bank preferences. In particular, I consider the possibility that reference dependence and the associated property of diminishing sensitivity is a realistic feature of a central banker which cares about inflation. Moreover, non-linear weighing of probabilities is also a feature which is very realistic to assume for central bankers. The model of the economy is a simple backward-

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[1] This chapter has been prepared under the close supervision of Ali al-Nowaihi, which has resulted in a co-authored journal submission.

looking IS-Phillips curve model with a non-Normal distributed additive shock to inflation representing central bank additive uncertainty over the level of inflation. The chapter finds that the interaction of a non-Normal additive disturbance to inflation and the set of non-quadratic preferences postulated leads to situations where the principle of certainty equivalence does not hold, i.e. where additive uncertainty matters. Only if the disturbance is Normally distributed is the usual result of certainty equivalence still valid. If one observes that non-Normally distributed shocks are an essential element which monetary policy-makers have to deal with, the overall policy message of this chapter is that additive uncertainty matters, and that the assumption of a quadratic loss function may not be as innocuous as is often thought. More generally, this chapter suggests that reference dependence (and diminishing sensitivity) and non-linear weighing of probabilities might affect the way monetary policy is carried out and thereby influence macroeconomic outcomes.

Chapter 3: The optimal allocation of risks under prospect theory, or, do put your eggs in the same basket

This chapter applies prospect theory to a classical problem of the finance literature, namely the optimal allocation of the agent's stakes (e.g., wealth) among $n > 1$ identically distributed and symmetric sources of risk. In particular, the chapter assumes that the optimal selection of allocation weights among n risky payoff functions represents a framed prospect. Hence, our representative agent assigns the allocation weights so as to maximize the prospective value function defined for the problem.

The most striking result of this simple analysis is that, under the fairly general assumption that the subjective probability of obtaining a *perfect* hedge is negligible, the optimal allocation of risks by a prospect theory agent requires risk concentration, rather than risk diversification (as held in mainstream finance and economics). Thus, the prospect theory agent, instead of "not putting the eggs in the same basket" as it is taught in economics textbooks, will optimally "put the eggs in the same basket". The features of prospect theory crucially driving this result are loss aversion and in particular diminishing sensitivity to gains and losses, which makes the value function convex for losses. Under the assumption of diminishing sensitivity, agents are supposed to care more - compared, for instance, with a mean-variance specification of preferences - about small shocks with high probability than about large shocks with low probability. While risk diversification reduces the likelihood of events very distant from the reference point, it actually increases the noise in its neighborhood, which has an important negative bearing on agents' welfare under prospect theory. As a consequence, the fact that diversification averages existing downside risks is welfare-reducing for a prospect theory agent (whilst it is welfare-improving for an agent with a standard concave utility function). This explains why risk concentration, as opposed to risk diversification popular in the finance literature, turns out to be optimal in the risk allocation problem, provided that the subjective probability of obtaining a perfect hedge is negligible.

A reluctance to diversify risks, resulting from reference dependence and diminishing sensitivity, is a feature which could have importance macroeconomic consequences. There is a large body of evidence showing that agents tend to refrain from diversification in a variety of contexts important for economics, and that, when they diversify, they do so in a naive manner. So, the idea that risk concentration might be optimal, at least in some contexts, for rational and risk averse agents, is broadly consistent with the available evidence. Moreover, the finding that refraining from diversification might reflect an essential feature of human preferences under risk might in turn help to explain why agents (as observed) do not seem to have, in a variety of situations, the strong incentives to diversify risks that economics textbooks attribute to them. This is a tendency which has so far found no convincing or sufficiently general explanation in the literature. Probably the most famous example of lack of risk diversification is the observed "home bias" in financial investment, which has been widely documented, and yet found no convincing explanation. Against this background, it might be argued that prospect theory might contribute, together with other factors, to solve the "home bias puzzle", which is one of the most enduring in the international finance literature.

Chapter 4: Loss aversion, disappointment aversion, and the equity premium puzzle

This chapter[2] takes a close look at the "behavioral finance" explanations of the equity premium puzzle, namely myopic loss aversion (Benartzi and Thaler, 1995) and disappointment aversion (Ang, Bekaert and Liu, 2000). It is noted that both theories are built on the assumption that preferences are formed against a reference point which gains salience for agents, an idea introduced most forcefully by Kahneman and Tversky (1979) and which

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2 This chapter has been prepared under the close supervision of David Fielding, which has resulted in a co-authored journal submission.

features prominently in prospect theory. Building on these theories, this chapter proposes a simple specification of preferences, which is able to capture the main idea behind loss and disappointment aversion and to highlighting the differences between the two approaches, the most important being the way the reference point is determined. Moreover, we bring these theories to the data with a view to understand the relationship between the degree of loss and disappointment aversion and the investment *time horizon*.

The main conclusion of the chapter is that a highly short-sighted investment horizon is required for the historical equity premium to be explained by loss aversion, while reasonable values for disappointment aversion are found also for long investment horizons. So, stocks may lose only in the short term, but may disappoint also in the long term.

Against the background of these results, this chapter also offers some speculations on which one of the two proposed "behavioral" explanations of the equity premium has to be preferred. Benartzi and Thaler (1995) put forward the idea that institutional factors and principal-agent relationships might lead to myopic loss aversion, but there are also arguments in favour of disappointment aversion based on similar grounds, as argued in this chapter. One intriguing possibility is that the two approaches are not alternative, and that a high equity premium can be explained by *both* myopic loss aversion at short horizons and disappointment aversion at longer horizons. This would suggest that the reference point might evolve according to the time horizon. This is an interesting possibility which we leave to further research.

The main conclusions of the dissertation, and suggestions for

further research

In short, the main message arising from this dissertation is that psychological factors have to be taken seriously and can affect macroeconomic outcomes. Therefore, we should be wary of relying exclusively on expected utility models if we want to understand macroeconomic behavior. In particular, the different chapters have identified the following channels through which agents' psychology can affect the macro-economy:

· In **Chapter 1**, I review the behavioral finance literature to see whether the "anomalies" that this literature has identified could affect market prices. I find many examples in which it could be the case. This is particularly true for anomalies which are widespread among economic agents, for instance because of a social basis.

· In **Chapter 2**, I look at how a central bank which is characterized by reference dependence and diminishing sensitivity with respect to deviations of inflation from its target, and which weighs probabilities of uncertain events non-linearly. I find that it can behave in a way which is different from what standard models suggest, in particular by violating the certainly equivalence principle. As monetary policy can certainly affect aggregate developments and prices, this seems to be an important channel through which behavioral factors can influence the macro-economy.

· In **Chapter 3**, I see how it is optimal, under quite general conditions, for a prospect theory agent to concentrate, rather than diversify risks. This could contribute to explain, for instance, the "home bias" in international financial investment.

· Finally, in **Chapter 4**, I show how loss and disappointment aversion may explain the equity premium puzzle if different assumptions are maintained on the representative agent's investment time horizon and reference point.

In a way, the common thread of these results is the importance of *reference dependence* and its closely related property of *diminishing sensitivity* in agents' psychology on economic outcomes. Mainstream economists may generally find it odd that agents' preference may *change* according to the circumstances and to the economic problem at hand, but experimental evidence, psychological research and common sense all suggest otherwise. For example, a central banker who is entrusted by society with the task of keeping inflation at a certain target will structure his preferences around this target, and not in abstract terms (Chapter 2). An agent facing the problem of how to allocate risks may structure his preferences around a reference point, which may be the status quo (Chapter 3). Finally, it is plausible that an agent who has to decide his portfolio allocation between a risky equity and a safe Treasury bill will also form a reference point around which to build his preferences (Chapter 4).

Looking ahead, the concept of reference dependence might have potentially many useful applications in studying economics problems, and there is already a large body of literature applying reference dependence and prospect theory for example in labour and financial market contexts. Clearly, a firm understanding of *both* agents' psychology and the nature of the economic problem under consideration are needed for reference dependence to be usefully incorporated in macroeconomic models. To my assessment, the profession would greatly benefit from re-thinking about how agents' preferences are formed rather than a-critically assuming a particular and restrictive definition of preferences given by the expected utility theory. It is important to point out that allowing for reference dependence is not the same as simply devising preferences in an ad hoc and ”reverse engineering” way, as suspected for example by Zin (2002). Indeed, reference dependence does not appear to be inconsistent with the concept of rationality as it is normally intended in a broader sense by economists.

To my view, one important direction of future research is whether reference dependence can be tested empirically, which implies that testable conditions would have to be developed and brought to the data. There is already encouraging evidence that reference dependence is a reality in many contexts and that it can be found in the data. For example, the downward nominal rigidity of wages widely documented in the literature is, after all, nothing different from reference dependence in wage bargaining. I believe that this kind of phenomena can be found in many other contexts and problems, and this might turn out to be a useful avenue for research in the future.

Chapter 1
Behavioral finance and market prices: where do we stand?

"A conventional valuation which is established as the outcome of the mass psychology of a large number of ignorant individuals is liable to change violently as a result of the sudden fluctuation of opinion due to factors which do not really make much difference to the prospective yield; since there will be no strong roots of conviction to hold it steady", Keynes (1936), *General Theory*, ch. 12, p. 154.

"Speculative excesses, referred to concisely as a mania, and revulsion from such excess in the form of a crisis, crash, or panic can be shown to be, if not inevitable, at least historically common.", Kindleberger (1978), p. 2.

"A drunk walking through a field can create a random walk, despite the fact that no one would call his choice of direction rational", Thaler (1999b), p. 14.

1.1 Introduction

Behavioral economics and finance is one of the most dynamic and promising fields of economic research by its scope and size. There is an increasingly long list of phenomena which, while inexplicable with the standard tools and approaches of mainstream economics, have found a satisfactory explanation in behavioral economics and finance.[3] Nonetheless, it is far from being a foregone conclusion that the behavioral methodology will come to dominate economic research and completely supplant the mainstream approach based on expected utility maximization and rationality, and opposing views have been expressed in this respect (in the behavioral camp, see Thaler, 2000, and Colisk, 1996; on the mainstream side, see for example Fama, 1998, and Rubinstein, 2000).

[3] The increasing popularity of behavioural economics and finance is confirmed by the attribution of the 2002 Nobel prize to Daniel Kahneman.

This chapter selectively touches upon recent contributions in the behavioral finance literature. The objective of this review is to provide an answer to two key issues, namely:

-What are the most important and systematic behavioral biases which characterize economic agents that we know of?

-Are they relevant to explain *aggregate* market behavior, namely do they affect aggregate prices and competitive markets?

Behavioral finance rejects a vision of economic agents' behavior based on the maximization of expected utility. At the root of this rejection is the overwhelming evidence available that agents, both in controlled experiments and in real life situations, behave in a way so as to violate the axioms of expected utility (Starmer, 2000). It should be emphasized that the focus of behavioral finance is on a *positive* description of human behavior especially under risk and uncertainty, rather than on a *normative* analysis of behavior which is more typical of the mainstream approach.

One of the key objectives of behavioral finance is to understand systematic market implications of agents' psychological traits. The stress on the market implications is very important because the analysis of large, competitive markets with a low level of strategic interaction is at the heart of economics (Mas-Colell, 1999). So far, the behavioral finance literature has not reached a level of maturity which would allow it to provide a coherent, unified theory of human behavior in market contexts in the same way expected utility and mainstream economics and finance have done.[4] Nevertheless, cumulative prospect theory

[4] Frankfurter and McGoun (2002) are sceptical about the role of behavioural finance as a paradigm alternative to the mainstream approach. The sole purpose of behavioural finance, they claim, is "to discredit" the standard approach based on the efficient market hypothesis.

introduced by Tversky and Kahneman (1992) is approaching a point where it can represent a unified theory of behavior of agents under risk which is alternative, and possibly (in some contexts) superior, to expected utility.

This chapter will be structured as follows. The next section will provide a framework of analysis which will serve as a basis for categorizing and interpreting the contributions in the literature. Section 1.3 provides the reader with a bird's eye view of the main "anomalies" identified in the behavioral finance literature. Section 1.4 summarizes and puts into perspective the debate between mainstream and behavioral finance theorists on the rationality (or lack thereof) of the market. Finally, Section 1.5 contains some suggestions for further research and some concluding remarks.

1.2 Rational pricing and anomalies: a framework of analysis

This section proposes a framework of analysis for the remainder of this chapter by first describing briefly and in a simplified way the standard, mainstream approach to asset pricing in an intertemporal setting, based on expected utility maximization and rational expectations, and then discussing the role of asset price bubbles and behavioral biases in this context. Against the background of this framework of analysis, the role of the anomalies and the challenge that they pose to the standard approach are then dealt with in the subsequent section. While I restrict my analysis to intertemporal asset pricing, the essential elements remain valid for the more general pricing of Arrow-Debreu securities (i.e., securities the payoff of which depends on the realization of an unknown state of nature).

The standard approach makes few assumptions about agents' psychology, and this should be considered as one of its strengths. It assumes that agents derive utility from consumption, normally in a time separable manner, and that the marginal utility of consumption is diminishing, so that the utility function is concave. At time t, agents are typically assumed to maximize:

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} U(c_s), \tag{1.1}$$

where $U(c_s)$ is the instantaneous utility of consumption at time s, and β is a constant discount factor. If future consumption is unknown, agents maximize the expectation of (1.1):

$$Max E_t \sum_{s=t}^{\infty} \beta^{s-t} U(c_s), \tag{1.2}$$

using either the "objective" (or "true") probability distribution for c_s, which they are assumed to know (expected utility, EU), or subjective probabilities (*subjective* expected utility, SEU). Agents may be heterogeneous in initial wealth and preferences, but heterogeneity does not matter as long as financial markets are complete (Constantinides, 1982), so we can think in terms of a representative agent in the continuation of this analysis.

For simplicity of exposition, we will refer to an exchange economy. As shown by Lucas (1978), the maximization of (1.2) leads to the following equilibrium condition:

$$E_t \beta \frac{U'(c_{t+1})}{U'(c_t)} R_t^i = 1, \tag{1.3}$$

where U' is the marginal utility of consumption, R_t^i is the one-period return on asset i, i.e. $R_t^i = \frac{P_{t+1}^i + D_t^i}{P_t^i}$, where P_t^i is the real price level of asset i and D_{t+1}^i its real payoff.

Another hallmark of the standard approach is that agents have *rational expectations*, i.e. they do not make systematic mistakes, and know the "true" probability distribution of p and d. This implies that if Ω_t is the information set available to the representative agent at time t, then:

$$x_{t+1} - E_t(x_{t+1}/\Omega_t) = \varepsilon_{t+1}, \tag{1.4}$$

where x is a stochastic variable and ε_{t+1} is i.i.d.. Overall, equations (1.3) and (1.4) characterize "rational" asset pricing in the standard approach and define what is commonly known as the *efficient market hypothesis*. Economic agents look at asset prices in the light of what matters for them (the marginal utility of consumption) – as equation (1.3) suggests – and process information in an optimal manner, in particular by not doing systematic (avoidable) mistakes – as equation (1.4) indicates.[5]

It is worth emphasizing that the equilibrium condition in (1.3) pins down the *required real return* on asset i, but not its *price level*, which remains generally indeterminate.[6] Taking a log-linear approximation (Campbell and Shiller, 1988):

$$r = k + \alpha E_t p^i_{t+1} - p^i_t + (1 - \alpha) E_t d^i_{t+1}, \tag{1.5}$$

where, generally, $0 < \alpha < 1$, k is a simple transformation of α and lowercase letters denote logs, it can be seen that if a shock hits the future expected price level at time $t+1$, it

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[5] It should be emphasised that rationality in the expected utility sense is made of *both* equations (1.3) and (1.4). There is a tendency in the literature to identify rationality in the expected utility sense with the absence of arbitrage opportunities. While the condition (1.4) indeed implies no arbitrage opportunity identifiable ex ante, rationality is a richer concept that includes also the marginal utility of consumption which appears in equation (1.3).

[6] As noted by Sunder (1995), the no-arbitrage condition typical in mainstream finance models implies the efficiency of price *changes*, but not necessarily of price *levels*.

is bound to affect the current price as well. *Only* if the transversality condition holds that:

$$\lim_{s \to \infty} E_t P^i_{t+s} = 0, \tag{1.6}$$

and agents have an infinite time horizon and unlimited resources, will the price of asset i exclusively depend on the (rational) expectation of future dividends, which is normally referred to as the *fundamental* or *no-bubble* solution. Under the more realistic assumption that agents have a finite horizon or limited resources, condition (1.6) will not be binding and "rational" pricing does not prevent the possibility of asset price bubbles. So, equations (1.3) and (1.4) do not rule out *sunspot fluctuations* in asset prices (Blanchard, 1979), i.e. which do not depend on economic fundamentals. In other words, a shock to the future expected asset price level unrelated to the fundamentals is bound to affect the current level of asset prices, at least for some time. So, a financial asset which is traded on a market, when far from maturity, it is generally a claim *both* on the fundamentals and on market beliefs, however the latter are formed. Therefore, if a psychological bias of the representative agent drives the asset price away from its no-bubble solution, there is generally no way, at least in the short and medium term, for the market price to be corrected back to the fundamental solution. The bottom line is that the standard approach based on expected utility maximization and rational expectations *cannot explain* why asset price bubbles arise in the first place, but it *cannot rule them out* either, in general.

The importance of future expected prices for current prices has been emphasized in particular in the literature on "extrinsic" or "endogenous" uncertainty, namely the uncertainty surrounding future market beliefs (Cass and Shell, 1983), as opposed to "intrinsic" or "exogenous" uncertainty which is related to future fundamentals. More recently, the theory of *rational beliefs* (Kurz, 1994; Kurz and Motolese, 2001) has focused on endogenous uncertainty and self-justifying expectations as the main source of uncertainty in the market.

Another important issue is whether "endogenous" market fluctuations can also *themselves* drive fundamentals, resulting in *self-fulfilling* fluctuations driven by arbitrary market beliefs (Evans, 1989). Assume, for instance, that:

$$d^i_{t+1} = \theta p^i_{t+1} + z^i_{t+1}, \tag{1.7}$$

where $\theta > 0$ and z is a purely exogenous component of dividends. Then, recalling (1.5):

$$r = k + (\alpha + (1-\alpha)\theta) E_t p^i_{t+1} - p^i_t + (1-\alpha) E_t z^i_{t+1} \tag{1.8}$$

If θ is large enough, it is possible that $\alpha + (1-\alpha)\theta > 1$ and that sunspot fluctuations drive the asset price on an explosive path. Such *feedback* of market prices on dividends is not unrealistic, at least for some assets and markets. For example, an endogenous increase in market confidence about the firm i can push the price of stock i higher and, at the same time, channel enthusiasm to the firm's products, self-justifying the initial market movement. Another plausible situation is one in which managed dividends depend on stock prices.[7] In the exchange market, a plausible example is a speculative attack on a certain currency which leads to a devaluation, then to a financial crisis (for example because debt is denominated in foreign currency), eventually self-fulfilling the expectations which initially moved the exchange rate. Clearly, the extent to which a feedback mechanism can exist depends on the nature of the considered asset. An asset which has a payoff related to an

[7] This is close in spirit to, although it should be distinguished from, the "intrinsic bubbles" of Froot and Obstfeld (1991), namely bubbles which affect only the fundamentals.

exogenous event like, say, that it rains on a certain day cannot be affected by any feedback mechanism.

To sum up, the standard approach to asset pricing based on expected utility maximization is able to identify a fundamental solution, which pins down the asset price level and has certain desirable properties from an economic perspective (for example, that assets with higher future dividends will have a higher price). At the same time, the standard approach cannot rule out, in general, the possibility of endogenous market fluctuations unrelated to news on fundamentals, the importance of which depends crucially on two factors, namely (i) the remaining time to maturity of the asset and (ii) the importance of the feedback mechanism.

In the next section, we will take a closer look at the behavioral factors which may determine endogenous market fluctuations. As we have seen, the standard approach is not able to rule out such fluctuations, but it requires that they are impossible to predict and to systematically make money out of them, lest they violate the rational expectations assumption in (1.4).

1.3 A bird's eye view of the anomalies

We define *anomalies* the systematic traits of behavior of economic agents, which cannot be explained by the expected utility model.[8] The list of anomalies identified in the behavioral finance literature, especially based on experimental evidence, is very long and only the

[8] I do not use the term "anomalies" to trivialise them, but to indicate phenomena which represent an important challenge to the mainstream approach based on the efficient markets hypothesis. On the possibly derogatory use of the term "anomaly", see Frankfurter and McGoun (2001).

main ones will be touched upon in this section. The stress on the *systematic* nature of these biases is crucial, as every sufficiently general theory in social sciences should be allowed to make mistakes, expected utility not excluded (Rubinstein, 2000). Moreover, what matters for aggregate market prices is the behavior of the representative agent, so we do not have to care, in principle, about behavioral biases that cancel out in the aggregate.

It may be useful to note at this point that the idea that psychological factors might be relevant for market prices and economic developments is not a prerogative of behavioral economics and finance. Indeed, this idea has a distinguished past going back at least to Keynes and to his emphasis on "animal spirits" and the role of uncertainty and confidence in shaping economic and employment growth. In the Keynesian view, economic agents' psychology can be easily disturbed and manipulated. Psychology is a key element of the economic system, in contrast with the emphasis on rationality which is typical of the (now) mainstream approach. So, it might be argued that the focus of behavioral finance on psychological factors ultimately represents a vindication of Keynesian ideas.[9] A classic, informal description of the "manias and panics" in financial markets, closely related to this Keynesian tradition, is Kindleberger (1978). The behavioral finance literature, however, contains some important innovative elements compared with the Keynesian approach, namely the stronger focus on experimental – and in general empirical – evidence and the larger use of formal models, which may lead to sharper predictions. So, one might conclude that while behavioral finance is close in spirit to the Keynesian tradition, it makes use of a different methodology and analytical framework.

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[9] See Harvey (1998) on the relationship between modern economic psychology and the Keynesian emphasis on uncertainty and non-ergodicity in economic life.

Moving closer to the anomalies, the presentation will be structured according to five separate categories – bearing in mind that this taxonomy is arbitrary, that many other categorizations are possible, and that there may be considerable overlaps among the categories. The categories considered in this chapter are as follows:

1. *Decision heuristics*: the representative agent does not (and cannot) solve the (complex) problem in (1.1) or (1.2), mainly because of deliberation and optimization costs (Colisk, 1996). It therefore makes use of shortcuts and simple rules of thumb in making decisions.

2. *Emotions and visceral factors* (Loewenstein, 2000) which interfere in decisions.

3. *Choice bracketing*: in addition to making recourse to decision heuristics, agents tend to frame decision problems more narrowly than in (1.1) and (1.2) (Read, Loewenstein and Rabin, 1999). For example, agents normally have a shorter time horizon than their lifetime in their decisions.

4. *Unknown preferences*: recent contributions claim that a set of well-defined and deterministic preferences, as the utility function which is maximized in (1.1) and (1.2), does not exist altogether. Rather, stochastic and context-dependent preferences should be considered.

5. *Reference dependent models*: agents' preferences for consumption and other variables (including risk) do not seem to be defined in abstract and general terms as in the standard approach, but rather depend on "reference points". So, the utility function is not defined simply over c_t, but rather on $c_t - z_t$, where z is a reference point for the represen-

tative agent. One prominent example of a model taking into account reference dependence is *prospect theory* by Kahneman and Tversky (1979).

To date, there is no comprehensive behavioral finance model which is able to take into account all these anomalies and be analytically tractable (Shleifer, 2000), although prospect theory seems to have become an important alternative to expected utility as a model of human behavior under risk.

1.3.1 Decision heuristics

Standard economics and finance models overlook the importance of deliberation / optimization costs and assume that agents possess extremely high computational capabilities (Colisk, 1996). In reality, agents make often recourse to mental shortcuts and "rules of thumb" when the problem to solve is particularly complex and far-reaching. This reflects deliberation costs as well as limited information processing capabilities, namely bounded rationality (Simon, 1986; Williamson, 1997). The shortcuts agents use are known in the behavioral finance literature as *decision heuristics* (Kahneman and Tversky, 1974). Such heuristics may lead to poor decision outcomes and involve "blunders" which might be eliminated with a more "rational" analysis (i.e., an analysis where less weight is attributed to optimization costs). The behavioral finance literature has identified a large number of systematic blunders that plague economic agents, and we will touch upon only a few, namely (i) the mis-perception of the laws of probability, (ii) the representativeness bias and anchoring effects, (iii) limited attention and saliency, (iv) credulity. Moreover, we also touch upon ambiguity aversion.

A very common blunder is to mis-perceive the laws of probability, for example by systematically over-inferring from small samples ("law of small numbers") and underrate the importance of population parameters. Framed in the context of the Bayes formula, agents tend systematically to overvalue the sample evidence and systematically undervalue the a priori probabilities. This tendency may have an aggregate market implication if agents mis-perceive fluctuations in prices which are simply due to chance with a reversion to a mean (Rabin, 2002a). For example, excessive extrapolation from the past performance may be the reason why superior returns are earned by portfolios based on publicly available data (Lakonishok, Shleifer, and Vishny, 1994).

More generally, decision heuristics may be influenced by factors such as *vividness* and *representativeness*, which should have little to do with an optimal decision. One such factor is the *anchoring* to representative values which make it easier for agents to solve decision problems even when, if looked at carefully, they should not have the influence they actually have. A prominent example of this tendency is the fact that in most speculative markets the prevailing price is often regarded as a "normal" or "equilibrium" price level, even if agents have no idea of what an "equilibrium" or "fair" price might be (Mullainathan and Thaler, 2000) and future developments show that the market price was plainly wrong. The same might be said of many quantities (for example, the price of any good or service vis-à-vis any other good or service), where the *status quo* is automatically taken as a "natural" value – the computation of a truly natural value would in fact involve excessively high deliberation costs.[10]

[10] A tendency to *hindsight bias* – i.e., the false perception that once an event is part of history, there is a tendency to interpret the sequence as unavoidable – may be justified on similar gounds (Kelman, Fallas and

A key element of bounded rationality models is also *limited attention*. Agents are confronted with a confusing array of (sometimes conflicting) information, which encourages them to focus only on *salient* information (Shiller, 2001).[11] This makes the average human being (the average investor) particularly subject to fads (Shiller, 2000b) and to manipulation by others (Daniel et al, 2002). At the same time, agents take time (due to limited processing capability) to digest new information, even when it is actually relevant, which may lead to *conservatism bias*. Barberis, Shleifer and Vishny (1998) have developed a model in which agents react in an exaggerated manner to new information due to representativeness bias, while the overreaction is tempered by conservatism.[12] As stressed by Shiller (1994, 1998, 2000a, 2001), attention and saliency may have a *social basis*, which is the reason why past price increases may attract attention to a certain financial asset and determine a self-fulfilling spiral of rising price and increased optimism.

Lack of attention may also lead to investor *credulity* (Daniel et al, 2002), where – owing to limited computational capabilities – agent do not adequately account for the incentives of others in manipulating and presenting information. For example, it has been documented that firms tend to present positive information in a salient way, while they normally report negative information in a highly non-salient manner, but investors do not

Folger, 1998). On hindsight bias in forecasting, see for example Fisher and Statman (2000).

[11] On the role of salient information and the irrelevance of a "rational" weighing of events and probabilities, see Shafir and Tversky (1993).

[12] It should be noted that the co-existence of conservatism and the "law of the small numbers" does not imply that agents are Bayesian on average (Camerer, 1995). In fact, the conditions under which agents are over- or under-responsive to sample evidence are predictable. In general, both the base rates and the sample evidence are overweighted if they are salient and highly representative, while they are underweighted if they are pallid and difficult to understand (Griffin and Tversky, 1992).

seem to take this factor into account (Klibanoff et al, 1999). In general, the way information is presented matters.[13]

Finally, the application of the standard expected utility maximization to real world problems is further complicated by the observation that "objective" probabilities are rarely known to decision-makers. The decision problem then becomes the "maximization over a probability distribution of the probability distribution", and so on again ad infinitum. There is evidence that agents dislike "ambiguous" situations (i.e., situations in which there the probability distribution is unknown) more than "risky" situations (where at least the probability distribution of the event is known). So, the distinction between clear and vague probabilities matters and agents are normally willing to pay to avoid ambiguity.[14] Camerer and Weber (1992) provided a very good review of the literature on *ambiguity aversion*.[15] Heath and Tversky (1991) have found that the aversion to ambiguity is particularly strong if agents feel that knowable information is missing, suggesting that ambiguity aversion is driven by feelings of incompetence (agents prefer to bet on events on which they feel competent about, and shy away from bets they feel to have little knowledge about). Related to this, Fox and Tversky (1995) showed that ambiguity aversion arises when agents evaluate clear and vague prospect jointly, but not when they evaluate prospects individually (*comparative ignorance* hypothesis).

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13 For instance, when attention and processing capabilities are limited disclosing information may actually turn out to be counterproductive and decrease transparency (Daniel et al, 2002, put it as "investors can lose the forest for the trees").

