Wavelet Theory and Its Applications in Economics and Finance

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Abstract

Wavelets orthogonally decompose data into different frequency components, and the temporal and frequency information of the data could be studied simultaneously. This analysis belongs within local nature analysis. Wavelets are therefore useful for managing time-varying characteristics found in most real-world time series and are an ideal tool for studying non-stationary or transient time series while avoiding the assumption of stationarity. Given the promising properties of wavelets, this thesis thoroughly discusses wavelet theory and adds three new applications of wavelets in economic and financial fields, providing new insights into three interesting phenomena. The second chapter introduces wavelet theory in detail and presents a thorough survey of the economic and financial applications of wavelets. In the third chapter, wavelets are applied in time series to extract business cycles or trend. They are useful for capturing the changing volatility of business cycles. The extracted business cycles and trend are linearly independent. We provide detailed comparisons with four alternative filters, including two of each detrending filters and bandpass filters. The result shows that wavelets are a good alternative filter for extracting business cycles or trend based on multiresolution wavelet analysis.

The fourth chapter distinguishes contagion and interdependence. To achieve this purpose, we define contagion as a significant increase in short-run market comovement after a shock to one market. Following the application of wavelets to 27 global representative markets' daily stock-return data series from 1996.1 to 1997.12, a multivariate GARCH model and a Granger-causality methodology are used on the results of wavelets to generate short-run pair-wise contemporaneous correlations and lead-lag relationships, respectively, both of which are involved in short-run relationships. The empirical evidence reveals no significant increase in interdependence during the financial crisis; contagion is just an illusion of interdependence. In addition, the evidence explains the phenomenon in which major negative events in global markets began to occur one month after the outbreak of the crisis. The view that contagion is regional is not supported.

The fifth chapter studies how macroeconomic news announcements affect the U.S. stock market and how market participants' responses to announcements vary over the business cycle. The arrival of scheduled macroeconomic announcements in the U.S. stock market leads to a two-stage adjustment process for prices and trading transactions. In a short first stage, the release of a news announcement induces a

sharp and nearly instantaneous price change along with a rise in trading transactions. In a prolonged second stage, it causes significant and persistent increases in price volatility and trading transactions within about an hour. After allowing for different stages of the business cycle, we demonstrate that the release of a news announcement induces larger immediate price changes per interval in the expansion period, but more immediate price changes per interval in the contraction period, from the old equilibrium to the approximate new equilibrium. It costs smaller subsequent adjustments of stock prices along with a lower number of trading transactions across a shorter time in the contraction period, when the information contained in the news announcement is incorporated fully in stock prices. We use a static analysis to investigate the immediate effects of news announcements, as measured by the surprise in the news, on prices, and adopt a wavelet analysis to examine their eventual effects on prices. The evidence shows that only 6 out of 17 announcements have a significant immediate impact, but all announcements have an eventual impact over different time periods. The combination of the results of both analyses gives us the time-profile of each news announcement's impact on stock prices, and shows that the impact is significant within about an hour, but is exhausted after a day.

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Chapter 1

Introduction

Most econometric methodologies are dominated by models that are rooted in the time domain. As a tool that is complementary to time domain analysis, frequency domain analysis also receives much attention. This type of analysis provides new insights into economic issues by decomposing economic data into sinusoidal components, with intensities that vary across the frequency spectrum. Here, the Fourier transform must be mentioned; this method decomposes time series into a combination of trigonometrical or complex exponential components. This approach has a long history and is used widely in many disciplines. However, the Fourier transform has many disadvantages. First, this method assumes that intensities are constant over time and requires stationary data, which are typically not available in practice. Second, the method has a global nature. The sine and cosine functions that are the base functions of the Fourier transform are localised in frequency but not localised in time and extend over the entire real line. These functions are trigonometric and are not well suited to capturing abrupt spikes or cusps in a time series. This property of the Fourier transform renders it ineffective for studying time series that contain irregularities, such as discontinuities and spikes.

To conquer the limitations of the Fourier transform, Gabor (1946) presents the short-time Fourier transform, which is also known as the windowed Fourier transform. The idea is to study the frequency of a time series by segments, multiplying the time series by a fixed window function. The window is typically a box of unit height that is zero outside of a finite interval and whose size is fixed throughout the sample. The window serves to isolate the time series, which can then be subject to the Fourier transform. When one segment has been analysed, the window slides along the time series to isolate another section for analysis until the entire time series has been analysed. However, this technology also has one large problem: the size of the window is fixed. A small window is appropriate when one is interested in high-frequency components, but a small window loses information with low-frequency components. Conversely, a wide window is associated with a similar problem. The windowed Fourier transform is a fixed resolution analysis and is thus unsuitable for time series that include irregularities.

Thus, there is a need for a methodology that can study temporal and frequency information simultaneously. This methodology is known as wavelet analysis, which are applied in a wide variety of disciplines, such as astronomy, engineering, geology, medicine, meteorology, physics and geography. In the 1990s, wavelet analysis were introduced to economics and finance fields. By definition, wavelets are small waves, as they have finite length (compactly supported) and are oscillatory. Wavelets with finite support begin at a point in time and die out at a subsequent point in time. They are particular types of functions that are localised in both the time and frequency domains, whereas sines and cosines, which are the base functions of the Fourier transform, are functions with a frequency-by-frequency basis. The wavelet transform utilises local base functions that can be stretched and translated with a flexible resolution in both frequency and time to capture features that are local in both frequency and time. Therefore, wavelets are useful for managing the time-varying characteristics found in most real-world time series and are an ideal tool for studying non-stationary or transient time series while avoiding the assumption of stationarity.

For example, the first-difference is a type of filter that performs mathematical operations to rearrange a data structure. In empirical works in economics and finance, the required frequency of observations is commonly not available because it is very expensive or not possible to collect data in the required frequency for particular variables. However, there is no reason to believe that data collected in the required frequency would be able to fully capture the movement of the economy. To solve this issue, a mathematical method referred to as temporal aggregation is required. The implicit assumption of this method is that the underlying stochastic process in continuous time is observed in discrete intervals. When the required frequency of observations is not available, the temporal aggregation is applied to obtain the ideal frequency of data.

Consider a case in which monthly observations of an economic variable are collected from the market. However, quarterly data for this variable are actually required to study an empirical issue. A simple and frequently used way of converting the observations would be to take the sums or the averages of successive sets of three months. This process is equivalent to subjecting the data to a three-point moving sum or average and then subsampling the resulting sequence by picking one in every three points. However, a problem called "aliasing" arises with this procedure. Aliasing refers to an effect that leads to different data sequences that are indistinguishable when sampled.

To avoid the aliasing problem, it is appropriate to use a filter that can better aggregate data without creating this problem and losing any data points in the process. Although several filters fulfil this condition, they cannot make the filtered data linearly independent on different time scales, which could simplify some economic issues. Fortunately, wavelet theory provides this type of filter.

Given the promising properties of wavelets, the economics and finance literature on wavelets has rapidly expanded over the past 20 years. A comprehensive discussion of the economic and financial applications of wavelets can be found in the survey articles by Ramsey (1999, 2002) and Crowley (2007). These applications provide different insights into the economic and financial fields and report fruitful results.

This thesis thoroughly discusses wavelet theory and adds three new applications of wavelets in these fields, providing new insights into three interesting phenomena. In the second chapter, we show that wavelets present orthogonal decomposition, maintain local features in decomposed data, and provide multiresolution analysis. According to these properties, chapter 3 shows that wavelets are a good alternative filter for extracting business cycles from quarterly data. Compared with four traditionally used filters that are estimated based on Fourier transform, wavelets provide a better resolution in the time domain and are more useful for capturing the changing volatility of business cycles. As far as we know, it is the first paper to systematically discuss the application of wavelets (DWT) in extracting business cycles or trend, and compare its performance with the other four filters'; chapter 4 distinguishes financial contagion from interdependence and draws evidence of no contagion in the 1997 Asian crisis through a methodology that combines wavelets with a multivariate GARCH model and a Granger-causality methodology. This chapter is the first paper to propose a more precise definition of contagion to distinguish it with interdependence. Moreover, we develop a new method that combines a multivariate GARCH model and a Granger-causality methodology with wavelets, respectively, to examine the existence of financial contagion across 27 markets during the 1997 Asian crisis and present economic policies to reduce the impact of the crisis in global markets; chapter 5 examines news announcements' impact on the U.S. stock market, and illustrates immediate effect by static analysis and eventual effect by wavelet analysis. In this chapter, we discover how market participants' reactions to macroeconomic news announcements vary over the business cycle. Due to the limitation of previous methods, we use wavelet analysis to investigate which announcements eventually affect the stock price.

1.1 Wavelet Theory and Literature Review

As a method of temporal aggregation, filters can be lowpass or highpass filters. In terms of frequency domain theory, a lowpass filter is defined as preserving low-frequency components and abandoning high-frequency components, whereas a high-pass filter has a reverse effect on time series and only conserves high-frequency components. Generally, the two filters are particular types of bandpass filters that pass components with frequencies within a certain range and reject those with frequencies outside of the range. Bandpass filters have been used widely in the business cycle literature for years (Canova (1998), Baxter and King (1999), Christiano and Fitzgerald (2003), Pollock (2000), Gomez (2001), Iacobucci and Noullez (2005), Estrella (2007)).

These two different types of filters are included in wavelet theory. The lowpass filter is called a scaling filter, whereas the highpass filter is a wavelet filter. Unlike other filters, whose amplitudes preserve only the frequency properties of data but discard the temporal properties, the amplitudes of wavelets retain both types of properties. Moreover, wavelets produce an orthogonal decomposition of economic and/or financial variables by time scale, which is closely related to the frequency and time horizon. As suggested by Ramsey and Lampart (1998a,b), the structure of decisions, the strength of relationships and the relative variables differ by time scale. Accordingly, some issues in economics and finance are difficult to solve using conventional econometric models, which account for only the temporal properties of data. The orthogonal property of wavelets allows them to provide new insights to solve some issues that were ignored or impossible to address in the past. Finally, wavelet analysis has a local nature and is thus more useful for detecting structural breaks or jumps in data. On the contrary, Fourier analysis, which is traditionally used to capture the frequency properties of data, has a global nature and is thus inappropriate for studying issues that include local episodes. Consequently, wavelets have attracted the attention of economists in recent years, and the related literature has grown rapidly over the last two decades.

1.2 Extracting Business Cycles and Detrending via Wavelets

Economic time series typically contain a trend that provides long-term information regarding the economy, business cycles that reveal short-term information regarding the economy, and noise that is affected by transitory shocks or/and measurement errors. According to spectral analysis theory, these three types of economic data correspond to specific frequency components: the high-frequency components are noise, the low-frequency components are trend, and the complementary components are business cycles. A linear filter is a useful tool for extracting the components of interest (typically trend or business cycles). An ideal filter that is able to extract the exact components in the required frequency bands from economic time series needs a double-infinite order of coefficients for an infinite time series, which is not possible in practice. Thus, Hodrick and Prescott (1997), Baxter and King (1999), Pollock (2000), and Christiano and Fitzgerald (2003) present their own approximate filters to extract business cycles.

The wavelet filter is an alternative filter based on multiresolution wavelet analysis.[1] This filter orthogonally decomposes economic time series into the trend, business cycles and noise. The synthesis of these components is the original time series through the perfect reconstruction of wavelets. Similar to other filters, including the Hodrick-Prescott filter, the Baxter-King bandpass filter and the digital butterworth filter, the wavelet filter is a symmetric filter. Therefore, the phase effect, which generates time differences between the filtered data and the original data, does not occur. This effect is usually not allowed to be present because in economics, it is important to preserve the temporal properties of data. The Christiano-Fitzgerald bandpass filter is an asymmetric filter that generates the phase effect in filtered data. Moreover, the base functions of the wavelet filter are both time- and scale-localised (frequency-localised), whereas the base functions of the Fourier transform on which the other four filters are estimated are only frequency-localised. Consequently, the wavelet filter, which provides better resolution in the time domain, is more useful for capturing the changing volatility of business cycles.

The extracted trend and business cycles based on the wavelet filter are orthogonal, which implies that they are linearly independent. The relationship between business cycles and trends is the subject of debate, but they are generally considered to be related. It is nearly impossible to accurately determine this relationship

[1] In this chapter, the wavelet filter is a general notion that represents wavelets, including scaling filter and wavelet filter.

in practice. To allow for ease and simplicity when studying economic issues, the business cycles and the trend are sometimes assumed to be linearly independent in empirical analyses. For example, it is interesting to examine the effect of a shock to an economy on the long- and short-run equilibrium of that economy. To simplify this investigation, the linear relationships between the shock and the trend and between the shock and the business cycles are estimated, respectively. The business cycles and the trend are thus required to be linearly independent; otherwise, the results would be ambiguous. Compared with other filters, only the wavelet filter is able to undertake this task. Given its attractive properties, we believe that the wavelet filter is a useful filter for isolating different frequency components of data.

Given the lack of an ideal filter as a benchmark, we use a Monte Carlo simulation to evaluate the wavelet filter and other filters when applying them to time series to extract business cycles or trends. These other filters consist of the Hodrick-Prescott filter, the Baxter-King bandpass filter, the digital butterworth filter and the Christiano-Fitzgerald bandpass filter. The results from the simulation indicate that the Baxter-King bandpass filter performs best with annual data, the wavelet filter with quarterly data and the digital butterworth filter with monthly data when extracting business cycles. For the purpose of extracting the trend, the Baxter-King bandpass filter outperforms the other filters with annual and quarterly data, and the digital butterworth filter is the optimal choice for monthly data. The Hodrick-Prescott filter is a detrending filter because its filtered data contain both business cycles and noise. Furthermore, the data at the end of an economic time series are important to the current analysis. Consequently, good processed data are needed at the end. However, the first K and the last K data are not processed in the Baxter-King bandpass filter.[2] Because of strong deviations at the end of the filtered data in the digital butterworth filter, it is recommended that these data be discarded. However, this issue does not arise for the wavelet filter. Overall, the wavelet filter is a good alternative filter for extracting the components of interest, particularly for business cycles from quarterly data.

1.3 No Contagion, Only Interdependence

In this chapter, we distinguish contagion from interdependence. A shock to one market typically imposes a negative effect and may even cause a crisis in other markets. The concept of contagion is used to describe the spreading of a shock to other markets. There is consensus that a new transmission mechanism through

[2] The width of the Baxter-King bandpass filter is $2K + 1$.

which a shock propagates across markets should be built during a crisis period in the presence of contagion. However, such a shock can also use a normal transmission mechanism to spread across markets because markets have become increasingly interdependent as globalisation increasingly occurs. Consequently, there is a debate as to whether a shock spreads across markets through a normal transmission mechanism or through a new transmission mechanism. This debate is similar to the debate regarding whether the spreading of a crisis across markets is contagion or just interdependence. It is therefore worth distinguishing between contagion and interdependence. Policy measures may be proposed to isolate a crisis in the case of contagion, whereas it is nearly impossible to eliminate the negative effects from a crisis in other markets as a result of interdependence because interdependence always exists in both tranquil and crisis periods.

In past empirical papers, a significant increase in linkages between a shock-hit market and other markets indicated the presence of contagion. However, this widely used concept of contagion is difficult to distinguish from interdependence. As Fratzscher (2003) notes, contagion is defined identical to financial interdependence in many early works. Consequently, it is necessary to propose a more precise definition of contagion.

Generally, a shock to one specific market propagates across other markets through two channels. These channels are trade linkages (or real linkages), which are estimated based on macroeconomic fundamentals such as trade or international business cycles, and financial linkages, which are estimated based on financial fundamentals. Some empirical works show that financial linkages outperform trade linkages and are able to explain many phenomena, such as the near simultaneity of a crisis in different markets.

Financial linkages are typically estimated using the following logic. Traders hold multiple assets in a broad range of markets. When a crisis erupts in a market, assets in the shock-hit market are devalued too substantially to satisfy marginal calls or to meet traders' liquidity requirements. These assets are difficult to sell because few traders are willing to accept assets whose prices have collapsed given the lemon problem.[3] Even when such assets are sold, the funds may be too small to satisfy requirements, and other assets in traders' portfolios must therefore be sold. In sum, traders sell the assets of other markets short to meet their personal needs (e.g., to manage risks or minimise losses). Such a shock propagates across markets during

[3] This problem is proposed by Akerlof (1970). Due to information asymmetry which indicates that seller knows more about a product than the buyer, the buyer is willing to pay for it only the price of a product of known average quality. Accordingly, the good product is withdrawn and only the bad product exists in the market.

this process. As suggested by Kaminsky *et al.* (2003), traders act rapidly. These behaviours are consistent with the argument of Moser (2003), when traders respond to a shock in the financial markets, asset prices are immediately corrected.

Consequently, the short-run relationships among markets substantially increase in the presence of contagion, which is consistent with the following proposal by Bekaert *et al.* (2005): contagion is a correlation between markets that is higher than the correlation accounted for by economic fundamentals. These relationships facilitate a more precise definition of contagion, and we therefore make a slight adjustment to the traditionally used definition in empirical works. "Contagion" occurs when the short-run relationships among markets increase significantly from tranquil periods to crisis periods.

In this chapter, we use the 1997 Asian crisis as an example to test our methodology.[4] Because a high-frequency relationship is classified as a short-run relationship, we use wavelets to orthogonally decompose 27 representative global markets' daily stock-return data series from 1996.1 to 1997.12 using a small time scale that is associated with high frequencies.[5] Given the number of monthly economic indicators that reflect economic fundamentals that are closely watched in financial markets, especially after the outbreak of a crisis, economic fundamentals appear to be relatively stable across a month. Accordingly, the short run is defined as "no greater than one month" for our purposes in this chapter. The fifth level decomposed by wavelets corresponds to a frequency interval of $(\pi/32, \pi/16]$, which is associated with a time interval of $[32, 64)$ days, which is slightly more than one month. Because this time series is associated with the longest time interval in the first five decomposed levels, it is decomposed into only five levels using wavelets. By definition, the time periods associated with the first five levels are linked to the "short run".

The simple correlation coefficients that are traditionally used to investigate contagion are biased because they are conditional on market volatility (Forbes and Rigobon (2002)). In addition, these coefficients measure only static relationships and thus are not appropriate for describing time-varying relationships during crisis periods. Consequently, a bivariate GARCH model (BEKK model, named after Baba, Engle, Kraft and Kroner, 1990) is used to generate conditional correlations.

[4]Kaminsky *et al.* (2003) indicate that the Asian crisis in 1997 was unanticipated. In the sample, the "fast and furious" actions of traders to the shock should induce the significant increases in short-run relationships between the shock-hit market and other markets, in the presence of contagion. Accordingly, the 1997 Asian crisis is a good example to illustrate the application of our methodology.

[5]Stock market rationally reflects economic fundamental values. Thus, stock market index is applied to investigate cross-market comovements. To investigate financial contagion, we should generate short-run relationships among markets. Because wavelets are able to achieve this aim, we use financial contagion to illustrate our methodology.

Because a lag effect from the shock on other markets may arise and because conditional correlations only reflect pair-wise contemporaneous correlations, a Granger-causality methodology is used to estimate pair-wise lead-lag relationships. Following the application of wavelets to 27 representative global markets' daily stock-return data series, the bivariate VAR-BEKK$(1, 1, 1)$ model and the Granger-causality test are used on the 27 subseries of stock returns at each level to generate short-run pair-wise contemporaneous correlations and lead-lag relationships between the shock-hit market and other markets, respectively, both of which are involved in short-run relationships. The empirical evidence reveals no significant increase in interdependence during the financial crisis; contagion is merely an illusion of interdependence. Moreover, the evidence explains the phenomenon in which major negative events in global markets began to occur one month after the outbreak of the crisis. In addition, the view of contagion as regional is not supported.

1.4 Stock Prices and Liquidity in the U.S. Stock Market

In the fifth chapter, we use tick-by-tick S&P 500 index futures price data to discover how scheduled macroeconomic news announcements affect the U.S. stock market and how the behaviour of market participants varies over the business cycle. This study reveals how information contained in news announcements spreads in financial markets, which is an important topic in financial economics.

In comparison with a number of papers studying macroeconomic news announcements and the responses of financial markets, this chapter primarily makes four contributions. First, to the best of our knowledge, this paper is the first to examine the effects of news announcements on the stock market using high-frequency (tick-by-tick) data, whereas previous papers have used low-frequency and high-frequency data to investigate this effect on the Treasury bond and foreign exchange markets and have examined this effect on the stock market using low-frequency data.

Second, news announcements generate immediate and eventual effects on the stock market. Some papers propose that the information contained in news announcements is incorporated into asset prices immediately, such that a sharp and instantaneous price change occurs at the time of news release. However, the implicit information from a news announcement is not fully learned during this process. Market participants need to adjust their initial analyses of news announcements after observing the market's performance. The subsequent adjustment of prices induces significant and persistent increases in price volatility and trading volume. These two effects are measured by price volatility and trading volume using one- and five-minute intervals, respectively.

Naturally, two questions arise as to which announcements immediately affect the stock price and which announcements eventually affect the stock price. To address the first question, we regress one-minute price changes on announcement surprises, as measured by the level of surprise in the news. Regarding the second question, previous papers use an OLS regression model of static price changes on announcement surprises to examine which announcements impose an eventual effect and the duration of that effect in financial markets. The static price changes are constructed by fixing the time before the announcement and shifting the examined time after the announcement. The largest time interval over which a price change is significantly affected by a news announcement indicates the market's response until that time. However, these results are not consistent with the results in many papers using price volatility, which persists over a longer time period. This issue stems from static changes in prices, which cannot fully reflect the eventual effect of a news announcement on the market. By contrast, wavelet-scale price changes can reveal this effect. Moreover, they maintain jumps in asset prices which are caused by news announcements (Andersen *et al.* (2003)). These data are regressed on announcement surprises to answer the second question. Because wavelet-scale price changes on different time scales are mutually orthogonal, which implies that they are linearly independent, the combination of estimation results from the OLS regression models of static and wavelet-scale price changes provides the time profile for a news announcement's effect on stock prices.

Third, it is of interest to examine the stability of the market's response over various stages of the business cycle. The same type of news is sometimes considered to be a positive signal for the economy during certain states of the business cycle and a negative signal for the economy during other states. Thus, the market responds in various ways. We study the different responses of the stock market to news announcements over various stages of the business cycle. In contrast to McQueen and Roley (1993), who investigate this question using daily stock price data, this chapter uses high-frequency data to investigate this issue.

Fourth, we examine the so-called "calm before the storm" effect on the stock market. Price volatility and trading volume decline before news announcements because market participants generally withdraw from the market prior to announcements to avoid the high risks that these announcements bring. Jones *et al.* (1998) find that this effect can be observed in the days prior to such announcements. We are deeply sceptical of this finding because the effect of news announcements in financial markets does not persist over the course of a day, as reported in a number of past papers. Consequently, in this chapter, we use high-frequency data to clarify this effect.

The following results are presented in this chapter. The arrival of scheduled macroeconomic announcements in the U.S. stock market leads to a two-stage adjustment process for prices and trading transactions. In a brief initial stage, the release of a news announcement induces a sharp and nearly instantaneous price change as well as a rise in the number of trading transactions. In a prolonged second stage, the release causes significant and persistent increases in price volatility and trading transactions within approximately one hour. After allowing for different stages of the business cycle, we demonstrate that the release of a news announcement induces larger immediate price changes per interval during an expansion period compared with more immediate price changes per interval during a contraction period as prices shift from the old equilibrium to the approximate new equilibrium. The announcement causes smaller subsequent adjustments of stock prices along with a lower number of trading transactions across a shorter time span during a contraction period as the information contained in the news announcement is fully incorporated into stock prices. These findings imply that markets are more efficient during contraction periods. We use a static analysis to investigate the immediate effects of news announcements on prices and adopt a wavelet analysis to examine their eventual effects on prices. The evidence shows that only 6 of 17 announcements have a significant immediate effect, but all announcements have an eventual effect over different time periods. The combination of the results of both analyses provides us with a time profile for the effect of each type of news announcement on stock prices and reveals that the effect is significant within approximately one hour but dissipates after one day. The "calm before the storm" effect is observed only a few minutes prior to announcements.

This thesis is organised as follows. Chapter 2 illustrates wavelet theory and literature review. Chapter 3 uses wavelets to extract business cycles or detrend. Chapter 4 investigates financial contagion based on wavelet analysis. Chapter 5 examines U.S. stock market's response to economic news across the business cycle. The conclusion of this thesis is shown in chapter 6.

Chapter 2

Wavelet Theory and Literature Review

2.1 Introduction

One of the first steps involved in empirical work is to collect data. However, the required frequency of observations is often unavailable because it is expensive or impossible to collect data in the required frequency. Furthermore, there is no reason to believe that data collected in the required frequency would be able to fully capture the movement of the economy. The solution to this problem is a mathematical method referred to as temporal aggregation. The implicit assumption of this method is that the underlying stochastic process in continuous time is observed in discrete intervals. When the required frequency of observations is not available, temporal aggregation is applied to obtain the ideal frequency of data.

For example, how can weekly data be generated from daily data? The simplest method is to sum or average all daily data within the same week. Recently, the model-free measurement of volatility from high-frequency financial data has received much attention (Andersen *et al.* (2003), Brownlees and Gallo (2006)). One approved measurement is realised volatility (RV), which is an estimator of integral (e.g., daily) volatility as the sum of squared intraday returns (Zhang *et al.* (2005), Bandi and Russell (2006, 2008, 2011)). These two applications of temporal aggregation are useful when the frequency of the required data is lower than that of the collected data.

To obtain the appropriate frequency of data, the required data are obtained by averaging the more frequently collected data over time. The moving average, which is a prevalent means of smoothing a time series, smooths out short-term fluctuations and highlights the long-term trend or cycles. The trend of a time series may be meaningful for researchers and policymakers, and the residuals or deviations of this time series as noise may be worthless. A moving average or sum is often applied to the technical analysis of financial data (e.g., stock prices and trading volumes) and is used in economics to examine the gross domestic product, money stock and other macroeconomic data.[6] In filter theory, a moving average is a type of lowpass filter that averages time series.

Similar to generating weekly data from daily data, it is natural to consider how to estimate daily data from weekly data. Although there is no method to estimate these data precisely, the simplest approximation involves using the average changes in adjacent weekly data, which implies that the averages of the differences in weekly data are the daily data. Because there is no apparent difference between the daily data within the same week, this approach is rather coarse. Another relative example is the return, which is generated by subtracting the previous price from the current price. The intuition regarding these two examples is that of a moving difference, which is frequently adopted to stationarise a time series. In filter theory, a moving difference is a type of highpass filter that selects residuals or deviations that are important for regression analysis while discarding the other components.

In conclusion, we briefly introduce the moving average and moving difference, which are types of lowpass filter and highpass filters, respectively, in filter theory. From the frequency domain theory perspective, the lowpass filter is defined as preserving low-frequency components and abandoning high-frequency components, whereas the highpass filter has a reverse effect on time series and only conserves high-frequency components. Generally, the two filters are particular types of bandpass filters that pass components with frequencies within a certain range and reject those with frequencies outside of that range. Bandpass filters have been widely used in the business cycle literature for many years (Canova (1998), Baxter and King (1999), Pollock (2000), Gomez (2001), Christiano and Fitzgerald (2003), Iacobucci and Noullez (2005), Estrella (2007)).

During the last two decades, wavelets have been gradually adopted in the economics and finance fields. Unlike other filters, for which amplitudes preserve only the frequency properties of data while discarding the temporal properties, the amplitudes of wavelets retain both properties. Moreover, wavelets produce an orthogonal decomposition of economic and financial variables by time scale, which is closely

[6]For stock data and flow data, a moving average and a moving sum, respectively, convert high-frequency data into lower-frequency data. For example, quarterly money stock as stock data and quarterly GDP as flow data are the results of taking the averages and sums, respectively, of successive sets of three monthly data points.

related to the frequency and time horizon.[7] As suggested by Ramsey and Lampart (1998a,b), the structure of decisions, the strength of relationships and the relative variables differ by time scale. Accordingly, some economics and finance issues are difficult to solve using conventional econometric models, which consider only the temporal properties of data. The orthogonal property of wavelets provides new insights into phenomena and solves some issues that have been ignored or that could not be addressed in the past. Finally, wavelet analysis has a local nature and is thus more useful for detecting structural break or jumps in data. On the contrary, Fourier analysis, which is traditionally used to capture the frequency properties of data, has a global nature and is thus inappropriate for studying issues that include local episodes. Consequently, wavelets have attracted the attention of economists in recent years.

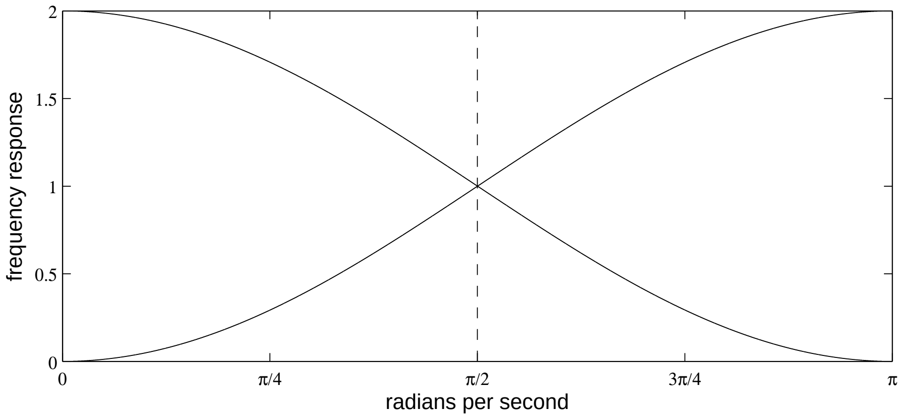


Figure 2.1: Squared gain functions for Haar (2) wavelet and scaling filters. The dotted line marks the frequency $\omega = \pi/2$ radians per second, which is the lower (upper) end of the nominal passband for the wavelet (scaling) filters.

In wavelet theory, there are two different types of filters: a lowpass filter (called a scaling filter) and a highpass filter (called a wavelet filter). The scaling filter eliminates components that lie within high-frequency bands and retains others, whereas the wavelet filter has a reverse effect, eliminating components that lie in low-frequency bands and preserving components in high-frequency bands. Figure [2.1] depicts the squared gain functions of Haar (2) scaling and wavelet filters, with the number in parentheses indicating the width of the filter.[8] The dotted line at the frequency value of $\pi/2$ radians per sample interval indicates the boundary between pass bands and stop bands. As shown, the scaling filter maintains the components

[7] Orthogonality implies that different frequency components are linearly independent and thus simplifies some economic issues.

[8] Haar wavelets, as the first and simplest wavelets, were proposed by Haar in 1909. The coefficients of the Haar (2) scaling filter are $g_0 = 1/\sqrt{2}$ and $g_1 = 1/\sqrt{2}$, while the coefficients of the Haar (2) wavelet filter are $h_0 = 1/\sqrt{2}$ and $h_1 = -1/\sqrt{2}$.

that lie in low frequencies, whereas the wavelet filter retains the components that lie in high frequencies.

Consider Haar (2) wavelets. Suppose that there is a data sequence of 8 points ($Y = [y_0, y_1, \cdots, y_7]'$); the moving average and the moving difference applied to this sequence can be achieved through matrix notation. In accordance with the 8-point size of the sample, the scaling filter ($g_0 = 1/\sqrt{2}$, $g_1 = 1/\sqrt{2}$) constitutes a 8×8 matrix G, which is represented by the following:

$$G = \begin{bmatrix} g_0 & 0 & 0 & 0 & 0 & 0 & 0 & g_1 \\ g_1 & g_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & g_1 & g_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & g_1 & g_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & g_1 & g_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & g_1 & g_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & g_1 & g_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & g_1 & g_0 \end{bmatrix}. \tag{2.1}$$

Correspondingly, the wavelet filter ($h_0 = 1/\sqrt{2}$, $h_1 = -1/\sqrt{2}$) constructs a 8×8 matrix H, which is expressed by the following:

$$H = \begin{bmatrix} h_0 & 0 & 0 & 0 & 0 & 0 & 0 & h_1 \\ h_1 & h_0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & h_1 & h_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_1 & h_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & h_1 & h_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_1 & h_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & h_1 & h_0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & h_1 & h_0 \end{bmatrix}. \tag{2.2}$$

To implement the moving average and the moving difference using the Haar (2) wavelet, the downsampling matrix $V = \Lambda' = [e_0, e_2, \cdots, e_{T-2}]'$ is required. This matrix is estimated by deleting alternate rows of the identity matrix I_T ($I_T = [e_0, e_1, \cdots, e_{T-2}, e_{T-1}]$) of order T. The result of VH or VG is a matrix in which the elements of H or G in odd-numbered rows are eliminated. This process is known as downsampling. In correspondence with downsampling, upsampling inserts zeros into alternate rows of a matrix, which is operated by Λ. Consequently, the result of ΛVH or ΛVG is a matrix in which the elements in odd-numbered rows are zeros.

The moving averages and moving differences of the data sequence are both equivalent to the first stage of the Haar discrete wavelet transform. From the matrix notation perspective, this stage is expressed by the following:

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} VH \\ VG \end{bmatrix} Y = QY = \begin{bmatrix} h_0 & 0 & 0 & 0 & 0 & 0 & 0 & h_1 \\ 0 & h_1 & h_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & h_1 & h_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & h_1 & h_0 & 0 \\ g_0 & 0 & 0 & 0 & 0 & 0 & 0 & g_1 \\ 0 & g_1 & g_0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & g_1 & g_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & g_1 & g_0 & 0 \end{bmatrix} \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \end{bmatrix}, \quad (2.3)$$

where the column vector α is constructed using the amplitude coefficients of the wavelet filter and the column vector β is constructed using the amplitude coefficients of the scaling filter. Thus,

$$\alpha = VHY = \begin{bmatrix} h_0y_0 + h_1y_7 \\ h_1y_1 + h_0y_2 \\ h_1y_3 + h_0y_4 \\ h_1y_5 + h_0y_6 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} y_0 - y_7 \\ y_2 - y_1 \\ y_4 - y_3 \\ y_6 - y_5 \end{bmatrix}, \quad (2.4)$$

$$\beta = VGY = \begin{bmatrix} g_0y_0 + g_1y_7 \\ g_1y_1 + g_0y_2 \\ g_1y_3 + g_0y_4 \\ g_1y_5 + g_0y_6 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} y_0 + y_7 \\ y_1 + y_2 \\ y_3 + y_4 \\ y_5 + y_6 \end{bmatrix}. \quad (2.5)$$

It is not difficult to discover that the amplitude coefficients (α) of the wavelet filter are the results of a moving difference of the data sequence, while the amplitude coefficients (β) of the scaling filter are identical to the results of a moving average.

It is easily confirmed that the matrix Q is an orthonormal matrix in which $QQ' = Q'Q = I$. Consequently, the vector Y can be synthesised by the vectors α and β as follows:

$$Y = Q' \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} VH \\ VG \end{bmatrix}' \begin{bmatrix} \alpha \\ \beta \end{bmatrix} = H'\Lambda\alpha + G'\Lambda\beta = H'\Lambda VHY + G'\Lambda VGY. \quad (2.6)$$

Normally, it is preferable to refer to $H'\Lambda\alpha = \mathbf{w_1}$ and $G'\Lambda\beta = \mathbf{v_1}$. Thus,

$$Y = \mathbf{w_1} + \mathbf{v_1}, \tag{2.7}$$

where the vector $\mathbf{w_1}$ comprises high-frequency components of the data sequence and the vector $\mathbf{v_1}$ contains low-frequency components. The vector $\mathbf{w_1}$ is mutually orthogonal with $\mathbf{v_1}$, as demonstrated subsequently. The relationship between $\mathbf{w_1}$ and $\mathbf{v_1}$ indicates that the corresponding components are also orthogonal. This simple MRA (multiresolution analysis) method separates each component of a data sequence into an individual frequency band for analysis. This type of analysis is an ideal method of studying a complex data sequence. It is widely accepted that the characteristics or relationships of economic or financial variables vary over frequencies, as verified by a number of studies. Accordingly, it is reasonable and efficient to study the variables in corresponding frequencies and then to identify which results are more convincing. For example, when one studies data that exhibit cyclical behaviour and/or that are affected by seasonal effects, it is preferable to remove seasonal components to estimate trend or residuals which are useful to capture some information, because seasonal effects can veil both the true underlying movement in the series and particular non-seasonal characteristics in which analysts may be interested.