14 In SEU, the distinction between known and unknown probabilities is not relevant, because subjective probabilities are always known. So, SEU implicitly assumes that agents are indifferent to the "risk of having the wrong belief" (Camerer and Weber, 1992).

15 The Keynesian definition of uncertainty and the related emphasis on confidence fit very well in this strand of literature. The Keynesian tradition sees aversion to ambiguity and confidence as having a major impact on market prices and on economic developments.

Overall, cognitive biases might affect asset prices to the extent that agents who demand a certain asset are incapable of processing the information underlying a rational pricing of that asset, i.e. to form the "correct" expectation $E_t(p^i_{t+1}, d^i_{t+1}/\Omega_t)$ based on the full information set available to them. If the cognitive biases are sufficiently systematic (e.g., the tendency not to discount for "window-dressing" firms' balance sheets), the market as a whole might be subject to biases.

It has been proposed that expected utility maximization might be amended, without changing its fundamental nature, by adding a deliberation cost to the utility function, and then proceed as in the standard approach (Colisk, 1996).[16] This way of casting bounded rationality in the standard approach, however, might be problematic for three reasons. First, it is unclear what precise form these deliberation costs should have. Second, even assuming that giving a determinate form to the deliberation costs may be possible, a problem of "infinite regress" may arise. If agents have deliberation costs, then they will also have deliberation costs in assessing their deliberation costs, and thus deliberation costs on the deliberation costs on the deliberation costs, and so on ad infinitum. While a practical solution might be to stop at the first deliberation cost and neglect higher order terms, this solution might be unsatisfactory. Third, there is the problem of specifying ex ante what the benefit of the deliberation would be.

[16] For a thorough review of how to model bounded rationality, see Lipman (1995).

1.3.2 Emotional and visceral factors

Emotional and visceral factors play an important role in individual decision-making (Loewenstein, 2000; Romer, 2000). A quite famous example is the evidence that the weather in the trading location and the time of the day influence equity prices (Saunders, 1993; Kamstra et al, 2000), presumably by affecting traders' emotional state.[17] The role of emotions may be particularly important in situations of risk and uncertainty, which are pervasive in finance (Loewenstein et al, 2001). A feature of expected utility is, instead, that agents face risk and uncertainty from a purely cognitive perspective, and their emotional state does not influence their decisions altogether, as we have seen in Section 1.2. In reality, emotional responses are ubiquitous and may depart significantly, sometimes dramatically, from cognitive responses. In general, factors such as vividness and proximity in time play a big role in emotional responses, while they should be irrelevant in purely cognitive decision processes.

Some anomalies related to emotional states are based on a trade-off between the need of the situation (i.e., making optimal decisions in a forward-looking manner) and the necessity to protect self-esteem and confidence as well as the emotional well-being. One anomaly relevant in a financial market context is the *disposition effect*, namely the reluctance to "declare" losses to oneself (fearing a loss of self-esteem), which pushes agents to hold losing assets too long (Shefrin and Statman, 1985; Odean, 1998a). A similar need to protect self-esteem may lead agents to *belief perseverance* and *confirmatory bias*: as there is an emotional cost associated to the recognition of having been wrong, agents tend

[17] However, the evidence that the weather influences trading and financial prices is not uncontroversial. For a criticism of the "weather effect", see Goetzmann and Zhu (2002).

to look for additional support for initial hypotheses (Rabin and Schrag, 1999) and to exaggerate correlations which might be due to chance, interpreting them in the light of a preconceived theory.[18] This form of *cognitive dissonance*[19] is sometimes labelled as the "curse of knowledge" (Thaler, 2000): when we know something, we cannot imagine ever thinking otherwise. Self-esteem may also lead to *overconfidence*, as agents draw some emotional gains from the perception of being smarter than others. Thus, the idea that people learn from past mistakes – a hallmark of the rational expectations school based on learning and evolutionary reasons (see Section 1.4 below) – may be doubted if learning implies a painful loss of self-esteem and the recognition not to be smarter than others (Griffin and Tversky, 1992). This form of self-enhancing bias may explain, for example, why trading is so large in financial markets: most market participants might think they are smarter than the average counterpart, and can make money from the folly of others (De Bondt and Thaler, 1994; Odean, 1998b and 2000). Of course, many of them are going to be disappointed (and to lose money due to transaction costs), but – again for the sake of their self-esteem – will attribute the disappointing outcome just to bad luck ("nature is against me") or malice from the part of others (this is, however, unlikely in a competitive market). Moreover, overconfidence may determine positive short-lag autocorrelations and negative long-lag autocorrelations, which are often observed in the data.[20] In this respect, it may af-

[18] The "law of small numbers" mentioned above might be partly related to these tendencies; again bounded rationality and emotions are closely connected.

[19] Cognitive dissonance may be defined as the bias of "fitting beliefs to convenience" (Rabin, 1994).

[20] Daniel, Hishleifer and Subrahmanyam (1998) and Hong and Stein (1999) have built models based on the assumption of traders' overconfidence in their private information, which leads to a (overconfident) misvaluation and, from an aggregate perspective, to both short-run momentum and long-run reversal. Statman and Thorley (1999) posit, and find empirical confirmation of the fact, that in a bull market, where the overconfidence of most investors is high, trading increases.

fect aggregate market prices. Above all, as emphasized by Odean (1998a), overconfidence leads agents to react in a distorted manner to information. In particular, information which is abstract, statistical, and difficult to interpret is generally dismissed; information which is salient, anecdotal and easy to interpret is overvalued.

One particularly important consequence of the fact that a decision may be emotionally loaded is agents' *weighing of probabilities*. The idea that agents weigh states according to objective or subjective probabilities in a linear manner is an essential feature of the expected utility theory, but it has been proved wrong in countless experiments, starting with the famous Allais paradox. In reality, agents seem to weigh objective probabilities subjectively, computing what is often referred to as the *subjective* expected value.[21] The probability weighing function may in turn depend to a significant extent on the agents' emotional state (Loewenstein et al, 2001), especially on whether events are "pallid" or "vivid" in agents' perception. For instance, Kahneman and Tversky (1979) noted that movements in probabilities around zero and one are normally given much more importance than movement between, say, .49 and .50, precisely because of vividness considerations (this is related to Allais' *certainty effect*). In general, the probability weighing function tends to be flatter (i.e., changes in probabilities count less when probabilities are high) for vivid outcomes, while it approaches the linear weighing for pallid outcomes (namely, events that do not prompt an emotional response by agents). Thus, a change from 0 to 0.01 in the probability, say, to die in a certain year (a very vivid and emotionally loaded outcome) may count much

[21] In principle, SEU is not inconsistent with this finding because it assumes that probabilities are always subjective. However, the subjective weighing of probabilities identified in this literature generally leads to sub-additive weighing, which is inconsistent with SEU.

more than a change from, say, .30 to .31, while the same .01 marginal change in probabilities would be weighted in the same manner if referred to, say, a change in government in a distant foreign country (a very pallid outcome).

Much experimental evidence has been gathered in the last decade on the functional form of the probability weighing function, say $w(p)$, and it has been generally found that such function is normally *sub-additive* (it integrates to a number strictly smaller than one), *regressive* ($w(p) > p$ for small p, and the opposite for high p) and *s-shaped* (first concave for small p, then convex).[22] Therefore, in most contexts small probabilities tend to over-weighted, while large probabilities tend to be under-weighted compared with the linear case. However, for very small probabilities, the function becomes indeterminate and both an over-weighting and an under-weighting are possible.[23]

Tversky and Kahneman (1992) and Prelec (1998), among others, have proposed quite general functional forms in which the degree of regressivity and s-shapeness depends on a single parameter or a small set of parameters. More research is needed, however, to assess to what extent the nature of a decision problem and its being emotionally loaded influence the parameters of the chosen probability weighing function.

One of the central tenets of expected utility is that "bygones are bygones" and utility maximization is always carried out in a forward-looking manner, where past experiences and risks taken do not matter at all (indeed, the maximization of utility in (1.1) and (1.2) is

[22] See in particular Tversky and Kahneman (1992), Tversky and Wakker (1995), and Prelec (1998). Wu and Gonzalez (1996) showed that the probability weighing function is nonlinear also away from the boundaries, i.e. from 0 and 1, suggesting that non-linearity is not only due to the certainty effect.

[23] In some cases very small probabilities are neglected altogher, so the decision problem is examined without regard to very unlikely events.

exclusively forward-looking). Conversely, the behavioral finance literature has identified a number of situations in which past developments and experiences do matter in determining agents' preferences and therefore their decisions.[24] For instance, the *endowment effect* (Kahneman, Knetsch and Thaler, 1991) postulates that the dis-utility of giving up an object (or an achievement, and so on) is greater than the utility of acquiring it. Therefore, agents' optimization not only concern utility from, say, wealth, but also utility of wealth vis-à-vis the status quo (by definition a backward-looking concept). In the same vein, risks born in the past may affect current decisions (Machina, 1989). The *"house money" effect* (Thaler and Johnson, 1990) suggests that agents are more risk averse following a loss, and more risk-loving (or less risk-averse) after a gain.[25] The behavioral explanation of such phenomenon is that when agents suffer a pain deriving from a loss, they have less "emotional reserves" to tolerate further losses, while they can "stockpile" a cushion of emotional strength after a gain.[26] The "house money" effect can affect aggregate market prices. For example, Barberis, Huang and Santos (2001) show that the house money effect, together with loss aversion (see Section 1.3.5 below) can explain both the equity premium puzzle and the predictability of equity returns at low frequency, phenomena that are difficult – albeit not impossible – to explain with the standard approach. *Regret theory* (Loomes and

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24 The importance of backward-looking considerations has been recently recognised also in mainstream finance and economics with the recent emphasis on *habit formation* (see for example Chapman, 1998, and Messinis, 1999).

25 The term "house money" comes from casino games.

26 By contrast, Gomes (2000) proposed a model in which investors are more willing to take risks after a loss, while being more conservative after a gain. After a loss, agents are willing to "gamble for resurrection", while after a gain, they want to protect their achievement. Thus, investors tend to sell winners and to hold on to losers, consistent with the disposition effect. According to Gomes (2000), heterogeneity in risk attitudes due to past history of investors (i.e., whether they have previously experienced gains or losses) can also explain trading in financial markets.

Sugden, 1982) and *disappointment aversion* (Gul, 1991) are both based on the idea that agents value the emotional cost of having taken a wrong decision (regret aversion) or of having their expectations disappointed (disappointment aversion).[27] The relevance of *sunk costs* (Thaler, 1991) is also related to this attitude: sometimes we think that we have "too much invested to quit", and this might lead to excessive risk-taking and, more in general, to sub-optimal choices (the relevance of sunk costs increases, of course, with the emotional investment associated to these costs).

Finally, *moral feelings* might also influence preferences and behavior. For instance, the role of feelings of reciprocation (when positive) and retaliation (when negative) have been studied in game theory contexts (Kahneman, Knetsch and Thaler, 1986).[28] However, while it is clear that such feelings may influence trading in strategic contexts with a low number of agents, it is doubtful that they might be relevant in the context of a market with a large number of participants and a low level of strategic interaction. The very tendency of agents to the representativeness heuristic (see above) – namely to consider the current market prices as "fair" – is likely to keep moral feelings out of the marketplace.[29] The same probably holds true for "caring about the others" (Rabin, 2002b).

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[27] Ang, Bekaert and Liu (2000) use disappointment theory to solve the puzzle of why agents find stocks disappointing but buy lottery tickets. Returns on stocks are likely to disappoint investors precisely because they have a positive expected value, which feeds through to agents' expectations. Therefore, the probability of being disappointed by stocks is high. In lotteries, agents expect to lose money with virtual certainty and may only be positively surprised by the outcome. This mechanism would explain why lottery tickets are so much in demand.

[28] For a review of reciprocity in economics, see Fehr and Gächter (1998).

[29] This is not necessarily true in other contexts, for example the labour market (for an important application of the concept of reciprocity to explain downward nominal wage rigidity in the labour market, see Bewley, 1995).

Overall, it is plausible that some emotional factors play an important role in the setting of financial market prices. Especially the emotional states associated to "feeling good" or "feeling bad" may be important to the extent that they influence *expectations formation* (including the weighting of probabilities with which expectations are computed). In a very interesting analysis, Abel (2002) shows how pessimism and doubt – features related to agents' emotional states – may both increase the average equity premium and, in general, influence market prices. The analysis by Cecchetti, Lam and Mark (2000) also goes in the same direction.

1.3.3 Choice bracketing

A key feature of the expected utility approach, including its applications in mainstream finance, is the *invariance axiom*: agents' preferences and their choices are independent of how a decision problem is described or presented. Conversely, the behavioral finance literature has found a number of important cases in which the way a certain decision problem is presented matters (namely, the independence axiom does not hold). *Framing and elicitation effects* (Tversky and Thaler, 1990) permeate the behavioral finance literature, and *narrow framing* is in particular one of its milestones. Framing may be a relevant factor not only at individual level, but also at a macro level; for instance, Shaffr, Diamond and Tversky (1997) explain money illusion as the tendency to frame economic quantities in nominal terms, which happens at low levels of inflation, reflecting the existence of computational costs. Conversely, at high levels of inflation agents find it optimal to measure economic phenomena in real terms. The fact that the adjustment for inflation is sometimes done incorrectly and that the error is systematic (low inflation is considered to be zero inflation) leads to the conclusion that money illusion can indeed affect market prices (in particular, nominal and real interest rates might be distorted downwards).

Choice bracketing can be defined as "a series of local choices that each appear to be advantageous but which collectively lead to a bad global outcome" (Read, Loewenstein and Rabin, 1999) and it is closely related to narrow framing as introduced by Thaler (1980). Under choice bracketing / narrow framing, agents maximize utility *locally* (for a narrowly defined decision problem) in an optimal manner, but by doing so they may come to a disastrous *global* outcome (in terms of overall welfare).

A main form of narrow framing is *procrastination*. Under procrastination, agents act on the basis of rational calculations at intervals that are irrationally short. Thus, while they maximize their utility in the short-term, they may end up in very unsatisfactory and sub-optimal situations over a long horizon.[30] One everyday life example of this tendency is the decision of when to quit smoking: on a given day, the sacrifice to refrain from smoking will always be greater than the (negligible) utility in terms of better health *on the same day*. Yet, after running this optimization over and over for thousands of days and always – locally, in an optimal manner – choosing not to quit smoking, the long-term consequences for health can become catastrophic.[31] Another key more finance-related example is the decision whether to consume or to save. In the short-term, consumption may give more satisfaction and saving decisions might be postponed indefinitely ("I will start saving tomorrow"). This

[30] Unless, of course, the maximisation reflects a very high discount rate, in which case being focused on the short-term is a fully rational behaviour, i.e. compatible with expected utility maximisation.

[31] Deciding when to start a diet is, of course, another classic example.

kind of behavior – all too familiar in everyday life – signals that human patience is not independent of the horizon and that preferences are not time-consistent.[32] Akerlof (1991) referred to this tendency as *hyperbolic discounting*. Under hyperbolic discounting, agents' impatience is steeper for near-term trade-offs than for long-term trade-offs. By contrast, the constant time discounting in standard expected utility models has been found to be descriptively invalid in a number of circumstances (Fredrick, Loewenstein and O'Donoghue, 2002).

A convenient two-parameter approximation of hyperbolic discounting is the "quasi-hyperbolic" proposed by Laibson (1997):

$$\beta_h = \begin{matrix} 1, \text{ if } h = 0 \\ \beta\delta^h, \text{ if } h > 0 \end{matrix}, \tag{1.9}$$

where β_h is the discount factor h periods ahead, β and δ are real scalars, both strictly smaller than one. This formulation can account for declining discounting rates, as the discount factor from tomorrow to today, $\beta\delta$, is smaller than the discount factor from tomorrow onwards, δ (Laibson, 1997). This leads to dynamically inconsistent preferences (I will not do tomorrow what I now assume I will do).[33] These preferences may certainly be undesirable from a normative perspective (agents should take their future preferences into account in maximizing their lifetime utility), but they are descriptively ubiquitous, and can be in-

[32] O'Donoghue and Rabin (1999b) report the example that agents may pay not to anticipate a certain unpleasant task from tomorrow to today, but they are indifferent between one day in six months time and the day before. While this behaviour is intuitively natural, it is in contrast with the expected utility model based on constant discounting. Moreover, O'Donoghue and Rabin show that small quantities are normally discounted more heavily than large quantities, and losses more than gains.

[33] However, time-inconsistent preferences does not necessarily imply that agents behave in a time-inconsistent manner, if they are sophisticated enough to take into account their future preferences in today's decision (Caillaud and Jullien, 2000).

terpreted as a form of diminishing sensitivity to time.[34] A quite large body of literature is developing on procrastination and on ways to overcome it (O'Donoghue and Rabin, 1999a, 1999b and 2001, Brocas and Carrillo, 2000, and Fischer, 2001).

While in some limited instances narrow bracketing may be optimal (for example, looking at a certain unpleasant task "a piece at the time" may increase the agent's determination to carry it out, without being scared off), it generally leads to sub-optimal outcomes. The next natural question is thus why agents tend to frame their decision problems so narrowly and to neglect the correlations among different aspects or time horizons in their lives. Presumably, cognitive limitations and deliberation costs play a major role in explaining narrow framing (Read, Loewenstein and Rabin, 1999).

Is narrow bracketing relevant from an aggregate market perspective? There are some indications that it might be so. First, *undersaving* may be an important consequence of hyperbolic discounting, as stressed by Laibson (1997). The clearest sign of undersaving is the observation that most people reduce their consumption level significantly after retiring, a phenomenon which is impossible to explain with the expected utility model. Laibson (1997) suggests that time-inconsistent preferences might make commitment devices – such as a highly illiquid financial asset – worthwhile, while the greater flexibility brought about by liquidity and financial innovation might turn out to be harmful. In addition, choice bracketing may also influence the choice of the composition of agents' financial portfolio. Benartzi and Thaler (1995) provided what is by now one of the most convincing explanations of the equity premium puzzle of Mehra and Prescott (1985), by relating the high risk

34 See Section 1.3.5 for more on diminishing sensitivity.

premium requested on equity to a *myopic loss aversion* of equity holders. Instead of focusing on their lifetime utility and noting that over the long-term equity is the most profitable investment by a wide margin (Siegel and Thaler, 1997), agents frame their investment decision more narrowly in an horizon of approximately one year, at which the risk that stocks under-perform bonds is indeed high. As agents are also highly averse to losses, this leads to a high risk premium and a sub-optimal under-investment in equity, a tendency with important consequence from a macroeconomic standpoint. Barberis and Huang (2001) provided a further refinement of this analysis, by distinguishing narrow framing on the equity portfolio and on individual stocks.[35]

1.3.4 Unknown preferences

Some contributions in the behavioral finance literature have pointed out that postulating the existence of predetermined, well-defined preferences underlying agents' decision in a variety of contexts and situations may be far-fetched, if not plainly false. In a number of experiments as well as in real life situations, *preference reversals* have been often observed. In general, preferences seem to depend to a large extent on the way a certain (economic) decision problem is presented to agents (Starmer, 2000). Preference reversals may imply that the principle of transitivity (if x is preferred to y and y is preferred to z, then x is

[35] Shefrin and Statman (1994, 2000) have proposed a "behavioural portfolio theory" based on the idea that people keep their portfolios in separate mental accounts: some money is retirement money, some is fun money, some is downside protection, some a shot at becoming rich. These mental accounts are considered in isolation and covariances among mental accounts are ignored. In this respect, there is no unified portfolio theory as in mainstream finance, but rather many portfolio theories according to the narrowly framed portfolio selection problem (Statman, 1999).

preferred to z) may be violated (x is preferred to y and y is preferred to z, but z is preferred to x, for instance if it is presented in a different manner than x).

The concept of utility in mainstream economics and finance is also sometimes seen as unclear. Kahneman (1994) in particular emphasized the need to distinguish at least between *hedonic experience* ex post and the ex ante concept of *decision utility*. Not necessarily, and actually quite seldom, is the latter a good predictor of the former because agents may be poor at forecasting their own tastes. One commonly observed tendency, for instance, is for agents systematically to underestimate the degree to which they will adapt to a new situation, leading them to exaggerate the utility gain or loss deriving from a certain outcome different from the status quo (hedonic mis-prediction).[36] Remembered utility may play an important role in forecasting future tastes (and so in decision utility), but memory can also play tricks. Furthermore, utility may be derived from memory in itself (Elster and Loewenstein, 1992), again imparting a backward-looking orientation to agents' decisions. In general, this literature emphasizes the linkages between the past (memory), the present (decision utility) and the future (future experienced utility). The expectation of future experienced utility is not always assessed only cognitively, but is also accompanied by strong anticipatory feelings such as anxiety (Caplin and Leahy, 2001).[37] Moreover, preferences evolve over time, for instance with age, but agents seldom take this factor into account in their decisions.

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[36] This was labelled *projection bias* by Loewenstein, O'Donoghue and Rabin (2000), i.e. the underestimation of future changes in tastes.

[37] Caplin and Leahy (2001) put forward the idea that anxiety might be the root of risk aversion. At the same time, anxiety can drive decisions in a very different way than in standard expected utility models, for instance by causing extreme forms of nonlinear weighing of probabilities.

One interesting approach is to postulate that preferences, especially future preferences, are not fully known to the agent who must take a decision (Hey, 1995; Loomes and Sugden, 1995), i.e. they are stochastic. However, it is likely that uncertainty over own preferences – especially future ones – is much more pervasive and deeply rooted than the mere inclusion of a random term would imply. Nonetheless, stochastic preferences represent an interesting step forward as they highlight the idea that forecasting future tastes and linking them to memory is a key element in individual decision-making, as basic psychological intuition would anyhow suggest.

Are stochastic and context-dependent preferences relevant in a market context? The evidence on preference reversals reviewed in Tversky and Thaler (1990) does suggest so. It has been found experimentally that different methods of eliciting preferences often give rise to systematically different orderings among possible alternatives. For instance, a systematic tendency has been observed to overprice low probability / high payoff lotteries over high probability / low payoff lotteries (compared with the ordering obtained through a direct comparison between these alternatives). As Tversky and Thaler (1990) put it, "if option A is priced higher than option B, we cannot always assume that A is preferred to B in direct comparison". In simpler words, market behavior does not necessarily reflect the maximization of well-defined preferences, as in the standard approach outlined in Section 1.2. Indeed, it is thinking *in monetary terms* which appears to change those very preferences. The consequences of these findings for economics and finance could be of great importance. For instance, the idea that the market allocates resources to their best possible use would be completely disrupted if agents' preferences are not pinned down and are affected by the market mechanism itself.

1.3.5 Reference dependence

Reference dependence incorporates and summarizes many of the ideas which have been touched upon in the previous Sections, and it is perhaps the most important single element of the behavioral finance literature. According to leading behavioral finance theorists such as Thaler (2000) and Camerer (1998), *prospect theory* – which is closely related to reference dependence – is a key contender to expected utility as a descriptive theory of behavior under risk, and it might be considered as the natural behavioral finance counterpart of the expected utility model. Developed by Kahneman and Tversky in the seventies, the theory was honed in the early nineties (Tversky and Kahneman, 1992) and has received a great deal of empirical support especially in experimental economics (Kahneman and Tversky, 2000). One major advantage of prospect theory over expected utility is seen that it has no aspirations as a normative theory of behavior; it simply describes in the most parsimonious and analytically tractable manner agents' observed behavior (Barberis and Thaler, 2001).

The origins of the theory come from basic psychological intuition. In particular, the theory is based on three foundations:

1. Organisms habituate to steady states (adaptation);
2. The marginal response to changes is diminishing;
3. Pain is more urgent than pleasure.

The first assumption states that agents do not look at wealth – or variables of similar economic significance – per se, but rather compared with a reference point. In particular, *gains* compared with the reference point are carriers of positive utility, while *losses* are carriers of negative utility.

The second key assumption of the theory, a consequence of its emphasis on reference dependence, is that agents evaluate departures from the reference point in either direction with *diminishing sensitivity*. For example, a 1% marginal change in wealth at the reference point is more important than a marginal change 30% away from the reference point (in other words, agents perceive more strongly a change from 0% to 1% – positively or negatively – than a change from 30% to 31% if the reference point is zero, irrespective of whether the change is a loss or a gain). In expected utility there is no reference point, but if one takes the status quo as a (pseudo-)reference point, the concavity of the utility function implies the opposite tendency for losses, namely a marginal loss from 30% to 31% is – unlike in prospect theory – more serious than a marginal loss from 0% to 1%.

Finally, the third assumption postulates than losses loom larger than gains in agents' utility, which is normally referred to as *loss aversion*. In many experiments, it has been found that losses imply a dis-utility approximately two times greater than the utility of a gain of the same size. In the standard approach, gains and losses cannot be defined because of the absence of a reference value against which to measure them.[38]

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[38] On the other hand, it is worth stressing that prospect theory may be rewritten as a function of the level of wealth (Ang, Bekaert and Liu, 2000). Moreover, disappointment aversion by Gul (1991) implies an *endogenous* reference point given by the expected value of the lottery. Under disappointment aversion, the idea that agents value differently gains and losses is maintained, but unlike in prospect theory the reference point is determined endogenously. Despite this attractive feature, disappointment aversion theory has not gained the same popularity of prospect theory thus far.

In prospect theory, the choice is represented by a two-stage process. First, the problem is "edited", possibly using a form of decision heuristic and in the context of a narrow framing. For example, the agent will *narrow-frame* the problem "how to invest a certain amount of money" and construct a reference point z_t around which to evaluate gains and losses (for instance, the initial level of wealth). The agent will not look at the correlations between this particular decision and other aspects of his life, because of deliberation costs or limited information processing capabilities. In a second stage, the agent takes the decision (e.g., how much wealth to invest in equity) so as to maximize the *prospective value function* (Kahneman and Tversky, 1979).

To build and maximize the prospective value function, the agent must first consider his *value function* $V(x)$, which is typically defined as follows:

$$V(x,z) = \begin{array}{c} (x-z)^{\gamma}, \text{ if } x-z \geq 0 \\ -\lambda(z-x)^{\gamma}, \text{ if } x-z < 0 \end{array}, \tag{1.10}$$

with $\lambda > 1$ (loss aversion) and $0 < \gamma < 1$ (diminishing sensitivity). The value function is concave on gains and convex on losses, and kinked at $x = z$, so it is not concave everywhere as the utility function in expected utility theory.

In order to obtain the prospective value function, the agent weighs the value function in different states of the world according to some measure of probability associated to these states. In the original version of the theory (Kahneman and Tversky, 1979), agents consider a nonlinear weighing function of the probability density of the outcome. The prospective value function (PVF) is obtained as follows:

$$PVF = \int V(x,z)w(p(x))dx \tag{1.11}$$

As mentioned above, the probability weighing function put forward in the behavioral finance literature ($w(p)$) is generally regressive and s-shaped. In the advanced version of prospect theory, *cumulative prospect theory* (Tversky and Kahneman, 1992), the weighing function is defined on the cumulative probability distribution of gains and losses separately, rather than on the probability density. Thus, events are rated according to their rank in the possible range of events, as suggested by Quiggin (1982). The probability weighing function is evaluated separately on gains and losses, and varies between 0 and 1 separately for gains and losses, integrating to one in the domain of gains and in the domain of losses separately. In experimental studies it has been often found that the probability weighing is approximately symmetric between gains and losses; namely, the weighed probability assigned to a gain with a certain cumulative probability over gains is approximately the same as that assigned to a loss with the same cumulative probability over losses (*reflection property*).

The curvature of the value function in (1.10), which reflects the assumption of diminishing sensitivity, together with a typical nonlinear weighing of probabilities tends to suggest a *four-fold pattern of risk aversion* (Kahneman and Tversky, 2000), whereby the representative agent is risk averse for large-probability gains and small-probability losses, but risk-loving for small-probability gains and large-probability losses. This creates a richer pattern of attitudes towards risk compared with the expected utility theory, which assumes risk aversion everywhere. This is an interesting feature of the theory which might be able to explain why agents tend to take risks in some contexts (e.g., lotteries) but to avoid risk in others (e.g., portfolio allocation).

It is interesting to observe that the property of diminishing sensitivity is conceptually similar to, although used in a different context from, the idea of "first order" risk aversion as put forward by Epstein and Zin (1990) and Segal and Spivak (1990). The common denominator of these two concepts is the fact that the utility function exhibits aversion to small shocks. In expected utility, agents are practically risk-neutral over small shocks and only care about large shocks ("second order" risk aversion).[39] It might be noted that diminishing sensitivity is a reasonable assumption in contexts where reference dependence is important, but it does not seem appropriate in *every* decision problem. In fact, there may be situations in which diminishing sensitivity becomes implausible. For instance, diminishing sensitivity is unlikely to hold in the domain of losses if the agent risks poverty – the marginal dollar lost which throws him into poverty is likely to carry a high dis-utility despite its being away from the agent's reference point.[40]

Reference dependence can affect market prices, to the extent that assets are priced with respect to gains and losses vis-à-vis an arbitrary reference point which gains salience for economic agents. So, returns on financial assets may be evaluated against an arbitrary reference point and not in relation to the marginal utility of consumption as equation (1.3) suggests. Reference dependence appears to matter also in a broader sense in macroeconomic developments, which tend to affect financial market prices (Akerlof, 2001). For

[39] Rabin (2000b) shows in a calibration theorem that under expected utility, assuming that agents are averse to lotteries with stakes of moderate size, which is in line with the experimental evidence, agents have to be absurdly risk averse towards lotteries involving large stakes. So, expected utility assumes that agents are practically risk neutral towards lotteries with moderate stakes. This might be appropriate in some contexts, but does not represent a good characterisation of preferences in general terms.

[40] As noted by Fennema and van Assen (1999), diminishing sensitivity "has nothing to do with our evaluation of money but it is purely a matter of perception of numbers". In the neighbourhood of poverty, it is likely that our perception of money becomes more important than our perception of numbers. In such a situation, a concave utility function over losses is presumably more appropriate.

example, habit formation in consumption can be interpreted as a form of reference dependence.

Is prospect theory really a serious challenger to expected utility, and does it help to explain market behavior better than expected utility theory? According to Camerer (1998), the empirical evidence in its favor is such that cumulative prospect theory should be put at least on an equal footing with expected utility, and at least for some decision problems this conclusion seems broadly correct. One important asset of prospect theory is that it is analytically tractable, although arguably somewhat less easily than expected utility. Preferences are well defined and can be maximized for the decision problem at hand (the same would not be true, for example, with models postulating preference reversals). Moreover, prospect theory is not inconsistent with agents having rational expectations, namely not making systematic mistakes in their forecasts. So, it is conceivable that asset pricing equations similar to those in Lucas (1978) might be developed by maximizing a function like:

$$PVF = E_t^w \sum_{s=t}^{\infty} V(c_s - z_s), \tag{1.12}$$

where E_t^w is a subjective "pseudo-expectation" derived using the w-weighted probabilities, possibly with the property that $\varepsilon_{t+1} = x_{t+1} - E_t^w x_{t+1}$ is i.i.d..[41] This should make it possible for the theory to be incorporated in asset pricing models based on no-arbitrage conditions that are pervasive in the finance literature.[42] Moreover, the theory is intuitively

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41 This is, indeed, the spirit of the analysis of Barberis, Huang and Santos (2000).

42 The value function of prospect theory, however, is not differentiable, which should make it more difficult to obtain explicit analytical results as in expected utility theory. For a differentiable utility function incorporating reference dependence and diminishing sensitivity, see Bray and Goodhart (2002).

appealing, as it is based on much stronger psychological foundations compared with expected utility and yet is mathematically tractable.

It is sometimes mentioned that a serious problem of the theory is that it assumes away how the reference point is determined. While the reference dependence feature of the theory certainly makes sense – reference points may be determined by non-economic factors such as social norms –, it should make it more difficult for advocates of prospect theory to build asset pricing models with the same degree of generality as mainstream finance theorists have done. This limitation, however, should not be overemphasized. Indeed, much of mainstream finance theory is built on the mean-variance utility function, which implicitly assumes the existence of a reference point, namely the current level of wealth. It should be feasible to develop asset pricing models based on prospect theory taking the same reference point of mainstream finance, current wealth.

Overall, prospect theory seems to be preferable to expected utility at least in contexts in which reference dependence seems important and where agents are significantly averse to lotteries with stakes of moderate size (which is closely related to reference dependence). For problems where reference dependence does not seem relevant, expected utility may remain preferable, mainly because of its superior analytical tractability.

1.4 Is the market "rational"? Some perspective to this debate

So far we have discussed about rationality, or lack thereof, mainly at the individual level. In this section, I concentrate on rationality at the aggregate, market level which is arguably the most relevant aspect for economists. In fact, few, if any, mainstream finance theorists contend that individual agents cannot behave in an irrational way and that the homo economicus is anything else than a gross simplification that does not describe accurately any human being (including the theorists themselves). At the same time, mainstream economists normally maintain that the functioning of markets may be well described and predicted "as if" agents were all homo economicus. This is most relevant, because the analysis of the functioning of markets is the core task of economics, and economics does not – and should not – deal with the psychology of economic agents as an objective per se (Mas-Colell, 1999), but only (or at least mainly) with the market implications of it.

The concept of rationality normally maintained by mainstream finance theorists is normally in the beat-the-market sense. Do the anomalies determine exploitable profit opportunities for a smart arbitraguer? Initially, the publication of the paper by De Bondt and Thaler (1985) – according to whom the stock market displays a systematic tendency to overreact to news – seemed to deal a blow to the market rationality even in the restricted (and favored by mainstream theorists) beat-the-market sense. However, in subsequent years several instances of market under-reaction were also detected. This has led Fama (1998) to claim that over- and under-reaction anomalies are simply due to chance, and that market efficiency prevails *on average* (thus, no ex ante exploitable excess profit opportunity arises). Moreover, Fama (1998) stressed that the evidence for most long-term abnormal returns is fragile and does not withstand a closer scrutiny and / or a reasonable change in the statistical methodology (Barber and Lyon, 1997). Today, there seems to be almost a consensus that the market is most of the times rational in this beat-the-market sense. The most solid proof of that is that portfolio managers, and in general active investment strategies, do not outperform passive investment strategies (especially when transaction costs are taken into account; Malkiel, 1995). In this beat-the-market sense, mainstream finance seems to have resisted the "attack" by behaviorists (as behavioral finance advocates such as Thaler, 1999b, and Statman, 1999, conceded). Homo economicus is still alive here.

It is important to stress, however, that market rationality in the beat-the-market sense is not necessarily inconsistent with the idea that anomalies are pervasive and that the systematic behavior of agents leads to departures from rational asset pricing. It simply signals that it is not easy to make money out of these anomalies, for example because there are limits to sustained arbitrage activity (Shleifer and Vishny, 1997; Mitchell, Pulvino and Stafford, 2002). As pointed out by Mullainathan and Thaler (2000) and Barberis and Thaler (2001), it is impossible to arbitrage away many instances of "irrationality", simply because there is no speculative market on such matters, or because arbitrage is risky, or because agents cannot wait too long before closing a position.[43] Recalling Section 1.2, the existence of a bias in market prices due to behavioral factors is indeed fully compatible with rational expectations and a random walk behavior of asset prices.

Moreover, the argument initially attempted by mainstream finance theorists to reconcile the overwhelming evidence in favor of the anomalies at the individual level with rationality of the market at the aggregate level on learning and evolutionary grounds has proved to be slippery.[44] If financial markets can be characterized as a long-lasting, repet-

[43] Colisk (1996) expressed this concept forcefully as follows: "... we commonly read in the financial pages that firms fail for lack of profits, but we seldom read in obituary pages that people die of suboptimisation" (p. 684). Barberis and Thaler (2001) state that "no free lunch can also be true in an inefficient market" (p. 6).

[44] For example, De Long, Shleifer, Summers and Waldman (1992) show that agents who fail to maximise their expected utility survive markets better than expected utility maximisers.

itive environment, then they would provide agents with good opportunities for learning and correcting behavioral biases over time. Learning is made easier by a number of conditions such as repeated opportunities for practice, small deliberation costs, availability of good feedback, and unchanging circumstances. That the financial market provides *all* these conditions is doubtful. For example, it can hardly be defined as an environment with unchanging circumstances (Thaler, 2000). So, the idea of a convergence to rational expectations via learning on the market is a difficult route for mainstream theorists.[45] Moreover, learning is closely related to experimentation. In some context of importance for finance, the cost of experimentation may be extremely high (Mullainathan and Thaler, 2000); for instance, deciding on whether to take on a house mortgage does not leave much space for experimentation (and learning).[46] In such situations, we should expect the behavioral biases to apply in full force. Overall, the evolution / learning argument has proved difficult for mainstream finance advocates.[47]

Furthermore, most advocates of behavioral finance contend that the beat-the-market definition of market rationality is too narrow and not relevant from a welfare perspective (Barberis and Thaler, 2001). The ultimate function of the financial market is not to allow agents to speculate over future movements in prices, but rather (over time) to allocate

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45 For example, Timmerman (1994) showed that it would have been virtually impossible for market participants to learn in real time the law of motion of the U.K. stock market.

46 Brav and Heaton (2002) refer to "rational structural uncertainty" to show that the law of motion of prices and dividends may not be learnable at all, even by rational agents with unbounded computational capabilities. In this respect, they point out that the distinction between behavioural and rational theories becomes blurred in the presence of structural uncertainty.

47 For example, Nyarko (1991) has shown that learning models can be used to explain price developments which are ex post inconsistent with rational expectations. On the other hand, some papers have explicitly dealt with the selection property of the market, suggesting market efficiency. Gode and Sunder (1993) have shown that the market can, under certain conditions, process information very efficiently even if simulated traders have very little rationality. In the same line, see also Sandroni (2000).

consumption in the lifetime in an optimal manner and (at a certain point in time) to allocate funds to the most productive investment opportunities. It should be emphasized that the absence of arbitrage opportunities and the fact that changes in prices and dividends are not predictable – which is the typical focus in the mainstream finance literature – does not necessarily imply that market prices are "rational" in the expected utility sense. Indeed, as we have seen in Section 1.2, the absence of arbitrage opportunities is only one of the two conditions defining rationality in the standard approach, the other one being that expected returns satisfy equation (1.3).[48] So, there may be anomalies which do not provide any scope for arbitrage, while still being inconsistent with rationality as defined in the standard approach. Overall, there is very little research available on whether behavioral biases lead to mis-allocations of capital and to lower economic growth and welfare in the long run, despite the obvious importance of this matter.[49]

At times, the evidence seems compelling that market prices are irrational. A famous case is the discrepancy of the share prices of the Royal Dutch-Shell group from their theoretical value. Although the interests of the Royal Dutch and the Shell corporations were merged on a 60-40 basis, the ratio between their share prices deviated by more than 35%

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48 It might be added at this point that the standard approach based on the maximisation of lifetime expected utility is related to the idea that agents trading in the market are mainly households, who care about consumption. The standard approach seems much less realistic in a market dominated by institutional investors, as it is now the case in most industrialised countries. Institutional traders are likely to be concerned above all with the short-term maximisation of profits and with reputational issues related to the principal-agent relationship in which they are normally the agent. This feature of financial markets may explain the stronger focus on the no-arbitrage condition with less emphasis on the other main foundation of the efficient market hypothesis, namely equation (1.3).

49 Wurgler (2000) provides interesting evidence in favour of market rationality defined as the ability to allocate funds to the most profitable investment opportunities, finding in a cross-country analysis that "financially developed countries boost investment more in their growing industries and cut it more in their declining industries". Gilchrist, Himmelberg and Huberman (2002) try to measure the effect of stock market "bubbles" on corporate investment and corporate finance decision by firms.

from the theoretical value of 60/40 depending on the location of trade (Froot and Dabora, 1999).[50] Another possible key example is the crash of the New York Stock Exchange on 19 October 1987, which occurred in the absence of any relevant news which might have justified a collapse of more of 20% of the stock index value. Given that the stock market index ultimately represents the value of the corporate sector, how could this value fall so dramatically in a matter of hours and without any new information?[51] More fundamentally, the "excess" volatility of equity prices as stressed by Shiller (1981) and the large amount of trading in financial markets world-wide are difficult (albeit nor impossible) to justify on purely "rational" grounds in the standard expected utility sense.

It would be desirable for research to focus on a proper definition of market rationality around which to structure the debate between advocates of behavioral and mainstream finance. A promising distinction is between *exogenous rationality* and *endogenous rationality* (Rubinstein, 2000). By exogenous rationality we may define a situation in which the market price optimally reflects some exogenous objective quantity (e.g., the profitability of the U.S. corporate sector), i.e. where there are no "extrinsic" fluctuations. The case of the Royal Dutch-Shell group (and possibly also the crash of the New York Stock Exchange in 1987) indicates that the market is not (always) exogenous-rational. This also underpins Shiller's (1981, 1998) claim that stock prices have moved too much to be explained by subsequent changes in dividends, although an explanation consistent with market efficiency (i.e., time-varying stochastic discount factors) cannot be entirely ruled out either. At the

[50] Lamont and Thaler (2001) report similar episodes.

[51] Of course, computer-based trading and stop-loss automatic rules are often quoted as the main curprit of the 1987 crash. However, it is doubtful that such rules may be considered as being consistent with rationality.

same time, there may be a form of endogenous rationality according to which each market participant possesses an unbiased estimate of the (future) market price, even if the market price is completely detached from fundamentals and is affected by behavioral biases which are impossible to arbitrage away.

The distinction between endogenous and exogenous rationality is, however, more complicated if the fundamentals are themselves affected by the market evaluation, i.e. if the feedback mechanism is strong and prophecies can become self-fulfilling. There is often a tendency (probably because economists are themselves affected by hindsight bias) to regards a certain development caused by market developments as unavoidable (supporting the idea of exogenous rationality). But it can sometimes be the result of a self-fulfilling spiral in which the prime mover is indeed an "endogenous" market whimsical move.

Reflecting a growing recognition of the role of fads and endogenous market fluctuations, much research has focused in recent years on why large deviations of market values from fundamentals occur in the first place and how "false" information can be disseminated in the market. Studying *herd behavior* (for a survey, see Devenow and Welch, 1995, and Bikhchandani and Sharma, 2000) has been the object of considerable effort in recent years for its possible role in amplifying fads and lead market prices astray from fundamentals. "Rational" herding behavior (i.e., rational in the sense of maximizing the individual market participant's utility) may create "information cascades" with market participants possibly transmitting false information, thus creating a negative externality (Banejeree, 1992). This may happen, and can be explained in an expected utility framework, when each agent thinks that the information that he receives from other traders is better than his own private information, and decides to discard the latter. If all agents behave in this way, private information is not transmitted at all and the informational efficiency of the market is disrupted (Sunder, 1995).

Several factors may reinforce a tendency to herding and conformity, including reputation in a principal-agent context if the performance of the portfolio manager (the agent) is costly to monitor (Scharfstein and Stein, 1990), and the fact that compensation is often computed comparing with other investors' performance, pushing risk-averse traders to conform to the "average" assessment of the market.[52] In spite of notable theoretical developments, the empirical literature has thus far failed to provide convincing evidence of herd behavior at least in financial markets in developed countries. This is not surprising as one should ideally separate price movements which reflect fundamentals from price movements merely reflecting the mood of the market, and this is very difficult to do (Lakonishock, Schleifer, and Vishny, 1992; Wermers, 1999).

Summing up, is the controversy about market rationality going to be sorted out any time soon? This is unlikely because, as Fama (1998) pointed out, market efficiency is per se un-testable. In fact, testing the hypothesis that the market is efficient requires a model of expected returns, which is actually tested together with the hypothesis. Only the evidence that it is possible *systematically* to beat the market would be a bullet-proof way to discredit the hypothesis of market efficiency. Thus far, behavioral finance has failed to provide such evidence.[53] At the same time, it is clear that endogenous market fluctuations driven

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52 Herding behaviour has been postulated also for investment analysts (Graham, 1999), again on reputational grounds. Risk-averse investment analysts will tend to cluster on the average and be very conformist, for the loss of being wrong may be higher if the other investment analysts were right.

53 On the other hand, it has to be noted that serial correlation tests typically used to test market efficiency

by behavioral factors, though unpredictable and impossible to arbitrage away, are widely regarded as a key feature of financial markets, and represent a key challenge for market efficiency in a broader sense.

A final remark is due on whether the alleged influence of behavioral biases on financial markets calls for a policy response. Daniel et al (2002) are the only ones to deal with this issue directly. According to these authors, governments are likely to be affected by behavioral biases as well, with the difference that they would not be subject to the powerful disciplinary force of competition. Thus, their involvement in setting market prices would probably be counterproductive (Wurgler, 2000, reports empirical evidence that government intervention reduces the economic efficiency of financial markets). At the same time, governments could make economic agents more aware of their psychological biases and of the incentives that others have to exploit them, creating some room for policy intervention in terms of reporting rules and disclosure. Moreover, policy-makers should be at least aware that markets may at times display irrational tendencies and that large deviations of market prices from fundamentals may exist. Apart from the difficulty in implementing policy measures aimed at correcting these biases, this awareness might at least increase policy-makers' understanding of the world, which would have a positive effect *per se*.

1.5 Conclusions

Behavioral finance is a rapidly growing area of research and one of the most promising fields of economics. The fertilization of finance (and economics in general) with psycho-

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have normally very low power (Akerlof, 2001).

logical ideas and evidence makes it a very interesting and lively field. At the same time, it could be argued that behavioral finance is running the risk of being un-parsimonious (Wachter, 2002; Tirole, 2002). While the list of anomalies discovered is now impressive, convincing evidence is still to be provided that expected utility is a flawed analytical framework for studying the behavior of agents in a (financial) market context, which is at the core of economics (Constantinides, 2002).[54] Bullet-proof evidence that the market is not rational in the mainstream finance, beat-the-market sense is yet to be provided, although many hints that the market may not be rational in other reasonable senses have indeed been provided.

Against this background, the key challenge for behavioral finance seems to be to study in more detail the market implications of the widely documented agents' behavioral biases. In particular, to study how prices are determined in large competitive markets more recourse to *social*, rather than *individual* psychology might be warranted. The work on synchronization of expectations, fads and the role of communication (see, e.g., Shiller, 2000a, 2000b) seems to be most promising in this respect.

In addition, a more thorough analysis of the possible definitions of market rationality which would be relevant from a *welfare perspective* would be greatly beneficial. Does it support social welfare that it is impossible to beat the market? Does it hamper welfare that a large stock market can fall by 20% in a matter of hours without any news? Is market volatility due to arbitrary beliefs good or bad?[55] The answers to these questions are likely

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[54] Moreover, the large number of approaches followed leaves it open to the criticism of "reverse engineering" (Zin, 2002). By making marginal utility state-dependent, behavioural theories could explain every phenomenon. Frankfurter and McGoun (2002) put it as follows: "no matter what happens in the market, there is a psychological effect that can be mustered to explain it". A good theory must instead be able to explain the moments that it was *not* designed to match (Wachter, 2002).

[55] That market volatility due to behavioural factors is necessarily negative is not a forgone conclusion. For

to shed some light on the relative usefulness of behavioral and mainstream finance. Indeed, the two approaches need not be seen necessarily as antagonists. Both may well be useful in explaining part of reality, depending on the problem under investigation. behavioral finance is a more suitable approach to explain endogenous market fluctuations and the formation of market beliefs and fads, while the mainstream approach may be preferable to study issues more closely related to the rational expectations assumption for *given* market beliefs, for example the pricing of derivatives.

If this line of reasoning is appropriate, the relevance of each approach will depend crucially on whether the fact that behavioral biases distort asset prices in large and competitive markets has a significant implication on the quality of the allocation of capital and ultimately on long-term economic growth and welfare, namely on the *economic efficiency* of financial market prices. The issue of the feedback mechanism seems most relevant in this respect. Thus far, there has been no systematic attempt to address the issue of the feedback from market prices to fundamentals, and only some informal speculations have been provided (see Shiller, 2000a, and Daniel et al, 2002).

Finally, one further intriguing area of research is represented by the study of possible behavioral biases of large actors such as policy-makers (for example central bankers; see al-Nowaihi and Stracca, 2003).[56] Because of their size and role, these actors have a direct influence on financial markets and their alleged behavioral biases may certainly have repercussions on market outcomes. In addition, learning and evolutionary forces are deemed to

example, it might be argued that increased volatility creates the possibility of profitable short-term speculation thereby increasing the *liquidity* of the markets.

[56] It is useful to note that a "behavioural" approach is starting to be applied to study monetary policy issues; see, for example, Rotemberg (2002) and Ball, Mankiw and Reis (2002).

apply less forcefully than for atomistic agents participating in a large, competitive market. However, an analysis of the systematic psychological traits of economic policy-makers is yet to be developed, and represents a challenge for future research.

Chapter 2
Non-standard central bank loss functions, skewed risks, and certainty equivalence

2.1 Introduction

The role of uncertainty in monetary policy-making has attracted considerable interest in the literature in recent years (see, e.g., the review by Goodhart, 1999). It has long been known that *multiplicative* uncertainty (namely, over the true "value" of the interest rate elasticity of output and inflation) is not neutral for the policy-maker and generally leads to caution and policy gradualism (Brainard, 1967).

There is, however, much less consensus on the role of *additive* uncertainty, which denotes the uncertainty over the true "state of the economy", despite the obvious importance of this matter for policy-makers, who are confronted with this type of uncertainty practically every day.[57] On the one hand, the view is prevailing in the academia that additive uncertainty should not matter, a principle known as "certainty equivalence" (Theil, 1958).[58] Svensson and Woodford (2003) characterize this situation as the "orthogonality of estimation and policy". Moreover, the principle of certainty equivalence does not seem to

[57] Data measurement problems, uncertainty over the economy's natural rate of employment (or the natural rate of interest) at any point in time, shocks to the inflation rate and/or to the output gap which occur *after* a certain monetary policy decision but *before* its impulse has fully worked through the economy, are all prominent examples of additive uncertainty.

[58] Formally, let x be a control variable and y a state variable, with $y = f(x) + e$, e being a zero mean additive disturbance and f a deterministic function. If certainty equivalence holds, the optimal value of x (for instance, the value which minimizes y) is independent of any moment of the probability distribution of e.

be a mere artefact of the use of a quadratic loss function. Chadha and Schellekens (1999) – henceforth CS – have shown that the certainty equivalence principle holds for a general class of convex loss functions, and the coefficients of the optimal policy rule are not affected by additive uncertainty even if the preferences of the central banker are asymmetric.[59] On the other hand, policy-makers generally do not seem to think that additive uncertainty is irrelevant (see for example Blinder, 1998). A casual look at central banks' external communication tends to lend support to this assessment. For instance, in the Sveriges Riiksbank's Inflation Report (quoted in Blix and Sellin, 2000), it was reported that:

> "The element of *uncertainty in the inflation assessment* can accordingly influence monetary policy's construction. A high degree of uncertainty can be a reason for giving policy a more cautious turn" [emphasis ours],

and in the Bank of England's Inflation Report (again quoted from Blix and Sellin):

> "in the light of the central projection and *the risks surrounding it*, the Bank continues to see the need for a moderate tightening of policy". [emphasis ours]

Finally, this statement can be retrieved from the European Central Bank's website:

> The European Central Bank (ECB) confirms its position of 'wait and see' with regard to its monetary policy stance. In an environment of *increased uncertainty over the global economy and its impact on the euro area*, the Governing Council is carefully assessing whether and to what extent upward risks to price stability will continue to decline." [emphasis ours]

In all cases, central banks seem to refer to additive uncertainty (i.e., uncertainty over the state of the economy, not over the effect of the monetary policy levers) as an important element in the determination of policy.[60] Thus, there seems to be an important discrepancy of views between policy-makers and the academia over this key aspect of monetary policy-making. This divergence, in turn, should lead one to wonder whether simple and plausible

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59 Orphanides and Wieland (2000) analyse the properties of inflation "zone" targeting using a locally quadratic central bank loss function which includes a concern about a target zone for inflation.

60 Building on their experience, those with practical experience in central banking will probably have no problem whatever in accepting that addititive uncertainty matters for policy; see for example Blinder (1998).

monetary policy models alternative to those traditionally used in the academic literature on optimal monetary policy can be worked out to give account of the seemingly important role of additive uncertainty in actual policy-making.

Against this background, this chapter sets out to analyze the interaction, if any, between non-standard and yet analytically tractable and behaviorally plausible central bank loss functions and *additive* uncertainty modelled as a non-Normal distributed additive shock to the inflation process. One particularly important example of such non-Normal additive uncertainty is the presence of *skewed risks*. The focus on skewed risks seems a natural starting point, as this kind of risks is encountered very often in central banking.[61] For example, Goodhart (2001) states that:

> When we (as MPC members) think of risk, we are generally concerned with asymmetric possible outcomes. Asymmetry implies skew, and skew drives the measures of central tendency apart. The question of which measure of central tendency one should focus upon depends on one's own individual loss function. [...] Help on how better to treat asymmetric risks would be much appreciated. (Goodhart, 2001, p. 179).

Overall, the analysis of this chapter seems to be a first step in this direction.

Throughout the chapter, the assumption will be maintained that the policy-maker has no multiplicative uncertainty, namely he has a perfect knowledge of the effect of the monetary policy instrument on the target variable(s). This chapter relaxes two commonly maintained assumptions on the central bank loss function. First, a curvature different from, and more general than, the quadratic is considered. Second, the effect of a non-linear weighing of probabilities by the central bank is analyzed. Ultimately, the objective of the analysis is to establish whether the principle of certainty equivalence carries through to the non-standard loss functions examined here, and hence whether the assumption of a

[61] Again, those directly involved in central banking will not have any difficulty in agreeing that this is indeed the case.

quadratic loss function evaluated according to the expected utility criterion – which permeates the bulk of the literature on optimal monetary policy – is indeed innocuous or not. That no assumption on the probability distribution of the additive shock is maintained is worth stressing, as it distinguishes the analysis in this chapter from that in CS.[62]

In devising behaviorally plausible and analytically tractable non-quadratic central bank loss functions, this chapter builds on the substantial body of evidence made available by the literature on economic psychology under uncertainty, especially by the strand linked to the names of Daniel Kahneman and Amos Tversky (see Kahneman and Tversky, 2000, for a review and assessment of this literature). With no evidence available thus far on the "typical central banker"'s psychology, the working assumption of this chapter is that the patterns and tendencies that the Kahneman-Tversky literature has identified in a number of experimental studies are also valid for those agents in charge of monetary policy.[63] Among the key elements identified in this literature, diminishing sensitivity to losses (i.e., non-convex loss functions) and non-linear weighing of probabilities (i.e., departures from the expected utility paradigm) seem to be plausible and interesting also as a characterization of central bank preferences.

In sum, the chapter finds that the interaction of a non-Normal additive disturbance to inflation and the set of non-quadratic preferences postulated here leads to situations where

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62 CS maintain the assumption of a Normal distributed additive shock to inflation throughout their paper.

63 It should be stressed that throughout the paper the emphasis will always be on the central bank's *positive* (i.e., descriptive) preferences. The analysis abstains from the determination of the *normative* preferences, for example those that would maximise society's welfare (see, e.g., Svensson, 2001). Of course, if it were possible for society to write down explicitly the central banker's loss function, this would lead us a long way towards the identification of the "true" central bank preferences. However, even in this (rather unrealistic) case the "ex post" preferences will not correspond to the "ex ante" preferences (those which matter for monetary policy-making), for the latter also involve the central banker's attitude towards risk.

the principle of certainty equivalence does *not* hold. Thus, additive uncertainty seems to matter. Instead, if the disturbance is Normal distributed (as assumed in CS), the usual result of certainty equivalence continues to be valid. If one observes that non-Normal distributed shocks are an essential element which monetary policy-makers have to deal with, the overall policy message of this chapter is that additive uncertainty matters, and that the assumption of a quadratic loss function may not be as innocuous as it is often regarded.

The chapter is organized as follows. In Section 2.2 the usual setting is outlined of a monetary policy-maker aiming at minimizing a loss function defined in terms of the inflation rate, with the structure of the economy acting as a constraint on behavior. The effect of considering non-quadratic central bank loss functions on the role of additive uncertainty is analyzed in Section 2.3. Finally, Section 2.4 concludes.

2.2 A simple optimal control model for discretionary monetary policy

This section lays down a standard optimal control problem for discretionary monetary policy.[64] The structure of the economy includes an IS curve, whereby the monetary authority can influence the output gap by steering the nominal interest rate, and a backward-looking Phillips curve, linking current inflation to past inflation and to past output gap (see, e.g., Clarida, Gali and Gertler, 1999, and Mankiw, 2001a).[65]

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[64] Throughout the paper, the assumption is always maintained that the central bank cannot credibly commit to follow a policy rule; monetary policy is thus carried out in a discretionary manner.

[65] The choice of a backward-looking specification of the Phillips curve is motivated by the fact that it squares better with the available empirical evidence (see in particular Mankiw, 2001a, and Rudebusch, 2001). The

The IS curve is specified as follows:

$$x_t = a_1 x_{t-1} - a_2(i_t - \pi_t) + u_t, \tag{2.13}$$

where the output gap x is affected by the real interest rate $(i - \pi)$, and u is a disturbance term, unknown to the central bank at time t, with $E_t u_t = 0$. Parameters in this equation are $0 < a_1 < 1$ and $a_2 > 0$, known to the central bank.