However, economics or financial variables undoubtedly evolve over time; thus, the time-domain methods and/or models have dominated economics and finance studies for many years. Consequently, the temporal and frequency properties of data are important. Because wavelet analysis considers both the time and frequency domains, this type of MRA is an excellent method for solving issues that are simultaneously related to time and frequency.

2.2 Wavelet Analysis

The above introduction to wavelets is the first stage of a dyadic decomposition. In fact, the data sequence could be decomposed further, depending on its length. Moreover, many other filters in wavelet theory are more advanced than Haar wavelets, such as Daubechies wavelets or Morlet wavelets, which also belong to bandpass filters.[9] Because the use of wavelets can clearly improve our work, it is important to learn about them and apply them to the fields of economics and finance. First, we introduce wavelet theory in detail below.

[9] Wavelets are generated by imposing additional constraints on a wavelet filter which will be introduced in the below sections. The Daubechies wavelets are discussed in detail in the appendix.

2.2.1 Wavelets

Wavelets literally mean small waves because they have finite length and are oscillatory. Wavelets on finite support begin at a certain point in time and then die out subsequently. The localised nature of wavelets enables them to be used in the analysis of episodic variations in the frequency composition of data, and they are thus referred to as a "mathematical microscope". There are two different functions in wavelet theory: a wavelet function ($\psi(t)$) and a scaling function ($\phi(t)$). By definition, the wavelet function $\psi_0(t)$ should satisfy the following two conditions:

$$1: \int_{-\infty}^{+\infty} \psi_0(t)\, dt = 0, \tag{2.8}$$

$$2: \int_{-\infty}^{+\infty} \psi_0(t-k)\psi_0(t-j)\, dt = \begin{cases} 1 & \text{if } k = j \\ 0 & \text{if } k \neq j \end{cases}. \tag{2.9}$$

The scaling function $\phi_0(t)$ also should fulfil two conditions:

$$1: \int_{-\infty}^{+\infty} \phi_0(t)\, dt = 1, \tag{2.10}$$

$$2: \int_{-\infty}^{+\infty} \phi_0(t-k)\phi_0(t-j)\, dt = \begin{cases} 1 & \text{if } k = j \\ 0 & \text{if } k \neq j \end{cases}, \tag{2.11}$$

where condition 2 of the wavelet and scaling functions guarantees that each is an orthonormal function.[10]

Suppose that there is a space $\mathcal{V}_0$. Under wavelet analysis, this space can be decomposed orthogonally into many different subspaces: $\mathcal{W}_1$, $\mathcal{W}_2$, $\mathcal{W}_3, \cdots$. More precisely, the space $\mathcal{V}_0$, which is in the range of frequencies $[0, \pi]$, is decomposed orthogonally into two subspaces $\mathcal{W}_1$ and $\mathcal{V}_1$, which correspond to the frequency bands $(\pi/2, \pi]$ and $[0, \pi/2]$, respectively. Because subspaces $\mathcal{W}_1$ and $\mathcal{V}_1$ are mutually orthogonal, the sum of $\mathcal{W}_1$ and $\mathcal{V}_1$ is $\mathcal{V}_0$: $\mathcal{W}_1 \oplus \mathcal{V}_1 = \mathcal{V}_0$. $\mathcal{V}_1$ is then decomposed into two mutually orthogonal subspaces $\mathcal{W}_2$ and $\mathcal{V}_2$, which belong to the frequencies $(\pi/4, \pi/2]$ and $[0, \pi/4]$. We repeat this process of decomposing the space of scaling functions J times. Finally, we obtain the Jth subspaces $\mathcal{W}_J$ and $\mathcal{V}_J$. This algorithm, known as the Pyramid Algorithm, reduces computation and improves efficiency.

[10]The subscript represents the decomposed level. The details are introduced as follows.

Because subspaces at the same level are mutually orthogonal, the algorithm may be expressed as follows from a mathematical perspective:

$$\begin{aligned} \mathcal{V}_0 &= \mathcal{W}_1 \oplus \mathcal{V}_1 \\ &= \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{V}_2 \\ &\vdots \\ &= \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{W}_3 \oplus \cdots \oplus \mathcal{V}_J, \end{aligned} \tag{2.12}$$

$\mathcal{W}_j$ and $\mathcal{V}_j$ are mutually orthogonal, which may be expressed as "$\mathcal{W}_j \perp \mathcal{V}_j$" and is known as "lateral orthogonal". $\mathcal{W}_j$ and $\mathcal{W}_k$ ($j \neq k$) are mutually orthogonal, expressed as "$\mathcal{W}_j \perp \mathcal{W}_k$" as well and known as "sequential orthogonal". However, $\mathcal{V}_j$ and $\mathcal{V}_k$ ($j \neq k$) are not mutually orthogonal. In sum, the scaling subspace $\mathcal{V}_j$ is not orthogonal across scales; orthogonality across scales comes from the wavelet subspace $\mathcal{W}_j$. Here, j ($1 \leqslant j \leqslant J$) is called the decomposed level and is related to scale (2^{j-1}), which is the inverse of the frequency band ($(\pi/2^j, \pi/2^{j-1}]$) for wavelet subspace $\mathcal{W}_j$.

For a function $f(t)$ ($t = 0, \cdots, T-1$, where $T = 2^J$) in $\mathcal{V}_0$, the projection of $f(t)$ in $\mathcal{W}_1$ is $\Delta f_1(t)$, and the projection of $f(t)$ in $\mathcal{V}_1$ is $f_1(t)$. Because $\mathcal{V}_0 = \mathcal{W}_1 \oplus \mathcal{V}_1$, $f(t) = \Delta f_1(t) + f_1(t)$. Given the Pyramid Algorithm, $f(t)$ could be a linear combination of $\Delta f_j(t)$ ($1 \leqslant j \leqslant J$) and $f_J(t)$: $f(t) = \sum_{j=1}^{J} \Delta f_j(t) + f_J(t)$. This approach is defined as multiresolution analysis (MRA). Note that the scaling function $\phi_j(t)$ constitutes an orthonormal basis of subspace $\mathcal{V}_j$, whereas the wavelet function $\psi_j(t)$ constructs an orthonormal basis of subspace $\mathcal{W}_j$. Therefore, $f_j(t)$ can be approximated by $\phi_j(t)$, and $\Delta f_j(t)$ can be expressed by $\psi_j(t)$. To clarify these functions, the first necessary step is to introduce the scaling function $\phi_j(t)$ and the wavelet function $\psi_j(t)$ in the following subsections.

2.2.2 The Dilation Equation

In subspace $\mathcal{V}_1 \in \mathcal{V}_0$, $\mathcal{V}_1$ is spanned by the basis scaling function $\phi_1(t)$, whereas $\mathcal{V}_0$ is constituted by the basis scaling function $\phi_0(t)$; therefore, these two functions are related. This relationship may be expressed as follows:

$$\phi_1(t) = 2^{-1/2}\phi_0(2^{-1}t). \tag{2.13}$$

Generally, the adjacent level scaling functions also have this relationship. $\phi_2(t) = 2^{-1/2}\phi_1(2^{-1}t)$, and $\phi_1(t)$ is replaced by $\phi_0(t)$ according to Equation (2.13): $\phi_2(t) = 2^{-2/2}\phi_0(2^{-2}t)$. By recursion of this procedure, the general version of the scaling function $\phi_j(t)$ can be achieved by the following:

$$\phi_j(t) = 2^{-j/2}\phi_0(2^{-j}t). \tag{2.14}$$

Because subspace $\mathcal{V}_1$ is half of $\mathcal{V}_0$, the time rate in the scaling function $\phi_1(t)$ is also halved and is equal to $2^{-1}t$. The factor $2^{-1/2}$ on the RHS of Equation (2.13) leads the scaling function $\phi_1(t)$ to fulfil condition 2 in Equation (2.11) because

$$\int \phi_1^2(t)\,dt = 2^{-1}\int \phi_0^2(2^{-1}t)\,dt = 2^{-1}\int \phi_0^2(\tau)2\,d\tau = 1, \tag{2.15}$$

where $\tau = 2^{-1}t$. It is not difficult to infer that condition 2 is valid for all scaling functions $\phi_j(t)$ $(0 \leqslant j \leqslant J)$.

Equation (2.13) could also be written as follows:

$$\phi_0(t) = 2^{1/2}\phi_1(2t). \tag{2.16}$$

Normally, we prefer the latter expression. $\phi_1(t)$ is the dilated version of $\phi_0(t)$, and its translated version is $\phi_1(t-k)$, which is a function of the orthonormal basis of $\mathcal{V}_1$. Consequently, $\phi_1(t-k) = 2^{-1/2}\phi_0(2^{-1}t-k)$, which is called the dilated and translated version of $\phi_0(t)$. The general version of the scaling function is as follows:

$$\phi_{j,k} = 2^{-j/2}\phi_0(2^{-j}t-k), \quad k = 0, \cdots, \frac{T}{2^j} - 1. \tag{2.17}$$

The time rate t in the scaling function $\phi_0(t)$ is from 0 to $T-1$ and is now halved in the scaling function $\phi_1(t)$, from 0 to $(T-1)/2$. The translated function $\phi_1(t-k)$ begins at time $t = k+0$ and ends at time $t = k+(T-1)/2$. In conclusion, the scaling function $\phi_0(t-k)$; $k = 0, ..., T-1$ is located at each sample point; alternatively, it is identical to each scaling function residing within a cell of unit width. During the first stage of decomposition, there is a wavelet function $\psi_{1,k}(t)$ at every second point; alternatively, the function is in successive cells of width two, together with a scaling function $\phi_{1,k}(t)$ on alternate points. Generally, at each level of decomposition, the successions of wavelet functions and scaling functions span the entire sample. More details will be provided in the following section associated with Figure [2.2].

Because subspace $\mathcal{V}_1$ is involved in $\mathcal{V}_0$, $\phi_1(t)$ is an element of the basis of space $\mathcal{V}_0$ as well as $\phi_0(t)$. The elements must be combined as the basis of $\mathcal{V}_0$. The appropriate expression is as follows:

$$\phi_1(t) = \sum_k g_k\phi_0(t+k) = 2^{1/2}\sum_k g_k\phi_1(2t+k), \tag{2.18}$$

where $g_k = \langle \phi_1(t), \phi_0(t+k) \rangle = \int_{-\infty}^{\infty} \phi_1(t)\phi_0(t+k)\, dt$, which is known as a scaling filter. Equation (2.18) is referred to as the Dilation Equation.[11] Corresponding with the scaling function satisfying two conditions, the scaling filter should fulfil three conditions. First, the sum of g_k is $\sqrt{2}$ because

$$\int \phi_1(t)\, dt = 2^{-1/2} \int \phi_0(2^{-1}t)\, dt = 2^{-1/2} \cdot 2 \int \phi_0(\tau)\, d\tau = \sqrt{2}, \tag{2.19}$$

where $\tau = 2^{-1}t$, and

$$\int \phi_1(t)\, dt = \int \sum g_k \phi_0(t+k)\, dt = \sum g_k \int \phi_0(t+k)\, dt = \sum g_k. \tag{2.20}$$

Thus, $\sum g_k = \sqrt{2}$. Second, the sum of g_k squared is one because

$$\int \phi_1^2(t)\, dt = \int \sum g_k^2 \phi_0^2(t+k)\, dt = \sum g_k^2, \tag{2.21}$$

according to Equation (2.15): $\int \phi_1^2(t)\, dt = 1$, $\sum g_k^2 = 1$. Because the integral of any scaling function $\phi_j(t)$ squared is equal to one, $\sum g_k^2 = 1$ is the condition applied to all scaling functions.

A more important condition that guarantees orthogonality is $\sum_k g_k g_{k+2m} = 0$, which is derived from condition 2 in Equation (2.11). Because $\int_t \phi_1(t)\phi_1(t-m)\, dt = 0$ $(m \neq 0)$ and $\int_t \phi_1(t)\phi_1(t)\, dt = 1$ $(m = 0)$, in accordance with the Dilation Equation, we obtain the following:

$$\begin{aligned}
\int_t \phi_1(t)\phi_1(t-m)\, dt &= \int_t \sum_k \sum_j g_k g_j \phi_0(t+k)\phi_0(t-m+j)\, dt \\
&= 2 \int_t \sum_k \sum_j g_k g_j \phi_1(2t+k)\phi_1(2(t-m)+j)\, dt \\
&= 0.
\end{aligned} \tag{2.22}$$

If $k = j - 2m$, then Equation (2.22) is written as follows:

$$2 \sum_k g_k g_{k+2m} \int_t \phi_1(2t+k)\phi_1(2t+k) dt = \sum_k g_k g_{k+2m} = 0. \tag{2.23}$$

If $k \neq j - 2m$, then Equation (2.22) is expressed as follows:

$$2 \int_t \sum_k \sum_j g_k g_j \phi_1(2t+k)\phi_1(2t+j-2m)\, dt = \sum_k \sum_j g_k g_j \cdot 0 = 0, \tag{2.24}$$

[11]The Dilation and Wavelet Equations are summarised in Strang (1989).

where $2\int \phi_1(2t+k)\phi_1(2t+j-2m)\,dt = 0$, which is the second condition that the scaling function must fulfil. Consequently, $\sum_k g_k g_{k+2m} = 0\ (m \neq 0)$ is obtained from the combination of two results. We refer to the equations $\sum_k g_k^2 = 1$ and $\sum_k g_k g_{k+2m} = 0\ (m \neq 0)$ collectively as the orthonormal properties of the scaling filter.

2.2.3 The Wavelet Equation

The relationship among the adjacent level wavelet functions corresponds to the relationship between the adjacent level scaling functions. Consequently, the wavelet function ψ_j at level j is expressed as follows:

$$\psi_j = 2^{-j/2}\psi_0(2^{-j}t). \tag{2.25}$$

A dilated and translated version of the wavelet function is the following:

$$\psi_{j,k} = 2^{-j/2}\psi_0(2^{-j}t-k), \quad k = 0, \cdots, \frac{T}{2^j} - 1, \tag{2.26}$$

which constitutes the orthonormal basis of the wavelet subspace $\mathcal{W}_j$. Because the wavelet function $\psi_1(t)$ constitutes the basis of $\mathcal{W}_1$ and because $\mathcal{W}_1$ is involved in $\mathcal{V}_0$, it is possible to express the wavelet function $\psi_1(t)$ as a linear combination of the elements of the basis of $\mathcal{V}_0$. The appropriate expression is the following:

$$\psi_1(t) = \sum_k h_k \phi_0(t+k) = 2^{1/2}\sum_k h_k \phi_1(2t+k), \tag{2.27}$$

where h_k is called a wavelet filter and $h_k = \langle \psi_1(t), \phi_0(t+k)\rangle = \int_{-\infty}^{\infty} \psi_1(t)\phi_0(t+k)\,dt$. Equation (2.27) is referred to as the Wavelet Equation.

The expression of the wavelet filter h_k in Equation (2.27) is similar to that of the scaling filter g_k in Equation (2.18). The difference is that $\phi_1(t)$ is replaced by $\psi_1(t)$. Because condition 2 in Equations (2.9) and (2.11) is always valid for wavelet functions and scaling functions, respectively, the wavelet filter h_k also fulfils two conditions: $\sum h_k^2 = 1$ and $\sum h_k h_{k+2m} = 0\ (m \neq 0)$, as was the case with the scaling filter g_k. Furthermore, $\psi_1(t) = \sum h_k \phi_0(t+k)$, and the integral of scaling function $\phi_0(t)$ is one; thus, $\int_t \psi_1(t)\,dt = \int_t \sum h_k \phi_0(t+k)\,dt = \sum h_k$. Because $\int_t \psi_1(t)\,dt = \int_t 2^{-1/2}\psi_0(2^{-1}t)\,dt = 0$, $\sum h_k = 0$.

In conclusion, the wavelet filter h_k and the scaling filter g_k should satisfy the three conditions below:

$$\begin{aligned}\sum h_k = 0,\ \sum h_k^2 = 1,\ \sum h_k h_{k+2m} = 0 \quad (m \neq 0),\\ \sum g_k = \sqrt{2},\ \sum g_k^2 = 1,\ \sum g_k g_{k+2m} = 0 \quad (m \neq 0),\end{aligned} \tag{2.28}$$

which guarantee that the wavelet function or the scaling function is orthogonal to itself at different displacements, and these functions construct the orthonormal basis of subspace $\mathcal{W}_j$ or $\mathcal{V}_j$.

Another condition for estimating the mutual orthogonality of $\mathcal{W}_j$ and $\mathcal{V}_j$ is the following:

$$\sum_k g_k h_{k+2m} = 0 \ \ (m \neq 0), \tag{2.29}$$

which restricts the scaling function $\phi_j(t)$ of the basis of $\mathcal{V}_j$ and the wavelet function $\psi_j(t-m)$ of the basis of $\mathcal{W}_j$; those at different displacements will be mutually orthogonal.

Another condition is imposed to guarantee that the scaling function $\phi_j(t)$ and the wavelet function $\psi_j(t)$ will be mutually orthogonal at the same displacement level. This condition is described as follows:

$$\sum_k g_k h_k = 0. \tag{2.30}$$

Generally, there is a relationship between the scaling filter and the wavelet filter:

$$g_k = (-1)^{k+1} h_{L-k-1}, \tag{2.31}$$

and the inverse relationship is as follows:

$$h_k = (-1)^k g_{L-k-1}, \tag{2.32}$$

where L, the width of filter, must be even. $\{g_k\}$ is referred to as the "quadrature mirror filter" (QMF), corresponding to $\{h_k\}$. The scaling filter g_k is a lowpass filter that retains only the low-frequency components of the signal, whereas the wavelet filter h_k is a highpass filter that preserves only the high-frequency components.

2.2.4 The Decomposition of a Function in Space $\mathcal{V}_0$

As noted earlier in the paper, a space $\mathcal{V}_0$ can be decomposed orthogonally into two subspaces $\mathcal{W}_1$ and $\mathcal{V}_1$, whose bases are constructed by the wavelet function $\psi_1(t)$ and the scaling function $\phi_1(t)$, respectively. One consequence is that a continuous function $f(t)$ that belongs to the space $\mathcal{V}_0$ can be decomposed into elements for the subspaces $\mathcal{W}_1$ and $\mathcal{V}_1$, respectively. It follows that

$$f(t) = f_1(t) + \Delta f_1(t), \tag{2.33}$$

where $f_1(t)$ is the projection of $f(t)$ on the subspace $\mathcal{V}_1$ and $\Delta f_1(t)$ is the projection of $f(t)$ on the subspace $\mathcal{W}_1$. Because the scaling function $\phi_1(t)$ and the wavelet function $\psi_1(t)$ constitute the bases of subspaces $\mathcal{V}_1$ and $\mathcal{W}_1$, respectively, $f(t)$ may also be decomposed as follows:

$$f(t) = \sum_k c_{1,k}\phi_{1,k}(t) + \sum_k d_{1,k}\psi_{1,k}(t), \tag{2.34}$$

where $k = 0, 1, \cdots, T/2 - 1$; $c_{1,k} = \langle f(t), \phi_{1,k}(t)\rangle = \int_{-\infty}^{\infty} f(t)\phi_{1,k}(t)\,dt$; $d_{1,k} = \langle f(t), \psi_{1,k}(t)\rangle = \int_{-\infty}^{\infty} f(t)\psi_{1,k}(t)\,dt$; $\phi_{1,k}(t) = \phi_1(t-k)$; and $\psi_{1,k}(t) = \psi_1(t-k)$. Nevertheless, $d_{1,k}$ are the amplitude coefficients of the wavelet function $\psi_{1,k}$ for the projection of $f(t)$ on the subspace $\mathcal{W}_1$, whereas $c_{1,k}$ are the amplitude coefficients of the scaling function $\phi_{1,k}$ for the projection of $f(t)$ on the subspace $\mathcal{V}_1$. Here, $\sum_k c_{1,k}\phi_{1,k}(t) = f_1(t)$ and $\sum_k d_{1,k}\psi_{1,k}(t) = \Delta f_1(t)$. In the Pyramid Algorithm, $f_j(t)$ $(1 \leqslant j < J)$ is decomposed as in $f(t)$ until J. For example, $f_1(t) = \sum_k c_{2,k}\phi_{2,k}(t) + \sum_k d_{2,k}\psi_{2,k}(t) = f_2(t) + \Delta f_2(t)$, where $k = 0, 1, \cdots, T/4 - 1$. Therefore,

$$f(t) = \sum_{j=1}^{J} \Delta f_j(t) + f_J(t) = \sum_{j=1}^{J}\sum_k d_{j,k}\psi_{j,k}(t) + \sum_k c_{J,k}\phi_{J,k}(t), \tag{2.35}$$

where $k = 0, 1, \cdots, T/2^j - 1$; $d_{j,k}$ are the amplitude coefficients of the wavelet function $\psi_{j,k}(t)$, $d_{j,k} = \int_{-\infty}^{\infty} f(t)\psi_{j,k}\,dt$; and $c_{J,k}$ are the amplitude coefficients of the scaling function $\phi_{J,k}(t)$, $c_{J,k} = \int_{-\infty}^{\infty} f(t)\phi_{J,k}\,dt$. Note that $\Delta f_j(t)$ belongs to the subspace $\mathcal{W}_j$ and is thus in the range of frequencies $(\pi/2^j, \pi/2^{j-1}]$ as well, whereas $f_j(t)$ is located in the subspace $\mathcal{V}_j$ and is thus in the frequency interval of $[0, \pi/2^j]$.

2.2.5 Discrete Wavelet Transform

In economics and finance, a time series is regarded as discrete, but the above expressions apply to continuous time. Accordingly, their discrete versions are needed. The sampling theorem indicates that if there is an upper limit to the frequencies of the components of a continuous process $f(t)$, then it is possible to convey information regarding that process via a discrete sample y_t, $t = 0, \cdots, T-1$, provided that the sampling is sufficiently rapid.[12] $f(t) \simeq \sum_k y_k \phi(t-k)$: observe that there are no limits on the index k, i.e., $k \in (-\infty, +\infty)$. In the basic statement of sampling theorem, $\phi(t)$ is a sinc function that has infinite support, but $f(t)$ has the bounded domain of the entire real line. Thus, the linkage between the LHS and RHS of the equation is "$\simeq$".

The theorem can be adapted to cater to periodic functions or, equally, to the periodic extension of a function defined on finite support, in which the sinc function is replaced by a Dirichlet kernel defined on a circle of circumference T (which is the number of samples obtained from a single cycle of the function). From a mathematical perspective, the continuous function $f(t)$ is the result of the convolution of a discrete time series $y(t)$ and a scaling function $\phi_0(t)$, which is expressed as follows:

$$f(t) = \sum_{t=0}^{T-1} y(\tau)\phi_0(t-\tau), \tag{2.36}$$

where $\phi_0(t)$ is viewed as a train of the impulse function in the sampling theorem and $y(\tau)$ is the inner production of $f(t)$ and $\phi_0(t-\tau)$: $y(\tau) = \langle f(t), \phi_0(t-\tau)\rangle = \int_t f(t)\phi_0(t-\tau)\,dt$.

It is already known that the dilation and wavelet equations are

$$\begin{aligned} \phi_1(t) &= 2^{1/2} \sum_k g_k \phi_1(2t+k), \\ \psi_1(t) &= 2^{1/2} \sum_k h_k \phi_1(2t+k). \end{aligned} \tag{2.37}$$

Any shifted versions of the scaling function $\phi_1(t-m)$ and wavelet function $\psi_1(t-m)$ are equal to the following:

$$\begin{aligned} \phi_1(t-m) &= 2^{1/2} \sum g_k \phi_1(2t-2m+k) \\ &= 2^{1/2} \sum g_k \phi_1(2t+k-2m) \\ &= \sum g_k \phi_0(t+k-2m), \end{aligned} \tag{2.38}$$

and

$$\begin{aligned} \psi_1(t-m) &= 2^{1/2} \sum h_k \phi_1(2t-2m+k) \\ &= 2^{1/2} \sum h_k \phi_1(2t+k-2m) \\ &= \sum h_k \phi_0(t+k-2m). \end{aligned} \tag{2.39}$$

[12] The sampling theorem is demonstrated in the appendix.

Accordingly, the inner production of $f(t)$ and $\phi_1(t-m)$ is

$$\langle f(t), \phi_1(t-m)\rangle = \langle f(t), \sum g_k \phi_0(t+k-2m)\rangle = \sum g_k y_{2m-k}, \tag{2.40}$$

which can also be explained by the sampling theorem. These coefficients are associated with the basic function $\phi_1(t-m)$.

Similarly, the coefficients associated with the basic function $\psi_1(t-m)$ are

$$\langle f(t), \psi_1(t-m)\rangle = \langle f(t), \sum h_k \phi_0(t+k-2m)\rangle = \sum h_k y_{2m-k}. \tag{2.41}$$

Thus, the projection of $f(t)$ on the level-one subspace $\mathcal{V}_1$ is

$$f_1(t) = \langle f(t), \phi_1(t-m)\rangle \phi_1(t-m) = \sum g_k y_{2m-k} \phi_1(t-m), \tag{2.42}$$

whereas the projection of $f(t)$ on the level-one subspace $\mathcal{W}_1$ is

$$\Delta f_1(t) = \langle f(t), \psi_1(t-m)\rangle \psi_1(t-m) = \sum h_k y_{2m-k} \psi_1(t-m). \tag{2.43}$$

The synthesis of $f(t)$ is

$$f(t) = f_1(t) + \Delta f_1(t) = \sum g_k y_{2m-k} \phi_1(t-m) + \sum h_k y_{2m-k} \psi_1(t-m). \tag{2.44}$$

Let $\alpha_{1,m}$ denote the coefficients of the level-one wavelet function $\psi_1(t)$ and $\beta_{1,m}$ denote the coefficients of the level-one scaling function $\phi_1(t)$. These functions are

$$\begin{aligned} \alpha_{1,m} &= \sum_{k=0}^{T-1} h_k y_{2m-k}; \quad m = 0, 1, \cdots, T/2-1, \\ \beta_{1,m} &= \sum_{k=0}^{T-1} g_k y_{2m-k}; \quad m = 0, 1, \cdots, T/2-1, \end{aligned} \tag{2.45}$$

where T is the length of the data sequence. Because the data sequence is finite, a problem arises when a filter is applied at the end of the sample and required data lie beyond the end point. To overcome this problem, the filter can be applied to the data via a process of circular convolution, thus applying the filter to the periodic extension of data. Specifically, a data sequence of infinite length that would support all wavelets could be generated by the periodic extension of the sample. It is more appropriate to envisage wrapping the data around a circle of circumference T, which is equal to the number of data points, such that the end of the sample becomes adjacent to its beginning.

Another strategy involves applying the periodic extension of the filter to finite data. The filter is typically supported on an interval of a width that is less than the length of the data sequence. The filter can be wrapped around the circle of circumference T, which is identical to the number of data points. If the width of the filter exceeds the circumference, then it can continue to be wrapped around the circle, and its overlying ordinates can be added. Thus,

$$
\begin{aligned}
\alpha_{1,m} &= \sum_{k=0}^{T-1} h_k y_{2m-k \bmod T} = \sum_{k=0}^{T-1} h_{2m-k \bmod T} y_k; \quad m = 0, 1, \cdots, T/2-1, \\
\beta_{1,m} &= \sum_{k=0}^{T-1} g_k y_{2m-k \bmod T} = \sum_{k=0}^{T-1} g_{2m-k \bmod T} y_k; \quad m = 0, 1, \cdots, T/2-1,
\end{aligned}
\tag{2.46}
$$

where $h_{2m-k \bmod T}$ and $g_{2m-k \bmod T}$, generated by wrapping h_{2m-k} and g_{2m-k} around the circle of circumference T, are periodic extensions of the wavelet filter h_{2m-k} and the scaling filter g_{2m-k}, respectively. Normally, the second strategy is preferred.

The procedure in the second stage is exactly the same as that in the first stage. The only difference is that y_t is replaced by the scaling coefficients $\beta_{1,m}$ as inputs. Thus, the coefficients associated with the wavelet filter and the scaling filter, respectively, at the second level are as follows:

$$
\begin{aligned}
\alpha_{2,m} &= \sum_{k=0}^{T/2-1} h_{2m-k \bmod \frac{T}{2}} \beta_{1,k}; \quad m = 0, 1, \cdots, T/4-1, \\
\beta_{2,m} &= \sum_{k=0}^{T/2-1} g_{2m-k \bmod \frac{T}{2}} \beta_{1,k}; \quad m = 0, 1, \cdots, T/4-1.
\end{aligned}
\tag{2.47}
$$

The remaining procedures are treated in exactly the same manner as in the first and second stages. The complete process is more apparent and intuitive when using matrix notation. Suppose that there is a T-dimensional column vector Y whose tth element is y_t. A $T \times T$ orthonormal matrix Ω ($\Omega\Omega' = I_{T\times T}$) defines a $T \times 1$ column vector α whose elements are wavelet coefficients: $\alpha = \Omega Y$. According to a MRA, the column vector α could be divided as follows:

$$\begin{aligned}
\alpha_{(1)} &= [\alpha_0, \alpha_1, \cdots, \alpha_{\frac{T}{2}-1}]' = [\alpha_{1,0}, \alpha_{1,1}, \cdots, \alpha_{1,\frac{T}{2}-1}]', \\
\alpha_{(2)} &= [\alpha_{\frac{T}{2}}, \alpha_{\frac{T}{2}+1}, \cdots, \alpha_{\frac{3T}{4}-1}]' = [\alpha_{2,0}, \alpha_{2,1}, \cdots, \alpha_{2,\frac{T}{4}-1}]', \\
&\vdots \\
\alpha_{(J-1)} &= [\alpha_{T-4}, \alpha_{T-3}]' = [\alpha_{J-1,0}, \alpha_{J-1,1}]', \\
\alpha_{(J)} &= [\alpha_{T-2}] = [\alpha_{J,0}], \\
\beta_{(J)} &= [\alpha_{T-1}] = [\beta_{J,0}],
\end{aligned} \tag{2.48}$$

where only two wavelet coefficients are in the last decomposed level and the last α_{T-1} is denoted by $\beta_{(J)}$. Scaling coefficients accompany the wavelet coefficients; although the scaling coefficients are not the objective of the analysis, their presence is necessary to obtain the wavelet coefficients. Therefore, a $(T-2) \times 1$ column vector β whose elements are scaling coefficients is introduced, and it is partitioned as follows:

$$\begin{aligned}
\beta_{(1)} &= [\beta_{1,0}, \beta_{1,1}, \cdots, \beta_{1,\frac{T}{2}-1}]', \\
\beta_{(2)} &= [\beta_{2,0}, \beta_{2,1}, \cdots, \beta_{2,\frac{T}{4}-1}]', \\
&\vdots \\
\beta_{(J-1)} &= [\beta_{J-1,0}, \beta_{J-1,1}]'.
\end{aligned} \tag{2.49}$$

The consequence is that $\alpha = [\alpha_0, \alpha_1, \cdots, \alpha_{T-1}]' = \left[\alpha'_{(1)}, \alpha'_{(2)}, \cdots, \alpha'_{(J)}, \beta'_{(J)}\right]'$. According to the property of orthonormal transform, $\Omega\Omega' = I_{T\times T}$, $Y = \Omega'\alpha$. The partition of α is identical to the following:

$$Y = \Omega'\alpha = \sum_{j=1}^{J} \Omega'_j \alpha_{(j)} + \nu'_J \beta_{(J)}, \tag{2.50}$$

where Ω_j and ν_j are portions of the matrix Ω. The $T/2 \times T$ matrix Ω_1 constitutes the elements of Ω from the $k = 0$ up to the $T/2 - 1$ rows, the $T/4 \times T$ matrix Ω_2 is formed from the $k = T/2$ up to $3T/4 - 1$ rows, and so forth until the $1 \times T$ matrices Ω_J and ν_J constitute the second-to-last and last row, respectively. In sum, $\Omega = [\Omega'_1, \Omega'_2, \cdots, \Omega'_J, \nu'_J]'$, where Ω_j is a $T/2^j \times T$ matrix for $j = 1, \cdots, J$ and ν_J is a T dimensional row vector that is the last row of the matrix Ω. This form of Ω corresponds to $J + 1$ portions of the DWT coefficient vector α.

At this point, $\Omega'_j \alpha_{(j)}$ is defined as $\mathbf{w}_j$: $\mathbf{w}_j = \Omega'_j \alpha_{(j)}$ for $j = 1, \cdots, J$, which is a T dimensional column vector whose elements are associated with changes in Y at scale

2^{j-1} and is the portion of the synthesis $Y = \Omega'\alpha$ attributable to scale 2^{j-1}. $\nu'_J\beta_{(J)}$ is defined as $\mathbf{v}_J$: $\mathbf{v}_J = \nu'_J\beta_{(J)}$. Thus, Equation (2.50) could be written as follows:

$$Y = \Omega'\alpha = \sum_{j=1}^{J} \Omega'_j\alpha_{(j)} + \nu'_J\beta_{(J)} = \sum_{j=1}^{J} \mathbf{w}_j + \mathbf{v}_J, \tag{2.51}$$

which forms a MRA of Y. $\mathbf{w}_j$ is referred to as the jth level wavelet detail. To clearly and intuitively demonstrate the discrete wavelet transform using matrix notation, a simple example is illustrated in which the width of filter L is assumed to be four and the length of sequence T is eight. In accordance with the first stage of wavelet decomposition in Equation (2.46), we have the following:

$$\begin{bmatrix} \alpha_{1,0} \\ \alpha_{1,1} \\ \alpha_{1,2} \\ \alpha_{1,3} \\ \beta_{1,0} \\ \beta_{1,1} \\ \beta_{1,2} \\ \beta_{1,3} \end{bmatrix} = \begin{bmatrix} h_0 & 0 & 0 & 0 & 0 & h_3 & h_2 & h_1 \\ h_2 & h_1 & h_0 & 0 & 0 & 0 & 0 & h_3 \\ 0 & h_3 & h_2 & h_1 & h_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & h_3 & h_2 & h_1 & h_0 & 0 \\ g_0 & 0 & 0 & 0 & 0 & g_3 & g_2 & g_1 \\ g_2 & g_1 & g_0 & 0 & 0 & 0 & 0 & g_3 \\ 0 & g_3 & g_2 & g_1 & g_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & g_3 & g_2 & g_1 & g_0 & 0 \end{bmatrix} \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \end{bmatrix}. \tag{2.52}$$

This stage actually contains a downsampling process (denote as "$\downarrow 2$") that retains even-numbered elements and discards odd-numbered elements. Define a 8 by 8 matrix H_1 that is constituted by the wavelet filter h_k ($k = 0, 1, 2, 3$):

$$H_1 = \begin{bmatrix} h_0 & 0 & 0 & 0 & 0 & h_3 & h_2 & h_1 \\ h_1 & h_0 & 0 & 0 & 0 & 0 & h_3 & h_2 \\ h_2 & h_1 & h_0 & 0 & 0 & 0 & 0 & h_3 \\ h_3 & h_2 & h_1 & h_0 & 0 & 0 & 0 & 0 \\ 0 & h_3 & h_2 & h_1 & h_0 & 0 & 0 & 0 \\ 0 & 0 & h_3 & h_2 & h_1 & h_0 & 0 & 0 \\ 0 & 0 & 0 & h_3 & h_2 & h_1 & h_0 & 0 \\ 0 & 0 & 0 & 0 & h_3 & h_2 & h_1 & h_0 \end{bmatrix}. \tag{2.53}$$

In terms of polynomial expression,

$$H_1 = H(K_8) = h_0K_8^0 + h_1K_8^1 + h_2K_8^2 + h_3K_8^3, \tag{2.54}$$

where $K_T = [e_1, e_2, \cdots, e_{T-1}, e_0]$ is established by shifting the first column of an identity matrix ($I = [e_0, e_1, \cdots, e_{T-1}]$) to the last column. Here, $T = 8$ is the length of the original time series, and the polynomial degree 3 ($L-1$) is determined by the width of filter (L). The downsampled matrix H_1 is the following:

$$(\downarrow 2)H_1 = \begin{bmatrix} h_0 & 0 & 0 & 0 & 0 & h_3 & h_2 & h_1 \\ h_2 & h_1 & h_0 & 0 & 0 & 0 & 0 & h_3 \\ 0 & h_3 & h_2 & h_1 & h_0 & 0 & 0 & 0 \\ 0 & 0 & 0 & h_3 & h_2 & h_1 & h_0 & 0 \end{bmatrix}, \tag{2.55}$$

which also can be expressed as VH_1, where $V = \Lambda' = [e_0, e_2, \cdots, e_{T-2}]'$, which is derived from the identity matrix ($I = [e_0, e_1, \cdots, e_{T-1}]$) by deleting the alternate rows.