The Phillips curve is:

$$\pi_{t+1} = b_1 \pi_t + b_2 x_t + v_{t+1}, \tag{2.14}$$

where v is an additive disturbance, for instance capturing a cost-push shock unknown to the policy-maker at time t, with $E_t v_{t+1} = 0$, and $0 < b_1 < 1$ and $b_2 > 0$ are parameters, of which the central bank has again full knowledge.[66],[67] No further assumption on the probability distribution of v is added at this stage. For simplicity, a zero drift is assumed; this, together with the assumption that $b_1 < 1$, implies that the steady state level of inflation is zero.

For notational simplicity, it is convenient to consolidate the IS and the Phillips curves to obtain a reduced form for inflation as follows:

$$\pi_{t+1} = c_1 \pi_t + c_2 x_{t-1} - c_3 i_t + \varepsilon_{t+1}, \tag{2.15}$$

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line of argumentation in the paper, however, would not be substantially changed with a forward-looking Phillips curve, as long as inflationary expectations at time t are exogenous for the central bank (to avoid the simultaneity problems discussed by Svensson and Woodford, 2000).

66 That the monetary policy instrument i affects inflation with a longer lag than it affects output is consistent with the bulk of the available empirical evidence.

67 More complex Phillips curve equations may be conceived (see, e.g, the non-linear specification in Clark et al, 2001), but such complications should *prima facie* be immaterial for the purpose of the present analysis.

where $c_1 = b_1 + b_2 a_2$, $c_2 = b_2 a_1$, $c_3 = b_2 a_2$, and $\varepsilon_{t+1} = b_2 u_t + v_{t+1}$ is a zero mean random disturbance (comprising output gap and cost-push shocks). It should be emphasised that the additive uncertainty term ε_{t+1} need not be Normally distributed and symmetric, as it is often assumed in the literature. For example, the probability distribution of ε_{t+1} may be characterised by *skew*.[68]

Simplifying further:

$$\pi_{t+1} = z_t - c_3 i_t + \varepsilon_{t+1}, \tag{2.16}$$

with $z_t = c_1 \pi_t - c_2 x_{t-1}$, known to the central bank at time t and exogenous to i_t, representing the "state of the economy" or equivalently the "inflationary pressures" at time t. In general, policy rules are specified as feedback rules $i_t = f(z_t)$, whereby the central bank sets its monetary policy instrument in reaction to changes in the state of the economy.

For simplicity of notation, let us consider the variable $\widetilde{i}_t$ defined as follows:

$$\widetilde{i}_t = c_3 i_t - z_t \tag{2.17}$$

If $\widetilde{i}_t = 0$, $i_t = \frac{z_t}{c_3}$ and $E_t \pi_{t+1} = 0$. Therefore, $\widetilde{i}_t$ is the deviation of the monetary policy instrument from the value which offsets at time $t+1$ the expected impact of the inflationary pressures observed at time t (i.e., z_t). There follows that the inflation process may also be expressed as:

$$\pi_{t+1} = \varepsilon_{t+1} - \widetilde{i}_t, \tag{2.18}$$

with ε_{t+1} independent of $\widetilde{i}_t$. Intuitively, the inflation process is equal to the difference between the realization of the additive shock at time $t+1$ and the deviation of the monetary

68 It should be clarified at this point that there is no reason to believe that additive uncertainty is characterised by a skew in a certain direction on a permanent basis, i.e. independently of t. What it is argued here is that additive uncertainty *at the particular time horizon* $t+1$ (which is relevant for the policy-maker at time t) may be skewed in a certain direction. The skew might well vary over different time horizons.

policy instrument from the value for which $E_t\pi_{t+1} = 0$. Thus, the value $-\widetilde{i}_t$ may also be interpreted as the inflation target of the central bank at time t. In the continuation of this chapter, we will often refer to $\widetilde{i}_t$ when speaking about monetary policy, without any loss of generality as the monetary policy instrument may be derived straightforwardly as $i_t = \frac{\widetilde{i}_t + z_t}{c_3}$.

The task of monetary policy is to select a value of $\widetilde{i}_t$ which minimizes an intertemporal loss function defined in terms of inflation levels (assuming for simplicity – and without loss of generality – that the inflation objective of the central bank is zero).[69] The principle of certainty equivalence – the main focus of the present analysis – stipulates that the probability distribution of ε should *not* matter in the determination of $\widetilde{i}_t$ (or, i_t).

The standard approach to deal with monetary policy problems is to specify the objective function of the central banker, with the structure of the economy that acts as a constraint on behavior. Given a period-by-period loss function $L(\pi)$, the central bank is normally assumed to minimize a time separable intertemporal loss function expressed as the expected value of a discounted sum Λ (where $0 < \gamma < 1$ is the discount factor):

$$\Lambda = E_t \sum_{j=1}^{\infty} \gamma^{j-1} L(\pi_{t+j}), \tag{2.19}$$

under the constraint $\pi_{t+1} = \varepsilon_{t+1} - \widetilde{i}_t$. It is clear from the way the problem is formulated that the central bank's task may be reduced straightforwardly to the minimization of

69 As in CS, we do not consider the possibility that the monetary authority may also care about the output gap. While this is certainly a restrictive and rather unrealistic assumption, it greatly simplifies the algebra. Moreover, the inclusion of the output gap may under certain conditions determine a *departure* from certainty equivalence (see in particular Smets, 1998). Thus, given that the focus of the present analysis is to ascertain whether additive uncertainty *on inflation* matters *per se* under non-quadratic central bank loss functions, we leave this complication aside in order to balance the odds in favour of certainty equivalence as much as possible.

Λ. In fact, the problem is specified recursively and the monetary policy instrument at time t only affects inflation one period ahead.

Under quadratic preferences, we have:

$$\Lambda = E_t \pi_{t+1}^2 = E_t(\varepsilon_{t+1} - \widetilde{i_t})^2, \tag{2.20}$$

and thus, after solving the first order condition:

$$\widetilde{i_t} = 0, \tag{2.21}$$

given that $E_t \varepsilon_{t+1} = 0$. Of course, this result may be derived also by force of the statistical law that the arithmetic mean (zero, in this case) is the measure of central tendency minimizing the average of the squared deviations from it. Hence, $i_t = \frac{z_t}{c_3} = \frac{c_1 \pi_t - c_2 x_{t-1}}{c_3}$, *irrespective* of the probability distribution of ε; this is the standard result of certainty equivalence. In general, a value of $\widetilde{i_t}$ different from zero depending on (some moments of) the probability distribution of ε_{t+1} would signal a departure from certainty equivalence.

Having laid down the framework and the notation for the central bank's optimal control problem, in the ensuing section we move to analyze the effect of imposing non-quadratic loss functions onto the optimal choice of the monetary policy instrument $\widetilde{i_t}$.

2.3 Non-standard central bank loss functions

In this section two assumptions underlying the standard central bank intertemporal loss function normally considered in the literature are relaxed with a view to studying the effect of these departures on the role of additive uncertainty. First, we consider the possibility that the loss function may have a non-quadratic functional form, possibly being non-convex over its domain. Second, we analyze the effect of a non-linear weighing of probabilities by the central banker, thus departing from the expected utility paradigm. For each departure from the standard setting, some behavioral justification will be provided.

In order to devise *reasonable* alternatives to the quadratic as a central bank loss function, we should first ponder over the behavioral assumptions underlying the quadratic function. A key element of the quadratic function is that "large" shocks are penalized proportionally more heavily than "small" shocks. However, it appears *prima facie* useful to analyze functional forms that do not penalize large shocks as much as the quadratic function does (or, equivalently, do not attribute so little weight to small shocks). For example, Goodhart (2001) criticizes the assumption that large shocks should be penalized proportionally more than small shocks and reports that "I could never see why a 2% deviation from desired outcome was 4x as bad as a 1% deviation, rather than just twice as bad". Moreover, Rabin (2000) and Rabin and Thaler (2001) have shown (albeit in a context unrelated to monetary policy) that with a convex (e.g., quadratic) loss function and assuming even a small aversion to small shocks (which is normally found in experiments and observing real life behavior), the aversion to moderate or large shocks may easily reach astronomical (behaviorally absurd) levels. As we have seen in previous chapters, an important strand of the economic psychology literature, mainly popularized by Daniel Kahneman and Amos Tversky, building on basic psychological intuition and on a substantial body of experimental evidence, points to the fact that agents generally show a tendency to a mildly *decreasing* (instead of *increasing* as implied by the quadratic function) sensitivity to shocks (computed relative to a "reference point") as the size of the shock rises (see Kahneman and Tversky, 2000, Rabin, 1998, and Thaler, 2000, for a review of this literature). If the results of this literature are to be taken seriously, this would suggest loss functions with a *curvature* different from the quadratic, and in particular functions that are *non-convex*.[70]

Another key element of this literature is the observed widespread tendency for agents to weigh probabilities in a non-linear manner, hence departing from the expected utility paradigm (see, e.g., Starmer, 2000). A non-linear weighing of probabilies by central banks is very realistic and sometimes even advocated from a normative perspective by central bankers. Cecchetti (2000), for instance, reports that "[...] we would expect policy-makers to take action when the mean and variance of forecast distributions are likely to stay the same, while the probability of some extreme bad event increases. [...] even if the variance is unchanged, an increase in the possibility of a severe economic downturn is likely to prompt action." While this is somewhat at odds with Goodhart (2001), who claims that the "[...] main characteristic of risks which policy should *not* try to pre-empt is that they are low-probability events with a high pay-off", a non-linear weighing of probabilities is suggested in both cases. From a descriptive standpoint, non-linear weighting of probabilities – such as a disproportionate weight attached to changes in probabilities around zero, a phenomenon known as the "certainty effect" – is sometimes associated with anticipatory feelings and anxiety (see, for instance, Caplin and Leahy, 2001). While central bankers may be cooler than normal human beings, it is a fair assumption that at times they can certainly become anxious and mis-calculate (or willingly distort) probabilities according to their emotional state.

[70] The Kahneman-Tversky literature favours loss functions that have a "kink" on the reference value, and a diffferent curvature above and below this value.

In sum, central bankers are, above all, humans; the possibility should be at least considered that they display the same attitudes towards risk that have been documented for other types of agents in the economic psychology literature.[71] Hence the main focus of this section of the chapter is evaluating the effect of incorporating these attitudes (decreasing sensitivity and non-linear weighing of probabilities) in the central bank's loss function on the optimal setting of the monetary policy instrument and, in particular, on whether certainty equivalence continues to hold.

Turning first to the issue of the curvature of the period-by-period loss function, a relatively general and simple functional form to be considered in this analysis can be the following (Tversky and Kahneman, 1992):[72]

$$L(\pi_{t+1}) = \left|\pi_{t+1}\right|^{\beta} \tag{2.22}$$

If $\beta = 2$, this is the standard quadratic loss function. In the Kahneman-Tversky literature (see in particular Tversky and Kahneman, 1992) $0 < \beta < 1$ (although β is normally estimated to be very close to one[73]); thus, the loss function is not convex and has a "kink" at zero (clearly the "reference value" in our setting). A loss function specified as in (2.22) is able to encompass a large number of functional forms and behavioral assumptions. The parameter β, in particular, drives the curvature of the loss function. Values $\beta < 1$ identify non-convex loss functions, namely functions where the value of the loss grows

71 Another key trait identified by the Kahneman-Tversky literature is the asymmetric treatment of gains and losses. This feature, however, should not matter for our central banker, who can only lose (and not gain) from a shock to inflation.

72 The same loss function is considered in Vickers (1998).

73 For example, Tversky and Kahneman (1992) provided the estimate $\beta = 0.88$.

less than proportionally with the size of the shock (as in the function postulated in Tversky and Kahneman, 1992).[74]

According to Kahneman and Tversky, diminishing sensitivity is a general trait and a basic element of human perception. In the case of monetary policy-making, some special circumstances have to be taken into consideration. If our central banker behaves in a Kahneman-Tversky manner (i.e., $\beta < 1$), he will be more upset by a 1% *marginal* change in inflation from 2% to 3% than by a change from, say, 15% to 16% (a quadratic central banker will be more upset by the latter). However, the presence of thresholds and "target zones" for inflation (Orphanides and Wieland, 2000), as is common in one way or another in many countries, is likely to impart severe non-linearities to the central bank loss function, because crossing the threshold(s) may be particularly costly. So, a central banker may be *more* concerned by a marginal move of inflation from 2% to 3% than from 1% to 2% if the target zone has an upper bound at, say, 2.5%. Nevertheless, *within* and *outside* the target zone the general principle of diminishing sensitivity should prevail.

These considerations lead us to think that a realistic central bank loss function, at least in those countries where the inflation target is specified in terms of a "target zone" of symmetric width $\overline{\pi}$, is the following:

$$L(\pi_{t+1}) = a(\pi_{t+1}) + \left|\pi_{t+1}\right|^{\beta}, \tag{2.23}$$

where $a(\pi_{t+1}) = a > 0$ if $\left|\pi_{t+1}\right| \geq \overline{\pi}$, and zero otherwise. Thus, there is a fixed cost a to be incurred if inflation goes outside the target zone. Chart 2.1 depicts this loss function

[74] It may be worth noting that the loss function in (2.22) is *symmetric* in its argument. By contrast, the bulk of the literature dealing with departures from certainty equivalence due to non-quadratic loss functions is generally based on *asymmetric* loss functions (see, e.g., Gerlach, 2000, and Ruge-Murcia, 2000). Overall, this seems an important distinctive feature of the analysis of this chapter.

for three key values of β, namely $\beta = 2$ (quadratic), $0 < \beta < 1$ (diminishing sensitivity) and $\beta = 0$ (perfectionism). The case $\beta = 2$ is studied in Orphanides and Wieland (2000). This loss function implies that larger deviations of inflation from the target have an increasingly greater weight, especially when outside the target zone. For the considerations outlined above, we do not regard this loss function as very plausible. Instead, we argue that psychological insight and evidence, together with the standard institutional setting of monetary policy, determines quite naturally a loss function as in (2.23) with $0 \leq \beta < 1$ (diminishing sensitivity), with β is plausibly somewhat less than, but close to, one.

In any case, it is important to stress that the analysis in this chapter is not limited to the case $0 < \beta < 1$ in the loss function in (2.22), even if there are quite strong behavioral grounds to support it, and a full range of possibilies is considered. At one extreme, $\beta = 0$ indicates that the central bank cares about *any* shock, independent of the size of the shock.[75] Alternatively, $\beta > 1$ indicates that losses rise *more than proportionally* with the size of the shock, i.e. the loss function is convex (and it is also differentiable in the whole domain, having no "kink" at zero). As $\beta \to \infty$, the central bank becomes much more concerned with large shocks; in the limit, it only cares about the largest possible shock. This type of central banker can be labelled as "minimax".

In this setting, abstaining from considerations related to the target zone for simplicity and analytical tractability, the central bank's problem may be expressed as the minimization of:

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[75] This is the "perfectionist" central banker referred to above and in Vickers (1998).

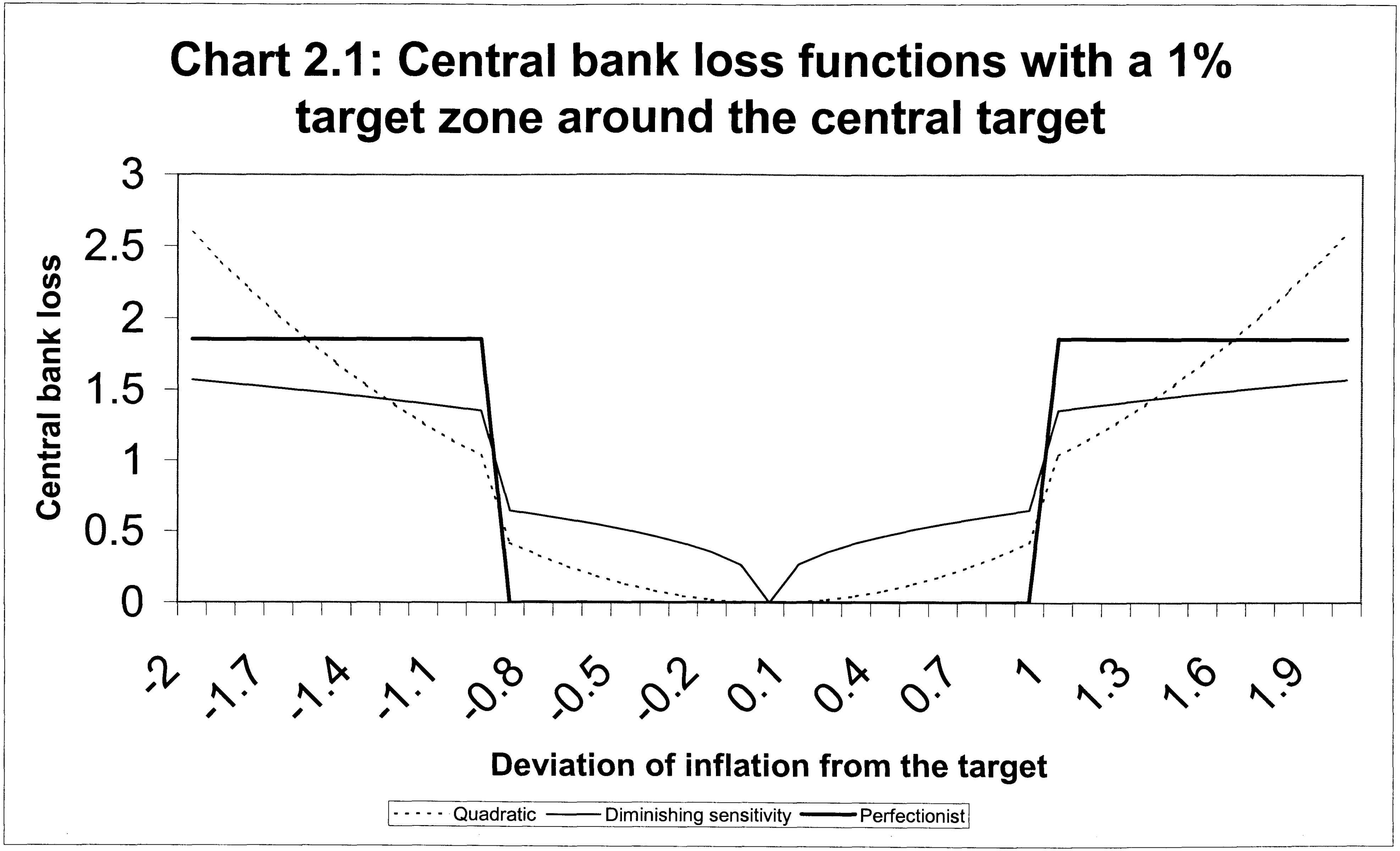
Chart 2.1: Central bank loss functions with a 1% target zone around the central target
Central bank loss
3
2.5
2
1.5
1
0.5
0
-2
-1.7
-1.4
-1.1
-0.8
-0.5
-0.2
0.1
0.4
0.7
1
1.3
1.6
1.9
Deviation of inflation from the target
Quadratic
Diminishing sensitivity
Perfectionist

$$E_t L(\pi_{t+1}) = E_t \left|\varepsilon_{t+1} - \widetilde{i}_t\right|^{\beta} \tag{2.24}$$

It is immediate to see that, in general, the measure of central tendency $\widetilde{i}_t$ that minimizes expression (2.24) will *differ* from the expected value of ε_{t+1} (i.e., zero), except under quadratic preferences (for it is well known in probability theory that $\widetilde{i}_t = E_t\varepsilon_{t+1} = 0$ minimizes (2.24) only if $\beta = 2$). In general, $E_t L(\pi_{t+1})$ will be minimized by the measure of statistical central tendency of order β.

Let us consider, for instance, the case $\beta = 1$.[76] The central bank's loss function collapses to the absolute value of the deviations of inflation from its target (zero):

$$E_t L(\pi_{t+1}) = E_t \left|\varepsilon_{t+1} - \widetilde{i}_t\right| \tag{2.25}$$

Under this specification, the optimal value for $\widetilde{i}_t$ is given by $\widetilde{i}_t = M_t\varepsilon_{t+1}$, where M represents the *median* value of the probability distribution of ε. Hence:

$$i_t = \frac{z_t}{c_3} + \frac{1}{c_3} M_t \varepsilon_{t+1} \tag{2.26}$$

In general, $M_t\varepsilon_{t+1}$ will be different from zero, and the optimal policy will deviate from that identified under the quadratic loss function ($i_t = \frac{z_t}{c_3}$, or $\widetilde{i}_t = 0$). In particular, if the probability distribution of ε_{t+1} is positively skewed, the median will be negative (i.e., smaller than the mean) and the optimal policy will imply a *smaller* value of i_t than it would have been the case under the quadratic loss function. The opposite, i.e. a *higher* value of i_t, will hold true if the probability distribution of ε_{t+1} is negatively skewed. Only if

[76] Goodhart (2001) describing his own experience at the Bank of England's MPC reports that "I believe that I could, more or less, interpret my loss function when I was at the MPC (*symmetrically linear* in the deviation from target at the six to eight quarter horizon)" [emphasis ours]. This would imply that for Goodhart $\beta = 1$.

the probability distribution of ε_{t+1} is symmetric is the standard result recovered ($i_t = \frac{z_t}{c_3}$). More generally, the value $\widetilde{i}_t$ – interpreted as a measure of central tendency minimizing the loss function in (2.24) for an arbitrary value of β – will certainly depend on the probability distribution of ε_{t+1}, i.e. *the principle of certainty equivalence will not hold (unless $\beta = 2$)*. In particular, if $\beta = 0$, the interest rate is set to be equal to the *mode* of ε_{t+1}.[77] The opposite case is $\beta \to \infty$ (i.e., only the largest shock matters), which leads to $\widetilde{i}_t = \varepsilon^*_{t+1}$, where ε^*_{t+1} is the value of ε for which $|\varepsilon_{t+1}|$ is largest. Clearly, this corresponds to a "robust control" or "minimax" solution to optimal monetary policy.

If, however, ε_{t+1} is Normally distributed, then all measures of central tendency collapse to the mean (at least for any finite β), and thus to zero. In this case, the principle of certainty equivalence holds and nothing is lost by using a quadratic loss function.

A second extension which appears to be both interesting and plausible (as discussed above) is a non-linear weighing of probabilities. Now, instead of minimizing $E_t L(\pi_{t+1})$, the central bank will aim at minimizing $E^\delta_t L(\pi_{t+1})$, with E^δ_t defined as follows:

$$E^\delta_t L(\pi_{t+1}) = \int L(\pi_{t+1}) \delta(P(\pi_{t+1})) d\pi_{t+1}, \tag{2.27}$$

where P is either the cumulative probability distribution of π_{t+1} or the probability density, and δ is a weighting function which satisfies $0 \leq \delta(P(\pi_{t+1})) \leq 1$. It should be noted that $\delta(P)$ is a function of P and not of π_{t+1}, with the consequence that if $P(\pi_{t+1}) = P(-\pi_{t+1})$, then $\delta(P(\pi_{t+1})) = \delta(P(-\pi_{t+1}))$. In plain words, the δ transformation preserves symmetry (the "reflection" property).

[77] The emphasis of many central banks on the mode of the probability distribution of future inflation (see, for instance, the fan chart in the Bank of England quarterly Inflation Report) might suggest a low (near-zero) underlying β. However, the emphasis on the mode might also simply reflect a presentational advantage (see Wallis, 1999).

If the δ function is the identity function and P is the probability density, the standard expected utility formulation is recovered. Otherwise, the δ function may be any non-linear transformation of either the cumulative probability distribution or of the probability density. The expression in (2.27) can be thought of as the mathematical expectation of $L(\pi_{t+1})$ computed according to the *transformed* "probability law" $\delta(P)$.[78] In the Kahneman-Tversky literature, P is normally the cumulative probability distribution (this property is often referred to as "rank dependence") and the function $\delta(\cdot)$ is normally found to give more weight to "small" probabilities and less weight to "large" probabilities compared with the linear case (see Kahmeman and Tversky, 2000). For instance, in Kahneman and Tversky (1992) the following function is postulated:

$$\delta(P) = \frac{P^{\omega}}{[P^{\omega} + (1-P)^{\omega}]^{\frac{1}{\omega}}}, \tag{2.28}$$

$\omega > 0$, which encompasses the linear weighing of expected utility models as a special case when $\omega = 1$.[79] If $0 < \omega < 1$, this weigthing function is first concave and then convex, crossing the linear weighing in a point which is also determined by the value of ω. This function is only an example, as many other weighing functions have been proposed in the literature.[80]

In order to deal with one complication at a time and to isolate the effect of a non-linear weighting of probabilities *in itself*, we assume that the period loss function is quadratic, i.e.

78 The caveat has to be borne in mind that the weighted probabilities do not necessarily add up to one over the domain of the variable. In particular, the weighted probabilities are normally sub-additive (i.e., $\int \delta(P(\pi_{t+1}))d\pi_{t+1} < 1$). Thus, the weighted probabilities cannot be interpreted *strictu sensu* as a probability distribution, although we adopt this simplification in loose terms for illustrative purposes.

79 In Kahneman and Tversky (1992) the parameter ω is estimated to be close to .6 for gains and to .7 for losses.

80 Another popular weighting function is the one proposed by Prelec (1998), $\delta(P) = \exp[-(-\ln P)]^{\alpha}$, $0 < \alpha < 1$.

$L(\pi_{t+1}) = \pi_{t+1}^2$. Our central banker is thus called to minimize:

$$E_t^\delta \pi_{t+1}^2 = E_t^\delta (\varepsilon_{t+1} - \widetilde{i}_t)^2 \tag{2.29}$$

Expanding the quadratic term in parentheses leads to:

$$E_t^\delta \pi_{t+1}^2 = E_t^\delta (\varepsilon_{t+1}^2 - 2\widetilde{i}_t \varepsilon_{t+1} + i_t^2) \tag{2.30}$$

The first order condition is thus:

$$\frac{\partial E_t^\delta \pi_{t+1}^2}{\partial \widetilde{i}_t} = 2\widetilde{i}_t - 2E_t^\delta \varepsilon_{t+1} = 0, \tag{2.31}$$

whereby:

$$\widetilde{i}_t = E_t^\delta \varepsilon_{t+1} \tag{2.32}$$

Recalling that $i_t = \frac{\widetilde{i}_t + z_t}{c_3}$, it follows that:

$$i_t = \frac{z_t}{c_3} + \frac{1}{c_3} E_t^\delta \varepsilon_{t+1}, \tag{2.33}$$

which corresponds to the canonical solution $i_t = \frac{z_t}{c_3}$ only if $E_t^\delta \varepsilon_{t+1} = 0$. If the probability distribution of ε_{t+1} is *symmetric*, then $E_t^\delta \varepsilon_{t+1} = E_t \varepsilon_{t+1} = 0$, due to the property of symmetry preservation of the $\delta(P)$ function, and the usual solution $i_t = \frac{z_t}{c_3}$ (thereby the irrelevance of additive uncertainty) is recovered.[81] In general, however, the non-linear weighing will not be neutral, i.e. $E_t^\delta \varepsilon_{t+1} \neq E_t \varepsilon_{t+1} = 0$, hence the probability distribution of ε_{t+1} will not be irrelevant. In other words, the principle of certainty equivalence will *not* hold. In particular, the interplay between a non-linear weighting of probabilities and a skewed probability distribution of ε_{t+1} determines a departure from the certainty equivalence principle.[82]

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81 Of course, a Normal distribution is symmetric and certainty equivalence, as in CS, is also recovered.

82 A very good example of this is in Goodhart (2001): "You should not run a systematically mildly inflation-

As a simple numerical example to illustrate the kind of situation to which the analysis in this section refers, consider a central banker who observes $z_t = 2$, $c_3 = \frac{1}{2}$ (expressed in percentage points). Assume further that the probability distribution of ε_{t+1} is the following:

ε_{t+1}	Prob
-2	.4
-1	.3
1	.1
5	.2

Of course, the assumption is satisfied that $E_t\varepsilon_{t+1} = 0$. Whilst this probability distribution is purely hypothetical, it is nevertheless representative at least of the *type* of uncertainty that central banks often have to deal with in practice. This probability distribution features high-probability and small-size downside risks (-2 and -1, respectively with probabilities .4 and .3), a low-probability, small-size upside risk (+1, with probability .1) and a low-probability, large-size upside risk (+5, with probability .2). For instance, the event "+5" might be associated with "extreme" circumstances such as, say, the collapse of an exchange rate peg. If our central bank has a quadratic loss function and weighs probabilities linearly, it will pick $\widetilde{i_t} = E_t\varepsilon_{t+1} = 0\%$ (and $i_t = \frac{z_t + c_3\widetilde{i_t}}{c_3} = 4\%$). However, central bankers with a different β will pick other values of i_t. For instance, Goodhart (2001) – for whom $\beta = 1$ – will select $i_t = \frac{z_t}{c_3} + \frac{1}{c_3}M_t\varepsilon_{t+1} = 4\% - 2\% = 2\%$ (the median value of ε_{t+1} is, in fact, -1). A "minimax" type of central banker ($\beta \to \infty$) will select $i_t = \frac{z_t}{c_3} + \frac{1}{c_3}\varepsilon^*_{t+1}$ (where ε^*_{t+1} is the value of ε for which $|\varepsilon_{t+1}|$ is largest); hence $i_t = 4\% + 10\% = 14\%$. Finally, a central banker for who $\beta = 0$ will pick $i_t = \frac{z_t}{c_3} + \frac{1}{c_3}\operatorname{mod}(\varepsilon_{t+1})$ – where $\operatorname{mod}(\varepsilon_{t+1})$ is the mode of ε_{t+1}, in this case -2 – and then $i_t = 4\% - 4\% = 0\%$. To sum up:

ary policy because there is a non-zero risk of a 1929 (or a Japanese) collapse in asset prices". This is clearly a recommendation for non-certainty equivalence behaviour based on a non-linear weighing of probabilities (the small probability of a 1929 collapse in asset prices is neglected).

Central banker type	**Optimal policy**
Quadratic ($\beta = 2$)	$i_t = 4\%$
Linear ($\beta = 1$)	$i_t = 2\%$
Minimax ($\beta \to \infty$)	$i_t = 14\%$
$\beta \to 0$	$i_t = 0\%$

A similar reasoning (and a similar discrepancy of policy outcomes) would follow by imposing non-linear methods of weighing probabilities. Assume, for instance, that our central banker weighs probabilities following the weighing function in Tversky and Kahnman (1992), as recalled above in (2.28), with $\omega = \frac{1}{2}$. The modified "probability distribution" (again bearing in mind the caveat that it does not necessarily add up to one) is the following:

ε_{t+1}	Prob
-2	.32
-1	.29
1	.20
5	.25

There follows that $E_t^\delta \varepsilon_{t+1} = 0.52 \neq 0$. Hence, even under the quadratic loss function $\widetilde{i_t} = 1.04\%$, and $i_t = 5.04\% \neq 4\%$. Another possibility is that the high-payoff, low probability event $\{5, .2\}$ is completely neglected by the central banker (Goodhart, 2001, mentions this explicitly, as already noted), while all other probabilities are weighted linearly. In that case, it is immmediate to find that $E_t^\delta \varepsilon_{t+1} = -1$, and thus $\widetilde{i_t} = -2\%$, $i_t = 2\% \neq 4\%$. Of course, many other realistic nonlinear weighing systems might be conceived.

Clearly, the large variation in the policy outcomes is the result of the particular set up of the example, but it is certainly not far-fetched – as already argued above – to think of a real-world probability distribution of future inflation resembling, at least *qualitatively*, to our hypothetical distribution. When the probability distribution of future inflation is very concentrated around the target and/or approximately Normal, however, the importance of the specification of the central bank loss function for actual policy outcomes becomes quite limited. One real world example might be drawn using the probability distribution for future inflation underlying the Bank of England's fan chart. The Bank of England, in particular, releases some moments of the distribution of the inflation forecast some time in the future, which is supposed to drive the MPC's policy decisions. For instance, in the November 2001 Inflation Report it is indicated that the probability distribution of RPIX inflation two years ahead (based on the assumption of unchanged short-term interest rates) was slightly skewed to the downside, with the mode of the distribution being very close to the Bank of England inflation target. Hence, a policy based on a loss function with $\beta = 0$ would somewhat depart from certainty equivalence, as the mode is different from the mean, albeit not by much. Similar considerations apply if $\beta = 1$ (i.e., targeting the median), and so on.

More generally, the probability distributions underlying the Bank of England's fan chart are an interesting tool from the point of view of the analysis of this chapter. With the complete probability distribution of future inflation available, it would certainly be possible to minimize even a non-differentiable loss function like the one proposed in (2.23) using numerical techniques. At the same time, the probability distribution underlying the fan chart is generally quite concentrated around 2.5% and approximately Normal, which makes the selection and analysis of the central bank loss function not very relevant from a practical perspective. However, it should be also noted that, first, the probability distribution of future inflation may not be so well-behaved in all countries, especially in those where inflation is on average still high, and, second, there may be "spurious" technical reasons which explain why the probability distribution is approximately Normal (for instance, because it is in part derived from econometric methods based on the assumption of Normality), while the MPC members' "true" subjective probability distribution of future inflation may be non-Normal. It is worth stressing here that, if certainty equivalence does not hold, estimation and policy cannot be separated, and the econometric technique used for forecasting should be optimally tailored to the user's loss function (see Granger, 1999). It is only in the particular case that certainty equivalence holds and the loss function is quadratic that forecasting methods based on standard inference are optimal. Thus, if the central banker has a non-quadratic loss function, he should invest in a forecasting technique tailored to his loss function, rather than relying on a (for his preferences) sub-optimal forecasting model based on standard inference.

In synthesis, this chapter has found that (i) a non-quadratic curvature of the loss function can determine a departure from the principle of certainty equivalence unless the probability distribution of ε_{t+1} is *Normal*, and (ii) a non-linear weighing of probabilities (alone) will bring about the same result if the probability distribution of ε_{t+1} is not *symmetric*. Therefore, additive uncertainty matters.

Finally, it is interesting to note that none of these departures from the principle of certainty equivalence suggests the optimality of policy gradualism: the optimal response to z_t (the "state of the economy" at time t) is independent of the probability distribution of ε_{t+1}, for $\frac{\partial i_t}{\partial z_t} = \frac{1}{c_3}$ in all the considered cases. It appears that some form of multiplicative uncertainty is necessary to explain monetary policy-makers' tendency to react cautiously to new information.

2.4 Conclusions

This chapter has investigated the robustness of the principle of certainty equivalence, i.e. the irrelevance of the uncertainty over additive shocks, to simple and behaviorally plausible departures of the central bank's loss function from the standard time separable expected quadratic loss. The analysis essentially follows up a paper by Chadha and Schellekens (1999). Compared to that study, this chapter relaxes a key assumption, namely that the additive shock to inflation is Normal distributed. This seems to be quite an important innovation because non-Normal probability distributions of future inflation (e.g., skewed risks as stressed by Goodhart, 2001) are arguably the "daily bread" of central bankers.

The analysis in this chapter has considered a main departure from the expected time separable quadratic loss, which is a main analytical tool of the literature on optimal monetary policy. Building on a substantial body of literature on economic psychology mainly linked to the names of Daniel Kahneman and Amos Tversky, this chapter has considered the possibility that the central bank loss function displays diminishing sensitivity to losses (namely, be non-convex over its argument) and that the central bank weighs probabilities in a non-linear manner (thus departing from the expected utility paradigm). This analysis appears to be interesting and to make sense, because these tendencies are firmly grounded in the economic psychology literature and have been tested and confirmed in a large number of experimental studies (Kahneman and Tversky, 2000). Therefore, it appears *prima facie* reasonable to assume that central bankers – who are humans after all – may also display such attitudes. From this analysis it is found that this form of non-quadratic loss functions generally brings about a departure from the principle of certainty equivalence, unless the probability distribution of the additive shocks to inflation is *Normal* (as far as the property of diminishing sensitivity is concerned, and in general if the curvature of the function is not of the quadratic type) or *symmetric* (as far as a non-linear weighing of probabilities is concerned). Overall, thus, the assumption of a quadratic loss function does not seem to be innocuous when considering the effect of non-Normal distributed additive shocks to inflation onto the determination of the optimal policy.

In sum, this chapter has shown that with additive uncertainty of a non-Normal type, the assumption of quadratic loss functions may not be completely innocuous. Thus, this chapter tends to limit the generality of the conclusions of Chadha and Schellekens (1999), who favored the idea that the assumption of a quadratic loss function is not so restrictive as far as the role of additive uncertainty is concerned. This result should caution against an un-critical reliance on simple policy rules, such as the Taylor rule, in which certainty equivalence holds and uncertainty about the state of the economy does not play any role in the optimal setting of monetary policy instruments.

In addition, this chapter provides an analytical framework for studying the role of the higher moments of (non-Normal) probability distribution of the additive shock to inflation, for example its skewness, for optimal policy-making. This seems to be a very interesting line of research because policy-makers are continuously confronted with this type of uncertainty.[83] At the same time, this chapter does not shed any light on explaining the alleged positive correlation between policy gradualism and uncertainty over the state of the economy in actual policy-making. It seems that imposing some form of multiplicative

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[83] Goodhart (2001) reports that "unlike uncertainty and variance, skew and risk mapped directly into the interest rate decision".

uncertainty is necessary to explain central banks' tendency to act gradually and to react cautiously to new information.

One important direction of further research is whether the kind of departures from certainty equivalence that this chapter has identified may be detected empirically. However, it has to be borne in mind that departures from certainty equivalence which are due to the interaction between a non-quadratic loss function and a non-Normal additive shock may fail to give rise to any empirical regularity over the long term. Indeed, the fact that, at a certain point in time, the probability distribution of future inflation is non-Normal in a certain way does not necessarily imply that it *always* presents that precise form of non-Normality. For instance, additive uncertainty might be one time positively skewed, and the next time negatively skewed; this is a fact of life. In both cases, this might affect policy outcomes, but not in a way that would be easily detectable in empirical analysis, as such analysis would need to know the precise nature of the non-Normality *at each point in time.* With the partial exception of the Bank of England's fan chart, this is not information that it is usually released by central banks around the world.

Chapter 3
The optimal allocation of risks under prospect theory

3.1 Introduction

Prospect theory, first introduced by Kahneman and Tversky (1979), stands out prominently in applied and theoretical research. It is fair to say that it ranks second only to expected utility as a positive theory of human attitudes towards risk. Derived initially from theoretical reasoning, it has found important confirmation in experimental studies. The advanced, rank-dependent version of the theory (Tversky and Kahneman, 1992) has been used by Benartzi and Thaler (1995), Barberis, Huang and Santos (2000) and Barberis and Huang (2001) to provide intriguing explanations to some of the most enduring puzzles in finance, such as the equity premium puzzle of Mehra and Prescott (1985) and the predictability of equity returns at low frequency. These papers pave the way to a promising research agenda in financial economics.[84]

This chapter builds on this strand of literature to apply cumulative prospect theory (hereafter for brevity CPT) to a classical problem of the finance literature, namely the optimal allocation of the agent's stakes (e.g., wealth) among $n > 1$ identically distributed and symmetric sources of risk.[85] According to CPT, losses matter more than gains in the

[84] On the potential of prospect theory in explaining existing puzzles in economics and finance, see Rabin (1998) and Thaler (2000).

[85] The optimal allocation between a safe and a risky asset for an investor whose preferences may be modeled with prospect theory has been already investigated quite extensively in the literature (see for example

agent's value function (loss aversion); the value function is mildly concave for gains and mildly convex for losses (i.e., there is diminishing sensitivity to gains and losses); and the agent weighs non-linearly probabilities of events according to their ranking, thus departing from expected utility theory. In this chapter, it is assumed that the optimal selection of allocation weights among n risky payoff functions represents a "framed prospect" in the sense of Tversky and Kahneman (1986), namely a self-contained decision problem which is analyzed independently by our representative agent. Hence, the agent assigns the allocation weights so as to maximize the prospective value function defined for the problem.

The most striking result of this simple analysis is that, under the fairly general assumption that the subjective probability of obtaining a *perfect hedge* is negligible, the optimal allocation of risks by our CPT agent requires risk *concentration*, rather than *diversification* as held in mainstream finance and economics (Samuelson, 1967). Thus, the CPT agent, instead of "not putting the eggs in the same basket" as is taught in economics textbooks, will optimally "put the eggs in the same basket". The features of CPT crucially driving this result are loss aversion and in particular diminishing sensitivity to gains and losses, which makes the representative agent risk-seeking over losses. Under the assumption of diminishing sensitivity, agents are supposed to care more – compared, for instance, with a mean-variance specification of preferences – about small shocks with high probability than about large shocks with low probability. While risk diversification reduces the likelihood of events very distant from the reference point, it may actually *increase* the

Barberis, Huang and Santos, 2000, Berkelaar and Kouwenberg, 2000a and 2000b, and Hwang and Satchell, 2001). The focus of the present paper is, instead, the optimal allocation of stakes among $n > 1$ risky payoff functions.

noise in its neighborhood, which has an important negative bearing on agents' welfare under CPT. As a consequence, the property of diversification of averaging existing downside risks is welfare-reducing for a CPT agent, whilst it is welfare-improving for an agent with a standard concave utility function. This explains why risk concentration, as opposed to risk diversification popular in the finance literature, turns out to be optimal in the risk allocation problem, provided that the subjective probability of obtaining a perfect hedge is negligible.

To grasp the intuition behind this result, consider this simple example. Assume that you have two credit cards and two wallets. You might decide to spread the two credit cards in the two wallets, or to keep both in one wallet. The probability of losing each of the two wallets is $\frac{1}{4}$, and the event of losing one wallet is independent of the event of losing the other one (thus, the probability that both wallets go lost is $\frac{1}{16}$). If you diversify your risks and you keep the two credit cards separate, you will have a probability of $\frac{1}{16}$ to lose *both* credit cards, and a probability of $\frac{6}{16}$ to lose *one* credit card (therefore, a total probability of $\frac{7}{16}$ of losing *at least one* credit card). If you, instead, put your eggs in the same basket (i.e., the two credit cards in the same wallet), you have a probability of $\frac{1}{4}$ of losing both credit cards, and zero probability of losing only one. Under expected utility theory, the event of losing two credit cards carries proportionally more dis-utility than the event of losing only one credit card; therefore, you will be better off by diversifying your risks, i.e. by putting your credit cards in separate wallets. By contrast, with diminishing sensitivity to losses as postulated in CPT, the dis-utility associated with the loss of both credit cards is *proportionally* smaller than that associated with the loss of one credit card (in other words, the marginal dis-utility of losing one credit card is smaller if you have already lost one credit card, for instance because you have to call your bank anyway). Thus, if you behave like a CPT agent, you will be better off by concentrating your risks (in one wallet), reflecting your risk-seeking attitude for losses. After all, you have a $\frac{3}{4}$ probability of getting off scot-free if you concentrate risks, compared with only $\frac{9}{16}$ if you diversify risks. Overall, keeping the two credit cards in the same wallet does not seem to be a counter-intuitive behavior, although it is certainly in contrast with economics textbooks and mean-variance optimization.

Is it a plausible idea that *rational* risk (loss) averse agents may want to concentrate, rather than diversify, risks, and is it relevant? The answer seems to be positive to both questions. First, there is indeed a large body of evidence showing that agents tend to refrain from diversification in a variety of contexts important for economics (French and Poterba, 1991; Shiller, 1998), and that, when they diversify, they do so in a naive manner (Benartzi, 2001). Thus, the idea that risk concentration might be optimal, at least in some contexts, for rational and risk averse agents, although it is inconsistent with mainstream finance and economics, is not necessarily in contrast with the available evidence. Second, the finding that refraining from diversification might reflect an essential feature of human preferences under risk might in turn help to *explain* why agents (as observed) do not seem to have, in a variety of situations, the strong incentives to diversify risks that economics textbooks attribute them, a tendency which has so far found no convincing and sufficiently general explanation in the literature. Thus, in this respect the results of this chapter appear to be of relevance. Probably the most famous example of lack of risk diversification is the observed "home bias" in financial investment, which has been widely documented, and yet found no convincing explanation, in the literature (Lewis, 1999). Against this background, it can be argued that prospect theory might contribute, together with other factors, to solve the home bias puzzle, which is one of the most enduring in the international finance literature.

The chapter is structured as follows. The optimal allocation of risks for a representative agent whose preferences may be modelled with prospect theory is laid down in Section 3.2. Section 3.3 generalizes the results of the analysis to the situation where the sources of risk are not identically and symmetrically distributed. The relevance of the results of this section is then discussed in Section 3.4. Section 3.5 concludes.

3.2 The optimal allocation of risks under cumulative prospect theory

3.2.1 The environment

In the ensuing analysis we consider a simple environment in which our agent has to allocate his stakes (e.g., wealth) among $n > 1$ separate and identical sources of risk, each of them having zero expected return compared with a reference point. While it would be straightforward to present a more realistic setting where the carriers of risk were also carriers of a non-zero expected return and the sources of risk were not identical, this would make the problem less easily tractable, without implying a real loss of generality. Moreover, a symmetric probability distribution around a reference point is arguably the most intuitive notion of "risk" that agents have in mind, and it is a situation traditionally prominent in finance.

Nonetheless, in Section 3.3 we consider the more general case of not identically and symmetrically distributed sources of risk, which is however slightly more cumbersome and less intuitive from an analytical perspective.

From now on, the n sources of risk have stochastic payoffs x_i^θ, $i = 1, ..., n$, vis-a-vis a reference point for the agent θ (the role of which will be discussed below), with $E(x_i^\theta) = 0$, x_i^θ identically distributed and the joint probability density $\Pr(x_1^\theta, ..., x_n^\theta)$ symmetric in its arguments, with finite variances and covariances.[86]

Our representative agent cares about the following variable y_θ :

$$y_\theta = \sum \alpha_i x_i^\theta, \tag{3.34}$$

where α_i are allocation weights set by the agent, $0 \leq \alpha_i \leq 1$ and $\sum \alpha_i = 1$. In vector notation, let $x' = [x_1^\theta, ..., x_n^\theta]$ and $\alpha' = [\alpha_1, ..., \alpha_n]$, whereby $y_\theta = \alpha' x$. Due to the symmetry of the probability distribution of x, y_θ is symmetrically distributed around zero (this assumption will be relaxed in Section 3.3).

This is a situation well known in the finance literature at least since the work of Samuelson (1967), where it is optimal for an agent with expected utility defined over y_θ to spread his stakes evenly over the n sources of risk (equal diversification, i.e. $\alpha_i = \frac{1}{n}$). Hence, the agent "should not put the eggs in the same basket". However, while this conclusion is mandatory for an agent maximizing expected utility, it is not always warranted for a prospect theory agent. Indeed, as it is shown later, under very general conditions

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[86] In the remainder of this paper, payoffs in excess of the reference point θ will generally have the superscript θ, to emphasize that they are defined in terms of that particular reference point and not in abstract terms.

a prospect theory agent with loss aversion will find it optimal to concentrate, rather than diversify risks.

3.2.2 The prospective value function for the problem

Prospect theory is a well established descriptive theory of human behavior under risk. Its success in explaining phenomena which are puzzling for the mainstream approach based on expected utility maximization is already impressive (see, e.g., Benartzi and Thaler, 1995).

Prospect theory postulates that agents form their decisions in two steps. First, a certain decision problem is "framed" (editing phase), i.e. considered as a self-contained decision problem, often in a very narrow setting. Subsequently, in a second step the decision is taken by maximizing the prospective value function defined for the problem (Kahneman and Tversky, 1979).

Four key features of the prospective value function distinguish it from expected utility. First, *changes* in wealth (or other economic variables), rather than levels, matter most for a prospect theory agent (levels have a second-order importance). In turn, changes are defined in terms of a *reference point* which is determined in the editing phase, and broadly reflects the (uniquely given) agent's expectations and norms. Second, changes are evaluated as gains and losses compared with the reference point, with losses looming larger than gains (*loss aversion*). Third, deviations from the reference point are evaluated with *diminishing sensitivity*, i.e. a marginal deviation from the reference point is more important close to the reference point than far away from it. Diminishing sensitivity reflects a general principle of human perception (Kahneman and Tversky, 2000) and it is the key property of prospect theory driving the results in this chapter. Finally, probabilities of events are weighted non-linearly, generally with large probabilities being undervalued and small probabilities being overvalued compared with the standard linear case.

In this analysis we assume that the allocation of the agent's stakes over the n sources of risk is a *framed prospect* in the sense of Tversky and Kahneman (1986), namely a self-contained decision problem for the agent. As in Kahneman and Tversky (1979), we assume the following *value function*:

$$V(y_\theta) = \{ \begin{array}{c} y_\theta^b, \text{ if } y_\theta \geq 0 \\ -a(-y_\theta)^b, \text{ if } y_\theta < 0 \end{array}, \tag{3.35}$$

where $0 < b < 1$ (diminishing sensitivity), $a > 1$ (loss aversion).[87] The *prospective value function* is the *subjective* expected value of the value function, where the word "subjective" signals that it is computed by using subjectively weighed probabilities, rather than the original probabilities. In line with the literature, we assume that the probability weighing function, $\pi(\cdot)$, is a function of the probability density of y_θ defined from $[0, 1]$ into $[0, 1]$, such that $\pi(\Pr(y_\theta)) \geq 0$, $\pi(0) = 0$, and $\pi(\Pr(y_\theta)) = \pi(\Pr(-y_\theta))$ if $\Pr(y_\theta) = \Pr(-y_\theta)$, due to the reflection property (Tversky and Kahneman, 1992).[88] In the original version of prospect theory (Kahneman and Tversky, 1979), $\pi(\cdot)$ is a function of the probability density, while in the more advanced, cumulative version of the theory (Tversky and Kahneman, 1992) $\pi(\cdot)$ is a function of the cumulative probability distribution. For simplicity

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[87] This value function has a "kink" at the reference point and it is concave over gains and convex over losses. In general, in this paper it is assumed $b < 1$ (Tversky and Kahneman, 1992), but the case $b = 1$ is also touched upon later on.

[88] The property of *reflection* may be defined as preferences between *negative* prospects (i.e., prospects involving only losses) being the "mirror image" of preferences between *positive* prospects (i.e., prospects involving only gains). Besides simplifying the notation considerably, the property of reflection has also found broad confirmation in experimental studies (Kahneman and Tversky, 2000).

of notation, we will refer to the original version of the theory where $\pi(\cdot)$ is defined over the probability density, which in our setting implies no loss of generality, as it will be shown later.

Against this background, the prospective value function for our problem (henceforth PVF) may be written as follows:

$$PVF = \int_{-\infty}^{\infty} V(y_\theta)\pi(y_\theta)dy_\theta, \tag{3.36}$$

where $V(y_\theta)$ is defined as in (2). It is immediate to show that, owing to the symmetric probability distribution of y_θ and to the property of reflection of $\pi(\cdot)$, the PVF can be written as a function of losses only. In fact, developing the PVF we obtain:

$$PVF = -a\int_{-\infty}^{0} (-y_\theta)^b\pi(y_\theta)dy_\theta + \int_{0}^{\infty} y_\theta^b\pi(y_\theta)dy_\theta, \tag{3.37}$$

whereby:

$$PVF = (1-a)\int_{-\infty}^{0} (-y_\theta)^b\pi(y_\theta)dy_\theta - \int_{-\infty}^{0} (-y_\theta)^b\pi(y_\theta)dy_\theta + \int_{0}^{\infty} y_\theta^b\pi(y_\theta)dy_\theta \tag{3.38}$$

Due to the property of reflection, $\pi(y_\theta) = \pi(-y_\theta)$, and y_θ^b for $y_\theta \geq 0$ is equal to $(-y_\theta)^b$ for $y_\theta < 0$. Thus, $[-\int_{-\infty}^{0}(-y_\theta)^b\pi(y_\theta)dy_\theta + \int_{0}^{\infty} y_\theta^b\pi(y_\theta)dy_\theta] = 0$. Therefore:

$$PVF = (1-a)\int_{-\infty}^{0} (-y_\theta)^b\pi(y_\theta)dy_\theta \tag{3.39}$$

Without loss of generality and in order to simplify slightly the notation, we assume $a = 2$ (a value broadly in line with much experimental evidence; Kahneman and Tversky, 2000), so $PVF = -\int_{-\infty}^{0}(-y_\theta)^b\pi(y_\theta)dy_\theta$.

The specification of the PVF in (3.39) makes it clear why considering the original version of prospect theory does not imply any loss of generality. For our CPT agent and given our assumptions about $\Pr(x)$, the framed prospect $\{y_\theta, \Pr(y_\theta)\}$ can be mapped into a one-sided prospect $\{y_\theta, \Pr(y_\theta)\}_{y_\theta<0}$. For one-sided prospects (i.e., prospects which involve only losses or only gains), it does not matter if one considers the original or the cumulative version of the theory (Tversky and Kahneman, 1992).[89]

To sum up, our agent has to select the weights α_i so as to maximize:

$$PVF = -\int_{\sum \alpha_i x_i<0} (-\sum \alpha_i x_i)^b \pi(\sum \alpha_i x_i) d \sum \alpha_i x_i \tag{3.40}$$

It is useful for the ensuing analysis to write down the PVF in (3.40) in vector notation and as a n-dimensional multiple integral defined over x:

$$PVF = -\int_{\alpha' x<0} (-\alpha' x)^b \pi(x) dx \tag{3.41}$$

After having laid down the environment and the assumptions underlying our analysis, the next section studies the CPT agent's attitude towards risk concentration or diversification, which is the key objective of this chapter.

3.2.3 Conditions for optimal risk concentration or diversification

In our simple setting with symmetric and identically distributed risks, only two allocations make sense, namely equal diversification ($\alpha_i = \frac{1}{n}$) and full concentration ($\alpha_j = 1$ for one j, $\alpha_i = 0 \; \forall i \neq j$). To assess the relative desirability of diversification and concentration,

[89] One might argue that the agent should wish to *withdraw* from a lottery involving only losses and no gains. However, this possibility is ruled out by the assumption that the allocation of risks is a framed prospect for the agent. In other words, for our agent the allocation of risks is a self-contained decision problem, which *has to be dealt with* (otherwise, it would not have survived the editing phase). It is implicitly assumed that other benefits (for example, a positive expected return on the lottery) may overcome the perceived losses associated to the allocation of risks, and that the agent analyzes these benefits separately from the allocation of risks.

let us consider the PVF in the two cases:

$$PVF_{div} = -\int_{\overline{x}<0} (-\overline{x})^b \pi(x) dx, \tag{3.42}$$

$$PVF_{con} = -\int_{x_j<0} (-x_j)^b \pi(x) dx, \tag{3.43}$$

where PVF_{div} is the PVF for diversification, PVF_{con} is the PVF for concentration, and $\overline{x} = \sum \frac{x_i}{n}$. Hence, diversification is optimal if $PVF_{div} > PVF_{con}$, while the opposite holds true if $PVF_{div} < PVF_{con}$.

PVF_{div} and PVF_{con} are statistics derived from the joint probability distribution of x. Thus, whether diversification or concentration is optimal depends on this probability distribution, $\Pr(x)$. As it is shown in two examples below, diversification and concentration can both be optimal, depending on $\Pr(x)$.

Before moving closer to the examples, however, it may be useful to explain the intuition behind the idea that diminishing sensitivity is likely to shift the balance in favor of risk concentration, rather than diversification. Essentially, risk diversification can have two consequences, namely *removing* (perfect hedging) or *averaging* existing risks (in particular, for any given probability density of x the averaging mechanism reduces the probability of "large" risks and increases the probability of "small" risks). In our context, only downside risks matter (as it is shown in equation (3.41), the PVF may be transformed into a function of losses only). Clearly, if diversification *removes* downside risks, our agent will be better off than he would be under risk concentration. In fact, a loss averse agent is also risk averse, i.e. he would prefer not to play a lottery with zero expected value. However, if diversification simply *averages* existing *downside* risks without eliminating them, it will reduce, rather than improve the agent's welfare. In fact, a distinct feature of prospect theory is risk-seeking behavior over losses, which is a consequence of the property of diminishing sensitivity.[90] Conversely, under expected utility the agent is risk averse everywhere and diversification is always welfare-improving, even in the absence of perfect hedging.

To pin down the intuition behind these considerations, let us consider two very simple examples. The first example is of optimal risk diversification. Suppose that our agent has to invest his financial wealth and can select two risky assets with payoffs x_1^θ and x_2^θ compared with the reference point (which is, say, the current level of wealth). The joint probability distribution is the following:

	$x_2^\theta = -9\%$	$x_2^\theta = 0\%$	$x_2^\theta = 9\%$
$x_1^\theta = -9\%$	1/9	1/9	1/9
$x_1^\theta = 0\%$	1/9	1/9	1/9
$x_1^\theta = 9\%$	1/9	1/9	1/9

Our agent has to select the decision weight α_1 in order to maximize the PVF defined over $y_\theta = \alpha_1 x_1^\theta + (1 - \alpha_1) x_2^\theta$. As in Tversky and Kahneman (1992), we posit $a = 2.25$ and $b = 0.88$. For the probability weighing function, we assume for simplicity a linear weighing, $\pi(p) = p$. It is immediate to find that the diversified portfolio strictly dominates the concentrated portfolio ($PVF_{div} = -2.00 > PVF_{con} = -2.88$). So, risk diversification is optimal.

In a second example, let us assume the same setting, but with the following probability distribution for $x = [x_1^\theta, x_2^\theta]$:

---

90 The non-linear weighing of probabilities postulated in CPT may also determine a risk-seeking behavior over small-probability losses (see, for example, Prelec, 1998).