The scaling filter g_k constructs a matrix G_1, whose format is the same as that for H_1:

$$G_1 = G(K_8) = g_0 K_8^0 + g_1 K_8^1 + g_2 K_8^2 + g_3 K_8^3. \tag{2.56}$$

The first stage of wavelet decomposition by matrix notation can be written as follows:

$$\begin{bmatrix} \alpha_{1,0} \\ \alpha_{1,1} \\ \alpha_{1,2} \\ \alpha_{1,3} \\ \beta_{1,0} \\ \beta_{1,1} \\ \beta_{1,2} \\ \beta_{1,3} \end{bmatrix} = \begin{bmatrix} VH_1 \\ VG_1 \end{bmatrix} \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \end{bmatrix}, \tag{2.57}$$

or

$$\begin{bmatrix} \alpha_{(1)} \\ \beta_{(1)} \end{bmatrix} = \begin{bmatrix} VH_1 \\ VG_1 \end{bmatrix} Y. \tag{2.58}$$

In the second stage of wavelet decomposition, the first-level wavelet coefficients are preserved. The first-level scaling coefficients as inputs are decomposed into wavelet coefficients associated with second-level wavelet filter and scaling coefficients associated with the second-level scaling filter. This stage is expressed by the following:

$$\begin{bmatrix} \alpha_{1,0} \\ \alpha_{1,1} \\ \alpha_{1,2} \\ \alpha_{1,3} \\ \alpha_{2,0} \\ \alpha_{2,1} \\ \beta_{2,0} \\ \beta_{2,1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_0 & h_3 & h_2 & h_1 \\ 0 & 0 & 0 & 0 & h_2 & h_1 & h_0 & h_3 \\ 0 & 0 & 0 & 0 & g_0 & g_3 & g_2 & g_1 \\ 0 & 0 & 0 & 0 & g_2 & g_1 & g_0 & g_3 \end{bmatrix} \begin{bmatrix} \alpha_{1,0} \\ \alpha_{1,1} \\ \alpha_{1,2} \\ \alpha_{1,3} \\ \beta_{1,0} \\ \beta_{1,1} \\ \beta_{1,2} \\ \beta_{1,3} \end{bmatrix}, \tag{2.59}$$

or

$$\begin{bmatrix} \alpha_{(1)} \\ \alpha_{(2)} \\ \beta_{(2)} \end{bmatrix} = \begin{bmatrix} I_{4\times 4} & \mathbf{0}_{4\times 4} \\ \mathbf{0}_{2\times 4} & VH_2 \\ \mathbf{0}_{2\times 4} & VG_2 \end{bmatrix} \begin{bmatrix} \alpha_{(1)} \\ \beta_{(1)} \end{bmatrix}, \tag{2.60}$$

where both H_2 and G_2 are 4 by 4 matrices. In terms of polynomial expression, $H_2 = H(K_4) = h_0K_4^0+h_1K_4^1+h_2K_4^2+h_3K_4^3$ and $G_2 = G(K_4) = g_0K_4^0+g_1K_4^1+g_2K_4^2+g_3K_4^3$, where K_4 is a 4×4 matrix that shifts the first column of an identity matrix to the last column.

In the last round, the inputs of the Pyramid Algorithm are only two pieces of data. With respect to the two filters of width 4, the consequence is that the precedent scaling coefficients (inputs) β_{20} and β_{21} must be used twice in the last round of computation. This round is represented by the following

$$\begin{bmatrix} \alpha_{3,0} \\ \beta_{3,0} \end{bmatrix} = \begin{bmatrix} h_0 & h_3 & h_2 & h_1 \\ g_0 & g_3 & g_2 & g_1 \end{bmatrix} \begin{bmatrix} \beta_{2,0} \\ \beta_{2,1} \\ \beta_{2,0} \\ \beta_{2,1} \end{bmatrix}. \tag{2.61}$$

To avoid using $\beta_{2,0}$ and $\beta_{2,1}$ twice, another expression may apply:

$$\begin{bmatrix} \alpha_{3,0} \\ \beta_{3,0} \end{bmatrix} = \begin{bmatrix} h_0+h_2 & h_3+h_1 \\ g_0+g_2 & g_3+g_1 \end{bmatrix} \begin{bmatrix} \beta_{2,0} \\ \beta_{2,1} \end{bmatrix}. \tag{2.62}$$

Note that $VH_3 = \begin{bmatrix} h_0+h_2 & h_3+h_1 \end{bmatrix}$ and $VG_3 = \begin{bmatrix} g_0+g_2 & g_3+g_1 \end{bmatrix}$, where $H_3 = H(K_2) = h_0K_2^0 + h_1K_2^1 + h_2K_2^2 + h_3K_2^3$ and $G_3 = G(K_2) = g_0K_2^0 + g_1K_2^1 +$

$g_2K_2^2 + g_3K_2^3$. Consequently, this expression is represented by the following:

$$\begin{bmatrix} \alpha_{3,0} \\ \beta_{3,0} \end{bmatrix} = \begin{bmatrix} VH_3 \\ VG_3 \end{bmatrix} \begin{bmatrix} \beta_{2,0} \\ \beta_{2,1} \end{bmatrix}. \tag{2.63}$$

In conclusion, the wavelet coefficients associated with the jth-level wavelet filter are written as follows:

$$\alpha_{(j)} = V_jH_jV_{j-1}G_{j-1}\cdots V_1G_1Y = \Omega_jY, \tag{2.64}$$

and the scaling coefficients associated with the jth-level scaling filter are expressed by the following:

$$\beta_{(j)} = V_jG_jV_{j-1}G_{j-1}\cdots V_1G_1Y, \tag{2.65}$$

where $V_j = \Lambda_j' = [e_0, e_2, \cdots, e_{T/2^{j-1}-2}]'$, which is established by deleting the alternate rows of an identity matrix ($I_{T/2^{j-1}} = [e_0, e_1, \cdots, e_{T/2^{j-1}-1}]$).[13] The matrices H_j and G_j are written in polynomial expressions as follows:

$$\begin{aligned} H_j &= H(K_{T/2^{j-1}}) = h_0K^0_{T/2^{j-1}} + h_1K^1_{T/2^{j-1}} + \cdots + h_{L-1}K^{L-1}_{T/2^{j-1}}, \\ G_j &= G(K_{T/2^{j-1}}) = g_0K^0_{T/2^{j-1}} + g_1K^1_{T/2^{j-1}} + \cdots + g_{L-1}K^{L-1}_{T/2^{j-1}}. \end{aligned} \tag{2.66}$$

It is not difficult to find that

$$\begin{aligned} V_jH_jV_{j-1}G_{j-1}\cdots V_1G_1 &= V_jV_{j-1}\cdots V_1H(K_T^{2j-2})G(K_T^{2j-4})\cdots G(K_T), \\ V_jG_jV_{j-1}G_{j-1}\cdots V_1G_1 &= V_jV_{j-1}\cdots V_1G(K_T^{2j-2})G(K_T^{2j-4})\cdots G(K_T), \end{aligned} \tag{2.67}$$

where the jth-level wavelet filter $\{h_{j,l}\}$ forms the following matrix:

$$H(K_T^{2j-2})G(K_T^{2j-4})\cdots G(K_T), \tag{2.68}$$

and the jth-level scaling filter $\{g_{j,l}\}$ constructs the following matrix:

$$G(K_T^{2j-2})G(K_T^{2j-4})\cdots G(K_T), \tag{2.69}$$

in which j is no smaller than 2. If $j = 1$, the first-level wavelet and scaling filters correspond to the matrices $H(K_T)$ and $G(K_T)$, respectively. Accordingly, the

[13] To unify the matrix expression, the matrices V and Λ in the jth decomposed level are rewritten as V_j and Λ_j, respectively.

wavelet amplitudes and the scaling amplitudes are also identical to the following:

$$\begin{aligned} \alpha_{(j)} &= V_j V_{j-1} \cdots V_1 H(K_T^{2j-2}) G(K_T^{2j-4}) \cdots G(K_T) Y, \\ \beta_{(j)} &= V_j V_{j-1} \cdots V_1 G(K_T^{2j-2}) G(K_T^{2j-4}) \cdots G(K_T) Y. \end{aligned} \tag{2.70}$$

It is apparent that both the wavelet filter and the scaling filter are involved in the jth-level wavelet filter because its matrix form contains the scaling filter matrices.

2.2.6 Reconstruction of a Time series Using Wavelet Coefficients

From the previous section, we already know how to estimate the wavelet coefficients associated with a specific level of wavelet filter; those coefficients can be used to construct wavelet variance, which is in turn used to identify the contribution of variance in a frequency band to the overall variance. Then attention is devoted to the components of the time series within this frequency band. Furthermore, some factors, such as earthquakes and war, occasionally affect economies and/or financial markets and produce noise in the data that reveals unusual or insignificant characteristics. As a result, it is necessary to remove this noise. Typically, when magnitudes of wavelet coefficients are below the threshold for many methods (Donoho and Johnstone (1994, 1995), Donoho *et al.* (1995), and Nason (1995)), they are set to zero according to four rules (Percival and Walden (2000)).[14] Subsequently, the zero wavelet coefficients are combined with the other unaffected wavelet coefficients to generate a new time series without noise through a recovery process. Eventually, an important task emerges: to reconstruct a time series using wavelet coefficients.

As introduced earlier in the paper, the process of generating wavelet coefficients compromises downsampling: the odd-numbered elements of the time series are discarded, and only the even-numbered elements are preserved. To recover the original time series, we do not need two full-length signals to replace one. It is not desirable to double the volume of data. The information is not doubled: the outputs from the two filters must be redundant. Consequently, another process, which is known as upsampling, is needed. This operation inserts zeros between each element to ensure that the odd-numbered elements of the upsampled wavelet coefficients sequence are zeros and to ensure that the even-numbered elements of the upsampled wavelet coefficients sequence are the wavelet coefficients. The matrix notation for this process is Λ.

The Shannon Sampling Theorem indicates that for a band-limited signal, the odd-

[14]See Nason (1995) to review various methods for selecting a threshold.

numbered elements are recovered from the even-numbered elements.[15] Note that the outputs corresponding to the wavelet filter are highpass components and that the outputs associated with the scaling filter are lowpass components. Consequently, the time series recovered by the wavelet coefficients or the scaling coefficients lacks some information (lowpass components or highpass components). These components are synthesised to generate the original time series. By matrix notation, $\mathbf{w}_j$ ($1 \leqslant j \leqslant J$) comprises the components of a time series associated with the frequency band $(\pi/2^j, \pi/2^{j-1}]$, and $\mathbf{v}_j$ contains the components of a time series associated with the frequency band $[0, \pi/2^j]$. Because $\alpha_{(j)} = V_j H_j V_{j-1} G_{j-1} \cdots V_1 G_1 Y = \Omega_j Y$ in Equation (2.64), $\mathbf{w}_j = \Omega_j' \alpha_{(j)}$, which is identical to the following:

$$\mathbf{w}_j = [V_j H_j V_{j-1} G_{j-1} \cdots V_1 G_1]' \alpha_{(j)}. \tag{2.71}$$

Similarly,

$$\mathbf{v}_j = [V_j G_j V_{j-1} G_{j-1} \cdots V_1 G_1]' \beta_{(j)}. \tag{2.72}$$

In accordance with Equation (2.67), these equations are identical to the following:

$$\begin{aligned} \mathbf{w}_j &= [V_j V_{j-1} \cdots V_1 H(K_T^{2j-2}) G(K_T^{2j-4}) \cdots G(K_T)]' \alpha_{(j)} \\ &= G(K_T^{-1}) \cdots G(K_T^{4-2j}) H(K_T^{2-2j}) \Lambda_1 \cdots \Lambda_{j-1} \Lambda_j \alpha_{(j)}, \end{aligned} \tag{2.73}$$

and

$$\begin{aligned} \mathbf{v}_j &= [V_j V_{j-1} \cdots V_1 G(K_T^{2j-2}) G(K_T^{2j-4}) \cdots G(K_T)]' \beta_{(j)} \\ &= G(K_T^{-1}) \cdots G(K_T^{4-2j}) G(K_T^{2-2j}) \Lambda_1 \cdots \Lambda_{j-1} \Lambda_j \beta_{(j)}, \end{aligned} \tag{2.74}$$

where $K_T^{-1} = K_T'$. According to the orthogonality conditions of the wavelet and scaling filters on Equations (2.28), (2.29) and (2.30), we have the following:

$$\begin{bmatrix} V_j H_j \\ V_j G_j \end{bmatrix} \begin{bmatrix} (V_j H_j)' & (V_j G_j)' \end{bmatrix} = \begin{bmatrix} V_j H_j H_j' \Lambda_j & V_j H_j G_j' \Lambda_j \\ V_j G_j H_j' \Lambda_j & V_j G_j G_j' \Lambda_j \end{bmatrix} = \begin{bmatrix} I_{T/2^j} & \mathbf{0}_{T/2^j} \\ \mathbf{0}_{T/2^j} & I_{T/2^j} \end{bmatrix}, \tag{2.75}$$

and

$$\begin{bmatrix} (V_j H_j)' & (V_j G_j)' \end{bmatrix} \begin{bmatrix} V_j H_j \\ V_j G_j \end{bmatrix} = H_j' \Lambda_j V_j H_j + G_j' \Lambda_j V_j G_j = I_{T/2^{j-1}}. \tag{2.76}$$

[15]The proof is given in the appendix.

Consequently, it is not difficult to infer that

$$\begin{aligned}
&\mathbf{v}_j'\mathbf{w}_j = 0, \\
&\mathbf{w}_j'\mathbf{w}_k = 0 \quad (j \neq k), \\
&\sum_{j=1}^{J} \mathbf{w}_j + \mathbf{v}_J = Y,
\end{aligned} \tag{2.77}$$

which illustrate lateral orthogonality ("$\mathbf{w}_j \perp \mathbf{v}_j$") and sequential orthogonality ("$\mathbf{w}_j \perp \mathbf{w}_k$"), respectively. Thus, a data sequence can be decomposed orthogonally into components by time scales using wavelets.

2.3 The Analysis of Two-Channel Filter Banks

The architecture of a dyadic wavelet analysis is easily understood by considering the nature of a two-channel quadrature mirror filter. This filter highlights the asymmetric characteristics of wavelet coefficients and the symmetric characteristics of wavelet components, and reveals the essence of wavelet analysis.

Consider a sequence $\{y_t, t = 0, 1, \cdots, T-1\}$ in which the tth element of a column vector Y is y_t. This sequence undergoes the highpass filter H_1 that is constructed by wavelet filters via a downsampling process ($\downarrow 2$) in which the odd-numbered elements of the filtered signal are discarded and the even-numbered elements are preserved. The filtered and downsampled signal, which holds half of the information of y_t, is then stored and transmitted. Subsequently, this signal undergoes an anti-imaging highpass filter C_1, which is constructed using wavelet filters. Prior to this procedure, upsampling ($\uparrow 2$) is performed by inserting zeros between each element of the filtered and downsampled signal. Finally, $\mathbf{w_1}$, involving a half component of the signal y_t in the specific frequency band, is obtained. This process is also applied to scaling filters, and $\mathbf{v_1}$, which contains the other half of y_t, is derived. The graphic of this flow path is thus the following:

$$Y \longrightarrow H_1 \longrightarrow \downarrow 2 \longrightarrow \simeq \longrightarrow \uparrow 2 \longrightarrow C_1 \longrightarrow \mathbf{w}_1 \ ,$$

$$Y \longrightarrow G_1 \longrightarrow \downarrow 2 \longrightarrow \simeq \longrightarrow \uparrow 2 \longrightarrow D_1 \longrightarrow \mathbf{v}_1 \ ,$$

where the structures of matrices G_1 and D_1 are the same as those of matrices H_1 and C_1, respectively. Both the lowpass filter G_1 and the anti-imaging lowpass filter D_1 are constructed using scaling filters. Here, filters H_1 and G_1 are called analysis filters, and filters C_1 and D_1 are called synthesis filters. The symbol $\simeq$ represents the storage and transmission of the signal. The output signals formed by the two-channel filter banks are $\mathbf{w}_1$ and $\mathbf{v}_1$, and their combination is the original signal: $\mathbf{w}_1 + \mathbf{v}_1 = Y$.

Normally, compared with temporal notation, frequency notation is preferred to express this flow path because it can show certain properties of these filters. Therefore, the highpass and lowpass flow paths are expressed, respectively, as follows:

$$y(z) \longrightarrow H_1(z) \longrightarrow \downarrow 2 \longrightarrow \simeq \longrightarrow \uparrow 2 \longrightarrow C_1(z) \longrightarrow \mathbf{w}_1(z) \ ,$$

$$y(z) \longrightarrow G_1(z) \longrightarrow \downarrow 2 \longrightarrow \simeq \longrightarrow \uparrow 2 \longrightarrow D_1(z) \longrightarrow \mathbf{v}_1(z) \ ,$$

where z could be ω or $e^{-\mathrm{i}\omega}$, and this expression has more generality. It has been proven that the Fourier transforms of $(\downarrow 2)y_t$ and $(\uparrow 2)y_t$ are $[\varepsilon(\omega/2)+\varepsilon(\omega/2+\pi)]/2$ and $\varepsilon(2\omega)$, respectively.[16] Because $e^{-\mathrm{i}\omega/2} = z^{1/2}$ and $e^{-\mathrm{i}(\omega/2+\pi)} = -z^{1/2}$ where $z = e^{-\mathrm{i}\omega}$, in terms of the z-transform, they can be written as follows:

$$(\downarrow 2)y_t \longleftrightarrow [\varepsilon(z^{1/2}) + \varepsilon(-z^{1/2})]/2 \qquad \text{and} \qquad (\uparrow 2)[(\downarrow 2)y_t] \longleftrightarrow [\varepsilon(z) + \varepsilon(-z)]/2, \tag{2.78}$$

where "$\longleftrightarrow$" denotes the Fourier transform, and the RHS term is the Fourier transform coefficient.

In conclusion, in terms of the z-transform, the highpass and lowpass flow paths are expressed by two equations, respectively:

$$\begin{aligned} \mathbf{w}_1(z) &= \frac{1}{2}C_1(z)[H_1(z)y(z) + H_1(-z)y(-z)], \\ \mathbf{v}_1(z) &= \frac{1}{2}D_1(z)[G_1(z)y(z) + G_1(-z)y(-z)]. \end{aligned} \tag{2.79}$$

It is presumed that the synthesis of $\mathbf{w}_1(z)$ and $\mathbf{v}_1(z)$ is $x(z)$; thus,

$$\begin{aligned} x(z) = &\frac{1}{2}[C_1(z)H_1(z) + D_1(z)G_1(z)]y(z) \\ &+ \frac{1}{2}[C_1(z)H_1(-z) + D_1(z)G_1(-z)]y(-z). \end{aligned} \tag{2.80}$$

As $y(-z)$ is a result of aliasing from the downsampling process, it must be elimi-

[16] The details are provided in the appendix: downsampling doubles frequency and upsampling halves frequency.

nated.[17] Here, we set $C_1(z) = -z^{-d}G_1(-z)$ and $D_1(z) = z^{-d}H_1(-z)$, where d is identical to $L-1$ and L is the width of the filter. Thus, Equation (2.80) becomes the following:

$$x(z) = \frac{z^{-d}}{2}[H_1(-z)G_1(z) - H_1(z)G_1(-z)]y(z). \tag{2.81}$$

Note that the aliasing term $y(-z)$ can be cancelled by any choice of $H_1(z)$ and $G_1(z)$ when the anti-imaging filters $C_1(z)$ and $D_1(z)$ are identical to $-z^{-d}G_1(-z)$ and $z^{-d}H_1(-z)$, respectively. However, a restriction on the choice of $H_1(z)$ and $G_1(z)$ is imposed such that the coefficients of the wavelet and scaling filters are mutually orthogonal, including being both sequentially orthogonal and laterally orthogonal. To demonstrate this restriction, we assume that the width of the filter is four. Thus,

$$\begin{aligned} G_1(z) &= g_0 + g_1 z + g_2 z^2 + g_3 z^3, \\ H_1(z) &= h_0 + h_1 z + h_2 z^2 + h_3 z^3. \end{aligned} \tag{2.82}$$

Because $\{g_k\}$ is referred to as the "quadrature mirror filter" (QMF), corresponding to $\{h_k\}$, $h_k = (-1)^k g_{L-k-1}$ indicates that Equation (2.82) could be written as follows:

$$\begin{aligned} G_1(z) &= -h_3 + h_2 z - h_1 z^2 + h_0 z^3 = z^3 H_1(-z^{-1}) = D_1(z^{-1}), \\ H_1(z) &= g_3 - g_2 z + g_1 z^2 - g_0 z^3 = -z^3 G_1(-z^{-1}) = C_1(z^{-1}), \end{aligned} \tag{2.83}$$

where

$$\begin{aligned} D_1(z) &= -h_3 + h_2 z^{-1} - h_1 z^{-2} + h_0 z^{-3} = z^{-3} H_1(-z) = G_1(z^{-1}), \\ C_1(z) &= g_3 - g_2 z^{-1} + g_1 z^{-2} - g_0 z^{-3} = -z^{-3} G_1(-z) = H_1(z^{-1}). \end{aligned} \tag{2.84}$$

$C_1(z) = H_1(z^{-1})$ and $D_1(z) = G_1(z^{-1})$ indicate that the synthesis filters are simply the reversed-sequence anti-causal versions of the analysis filters. Equations (2.83)

[17] Aliasing may occur when a temporal sequence is Fourier transformed in terms of its frequency. Aliasing occurs because the basis of the Fourier analysis is the cosine and sine functions. Suppose that the angular velocity ω exceeds the Nyquist value (π), $\omega \in (\pi, 2\pi]$, and define $\omega_* = 2\pi - \omega$, $\omega_* \in [0, \pi)$. Thus, for all values of $t = 0, \ldots, T-1$, $\cos(\omega t) = \cos(2\pi t - \omega_* t) = \cos(2\pi t)\cos(\omega_* t) + \sin(2\pi t)\sin(\omega_* t) = \cos(\omega_* t)$. Thus, the cosine functions at the frequencies ω and ω_* are identical; this problem is called "aliasing". It is not possible to distinguish the angular frequency ω from the value of the cosine (or sine) function. To avoid aliasing, the positive frequencies in the spectral analysis of a discrete time process are limited to the interval $[0, \pi]$. Whether an aliasing problem exists depends on the structure of the particular time series. For many econometric time series, the problem does not arise because their positive frequencies are limited to the average $[0, \pi]$.

and (2.84) indicate that Equation (2.81) can be rendered as follows:

$$\begin{aligned} x(z) &= \frac{1}{2}[H_1(z)H_1(z^{-1}) + G_1(z)G_1(z^{-1})]y(z) \\ &= \frac{1}{2}[D_1(-z)G_1(-z) + D_1(z)G_1(z)]y(z) \\ &= \frac{1}{2}[P(-z) + P(z)]y(z), \end{aligned} \tag{2.85}$$

where

$$\begin{aligned} P(-z) = D_1(-z)G_1(-z) = H_1(z)H_1(z^{-1}), \\ P(z) = D_1(z)G_1(z) = G_1(z)G_1(z^{-1}). \end{aligned} \tag{2.86}$$

To achieve the perfect reconstruction in which $x(z)$ is equal to $y(z)$, the following condition is imposed:

$$H_1(z)H_1(z^{-1}) + G_1(z)G_1(z^{-1}) = 2. \tag{2.87}$$

This condition guarantees the perfect reconstruction of the original sequence $y(t)$ from outputs using two-channel filter banks. The terms in $H_1(z)H_1(z^{-1})$ and $G_1(z)G_1(z^{-1})$ with an odd power of z are cancelled because of the relationship between the wavelet filter $\{h_l\}$ and the scaling filter $\{g_l\}$ (Equations (2.29) and (2.30)). The orthogonality conditions of $\{h_l\}$ and $\{g_l\}$ (Equation (2.28)) make the terms that have an even power of z equal to zero and make the terms associated with a zero power of z identical to 2. Consequently, Equation (2.87) is always valid in wavelet theory.

It is of interest to use the circulant matrix K_T to replace z in Equation (2.82). The results are matrix representations of the filters H_1 and G_1, which are shown in Equations (2.54) and (2.56). Because $K_T^{-1} = K_T'$, the filter matrices H_1' and G_1' are associated with $H_1(z^{-1})$ and $G_1(z^{-1})$, respectively. This result is also applied to the further decomposition and reconstruction in DWT. Thus, the sum of the component signals is the original signal: $\sum_{j=1}^{J} \mathbf{w}_j + \mathbf{v}_J = Y$. In conclusion, we briefly introduce the deconstruction and perfect reconstruction of a time series using two-channel filter banks. These two-channel filter banks offer us the entire architecture for the dyadic wavelet analysis and facilitate our interpretation of this analysis.

2.4 The Symmetric and Asymmetric Properties

In terms of the z-transform, the gain function of the jth-level wavelet filter $\{h_{j,l}\}$ is as follows:

$$H(z^{2j-2})G(z^{2j-4})\cdots G(z), \tag{2.88}$$

and the gain function of the jth-level scaling filter $\{g_{j,l}\}$ is as follows:

$$G(z^{2j-2})G(z^{2j-4})\cdots G(z), \tag{2.89}$$

which are derived easily using z rather than K_T in Equations (2.68) and (2.69), respectively. These equations show that the wavelet and scaling filters are asymmetric. Consequently, the wavelet and scaling amplitudes have phase shifts with the original time series. However, the component signal does not have phase displacements with the original signal because the term z^{-d} that is involved in filters $C_1(z)$ and $D_1(z)$ in Equation (2.84) has the ability to compensate for time lags or time advances caused by the asymmetric filters $H_1(z)$ and $G_1(z)$. For instance, following the previous example, the component signals $\mathbf{w}_1$ and $\mathbf{v}_1$ are as follows:

$$\begin{aligned} \mathbf{w}_1 &= H_1'\Lambda V H_1 Y, \\ \mathbf{v}_1 &= G_1'\Lambda V G_1 Y, \end{aligned} \tag{2.90}$$

where $H_1'\Lambda V H_1$ is

$$\left[\begin{array}{cccc|cccc}
h_0^2+h_2^2 & h_1h_2 & h_0h_2 & 0 & 0 & h_0h_3 & h_0h_2 & h_0h_1+h_2h_3 \\
h_1h_2 & h_1^2+h_3^2 & h_0h_1+h_2h_3 & h_1h_3 & h_0h_3 & 0 & 0 & h_1h_3 \\
h_0h_2 & h_0h_1+h_2h_3 & h_0^2+h_2^2 & h_1h_2 & h_0h_2 & 0 & 0 & h_0h_3 \\
0 & h_1h_3 & h_1h_2 & h_1^2+h_3^2 & h_0h_1+h_2h_3 & h_1h_3 & h_0h_3 & 0 \\
\hline
0 & h_0h_3 & h_0h_2 & h_0h_1+h_2h_3 & h_0^2+h_2^2 & h_1h_2 & h_0h_2 & 0 \\
h_0h_3 & 0 & 0 & h_1h_3 & h_1h_2 & h_1^2+h_3^2 & h_0h_1+h_2h_3 & h_1h_3 \\
h_0h_2 & 0 & 0 & h_0h_3 & h_0h_2 & h_0h_1+h_2h_3 & h_0^2+h_2^2 & h_1h_2 \\
h_0h_1+h_2h_3 & h_1h_3 & h_0h_3 & 0 & 0 & h_1h_3 & h_1h_2 & h_1^2+h_3^2
\end{array}\right], \tag{2.91}$$

and $G_1'\Lambda VG_1$ is

$$\left[\begin{array}{cccc|cccc}
g_0^2+g_2^2 & g_1g_2 & g_0g_2 & 0 & 0 & g_0g_3 & g_0g_2 & g_0g_1+g_2g_3 \\
g_1g_2 & g_1^2+g_3^2 & g_0g_1+g_2g_3 & g_1g_3 & g_0g_3 & 0 & 0 & g_1g_3 \\
g_0g_2 & g_0g_1+g_2g_3 & g_0^2+g_2^2 & g_1g_2 & g_0g_2 & 0 & 0 & g_0g_3 \\
0 & g_1g_3 & g_1g_2 & g_1^2+g_3^2 & g_0g_1+g_2g_3 & g_1g_3 & g_0g_3 & 0 \\
\hline
0 & g_0g_3 & g_0g_2 & g_0g_1+g_2g_3 & g_0^2+g_2^2 & g_1g_2 & g_0g_2 & 0 \\
g_0g_3 & 0 & 0 & g_1g_3 & g_1g_2 & g_1^2+g_3^2 & g_0g_1+g_2g_3 & g_1g_3 \\
g_0g_2 & 0 & 0 & g_0g_3 & g_0g_2 & g_0g_1+g_2g_3 & g_0^2+g_2^2 & g_1g_2 \\
g_0g_1+g_2g_3 & g_1g_3 & g_0g_3 & 0 & 0 & g_1g_3 & g_1g_2 & g_1^2+g_3^2
\end{array}\right], \tag{2.92}$$

which are symmetric block-Toeplitz matrices. This result implies the absence of any phase effects. Because the component signals $\mathbf{w}_1$ and $\mathbf{v}_1$ are the results of filtering signal Y with the symmetric filter matrices $H_1'\Lambda VH_1$ and $G_1'\Lambda VG_1$, respectively, they are perfectly aligned with the original signal. From another perspective, the synthesis of the component signals is the original signal, also illustrating this view. In conclusion, the evidence indicates that the component signal does not have phase displacements from the original signal.

As a consequence of the downsampling (V) and upsampling (Λ) operations, the filter matrix $H_1'\Lambda VH_1$ or $G_1'\Lambda VG_1$ is not a matrix representation of any linear symmetric filter. Although $H_1'H_1$ or $G_1'G_1$ is related to a linear symmetric filter whose gain function is the squared gain function of the wavelet filter $\{h_l\}$ or the scaling filter $\{g_l\}$, we cannot find an ordinary linear symmetric filter associated with the symmetric filter matrix $H_1'\Lambda VH_1$ or $G_1'\Lambda VG_1$. This result implies that the filter that is used to generate the component signal in DWT has no gain function. In addition, the $z-$transform functions of the component signals also demonstrate this point: $\mathbf{w}_1(z) = \frac{1}{2}H_1(z^{-1})[H_1(z)y(z) + H_1(-z)y(-z)]$ or $\mathbf{v}_1(z) = \frac{1}{2}G_1(z^{-1})[G_1(z)y(z) + G_1(-z)y(-z)]$. We can use the squared gain function of the wavelet filter or the scaling filter to approximate the gain function of this filter. Accordingly, the squared gain function of the jth-level wavelet filter $\{h_{j,l}\}$ or the jth-level scaling filter $\{g_{j,l}\}$ yields approximate changes in the amplitudes of a time series in the specific frequencies, and the function accounts for the coefficient $1/2^j$. In fact, only the first term in the $z-$transform function of the component signal shows this effect. Regarding the gain functions of the jth-level wavelet and scaling filters in Equations (2.88) and (2.89), the squared gain functions of the jth-level wavelet and scaling filters are denoted by Γ_j and Θ_j, respectively, as follows:

$$\begin{aligned} \Gamma_j &= H(z^{2j-2})G(z^{2j-4})\cdots G(z)H(z^{2-2j})G(z^{4-2j})\cdots G(z^{-1}), \\ \Theta_j &= G(z^{2j-2})G(z^{2j-4})\cdots G(z)G(z^{2-2j})G(z^{4-2j})\cdots G(z^{-1}), \end{aligned} \tag{2.93}$$

$\Gamma_j/2^j$ and $\Theta_j/2^j$ are used to show the approximate changes in the data components in the specific frequencies by the corresponding filters in DWT.

2.5 Structure of Wavelet Analysis

Generally, wavelet analysis decomposes a signal into shifted (translated) and scaled (dilated or compressed) versions of a wavelet function. All of the basis functions (wavelet functions or scaling functions) are self-similar, which is to say that they differ from one another only in the translations and the changes of scale. Wavelets are particular types of functions that are localised both in time and frequency domain, whereas each of the sines and cosines that compose the basis function of Fourier transform is itself a function of frequency-by-frequency basis. The wavelet transform utilises a basic function (called the wavelet function or mother wavelet), which is shifted (translated) and scaled (dilated or compressed) to capture features that are local in time and in frequency. Therefore, wavelets are good at managing the time-varying characteristics found in most real-world time series and are an ideal tool for studying non-stationary or transient time series while avoiding the assumption of stationarity.

Figure [2.2] provides a good explanation of the effectiveness of wavelets. The horizontal axis is time, whereas the vertical axis is scale (frequency). It is easily found that scale decreases further along the vertical axis. As the scale declines, it reduces the time support, increases the number of frequencies captured, and shifts towards higher frequencies, and vice versa. In the dyadic wavelet case, from one scale to the next scale, the bandwidth of frequency is halved and the temporal dispersion of wavelet is doubled. The frequency band of the original time series is $[0, \pi]$, and the time horizon is its time, such as 1 minute for per-minute data, 1 day for daily data, or 1 month for monthly data. Regarding scale 2^{j-1} ($1 \leqslant j \leqslant J$), the corresponding frequency band is $(\pi/2^j, \pi/2^{j-1}]$ and the time horizon is 2^j times of the original time series, such as the time interval $[2^j, 2^{j+1})$ minutes for per-minute data, the time interval $[2^j, 2^{j+1})$ days for daily data, or the time interval $[2^j, 2^{j+1})$ months for monthly data. This treatment indicates that scale is related to time horizon and the inverse of frequency band.

Consequently, the wavelet transform provides good frequency resolution (and poor time resolution) at low frequencies and good time resolution (and poor frequency resolution) at high frequencies. It maintains a balance between frequency and time. From the top to the bottom of Figure [2.2], the frequency resolution improves and the time resolution worsens, which implies that wavelets provide a flexible framework for the time series. By combining several functions of shifted and scaled mother wavelet, the wavelet transform is able to capture all information contained in a time series and associate it with specific time horizons and locations in time.

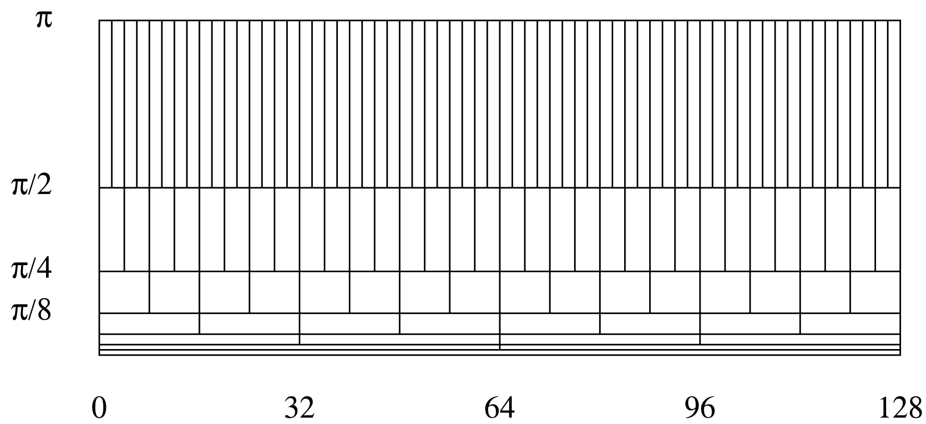


Figure 2.2: The partitioning of the time-frequency plane according to a multiresolution analysis of a data sequence of $128 = 2^7$ point.

The resulting time-frequency partition corresponding to the wavelet transform is long in time when capturing low-frequency events and thus has good frequency resolution for these events, and it is long in frequency when capturing high-frequency events and thus has good time resolution for events where the wavelet transform has the ability to capture events that are localised in time. The wavelet transform intelligently adapts itself to capture features across a wide range of frequencies, making it an ideal tool for studying non-stationary or transient time series.

2.6 Literature Review

In the markets, different agents have various investing purposes, such as speculative and value investments. As a consequence, some agents concentrate on daily market movement, whereas others are concerned with yearly movement. Moreover, some agents' interest in market movement is based on shorter or longer time horizons. Thus, the actions of agents that determine economic or financial processes differ. Consequently, economic or financial processes vary over time in a way that is closely related to frequency. An economic or financial time series is a combination of components that operate on different frequencies. High-frequency components capture short-term behaviours, whereas low-frequency components reflect long-term fluctuations. It is useful to study time series from a frequency domain perspective, which provides new insights into economic and/or financial issues. Differing results or differing phenomena stemming from issues in the short and long run are always puzzling. The analysis from the frequency domain provides a novel description to explain these puzzles and presents an improvement on the time domain approach (Lucas (1980), Knif *et al.* (1995), Barucci and Reno (2002a,b), Kanatani (2004), Rua and Nunes (2005), Mancino and Sanfelici (2008)).[18]

The frequency domain is typically related to Fourier analysis, which has the ability to reveal joint characteristics that are not obvious in the time domain.[19] For example, Lucas (1980) finds that the frequency level determines the relationship between monetary growth and inflation. The Fourier transform involves the projection of a time series onto an orthonormal set of trigonometric components: cosine and sine functions. The Fourier transform disregards temporal properties and yields only the frequency properties of data. The temporal property is important for any data. Accordingly, Fourier analysis is unable to capture time-varying features, which are common in economics and finance. For example, the link between monetary growth and inflation evolves over time as well as across frequencies (see, e.g., Rolnick and Weber (1997), Christiano and Fitzgerald (2003), Sargent and Surico (2008), Benati (2009)). Given this limitation, Fourier analysis is less appealing. Moreover, Fourier analysis belongs within global nature analysis because cosine and sine functions do not fade or change over time. Any slight disturbance will change all Fourier coefficients; thus, it is not effective for studying a time series that contains irregularities, such as discontinuities and spikes. A time series must be stationary for Fourier analysis, but this strong assumption is not often applicable in practice.

To overcome this limitation, the windowed Fourier transform, which combines the Fourier transform with a window of fixed length as it slides across all the data, is proposed in the literature. Accordingly, the assumption of homogeneous data may be relaxed, and the time dependent feature may be captured. However, the windowed Fourier transform generates a uniform partition of the time-frequency plane because of a constant length window. An overrepresentation of high-frequency components and an underrepresentation of low-frequency components may be the result. Hence, the windowed Fourier transform does not yield an adequate resolution for

[18]Phase, gain, coherency and spectrum are common indicators for research in the frequency domain.

[19]In the time domain, it is difficult to identify lead/lag structures if they are different in various frequency bands and/or if they are not stable over time.

all frequencies. By contrast, the wavelet transform provides a flexible resolution in both the frequency and time domains, as shown in Figure [2.2], because the local base functions can be dilated and translated. Given this feature, the wavelet transform becomes more attractive than the windowed Fourier transform for non-stationary time series. Wavelet analysis is localised in time and frequency simultaneously because wavelets oscillate and decay in a limited time period. Wavelet analysis therefore belongs within local nature analysis. Wavelet analysis is more useful than other methods for detecting structural breaks or jumps and for simultaneously assessing how variables are related on different frequencies and how this relationship has evolved over time, among other patterns.

Although wavelets are primarily used in signal and image processing, meteorology, astronomy and physics, the body of literature on wavelet applications in economics and finance has grown rapidly over the last two decades. Ramsey and his coauthors present a series of pioneering works. More specifically, Ramsey *et al.* (1995) use wavelets to study U.S stock price behaviour. The wavelet transform localised in time indicates how the power of the projection of a time series onto the kernel varies according to the scale of observation. Ramsey and Zhang (1997) use waveform dictionaries to decompose the time series contained within three tick-by-tick foreign exchange rates. The waveform dictionary is a class of transforms that generalises both windowed Fourier transforms and wavelets. The authors conclude that waveform dictionaries are most useful for analysing data that are not stationary and even non-stationary up to the second order. Ramsey and Lampart (1998a,b) investigate the relationships between money supply and output and between income and consumption in terms of restrictions on the given time scales using wavelet MRA.

According to the frequency-dependent relationship between money supply and output, Dalkir (2004) uses the Wald variant of the Granger test on the wavelet time-scale components of the two variables above to examine their relationship in different time scales. Lee (2004) utilises wavelets to orthogonally decompose the KOSPI and DJIA daily stock market indices into different respective time-scale components. The regression of the time-scale components of KOSPI on the corresponding time-scale components of the DJIA and the reverse regression show a positive and significant transmission from the U.S. stock market to the Korean stock market, but the reverse was not found. Fernandez (2005) employs wavelet analysis to quantify price spillovers among a wide range of regional stock markets on different time horizons. Yogo (2008) decomposes an economic time series (GDP data) into a trend, cycles and noise according to MRA and then measures business cycles. Based on wavelet multiresolution analysis, Gallegati *et al.* (2014) examine the relationship between future output and a variety of financial indicators using 'scale-by-scale' and 'double residuals' regression analysis. Maslova *et al.* (2014) use wavelets to study growth and volatility of GDP series over different time horizons, where the focus is on changes in the growth rates as well as the levels of GDP.

The discounted cash flow valuation model indicates that financial stock prices are related to aggregate economic activity. A number of empirical works show that stock returns have predictive power with respect to real economic activity and that this power increases as the time horizon lengthens. Empirical studies employ OLS regression, vector autoregressive and vector error correction models to investigate the interactions between stock returns and real economic activity. However, because of a lack of appropriate tools, the time scales are typically decomposed into the short and long run. Different agents in the financial markets have various investment purposes, ranging from speculative to investment activity, which implies that the relevant time scales are not only short and long run but also include time from minutes to years. In the presence of wavelet analysis, Kim and In (2003, 2005) study the lead-lag relationship between financial variables and real economic activity and the relationship between stock returns and inflation on different time scales, whereas Gallegati (2008) constructs wavelet correlation and cross-correlation to examine the lead-lag relationship between stock returns and real economic activity.

Consider the issue of forecasting. Some empirical works provide evidence that wavelet analysis improves forecast accuracy. In particular, Wong *et al.* (2003) provide an example in which wavelet-based methods are used to forecast foreign exchange rates. Arino *et al.* (2004) use the DWT and the scalogram, which is a DWT analogue of the well-known periodogram in spectral analysis, to detect and separate periodic components in time series. These researchers forecast each component using a regular ARIMA formulation and then combine the results to obtain the forecast for the complete time series. The proposed method is then used to forecast a Spanish concrete production data set. Yousefi *et al.* (2005) illustrate an application of wavelets to investigate the issue of market efficiency in the futures market for oil. A wavelet-based prediction procedure is introduced and used to forecast market data for crude oil over different forecasting horizons. Using MRA, Rua (2011) proposes the wavelet approach with factor-augmented models to assess the short-term forecasting of quarterly GDP growth in the major Euro area countries, namely, Germany, France, Italy and Spain. The results show that this approach outperforms other models, including AR models, factor-augmented models, and the wavelet approach with an AR model, for forecasting purposes.

These papers consider the time scale (frequency) and show obvious improvements compared with past empirical works using wavelets. Economists realised very early that time scale is relevant to the structure of decision-making processes and the strength of relationships among variables. Unfortunately, an appropriate tool for decomposing economic time series into orthogonal components has historically been lacking; thus, this important feature is not considered in conventional econometric models. The major innovation in most of these papers stems from decomposing time series into their time-scale components in an attempt to unravel the characteristics or relationships among economic or financial variables that vary over time and across frequencies simultaneously. The core strategy of this methodology is to employ standard econometric models to study the wavelet-decomposed time-scale components of these variables. Accordingly, the primary contribution of these papers is attributed to wavelets. These papers confirm that the characteristics, structures, and relationships of economic or financial variables vary over both time and frequency simultaneously, which aids in resolving some anomalies in the empirical literature. Moreover, this methodology clarifies that an individual's decision-making process is generated over time horizons that are more complex than simply the short and long run that are usually considered by economists.

Another important application of wavelets is related to wavelet coefficients. Given that wavelets are localised in time and scale (frequency), wavelet coefficients are accordingly concentrated in time and scale. It is intuitive to construct wavelet variance or wavelet covariance on a scale-by-scale basis to study the scaling properties or relationships between economic and/or financial variables. Variance is an important statistic and indicator in economics and finance, and it is a common tool for measuring risks in risk management. Variance refers to the degree to which variables or data fluctuate from the mean of the entire sample. A larger variance suggests that the data at that time are farther from the mean. Therefore, variance is the prevalent method of detecting risks and certain abnormal events.

Conventionally, it has always been believed that variance is only associated with time. However, the time domain variance, which is considered the overall or aggregate variance, is incapable of revealing all information in the data related to market turmoil that may be shadowed in the individual variance. For example, there are many different types of financial market crashes. Through the graphics of overall variances, these types are quite similar: preceding a crash, variance is quite stable; it rises dramatically and oscillates at a high level during the crash; and it then decays to a normal level after the crash. It is difficult to distinguish different financial market crashes from the aggregate variance that veils their personal characteristics.

However, the frequencies of observables in economics and finance related to time horizons are distinct. Hence, variance changes across frequencies and over time, and this change may thus provide another angle from which to study several issues. Moreover, the sample variance that is used as the estimator of variance may have poor bias properties for particular stationary processes because of the need to estimate the process mean μ_x using the sample mean (μ_x is rarely known a priori when a stationary process is used as a model for an observed time series). In addition, there are certain non-stationary processes with stationary differences for which the sample variance is not a particularly useful statistic because of the inability to define process variance that is both finite and time invariant.

The statistical properties of non-stationary time series, such as their mean, variance, and autocorrelation, vary over time (Giovannini and Jorion (1989), Loretan and Phillips (1994), Karuppiah and Los (2005), Kyaw *et al.* (2006), Los and Yalamova (2006)). However, some econometric models implicitly assume that these statistical properties are constant over time. Thus, non-stationary data must be converted into stationary data. Wavelets can transform non-stationary time series into stationary time series using the stationary differencing process. This process is identical to the differencing method, which is widely used to stationarise time series. For example, any Daubechies wavelet filter involves a $L/2$th order backward difference filter in which L is the width of the filter; thus, the corresponding wavelet coefficients are stationary.

If wavelet coefficients are independent (simulation studies by Whitcher (1998) and Whitcher *et al.* (2000) demonstrate that the decorrelation is good in terms of the test statistic), then the wavelet variance constructed by them will be time independent. This property of wavelet variance is appealing to economics and finance scholars because many econometric models implicitly assume time-independent variance. Wavelets have another advantage in that they provide a zero mean for wavelet coefficients. The zero mean avoids the issue of the bias properties of sample variance, which occurs because the mean is rarely known a priori when estimating the sample variance. In sum, it is reasonable to use wavelet coefficients to construct wavelet variance on a scale-by-scale basis.

Wavelet variance is viewed as an individual variance because the sum of all wavelet variances over scales is the overall variance estimated by the sample variance. The wavelet variance measures the variability over a certain scale, whereas the overall variance assesses global variability. Thus, the wavelet variance more precisely reflects the local characteristics of observables in one time period; it records the contribution of components in a given scale (or frequency) to the overall variance of the process. If a scale contributes a larger proportion of the overall variance, then this scale is thought to be more important, and the corresponding components are more meaningful compared with a scale that contributes a smaller amount of the overall variance. These components, which are believed to contain significant information, could then be extracted from the entire time series and provide important subjects for analysis. In conclusion, wavelet analysis provides several new and different insights into economic and/or financial issues.

A number of papers demonstrate the successful application of wavelet variance in many areas, including meteorology (e.g., Torrence and Compo (1998)), and economics and finance (e.g., Jensen (1999, 2000), Whitcher and Jensen (2000), Gencay *et al.* (2001a,b, 2003, 2005), Connor and Rossiter (2005), Crowley and Lee (2005), Fernandez (2006a,b), Gallegati and Gallegati (2007), Gallegati (2008), Fan and Gencay (2010), Gencay *et al.* (2010)). Jensen (1999) estimates the differencing parameter d of the ARIMA (p, d, q) long-memory process using the log-linear relationship between the wavelet variance and the scaling parameter, with a small sample bias and variance. Jensen (2000) uses the wavelet transform to decompose the variance of a long-memory process and to construct a wavelet covariance matrix; he then develops a wavelet maximum likelihood estimator alternative to the frequency domain estimators of the long-memory parameter, but only for globally stationary long-memory processes. Whitcher and Jensen (2000) devise an estimator of the time-varying long memory parameter using the log-linear relationship between the local variance of the maximum overlap discrete wavelet transform's (MODWT) coefficients and the scaling parameter for locally stationary long-memory processes.

Gencay *et al.* (2001b) investigate the scaling properties of foreign exchange volatility using the wavelet multiscaling approach, and they detect the degree of persistence in the volatility and the correlation between foreign exchange volatilities. Regarding the features of beta (systemic risk) in the capital asset pricing model (CAPM), which are affected by the return interval, Gencay *et al.* (2003, 2005) introduce a new method using the wavelet multiscaling approach to estimate the beta on a scale-by-scale basis. Fernandez (2006a) employs wavelets to estimate the CAPM and the value at risk (VaR) on different time scales for the Chilean stock market. Connor and Rossiter (2005) use wavelets to estimate price correlations on a scale-by-scale basis as well as long memory in the volatility of commodity prices to investigate heterogeneous trading in such markets. Crowley and Lee (2005) employ MODWT to construct static wavelet variance, correlation and co-correlation (to measure the phase) by time scale, and they apply the DCC-GARCH model on the wavelet time-scale components to estimate dynamic correlations by time scale and to study the comovements among the European business cycles. Fernandez (2006b) adopts wavelet-based variance analysis to detect structural breaks in volatility during the 1997-2002 period, including the Asian crisis and the terrorist attacks of September 11. In and Kim (2006) propose three ways to use the wavelet-based approach to examine the relationship between stock and futures markets over different time scales: the lead-lag relationship using the Granger causality test on time-scale components, the correlation by measuring the ratio of wavelet covariance over wavelet variance, and the hedge ratio by computing wavelet covariance and variance.

Gallegati and Gallegati (2007) apply the estimator of MODWT wavelet variance to detect the occurrence and sources of decline in output volatility over the past forty years based on data from the industrial production index of the G-7 countries between 1961:1 and 2006:10. Gallegati (2008) employs the wavelet coefficients of stock returns and the growth rates of industrial production to develop wavelet variances and to identify their scaling properties. They then adopt these coefficients to construct a wavelet cross-correlation to study the lead-lag relationship between these two variables on a scale-by-scale basis. Fan and Gencay (2010) present a wavelet spectral approach to test the presence of a unit root in a stochastic process by measuring the test statistic, which is equal to the ratio of the variance of unit-scale scaling coefficients to the total variance of the time series via the DWT.

Note that there are two approaches for detecting the relationship between economic and/or financial variables on a scale-by-scale basis. The first method constructs the correlation between the time-scale components of these variables, whereas the second method employs the wavelet coefficients of these variables to estimate the wavelet covariance or correlation. Some researchers, such as Kim and In (2003, 2005) use both methods with consistent results.

Early works have explored the DWT in economics and finance contexts. Recently, the continuous wavelet transform (CWT) has drawn much attention from economists as well. In practice, economic and financial data are observed at pre-ordained points in time and represent stochastic processes in continuous time. Hence, it is intuitive to apply the CWT to economics and finance. The CWT avoids one particular problem: in most of the literature from the frequency domain, the cut-off of the frequency band is arbitrary for the analysis. The CWT provides a continuous assessment of relationships or structures, as well as other observations.

Consider the case of spectral analysis, which can identify periodicities in data. The power spectrum is estimated using the Fourier transform; therefore, spectral analysis has the same problems as the Fourier transform. The results based on spectral analysis are misleading when the time series is not stationary. Consequently, spectral analysis is unable to detect transient and irregular cycles and structural breaks in the periodicity of those cycles. Fortunately, wavelet spectral analysis can serve this purpose. Wavelet spectral analysis is analogous to spectral analysis but uses the CWT rather than the Fourier transform. Because wavelets yield frequency and time information simultaneously, the wavelet power spectrum varies over time and across frequencies. Wavelet spectral analysis measures the variance distribution of a time series in the time-frequency space. Changes in periodicity across time may be recorded in the wavelet power spectrum; thus, we can easily capture irregular cycles and identify time periods of different predominant cycles in the time series.

The tools within the CWT used by economists include not only the wavelet power spectrum but also cross-wavelet power, cross-wavelet coherency, the wavelet phase and the wavelet phase-difference. These tools have analogous concepts in Fourier analysis but are based on the CWT rather than the Fourier transform. Specifically, cross-wavelet power depicts the local variance between two time series. Cross-wavelet coherency describes the local correlation between two time series at each time and scale. The wavelet phase provides time information regarding the position of a time series in the cycle, and the wavelet phase-difference provides information regarding the possible lead-lag relationship of the oscillations of two series as a function of time and scale. These tools within wavelet analysis enable us to study the time-frequency dependencies between two time series, which are considered to be important features of economic and financial data. Consider the link between inflation and interest rates. If inflation rises, then the central bank may increase interest rates to dampen this increase, which implies that the short-run relationship between inflation and interest rates is positive, while the long-run relationship is negative. Consequently, their relationship differs in frequency in relation to the time horizon. In addition, this relationship varies over time, which is widely acknowledged; for example, some factors do not fundamentally change but affect the relationship across time. To study this case, a tool to simultaneously examine features of time and frequency is required. Wavelets provide such a tool.

To overcome the limitations of the two approaches measuring comovement, including one based on the time domain and the other based on the frequency domain, Rua and Nunes (2009) define wavelet squared coherency. This tool plays a role as the correlation coefficient around each moment in time and frequency; it is the absolute value squared of the smoothed cross-wavelet power normalised by the smoothed wavelet power; and it is used to measure the extent to which two time series move together over time and across frequency and to study the VaR based on wavelet covariance and wavelet variance. Rua (2010) proposes a wavelet-based measure of comovement that combines the analyses from the time and frequency domains. This measure assesses the changes in the degree of comovement across frequencies and over time. Regarding the evidence that the link between monetary growth and inflation varies over time and across frequencies, Rua (2012) employs wavelet squared coherency and the wavelet phase-difference to simultaneously measure the strength of the contemporaneous relationship and the lead-lag relationship over time and across frequencies, respectively. Similarly, given the time-frequency varying feature of beta (market risk), Rua and Nunes (2012) construct a wavelet-based estimator of beta from the CAPM to measure systematic risk, and they use a wavelet-based R^2 to assess the importance of systematic risk to the total risk in the time-frequency space. These approaches provide additional insights into risk assessment issues, including the consideration of emerging markets as a case in their paper.

Aguiar-Conraria and Soares (2011) design a metric using the wavelet transform to measure and test business cycle synchronisation across the EU-15 and EU-12 countries, and they use the wavelet coherency and the wavelet phase-difference to examine the strength of the contemporaneous relationship and the lead-lag relationship, respectively. Aguiar-Conraria *et al.* (2012a) apply three wavelet tools, including the wavelet power spectrum, the cross-wavelet coherency, and the phase-difference, to study cycles in American elections and war severity. These researchers detect transient and irregular cycles and structural breaks in the periodicity of those cycles. Likewise, Aguiar-Conraria *et al.* (2012b) employ the same three wavelet tools to study the relationship between three Nelson-Siegel latent factors of the yield curve (the level, slope and curvature) and four macroeconomic variables (unemployment, an index of macroeconomic activity, inflation and the monetary policy interest rate) in the U.S. across time and frequencies.

Wavelets also provide insight into econometrics and other topics in the economic and financial fields. Lee and Hong (2001) design a wavelet-based consistency test for a serial correlation of unknown form in univariate time series models. The proposed test statistic is constructed by comparing an estimator of the wavelet spectral density function and the null spectral density.[20] The simulation study shows that this test outperforms a kernel-based test when the time series has distinctive local spectral features but underperforms the kernel-based test when the time series is smooth with no peaks or spikes. Duchesne (2006a) devises a similar wavelet-based test for serial correlation that is used for multivariate time series models and obtains results similar to those of Lee and Hong (2001). Hong and Kao (2004) use the wavelet

[20]The wavelet spectral density function is the orthogonal projection of the spectral density function on wavelet bases.

spectral density estimator to construct consistency tests for a serial correlation of unknown form in the estimated residuals of a panel regression model in the presence of substantial inhomogeneity in serial correlation across individuals. The simulation study demonstrates that these tests perform well in small and finite samples relative to some existing tests.

Hong and Lee (2001) devise a one-sided test for ARCH effects using a wavelet spectral density estimator at frequency zero for a squared regression residual series. The simulation study shows that the wavelet-based test is more powerful than a kernel-based test in small samples when ARCH effects are persistent or when ARCH effects have a long distributional lag. Duchesne (2006b) extends Duchesne (2006a)'s approach to the ARCH effects of multivariate time series and advocates for a test statistic by comparing a multivariate wavelet-based spectral density estimator of the squared and cross-residuals to the spectral density under the null hypothesis of no ARCH effects. This method is a useful complement to the existing tests for vector ARCH effects, particularly for spectral density that exhibits nonsmooth features. Boubaker and Peguin-Feissolle (2013) develop a Wavelet Exact Local Whittle estimator and a Wavelet Feasible Exact Local Whittle estimator of memory parameter d in the fractionally integrated process $I(d)$. Simulation experiments show that the new estimators outperform traditional ones under most situations in the stationary and nonstationary cases. Barunik and Kraicova (2014) construct wavelet-based Whittle estimator of the Fractionally Integrated Exponential Generalized Autoregressive Conditional Heteroscedasticity (FIEGARCH) model, and find wavelet-based estimator may become an attractive robust and fast alternative to the traditional methods of estimation for data in presence of jumps. Xue *et al.* (2014) propose a wavelet-based test to identify jump arrival times in high frequency financial time series data. The test is robust for different specifications of price processes and the presence of the microstructure noise. Among others, Jensen (1999, 2000) and Whitcher and Jensen (2000) study the long-memory parameter using wavelet variance analysis.

Davidson *et al.* (1997) present a semi-parametric approach using wavelets to study commodity prices. These researchers conclude that wavelet analysis is particularly useful for describing the general features of commodity prices, such as structural breaks, comovements of prices, unstable variance structure and time-dependent volatility. Because of contamination in time series, which are highly perturbed by exogenous forces and factors, Capobianco (1999) employs wavelet shrinkage to denoise data and finds that this approach is useful for improving volatility prediction power through the GARCH model. Struzik (2000) uses the wavelet transform to display the local spectral (multifractal) contents of the S&P 500 index and studies the collective properties of the local correlation exponent as perceived by traders, exercising various time horizon analyses of the index. Struzik and Siebes (2002) propose wavelet-based multifractal formalism to detect and localise outliers in financial time series and other stochastic processes. Los and Yalamova (2006) introduce the notion of a scalogram, which is a 2-dimensional array that yields normalised risks or variances across time and scale, and they propose Gibb's partition function, which effectively computes the moments of the absolute values of wavelet coefficients. Gencay *et al.* (2010) apply the wavelet-based hidden Markov tree models to the volatilities of high-frequency data and report the asymmetric volatility dependence across different time horizons. Grane and Veiga (2010) develop a wavelet-based procedure to detect and correct outliers in financial time series. The intensive Monte Carlo study shows that this approach is effective for detecting isolated outliers and outlier patches and is significantly more reliable than other alternatives.

A thorough discussion of the economic and financial applications of wavelets can be found in the survey articles by Ramsey (1999, 2002). Moreover, Crowley (2007) briefly introduces different types of tools in wavelet analysis related to economics and finance and provides a survey of their application in the literature. Abramovich *et al.* (2000) review wavelet analysis in statistical applications, including its use in nonparametric regression, density estimation, and certain aspects of time series. These applications provide different insights into the economic and financial fields.

Note that in the last decade, the main literature of wavelets in economics and finance is focused on multiresolution analysis (MRA). There are still many benefits of wavelets for these fields to explore. For example, wavelets are able to extract business cycles from a data sequence the way as the commonly used filters, and to rearrange the structure of data, to unveil economic natures or phenomena that are always ignored. Moreover, wavelets remain local features of time series caused by events in decomposed components, which are useful for studying local events. As far as we know, there are not many papers to serve this purpose using wavelets. Therefore, our thesis fills these gaps by studying three interesting phenomena and providing new insights into them.

Chapter 3

Extracting Business Cycles and Detrending via Wavelets

3.1 Introduction

In macroeconomics, the cyclical fluctuations surrounding a secular trend are believed to be business cycles. The secular trend provides long-term information, whereas the business cycles offer short-term information. Economists have long been concerned about isolating these cycles; the tool used in this task is commonly called a "filter". The Hodrick-Prescott filter is the best known and, to the best of my knowledge, is still widely used. Originally, the Hodrick-Prescott filter was applied in detrending quarterly data. However, according to one argument, the detrended data from the Hodrick-Prescott filter, which are considered to represent business cycles, appear to be volatile. Because business cycles are required to be smooth, the Hodrick-Prescott filter is widely criticised.

Economists have presented several other filters for extracting business cycles from a time series. These filters are designed based on spectral analysis theory, which states that data are the sum of different unrelated frequency components. These frequency components are linked with three types of economic data: the high-frequency components are noise; the low-frequency components are the trend; and the complementary components are business cycles, which are defined as cyclical components ranging from 6 quarters to 32 quarters.[21] With respect to this definition, the business cycles lie in the intermediate range of frequencies $[2\pi/32, 2\pi/6]$.

An ideal filter should serve two purposes. First, the filter should fully preserve all components in the frequencies between a specific lower and upper frequency,

[21] This definition, initially proposed by Baxter and King (1999), is widely quoted in recent literature.

which is referred to as the passband. Second, the filer should completely eliminate components in other frequencies, which is known as the stopband. However, an ideal filter is possible only when applied to an infinite time series because the order of its coefficients is doubly infinite. Because a limitation in the length of time series in practice, we must approximate the ideal filter. Accordingly, Baxter and King (1999), Pollock (2000), and Christiano and Fitzgerald (2003) present their own filters to extract business cycles.

There is no consensus on the relationship between business cycles and trends. If we suppose that they are related, it is still not possible to accurately understand this relationship in practice. To facilitate the study of economic issues, business cycles and trends are sometimes assumed to be linearly independent in empirical analyses. For instance, consider a shock to an economy. It is of interest to examine the effect of the shock on the trend and business cycles because such a significant effect indicates that the long- or short-run equilibrium, respectively, of the economy is affected. To simplify this investigation, the trend and business cycles are assumed to be linearly independent. As a result, the linear relationship between the shock and the trend does not affect the relationship between the shock and the business cycles. Otherwise, the results are ambiguous.

Consequently, a filter that can produce linearly independent frequency components is required, but the above four filters cannot meet this requirement. Fortunately, the wavelet filter, which orthogonally decomposes data into different frequency components, is a potential solution.[22] Orthogonality implies that different frequency components are linearly independent and thus simplifies some economic issues. Because the wavelet filter is a symmetric filter, it does not result in the phase effect or the corresponding phase shift in time. The temporal properties of data are important in economics, and a phase shift in time is thus not allowed. Moreover, the base functions of the wavelet filter are time- and scale-localised (frequency-localised), whereas the base functions of the Fourier transform on the which other four filters are estimated are only frequency-localised. Consequently, the wavelet filter provides better resolution in the time domain, which is more useful for capturing the changing volatility of business cycles. Given the attractive properties of the wavelet filter, it is believed to be a good alternative filter for isolating different frequency components of data.

Lacking an ideal filter as a benchmark, it is difficult to evaluate the wavelet filter

[22]In this paper, the wavelet filter is a general notion that represents wavelets, including scaling filters and wavelet filters. We apply the Daubechies least asymmetric (LA) wavelet filter of width 8 in this paper. This filter is recommended by Percival and Walden (2000) and is widely used in the literature of economic and financial applications of wavelets.

and other filters when they are applied to real data. These other filters consist of the Hodrick-Prescott filter, the digital butterworth filter, the Baxter-King bandpass filter and the Christiano-Fitzgerald bandpass filter. To compare the different filters, this paper uses the root-mean-square deviation (RMSD), which is derived using a Monte Carlo simulation. A smaller RMSD value indicates better performance of the filter. Generally, we find that the Baxter-King bandpass filter, the wavelet filter and the digital butterworth filter outperform the other filters for annual data, quarterly data and monthly data to extract business cycles, respectively. Furthermore, when a filter is used to extract the trend from data, the Baxter-King bandpass filter outperforms the other filters for annual and quarterly data, and the digital butterworth filter is the optimal choice for monthly data. It is important to note that the first K and the last K data are not processed in the Baxter-King bandpass filter.[23] Moreover, there is a high deviation at the end of the filtered data in the digital butterworth filter. It is recommended that these data be discarded. However, the data at the end are important to current analysis in economics. These issues reflect major disadvantages of these two filters, but they are not associated with the use of the wavelet filter.

This paper is organised as follows. Section 2 provides an overview of the classical Wiener-Kolmogorov filters (consisting of the Hodrick-Prescott filter and the digital butterworth filter) and the bandpass filters (consisting of the Baxter-King bandpass filter and the Christiano-Fitzgerald bandpass filter). Section 3 describes the performance of these filters, compares them by frequency response functions, and demonstrates the gain and phase effects of the Christiano-Fitzgerald bandpass filter in a three-dimensional figure. As a result of the downsampling and upsampling process, the wavelet filter does not have its own gain function; hence, it is not adopted for comparison with other filters here. Section 4 constructs artificial data for an experiment, including annual data, quarterly data and monthly data. Section 5 summarises the results of these filters as applied in the experiment. Section 6 concludes the paper and illustrates the effect of the wavelet filter's drawbacks in extracting business cycles or trends from data.

[23] $2K + 1$ is the width of the Baxter-King bandpass filter.

3.2 Overview of Four Commonly Used Filters

3.2.1 The Classical Wiener-Kolmogorov Filter

This section briefly introduces four commonly used filter models. Many filters involving the Baxter-King bandpass filter (abbreviated as the BK filter) and the Christiano-Fitzgerald bandpass filter (abbreviated as the CF filter) have been proposed over the past two decades to extract business cycles from a trended data sequence, but the Hodrick-Prescott filter (abbreviated as HP filter below) dating back to 1980 is still used widely in economics. Because the HP filter is an example of a classical Wiener-Kolmogorov filter, which is a signal extracting device that minimises the mean squared errors (MSE) of data estimates with observations, it is natural to introduce this filter first.

Here, a signal and noise generate an observed data sequence expressed as follows:

$$y(t) = x(t) + \varepsilon(t), \tag{3.1}$$

where the signal $x(t)$ and noise $\varepsilon(t)$ are mutually independent white noise processes. Consequently, the autocovariance generating function $\gamma^{yy}(L)$ of $y(t)$ is a combination of those of $x(t)$ and $\varepsilon(t)$, denoted as $\gamma^{xx}(L)$ and $\gamma^{\varepsilon\varepsilon}(L)$, respectively:

$$\gamma^{yy}(L) = \gamma^{xx}(L) + \gamma^{\varepsilon\varepsilon}(L), \tag{3.2}$$

where L is the lag operator as $Lx_t = x_{t-1}$ and the autocovariance generating function is $\gamma(L) = \sum_{-\infty}^{\infty} \gamma_h L^h$, in which γ_h is the covariance. $x(t)$ and $\varepsilon(t)$ are mutually independent, which implies that

$$\gamma^{yx}(L) = \gamma^{xx}(L). \tag{3.3}$$

A Wiener-Kolmogorov filter $B_x(L)$ extracts the signal from observations while minimising the MSE of the estimates with the observations, which can be expressed as follows:

$$\begin{gathered} \hat{x}(t) = B_x(L)y(t) = \sum\nolimits_j B_j L^j y(t) \\ \text{within} \quad \min \sum_{t=-\infty}^{\infty} [x(t) - \hat{x}(t)]^2, \end{gathered} \tag{3.4}$$

here, the number of observations is infinite, and the filter is an infinite impulse response (IIR).

$\hat{x}(t)$ is estimated by minimising the MSE criterion, which implies that the errors of $x(t)$ with $\hat{x}(t)$ should be uncorrelated with the observations. Otherwise, given the current information, we can improve the accuracy of the estimates by the observations. Accordingly,

$$\begin{aligned}
0 &= E[y(t-k)(x(t)-\hat{x}(t))] \\
&= E[y(t-k)x(t) - y(t-k)\hat{x}(t)] \\
&= E[y(t-k)x(t) - y(t-k)\sum\nolimits_j B_j L^j y(t)] \\
&= \gamma_k^{yx}(L) - \sum\nolimits_j B_j \gamma_{k-j}^{yy}(L) \qquad \text{for all } k.
\end{aligned} \tag{3.5}$$

The $z-$transform function is written as follows:

$$\gamma^{yx}(z) = B_x(z)\gamma^{yy}(z). \tag{3.6}$$

According to Equations (3.2) and (3.3), $B_x(z)$ is identical to the following:

$$B_x(z) = \frac{\gamma^{yx}(z)}{\gamma^{yy}(z)} = \frac{\gamma^{xx}(z)}{\gamma^{xx}(z) + \gamma^{\varepsilon\varepsilon}(z)}. \tag{3.7}$$

The complement of this filter is $B_\varepsilon(z)$:

$$B_\varepsilon(z) = 1 - B_x(z) = \frac{\gamma^{\varepsilon\varepsilon}(z)}{\gamma^{xx}(z) + \gamma^{\varepsilon\varepsilon}(z)}. \tag{3.8}$$

Because the autocovariance generating functions are positive-definite, they are factorised by the following:

$$\gamma^{yy}(z) = \psi(z)\psi(z^{-1}) \quad \text{and} \quad \gamma^{\varepsilon\varepsilon}(z) = \phi(z)\phi(z^{-1}). \tag{3.9}$$

Consequently, the filter $B_\varepsilon(z)$ is identical to the following:

$$B_\varepsilon(z) = \frac{\phi(z)\phi(z^{-1})}{\psi(z)\psi(z^{-1})}. \tag{3.10}$$

Equation (3.10) tells us that a detrended data sequence $\varepsilon(t)$ can be estimated from a data sequence $\{y(t); t = 0, \pm 1, \pm 2, \cdots\}$ using two recursive operations. First, an intermediate sequence $q(t)$ is the result of the forward pass of the filter: $q(z) = \phi(z^{-1})/\psi(z^{-1})y(z)$. Second, the detrended data sequence $\varepsilon(t)$ is the output generated by the backward pass in $q(t)$: $\varepsilon(z) = \phi(z)/\psi(z)q(z)$.