	$x_2^\theta = -9\%$	$x_2^\theta = 0\%$	$x_2^\theta = 9\%$
$x_1^\theta = -9\%$	1/9	1/9	0
$x_1^\theta = 0\%$	1/9	1/3	1/9
$x_1^\theta = 9\%$	0	1/9	1/9

Chart 3.1 reports the probability distribution of the payoff on the diversified portfolio ($y_\theta^{div} = \frac{x_1^\theta + x_2^\theta}{2}$) and of the concentrated portfolio ($y_\theta^{con} = x_1^\theta$). In this example the property of averaging risks of diversification stands out clearly. The concentrated portfolio has a downward risk $\{-9\%, \frac{2}{9}\}$; if one multiplies the size of the risk (-9%) by its probability ($\frac{2}{9}$), the total expected downside risk is equal to 2%. The diversified portfolio "averages" this total downside risk of 2% as follows: $\{-9\%, \frac{1}{9}; -4.5\%, \frac{2}{9}\}$. Owing to the property of risk-seeking over losses, the CPT agent does not like such averaging of the total downside risk. In fact, under the same assumptions for the parameters of the PVF of the above example, it is found that risk concentration is optimal ($PVF_{con} = -1.92 > PVF_{div} = -2.00$).

This latter example is an illustration that risk concentration may be optimal under CPT, in contrast with the standard approach based on expected utility maximization. A general sufficient condition for optimal risk concentration is laid down in the following Proposition:

Proposition 1 *If:*

$$\int_{All\ x} \pi(\alpha x = 0)dx = 0 \, \forall \alpha, \tag{3.44}$$

then risk concentration ($\alpha_j = 1$ for one j, and $\alpha_i = 0$ for $i \neq j$) is always optimal for a cumulative prospect theory agent with PVF defined as in (3.41).

Proof. Let D_α^+, D_α^0 and D_α^- be subsets of the domain D of x defined as follows:

$$D_\alpha^+ : \{x / \alpha' x > 0\} \tag{3.45}$$

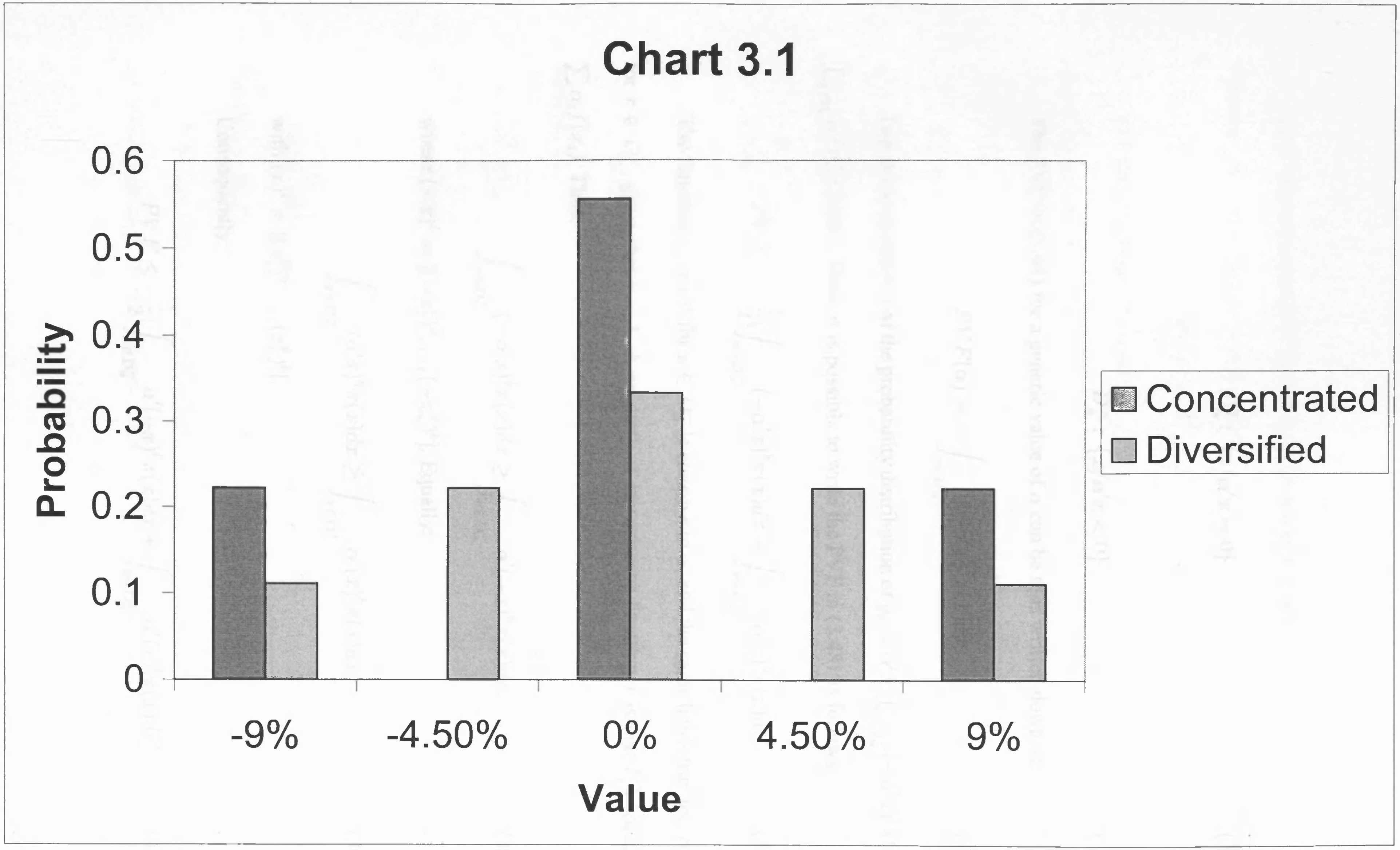
Chart 3.1
Probability
0.6
0.5
0.4
0.3
0.2
0.1
0
-9%
-4.50%
0%
4.50%
9%
Value
Concentrated
Diversified

$$D_\alpha^0 : \{x/\alpha' x = 0\} \tag{3.46}$$

$$D_\alpha^- : \{x/\alpha' x < 0\} \tag{3.47}$$

The PVF in (3.41) for a generic value of α can be thus written down as:

$$PVF(\alpha) = -\int_{x\in D_\alpha^-} (-\alpha' x)^b \pi(x) dx \tag{3.48}$$

Due to the symmetry of the probability distribution of $y_\theta = \alpha' x$, $\int_{x\in D_\alpha^-}(-\alpha' x)^b \pi(x) dx = \int_{x\in D_\alpha^+}(\alpha' x)^b \pi(x) dx$. Thus, it is possible to write the PVF in (3.48) as follows:

$$PVF = -\frac{1}{2}[\int_{x\in D_\alpha^-} (-\alpha' x)^b \pi(x) dx + \int_{x\in D_\alpha^+} (\alpha' x)^b \pi(x) dx] \tag{3.49}$$

The function $(-\alpha' x)^b$ for $x \in D_\alpha^-$ is *concave* in α, and the same holds true for $(\alpha' x)^b$ for $x \in D_\alpha^+$, given that $b < 1$. A property of any concave function f is that $f(\sum \alpha_i x_i) \geq \sum \alpha_i f(x_i)$. Thus:

$$\int_{x\in D_\alpha^-} (-\alpha' x)^b \pi(x) dx \geq \int_{x\in D_\alpha^-} \alpha' (-x)^b \pi(x) dx, \tag{3.50}$$

where $(-x)^b = [(-x_1^\theta)^b, ..., (-x_n^\theta)^b]$. Equally:

$$\int_{x\in D_\alpha^+} (\alpha' x)^b \pi(x) dx \geq \int_{x\in D_\alpha^+} \alpha' (x)^b \pi(x) dx, \tag{3.51}$$

with $(x)^b = [(x_1^\theta)^b, ..., (x_n^\theta)^b]$.

Consequently:

$$PVF \leq -\frac{1}{2}[\int_{x\in D_\alpha^-} \alpha' (-x)^b \pi(x) dx + \int_{x\in D_\alpha^+} \alpha' (x)^b \pi(x) dx] \tag{3.52}$$

This expression may be rewritten as an integral over the whole domain D of x as follows:

$$PVF \leq -\frac{1}{2}\alpha' \int_{x \in D} \mid x \mid^b \pi(x)dx, \tag{3.53}$$

owing to the assumption that $\int_{x \in D_\alpha^0} \pi(x)dx = 0$. Writing the right hand side term of expression (3.53) differently:

$$-\frac{1}{2}\alpha' \int_{x \in D} \mid x \mid^b \pi(x)dx = -\frac{1}{2}\sum \alpha_i \int_{x \in D} \mid x_i^\theta \mid^b \pi(x)dx \tag{3.54}$$

Due to the assumption that the x_i^θ are identically distributed and noting that $\int_{x \in D} \mid x_j^\theta \mid^b \pi(x)dx$ is a moment of the probability distribution of a generic x_j^θ, we can define:

$$x_b^* = \int_{All\ x} \mid x_j^\theta \mid^b \pi(x)dx, \tag{3.55}$$

which takes the same value for every j. Thus, the following holds:

$$PVF \leq -\frac{1}{2}\sum \alpha_i \int_{x \in D} \mid x_i^\theta \mid^b \pi(x)dx = -\frac{1}{2}\sum \alpha_i x_b^* = -\frac{1}{2}x_b^* \tag{3.56}$$

It is immediate to show that the right hand side term is the PVF of the concentrated solution, as:

$$-\frac{1}{2}x_b^* = -\frac{1}{2}\int_{x \in D} \mid x_j^\theta \mid^b \pi(x)dx = -\int_{-\infty}^{0} \mid x_j^\theta \mid^b \pi(x_j^\theta)dx_j^\theta = PVF_{con} \tag{3.57}$$

Thus, taking into account (3.52), it has been shown that:

$$PVF(\alpha) \leq PVF_{con}\ \forall \alpha, \tag{3.58}$$

which leads necessarily to the conclusion that risk concentration is optimal, as it maximizes the PVF of the agent for α. ■

A special case which is worth investigating is *linear* sensitivity to gains and losses, i.e. $b = 1$. Although prospect theory models normally postulate $b < 1$, in empirical studies

b is found to be close to one (Tversky and Kahneman, 1992), so studying the case $b = 1$ makes sense. As the following Proposition shows (and as it is quite intuitive in this setting), if our agent has a linear sensitivity to gains and losses, he will be indifferent between concentrating and diversifying risks:

Proposition 2 *If:*

$$\int_{All\ x} \pi(\alpha x = 0)dx = 0\ \forall\alpha, \tag{3.59}$$

then, if $b = 1$, risk concentration ($\alpha_j = 1$ for one j, and $\alpha_i = 0$ for $i \neq j$) and risk diversification ($\alpha_i = \frac{1}{n}, \forall i$) provide the same utility to the CPT agent.

Proof. Let us write the PVF as in (3.48), with $b = 1$:

$$PVF = \int_{\alpha' x<0} (\alpha' x)\pi(x)dx = \alpha' \int_{\alpha' x<0} x\pi(x)dx = \sum \alpha_i \int_{\alpha' x<0} x_i^\theta \pi(x)dx \tag{3.60}$$

Due to the symmetry of the x_i^θ and the assumption that $\int_{All\ x} \pi(\alpha x = 0)dx = 0\ \forall\alpha$, the following condition holds:

$$\int_{\alpha' x<0} x_j^\theta \pi(x)dx = -\frac{1}{2}\int_{All\ x} \mid x_j^\theta \mid \pi(x)dx,\ \forall j, \tag{3.61}$$

and the PVF is equal to:

$$PVF = -\frac{1}{2}\sum \alpha_i \int_{All\ x} \mid x_i^\theta \mid \pi(x)dx \tag{3.62}$$

As noted above, due to the assumption that the x_i^θ are identically distributed and noting that $\int_{All\ x} \mid x_j^\theta \mid \pi(x)dx$ is a moment of the probability distribution of the generic x_j^θ, we can define:

$$x^* = -\int_{All\ x} \mid x_j^\theta \mid \pi(x)dx, \tag{3.63}$$

which is independent of j. Thus:

$$PVF = \frac{1}{2}x^* \sum \alpha_i = \frac{1}{2}x^*,\ \forall \alpha_i \tag{3.64}$$

The conclusion is that the PVF does not depend on the values chosen for α_i, i.e. the agent is indifferent between risk concentration and diversification. ■

In intuitive terms, Proposition 1 states that if perfect hedging is not possible with strictly positive weighted probability (assumption in (3.44)), it is always optimal for a CPT agent to concentrate risks. Only if $b = 1$ (linear sensitivity, i.e. neither decreasing nor increasing) is the agent indifferent between risk concentration and diversification (Proposition 2). This seems to be a rather striking result, as (at least some) risk diversification is instead *always* optimal in the standard setting, i.e. under expected utility theory (Samuelson, 1967). Moreover, the assumption in (3.44) is far from being a mere theoretical *curiosum*. There are indeed many examples in real life where the weighted probability of obtaining a *perfect* hedge is negligible. Notably, the probability of having a perfect hedge is always zero with continuous probability distributions. Assume, for instance, that a CPT investor has to allocate his wealth among n identical risky assets, the payoff on each of them Normally distributed with zero mean. This is a standard finance textbook problem, which has found solution in mean-variance diversification since the work of Markowitz (1952). Under prospect theory, irrespective of the correlation matrix among the n returns[91], the agent will be better off by concentrating risk on only one asset. Another example is, of course, the situation facing you with your two credit cards mentioned in the Introduction of this chapter.

[91] Unless, of course, two returns are perfectly negatively correlated – a situation very uncommon in practice.

It has to be emphasized that (3.44) is a *sufficient*, but not a *necessary* condition for optimal risk concentration. There might be cases in which perfect hedging has a strictly positive weighted probability and yet risk concentration remains optimal. To show this, assume that (3.44) does not hold and, therefore, that:

$$\int_{-\infty}^{\infty} \pi(\overline{x} = 0)dx > 0, \tag{3.65}$$

which shifts the balance in favor of risk diversification (now there is a strictly positive probability that diversification eliminates downside risks, which is clearly beneficial for our CPT agent exactly as for an agent maximizing expected utility). The difference between the concentrated and the diversified PVF may be derived straightforwardly as follows:

$$PVF_{con} - PVF_{div} = PVF_{AVE} - PVF_{HED}, \tag{3.66}$$

where:

$$PVF_{AVE} = \int_{x/\overline{x}<0} (-\overline{x})^b \pi(x)dx - \int_{x/x_j<0} (-x_j)^b \pi(x)dx, \tag{3.67}$$

$$PVF_{HED} = \int_{x/\overline{x}=0, x_j<0} (-x_j)^b \pi(x)dx \tag{3.68}$$

From Proposition 1, we know that $PVF_{AVE} \geq 0$ (i.e., the property of diversification of averaging downside risks decreases the agent's utility), while $PVF_{HED} \leq 0$ (i.e., the perfect hedging property of diversification increases the agent's welfare). The relative desirability of risk concentration or diversification depends on whether the downside risks averaging effect (PVF_{AVE}) prevails over the perfect hedging effect (PVF_{HED}). If $\mid PVF_{AVE} \mid > \mid PVF_{HED} \mid$, risk concentration remains optimal even if perfect hedging has a strictly positive weighted probability. However, given that b is normally to found to be close to (albeit slightly smaller than) one (Kahneman and Tversky, 2000), the term PVF_{AVE} is unlikely to be large (because the value function is only mildly convex over losses, i.e. it is very close to a piecewise linear function). Thus, even a small probability of having a perfect hedge is likely to put the odds in favor of risk diversification.

3.3 The more general case

In this section the assumptions regarding the distribution of x previously imposed are relaxed, so as to consider a more general setting than in the previous section. From now on, the x_i^θ can have whatever probability distribution, i.e. also non-symmetric and non-identically distributed. The purpose of this section is to identify the conditions under which risk concentration or diversification is optimal for our CPT agent in this general case. As this setting is slightly more complicated to deal with from an analytical perspective, the results will be less neat and general than in the previous section. Nonetheless, the basic intuition behind the results remains the same.

Using the same notation as in the proof of Proposition 1, we can write the PVF as follows:

$$PVF(\alpha) = PVF_+ - aPVF_-, \tag{3.69}$$

where $PVF_+ = \int_{x\in D_\alpha^+}(\alpha'x)^b\pi(x)dx$, $PVF_- = \int_{x\in D_\alpha^-}(-\alpha'x)^b\pi(x)dx$.

Let us consider:

$$PVF^* = \sup_\alpha PVF, \tag{3.70}$$

which is the maximum value for α of the PVF. In general, the optimal α cannot be derived analytically as the first and second order conditions cannot be solved in closed form, but it is easy to compute it using numerical methods. At the optimal value of α, say α^*, one obtains:

$$PVF^* = PVF_+^* - aPVF_-^* \tag{3.71}$$

There follows:

Proposition 3 *If:*

$$\int_{All\ x} \pi(\alpha x = 0)dx = 0\ \forall\alpha, \tag{3.72}$$

then risk concentration ($\alpha_j = 1$ for one j, and $\alpha_i = 0$ for $i \neq j$) is optimal for a cumulative prospect theory agent with PVF defined as in (3.41) if:

$$PVF_+^* - aPVF_-^* < 0, \tag{3.73}$$

while at least some risk diversification (there are at least two i and j for which $\alpha_i > 0$ and $\alpha_j > 0$) is optimal if:

$$PVF_+^* - aPVF_-^* > 0 \tag{3.74}$$

Proof. If (3.73) holds, then:

$$PVF^* = -a'PVF_-^*, \tag{3.75}$$

with $a' > 0$. Hence, the PVF^* may be written as a function of losses only as in (3.41) and it is therefore a convex function in α, at least in the neighbourhood of $\alpha = \alpha^*$. Thus, the maximum value for the PVF must be necessarily found at the boundary of the parameter space of α, implying that risks concentration has to prevail.

By contrast, if (3.74) holds, then:

$$PVF^* = a'PVF^*_+, \tag{3.76}$$

with $a' > 0$. In this case, the PVF^* may be written as a function of gains only and it is therefore a concave function in α, at least in the neighbourhood of $\alpha = \alpha^*$. Under these conditions, Theorem II of Samuelson (1967) applies and at least some diversification is mandatory. ■

It should be emphasized that Proposition 3 is a generalization of Proposition 1, where the assumption of identically distributed and symmetric x_i^θ ensures that the condition (3.73) holds, leading to the result that risk concentration is optimal for the CPT agent.

In intuitive terms, whether risk concentration or diversification prevails depends on the position of the reference point relative to the probability distribution of the n sources of risk. If the reference point is "high" and losses tend to prevail, agents are willing to be risk-seeking and therefore prefer risk concentration. For example, in the case of the credit cards mentioned in the Introduction the (implicitly assumed) reference point (no credit card lost) is an upper bound for the possible states of the world, and any event different from it would be perceived as a loss, prompting risk-seeking behavior by our CPT agent. Conversely, if the reference point is "low" and most states of the world are perceived as a gain, risk-averse behavior would prevail and risk diversification with it. Suppose, for instance, that in the same example of the credit cards our agent chose the worst possible outcome (both credit cards lost) as the reference point. In such case, all events different from the reference point would be perceived as a gain, for which a CPT agent is risk averse. Hence, it would be optimal for this agent not to keep the eggs in the same basket, namely to spread the credit cards between the two wallets.[92]

It might be argued that, in most circumstances, the condition in (3.73) is more likely to hold than that in (3.74), because losses are more important than gains to a CPT agent ($a > 1$). This is the reason why, for instance, the condition (3.73) prevails if the probability distributions of the x_i^θ are symmetric around zero. Ultimately, the likelihood that condition (3.73) and thereby risk concentration hold hinges on whether the assumption that the allocation of risks is treated as an independent decision problem (framed prospect) is justified or not. Were this assumption not justified, it would not be possible to postulate that the agent willingly enters in a lottery involving mainly losses.[93] Nevertheless, while the psychological process leading to any decision depends uniquely on the nature of the problem and no generalization can be made, it is fair to say that the assumption that the allocation of risks can be treated as a framed prospect makes sense in many circumstances (for instance, where the agent cannot avoid to take risks, a situation very common in life). Overall, the conclusion seems warranted that risk concentration is likely to be optimal at least in some circumstances under CPT, and it therefore deserves serious consideration as far as its consequences for economic behavior are concerned.

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92 As the value function is only mildly concave for gains, the incentives for risk diversification would in most cases be small, e.g. much smaller than with a mean-variance specification of preferences. Of course, also the incentives towards concentration (when condition (3.73) holds) will be rather small. With the value function close to a piecewise linear function, the issue of risk concentration or diversification loses much of its importance, as Proposition 2 suggests, unless diversification makes perfect hedging possible.

93 It is interesting to note that risk concentration can never be preferable to not entering the lottery at all, i.e. no gain and no loss (and $PVF = 0$), as expressions (3.73) and (3.74) in Proposition 3 show.

3.4 Prospect theory: a solution to the "home bias" puzzle?

The key result of the previous sections is that risk concentration may be optimal for a *rational* loss averse agent displaying risk-seeking behavior on losses, provided that the probability of obtaining a perfect hedge is negligible and the agent sees the allocation of risks as a self-contained decision problem. In other words, risk diversification does not lead to risk (loss) minimization for our CPT agent, in contrast with a standard expected utility agent. The two next interesting questions are, first, whether this result is plausible, and, second, whether it is relevant.

As to the plausibility of the result that rational loss averse agents may prefer risk concentration to risk diversification, it should be noted that lack of risk diversification is a tendency which is often found among economic agents and has been widely documented in the literature. For example, French and Poterba (1991) and Shiller (1998) reported that agents seem to have no appetite for diversification, be it in financial investment or in real estate acquisition. Rode (2000) reported survey data indicating that diversification is normally seen by agents as being a different thing from risk minimization. Indeed, while agents in some occasions appear to have a vague feeling that diversification is beneficial (for instance in financial investment), they fail to diversify appropriately (Goetzmann and Kumar, 2001). Moreover, when they actually diversify risks, they tend to do so in a very naive manner, for instance by following the $1/n$ heuristic (Benartzi, 2001). Overall, it appears that the idea that agents in a variety of contexts do not regard risk diversification as contributing to risk minimization, in contrast with the normative implications of expected utility theory, is broadly in keeping with the available empirical evidence. It is therefore an interesting and plausible result that prospect theory (namely, a theory which is based on strong psychological foundations and which has received a substantial amount of empirical support) implies that risk diversification is not mandatory, but rather predicts that its opposite, risk concentration, is likely to emerge in a variety of situations. All in all, the analysis in this chapter seems to have identified a key difference between the implications of prospect theory and those of standard expected utility theory.

As to the relevance of the results in this study to explain real world phenomena, perhaps the most famous example of a lack of diversification is the "home bias" in international financial investment (French and Poterba, 1991). As noted by Lewis (1999), the so-called "home bias puzzle" is still far from having received a satisfactory explanation despite repeated efforts in the literature. It has to be noted that one of the key alleged benefits of international diversification is the minimization of risk for a given expected return, which emerges naturally in a mean-variance context (Markowitz, 1952; Grubel, 1968). In other words, international diversification should be no more than an application of the general law "do not put your eggs in the same basket" (in this case, country). Noting that in financial investment the probability of obtaining a perfect hedge is probably nil and that the expected return on the portfolio may be considered as an appropriate reference point for the normal investor, loss aversion with diminishing sensitivity suggests that concentrating risks on a *single* asset might be optimal from the point of view of risk (loss) minimization, as noted in the previous sections. Hence, the investor should "put all his eggs in one basket", i.e. country. Thus, if prospect theory is an accurate description of human attitudes towards risk, the benefits of international diversification would be reduced to a significant extent. In other words, holding internationally diversified portfolios might not be as "desirable" for economic agents as it is commonly regarded.

Yet, while prospect theory could explain the tendency to concentrate risks on a single asset rather than to hold a well diversified portfolio, it cannot explain why the single asset the investor chooses is a *domestic* one. Clearly, the home bias in financial investment is probably a complex and multi-faceted phenomenon, which reflects the influence of various factors. For instance, transaction costs (Tesar and Werner, 1992), a greater familiarity with domestic assets (Gehrig, 1993), and the fact that holding a portfolio concentrated on domestic assets was mandatory in the past due to restrictions to international capital flows, are all likely to play an important role. However, it is important to stress that, as shown in this chapter, risk diversification aimed at risk minimization (for a given level of expected return) could be far from being the powerful force to remove these obstacles to international diversification that has been hypothesized in the past (Grubel, 1968), if investors indeed behave like prospect theory agents. Thus, even if prospect theory *per se* cannot account for the home bias, it might be a key element in its overall explanation.

3.5 Conclusions

Cumulative prospect theory posits that agents care about losses against a reference point comparatively more than about gains of equal size (loss aversion), and that the importance of marginal gains and losses is higher close to the reference point than away from it.

This chapter has studied the optimal allocation of a representative CPT agent's stakes among n identical sources of risk (which it is assumed to be a "framed prospect" for the agent in the definition of Tversky and Kahneman, 1986). This is a classical problem of the finance literature since at least Markowitz (1952). The key result of this analysis is that, due to the prevalence of losses in the agent's value function and to risk-seeking behavior for losses, the property of diversification of averaging downside risks is welfare-reducing rather than welfare-improving. Therefore, provided that the subjective probability of obtaining a perfect hedge is negligible, our CPT agent is better off by *concentrating* rather than by *diversifying* risks, which is in contrast with the normative prediction of standard expected utility models normally found in economics and finance textbooks. Noting that there is ample evidence that agents refrain from diversifying risks in a variety of contexts (French and Poterba, 1991; Shiller, 1998), the overall conclusion of the chapter is that optimal risk concentration is an interesting and rather realistic feature of prospect theory, and an important point of departure of this theory from expected utility theory. The chapter has also argued that the optimality of risk concentration, at least in some circumstances, might be one of the factors (albeit certainly not the only one) which explain the widely observed "home bias" in international financial investment.

The analysis in this chapter might be expanded in several directions, and two of them might be mentioned here. First, it might be useful to link prospect theory with other observed behavioral biases to assess the overall importance of psychological factors for the incentives for (or against) risk diversification. For example, the "event-splitting" heuristic described in Starmer and Sugden (1993) – namely, the tendency for risks to appear bigger under a disaggregated description – is likely to further contribute to shift the balance against risk diversification. Conversely, the naive diversification strategies described in Benartzi

(2001) are expression of an equally naive agents' preference for diversification as an "end in itself". Second, it would be interesting to study models in which the reference point evolves endogenously depending on the allocation of risks. In this respect, disappointment aversion introduced by Gul (1991), with its focus on the endogenous formation of reference points, seems to be a good place to begin.

Chapter 4
Myopic loss aversion, disappointment aversion, and the equity premium puzzle

4.1 Introduction

The equity premium puzzle introduced by Mehra and Prescott (1985) is still far from having received a fully-fledged and convincing explanation in the literature (Kochelarkota, 1996; Siegel and Thaler, 1997; Mehra, 2001). A puzzle arises in the first place because, according to Mehra and Prescott, the magnitude of the covariance between the marginal utility of consumption and equity returns is not large enough to justify the 6% (or so) historical equity premium observed in the United States over the last century. Several possible explanations to this puzzle have been proposed in the literature. These include first order risk aversion (Epstein and Zin, 1990), habit formation (Costantinides, 1990; Otrok, Ravikumar, and Whiteman, 2002), fear of disaster (Reiz, 1988), survivorship bias (Brown, Goetzmann and Ross, 1995), borrowing constraints coupled with consumer heterogeneity (Constantinides, Donaldson and Mehra, 2001), and, notably, myopic loss aversion (Benartzi and Thaler, 1995; Barberis, Huang and Santos, 2000) and disappointment aversion (Ang, Bekart and Liu, 2000). In spite of the sheer research effort, however, the profession has still to reach a consensus on the explanation of the large equity premium observed historically in the United States and in other industrialized countries.

Against this background, this chapter takes a closer look at the "behavioral finance" explanations of the equity premium puzzle proposed thus far in the literature, namely *myopic loss aversion* (Benartzi and Thaler, 1995) and *disappointment aversion* (Ang, Bekaert and Liu, 2000). The two "behavioral" explanations have something in common, namely the fact that agents' preferences are defined against a reference point (*reference dependence*), and not in absolute terms as in the standard approach.[94] Moreover, the maintained assumption in both approaches is that agents *narrow-frame* the problem of how to allocate wealth between safe and risky assets and consider this problem in an independent manner, focusing on the prospects for returns without considering the co-variability with consumption. This is in contrast with the expected utility approach used by Mehra and Prescott (1985) and many other subsequent papers.

Myopic loss aversion, proposed by Benartzi and Thaler (1995) – henceforth BT – posits that economic agents are averse to losses at an irrationally short horizon, due to institutional reasons or because they are affected by a behavioral bias (in particular, because they are too anxious to evaluate the performance of their portfolio on a short-term basis). BT showed that the observed equity premium is consistent with a moderate degree of loss aversion at an investment horizon of approximately one year, which BT regard as intuitively reasonable. Under loss aversion, agents have a *fixed* reference point (which BT assume to be the current level of wealth) against which they evaluate gains and losses.