3.2.2 The Hodrick-Prescott Filter

The HP filter is designed to remove a smooth trend from a data sequence that follows a second-order random walk. The data sequence $y(t)$ is postulated as the sum of a trend $x(t)$ that follows a second-order random walk and a white noise process $\varepsilon(t)$:

$$y(t) = x(t) + \varepsilon(t). \tag{3.11}$$

To use the Wiener-Kolmogorov formulation, the second difference is applied in the data sequence:

$$\begin{aligned}(1-L)^2 y(t) &= (1-L)^2 x(t) + (1-L)^2 \varepsilon(t) \\ &= \kappa(t) + \eta(t).\end{aligned} \tag{3.12}$$

The following are the autocovariance generating functions of the differenced components:

$$\begin{aligned}\gamma^{\kappa}(L) &= \sum_{h=-\infty}^{\infty} \gamma_h^{\kappa} L^h = \sigma_\kappa^2, \\ \gamma^{\eta}(L) &= \sum_{h=-\infty}^{\infty} \gamma_h^{\eta} L^h = \sigma_\eta^2 (1-L)^2 (1-L^{-1})^2,\end{aligned} \tag{3.13}$$

where $\gamma^{\kappa}(L)$ and $\gamma^{\eta}(L)$ are the autocovariance generating functions of the sequence $\kappa(t)$ with variance σ_κ^2 and of the sequence $\eta(t)$ with variance σ_η^2, respectively.

In accordance with the Wiener-Kolmogorov principle, the detrending filter is established from the ratio of the autocovariance generating functions in terms of the $z-$transform function:

$$HP(z) = \frac{\gamma^{\eta}(z)}{\gamma^{\eta}(z) + \gamma^{\kappa}(z)} = \frac{\sigma_\eta^2 (1-z)^2 (1-z^{-1})^2}{\sigma_\eta^2 (1-z)^2 (1-z^{-1})^2 + \sigma_\kappa^2} = \frac{\lambda (1-z)^2 (1-z^{-1})^2}{\lambda (1-z)^2 (1-z^{-1})^2 + 1}, \tag{3.14}$$

where $\lambda = \sigma_\eta^2 / \sigma_\kappa^2$ is a parameter that simultaneously controls the cut-off frequency and the rate of transition from the stopband to the passband. Hodrick and Prescott (1997) recommend $\lambda = 1600$ for quarterly data. $\lambda = 100$ and $\lambda = 14400$ are consistently applied to the annual data and monthly data, respectively. However, Ravn and Uhlig (2002) argue that $\lambda = 6.25$ for annual data and $\lambda = 129600$ for monthly data are better via the equation $\lambda = s^4 \lambda_q$, where $s = 1/4$ for annual data, $s = 3$ for monthly data and $\lambda_q = 1600$.

Equation (3.14) is the $z-$transform function of the HP filter. $HP(0) = 0$ and

$HP(-1) \approx 1$ indicate that it is a highpass filter. Moreover, the HP filter is a linear time-invariant symmetric filter, which implies that there is no phase displacement in the estimates with observations because $HP(z) = HP(z^{-1})$. The detrended data that are deemed business cycles are generated by filtering the data sequence $y(t)$ using $HP(z)$. The complementary lowpass filter $1 - HP(z)$ extracts a smooth trend from the observations.

Based on Equation (3.14), the frequency response function of the infinite version of the HP filter is obtained using $e^{\mathrm{i}\omega}$ to replace z, which is stated as follows:

$$HP(\omega) = \frac{\lambda(1-e^{\mathrm{i}\omega})^2(1-e^{-\mathrm{i}\omega})^2}{\lambda(1-e^{\mathrm{i}\omega})^2(1-e^{-\mathrm{i}\omega})^2+1} = \frac{4\lambda(1-\cos\omega)^2}{4\lambda(1-\cos\omega)^2+1}, \tag{3.15}$$

the term $(1-e^{\mathrm{i}\omega})^2(1-e^{-\mathrm{i}\omega})^2$ in the numerator has 4 zeros at zero frequency; it is thus able to stationarise an $I(4)$ process. However, this term in the denominator exacerbates this effect, indicating that the HP filter can remove non-stationary components that are integrated of an order less than 4 (King and Rebelo (1993)).

As the infinite impulse response is associated with an infinite time series, which is not realistic in economics, it is necessary to construct a finite version of the filter. In terms of the inverse Fourier transform, the finite impulse response HP_j is

$$HP_j = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{4\lambda(1-\cos\omega)^2}{4\lambda(1-\cos\omega)^2+1} e^{\mathrm{i}\omega j} d\omega. \tag{3.16}$$

The cut-off frequency ω_c associated with the parameter λ is calculated when the transition occurs at a frequency for which the frequency response equals 0.5. From the equation $HP(\omega_c) = 0.5$, we obtain the following:

$$\begin{aligned} &\omega_c = 2\arcsin(\frac{\lambda^{-1/4}}{2}), \\ \text{or}\qquad &\lambda = [2\sin(\frac{\omega_c}{2})]^{-4}. \end{aligned} \tag{3.17}$$

Therefore, for quarterly data, the cut-off frequency is 0.1582, which is related to 39.7 quarters. In other words, the periodicity of the business cycle is less than 39.7 quarters based on the suggestion of Hodrick and Prescott (1997). According to the definition of U.S. business cycles from Baxter and King (1999), the upper cut-off frequency $\omega_c^u = 2\pi/6 = 1.0467$ is associated with the upper parameter $\lambda^u = 1$, and the lower cut-off frequency $\omega_c^l = 2\pi/32 = 0.1963$ corresponds to the lower parameter $\lambda^l = 677.13$.[24]

[24] These frequencies indicate that the duration of U.S. business cycles lasts no fewer than 6 quarters (18 months) but fewer than 32 quarters (8 years).

One problem is always ignored when adopting the HP filter to detrend an economic time series. As noted earlier, the results from filtering a data sequence using the HP filter are detrended data, which are typically regarded as business cycles. However, detrending and extracting business cycles are two different concepts. A macroeconomic time series is regarded as a combination of a trend, business cycles and noise. Thus, the estimates using the HP filter contain not only business cycles but also noise. These estimates are not smooth, as Baxter and King (1999) argue. At a theoretical level, this result implies that the HP filter performs worse than the bandpass filters, which are able to separate business cycles from a data sequence. Given the inferences on the parameters λ for the lower and upper cut-off frequencies from Equation (3.17), we can use the following procedure to extract business cycles using a highpass filter such as the HP filter. It is supposed that business cycles lie in the range of frequencies $\left[\omega^l, \omega^u\right]$. First, we use the highpass filter with the lower cut-off frequency $\omega_c = \omega^l$ that is associated with λ^l via Equation (3.17) for the HP filter to generate data estimates that include business cycles and noise. We then adopt the filter with the upper cut-off frequency $\omega_c = \omega^u$ that is related to λ^u via Equation (3.17) for the HP filter to estimate the noise. Second, the business cycles are separated by subtracting the noise from the data estimates.

3.2.3 The Digital Butterworth Filter

Pollock (2000) develops a more general version of this type of filter to detrend an economic time series; this version is known as a digital butterworth filter (abbreviated to the BW filter). The BW filter is well known in electrical engineering. Based on the specific properties of the BW filter,

$$\begin{aligned} \psi(z)\psi(z^{-1}) &= \phi_L(z)\phi_L(z^{-1}) + \lambda\phi_H(z)\phi_H(z^{-1}), \\ \phi(z)\phi(z^{-1}) &= \lambda\phi_H(z)\phi_H(z^{-1}), \end{aligned} \tag{3.18}$$

in Equation (3.9), where

$$\phi_L(z) = (1+z)^n \qquad \text{and} \qquad \phi_H(z) = (1-z)^n. \tag{3.19}$$

Accordingly, the highpass BW filter is

$$BW_H(z) = \frac{\lambda(1-z)^n(1-z^{-1})^n}{\lambda(1-z)^n(1-z^{-1})^n + (1+z)^n(1+z^{-1})^n}, \tag{3.20}$$

where $\lambda = (1/\tan\frac{\omega_c}{2})^{2n}$ and ω_c is the cut-off frequency that $BW_H(e^{-\mathrm{i}\omega})|_{n\to\infty} = 1$ if $\omega > \omega_c$; otherwise, $BW_H(e^{-\mathrm{i}\omega})|_{n\to\infty} = 0$. The lowpass BW filter ($BW_L(z)$) is the complement of the highpass BW filter ($BW_H(z)$): $BW_L(z) = 1 - BW_H(z)$. It appears that the BW filter is a linear time-invariant symmetric filter that can remove or extract the trend from a data sequence. Here, n is the order of the filter that determines the rate of transition from the stopband to passband within the cut-off frequency ω_c. The role of λ in the HP filter is implemented by ω_c and n separately in the BW filter. A higher value for n and a shift in the cut-off frequency ω_c away from the mid-point $\pi/2$ increase the rate of transition from the stopband to passband but also cause a stability problem.

As shown by the $z-$transform function of the HP filter, Equation (3.20) indicates that the BW filter is also an IIR. In economics, because of a sample size limitation, Pollock (2000) presents a finite-sample version of the BW filter to detrend an economic time series. A so-called transient effect, which is liable to affect all processed values, arises when a discernible disjunction appears where the beginning and the end of the sample are joined. Accordingly, proper values for the forward and backward pass are interpolated by forecasting and backcasting. One approach to address this issue involves converting the non-stationary data sequence into a stationary sequence by differencing or by extracting a polynomial trend. The start-up and end-sample values are zeros, which are the unconditional expectations of the stationary data sequence.

According to Equation (3.12), a procedure that converts a non-stationary data sequence into a stationary sequence using second-order differencing is involved in the HP filter. Thus, the transient effect does not arise when we use the HP filter. For the BW filter, the model under this consideration is expressed by the following:

$$y(t) = s(t) + c(t), \tag{3.21}$$

where $s(t)$ is a stochastic trend, $c(t)$ is a cycle, and $y(t)$ is a non-stationary data sequence integrated by order d and denoted by $I(d)$. Here, the $d-$order difference is adopted to convert the data sequence $y(t)$ into a stationary sequence as follows:

$$(1-L)^d y(t) = (1-L)^d s(t) + (1-L)^d c(t). \tag{3.22}$$

Accordingly,

$$y(t) = \frac{(1+L)^n}{(1-L)^d} v(t) + \frac{(1-L)^n}{(1-L)^d} \eta(t), \tag{3.23}$$

where $v(t)$ and $\eta(t)$ are white noise processes with variances $V\{v(t)\} = \sigma_v^2$ and $V\{\eta(t)\} = \sigma_\eta^2$, respectively, $(1-L)^d s(t) = (1+L)^n v(t)$ and $(1-L)^d c(t) = (1-$

$L)^n\eta(t)$. In terms of matrix notation, the estimate of a cycle from a sample with T observations is then given by the following:

$$\hat{c} = \lambda\Sigma Q(\Omega_L + \lambda\Omega_H)^{-1}Q'y, \tag{3.24}$$

where $\hat{c}$ and y are $T\times 1$ vectors, respectively; Σ is a $T\times T$ Toeplitz matrix generated by $[(1-L)(1-L^{-1})]^{n-d}$; Q represents the d-order difference operator $(1-L)^d$ in the form of a matrix, which is a $T\times(T-d)$ matrix with the coefficients of the polynomial $(1-L)^d$ in the elements with index (k, j), $k = j, \cdots, j+d$ and $j = 1, \cdots, T$; and Ω_L and Ω_H are $(T-d)\times(T-d)$ Toeplitz matrices generated by $(1+L)^n(1+L^{-1})^n$ and $(1-L)^n(1-L^{-1})^n$, respectively.

As a highpass filter, the BW filter confronts the same problem as the HP filter in that the data estimates contain both noise and business cycles. Accordingly, the approach to addressing this problem above is applied when using the BW filter to extract business cycles.

3.2.4 The Bandpass Filters

A macroeconomic time series is typically assumed to be the sum of a trend, business cycles and noise. As noted earlier, the HP and BW filters are detrending filters. Researchers and economists normally aim to extract business cycles from a time series. However, the detrended data still have irregular components. Consequently, a more straightforward idea is proposed: using a filter to separate business cycles from the other components directly. Because business cycles lie in the intermediate range of frequencies, the type of filter needed for this task is referred to as a bandpass filter. The components in a specific frequency interval can completely pass through a bandpass filter, while the other components are blocked. Actually, the highpass and lowpass filters are special types of bandpass filters.

Consider the following structural time series:

$$y(t) = x(t) + c(t) + \varepsilon(t), \tag{3.25}$$

where $x(t)$ is a trend, $c(t)$ is a business cycle, and $\varepsilon(t)$ is a white noise process with variance σ_ε^2. From a frequency perspective, the cycle $c(t)$ has the power only in the frequency interval $\{[\omega^l, \omega^u] \cup [-\omega^u, -\omega^l]\} \in (-\pi, \pi)$, the trend $x(t)$ has the power only in the frequencies $\{(0, \omega^l) \cup (-\omega^l, 0)\} \in (-\pi, \pi)$, and the power of noise $\varepsilon(t)$ works in terms of the complementarity of these frequencies in $(-\pi, \pi)$. For an ideal bandpass filter $B(L)$,

$$c(t) = B(L)y(t) = \sum_{j=-\infty}^{\infty} B_j L^j y(t), \tag{3.26}$$

where $B_j = B_{-j}$ implies that it is a symmetric filter. The transfer function of $B(L)$ is $B(\omega) = \sum_j B_j e^{-\mathrm{i}\omega j}$ and is characterised by the following:

$$B(\omega) = \begin{cases} 1 & |\omega| \in [\omega^l, \omega^u] \\ 0 & \text{otherwise} \end{cases}, \tag{3.27}$$

where $|B(\omega)|$ is the gain function of the "ideal" filter with the passband $|[\omega^l, \omega^u]|$. In terms of the inverse Fourier transform, the filter's weights are as follows:

$$B_j = \frac{1}{2\pi} \int_{-\pi}^{\pi} B(\omega) e^{\mathrm{i}\omega j} d\omega. \tag{3.28}$$

These weights are identical to the following:

$$\begin{aligned} B_j &= \frac{\sin(j\omega^u) - \sin(j\omega^l)}{j\pi}, \quad j = \pm 1, \pm 2, \cdots \\ B_0 &= \frac{\omega^u - \omega^l}{\pi}. \end{aligned} \tag{3.29}$$

3.2.5 The Baxter-King Bandpass Filter

Because the ideal bandpass filter is an IIR, a finite sample approximation of the ideal filter is devised for most finite macroeconomic time series in practice. It is already known that phase shifts in filtered data cause time lags or time advances for observations. These shifts are problematic in economics because the temporal properties of data are important and should be retained. Consequently, a symmetric filter whose phase function is zero and thus does not introduce phase shifts in the filtered data is recommended. Given this consideration, Baxter and King (1999) adopt a symmetric window to truncate the ideal bandpass filter to establish an optimal symmetric filter.

The symmetric property indicates that the filter is expressed by $A(L) = \sum_{j=-K}^{K} A_j L^j$, where $A_j = A_{-j}$. The objective is to find the appropriate filter's weights A_j to make $A(L)$ converge with the ideal filter. Accordingly, a loss function Q should be minimised:

$$Q = \frac{1}{2\pi} \int_{-\pi}^{\pi} |B(\omega) - A(\omega)|^2 d\omega, \tag{3.30}$$

where $A(\omega)$ is the frequency response function of $A(L)$: $A(\omega) = \sum_{j=-K}^{K} A_j e^{-\mathrm{i}\omega j}$. If this approximating bandpass filter is used as a lowpass filter, then $A(\omega) = 1$ at $\omega = 0$ indicates that the filter's weights sum to unity. The constraint $A(0) = 1$ as a slide condition and the minimisation problem in Equation (3.30) yield the following results:

$$\begin{aligned} A_j &= B_j + \theta \ , \qquad j = 0, \pm 1, \cdots, \pm K, \\ \theta &= \frac{1 - \sum_{j=-K}^{K} B_j}{2K + 1}, \end{aligned} \tag{3.31}$$

where B_j are the ideal filter's weights, as previously demonstrated in detail. Based on the summary statistics for several U.S. quarterly macroeconomic time series and the definition of business cycles that has evolved from that of Burns and Mitchell (1946), Baxter and King (1999) recommend the following parameters for quarterly data: $K = 12$, $\omega^l = 2\pi/32$, and $\omega^u = 2\pi/6$.[25]

3.2.6 The Christiano-Fitzgerald Bandpass Filter

Because filtered data are generated by $\hat{y}(t) = \sum_{j=-K}^{K} A_j L^j y(t)$ in the BK filter, it is apparent that the first K sample values and the last K sample values remain unprocessed, which avoids the so-called end-sample problem that will be discussed in detail subsequently. However, these unprocessed values create a disadvantage for the BK filter when it is used to execute the current analysis: the data at the end of the sequence are often important in economics. With respect to this issue, Christiano and Fitzgerald (2003) propose another approach to truncate the ideal bandpass filter to construct an optimal filter. In their opinion, the phase shift issue for filtered data with observations is not a serious issue. The restriction on the symmetric property of the filter could be released, and an asymmetric approximation to the ideal bandpass filter could thus be constructed.

The finite impulse sequence of the approximating filter $a(L)$ is $\{a_j\}_{j=-n_1}^{n_2}$ with $n_1 \neq n_2$. Based on the CF filter, the estimated business cycles are as follows:

$$\hat{c}(t) = a(L)y(t) = \sum_{j=-n_1}^{n_2} a_j L^j y(t). \tag{3.32}$$

[25] Corresponding to the parameters for quarterly data, $K = 36$, $\omega^l = 2\pi/96$, and $\omega^u = 2\pi/18$ are applied to monthly data, whereas $K = 3$, $\omega^l = 2\pi/8$, and $\omega^u = 2\pi/2$ are applied to annual data. It is noted that the duration of business cycles is shorter than 8 years but greater than 1.5 years. Correspondingly, the upper cut-off frequency ω^u is $2\pi/1.5$ for annual data. However, this cut-off exceeds the maximum frequency π and is thus adjusted to $2\pi/2$.

The filter's weights a_j are generated by minimising the MSE criterion,

$$a_j = \arg\min E[(c(t) - \hat{c}(t))^2 | y] \ , \qquad y = [y_1, \cdots, y_T]', \tag{3.33}$$

and thus, they are

$$\begin{aligned} a_j &= B_j, \qquad \text{for } j = -n_1 + 1, \cdots, n_2 - 1, \\ a_{-n_1} &= \frac{a_0}{2} - \sum_{j=0}^{n_1-1} a_j \qquad \text{and} \qquad a_{n_2} = \frac{a_0}{2} - \sum_{j=0}^{n_2-1} a_j, \end{aligned} \tag{3.34}$$

where $n_1 = T - t$ and $n_2 = t - 1$. From $a_{n_2} = a_0/2 - \sum_{j=0}^{t-2} a_j$, we know that t should be not less than 3; thus, the first 2 processed data are distorted, and it is recommended that they be discarded. The variables n_1 and n_2 associated with time t indicate that the finite impulse sequence $\{a_j\}_{j=-n_1}^{n_2}$ varies with time, illustrating that the CF filter is time-variant. Moreover, the CF filter is clearly asymmetric. In matrix notation, Equation (3.32) is written as follows:

$$C = AAy, \tag{3.35}$$

where C is a $T \times 1$ vector containing business cycle information, observation y is a column vector with T dimensions, and the matrix form of the corresponding CF filter is as follows:

$$AA = \begin{bmatrix} \frac{1}{2}B_0 & B_1 & B_2 & B_3 & \cdots & B_{T-3} & B_{T-2} & -\frac{1}{2}B_0 - \sum\limits_{j=1}^{T-2} B_j \\ -\frac{1}{2}B_0 & B_0 & B_1 & B_2 & \cdots & B_{T-4} & B_{T-3} & -\frac{1}{2}B_0 - \sum\limits_{j=1}^{T-3} B_j \\ -\frac{1}{2}B_0 - B_1 & B_1 & B_0 & B_1 & \cdots & B_{T-5} & B_{T-4} & -\frac{1}{2}B_0 - \sum\limits_{j=1}^{T-4} B_j \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ -\frac{1}{2}B_0 - \sum\limits_{j=1}^{T-4} B_j & B_{T-4} & B_{T-5} & B_{T_6} & \cdots & B_0 & B_1 & -\frac{1}{2}B_0 - B1 \\ -\frac{1}{2}B_0 - \sum\limits_{j=1}^{T-3} B_j & B_{T-3} & B_{T-4} & B_{T-5} & \cdots & B_1 & B_0 & -\frac{1}{2}B_0 \\ -\frac{1}{2}B_0 - \sum\limits_{j=1}^{T-2} B_j & B_{T-2} & B_{T-3} & B_{T-4} & \cdots & B_2 & B_1 & \frac{1}{2}B_0 \end{bmatrix}_{T \times T}. \tag{3.36}$$

Actually, every row in matrix AA corresponds to a linear asymmetric filter, evidently indicating that every observation has its own CF filter.

The frequency response function of the filter is $a(\omega) = \sum_{j=-n_1}^{n_2} a_j e^{-\mathrm{i}\omega j}$. $a(\omega) = \sum_{j=-n_1}^{n_2} a_j = 1$ at $\omega = 0$ is analogous to the constraint on the BK filter that the frequency response function at zero frequency should be equal to one when the CF filter is used as a lowpass filter. It is easy to adjust the CF filter to a symmetric filter, as in the work of Baxter and King (1999). Accordingly, the BK filter appears to be a special type of CF filter. Unlike the BK filter, the data at the beginning and end of sequence are processed, which is a promising property of the CF filter for the current analysis. In addition, the CF filter is derived from a model-based methodology, with a priori knowledge regarding the spectral density of a time series. The CF filter is optimal when the process $y(t)$ is a random walk with a constant drift. To estimate business cycles, it is recommended that a linear trend be extracted from the non-stationary process $y(t)$ prior to filtering.

We should be cautious in addressing the end-sample problem when detrending a finite time series. If the appropriate approach is not used to resolve this problem, then all of the processed values would be affected. To detrend a data sequence at the beginning or at the end as well as in the middle, we should supply pre-sample and post-sample values for the symmetric filter, which can be either finite or infinite. Otherwise, the symmetric filter becomes a one-sided filter that is asymmetric when it is applied at the beginning or end of a data sequence, leading to phase displacement in these processed data. Note that the transient effect is generated by choosing inappropriate values for the forward or backward pass. There should be no distinguishable disjunction where the beginning and end of the sample are joined.

Consequently, a data sequence should be detrended to the extent that the apparent disjunction is diminished, which can be accomplished by differencing or by applying a polynomial regression of degree d. The data values interpolated at the beginning and at the end are zeros, which are the unconditional expectations of a stationary data sequence according to the HP and BW filters. The HP filter converts a non-stationary data sequence into a stationary sequence by second-order differencing, whereas the BW filter stationarises a data sequence by $d-$order differencing. However, this approach leads to a strong distortion at the end of the processed data sequence when it is applied to the BW filter, as will be shown subsequently. To further smooth the transition between the end and the beginning, we can interpolate a piece of pseudo-data at that location. In the wavelet filter, these data are estimated by backcasting and forecasting the residuals, which are results in removing the polynomial trend from a data sequence.

Furthermore, two other methods are recommended to solve the end-sample problem. Regarding the symmetric filter, a more straightforward method involves eliminating some sample values at the beginning and end of the sequence, as the BK filter does. From a different perspective, an asymmetric filter such as the CF filter that accounts for all observations when estimating filtered data may also be recommended.

3.3 Comparison of These Filters Using the Gain Function

The gain function yields changes in the amplitude of a time series over specific frequency intervals. Therefore, we compare these filters from a theoretical perspective. The wavelet filter is interesting in that its gain function is difficult to derive because of downsampling and upsampling. Moreover, the CF filter is an asymmetric and time-variant filter, which implies that it actually consists of many asymmetric filters. It is meaningless to show the gain effect of only one of its filters. Consequently, we will compare only the HP, BW and BK filters below. In addition, the gain effect and the phase effect of the CF filter will be discussed.

As noted previously, for an ideal bandpass filter, the desired components are able to completely pass, while other components are blocked. In addition, economics research requires that no time delay or time advance be produced during this process. From a frequency domain perspective, the amplitudes of the desired components are not altered while those of the other components are set to zero: the implication is that the ideal filter's gain function in the desired frequency intervals has a value of one while the other values are zero. Accordingly, for an ideal bandpass filter, the shape of the gain function is a square wave with an instantaneous transition between the stopband and passband. Moreover, the values of the filter's phase function should be identical to zero, suggesting that it is a symmetric filter. In this paper, we have revealed the symmetric filters involving the HP filter, the BW filter, the BK filter and the wavelet filter, noting an exception for the CF filter. Following the definition of business cycles suggested by Baxter and King (1999), the frequency band for the BW filter with $n = 8$ and the BK filter with $K = 12$ is $[2\pi/32, 2\pi/6]$, and the parameter λ is identical to 1600 for the HP filter in quarterly data.

When these filters are applied to a time series, three phenomena involving leakage, compression and exacerbation arise because these approximating filters are not ideal. As shown in Figure [3.1], some elements in the range of frequencies below the lower cut-off frequency or above the upper cut-off frequency (which should be prevented) pass through the filter; this phenomenon is called "leakage". Some elements in the desired range of frequencies do not fully pass the filter, which is called "compression". Finally, when the amplitudes of the gain function are greater than one, some elements are amplified, which is referred to as "exacerbation". Because

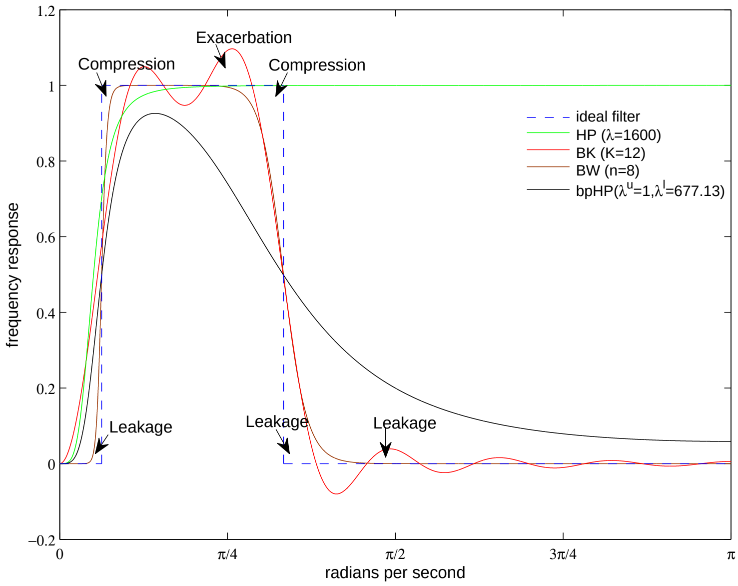


Figure 3.1: the gain functions of different filters

the ideal filter is not realistic, some of these phenomena always exist. These issues demonstrate that the extracted business cycles contain some elements of trend and/or noise but do not thoroughly capture real business cycles.

In contrast to the HP and BW filters, only the BK filter introduces the exacerbation effect. These three phenomena in the BK filter are caused by truncating the ideal filter; furthermore, the constraint condition imposes a constant on the ideal filter's weights, worsening the leakage at high frequencies. The negative frequency responses are associated with phase shifts, although Baxter and King (1999) regard them as minor issues. There is still no phase displacement within the entire filtered data sequence.

The most rapid transition from stopband to passband or from passband to stopband occurs in the BW filter, which means that it has the least leakage and compression issues. The steepness from stopband to passband or from passband to stopband is weaker in the HP and BK filters, which indicates that the leakage and compression effects are stronger in those filters. Figure [3.1] indicates that the BW filter is more advanced on a theoretical level. The gain functions of the HP and BW filters derive from their infinite versions, as does the approximating filter's gain function for the BK filter. In practice, the results estimated by these filters may not be what we find in Figure [3.1]. Furthermore, the performance of the CF filter and the wavelet filter is unclear from the gain function perspective. Therefore, a simulation study is conducted to assess the values of these five filters applied in an artificial series.

As discussed in the section above, the HP filter is a highpass filter. The filtered data, which are always considered to be business cycles, actually involve noise in addition to business cycles, as illustrated in Figure [3.1]. Thus, the HP filter is a detrending filter, and it would therefore not be consistent to compare the results for this filter with those of the bandpass filters. Generally, the HP filter could be constructed as a bandpass filter with the lower parameter $\lambda^l = 677.13$ and the upper parameter $\lambda^u = 1$ for quarterly data, as denoted by the bpHP filter. Figure [3.1] indicates that this approximation is poor because the leakage and compression effects are too strong. Because economists conventionally use the HP filter to extract business cycles and then critically analyse it using bandpass filters, we will compare the traditional HP filter to other filters below in the simulation study.

Figures [3.2] and [3.3] display the gain and phase effects of the CF filter, respectively. The asymmetric and time-variant CF filter actually has a corresponding filter for every observation. Here, the size of the underlying data sample is 150. The passband is consistent with that for the BW and BK filters, $[2\pi/32, 2\pi/6]$, which corresponds to a periodicity ranging from 6 quarters to 32 quarters. Leakage, compression and exacerbation arise in both the CF and BK filters. In particular, the leakage in the frequencies above the passband is pronounced, at least for the first 12 and last 12 data points. At the end of the data sequence, some of the exacerbation is substantial. Consequently, they suggest that these data be discarded, which is consistent with the first and last data points in the BK filter.

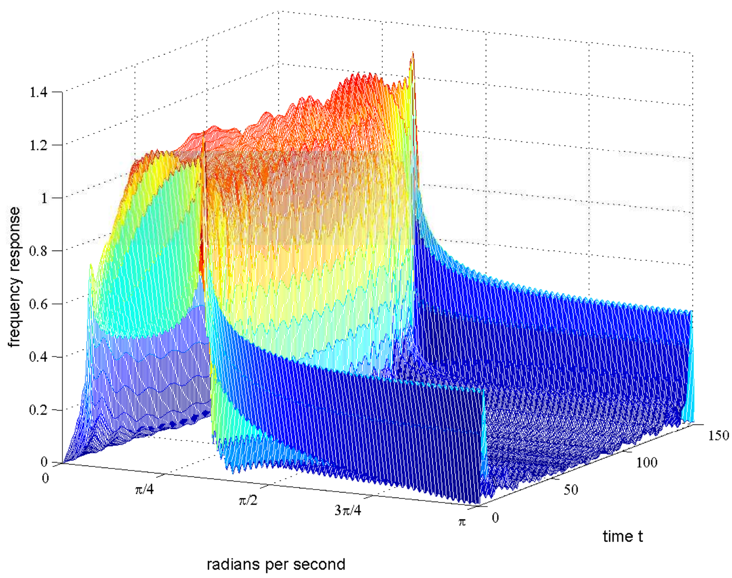


Figure 3.2: the gain effect of the CF filter

As implied by the transfer function, phase shifts are introduced by the CF filter. Figure [3.3] shows only the phase effect in the passband. The phase effect at the beginning or end of the data sequence is obvious. The absolute maximum value of the phase is approximately 1.4 quarters. As a result, some elements shift upward ±1.4 quarters, causing a maximum 2.8 quarters shift from the original data. Thus, the phase effect on the filtered data is strong, which contradicts the argument of Christiano and Fitzgerald (2003). Moreover, in a simulation study, Iacobucci and Noullez (2005) show a strong phase effect in data processed using the CF filter. The phase shifts in the filtered data cause time delays or time advances from the original data. As a consequence, the temporal and correlation properties among different frequency components within the series or among different filtered data sequences are meaningless, and the CF filter thus becomes less attractive.

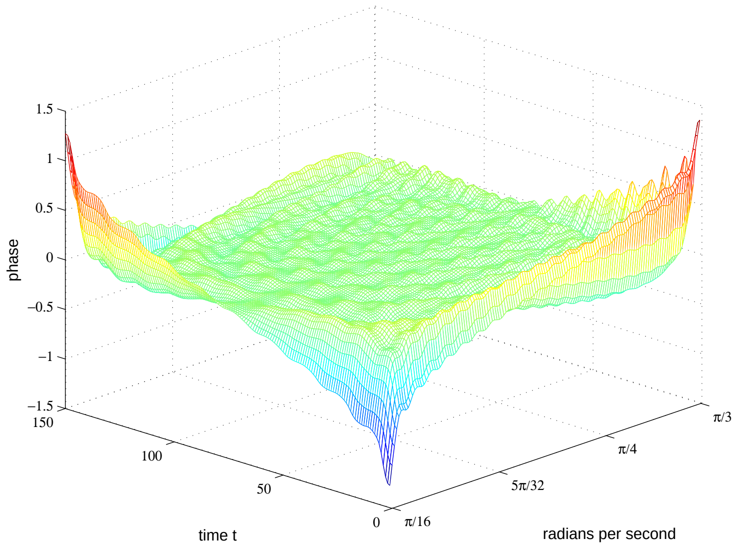


Figure 3.3: the phase effect of the CF filter

3.4 Monte Carlo Simulation

In spectral analysis theory, a macroeconomic time series is believed to be a combination of a trend at low frequencies, business cycles at intermediate frequencies, and noise at high frequencies. The various components are unrelated in their own specific frequency intervals. According to this theory, an ideal filter is required to isolate these components. Unfortunately, it is not realistic to apply the ideal filter, which is an IIR filter, to a macroeconomic time series because of the short length of the series. As a result, a filter to approximate the ideal filter is applied to a data sequence instead. Some leakage, compression and exacerbation effects arise and are attributed to the approximations. It is not easy to illustrate whether a filter is desirable for extracting a trend or business cycles, as the benchmark ideal filter does not exist in practice.

Accordingly, an artificial series is constructed to investigate the performance of a filter in extracting the trend or business cycles. Following the finding that important macroeconomic variables tend to be series with a unit root by Nelson and Plosser (1982), the trend $x^A(t)$ is postulated as a random walk with drift in the simulation:

$$x_t^A = x_{t-1}^A + 0.02 + \varepsilon_t^A, \qquad \varepsilon_t^A \sim N(0, 0.0004). \tag{3.37}$$

Because of the common application of the natural logarithm to data in practice, the trend here is interpreted as 2% annual economic growth with fluctuations in the range of $[-0.02, 0.02]$.

Baxter and King (1999) propose a definition of U.S. business cycles in which the duration of cyclical components ranges from 6 quarters (18 months) to 32 quarters (8 years). With respect to this definition, the simplest model to measure business cycles in annual data is the following:

$$c_t^A = 0.008[\sin(\frac{2\pi t}{8}) - 0.15\sin(\frac{2\pi t}{2})], \tag{3.38}$$

where the periodicities of the function are 2 and 8. To remove this cyclical component, the passband should be $[2, 8]$, which is consistent with the definition of business cycles. The 1.5-year periodicity of cycles should be adjusted to 2 years because of the maximum frequency π. For annual data, the passband for a filter is thus $[2, 8]$ years in time duration, which is associated with cut-off frequencies of $[\pi/4, \pi]$.

Overall, an annual data sequence $y^A(t)$ is a sum of the trend $x^A(t)$, the business cycles $c^A(t)$ and the noise $\eta^A(t)$:

$$y_t^A = x_t^A + c_t^A + \eta_t^A, \tag{3.39}$$

where $\eta^A(t)$ is a white noise process with a variance of 0.0002. To ensure consistency when constructing an artificial series, the quarterly data sequence $y^Q(t)$ is as follows:

$$y_t^Q = x_t^Q + c_t^Q + \eta_t^Q, \tag{3.40}$$

where the trend is a random walk with drift: $x_t^Q = x_{t-1}^Q + 0.02/4 + \varepsilon_t^Q$ with $\varepsilon_t^Q \sim N(0, 0.0004/16)$; the cycles are $c_t^Q = 0.0008[\sin(2\pi t/32) - 0.15\sin(2\pi t/6)]$; and the white noise process $\eta^Q(t)$ has a variance of 0.0002/16.

The monthly data sequence $y^M(t)$ is

$$y_t^M = x_t^M + c_t^M + \eta_t^M, \tag{3.41}$$

where the trend is $x_t^M = x_{t-1}^M + 0.02/12 + \varepsilon_t^M$ with $\varepsilon_t^M \sim N(0, 0.0004/144)$; the cycles are $c_t^M = 0.001[\sin(2\pi t/96) - 0.15\sin(2\pi t/18)]$; and the white noise process $\eta^M(t)$ has a variance of 0.0002/144.

In sum, the periodicities of the cycles are 2 years and 8 years for annual data, 6 quarters and 32 quarters for quarterly data, and 18 months and 96 months for monthly data. Correspondingly, the bands in time duration for the filter are $[2, 8]$ years, $[6, 32]$ quarters and $[18, 96]$ months, respectively. Thus, the cut-off frequencies are submultiples of the signal duration. Because the decomposed frequency is dyadic in the wavelet filter, the cut-off frequencies are adjusted to $[\pi/16, \pi/4]$ for quarterly data and $[\pi/32, \pi/8]$ for monthly data, whereas the cut-off frequencies remain the same, $[\pi/4, \pi]$, for annual data. Furthermore, the traditional value of the parameter λ in the HP filter is used here: 100 for annual data, 1600 for quarterly data and 14400 for monthly data.

The RMSD is a measure of the average error between actual observation and estimated observation. A smaller value of RMSD indicates that the estimated observation is closer to the actual value, which illustrates that the filter performs better than other filters. Because business cycles are a common concern in economics, demonstrating whether a filter is effective in extracting these cycles is the primary purpose of our investigation. In addition, estimating the trend is often a secondary goal. To evaluate the value of the filter used to extract the trend or business cycles, we build two types of RMSD:

$$\begin{aligned} \text{RMSD}^t &= \sqrt{\frac{1}{T}\sum_{t=1}^{T}(x_t - \hat{x}_t)^2}, \\ \text{RMSD}^b &= \sqrt{\frac{1}{T}\sum_{t=1}^{T}(c_t - \hat{c}_t)^2}, \end{aligned} \tag{3.42}$$

where RMSD^t and RMSD^b measure the estimated trend and the estimated business cycles, respectively. Lower values for RMSD^t or RMSD^b indicate that the filter is better able to extract the trend or business cycles. To provide a reliable result, we run the model 10000 times and obtain the sum of RMSD^t or RMSD^b.

3.4.1 The Simulation Findings

Tables [B.1] and [B.2] sum the results of the five filters used to extract the trend and business cycles, respectively. The remaining tables record the corresponding RMSD^t and RMSD^b values when each parameter in the artificial series is altered. Because the first K sample values and the last K sample values are not filtered in the BK filter, we also list the results obtained when these processed data are excluded in other filters for a consistent comparison between these filters. Moreover, the filtered data at the beginning or end are affected by extrapolation; thus, the RMSD values in the middle segment reflect the true consequences of these approximating filters applied to data at various sampling frequencies without the influence of the end-sample problem. As a result, the entire data sequence is divided into three segments to show the corresponding performance of the filters.

The results are quite robust with a few exceptions, regardless of the parameter of the artificial series. As the sampling frequency of data increases, the RMSD values decrease. This result is partly caused by eliminating irregular components of the quarterly or monthly data from the estimated data, which also contributes to declines in the oscillations of the trend. When these five filters are applied to an annual data sequence, they are actually highpass filters and are thus detrending filters. The frequency band for annual data is $[\pi/4, \pi]$, where π is the maximum frequency, referred to as the Nyquist frequency. Thus, the bandpass filters, such as the BK filter, the CF filter and the wavelet filter, become highpass filters. The data estimated by these filters consist of business cycles and noise. In our simulation, the value of the variance of noise in the annual data sequence is highest in the data at three sampling frequencies. This result explains why RMSD values are the highest for the annual data. The comparisons of the estimated trend are identical to those of the estimated business cycles, which are illustrated by the same values of RMSD^t and RMSD^b for annual data.

With regard to estimating the trend from a time series, the BK filter is the optimal choice and the HP filter is the worst choice for annual data in most cases. The results for the BK, BW and CF filters are similar. For quarterly data, the RMSD^t values for the BK filter are the smallest, whereas the values for the BW filter are the largest. In fact, the values for the BK, CF, HP and wavelet filters are close. The results indicate that the BK filter is the optimal choice and that the BW filter the worst choice for extracting the trend from a quarterly data sequence. In addition, the BW filter dominates for monthly data, whereas the CF and BK filters perform poorly for such data. The RMSD^t values in the HP wavelet filters show that they are also reasonable choices. Tables [B.23] and [B.25] indicate that all of the filters construct a smooth trend. When the data fluctuate greatly, which is evidenced by high values for the parameters of the variance in the trend in the artificial series, the quality of results from these filters declines substantially.

For the purpose of extracting business cycles, these five filters show the same results as when they are used to extract the trend from an annual data sequence: the BK filter is the optimal choice, and the HP filter is the worst choice. However, in some cases, the BW filter outperforms the BK filter. The smallest values for RMSD^b in the wavelet filter indicate that this method is optimal for extracting business cycles from a quarterly data sequence, whereas high values indicate that the BW filter is the worst choice. For monthly data, the BW filter is dominant in extracting business cycles. The BK and CF filters yield the highest RMSD^b values when they extract business cycles from a monthly data sequence, which indicates that they perform poorly with respect to extracting business cycles. The wavelet filter is nearly the second-best choice, and the HP filter is also a reasonable choice.

As introduced in the section above, the RMSD values in the middle segment of the data sequence capture the real performance of these approximating filters without the influence of the end-sample problem. The results are nearly identical to those for the entire data sequence without the first and last K sample values. In accordance with the RMSD values for the three segments, we find that the results are similar, with the exception of the BW filter. When the last K processed sample values are involved at the end of the sequence, the values for the RMSD either at the end or in the entire sequence increase substantially. As a result, it is advisable to eliminate these values. For the other three filters, the last K processed sample values do not affect the results. In some cases, the RMSD values at the end of the sequence are smaller than those for the middle segment. Therefore, it is recommended that these values be preserved. In addition, this result implies that the appropriate approach is being used to address the end-sample problem in these filters.

Indeed, it is not fair to compare the HP filter with the three bandpass filters involving the BK filter, the CF filter and the wavelet filter when used to extract business cycles. The data estimated using the HP filter as a highpass filter contain business cycles and noise, whereas the bandpass filters extract only business cycles. We can construct a bandpass version of the HP filter using a method that is adopted for the BW filter as well. For the BW filter, which is also a highpass filter, the noise is removed when the estimated data with an upper cut-off frequency are subtracted from those with a lower cut-off frequency. However, it is not common to extract business cycles using the HP filter, and the results are poor, as shown in Figure [3.1], from a theoretical perspective. Accordingly, this method for the HP filter is not employed here.

Consequently, the RMSD^b values should be large when the HP filter is used to extract business cycles, as shown in many cases. However, it is surprising to find that the RMSD^b values are small when the HP filter is applied to quarterly or monthly data to extract business cycles. The small oscillations in the irregular components explain this finding. Although these oscillations are involved in the estimated data, they cannot affect the results substantially. When irregular components fluctuate greatly, which is reflected in large values of the noise parameter, the HP filter yields the worst results because the irregular components are heavily weighted in the estimated data. Tables [B.12], [B.14], [B.16] and [B.18] justify our inferences here.

It is acknowledged that the trend is not linearly dependent on the business cycles in the simulations. Although linear independence is not identical to orthogonality, the results for the wavelet filter in the simulation may somehow contribute to this set. From another perspective, if business cycles and trends are required to be linearly independent in empirical works, then the wavelet filter would be a good choice.

3.5 Conclusion

In sum, the consequences of applying these five filters at various sampling frequencies are different when estimating trends and extracting business cycles. No filter has the ability to dominate in all situations, with the exception of the wavelet filter, which always extracts the best business cycles from a quarterly data sequence. Given economists' concerns regarding business cycles, the BK filter, the wavelet filter and the BW filter show the smallest RMSD^b values for annual data, quarterly data, and monthly data, respectively, implying that these filters are the optimal choices for data at these specific sampling frequencies. Furthermore, from the perspective of estimating a trend, the BK filter dominates for annual and quarterly data, and the BW filter generates the best trend from a monthly data sequence.

In general, the BK, CF and wavelet filters provide similar results in many cases, particularly the former two filters, as their construction principles are similar. Because of the asymmetric property of the CF filter, the BK filter is preferred. However, the first K sample values and the last K sample values are not processed by the BK filter. The data at the end of the sequence are important in economics, especially for the current analysis; therefore, this shortcoming of the BK filter makes it less appealing to economists and researchers.

A so-called distortion problem is identified by eliminating the first and last K sample values, as implemented by the BK filter. The majority of tables show high deviations at the end of the estimated data sequence using these data in the BW filter. This result suggests that it is preferable to discard these data. In other filters, the distortion problem does not appear to arise. This problem is naturally avoided by the BK and the CF filters, which use all data to estimate the filtered data. For the wavelet filter, forecasting and backcasting data are based on residuals by removing a polynomial trend from the data sequence; they are then interpolated at the end and at the beginning, respectively. This approach thus minimises the effects of this issue. It is interesting to observe the declines in RMSD values when the first and last K sample values are included. In comparison with the middle segment of the data sequence, the RMSD values at the end are even smaller. This result implies that the last K processed sample values could be retained in the wavelet filter, which is a promising feature of the wavelet filter for the current analysis.

Moreover, the base functions of the wavelet filter are localised in time and in frequency, which can be stretched and translated using a flexible resolution (in both time and frequency) to capture features that are local in both time and frequency. The sine and cosine functions that are the base functions of the Fourier transform are localised in frequency but not in time, although they extend over the entire real line. Accordingly, compared with the other four filters that are estimated using the Fourier transform, the wavelet filter provides a better resolution in the time domain and is more useful for capturing the changing volatility of business cycles. The wavelet filter is thus believed to be a good alternative filter for isolating different frequency components of data.

In addition, the main attractive property of the wavelet filter is that it orthogonally decomposes data into different components with various frequencies that are associated with time horizons. Orthogonality implies that different frequency components, including the business cycles and the trend, are linearly independent. Sometimes this relationship is required because it eases and simplifies the study of certain economic issues. According to the results of the above Monte Carlo simulation, using the wavelet filter to estimate the trend or business cycles is quite reasonable, especially when using it to extract business cycles from a quarterly data sequence. Consequently, if the business cycles and trend are required to be linearly independent, then the wavelet filter will provide promising results. Over the last two decades, a growing number of papers have studied economic or financial issues using the wavelet filter; see the literature review section.

A disadvantage of the wavelet filter is its strict dyadic frequency. The cut-off frequencies are adjusted for the wavelet filter when it is applied to quarterly or monthly data. However, this problem is not believed to be sufficiently serious to require abandoning the wavelet filter. First, this strategy has been used with annual data for all bandpass filters. The duration of business cycles is defined as being between 1.5 and 8 years. For the bandpass filter, the passband is not $[1.5, 8]$ years but is adjusted to $[2, 8]$ years because the Nyquist frequency is π. Second, the approximating filter is not ideal, such that the leakage effect is not prevented. An adjustment in the passband that causes the leakage can thus be tolerated. Third, in our simulation study, the periodicity of cycles is already known. The results for the wavelet filter are quite good. Especially for quarterly data, the wavelet filter is the dominant method for extracting business cycles. Thus, an adjustment in the passband for the filter is acceptable.

Finally, the duration of business cycles is not precisely defined. In the empirical literature, different business cycle lengths are used, e.g., 4 years by Croux *et al.* (2001), between 12 and 32 quarters by Levy and Dezhbakhsh (2003), between 8 and 32 quarters by Crowley and Lee (2005), between 16 and 128 months by Gallegati and Gallegati (2007), between 4 and 32 quarters by Yogo (2008), between 2 and 8 years by Aguiar-Conraria and Soares (2011), and between 4 and 12 years by Aguiar-Conraria *et al.* (2012b). By contrast, the trend is defined as fluctuations longer than 8 years by Jaeger (2003), 4 years by Assenmacher-Wesche and Gerlach (2008a,b), or 30 years by Benati (2009). This evidence appears to suggest possible durations for business cycles. Moreover, Aguiar-Conraria *et al.* (2012b) report that the duration of business cycles varies among countries. As shown by Bergman *et al.* (1998), business cycles last for 4.8 years on average during the post-war period, with Finland having longer average cycles (5.8 years) and Norway having shorter cycles (3.6 years).

In this paper, we use the definition proposed by Baxter and King (1999), who further develop the definition of Burns and Mitchell (1946). Burns and Mitchell (1946) offer the following definition of business cycles: "in duration business cycles vary from more than one year to ten or twelve years". Accordingly, there is no precise definition regarding the duration of business cycles; the definite is only approximate and can be adjusted to ensure good results. In many studies addressing the proper design of a detrending filter, the objective is to work with U.S. data. However, for many other countries, we do not even know the rough periodicity of business cycles. In conclusion, the insufficient prior knowledge of the periodicity of business cycles suggests that the cut-off frequencies should not be fixed when a filter is adopted to extract business cycles. From another perspective, this lack of knowledge also demonstrates that the drawback of the wavelet filter with its fixed cut-off frequencies does not affect our considerations when using a filter to extract the trend or business cycles from a data sequence.

Chapter 4

No Contagion, Only Interdependence: New Insights from Stock Market Comovements by Wavelet Analysis

4.1 Introduction

The 1997 Asian crisis, the 2007-2008 financial crisis, and the recent Euro sovereign debt crisis are cases in which extreme instability hit not only the original market but also the entire region, and even the global markets. Economists use the concept of contagion to describe the spreading of crisis from one market to other markets, such as a contagious disease. However, it is debated whether this propagation of a shock from one specific market across other markets is, in fact, contagion. Although there is some support for this argument, others believe that this phenomenon is merely a continuation of the same cross-market linkages that exist during more tranquil periods and is not a contagion of a crisis. For instance, southeast Asian markets or Euro markets in the same region have similar economic structures and histories. They have strong economic linkages, which implies that they are correlated in tranquil periods. A shock to one market could propagate to other markets through a normal transmission mechanism. This process is not contagion but merely interdependence. To clarify this debate, it is necessary to understand contagion thoroughly.

Generally, the concept of contagion is intended to describe incidents in which a financial crisis in one country imposes a negative impact on another country and induces a crisis in that country as well. However, this concept is difficult to distinguish from interdependence, which arises among countries because their economic fundamentals are linked through the balance of payments. As in the above argument, contagion was identical to “financial interdependence” in many early works (Fratzscher (2003)). Is it contagion if a simultaneous occurrence of financial crises in many countries is derived from a common shock? Consequently, a more precise definition of contagion is needed to distinguish these incidents. Unfortunately, there remains no consensus on this issue.

Many economists advocate that “abnormal” excess comovements should accompany contagion. Consistent with this suggestion, Kaminsky *et al.* (2003) refer to contagion as “an episode in which there are significant immediate effects in a number of countries following an event-that is, when the consequences are fast and furious and evolve over a matter of hours and days”.[26] This definition is appealing because the phrase “fast and furious” distinguishes contagion and interdependence. The gradual response to a shock is considered a continuation of a transmission mechanism in tranquil periods. This response is labelled “spillovers” by Kaminsky *et al.* (2003). Before proposing our definition of contagion, it is necessary to introduce the channels by which a shock to one market initially propagates across other markets. Without a clear understanding of this phenomenon, we cannot evaluate the issue precisely or propose appropriate policy measures to limit it.

The way that a shock to one specific market propagates across other markets is a core research topic in the study of “contagion”. Kaminsky *et al.* (2003) review the relative literature. Generally, there are two channels by which a shock propagates across markets. Initially, economists thought that it spread through trade linkages (or real linkages), which are estimated on macroeconomic fundamentals, such as trade or international business cycles. For instance, a country trades with another country not only through bilateral trade links but also through indirect trade links resulting from an intermediate country. A crisis in the first country induces the devaluation of currency and reduces the demands for imports. To remain competitive and stimulate exports, the second country devalues the currency as well. The following negative events cause a crisis in the second country.

Considerable attention has been devoted to trade linkages in the early period. At the theoretical level, Helpman and Razin (1978), Cole and Obstfeld (1991), Backus *et al.* (1992), Baxter and Crucini (1993), Cass and Pavlova (2004), and Pavlova and Rigobon (2007) utilise macroeconomic theory to explain how a shock to one market affects others. Under the analysis of trade, policy coordination, country reevaluation, and random aggregate shocks, the empirical literature (Eichengreen

[26]Forbes and Rigobon (2002) propose a similar definition of contagion but adopt “shift-contagion” instead.

et al. (1996), Glick and Rose (1999), Kaminsky and Reinhart (2000)) illustrates that trade linkage is an efficient transmission mechanism for shocks.

However, a growing number of economists question this channel and believe that the transmission mechanism that exists during more tranquil periods does not change. Kindleberger (1985) has argued that such trade linkages are "too strung out with lags to explain the near simultaneity of crisis". Moreover, this approach is not able to explain many phenomena arising in these two decades. For example, the fact that the currency devaluations in Turkey and Argentina in 2001 did not affect neighbouring countries illustrates the invalidity of this channel. Brazil's currency and equity prices declined in 1999 after the Russian default in 1998, but there was nearly zero trade between two countries: only 0.2 percent of Brazil's exports were destined for the Russian market. Consequently, it was eventually realised that trade linkages were not sufficient for explaining contagion.

An increasing number of theoretical and empirical studies present financial linkages to explain how a shock propagates across markets. As globalisation deepens, capital flows are relatively easier; consequently, the amount of capital flows is larger than ever before. Table [C.1] lists the relative data in the late 1980s and 1990s. In global markets, annual average capital flows increased from 15.0 billion U.S. dollars in the late 1980s to 151.1 billion U.S. dollars during 1990-1996. Among these capital flows, net direct investment and net portfolio investment also substantially increased, from 13.1 U.S. dollars to 61.7 U.S. dollars and from 3.6 billion U.S. dollars to 54.9 U.S. dollars since the late 1980s. According to these data, the percentage of net portfolio investment in net private capital flows has risen from 24 percent to 36.3 percent, which illustrates the importance of net portfolio investment strengthening. Specifically, net private capital flows consisting of net direct investment and net portfolio investment in every region have increased substantially.

Consequently, a greater number of investors (in particular, commercial banks, mutual funds, and hedge funds) hold multiple types of assets in a diverse range of markets. When there is a shock in one market, arbitrageurs fear the outflow of funds. To satisfy marginal calls or to meet liquidity requirements, they will sell another market's assets in their portfolios. It is very difficult to sell assets whose prices have already collapsed given the lemon problem. Even when these assets are sold, the withdrawn fund is too small to meet arbitrageurs' purposes. Consequently, investors prefer to sell other assets in portfolios. This action propagates the shock to a second market. It is observed that these are implemented by portfolio rebalancing activity. Furthermore, to meet capital constraints, banks withdraw foreign loans, which hurts another country's economy and induces a crisis in that country. Therefore, financial integration leads to a significant increase in global leverage, doubles the probability of balance sheet crises for any one country, and dramatically increases the degree of 'contagion' across countries. (Devereux and Yu (2014))

At the theoretical level, King and Wadhwani (1990) propose a correlated information channel, Fleming *et al.* (1998) and Calvo (1999) introduce a portfolio rebalancing activity channel, and Kyle and Xiong (2001) and Yuan (2005) propose a correlated liquidity channel to illustrate financial linkages. Moreover, Kodres and Pritsker (2002) present a rational expectation model to explain financial contagion.

Kaminsky and Reinhart (2000) show that trade linkage (trade in goods and services) through a third party is a transmission mechanism for some cases, but financial linkage through common bank creditors has recently been more prominent than trade linkage in driving the propagation of shocks for most markets. To identify a set of underlying variables that contribute to vulnerability to contagion and provide some useful suggestions for policymakers, Mody and Taylor (2003) examine the vulnerability of a region to the occurrence of a crisis and cast doubt on the role of trade linkages in the propagation process. Compared with lag effects in trade linkages, financial linkages are associated with nearly simultaneous effects. The authors find this result because the responses of traders in financial markets are able to immediately adjust asset prices. Consequently, the latter driver is better able to explain the propagation of a shock across markets. In conclusion, financial linkage dominates in the propagation mechanism of a shock, which is a main channel by which a shock to one market spreads to other, unrelated fundamental macroeconomic markets.

4.2 Heterogeneity of Asymmetric Information and Contagion

Pasquariello (2007) proposes a model that rules out the above three financial linkage channels (correlated information, portfolio rebalancing activity, and correlated liquidity channel) to explain financial contagion across unrelated markets from a new insight: heterogeneity of asymmetric information.

In his model, three types of investors, including informed speculators, uninformed market makers (MMs), and liquidity traders, trade multiple asset types in various markets on three dates and over two time periods. At time $t = 0$ (first date), informed speculators do not have any asymmetric information about assets. In the first time period between time $t = 0$ and $t = 1$, each speculator receives two sets of private and noisy signals about idiosyncratic and systematic shock. Then, the speculators utilise their informational advantage to trade at the end of time $t = 1$.

In the second time period, between time $t = 1$ and $t = 2$, the MMs observe the order flow, learn the activity of speculators, and rationally update their beliefs about the terminal payoffs of the trading assets before setting equilibrium prices. Based on their cross-inferences, they set the payoffs of multiple assets at the end of time $t = 2$.[27]

The multiple asset types include N risky assets and a riskless asset (the numeraire), whose gross return is normalised to one. An $N \times 1$ vector v that is multivariate normally distributed (MND) with mean $\bar{v}$ and nonsingular covariance matrix Σ_v denotes the $N+1$ assets. At the end of time $t = 2$, the terminal payoffs of the risky assets are gained. The vector v is linearly dependent on idiosyncratic shock ($N \times 1$ random vector μ) and systematic shock ($F \times 1$ random vector ϑ) and may be expressed by

$$v = \mu + \beta\vartheta, \tag{4.1}$$

where β is an $N \times F$ matrix of factor loadings.[28] μ and ϑ are assumed to be MND with means $\bar{\mu}$ and $\bar{\vartheta}$ and covariance matrices Σ_μ and Σ_ϑ (diagonal and nonsingular); thus, $\bar{v} = \bar{\mu} + \beta\bar{\vartheta}$ and $\Sigma_v = \Sigma_\mu + \beta\Sigma_\vartheta\beta'$ are nondiagonal and nonsingular.[29] Here, μ is interpreted as future realisations of domestic risk factors and ϑ as future realisations of global risks.

At time $t = 0$, all traders share information and do not have informational advantage. The prices of risky assets (P_0) are set to be the unconditional means of their terminal payoffs: $P_0 = \bar{v}$. In the first time period during time $t = 0$ and $t = 1$, each speculator (k) receives two sets of private (μ and ϑ) and noisy signals ($\varepsilon_{\mu k}$ and $\varepsilon_{\vartheta k}$) about future realisations of μ and ϑ: $S_{\mu k} = \mu + \varepsilon_{\mu k}$ and $S_{\vartheta k} = \vartheta + \varepsilon_{\vartheta k}$, where $\varepsilon_{\mu k} \sim MND(0, \Sigma_{\varepsilon_{\mu k}})$ and $\varepsilon_{\vartheta k} \sim MND(0, \Sigma_{\varepsilon_{\vartheta k}})$. Here, μ, ϑ, all $\varepsilon_{\mu k}$ and $\varepsilon_{\vartheta k}$ are mutually independent and induce $\Sigma_{\varepsilon_{\mu k}} = \Sigma_{\varepsilon_\mu}$ and $\Sigma_{\varepsilon_{\vartheta k}} = \Sigma_{\varepsilon_\vartheta}$ (Σ_{ε_μ} and $\Sigma_{\varepsilon_\vartheta}$ are diagonal). Consequently, before trading with the MMs, each speculator's expectation

[27] The uninformed MMs may use the observed demand for assets of one market to learn about terminal payoffs of other markets' assets. This learning activity is referred to as "cross-inference".

[28] In the model, β measures the relationships among markets. $\beta(n, j) = 0$ indicates that two countries (n and j) are fundamentally unrelated. $\beta \neq 0$ means that some countries share the same factors. For example, two emerging markets (Thailand and Brazil) are fundamentally unrelated, but one developed market (Germany) is economically interconnected with both emerging markets. When $\beta = 0$, these three countries are fundamentally macroeconomically unrelated.

[29] To remove the impact of correlated information, portfolio rebalancing activity, and correlated liquidity channel on the model, Σ_μ and Σ_ϑ are diagonal. Therefore, in the model, only the heterogeneity of asymmetric information is considered.

of v at time $t = 1$, is

$$E(v|S_{\mu k}, S_{\vartheta k}) = E_1^k(v) = P_0 + \Sigma_v \Sigma_{S_v}^{-1}(S_{v_k} - P_0)$$
$$= \bar{v} + \Sigma_\mu \Sigma_{S_\mu}^{-1}(S_{\mu k} - \bar{\mu}) + \beta \Sigma_\vartheta \Sigma_{S_\vartheta}^{-1}(S_{\vartheta k} - \bar{\vartheta}), \qquad (4.2)$$

where $S_{vk} = S_{\mu k} + \beta S_{\vartheta k}$, $\Sigma_{S_v} = \Sigma_{S_\mu} + \beta \Sigma_{S_\vartheta} \beta'$, $\Sigma_{S_\mu} = \Sigma_\mu + \Sigma_{\varepsilon\mu}$ and $\Sigma_{S_\vartheta} = \Sigma_\vartheta + \Sigma_{\varepsilon\vartheta}$. Thus, the benefits from informational advantage that are obtained from speculators trading with the uninformed MMs are δ_k: $\delta_k = E_1^k(v) - \bar{v} \sim MND(0, \Sigma_\delta)$, and

$$\text{var}(\delta_k) = \Sigma_\delta = \Sigma_\mu \Sigma_{S_\mu}^{-1} \Sigma_\mu + \beta \Sigma_\vartheta \Sigma_{S_\vartheta}^{-1} \Sigma_\vartheta \beta' \qquad (4.3)$$

is nonsingular. Therefore, for any two δ_k and δ_i,

$$\text{cov}(\delta_k, \delta_i) = \Sigma_c = \Sigma_\mu \Sigma_{S_\mu}^{-1} \Sigma_\mu \Sigma_{S_\mu}^{-1} \Sigma_\mu + \beta \Sigma_\vartheta \Sigma_{S_\vartheta}^{-1} \Sigma_\vartheta \Sigma_{S_\vartheta}^{-1} \Sigma_\vartheta \beta' \qquad (4.4)$$

is an symmetric positive definite (SPD) matrix. Consequently, $E_1^k(\delta_i) = \Sigma_c \Sigma_\delta^{-1} \delta_k$. In this setting, if $S_{\mu k} = S_\mu$ and $S_{\vartheta k} = S_\vartheta$, which implies $\Sigma_c = \Sigma_\delta$, the informed speculators receive the same or similar information; if $S_{\mu k} \neq S_{\mu i}$ and $S_{\vartheta k} \neq S_{\vartheta i}$ but $\Sigma_c = \rho \Sigma_\delta$ ($\rho \in (0, 1)$), it is called information homogeneity ($\Sigma_c = \rho \Sigma_\delta$). Accordingly, $\Sigma_c \neq \rho \Sigma_\delta$ indicates that the speculators receive heterogeneous information. $\Sigma_c \neq \rho \Sigma_\delta$ is referred to as information heterogeneity.

Homogeneously informed speculators trade more aggressively and act noncooperatively to maximise their benefits from information advantage because they are worried that the MMs will learn their information, which dissipates their advantage. Heterogeneously informed speculators trade as a monopoly and less aggressively because they do not want the MMs and other investors to learn their information. They should protect their informational advantage to maximise their benefits. In accordance with their behaviours, the MMs correctly anticipate homogeneously informed speculators' strategic trading activities from their aggressive behaviours, which indicate that their informational advantage is dissipated. However, the MMs cannot correctly anticipate heterogeneously informed speculators' behaviours because it is very difficult for the MMs to find the information that heterogeneously informed speculators hold from the observed order flow. In addition, liquidity traders prevent the aggregate order flow from becoming sufficient information for the MMs to learn the speculators' trading strategies. In conclusion, heterogeneously informed speculators and liquidity traders induce MMs' inferences about payoffs of multiple assets that are not correct. As a result, the MMs lose to heterogeneously informed speculators but compensate their losses from liquidity traders.

At time $t = 1$, the MMs set the equilibrium prices of multiple assets after speculators and liquidity traders submit their orders. The demand of liquidity traders is z, with mean $\bar{z}$ and nonsingular covariance matrix Σ_z. Each speculator's utility function of the net asset value (NAV) of his portfolio at time $t = 2$ is

$$U_k = U(NAV_{2k}) = NAV_{0k} + X_k'(v - P_1), \tag{4.5}$$

where X_k ($N \times 1$ vector) is the demand of speculator k for the multi-asset portfolio, NAV_{0k} is the amount of the riskless asset, and NAV_{2k} is announced at the end of the second period, after v is realised. Accordingly, the total demand of all K speculators is $\sum_{i=1}^{K} X_i$. The order flow of demand for the multi-asset portfolio (w_1) that the MMs receive is $w_1 = \sum_{i=1}^{K} X_i + z$, and then the MMs set the market-clearing price $P_1(w_1)$ based on it. Because $X_k = \arg\max_X E_1^k(U_k)$, the optimal demand on risky assets is estimated on informational advantage δ_k: $X_k = X_k(\delta_k)$. The competition among the investors drives individual investors' expectations of long-term profits based on the signal that they observe w_1 to zero: $w_1[E(v|w_1) - P_1] = 0$; thus,

$$E(v|w_1) = P_1 \tag{4.6}$$

is applied to the semi-strong market efficiency hypothesis. Suppose that the price P_1 and the demand X_k are linearly dependent on w_1 and δ_k, respectively, given the maximal expectation of utility function $E_1^k(U_k)$, Equations (4.5) and (4.6) tell us

$$P_1 = P_0 + \frac{\sqrt{K}}{2}\Lambda(w_1 - \bar{z}) = P_0 + H\sum_{i=1}^{K}\delta_i + \frac{\sqrt{K}}{2}\Lambda(z - \bar{z}), \tag{4.7}$$

and the optimal demand for each speculator

$$X_k = C\delta_k, \tag{4.8}$$

where $\Lambda = \Sigma_z^{-1/2}\Psi^{1/2}\Sigma_z^{-1/2}$ is an SPD matrix, $\Psi = \Sigma_z^{1/2}\Gamma\Sigma_z^{1/2}$, $\Gamma = 2[\Sigma_\delta - (K-1)H\Sigma_c][H^{-1} + (K-1)(\Sigma_\delta^{-1}\Sigma_c - \Sigma_c\Sigma_\delta^{-1})]^{-1}$ is an SPD matrix, $C = \frac{2}{\sqrt{K}}\Lambda^{-1}H$, and $H = [2I + (K-1)\Sigma_c\Sigma_\delta^{-1}]^{-1}$.[30] If there is only one speculator (K=1), then $H = \frac{1}{2}I$; if there are many homogeneously informed speculators ($K > 1$ and $\Sigma_c = \rho\Sigma_\delta$), then $H = \frac{1}{2+\rho(K-1)}I$ is diagonal; and if there are many heterogeneously informed

[30] It is presumed that $P_1 = A_0 + A_1 w_1$ and $X_k = B_0 + B_1\delta_k$.

speculators ($K > 1$ and $\Sigma_c \neq \rho\Sigma_\delta$), then the matrix H is nondiagonal. Therefore,

$$\begin{aligned} \operatorname{var}(X_k) &= \frac{1}{K}\Sigma_z, \\ \operatorname{var}(P_1) &= KH\Sigma_\delta. \end{aligned} \tag{4.9}$$

For homogeneously informed speculators, H is diagonal and $\operatorname{var}(P_1) = \frac{K}{2+\rho(K-1)}\Sigma_\delta$; therefore, the fundamental structure of $\operatorname{var}(P_1)$ is similar to Σ_v implanted in covariance Σ_δ (Equation (4.3)). For heterogeneously informed speculators, H is nondiagonal, and the fundamental structure of $\operatorname{var}(P_1)$ is different from Σ_v.

This is the general description of the model. Financial contagion is defined as a phenomenon whereby "a shock to one market affects prices of other markets fundamentally unrelated either to that shock or to that market". (Pasquariello (2007)) Thus, Pasquariello uses the following definition to make it operational in the framework.

Definition of contagion: In equilibrium, financial contagion from country j to country n occurs if, as a result of a real shock (to μ or ϑ) or an information noise shock (to ε_{uk} or $\varepsilon_{\vartheta k}$),

$$\frac{\partial P_1(n)}{\partial S_{\mu k}(j)} \neq 0 \qquad \text{or} \qquad \frac{\partial P_1(n)}{\partial S_{\vartheta k}(j)} \neq 0, \qquad \text{but } \beta(n,j) = 0, \tag{4.10}$$

or if, as a result of a noise trading shock (to z),

$$\frac{\partial P_1(n)}{\partial z(j)} \neq 0. \tag{4.11}$$

Conversely, interdependence between country n and country j occurs if

$$\frac{\partial P_1(n)}{\partial S_{\mu k}(j)} \neq 0 \qquad \text{or} \qquad \frac{\partial P_1(n)}{\partial S_{\vartheta k}(j)} \neq 0, \qquad \text{and } \beta(n,j) \neq 0. \tag{4.12}$$

Regarding this model, there are three types of shocks: real shocks (μ or ϑ), information noise shocks ($\varepsilon_{\mu k}$ or $\varepsilon_{\vartheta k}$), and noise trading shocks (z). In accordance with these three types, contagion can be expressed by the following:

For real shocks μ or ϑ, the impact of shocks to u on P_1 is given by the $N \times N$ matrix

$$\frac{\partial P_1}{\partial \mu'} = KH\Sigma_\mu\Sigma_{S_\mu}^{-1}, \tag{4.13}$$

whereas the impact of shocks to ϑ on P_1 is given by the $N \times F$ matrix

$$\frac{\partial P_1}{\partial \vartheta'} = KH\beta\Sigma_{\vartheta}\Sigma_{S_{\vartheta}}^{-1}. \tag{4.14}$$

For information noise shocks $\varepsilon_{\mu k}$ or $\varepsilon_{\vartheta k}$, the impact of shocks to any $\varepsilon_{\mu k}$ on P_1 is given by the $N \times N$ matrix

$$\frac{\partial P_1}{\partial \varepsilon'_{\mu k}} = H\Sigma_{\mu}\Sigma_{S_{\mu}}^{-1}, \tag{4.15}$$

whereas the impact of shocks to any $\varepsilon_{\vartheta k}$ on P_1 is given by the $N \times F$ matrix

$$\frac{\partial P_1}{\partial \varepsilon'_{\vartheta k}} = H\beta\Sigma_{\vartheta}\Sigma_{S_{\vartheta}}^{-1}. \tag{4.16}$$

When $\beta \neq 0$, there is financial contagion from those shocks if and only if speculators are heterogeneously informed; otherwise, it is only dependence if speculators are homogeneously informed. When $\beta = 0$, there is no such contagion.