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[94] The recent focus on habit formation as a possible explanation of the equity premium appears to be closing the gap between the "standard" and the "behavioural" approaches (indeed, habit formation can be interpreted as a form of reference independence). For example, Otrok, Ravikumar and Whiteman (2002) show that habit agents are much more averse to high frequency fluctuations than to low frequency fluctuations, which is a result in some sense very close to the analysis in Benartzi and Thaler (1995).

Disappointment aversion, introduced by Gul (1991) and applied to explain the high premium required on equity by Ang, Bekaert and Liu (2000) – henceforth ABL – is based on the idea that reference points evolve *endogenously*. In particular, the *certainty equivalence* of a lottery may become a reference point for agents, and outcomes in excess (short of) the certainty equivalence are a source of elation (disappointment) for the agent. Reflecting the idea that pain is more urgent than pleasure, the disappointment related to outcomes below expectations is assumed (and normally found) to be stronger than the elation related to outcomes exceeding expectations. Unlike under loss aversion, a lottery with a higher certainty equivalence is not necessarily an improvement compared with a lottery with a smaller certainty equivalence, because higher expectations can result in a stronger disappointment (Jia, Dyer and Butler, 2001).

The main objective of this chapter is to take a close look at, and in particular carry out a sensitivity analysis of, the two behavioral finance explanations of the equity premium puzzle. We concentrate especially on the role of the *time horizon* in determining the size of the equity premium. This seems a crucial dimension of the problem because it is quite easy to explain a high equity premium with a short time horizon, at which stocks are very volatile, while it may be more difficult at longer time horizons. The key questions of this chapter are, first, how dependent the explanation proposed by BT is on a very short time horizon (how myopic agents have to be, assuming a reasonable degree of loss aversion) and, second, at what horizons reasonable parameters for the degree of disappointment aversion can explain the historical equity premium, which is an issue not directly addressed by ABL.

The analytical approach proposed in this chapter is a very simple one, inspired by, but not identical to, the analyses by BT and ABL. We posit that the choice between a safe Treasury bill and a risky equity portfolio represents a *framed prospect* for our representative agent, namely a self-contained decision problem which is analyzed independently. The agent has a certain time horizon in mind when editing the decision problem. The value function is defined in terms of *excess* returns on the risky asset at the relevant time horizon for both loss aversion and disappointment aversion (for the latter, we use the simple risk-value generalized disappointment utility function proposed by Jia, Dyer and Butler, 2001). It is important to emphasize the difference with the analysis in BT and ABL who focus on the *absolute* return on equity and bonds (or Treasury bills). Moreover, the chapter derives a model of expected returns and risk based on an equilibrium condition requiring stocks to be held in positive amount. Assuming rational expectations (the agent not making systematic mistakes in expectations), a testable condition linking the degree of loss and disappointment aversion respectively and the time horizon can be derived and tested on the data.

The empirical analysis, based on data for excess returns on stocks in the United States from 1871 onwards, shows that an explanation based on loss aversion is crucially dependent on a *very short* time horizon, and already a horizon of three years or so seems too long for loss aversion to be a satisfactory explanation of the historical equity premium. So, loss aversion requires a high degree of "myopia" and BT's results are crucially dependent on this assumption. By contrast, disappointment aversion appears to be a satisfactory explanation of the historical equity premium no matter the time horizon. In fact, realistic values for the degree of disappointment aversion can be found for long time horizons such as ten years.

This reflects the empirical finding that, while it is almost impossible to lose on stocks compared with safe assets if the time horizon is relatively long, stocks may disappoint even at long horizons. Overall, we suggest that this feature of disappointment aversion makes it an interesting explanation of the equity premium, possibly more robust than loss aversion because it can accommodate different time horizons. Moreover, we offer some speculations which point to the idea that disappointment aversion can be considered as a quite realistic representation of preferences especially in the context of delegated portfolio management with a principal-agent relationship affecting the nature of the portfolio selection problem. Still, taking into account that the arguments put forward by BT in favour of myopic loss aversion are very convincing, we prefer to think of the results of this chapter as indicating that a *combination* of myopic loss aversion at short horizons and disappointment aversion at longer horizons is an attractive overall explanation of the equity premium puzzle. In fact, the idea that agents may have a *multiple* time horizon and consequently multiple reference points in making portfolio allocation decisions seems interesting and plausible, as is also suggested by BT.

The chapter is organized as follows. After briefly describing the related approaches followed by BT and ABL in Section 4.2, we derive expected returns under loss aversion and disappointment aversion in Section 4.3, and derive a testable condition. In Section 4.4, we bring this condition to the data and find results which are then discussed in Section 4.5. Section 4.6 concludes.

4.2 The "behavioral finance" explanations of the equity premium puzzle

4.2.1 The Benartzi and Thaler (1995) approach

Loss aversion is based on psychological insight as well as experimental evidence (Kahneman and Tversky, 2000). It is a prominent feature of prospect theory, first introduced by Kahneman and Tversky (1979). Key elements of prospect theory are reference dependence (outcomes are evaluated not in absolute terms, but rather compared with a reference point), diminishing sensitivity (marginal departures from the reference point count more if they are close to it), loss aversion (losses compared with the reference point loom larger than gains) and non-linear weighing of probabilities (thus departing from the linear weighing as in expected utility theory). Moreover, the decision problem is analyzed in two steps. First, the problem is "edited" in a certain (narrow) frame. Second, the agent takes his decision by maximizing his prospective value function defined for the problem.

BT held the view that, especially with the increasing institutionalization of financial markets, loss aversion, combined with a myopic behavior of agents (very often financial intermediaries), might explain the equity premium puzzle. At time t, agents are concerned about *returns* (and not wealth *levels*) at time $t+h$, where h is the investment horizon.[95] The value function used by BT is the following:

$$V(x_{t+h}) = \{ \begin{array}{c} x_{t+h}^b, \text{ if } x_{t+h} \geq 0 \\ -a(-x_{t+h})^b, \text{ if } x_{t+h} < 0 \end{array}, \tag{4.77}$$

[95] BT point out that what matters is the *evaluation period* – which is the *implicit* relevant time horizon – rather than the original time horizon for the agent.

where x is either the nominal or the real return on equity or bonds, the reference point for the agent being the current level of wealth, and $a = 2.25$, $b = 0.88$ (which are estimates drawn from Tversky and Kahneman, 1992). As $2.25 > 1$, the representative agent is loss averse.[96]

BT bootstrap from the historical time series of equity and five-year bond returns over the sample period 1926-1990, and compute the horizon h for which the representative investor with value function as in (4.77) is indifferent between investing in equity and in bonds. They find that if h is approximately one year, investing in bonds and equity provides the same prospective value. Thus, if the agent's investment horizon is approximately one year, loss aversion can explain the equity premium puzzle. BT also report that considering a non-linear weighing of probabilities and a piecewise linear function (i.e., $b = 1$ in (4.77)) does not change the substance of the results. Hence, BT interpret this finding as suggesting that agents may forgo superior returns on equity due to their "myopia", i.e. the irrationally short time horizon at which they evaluate gains and losses. So, agents are "willing" to pay a high price for their "excessive vigilance".

In this chapter, we consider the simple linear case $b = 1$, which is broadly consistent with the available empirical evidence (Tversky and Kahneman, 1992) and greatly simplifies the notation. So, we shall consider the following piecewise linear loss aversion value function:

$$V_{LA}(x_{t+h}) = \{ \begin{array}{c} x_{t+h}, \text{ if } x_{t+h} \geq 0 \\ a_{LA} x_{t+h}, \text{ if } x_{t+h} < 0 \end{array} , \tag{4.78}$$

where x_{t+h} is the variable of interest for the agent, and LA stays for "loss aversion".

96 Barberis, Huang and Santos (2000) use the current level of wealth plus the risk-free rate as the reference point.

4.2.2 The Ang-Bekaert-Liu (2000) approach

Disappointment aversion, introduced by Gul (1991), is based on the idea that agents are disappointed if the outcome of a lottery falls short of the certainty equivalence, while they are elated if the outcome exceeds the certainty equivalence. In both disappointment aversion and loss aversion, a *reference point* plays a key role, but there is an important difference between the two theories. Under disappointment aversion, the reference point is endogenous to the lottery, i.e. it may change for different lotteries. By contrast, under loss aversion the reference point is generally given, so exogenous to the lottery.

ABL examine the role of disappointment aversion in the determination of the equity premium, by introducing an otherwise standard power utility function $U(w)$, where w is wealth, in which outcomes are weighted differently according to whether they exceed or fall short of the certainty equivalence. They show that a reasonable value for the degree of disappointment aversion is consistent with the historical equity premium if the investment horizon of the agent is one quarter or one year.

To illustrate ABL's explanation in a simple way, a convenient representation of disappointment aversion is the risk-value generalized disappointment aversion utility function proposed by Jia, Dyer and Butler (2001) written in terms of returns:

$$V_{DA}(x_{t+h}) = x_{t+h} + \left\{ \begin{array}{l} e(x_{t+h} - E_t x_{t+h}), \text{ if } x_{t+h} \geq E_t x_{t+h} \\ d(x_{t+h} - E_t x_{t+h}), \text{ if } x_{t+h} < E_t x_{t+h} \end{array} \right. , \tag{4.79}$$

where DA stays for "disappointment aversion", and $d > e > 0$, reflecting the idea that disappointment is more important than elation (this is closely related to the concept that losses loom larger than gains and that agents are loss averse). So, stocks may disappoint exactly because their rate of return has a high expected value (i.e., $E_t x_{t+h}$ is "high"). If d is large and stock returns are very volatile, the equity premium required to compensate for a high probability of disappointment will have to be high.

4.3 Loss aversion, disappointment aversion, and the investor time horizon

4.3.1 A simple specification of preferences for loss and disappointment aversion

In this chapter we build on the "behavioral" theories of the equity premium and propose a simple approach to map combinations of, respectively, loss aversion and disappointment aversion with the time horizon, with the objective of assessing their overall plausibility from an empirical perspective. In particular, the analysis of this chapter is built on the following assumptions. First, we posit that the allocation of a representative agent's wealth between a safe and a risky asset constitutes a *framed prospect* in the sense of Tversky and Kahneman (1986), i.e. a self-contained decision problem. As in BT and ABL, we assume that the representative agent considers his portfolio choice problem in isolation, and does not look at the correlation with other sources of variability in consumption. Second, we assume that the agent has only two assets available, namely a risk-free Treasury bill and a risky equity.[97]

[97] BT consider a five-year bond as a "safe" asset. In this chapter, we prefer a one-year Treasury bill because it does not have practically *any* risk, at least in nominal terms, making it a plausible reference point for our representative agent. By constrast, five-year bonds bear some risk at horizons shorter than five years.

There is almost a consensus in the literature that a is a number of the order of magnitude of 2. Tversky and Kahneman (1992) estimated a to be 2.25, and this number has been later broadly confirmed in several experimental studies (Kahneman and Tversky, 2000). A number of this magnitude also makes much sense from an intuitive, everyday life perspective; it indicates that agents are more or less twice more upset for a loss than they are happy for a gain. However, there seem to be no compelling reasons to assume a certain time horizon for the representative investor. BT put forward some arguments in favour of the one year horizon (e.g., portfolio managers often report to their clients on a yearly basis), but they did not provide definitive answers on this matter. In addition, BT treat myopic behavior as an essentially "irrational" behavior, explaining the equity premium as the "cost of impatience". So, it should not be ruled out *a priori* that the representative agent has a time horizon different from one year, especially a longer one which might arguably be interpreted as being more "rational" from a normative standpoint.

Even if we have the same question in mind, our approach is different from that of BT and ABL in two main respects. First, as noted, our value function is defined on the *excess* return on the risky asset, rather than on its absolute return. We believe that this measure makes more sense when analyzing the allocation of wealth between a safe and a risky asset, as the agent is likely to be concerned above all by the *relative* performance of the two (Cochrane, 1997).[98] Second, BT's aim was to find out the time horizon h at which the representative investor with value function as in (4.77), and *given* a_{LA}, is indifferent between investing in equity and bonds. Our approach is different and slightly more general.

[98] A value function defined on the excess return on the risky asset is also used in Barberis, Huang and Santos (2000).

We seek to look at the combinations $\{a_{LA}, h\}$ for which the degree of loss aversion derived from the data is a realistic number (i.e., a small number possibly not too far from 2). BT found that for a_{LA} to be approximately equal to 2, h must be approximately one year; they could not say anything about what happens to a_{LA} if h is assumed to be longer, say ten years or so. This sensitivity analysis is the main objective of this study.

Turning to disappointment aversion, the analysis in ABL does not really deal with the problem of the time horizon. ABL only find that at a one-quarter or one-year horizon disappointment aversion seems to be a good explanation of the historical equity premium. This does not seem surprising given the close similarity between disappointment aversion and loss aversion and BT's results on myopic loss aversion and the equity premium. In this chapter, we seek to look at the parameters of a simple disappointment aversion model if the investment horizon is progressively increased beyond one year.

To pin down our simple model of preferences under loss aversion (henceforth LA) and disappointment aversion (henceforth DA), we shall consider the following measure of *departure* of the outcome from the reference point relevant for each specification of preferences. We call this *ex post* measure ρ_j, where $j = LA, DA$. If evaluated *ex ante* in expectation and in absolute value, this measure can be interpreted as a measure of *risk*. The variable ρ_j can be defined in a compact way for both LA and DA as follows:

$$\rho_{j,t+h} = (x_{t+h} - z^j_{t+h})(1 - I^-_j) + a_j I^-_j (x_{t+h} - z^j_{t+h}), \tag{4.80}$$

where z^j_{t+h} is the reference point, $z^{LA}_{t+h} = 0$, $z^{DA}_{t+h} = E_t x_{t+h}$, $I^-_j = 1$ if $x_{t+h} < z^j_{t+h}$ and zero otherwise, and $a_{LA} > 1$, $a_{DA} = \frac{d}{e} > 1$. Under loss aversion, the ex post value function is simply given by $V_{LA,t+h} = \rho_{LA,t+h}$, while under generalized disappointment aversion as in Jia, Dyer and Butler (2001) – which is a risk-value utility function – it is given by:

$$V_{DA,t+h} = x_{t+h} + e\rho_{DA,t+h} \tag{4.81}$$

Ex ante, the expected value function under loss aversion is:

$$E_t V_{LA,t+h} = E_t \rho_{LA,t+h}, \tag{4.82}$$

while for disappointment aversion it is obtained:

$$E_t V_{DA,t+h} = E_t x_{t+h} + eE_t \rho_{DA,t+h} \tag{4.83}$$

It should be noted that under this specification the coefficient e can be interpreted as a measure of the overall risk aversion (measuring the importance of the expected value vis-a-vis deviations from it), while a_{DA} measures the relative importance of disappointment (negative feeling) compared with elation (positive feeling) in this preference specification. Reflecting previous results in the literature, one should expect e to be quite a small number, perhaps not too different from 1, and a_{DA} (by analogy with loss aversion) not too distant from 2. In the continuation, we shall assume for simplicity (and quite realistically) $e = 1$ and concentrate the estimation effort on a_{DA}. Given that $E_t(x_{t+h} - E_t x_{t+h})(1 - I_{DA}^-) = E_t(x_{t+h} - E_t x_{t+h}) I_{DA}^-$, the disappointment aversion specification in (4.79) can be rewritten in a simplified format as:

$$E_t V_{DA,t+h} = E_t x_{t+h} + (a_{DA} - 1) E_t (x_{t+h} - E_t x_{t+h}) I_{DA}^- \tag{4.84}$$

Under both specifications of preferences, as argued above, we consider the excess return on equity between t and $t + h$ to be the variable of interest, x_{t+h}. In the next section, we derive an equilibrium condition between investing in the safe and the risky asset for our either loss or disappointment averse agent, which will serve as a basis for the empirical estimation carried out thereafter.

4.3.2 A model of expected returns under loss aversion and disappointment aversion

We assume that in every period t the investor evaluates the investment prospects based on the expected value functions as in (4.82) for loss aversion and (4.83) for disappointment aversion. As far as the mechanism for expectation formation is concerned, we allow expectations to be formed under a non-linear weighing of probabilities, which is in line with most experimental evidence on decision-making under risk (Kahneman and Tversky, 2000).

The expected (or "prospective") value function for investing in the (risky) equity portfolio is the following:

$$EV_{j,t} = E_t^w V_{j,t+h} = \int_{-\infty}^{\infty} V_{j,t+h} w(p(x_{t+h}))dx_{t+h}, \tag{4.85}$$

where $w(\cdot)$ is a function used to weigh the probabilities p (Tversky and Kahneman, 1992), $E_t^w V_{j,t+h}$ is the *subjective* expected value of $V_{j,t+h}$ at time t (where "subjective" signals that it is computed with the probabilities weighted with the function w), and $j = LA, DA$.

Having normalized the expected value function for investing in the safe asset at zero, letting α be the share of wealth invested in the risky asset, the expected value function for the portfolio will be equal to αEV_j (where EV_j is defined as in (4.85)). In equilibrium, for any $\alpha \neq 0$ this implies that $EV_j = 0$ (otherwise, if $EV_j > 0$ it would be convenient for our investor to be infinitely short in the safe asset and long in the risky asset, and the opposite would be true if $EV_j < 0$).[99] So:

$$E_t^w V_{j,t+h} = 0 \tag{4.86}$$

Recalling the results of the previous section, in the case of loss aversion this implies that:

$$E_t^w x_{t+h}(1 - I_{LA}^-) + a_{LA} E_t^w x_{t+h} I_{LA}^- \tag{4.87}$$

For disappointment aversion, assuming that also for the subjective expectation $E_t^w(x_{t+h} - E_t^w x_{t+h})(1 - I_{DA}^-) = E_t^w(x_{t+h} - E_t^w x_{t+h}) I_{DA}^-$, equation (4.86) implies:

$$E_t^w x_{t+h} + (a_{DA} - 1) E_t^w (x_{t+h} - E_t^w x_{t+h}) I_{DA}^- = 0 \tag{4.88}$$

Further, we assume that agents have rational expectations and do not make systematic mistakes in their subjective expectations. Hence:

$$E_t^w V_{j,t+h} = V_{j,t+h} + \varepsilon_{t+h}, \tag{4.89}$$

with $\varepsilon_{t+h} \sim MA(h-1)$ with all roots outside the unit circle, i.e. a stationary process. Let us consider a sample period $t = 1, ..., T$, with T large (in particular, $T >> h$). Asymptotically, we have:

$$\sum_{t=1}^{T} (E_t^w \frac{V_{j,t+h}}{T} - \frac{V_{j,t+h}}{T}) = \sum_{t=1}^{T} \frac{\varepsilon_{t+h}}{T} \simeq 0, \tag{4.90}$$

owing to our assumption on the stochastic properties of ε_{t+h}. Because ε_{t+h} is a stationary process, the unconditional mean of $V_{j,t+h}$ is an unbiased estimate of $E_t^w V_{j,t+h}$. Therefore, the equilibrium condition (4.86) requires that the value function is ex post ap-

99 This implies that our method does not allow to identify α, as it may be the case with the approach followed by BT and ABL. So, the optimal portfolio allocation remains *indeterminate*, exactly because we assume that agents are indifferent between investing in equity and Treasury bills.

proximately zero on average:

$$\sum_{t=1}^{T} E_t^w \frac{V_{j,t+h}}{T} = \sum_{t=1}^{T} \frac{V_{j,t+h}}{T} = 0 \tag{4.91}$$

This equation is the basis for our empirical analysis. For loss aversion, this implies:

$$\frac{1}{T}\sum_{t=1}^{T}(x_{t+h}(1 - I_{LA}^{-}) + a_{LA}x_{t+h}I_{LA}^{-}) = 0, \tag{4.92}$$

or:

$$a_{LA} = \frac{\sum_{t=1}^{T} x_{t+h}(1 - I_{LA}^{-})}{\sum_{t=1}^{T} x_{t+h}I_{LA}^{-}} \tag{4.93}$$

For disappointment aversion, expression (4.91) leads to:

$$\frac{1}{T}(\sum_{t=1}^{T} x_{t+h} + (a_{DA} - 1) + (x_{t+h} - \frac{1}{T}\sum_{t=1}^{T} x_{t+h})I_{DA}^{-}) = 0, \tag{4.94}$$

whereby:

$$a_{DA} - 1 = \frac{\sum_{t=1}^{T} x_{t+h}}{\sum_{t=1}^{T}(x_{t+h} - \frac{1}{T}\sum_{t=1}^{T} x_{t+h})I_{DA}^{-}} \tag{4.95}$$

From equations (4.93) and (4.95), for given values of the time horizon h, a_{LA} and a_{DA} can be estimated from the data. The key objective of our analysis is to see which combinations $\{a_j, h\}$ deliver plausible values for both variables under loss and disappointment aversion.

4.4 The empirical analysis

4.4.1 The data

In the empirical analysis of this chapter we use annual observations for the returns on the US stock market as proxied by the Standard and Poor composite index from 1871 to 2001, which gives us 130 annual observations. The data are drawn from Global Financial Data.

Stock returns include both dividends and capital gains. For the return on the safe asset, we consider the return on the one-year Treasury bill, also available from 1871 onwards in the database.

It should be noted that our sample period is different from that of BT (1926-1990). In particular, the last part of the sample period, after 1990, seems particularly interesting given the large swings in equity prices and the boom-bust of the dot.com bubble (Shiller, 2000). In this respect, an interesting question on its own is whether the results of BT carry through to our sample period. As we show later, we broadly find this to be the case.

We make the assumption that our representative investor has one US dollar at the end of 1871, and may invest it either in the U.S. stock market (i.e., in the Standard and Poor composite index) or in one-year Treasury bills. Then, we compute the value outstanding in each year t in dollars for the investment in equity (which we call $RISKY$) and for the 1-year Treasury bill (which we call $SAFE$). Chart 4.1 reports the value of the investments $RISKY$ and $SAFE$ over the whole sample period. Consistent with many other contributions in the literature (see, e.g., Siegel and Thaler, 1997), we find that over our sample period, which covers more than a century, stocks outperformed Treasury bills by a very wide margin. Indeed, the same dollar invested in 1871 in the stock market would have been worth more than 100,000 dollars, against only slightly more than 250 dollars if invested in Treasury bills (Chart 4.1). It is interesting to observe that the difference in performance is particularly striking in the postwar period, and much more contained beforehand (Chart 4.2).

However, it is highly doubtful that there is any investor having such a long investment horizon. Therefore, we look at shorter, more realistic time horizons, in particular between *one year* and *ten years*. The upper panel of Chart 4.3 reports the one-year *excess* return on the Standard and Poor composite compared with the one-year Treasury bill, and – for a comparison – the lower panel of the chart reports the ten-year excess return. It stands out in these charts that excess returns often turn out to be negative at the one-year horizon, but hardly so at the ten-year horizon (in the postwar period, ten-year excess returns have been negative only in the seventies). Nonetheless, at both the one-year and ten-year horizons equity excess returns are very volatile, ranging between losses of around 40% for both horizons and gains of more than 40% for the one-year and almost 500% for the ten-year horizon. So, already a first look at this evidence suggests that if the relevant horizon h is short, loss aversion and disappointment aversion are both plausible explanations of the historical equity premium. In fact, excess returns on stocks at a short horizon imply both the possibility of losses (relevant for LA) and large volatility around the mean (relevant for DA). By contrast, at longer horizons such as ten years excess returns remain very volatile, but the probability and size of the losses declines dramatically. This is reflected in the key statistics for the returns at one-year and ten-year horizons reported in Table 4.I. At the short horizon and over the full sample period from 1881 onwards, excess returns on equity have a mean of 6.6% and a standard deviation of 19.5%, which implies that positive returns are 2.27 times greater than negative returns on average.[100] At the long horizon, excess returns on equity are positive by 115.5% on average and have a high standard deviation, 132.0%,

100 These are values close to those analysed in Mehra and Prescott (1985) and subsequent studies.

but the positive excess returns are as much as 24.67 times greater on average than negative excess returns.

In the empirical analysis, we consider ten investment horizons, from one to ten years. Ex post *excess* returns at time $t+h$ are computed as follows:

$$x_{t+h} = \frac{RISKY_{t+h} - RISKY_t}{RISKY_t} - \frac{SAFE_{t+h} - SAFE_t}{SAFE_t}, \tag{4.96}$$

for $h = 1, ..., 10$ years. For each h, we then compute a_{LA} and a_{DA} as implied by equations (4.93) and (4.95).

4.4.2 Results

Starting with loss aversion, we run the analysis on the full sample period 1881-2001 first, and then on the restricted sample period 1926-2001, which partly overlaps that used by BT. The upper panel of Chart 4.4 reports the combinations of the estimated degree of loss aversion, a_{LA}, and the time horizon h which satisfy equation (4.93).

One first striking result of this analysis is that, despite the use of a different methodology and sample period compared with BT, the combination $\{a_{LA}, h\}$ identified in BT is broadly supported. In fact, if $h = 1$, a_{LA} is very close to 2.25 found by BT. In this respect, our analysis is in keeping with the results of BT, even if we only look at a different sample period and include the dot.com boom-bust in the second part of the nineties and early 2000s, which might have significantly affected the results.

The novel element of our analysis, which could have been easily anticipated by just looking at the excess returns at the different horizons in the previous section, is what happens with longer time horizons. The upper panel of Chart 4.4 shows that the estimated a_{LA}

increases quite dramatically with h, and only at horizons of less than three years is this parameter of acceptable size. At the longer time horizon, ten years, the loss aversion parameter is close to 25, which seems to be a exaggeratedly high value. Such a large value for a_{LA} would imply, for instance, that the representative agent would possibly turn down a lottery paying, say, 24 dollars with probability $\frac{1}{2}$ and losing 1 dollar with probability $\frac{1}{2}$. This is clearly unrealistic and at odds with the experimental evidence suggesting that a_{LA} is a small number (Kahneman and Tversky, 2000).

The conclusion of this analysis does not change if we look at the sample period starting from 1926, reported in the lower part of Chart 4.4. Indeed, the increase in the estimated loss aversion if the time horizon gets longer is even more pronounced, reflecting the larger weight of the postwar period in the sample when stock market developments have been particularly favorable. The conclusion of this analysis is that loss aversion is a good explanation of the equity premium puzzle only if the representative agent's time horizon is very short – agents must have a high degree of myopia, so to speak.

In Chart 4.5, we repeat the same analysis on disappointment aversion, making use of equation (4.95). In the upper part of the chart, we refer to the full sample period, while results for the restricted sample period starting from 1926 are in the lower panel. It can be observed that the degree of disappointment aversion, $a_{DA} - 1$, rises only very mildly with the time horizon. In the full sample period, it is close to 1 at short horizons, and rises to close to 2 at the longer horizons. In the shorter sample period, it is again close to 1 at very short horizons, and rises to somewhat above 2 at longer horizons – again, reflecting a greater weight of the good stock market performance in the postwar period.

These values for disappointment aversion seem very reasonable and in keeping with the experimental evidence on loss aversion. So, this analysis suggests that an explanation of the historical equity premium based on disappointment aversion is valid no matter the length of the investment horizon (within reasonable limits). In particular, an explanation built on disappointment aversion does not necessarily hinge on the assumption that agents are highly myopic. Stocks may lose compared with Treasury bills (or similarly safe assets) only at short horizons, but may strongly disappoint at *both* short and long time horizons.

One *caveat* surrounding these results is the size of our sample period compared with the longest time horizon we look at. In fact, the assumption that $\sum_{t=1}^{T} \frac{\varepsilon_{t+h}}{T} \simeq 0$ is warranted only if $T >> h$. Although our sample period covers more than a century, for the longest horizon that we consider, $h = 10$, the sample has only 12 independent observations, and even fewer when we look at the sample period starting from 1926. This might raise the concern that the results that we obtain might be spurious and distorted by small sample bias.

To take into account this possibility, we check the robustness of the results by doing a simulation exercise as follows. A very simple model is estimated for the *one-year* excess return on equity, $x_{t+1} = \frac{RISKY_{t+1} - RISKY_t}{RISKY_t} - \frac{SAFE_{t+1} - SAFE_t}{SAFE_t}$, for which 129 non-overlapping observations are available. The model estimated is very simple:

$$x_{t+1} = \beta + \eta_{t+1}, \tag{4.97}$$

where β is a real scalar and η_t is a white noise disturbance term. The coefficient β is the average annual premium on equity, while the variance of the shock η indicates the degree of uncertainty surrounding one-year excess returns on equity. The purpose of estimating the model in (4.97) is to identify a very simple stochastic process driving *excess* returns on equity, in order to find out what configuration of the parameters of the representative agent's value function (namely a_j and h) makes the agent indifferent between investing in a hypothetical asset with annual excess returns given by (4.97) and holding financial wealth in safe assets. The simplicity of the model makes it plausible that it may have been perceived as the "approximate" model driving annual excess returns by a relatively unsophisticated representative investor.