In contrast, for noise trading shocks (z), the impact of shocks to z on P_1 is given by the $N \times N$ matrix

$$\frac{\partial P_1}{\partial z'} = \frac{\sqrt{K}}{2}\Lambda. \tag{4.17}$$

When $\beta \neq 0$, contagion always exists regardless of whether speculators are heterogeneously informed or homogeneously informed. When $\beta = 0$, there is no such contagion.

The off-diagonal terms in Equations (4.13), (4.14), (4.15), (4.16), and (4.17) measure the magnitude of contagion by real shocks or information noise shocks or noise trading shocks. Accordingly, the number of informed speculators (K) and the degree of heterogeneity of asymmetric information (α) assess the vulnerability to contagion and the magnitude of contagion.[31] Heterogeneous asymmetric information prevents the MMs from learning whether the shock is idiosyncratic or systematic and whether it is caused by news or noise. Consequently, μ, ϑ, $\varepsilon_{\mu k}$, and $\varepsilon_{\vartheta k}$ have the same effect on a speculator's informational advantage δ_k, and the only difference is that K speculators observe the news (μ or ϑ), but only one speculator k has the noise signal ($\varepsilon_{\mu k}$ or $\varepsilon_{\vartheta k}$). Therefore, Equations (4.13) and (4.14) are K times Equations (4.15) and (4.16), respectively. This relationship implies that not only the real shock but also the personal misleading information about a country affect contagion between

[31] $\alpha \in [0, 1]$. When $\alpha = 0$, the information is homogeneous; when $\alpha = 1$, the information is maximally heterogeneous.

countries.

It is clear that the degree of heterogeneity of asymmetric information (α) increases the vulnerability and magnitude of contagion (through H) because all of $|\frac{\partial P_1}{\partial \mu'}|$, $|\frac{\partial P_1}{\partial \vartheta'}|$, $|\frac{\partial P_1}{\partial \varepsilon'_{\mu k}}|$, $|\frac{\partial P_1}{\partial \varepsilon'_{\vartheta k}}|$, and $|\frac{\partial P_1}{\partial z'}|$ increase in α. In contrast, only $|\frac{\partial P_1}{\partial \mu'}|$ and $|\frac{\partial P_1}{\partial \vartheta'}|$ increase in K. This relationship suggests that a higher number of informed speculators (K) increases the vulnerability and magnitude of contagion by real shocks. In addition, the idiosyncratic shock (μ) to one specific country initially increases this country's vulnerability to contagion ($\frac{\partial P_1}{\partial \mu'}$ higher) but improves the perceived quality of speculators' signal on this country, indicating that the order flow (w_1) for the MMs is more reliable. Therefore, the multi-asset portfolio's prices set by the cross-inference of the MMs are closer to real values, and the vulnerability and magnitude of contagion decrease. In conclusion, a greater idiosyncratic shock (μ) initially leads to greater contagion, but the latter force eventually dominates and reduces the vulnerability and magnitude of this contagion between countries.

For information noise shocks, the presence of a greater number of informed speculators (K) has two contrasting effects. A greater number of informed speculators (K) makes it more difficult for MMs to learn their behaviours, which increases the vulnerability and magnitude of contagion; however, it makes the order flow of multiple assets more informative about their strategic trading activities and the equilibrium prices (P_1) less sensitive to individual trades ($|\Lambda|$ smaller), which limits contagion. Finally, the latter channel has a greater effect on contagion, which implies that the rising participation of informed speculators on the various markets for multiple assets reduces the vulnerability of all countries to contagion through information noise shocks.

Although the noise trading shocks lead to the incorrect cross-inference about the order flow (w_1) that causes contagion, neither the informed speculators nor the MMs are aware of the noise trading shocks, which indicates that the number of informed speculators (K) and H do not affect whether contagion exists. However, a greater number of informed speculators (K) makes each market more liquid, which reduces the impact of $d(z)$ on P_1 ($\lim_{k\to\infty} \frac{\partial P_1}{\partial z'} = 0$); thus, it reduces the magnitude of contagion by noise trading shocks. In contrast, the greater number of speculators (K) has two contrasting effects for the MMs to learn the order flow (w_1): learning the shared portions of speculators' signals more easily is accompanied by greater difficulty in learning the private portions of their signals. In conclusion, the more informed speculators initially increase the magnitude of contagion and eventually reduce it.

This model is particularly suited for emerging markets. Suppose that there are two emerging markets (Thailand and Brazil, as examples in the paper) that are fundamentally unrelated and one developed market (Germany) that is economically interconnected to both emerging markets. This model explains how a shock to one emerging market (Thailand) propagates to another emerging market (Brazil) through an intermediate market (Germany). For example, Equation (4.1) could be expressed by

$$\begin{aligned} v(1) &= \mu(1) + \vartheta(1), \\ v(2) &= \mu(2) + 0.5\vartheta(1) + 0.5\vartheta(2), \\ v(3) &= \mu(3) + \vartheta(2). \end{aligned} \tag{4.18}$$

Country 1 (Thailand) and country 3 (Brazil) are autarkic and fundamentally unrelated ($\text{cov}[v(1), v(3)] = 0$) but share exposure to the "core" country 2 (Germany) through the systematic factors $\vartheta(1)$ and $\vartheta(2)$. We find that country 2 is economically interconnected with country 1 and country 3. When a negative idiosyncratic shock hits country 1 ($d\mu(1) < 0$), in the short run (at time $t = 1$), K, informed speculators receive this signal and reduce their optimal demands for that security ($dw_1(1) < 0$). The MMs observe the resulting outflow from country 1 ($dw_1(1) < 0$) and downgrade their beliefs about $v(1)$, which leads to an equilibrium price $P_1(1)$ that is lower ($dP_1(1) < 0$). To attenuate the negative impact of decreasing price in country 1 and maintain expected profits from trading the multi-asset basket, the informed speculators also buy more (sell fewer) units of country 2's index, leading to a price increase in country 2.

This trade leads the MMs to make an incorrect cross-inference about the order flow: good news for country 2 or good news leading to a positive systematic shock ($d\vartheta(1) > 0$) because both country 1 and country 2 are exposed to this factor ($\beta(1, 1) > 0$ and $\beta(2, 1) > 0$). This supposition increases the price in country 2 ($dP_1(2) > 0$) and attenuates the drop in $P_1(1)$. Moreover, the increase of price in country 2 prompts the informed speculators to buy more assets in country 2 ($dX_k(2) > 0$).

However, country 2 is also exposed to $\vartheta(2)$ ($\beta(2, 2) > 0$), which ultimately mitigates the impact of these trades on the dealers' beliefs about $\vartheta(1)$. Regarding the positive factor ($\beta(3, 2) > 0$) for $\vartheta(2)$ in country 3, the speculators sell more (buy fewer) units of country 3's index ($dX_k(3) < 0$) to induce the MMs to adjust their beliefs about $\vartheta(2)$ downward ($d\vartheta(2) < 0$) and about $\vartheta(1)$ upward ($d\vartheta(1) > 0$), thus mitigating both $dP_1(2) > 0$ and $dP_1(1) < 0$.

During this process, the MMs gradually learn the economic structure of Equation

(4.18) and then rationally cross-infer the new information about other countries' assets from the order flow for one country's assets. To prevent the MMs from learning this process, imperfectly competitive informed speculators strategically trade in these three countries ($dX_k(1) < 0$, $dX_k(2) > 0$, and $dX_k(3) < 0$) to protect their informational advantage on the idiosyncratic shock to country 1 as much as possible. $d\vartheta(1) > 0$ and $d\vartheta(2) < 0$ lead the MMs to set a smaller price P_1 ($dP_1(1) < 0$) and price P_3 ($dP_1(3) < 0$) but a larger price P_2 ($dP_1(2) > 0$), thus inducing the speculators to obtain greater benefit from trading in three markets than exclusively in country 1. If there is only one or many homogeneously informed speculators, the MMs may correctly anticipate their information from their aggressive behaviours. This result implies that the shock to country 1 cannot propagate across the other two countries: $dP_1(2) = 0$ and $dP_1(3) = 0$.

In other words, a developed market plays an intermediate role in the contagious propagation process. Without this role, the shock to emerging country 1 cannot spread to emerging country 3. Although country 1 and country 3 are fundamentally uncorrelated, they are economically interconnected with country 2 (developed country).

This finding tells policymakers that in the short run, globalisation increases contagion because of an increasing number of informed speculators. However, in the long run, the dissipation of heterogeneous asymmetric informational advantage reduces contagion. Therefore, the finding suggests that financial capital controls are a temporary measure. Globalisation implies greater participation of common creditors (e.g., commercial banks, mutual funds, and hedge funds) in the developed and emerging markets, and the use of uniform and stringent information disclosure rules is an effective means of reducing vulnerability to contagion and the magnitude of contagion.

In conclusion, the heterogeneity of asymmetric information plays a key role in the model. The degree of heterogeneity and the number of informed speculators determine the susceptibility to contagion and the magnitude of contagion. This conclusion emphasises that "the model is best suited to capture the immediate, 'fast and furious' propagation of shocks across uncorrelated markets that, according to Kaminsky *et al.* (2003), take place over a matter of hours or days during most episodes of financial contagion" (Pasquariello (2007)).

Generally, many theories about financial linkages, such as Pasquariello's model, are generated from the point of view of the trader. When traders realise the occurrence of financial turmoil, they will react as soon as possible to maximise their benefits or minimise their losses; thus, their actions are "fast and furious". This behaviour is consistent with the argument of Moser (2003): traders in financial markets respond to a shock causing immediate corrections in asset prices.

In Pasquariello's model, two time periods (two intervals of three dates) are very short for the fundamentals and factor loadings (β) in the model to be taken as given and thus are sufficiently short to enable us to reach a meaningful comparative statics analysis. The MMs set the payoffs of multiple assets, but the asymmetric information about shocks that changes economic fundamentals remains unpublished. Due to monopoly and the less aggressive trading behaviours of heterogeneous informed speculators, the MMs are unable to distinguish them from all traders. Consequently, the MMs set equilibrium prices based solely on the order flow. At this point, they believe economic fundamentals are still the same as in the past. Otherwise, they are aware of changes in economic fundamentals and update their beliefs about equilibrium prices of the multi-asset basket. As a result, the equilibrium prices reflect the true values of multiple assets and the informed speculators' asymmetric informational advantage dissipates, contradicting those illustrated by the model. This finding implies that economic fundamentals are assumed to not change in the model. In accordance with economic theories, the relationships among markets that change due to trading behaviours of informed speculators and liquidity traders hold in the short run. "It is consistent with extant empirical evidence suggesting that, in proximity of most financial crises, conjoined asset price changes are often not only sudden and excessive but also short-lived" (Pasquariello (2007)).

It is a stylised fact about speculative markets (especially emerging markets) that better-informed traders (if large enough) use their informational advantage to influence prices instead of taking them as given. It would be "irrational" (Grinblatt and Ross (1985)) if we take prices as given because they respond to traders' actions. It is very difficult for informed speculators to hold their information advantage for a long time. To maximise their benefits in a short time and avoid doubts over whether others learn their information, informed speculators like to trade several assets through a series of trading transactions in a short time. Thus, the short-run relationships among markets substantially increase in the presence of contagion, which is consistent with a proposal by Bekaert *et al.* (2005): contagion is referred to as a correlation between markets that is higher than what is accounted for by economic fundamentals.

This relationship facilitates a more precise detection of contagion within short time horizons. Financial linkages as well as trade linkages already exist in tranquil periods. There is consensus that contagion occurs when the transmission mechanism changes during crisis periods. Many empirical works study financial contagion based on the idea that the relationships among markets should increase significantly from tranquil periods to crisis periods. In this manner, the definition of contagion in this study is similar. However, the relationships should occur in the short run, which was introduced early. Generally, as technology develops, information spreads across markets faster than ever before. An event that occurs in one country could be reported synchronously in another faraway country. Traders' reactions to a financial turmoil are "fast and furious". The repercussions of the turmoil in other markets are generated in a short time, such as hours or days. Accordingly, "contagion" is when the short-run relationships among markets increase significantly from tranquil periods to crisis periods. Correspondingly, a significant increase in long-run relationships among markets is not an indication of contagion and is thus called "interdependence".[32] This definition is consistent with those proposed by Masson (1998), Kaminsky *et al.* (2003), Bekaert *et al.* (2005), and Pasquariello (2007). This definition precisely captures the characteristics of responses of a diverse range of traders on an abnormal event, which leads to a significant increase in short-run relationships among markets. Our definition only considers the propagation mechanism that is estimated on traders' behaviours, particularly for liquidity traders and informed speculators. The results provide evidence to implicitly illustrate whether liquidity traders and informed speculators have the ability to cause financial contagion.

It is acknowledged that the economic and financial process is a combination of various traders' actions. Consequently, there are more than two time periods involved in an economic decision-making process, including the short run and the long run. Heterogeneous traders make decisions over different time horizons and/or operate at each moment on various time scales, with purposes ranging from speculation to investment activity.[33] Accordingly, the structure of the decision-making process, the strength of relationships among relevant variables, and even their characteristics differ by frequency, which is associated with time scale. High frequency is associated with a short time horizon, and low frequency corresponds to a long time horizon. With regard to the relationship between the two variables, high frequency shows the short-run relationship, whereas low frequency illustrates the long-run relationship.

The importance of dynamic adjustments was realised very early by economists. Unfortunately, there has been a historical lack of an appropriate tool to decompose economic time series into orthogonal components. Wavelets, as a relevantly new

[32]It simply increases the linkages across markets that already exist in tranquil periods. The linkages may be small in tranquil periods, but an abnormal event broadens and makes them visible.

[33]As mentioned earlier, the time scale is closely related to the investment horizon.

tool, are adopted to produce an orthogonal decomposition of some economic and financial variables by time scale in an attempt to unveil some previously ignored phenomena. A number of studies (Ramsey and Lampart (1998a,b), Kim and In (2003, 2005), Yousefi *et al.* (2005)) in these two decades consider this feature and clearly improve past empirical works using wavelets.

Wavelets are particular types of functions that are localised both in time and frequency domains. The wavelet transform utilises a basic function (called the wavelet function or mother wavelet) that is shifted (translated) and scaled (dilated or compressed) to capture features that are local in time and in frequency. Therefore, wavelets are good at managing the time-varying characteristics found in most real-world time series and are an ideal tool for studying non-stationary or transient time series while avoiding the assumption of stationarity. It is a stylised fact that financial crises lead to jumps in asset prices in financial markets. Due to the localised nature of wavelets, the local characteristics of a sequence of asset returns are maintained in orthogonal components.

Because a high-frequency relationship is classified as being in the short run, we use wavelets to orthogonally decompose the sequence of asset returns by small time scale, which is associated with high frequency. Many important economic indicators that reflect economic fundamentals are published monthly. They are closely watched indicators in financial markets, especially after the outbreak of a crisis. Economic fundamentals are believed to be relatively stable in a month. Accordingly, for our purposes, we are going to define the short run as "no greater than one month". The maximum decomposed level by wavelets is five, which corresponds to a frequency interval of $[\pi/32, \pi/16]$. This figure is associated with a time interval of $[32, 64)$ days, which is just over one month.[34] By definition, the time periods associated with the first five levels are linked to the "short run".

The 1997 Asian crisis is a good case for investigating contagion because the crisis is unexpected that only informed speculators have informational advantage about the shock. A total of 27 global representative markets including Thailand, which is the source of the crisis, are selected in the sample to be analysed. Given the stock market index as an important economic indicator, the sequences of every market's stock returns, which are differences in the natural logarithms of the stock market indexes, are decomposed orthogonally into five subseries in corresponding frequencies associated with time horizons. Next, the subseries of Thailand and other 26 markets

[34] The relationship between the decomposed level, scale, and frequency is described as follows: the jth decomposed level is related to the 2^{j-1} scale, where the frequency is in the interval $\left(\pi/2^j, \pi/2^{j-1}\right]$. Because the frequency ω has a relationship with the time horizon T: $\omega = 2\pi/T$, the jth decomposed level denotes the time interval $[2^j, 2^{j+1})$.

at the same level as inputs are used to perform further analysis to estimate short-run pair-wise relationships.

Forbes and Rigobon (2002) show that heteroscedasticity in market returns exaggerates simple correlation coefficients between two market returns, which leads to incorrect conclusions. In addition, because simple correlation coefficients only reflect static relationships, it is not appropriate to establish time-varying relationships during the crisis period using them. To overcome the problem of biased simple correlation coefficients and to capture the dynamic characteristics of correlations, we use a multivariate GARCH model to estimate short-run conditional correlation coefficients. The BEKK model, as a type of this model that guarantees positive conditional variances, reduces the computational demand, and improves the efficiency of parameters for a small size sample, is applied on subseries of 27 markets' stock returns to capture the dynamic feature of short-run correlations. Furthermore, because conditional correlations only measure contemporaneous relationships among markets and because a shock from one market may take time to spread across other markets, it is natural to adopt a Granger causality test, which is also applied on a subseries of 27 markets' stock returns, to assess short-run lead-lag relationships among markets. The combinations of both findings of short-run pair-wise contemporaneous correlations and lead-lag relationships between Thailand and 26 other markets identify whether this is financial contagion or merely interdependence.

Our empirical analysis provides new insight into the study of financial contagion from short-run relationships involving contemporaneous correlations and lead-lag relationships among markets perspective. This new insight reflects traders' reactions to a shock in a market, which are "fast and furious". It is believed that the results are more accurate for revealing financial contagion and offer some useful suggestions to policymakers and portfolio managers.

The empirical findings in this paper consistently show that the short-run pair-wise relationships consisting of contemporaneous correlations and lead-lag relationships between Thailand and the majority of markets in the sample do not increase significantly, particularly when the short run is related to a time interval of $[2, 4)$ days, $[4, 8)$ days, $[8, 16)$ days, or $[16, 32)$ days. Even when it is extended to a time interval of $[32, 64)$ days, which is longer than one month, a significant increase in the short-run relationship between Thailand and only one market is found. These findings implicitly refute the implications of Pasquariello's model. Because the heterogeneity of asymmetric information is a driver of financial contagion, his model implies significant increases in short-run relationships among markets. However, our findings provide evidence to reject this.

Accordingly, financial contagion does not arise in a majority of markets when a crisis erupts in a market. Because the short run is regarded as "no greater than one month", our results explain a phenomenon in which major negative events in global markets began to occur a month after the outbreak of the crisis. Moreover, because there are at most 2 markets out of 7 in the same region as Thailand for which the short-run relationships increase significantly, the view that contagion is regional is not supported. The economic changes reflected by stock market returns in this study are attributed to interdependence, which already exists during normal period by trade linkages and/or financial linkages. This finding implies that the changes are not avoided. However, many measures are recommended to minimise the impact of crisis in one market on other markets.

The remainder of the paper is organised as follows. Section 3 explains the reason we use the 1997 Asian crisis as the sample and describes the data. Section 4 introduces the methodologies, including brief wavelet analysis, conditional correlation analysis, and a Granger causality test. Section 5 presents the estimation results of the bivariate VAR-BEKK model, measures the conditional correlation coefficients at two different phases of the crisis, reports the results of the Granger causality test, and summarises the results of both methodologies. Section 6 concludes the empirical findings and provides some suggestions for policymakers and portfolio-makers.

4.3 Data and Descriptive Statistics

Kaminsky *et al.* (2003) conclude that anticipated crises are preceded by credit rating downgrades and widening interest spreads, whereas for unanticipated crises, the downgrades and widening spreads occur during or after crises. The authors indicate that the tequila crisis of Mexico in 1994, the Asian crisis in 1997, and the Russian default in 1998 were unanticipated. Table [C.2] reports capital flows in percent of GDP in selected Asian economies in the 1980s and 1990s. Although fluctuations exist, net private capital flows consisting of net direct investment and net portfolio investment in Thailand, Philippines, Malaysia, South Korea, and Indonesia from 1992 to 1996, hard-hit markets during the crisis, are positive with the exception of Singapore and Taiwan, which implies capital inflows to these markets. There is no sign of an onset of the Asian crisis before 1997. Accordingly, the 1997 Asian crisis is a good example to study the effectiveness of Pasquariello's model because in the sample, only informed speculators have informational advantage about the shock. The "fast and furious" actions of traders to the shock should induce the significant increases in short-run relationships between the shock-hit market and other markets, in the presence of contagion.

The 1997 Asian crisis begins with the collapse of the Thai Baht on July 2, 1997. To remove the impact of the 1998 Russian default (August) in other markets, the sample period is from January 1, 1996 to December 31, 1997. Accordingly, the tranquil period is from January 1, 1996 to July 1, 1997, and the remaining time period is referred to as the crisis period. The daily data on main stock market indices are collected from Datastream, including 8 East Asian emerging markets (Indonesia, South Korea, Malaysia, Philippines, Singapore, Taiwan, Thailand, and Hong Kong), 5 Latin American markets (Argentina, Brazil, Chile, Mexico, and Peru), 12 developed markets (Austria, Australia, Canada, France, Germany, Italy, Japan, Netherlands, Spain, Sweden, the UK, and the USA), and two other emerging markets (China and India). The preliminary step in managing the data is to obtain a market's stock return (r_t) that is identical to a daily log-difference of stock market closing prices. This is expressed by: $r_t = \ln(p_t) - \ln(p_{t-1})$, where p_t is the stock market closing quote at date t, and p_{t-1} is the quote at date $t-1$.

Table [C.3] reports descriptive statistics on daily stock market indices returns for 27 markets from January 1, 1996 to December 31, 1997. Due to differences in national holidays, bank holidays, and other holidays, data in some markets on some days are not available. To maintain consistency, the data are assumed to be the same as the previous trading data. The data are summarised for 519 observations in the sample, including 389 observations in the tranquil period and 130 observations in the crisis period. The highest average return is 0.0017 for MSCI Brazil of Brazil, whereas the lowest average return is -0.0024 for Bangkok S.E.T of Thailand, which is not surprising given that Thailand was the source of the 1997 Asian crisis. The most volatile data are observed for SEE of China with a standard deviation of 0.0241, whereas the least volatile data are recorded for the IGPA of Chile, with a standard deviation of 0.0057.[35] The skewness and kurtosis of the data indicate empirical distributions with heavy tails and sharp peaks at the centre compared to the normal distribution for most time series.

4.4 Methodology

It is a stylised fact that abnormal events, including financial crises, cause jumps in asset prices. Wavelets, which are called a "mathematical microscope", are an ideal tool to orthogonally decompose a time series into many subseries in a diverse

[35]The Bangkok S.E.T and SEE represent the Stock Exchange of Thailand and Shanghai Se Composite, respectively. MSCI Brazil, Bangkok S.E.T, SEE, and IGPA are the names of stock market indices, which are introduced in detail in Table [C.3].

range of frequencies that are associated with time horizons under the premise that the local characteristics of the time series remain. The next subsections present a brief introduction to wavelet functions, multiresolution analysis, and the structure of wavelet analysis. The bivariate VAR-BEKK model and Granger Causality test are demonstrated as well.

4.4.1 Wavelet Analysis

4.4.1.1 The Wavelets

Wavelets literally mean small waves because they have finite length and are oscillatory. Wavelets on a finite support begin at a point in time and then die out at a later point in time. Their localised nature enables them to be used in analysing episodic variations in the frequency composition of data and thus are referred as a "mathematical microscope". There are two different functions in the wavelet theory: wavelet function ($\psi(t)$) and scaling function ($\phi(t)$). According to the definition, the wavelet function $\psi_0(t)$ should satisfy the below two conditions:

$$1: \int_{-\infty}^{+\infty} \psi_0(t)\, dt = 0, \tag{4.19}$$

$$2: \int_{-\infty}^{+\infty} \psi_0(t-k)\psi_0(t-j)\, dt = \begin{cases} 1 & \text{if } k = j \\ 0 & \text{if } k \neq j \end{cases}, \tag{4.20}$$

and the scaling function $\phi_0(t)$ also should fulfil the two conditions:

$$1: \int_{-\infty}^{+\infty} \phi_0(t)\, dt = 1, \tag{4.21}$$

$$2: \int_{-\infty}^{+\infty} \phi_0(t-k)\phi_0(t-j)\, dt = \begin{cases} 1 & \text{if } k = j \\ 0 & \text{if } k \neq j \end{cases}, \tag{4.22}$$

where condition 2 of the wavelet and scaling functions guarantees that each of them is an orthonormal function.[36]

Suppose that there is a space $\mathcal{V}_0$. Under wavelet analysis, it can be decomposed orthogonally into many different subspaces: $\mathcal{W}_1$, $\mathcal{W}_2$, $\mathcal{W}_3$,$\cdots$. More precisely, the space $\mathcal{V}_0$, which is in the range of frequencies $[0, \pi]$, is decomposed orthogonally

[36]The subscript represents the decomposed level. The details are introduced as follows.

into wavelet subspace $\mathcal{W}_1$ and scaling subspace $\mathcal{V}_1$, which correspond to frequency bands $(\pi/2, \pi]$ and $[0, \pi/2]$, respectively. Because subspaces $\mathcal{W}_1$ and $\mathcal{V}_1$ are mutually orthogonal, the sum of $\mathcal{W}_1$ and $\mathcal{V}_1$ is $\mathcal{V}_0$: $\mathcal{W}_1 \oplus \mathcal{V}_1 = \mathcal{V}_0$. Then, $\mathcal{V}_1$ is decomposed into two mutually orthogonal subspaces $\mathcal{W}_2$ and $\mathcal{V}_2$, which belong to frequencies $(\pi/4, \pi/2]$ and $[0, \pi/4]$. We repeat this process of decomposing the scaling space J times. Finally, we obtain Jth subspaces $\mathcal{W}_J$ and $\mathcal{V}_J$. It is noticed that the wavelet function $\psi_j(t)$ constructs an orthonormal basis of subspace $\mathcal{W}_j$, whereas the scaling function $\phi_j(t)$ constitutes an orthonormal basis of subspace $\mathcal{V}_j$. This algorithm is called the Pyramid Algorithm, which reduces computation and improves efficiency. Because the subspaces at the same level are mutually orthogonal, from a mathematical perspective, it may be expressed as

$$
\begin{aligned}
\mathcal{V}_0 &= \mathcal{W}_1 \oplus \mathcal{V}_1 \\
&= \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{V}_2 \\
&\vdots \\
&= \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{W}_3 \oplus \cdots \oplus \mathcal{V}_J,
\end{aligned}
\tag{4.23}
$$

$\mathcal{W}_j$ and $\mathcal{V}_j$ are mutually orthogonal, which may be expressed as "$\mathcal{W}_j \perp \mathcal{V}_j$" and is called "lateral orthogonal". $\mathcal{W}_j$ and $\mathcal{W}_k$ $(j \neq k)$ are mutually orthogonal "$\mathcal{W}_j \perp \mathcal{W}_k$" as well, called "sequential orthogonal". However, $\mathcal{V}_j$ and $\mathcal{V}_k$ $(j \neq k)$ are not mutually orthogonal. In a word, scaling subspace $\mathcal{V}_j$ is not orthogonal across scales; orthogonality across scales comes from wavelet subspace $\mathcal{W}_j$. Here, j $(1 \leqslant j \leqslant J)$ is called the decomposed level and is related to scale (2^{j-1}), which is the inverse of the frequency band $((\pi/2^j, \pi/2^{j-1}])$ for wavelet subspace $\mathcal{W}_j$.

Because subspace $\mathcal{V}_1 \in \mathcal{V}_0$ where $\mathcal{V}_1$ is spanned by the basis scaling function $\phi_1(t)$ whereas $\mathcal{V}_0$ is constituted by the basis scaling function $\phi_0(t)$, there exists a relationship between these two functions. This relationship may be expressed as

$$\phi_1(t) = 2^{-1/2}\phi_0(2^{-1}t). \tag{4.24}$$

It is expected that the adjacent level scaling functions have this relationship as well. The scaling function $\phi_2(t) = 2^{-1/2}\phi_1(2^{-1}t)$, which may be replaced by $\phi_0(t)$: $\phi_2(t) = 2^{-2/2}\phi_0(2^{-2}t)$. By recursion of this procedure, the general version of $\phi_j(t)$ is achieved by

$$\phi_j(t) = 2^{-j/2}\phi_0(2^{-j}t). \tag{4.25}$$

In correspondence with the relationship between each-level scaling functions, wavelet functions at different levels have this relationship as well. Generally, wavelet function ψ_j at level j may be expressed as

$$\psi_j = 2^{-j/2}\psi_0(2^{-j}t). \tag{4.26}$$

4.4.1.2 Multiresolution Analysis

For a function $f(t)$ ($t = 0, \cdots, T-1$, where $T = 2^J$) in $\mathcal{V}_0$, the projection of $f(t)$ in $\mathcal{W}_1$ is $\Delta f_1(t)$, and the projection of $f(t)$ in $\mathcal{V}_1$ is $f_1(t)$. Because $\mathcal{V}_0 = \mathcal{W}_1 \oplus \mathcal{V}_1$, $f(t) = \Delta f_1(t) + f_1(t)$. Given the Pyramid Algorithm, $f(t)$ could be a linear combination of $\Delta f_j(t)$ ($1 \leqslant j \leqslant J$) and $f_J(t)$: $f(t) = \sum_{j=1}^{J} \Delta f_j(t) + f_J(t)$. This approach is defined as multiresolution analysis (MRA).

Because scaling function $\phi_1(t)$ and wavelet function $\psi_1(t)$ constitute the basis of subspace $\mathcal{V}_1$ and $\mathcal{W}_1$, respectively, $f(t)$ may also be decomposed as follows:

$$f(t) = \sum_k c_{1,k}\phi_{1,k}(t) + \sum_k d_{1,k}\psi_{1,k}(t), \tag{4.27}$$

where $k = 0, 1, \cdots, T/2-1$; $c_{1,k} = \langle f(t), \phi_{1,k}(t)\rangle = \int_{-\infty}^{\infty} f(t)\phi_{1,k}(t)\,dt$; $d_{1,k} = \langle f(t), \psi_{1,k}(t)\rangle = \int_{-\infty}^{\infty} f(t)\psi_{1,k}(t)\,dt$; $\phi_{1,k}(t) = \phi_1(t-k)$; and $\psi_{1,k}(t) = \psi_1(t-k)$. Nevertheless, $d_{1,k}$ are the amplitude coefficients of wavelet function $\psi_{1,k}$ for the projection of $f(t)$ on the subspace $\mathcal{W}_1$, whereas $c_{1,k}$ are the amplitude coefficients of scaling function $\phi_{1,k}$ for the projection of $f(t)$ on the subspace $\mathcal{V}_1$. Here, $\sum_k c_{1,k}\phi_{1,k}(t) = f_1(t)$ and $\sum_k d_{1,k}\psi_{1,k}(t) = \Delta f_1(t)$. In the Pyramid Algorithm, $f_j(t)$ ($1 \leqslant j < J$) is decomposed as in $f(t)$ until J. For example, $f_1(t) = \sum_k c_{2,k}\phi_{2,k}(t) + \sum_k d_{2,k}\psi_{2,k}(t) = f_2(t) + \Delta f_2(t)$, where $k = 0, 1, \cdots, T/4-1$. Therefore,

$$f(t) = \sum_{j=1}^{J} \Delta f_j(t) + f_J(t) = \sum_{j=1}^{J}\sum_k d_{j,k}\psi_{j,k}(t) + \sum_k c_{J,k}\phi_{J,k}(t), \tag{4.28}$$

where $k = 0, 1, \cdots, T/2^j - 1$; $d_{j,k}$ are the amplitude coefficients of wavelet function $\psi_{j,k}(t)$, $d_{j,k} = \int_{-\infty}^{\infty} f(t)\psi_{j,k}\,dt$; $c_{J,k}$ are the amplitude coefficients of scaling function $\phi_{J,k}(t)$, $c_{J,k} = \int_{-\infty}^{\infty} f(t)\phi_{J,k}\,dt$. Note that $\Delta f_j(t)$ belongs to the subspace $\mathcal{W}_j$ and thus is in the range of frequencies $(\pi/2^j, \pi/2^{j-1}]$ as well, whereas $f_j(t)$ is located in the subspace $\mathcal{V}_j$ and thus is in the frequency interval of $[0, \pi/2^j]$.

4.4.1.3 Structure of Wavelet Analysis

Generally, wavelet analysis decomposes a signal into shifted (translated) and scaled (dilated or compressed) versions of a wavelet function. All of the basis functions (wavelet functions or scaling functions) are self-similar, which is to say that they differ from one another only in the translations and the changes of scale. Wavelets are particular types of functions that are localised both in time and frequency domains, whereas each of the sines and cosines that compose the basis function of Fourier transform is itself a function of frequency-by-frequency basis. The wavelet transform utilises a basic function (called the wavelet function or mother wavelet), which is shifted (translated) and scaled (dilated or compressed) to capture features that are local in time and in frequency. Therefore, wavelets are good at managing the time-varying characteristics found in most real-world time series and are an ideal tool for studying non-stationary or transient time series while avoiding the assumption of stationarity.

Figure [2.2] provides a good explanation of the effectiveness of wavelets. The horizontal axis is time, whereas the vertical axis is scale (frequency). It is easily found that scale decreases further along the vertical axis. As the scale declines, it reduces the time support, increases the number of frequencies captured, and shifts towards higher frequencies, and vice versa. In the dyadic wavelet case, from one scale to the next scale, the bandwidth of frequency is halved and the temporal dispersion of wavelet is doubled. The frequency band of the original time series is $[0, \pi]$, and the time horizon is its time, such as 1 minute for per-minute data, 1 day for daily data, or 1 month for monthly data. Regarding scale 2^{j-1} $(1 \leqslant j \leqslant J)$, the corresponding frequency band is $(\pi/2^j, \pi/2^{j-1}]$ and the time horizon is 2^j times of the original time series, such as the time interval $[2^j, 2^{j+1})$ minutes for per-minute data, the time interval $[2^j, 2^{j+1})$ days for daily data, or the time interval $[2^j, 2^{j+1})$ months for monthly data. This treatment indicates that scale is related to time horizon and the inverse of frequency band.

Consequently, the wavelet transform provides good frequency resolution (and poor time resolution) at low frequencies and good time resolution (and poor frequency resolution) at high frequencies. It maintains a balance between frequency and time. From the top to the bottom of Figure [2.2], the frequency resolution improves and the time resolution worsens, which implies that wavelets provide a flexible framework for the time series. By combining several functions of shifted and scaled mother wavelet, the wavelet transform is able to capture all information contained in a time series and associate it with specific time horizons and locations in time.

The resulting time-frequency partition corresponding to the wavelet transform is long in time when capturing low-frequency events and thus has good frequency resolution for these events, and it is long in frequency when capturing high-frequency events and thus has good time resolution for events where the wavelet transform has the ability to capture events that are localised in time. The wavelet transform intelligently adapts itself to capture features across a wide range of frequencies, making it an ideal tool for studying non-stationary or transient time series.

It is widely accepted that financial crises lead to jumps in asset prices in financial markets. A signal or a time series may be decomposed orthogonally by time scale using wavelets while its local characteristics are maintained. As a result, it is appropriate to use wavelets to estimate short-run relationships among markets affected by financial crises. According to Equation (4.28), two time series $\{X(t)\}$ and $\{Y(t)\}$ as sequences of Thailand's stock returns and another market's stock returns are decomposed orthogonally into many components in various frequencies. The wavelets used in the decomposition are the Daubechies least asymmetric (LA) wavelet filter of width 8, which is recommended by Percival and Walden (2000) and is widely used in the literature on wavelet analysis (Kim and In (2003), Gallegati (2008)). Components at the first five decomposed levels contain information within time intervals of $[2, 4)$ days, $[4, 8)$ days, $[8, 16)$ days, $[16, 32)$ days, and $[32, 64)$ days. The relationship between the components of two time series at the same level is associated with the corresponding time period.