The estimate for the simple model in (4.97) over the whole sample period 1871-2001 is reported in Table 4.II. It is found, in particular, that $\beta = 6.4\%$ and $\sigma_\eta^2 = 19.4\%$. The diagnostic statistics for the model are good and tend to indicate that the model is well specified and stable, despite its simplicity. For instance, recursive residuals and the recursive estimate of β (see the charts reported for illustrative purposes underneath Table 4.II) do not signal any significant instability in the model over the considered sample period. Interestingly, there is no sign of serial correlation in the residuals (for example as measured with the Q-test). Overall, these results indicate that we cannot reject the hypothesis that excess returns on equity are constant and i.i.d. in our sample period.

With this simple model at hand, we simulate 10,000 annual observations of x_{t+1}, and compute excess returns at various horizons, i.e for $h = 1, ..., 10$, by cumulating one-year excess returns. Subsequently, we estimate equations (4.93) and (4.95) on the simulated data. The results of this analysis (not reported here for brevity) confirm those based on the historical data and lead to the same conclusions as regards the relationship between loss and disappointment aversion and the investment time horizon.

Summing up, the results of the empirical analysis in this study (i) confirm that loss aversion and myopic behavior, if combined, represent a good explanation of the historical equity premium, as argued by BT, but also that (ii) disappointment aversion, at *both* short and long horizons, is also a valid explanation of the equity premium, as suggested by ABL. In the next section we provide some speculations on which one of the two explanations of the equity premium considered in this chapter is more plausible and interesting.

4.5 Discussion

Both "behavioral finance" explanations put forward to solve the equity premium puzzle involve some departure from rationality, at least as defined in the expected utility sense. Under myopic loss aversion, agents are irrationally short-sighted and forgo superior returns for being too anxious about short-term outcomes, as pointed out by BT. However, they are "rational" in the sense that they treat safe and risky returns in the same way, by having the same reference point for both types of investment. Under disappointment aversion, there is an element of "irrationality" (again, defined in terms of departure from standard preferences) which is related to the fact that agents' preferences depend on, and vary with, the lottery they are confronted with. So, there can be a lottery A displaying weak stochastic dominance vis-a-vis a lottery B, but agents might still prefer lottery B if this is less likely to disappoint their expectations. Therefore, agents may be more disappointed by stocks even if they are better than bonds in absolute terms *in every state of nature*.

The key question here is what form of departure from standard preferences is more plausible to describe financial investment behavior and thereby to be a good explanation of the historical equity premium.

BT put forward some strong arguments in favour of myopia in financial markets, mostly related to institutional features of the financial market and in particular to agency costs. While households should care about long-term outcomes, there is usually a principal-agent relationship between their money and investment decisions, and this relationship might work in favour of shortsightedness. Noting that delegated portfolio management and institutional investment are now largely prevalent in financial markets, the arguments proposed by BT are *prima facie* convincing.

It should be emphasized, however, that the same trend towards institutional trading and delegated portfolio management might also underpin a disappointment aversion specification of preferences. In a principal-agent relationship, the agent is often assessed in terms of performance against a certain benchmark, due to information asymmetries (Bray and Goodhart, 2002). The agent proposing an investment in stocks to the principal is likely to set a higher benchmark for returns compared with a safe investment strategy based on fixed income securities. From the perspective of the agent (who actually decides and implements the investment strategy) an outcome which falls short of the expectation is likely to lead to disappointment by the principal and to a reputation loss for himself. Conversely, an outcome exceeding expectations might lead to elation by the principal and to a reputation gain for the agent. This mechanism, which seems intuitively reasonable and realistic, would suggest that disappointment aversion is a good characterization of preferences in financial markets, at least approximately.

In addition, the finding that disappointment aversion works well to explain the equity premium at an horizon of approximately ten years seems to be quite interesting. Arguably, the single most important reason to invest in the US financial markets is saving for retirement. If this is true, the most relevant and "rational" investment horizon for each investor should be the time span before retirement, as this would maximize the agent's utility in terms of living standards after retirement.[101] Noting that peak saving years occur in mid and late career, ten years or so do not seem unreasonable as a time span before retirement for the "median" investor in the financial market (where the median is calculated taking into account each agent's stock of wealth).

Another argument which can be brought forward in favor of a disappointment aversion characterization of preferences, seen from the perspective of consumers (the principal), is *habit formation in consumption*. If consumption is characterized by habit formation, and if agents are rational and anticipate that future changes in consumption will also change the habit level, the higher future long-run level of consumption – that investing in risky stocks makes possible – will be discounted more heavily than in the case where consumption preferences have no habit. In this case, the risk of seeing consumption fall unexpectedly below the habit level, which investing in stocks also implies, will also be highly disliked by economic agents, leading to a high equity premium (Otrok, Ravikumar and Whiteman, 2002).

[101] This can be rationalised in an overlapping generations economy, where major investors in the market are middle-aged households (Constantinides, 2002).

All in all, we argue that disappointment aversion is a very interesting "behavioral" candidate for the explanation of the equity premium puzzle, which becomes particularly relevant at long time horizons. It should be stressed that myopic loss aversion and disappointment aversion might well be considered *in conjunction* for the overall explanation of the historical equity premium. Myopic loss aversion would imply that stocks can be quite painful in the short term but are a very good choice (too good to be true) if the investment horizon is long. Disappointment aversion would work to reduce the net benefit of investing in stocks if seen from the perspective of long-horizon returns. As pointed out by BT, agents may actually have *many* relevant investment horizons, which makes a multi-faceted explanation of the equity premium quite reasonable. Overall, we surmise that *both* loss and disappointment aversion might contribute to raise the equity premium to the high levels observed historically in the United States (and other industrialized economies).

4.6 Conclusions

This chapter takes a close look at the "behavioral finance" explanations of the equity premium puzzle, namely myopic loss aversion (Benartzi and Thaler, 1995) and disappointment aversion (Ang, Bekaert and Liu, 2000). Building on these ideas, this chapter proposes a simple specification of preferences, which is able to capture the main idea behind loss and disappointment aversion and to highlighting the differences between the two approaches, the most important being the way the reference point is determined. Moreover, we have brought these theories to the data with a view to understand the relationship between the degree of loss and disappointment aversion and the investment time horizon.

The main conclusion of the chapter is that a highly short-sighted investment horizon is required for the historical equity premium to be explained by loss aversion, while reasonable values for disappointment aversion are found also for long investment horizons. So, stocks may lose only in the short term, but may disappoint also in the long term.

Which of the two "behavioral" explanations of the equity premium has to be preferred? Benartzi and Thaler (1995) put forward the idea that institutional factors and principal-agent relationships might lead to myopic loss aversion, but there are also arguments in favour of disappointment aversion based on similar grounds, as argued in this chapter. One intriguing possibility is that the two approaches are not alternative, and that a high equity premium can be explained by *both* myopic loss aversion at short horizons and disappointment aversion at longer horizons. This would imply that the reference point evolves according to the time horizon. This is an interesting possibility which we leave to further research.

Finally, extending this analysis to data from other countries and periods would be an interesting topic for future research. Due to the observed "home bias" in equity investment, it is possible that participants in individual stock markets display country-specific cultural and psychological traits, which might lead to different degrees of loss and disappointment aversion as well as time horizons for investment.

Chart 4.1 – Value of one dollar invested in equity and Treasury bills in 1871

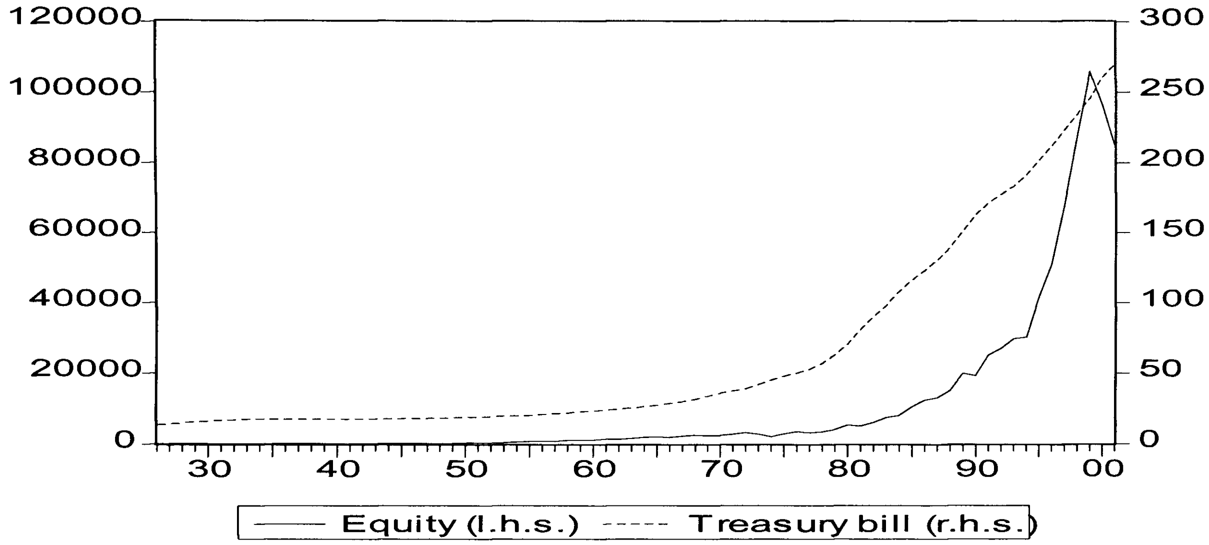


Chart 4.2 – Value share of equity and Treasury bill investment, in USD

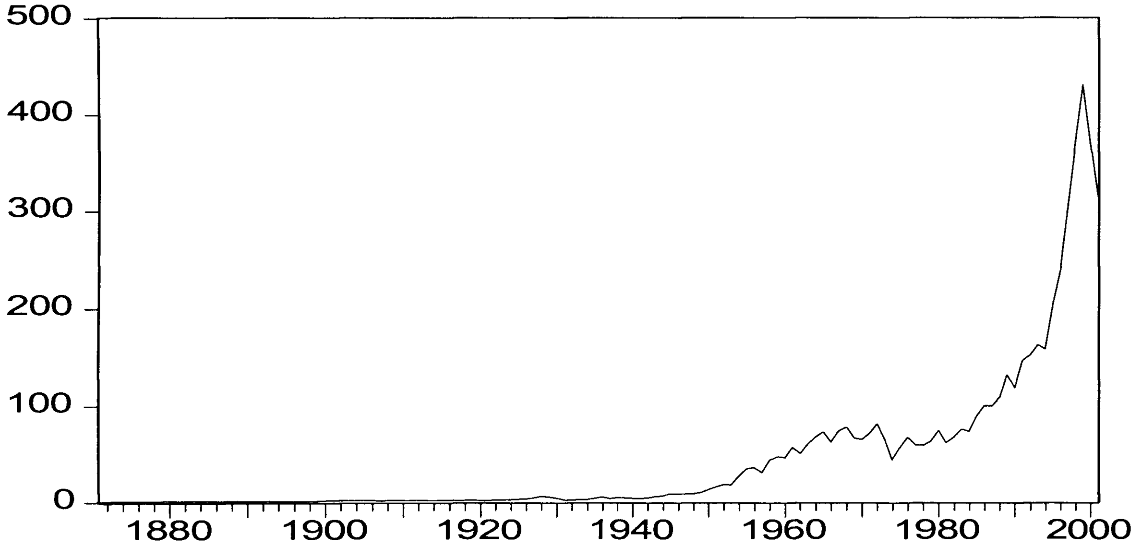
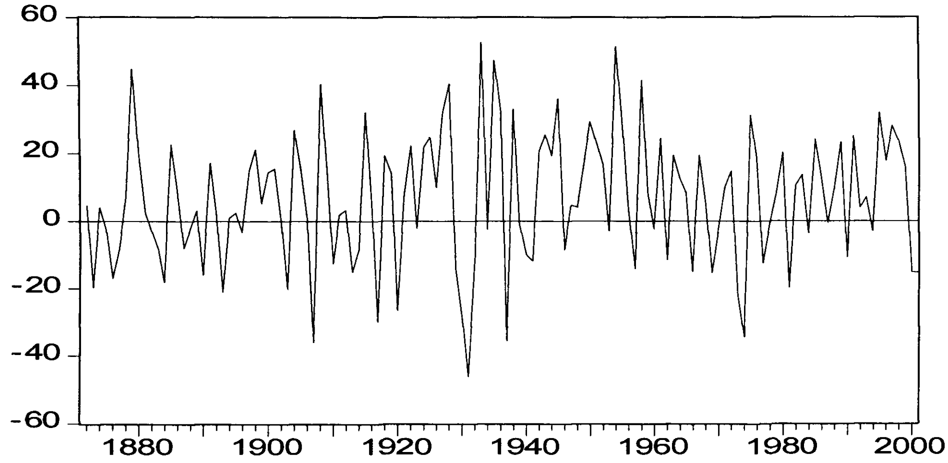


Chart 4.3 – Excess returns on equity, 1871-2001

1-year horizon



10-year horizon

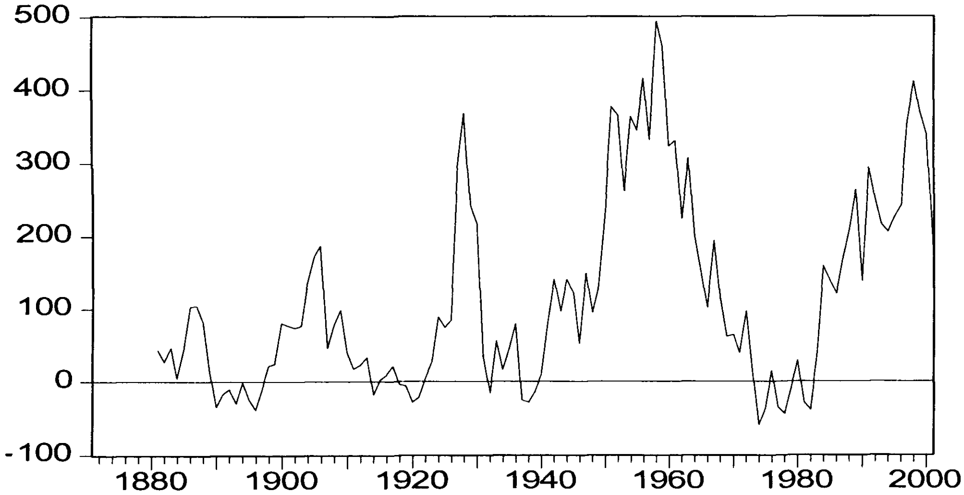


Table 4.I – Excess returns on equity at a 1-year and 10-year horizon

Sample period: 1881-2001

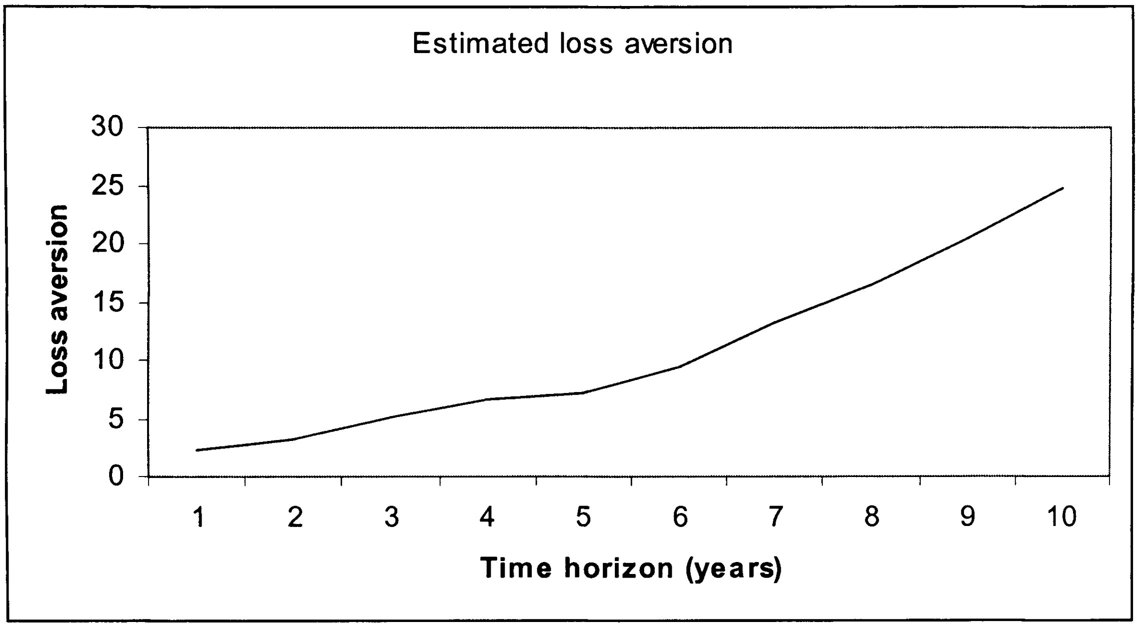
	ER1	*ER10*
Mean	6.590311	115.5088
Median	7.015681	78.71211
Maximum	52.65015	494.6028
Minimum	-46.19185	-59.84369
Std. Dev.	19.46995	132.0429
Skewness	-0.129030	0.896184
Kurtosis	2.815639	2.872475
Jarque-Bera	0.507110	16.27876
Probability	0.776037	0.000292
Pos./Neg.	**2.27**	**24.67**
Observations	121	121

Sample period: 1926-2001

	ER1	*ER10*
Mean	8.519619	162.4898
Median	9.777976	140.4511
Maximum	52.65015	494.6028
Minimum	-46.19185	-59.84369
Std. Dev.	20.68226	142.1342
Skewness	-0.195230	0.336087
Kurtosis	2.768986	2.120308
Jarque-Bera	0.651786	3.881302
Probability	0.721883	0.143610
Pos./Neg.	**2.74**	**37.2**
Observations	76	76

Chart 4.4 – Loss aversion and the time horizon

Sample period: 1881–2001



Sample period: 1926–2001

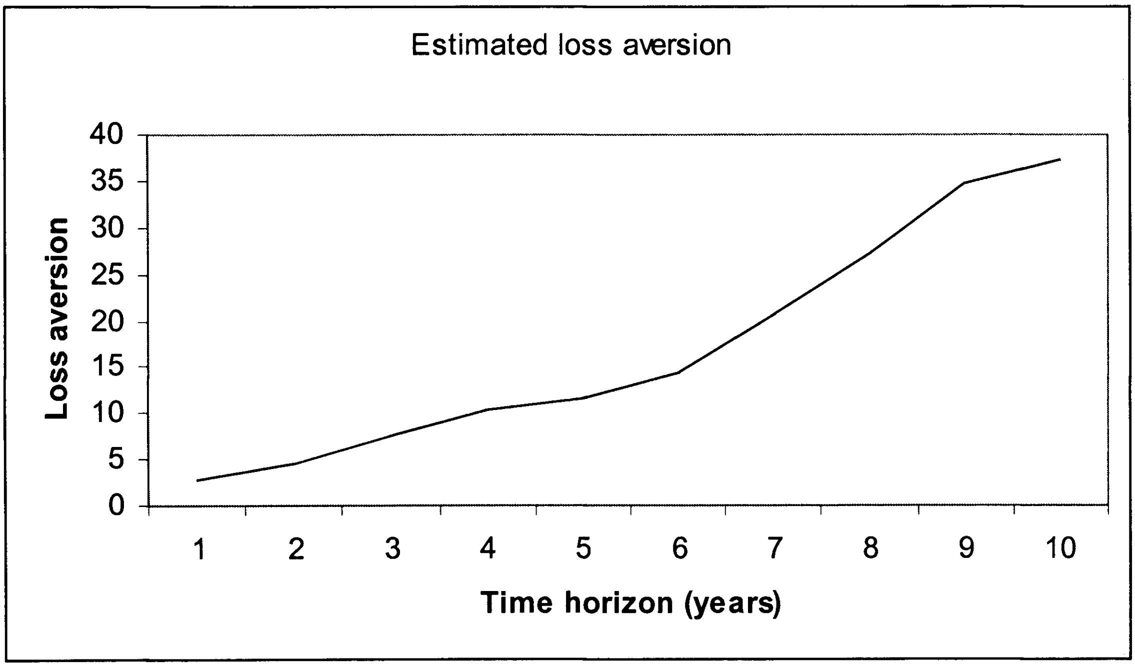
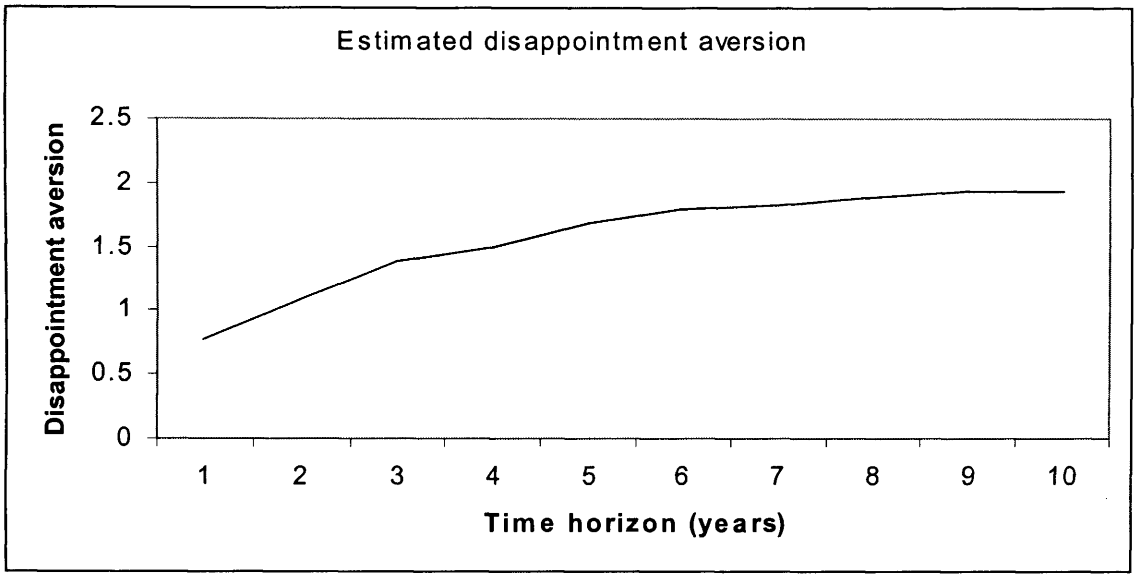


Chart 4.5 – Disappointment aversion and the time horizon

Sample period: 1881-2001



Sample period: 1926-2001

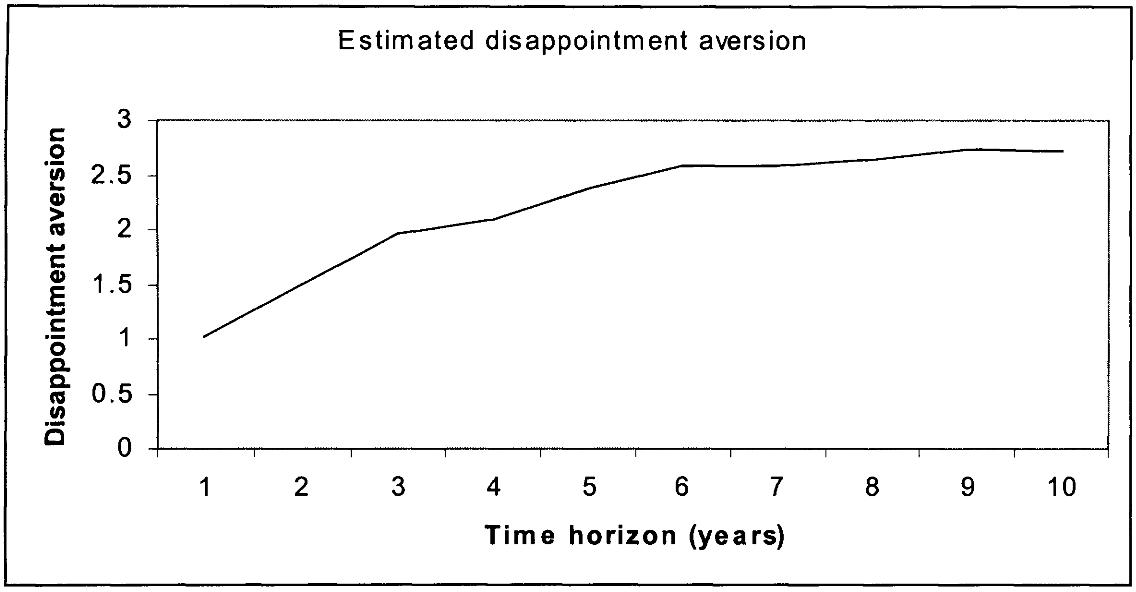
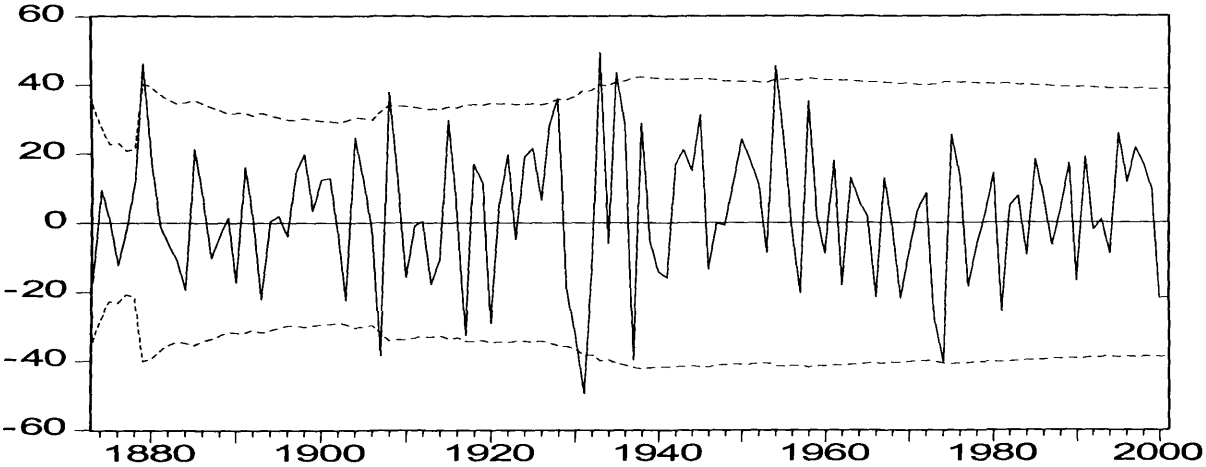
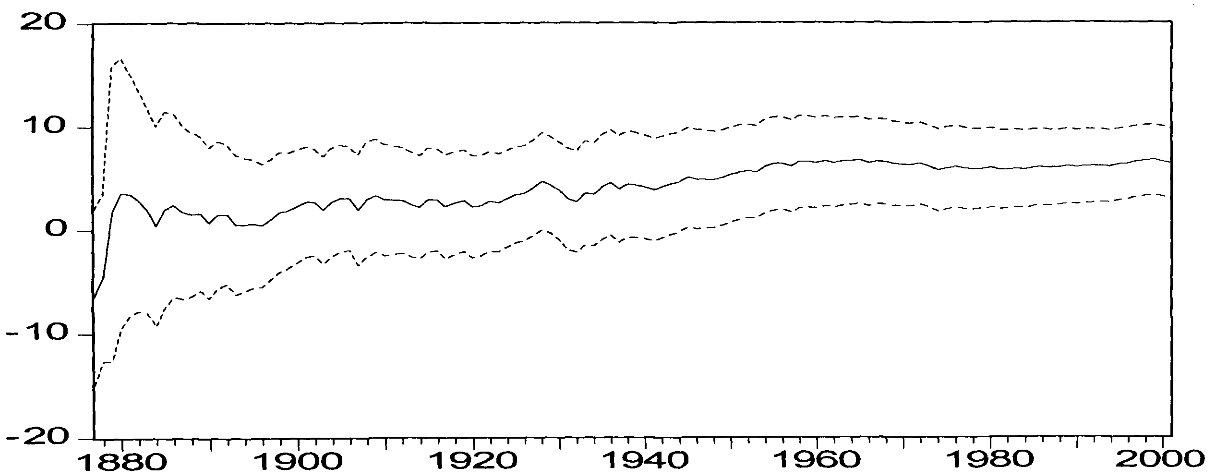


Table 4.II – Excess returns on equity, 1872-2001

Dependent Variable: ER1				
Method: Least Squares				
Sample(adjusted): 1872 2001				
Included observations: 130 after adjusting endpoints				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	6.381532	1.703692	3.745708	0.0003
R-squared	0.000000	Mean dependent var		6.381532
Adjusted R-squared	0.000000	S.D. dependent var		19.42507
S.E. of regression	19.42507	Akaike info criterion		8.778669
Sum squared resid	48676.02	Schwarz criterion		8.800727
Log likelihood	-569.6135	Durbin-Watson stat		1.944997



Recursive Residuals ----- ± 2 S.E.



Recursive C(1) Estimates ----- ± 2 S.E.

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