In this case, it is only decomposed into five levels. On the one hand, the sample size and the filter width restrict the decomposed level. The maximum decomposed level in this sample with the selected filter is six; therefore, the data decomposed at a further level are biased. On the other hand, the fifth level is associated with periods over one month (time interval of $[32, 64)$ days), which is the longest time interval in the first five levels. Many important economic indicators (e.g., CPI, PPI, RSI, CCI, CES) that reflect economic fundamentals are published monthly.[37] Traders in financial markets make the interpretation that economic fundamentals are relatively consistent within a month during the crisis. Regarding economic theories, it is meaningful that the components at the first five levels are used to describe short-run relationships, especially for the components at the first four levels. This finding is consistent with Moser (2003)'s view that traders in financial markets respond to a shock, causing immediate corrections in asset prices, and the argument of Kaminsky *et al.* (2003) that the propagation of a shock to a specific market across markets should be fast and furious and should evolve in a matter of hours and days. This finding is also consistent with that implied by the near simultaneity of crisis, which Kindleberger (1985) believes to be unexplainable by trade linkage. In a word, $\{X(t)\}$

[37] CPI, PPI, RSI, CCI, and CES represent the consumer price index, producer price index, retail sales index, consumer confidence index, and current employment statistics, respectively.

and $\{Y(t)\}$ are decomposed as follows:

$$\begin{aligned} X(t) &= \sum_{j=1}^{5} \Delta X_j(t) + X_5(t), \\ Y(t) &= \sum_{j=1}^{5} \Delta Y_j(t) + Y_5(t), \end{aligned} \tag{4.29}$$

where $\Delta X_j(t)$ and $\Delta Y_j(t)$ $(1 \leqslant j \leqslant 5)$ in the same level are adopted to conduct a further analysis in the next subsections.

4.4.2 Conditional Correlation Analysis

Cross-market correlations are widely used to investigate financial contagion in the literature. However, Forbes and Rigobon (2002) state that heteroskedasticity in market returns increases the correlation coefficient between two markets returns because the increase in the correlation coefficient is attributed to the high volatility in the crisis period even when the relationship between two markets does not change. Thus, cross-market correlation analysis misleads us to infer incorrect results. After adjusting for heteroskedasticity in market returns, the correlation coefficients during the crisis period are smaller, and no contagion is found. However, Corsetti *et al.* (2001) indicate that this methodology is based on a single-factor model that reduces the correlation coefficients. Furthermore, in contrast to time-invariant simple correlation coefficients, it is better to use time-varying correlation coefficients to capture the cross-market relationships that vary with sustained shocks.

Regarding time-varying correlation coefficients and heteroskedasticity in market returns, it is natural to take GARCH models into consideration. Univariate models only estimate a structure that the conditional variance of series X is an explicative variable in the conditional variance of series Y, or vice versa, but ignore the possibility of an interaction between both variances and do not consider the covariance between both series. To address these issues efficiently, multivariate GARCH models were developed to estimate interactions among variances of N different time series together. Based on maximum likelihood (ML), variances and covariances of the N series are simultaneously estimated. An $N \times 1$ vector stochastic process $\{y_t\}$ is expressed as follows:

$$y_t = \mu_t(\theta) + \varepsilon_t, \tag{4.30}$$

where $\mu_t(\theta)$ is the conditional mean vector: $\mu_t(\theta) = E(y_t|I_{t-1})$; I_{t-1} is information about y_t generated until time $t-1$; θ is a vector of finite parameters; and

$$\varepsilon_t = H_t^{1/2}(\theta) z_t, \tag{4.31}$$

where z_t is an $N \times 1$ random vector whose mean is the zero vector and variance is the identity matrix (I_N). $H_t(\theta)$ is an $N \times N$ positive definite matrix: the diagonal elements are conditional variances, and the off-diagonal elements are conditional covariances. In practice, the issues of multivariate GARCH models involve how to ensure that H_t is positive definite and reduce the computational demand of parameters. It is necessary to impose some restrictions on models and thus develop many types of multivariate GARCH models, including VEC models, BEKK models, constant conditional correlation (CCC) models, and dynamic conditional correlation (DCC) models.

In this study, we use the BEKK model to capture the dynamic feature of correlations and then detect financial contagion. The BEKK model is a special type of VEC model. The number of parameters in the VEC model is $N(N+1)(N(N+1)+1)/2$, whereas the number of parameters in the BEKK$(1,1,K)$ model is $2KN^2 + N(N+1)/2$. Compared with the VEC model, the BEKK model reduces the computational demand and improves the efficiency of parameters for a small size sample. Furthermore, it guarantees that H_t is positive definite. However, it is seldom used when the number of series (N) is larger than 3 or 4 due to high computational demand. In this paper, we only consider the pair-wise correlation between Thailand and another market. Therefore, the BEKK model is a case of bivariate GARCH models and avoids the issue of heavy computation. The bivariate BEKK$(1,1,K)$ model is defined as follows:

$$H_t = C'C + \sum_{k=1}^{K} A_k' \varepsilon_{t-1} \varepsilon_{t-1}' A_k + \sum_{k=1}^{K} G_k' H_{t-1} G_k, \tag{4.32}$$

where A_k and G_k are 2×2 matrices, C is a 2×2 upper triangular matrix, and the parameter K determines the general representation of a BEKK-type model.[38] An identification problem arises when $K > 1$ because a representation of the model could have several different groups of parameters. As a consequence, K is usually defined as 1 in practice. Due to the heavy computation in the full BEKK$(1,1,1)$

[38]To increase the generality of the BEKK model, Engle and Kroner (1995) present some propositions to govern K.

model, the diagonal version is applied here, which is expressed as

$$H_t = C'C + A_1'\varepsilon_{t-1}\varepsilon_{t-1}'A_1 + G_1'H_{t-1}G_1, \tag{4.33}$$

where

$$H_t = \begin{bmatrix} h_{11,t} & h_{12,t} \\ h_{12,t} & h_{22,t} \end{bmatrix} \quad \varepsilon_{t-1} = \begin{bmatrix} \varepsilon_{1,t-1} & \varepsilon_{2,t-1} \end{bmatrix}',$$

$$C = \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix} \ A_1 = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} \end{bmatrix} \ G_1 = \begin{bmatrix} g_{11} & 0 \\ 0 & g_{22} \end{bmatrix}.$$

It is obvious that the conditional covariance $h_{12,t}$ is a linear combination of lagged cross-products of errors $\varepsilon_{12,t-1}$ and lagged conditional covariance $h_{12,t-1}$. In this case, 1 represents Thailand, and 2 represents another market in the paper. The conditional correlation is measured by

$$\rho_{12,t} = \frac{h_{12,t}}{\sqrt{h_{11,t}}\sqrt{h_{22,t}}}, \tag{4.34}$$

where $\rho_{12,t}$ is the conditional correlation coefficient between market 1 (Thailand) and market 2 (another market); $h_{11,t}$ is the conditional variance of Thailand's stock returns; $h_{22,t}$ is the conditional variance of market 2 returns; and $h_{12,t}$ is the conditional covariance of Thailand and market 2 returns.

As in the univariate case, the parameters of the diagonal BEKK$(1, 1, 1)$ model are estimated by maximising the Gaussian log-likelihood function. The log-likelihood of a sample is given by

$$L = \sum_{t=1}^{T} l_t = -\frac{NT}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\ln(|H_t|) - \frac{1}{2}\sum_{t=1}^{T}\varepsilon_t' H_t^{-1}\varepsilon_t, \tag{4.35}$$

where l_t is the contribution of a single observation to the log-likelihood. The variance equation is illustrated above, and the mean equation is generated on a vector autoregressive (VAR) model. The reason for this treatment is that current market returns are affected not only by past market returns but also by another market past returns, especially in the crisis period. Based on the purpose of our study, the VAR model is a bivariate one as well:

$$r_t = \alpha_0 + \sum_{j=1}^{p} \alpha_j r_{t-j} + \varepsilon_t, \tag{4.36}$$

where market returns $r_t = [r_{1,t}, r_{2,t}]'$, errors $\varepsilon_t = [\varepsilon_{1,t}, \varepsilon_{2,t}]'$ with $\varepsilon_t | I_{t-1} \sim N(0, H_t)$, and H_t follows the diagonal BEKK$(1,1,1)$ process. The lag length p is determined by the Akaike information criterion (AIC) and Lagrange multiplier test (LM). Based on Equations (4.36) and (4.33), the positive definite matrix H_t is established.

The subseries of Thailand's stock returns and market 2 stock returns ($\Delta X_j(t)$ and $\Delta Y_j(t)$, $1 \leqslant j \leqslant 5$) at the same level are regressed on Equations (4.36), (4.33), and (4.34). The specific short-run conditional correlations are generated, which are associated with time intervals of $[2, 4)$ days, $[4, 8)$ days, $[8, 16)$ days, $[16, 32)$ days, and $[32, 64)$ days, respectively.

4.4.3 Granger Causality Test

The conditional correlations in the above subsection are used to measure the pairwise contemporaneous relationships between Thailand and other markets. Because the repercussions of the shock to Thailand on other markets may take time, the relationships between Thailand and other markets have a lag effect. It is necessary to analyse the lead-lag relationships between them. The Granger causality test is able to investigate the lead-lag relationship between two time series. In the paper, this test is used to identify whether a shock to one market has an impact on another market. If so, the market is causally linked to another, which implies that the shock propagates to that market. From the point of view of numerical analysis, a time series $\{r_{1,t}\}$ for the shock-hit market should precede a time series $\{r_{2,t}\}$ for another market. This phenomenon is called "Granger causality" and was proposed by Granger (1969). Notice that Granger causality does not capture a true causality but instead reflects that one endogenous variable precedes another endogenous variable. Because there is a problem in distinguishing whether the relationship of two sequences entails a lead or a lag, the Granger causality is double-directional. This test is easily incorporated into a VAR model (Equation (4.36)):

$$r_t = \alpha_0 + \sum_{j=1}^{p} \alpha_j r_{t-j} + \varepsilon_t, \tag{4.37}$$

where market returns $r_t = [r_{1,t}, r_{2,t}]'$, errors $\varepsilon_t = [\varepsilon_{1,t}, \varepsilon_{2,t}]'$ with $\varepsilon_t | I_{t-1} \sim N(0, H_t)$. In the standard Granger causality test, H_t is assumed to be a diagonal matrix with constants, which implies that the errors are independent and identically distributed. However, it contradicts the set of H_t in the conditional correlations that follows the diagonal BEKK$(1,1,1)$ process. To maintain consistency, investigating whether the H_t is diagonal is necessary prior to performing the Granger causality test. The null hypothesis H_0 that some specific parameters of the diagonal BEKK$(1,1,1)$ model are jointly zeros ($c_{12} = a_{11} = a_{22} = g_{11} = g_{22} = 0$) is proposed to be tested. The rejection of H_0 indicates the covariance matrix H_t is generated by the diagonal BEKK$(1,1,1)$ model. Otherwise, the covariance matrix H_t is diagonal with constant elements. In this study, we find the null hypothesis H_0 is rejected in every case.[39] Accordingly, the parameters in Equation (4.37) are generated by the bivariate VAR-BEKK$(1,1,1)$ model.

With respect to the Granger causality test, if it rejects the zeros coefficients of lagged r_2 values on r_1 that $\alpha_{1,12} = \alpha_{2,12} = \cdots = \alpha_{p,12} = 0$ and fails to reject the zeros coefficients of lagged r_1 values on r_2 that $\alpha_{1,21} = \alpha_{2,21} = \cdots = \alpha_{p,21} = 0$, the increase in $r_{2,t}$ increases $r_{1,t}$, so the Granger causality from $\{r_{2,t}\}$ to $\{r_{1,t}\}$ is unidirectional. If it rejects $\alpha_{1,21} = \alpha_{2,21} = \cdots = \alpha_{p,21} = 0$ and fails to reject $\alpha_{1,12} = \alpha_{2,12} = \cdots = \alpha_{p,12} = 0$, $r_{1,t}$ precedes $r_{2,t}$. Consequently, $\{r_{1,t}\}$ Granger causes $\{r_{2,t}\}$. Notice that the coefficients in the null hypothesis must always be the off-diagonal elements of α_j, which capture the effect of one lagged variable on a different variable.

It is noted that $\{r_{1,t}\}$ and $\{r_{2,t}\}$ should be stationary–that is, $I(0)$. If they are nonstationary–that is, the difference (Δ) is used to convert them into stationary series, which are available for the Granger causality test. They are expressed by

$$\Delta r_t = \alpha_0 + \sum_{j=1}^{p} \alpha_j \Delta r_{t-j} + \varepsilon_t. \tag{4.38}$$

However, MacDonald and Kearney (1987) note that this procedure is not correct if these two time series are cointegrated. Moreover, Engle and Granger (1987) note that the causality test is misspecified in the presence of cointegration and thus include an error correction term in the model to resolve this issue. First, the error terms ϵ_t are estimated when two time series $\{r_{1,t}\}$ and $\{r_{2,t}\}$, which are nonstationary and cointegrated, are regressed on equation $r_{1,t} = \theta r_{2,t} + \epsilon_t$. Second, the estimators of error terms ($\hat{\epsilon}_t$) as error correction terms (ECT) are added in Equation (4.38), which may be expressed as follows:

$$\Delta r_t = \alpha_0 + \sum_{j=1}^{p} \alpha_j \Delta r_{t-j} + \lambda \beta' r_{t-1} + \varepsilon_t, \tag{4.39}$$

where ECT $= \lambda\beta' r_{t-1}$, $\lambda = [\lambda_1, \lambda_2]'$ and $\beta = [1, -\theta]'$. In Equation (4.39), cointegration between $\{r_{1,t}\}$ and $\{r_{2,t}\}$ does not cause "spurious Granger causality".

[39] Due to the space limitations, the relative data on this Wald test are not attached to the paper.

Consequently, before testing Granger causality, the first step is to use the Augmented Dickey-Fuller (ADF) test to examine whether the two time series are stationary and to investigate whether they are cointegrated using the Johansen cointegration test. If they are stationary, Equation (4.37) is used to test Granger causality. If they are nonstationary and not cointegrated, the stationary outputs from differencing them are applied on Equation (4.38) to test Granger causality. If they are nonstationary and cointegrated, they are converted to stationary by difference, and the outputs are used to detect Granger causality by Equation (4.39). The lag length p is determined by the AIC and LM criteria.

Generally, the Granger causality test in this study, like the analysis of conditional correlation, is based on a bivariate VAR model. It is adopted to investigate a lead-lag relationship between the shock-hit market and another market, which is an indicator of financial contagion. The rejection of the null hypothesis indicates that an endogenous variable Granger causes another endogenous variable, which shows existence of a lead-lag relationship between two variables. Otherwise, there is no Granger causality between two variables, which is identical to no lead-lag relationship. The standard Wald test is used to examine the hypothesis. By definition, if two variables cointegrate, there must be Granger causality in at least one direction.

The examination is implemented separately in both time periods, including the tranquil period and the crisis period. Suppose $\{r_{1,t}\}$ is a sequence of Thailand's stock returns and $\{r_{2,t}\}$ is a time series of another market's stock returns. $\{r_{1,t}\}$ Granger causing $\{r_{2,t}\}$ only in the crisis period implies the shock from Thailand propagates to the other market, which is an indication of contagion; $\{r_{1,t}\}$ Granger causing $\{r_{2,t}\}$ and $\{r_{2,t}\}$ Granger causing $\{r_{1,t}\}$ simultaneously in the crisis period represent an interaction of two markets, which is not an indication of contagion but merely interdependence; and $\{r_{1,t}\}$ Granger causing $\{r_{2,t}\}$ in the crisis period as well as in the tranquil period indicates the existence of a lead-lag relationship between two markets throughout the time period. However, contagion is elusive because it is not certain that $\{r_{1,t}\}$ Granger causing $\{r_{2,t}\}$ in the crisis period attributes to the shock spreading to that market; rather, the causation in the crisis period could merely be a continuation of the relationship during the tranquil period. The other cases of causation do not provide evidence of contagion.

The subseries of Thailand's stock returns and another market's stock returns ($\Delta X_j(t)$ and $\Delta Y_j(t)$, $1 \leqslant j \leqslant 5$) at the same level as well as applied on the conditional correlation analysis are used to test Granger causality. The specific short-run lead-lag relationships are obtained, which are associated with time intervals of $[2, 4)$ days, $[4, 8)$ days, $[8, 16)$ days, $[16, 32)$ days, and $[32, 64)$ days, respectively.

In conclusion, two different methodologies are proposed to investigate financial contagion in this paper. The conditional correlation is analysed from the perspective of pair-wise contemporaneous relationships between Thailand and other markets, and the Granger causality test is based on the pair-wise lead-lag relationships between Thailand and other markets. These tests investigate financial contagion from two different aspects and provide evidence to support our arguments.

4.5 Empirical Findings

4.5.1 Estimation of the Bivariate VAR-BEKK Model

As financial contagion is linked with "fast and furious" spreading, investigating the short-run relationships between the shock-hit market and other markets is more reasonable for identifying whether it is a contagion. Wavelets could orthogonally decompose the time series over various frequencies that are associated with time horizons. Because many important economic indicators which reflect economic fundamentals are published monthly, the short run is regarded as "no greater than one month". Under the orthogonal decomposition on the sequence of stock market returns by wavelets, the short-run pair-wise correlations between Thailand and other 26 markets are analysed based on five groups: at the first level, associated with a time interval of $[2, 4)$ days; at the second level, associated with a time interval of $[4, 8)$ days; at the third level, associated with a time interval of $[8, 16)$ days; at the fourth level, associated with a time interval of $[16, 32)$ days; and at the fifth level, associated with a time interval of $[32, 64)$ days. The coefficients in the variance equations are reported in Tables [C.4], [C.5], [C.6], [C.7], and [C.8].[40] Figures [C.1], [C.2], [C.3], [C.4], and [C.5] show pair-wise conditional correlation coefficient series between the stock returns of Thailand and the other 26 markets during the time period $1996 - 1997$. The dotted line, associated with the day 07/02/1997, divides the entire time period into a tranquil period and a crisis period.

It appears that almost pair-wise conditional correlation coefficients at the first three levels fluctuate more widely than at the fourth level in the range of $[-1, 1]$. Moreover, they oscillate in a smaller range at the fifth level. In comparison with in the tranquil period, all of the conditional correlation coefficients appear to have no substantial difference in the crisis period. As a result, this finding roughly indicates that the short-run contemporaneous relationships between Thailand and the other 26 markets do not change significantly, which implies no contagion but merely

[40]Because the lag length p in every case is different, the coefficients in the mean equations are not attached here.

interdependence during the crisis period.

It is noted that in either the tranquil period or the crisis period, the fluctuations of the most conditional correlation series at each level are large, which implies that they are volatile. This finding suggests that adopting constant correlation coefficients to measure the vulnerability of contagion is not appropriate and is unable to capture the dynamic characteristics of correlation coefficients. Consequently, it is reasonable to use conditional correlation coefficients to study contagion.

4.5.2 Statistical Analysis of Conditional Correlation Coefficients in Two Different Phases of the Crisis

As illustrated by the graphics of the conditional correlation series, there appears to be no contagion. Next, the statistical analysis of conditional correlations is introduced to provide more convincing evidence. Because the sample time period consists of a tranquil period and a crisis period, a dummy variable ($DM_{1,t}$) is used to distinguish between them. The dynamic feature of conditional correlations associated with two different phases of the crisis is captured by a model, which is expressed by

$$\rho_{12,t} = c + \sum_{k=1}^{p} \lambda_k \rho_{12,t-k} + \gamma_0 DM_{1,t} + \varepsilon^*_{12,t}, \tag{4.40}$$

where the errors have a time-varying variance $h^*_{12,t}$ with $\varepsilon^*_{12,t}|I_{t-1} \sim N(0, h^*_{12,t})$; the optimal lag length p in Equation (4.40) is determined by the AIC and LM criteria; and the dummy variable $DM_{1,t}$ is set to 0 from time period 01/01/1996 to 07/01/1997 and is equal to 1 from time period 07/02/1997 to 12/31/1997. Because the mean equation has been associated with a dummy variable, the variance equation is a GARCH(1,1) process associated with the dummy variable $DM_{1,t}$ as well:

$$h^*_{12,t} = A_0 + A_1 \varepsilon^{*2}_{12,t-1} + B_1 h^*_{12,t-1} + \gamma_1 DM_{1,t}. \tag{4.41}$$

The significance of the estimated parameters on the dummy variable (γ_0 and γ_1) indicates the structure change in mean or/and variance shifts of the correlation coefficients in two different phases of the crisis, which implies that the shock significantly affects the cross-market linkages.

Here, the dummy variable $DM_{1,t}$ is used to distinguish between two phases: a tranquil period and a crisis period, providing more reliable evidence from a statistical perspective. Because the optimal lag length p in the mean equation (4.40) varies over different pair-wise correlations, Tables [C.9], [C.10], [C.11], [C.12], and [C.13] only list parameters and the corresponding $z-$statistic values (in parentheses) of the constant and the dummy variable $DM_{1,t}$ in the mean equation (4.40) and of all variables in the variance equation (4.41) for the conditional correlation series at the first level, at the second level, at the third level, at the fourth level, and at the fifth level, which are associated with time intervals of $[2, 4)$ days, $[4, 8)$ days, $[8, 16)$ days, $[16, 32)$ days, and $[32, 64)$ days, respectively. $*$ and $**$ represent statistical significance at the 5% and 10% levels, respectively.

At the first level, which is related to a time interval of $[2, 4)$ days, the dummy variable $DM_{1,t}$ is significant only in the conditional correlations between Thailand and the USA. This variable denotes only the structure of its mean equation changes, and its conditional correlations in the crisis period are significantly different from those in the tranquil period, whereas the other conditional correlations are not significantly distinct in the two periods. In particular, the parameter of its dummy variable $DM_{1,t}$ is negative, which indicates that the conditional correlations between Thailand and the USA significantly decrease from the tranquil period to the crisis period. Moreover, the dummy variables $DM_{1,t}$ in the pair-wise conditional correlations for the Thailand-Peru and Thailand-Korea pairs at the second level are statistically significant, and the corresponding parameters are positive, indicating that their correlations associated with the time interval of $[4, 8)$ days significantly increase from the tranquil period to the crisis period.

With respect to the coefficients and statistical significance of $DM_{1,t}$ in the mean equations for the conditional correlation series at the third level, which corresponds to the $[8, 16)$ day time interval, the pair-wise correlations for the Thailand-Austria, Thailand-Japan, Thailand-Hong Kong, and Thailand-Taiwan pairs increase significantly, whereas the pair-wise correlations for the Thailand-Canada, Thailand-Netherlands, Thailand-UK, Thailand-USA, and Thailand-Singapore pairs decrease significantly. Furthermore, the pair-wise correlations for the Thailand-Chile, Thailand-Mexico, Thailand-Italy, and Thailand-Sweden pairs at the fourth level, which is associated with the $[16, 32)$ day interval, increase significantly. At the fifth level, which is related to the $[32, 64)$ day interval, we only find significant increases in the pair-wise correlations for Thailand-Australia and significant decreases in the pair-wise correlations for the Thailand-Argentina, Thailand-Brazil, Thailand-German, Thailand-Singapore, and Thailand-Hong Kong pairs from the tranquil period to the crisis period.

To summarise, the short-run international stock markets' comovements are analysed based on five categories: the conditional correlation coefficients at the first level, at the second level, at the third level, at the fourth level, and at the fifth level, which are associated with time intervals of $[2, 4)$ days, $[4, 8)$ days, $[8, 16)$ days,

$[16, 32)$ days, and $[32, 64)$ days, respectively. Only in 2 cases at the second level, 4 cases at the third level, 4 cases at the fourth level, and 1 case at the fifth level do the pair-wise correlations significantly increase from the tranquil period to the crisis period, and no case is reported to show a significant increase in the 26 pairs of relationships at the first level. In comparison, 1 case at the first level, 5 cases at the third level, and 5 cases at the fifth level show that pair-wise conditional correlations between Thailand and other markets decrease significantly. Few cases suggest the presence of contagion among the total 26 cases of pair-wise correlations at each level. The highest number at the five levels is only 4 at the third or fourth level, which only occupies 15.38 percent in the total sample. Consequently, the findings provide supportive evidence of no contagion in the majority of markets. From the short-run contemporaneous correlations point of view, it is argued that only interdependence is found in the crisis period as well as in the tranquil period for the majority of markets. This finding implies that the shock to Thailand propagates across markets through an existing channel in the tranquil period and that no new channel is built that enables the spreading of shocks across markets.

It is of interest to study the volatility of pair-wise conditional correlation series. According to the parameters of the dummy variable $DM_{1,t}$ in the variance equation (4.41) for the conditional correlation series, 14 cases at the first level, 3 cases at the second level, 9 cases at the third level, 4 cases at the fourth level, and 7 cases at the fifth level show that the volatility in conditional correlation coefficients significantly increases from the tranquil period to the crisis period, whereas 3 cases at the third level, 9 cases at the fourth level, and 11 cases at the fifth level illustrate that the volatility in conditional correlation coefficients significantly decreases from the tranquil period to the crisis period.

Compared with the dummy variable in the mean equation, there are more cases presenting a statistically significant dummy variable in the variance equation, including positive and negative values. To be specific, among the all 26 conditional correlation series, 53.85% at the first level, 11.54% at the second level, 46.15% at the third level, 50.00% at the fourth level, and 69.23% at the fifth level show that the volatility of conditional correlation series changes significantly. The evidence suggests that correlation coefficients change substantially when a crisis occurs in a market, and this variability could be prolonged for a significant period of time. Accordingly, the estimates and the inferences could be misleading if constant correlation coefficients are used in the model that takes the crisis into account.

4.5.3 Results of Granger Causality Test

Before performing the Granger causality test, the time series as inputs should be examined regardless of whether they are stationary. According to the ADF test, all the time series are stationary-$I(0)$, and thus Equation (4.37) is adopted in the paper. The AIC and LM criteria indicate the optimal lag length p in the bivariate VAR model. Because the optimal lag length p is different in each case, those are not reported here. Next, the standard Wald test is implemented. If the absolute value of χ^2-statistic is larger than its absolute value for the significance levels, including 5% and 10%, the null hypothesis that each coefficient of the lagged values of a variable on another variable is zero should be rejected. It is interpreted that the former endogenous variable Granger causes the latter one, which is called "Granger causality". Tables [C.14], [C.15], [C.16], [C.17], and [C.18] report the χ^2-values of the Wald test, the corresponding probability values, and the results of the Granger causality test in the tranquil period and in the crisis period at the five levels. Furthermore, $*$ and $**$ reflect the statistical significances of the χ^2-values for Granger causality test at the 5% and 10% levels, respectively.

In general, one market's stock returns preceding another market's stock returns is interpreted as the existence of a lead-lag relationship between two markets. As mentioned earlier, if financial contagion exists, Thailand, as the source of the crisis, has the ability to cause instability in other markets. The market's stock return as an indicator may reflect this phenomenon. Because Thailand's stock returns may affect other markets' stock returns, the lagged values of Thailand's stock returns impose the impact on the current values of other markets' stock returns in the crisis period. According to the definition of financial contagion, whereby the short-run relationship increases significantly, only Thailand's stock returns Granger causing another market's stock returns in the crisis period indicates contagion. In the other cases, contagion does not exist, which was explained in detail in the above section.

The appearances of a causality pattern are mixed in both periods. In the tranquil period, the highest number of causality pattern cases, 51, is found at the fifth level, whereas the lowest number of causality pattern cases, 21, is reported at the fourth level. In the crisis period, the majority are at the fifth level, with 48 cases, whereas the minority are at the second level, with 30 cases. The crisis-contagion is interpreted such that the causality pattern that is absent in the tranquil period emerges in the crisis period. Regarding the causality pattern only in the crisis period as an indication of contagion, no case illustrates the propagation of the crisis from Thailand across other markets at all five levels.

As a result, no contagion is indicated in all global representative markets based on the lead-lag relationships associated with time intervals of $[2, 4)$ days, $[4, 8)$ days, $[8, 16)$ days, $[16, 32)$ days, and $[32, 64)$ days (over one month). Turning to the short-run lead-lag relationships, the findings claim that the propagation of the crisis from Thailand to all markets is attributed as occurring through an existing channel in normal time rather than through a new channel, which was initially thought to be established in the crisis period.

4.5.4 Summary of Findings in Conditional Correlation Analysis and Granger Causality Test

The conditional correlation analysis and Granger causality test investigate financial contagion from two different aspects: short-run pair-wise contemporaneous relationships and lead-lag relationships between Thailand and other 26 markets. When the findings on contemporaneous relationships and on lead-lag relationships at the same level are combined, zero cases at the first level, 2 cases at the second level, 4 cases at the third level, 4 cases at the fourth level, and 1 case at the fifth level denote significant increases in the short-run pair-wise relationships between Thailand and those markets. According to the definition of financial contagion, the evidence does not support the view of contagion in the majority of markets. Even at the third or fourth level, where the most cases are found, the minority of markets show the existence of contagion in 4 markets. As a result, it is argued that there is no contagion and only interdependence in the majority of markets when the short run is associated with a time interval of $[2, 4)$ days, $[4, 8)$ days, $[8, 16)$ days, or $[16, 32)$ days. Even when the short run is increased to a time interval of $[32, 64)$ days (over one month), it is found that contagion arises only in one global representative market in the sample.

It is of interest to study contagion in the same region with Thailand. For the other 7 East Asian emerging markets in the sample, only 1 case at the second level and 2 cases at the third level are found to support the view of contagion, with 14.29% at the second level and 28.57% at the third level. However, 1 case at the third level and 2 cases at the fifth level show significant decreases in the relationships between Thailand and those markets from the tranquil period to the crisis period. Consequently, these findings refute the argument that contagion is usually associated with "regional" markets; the other markets in the same region are easily affected by a crisis that erupts in a market. In addition, only interdependence is identified for the majority of markets in the same region. Markets in the same region have a strong economic linkage in normal time because of similar economic fundamentals.

Consequently, the shock to Thailand propagates across the majority of markets in the region through an existing channel, and no new channel is built during this process for the majority of markets in the region.

4.6 Conclusion

We examine financial contagion based on wavelets, from a new insight: short-run relationships among markets. These relationships capture the characteristics of international stock markets' comovements during a crisis. Therefore, contagion and international stock markets' comovements during a crisis can be better understood in accordance with the frequency-domain-based methodology. Thus, researchers and policymakers can propose appropriate measures to reduce the negative impact of a crisis in an individual market, and portfolio-makers can reduce risks when they invest in global markets.

Based on the analysis of short-run pair-wise relationships between Thailand and the other 26 markets from the perspective of contemporaneous correlations and lead-lag relationships, our findings indicate that there is no contagion and only interdependence in the majority of markets, especially for markets in the same region as Thailand, the source of the crisis. The changes in short-run pair-wise relationships between Thailand and these markets are attributed to a normal transmission mechanism, which already exists in the tranquil period. Regarding interdependence between Thailand and these markets at the level of financial linkage, these markets' asset prices change when a shock hits Thailand and devalues its assets. Consequently, the shock to Thailand leads to repercussions in these markets. An extremely negative and abnormal event gives the illusion that a shock to Thailand, such as a disease, spreads across markets and is thus called "contagion". However, our findings indicate no new propagation mechanism established in these markets. The propagation of the shock to Thailand across these markets is only a continuation of a normal transmission mechanism in the tranquil period.

The shock enlarges the transmission mechanism between Thailand and the majority of markets and makes their interdependence more visible. For example, it is very difficult to believe that Thailand is correlated to Argentina from a macroeconomic perspective; there is almost no bilateral trade between two markets. However, as international financial markets integrate and capital flows across markets are relatively easier, Thailand's and Argentina's assets are held in traders' (e.g., mutual funds, hedge funds) portfolios together. Therefore, the relationship between two markets is enhanced at the financial linkage level. This link is always ignored, so an increase in the short-run relationship between two markets may be attributed to a new propagation mechanism. In fact, the responses of liquidity traders and informed speculators to the shock in Thailand drive the impact on other markets and thus change the short-run relationships between Thailand and other markets.

Table [C.19] lists the major events in the world from 07/02/1997 to 12/31/1997, following the depreciation of Thailand's Baht on July 2, 1997. Although some negative events are reported in July, the majority of events begin to occur a month after the outbreak of the crisis. The changes in short-run relationships consisting of contemporaneous correlations and lead-lag relationships between Thailand and the majority of markets indicate no contagion and merely indicate interdependence in these markets, implying that the crisis in Thailand propagates across these markets through an existing channel that already exists in normal time, especially for relationships associated with a time interval of $[2, 4)$ days, $[4, 8)$ days, $[8, 16)$ days, $[16, 32)$ days, or even $[32, 64)$ days, which is longer than one month. Because contagion is connected with "fast and furious" propagation, in this phenomenon, major negative events in the global markets would not occur a month after the crisis erupts, which is consistent with the finding in Table [C.19]. Rather, they arise long after the crisis occurs in a market, which may be attributed to the incremental results of the negative impact of the crisis in these markets through a normal transmission mechanism.

Consequently, our findings are implicitly inconsistent with the implications of Pasquariello's model. The model claims that contagion is caused by the heterogeneity of asymmetric information. As introduced earlier, the short-run correlations between the shock-hit market and other markets should increase significantly, which is implied by this model. However, our finding that there is no significant increase in short-run relationships between Thailand and the majority of markets implicitly refutes it. The changes in correlations among markets due to heterogeneity do not support the presence of contagion in a majority of markets. In addition, heterogeneity as a driver of contagion, financial linkage channels, such as the correlated information channel, the portfolio rebalancing activity channel, and the correlated liquidity channel are ruled out in the model. This assumption implies that markets are assumed to be unrelated in terms of financial linkage during normal period, which contradicts real life. Consequently, the model overestimates the role of heterogeneity in contagion, which is identified in our findings. However, the heterogeneity of asymmetric information may make the correlation coefficients more volatile, which is verified by significant changes in the volatility of conditional correlation coefficients. Accordingly, we believe that informed speculators and liquidity traders do not have the ability to cause contagion but have the ability to change cross-market comovements in a majority of markets.

Our results offer some useful suggestions for policymakers and portfolio managers. First, to some extent, it is not necessary to worry about liquidity traders and informed speculators. Although these parties have an impact on many markets, the transmission mechanism always exists either in the tranquil period or in the crisis period. Shocks only make these links more visible. Second, eliminating this impact is nearly impossible because international financial markets are integrated and capital flows in current global economies are difficult to halt. One way for policymakers to reduce this impact or to isolate contagion in some markets is to allow more liquidity in markets. When traders do not worry about liquidity issues, they do not need to sell other assets from their portfolios. Therefore, limiting liquidity concerns reduces the impact of the shock to Thailand on many markets or may isolate contagion in some markets. One problem for this measure is that government as an institution is too slow to respond to shocks. Rather, it should spend some time injecting liquidity into markets. However, this impact is so "fast and furious" that government may miss the optimal time to inject. Moreover, as Pasquariello suggests, another way to restrict this impact is to strengthen information disclosures. A more uniform, consistent, and complete disclosure of information reduces the number of informed traders. Consequently, the impact of the shock on other markets declines.

Third, governments should pay more attention to net private capital flows. In contrast to foreign direct investment (FDI), net portfolio investment is relatively easier to withdraw. During the withdrawal process, crisis propagates across markets. Data on the different types of net capital flows show that FDI dominates net capital inflows in China, which is not affected significantly by the crisis. Net portfolio investments are substantial in Indonesia, Korea, Philippines, and Taiwan, which are hit seriously by the crisis. In particular, net portfolio investments in Thailand, which is the source of the crisis, are dominant, amounting to $0.5\% - 3.2\%$ of the GDP in each of the years $1992 - 1996$, whereas FDI is only approximately 1% of GDP (Table [C.2]). Accordingly, governments should regulate net private capital flows, especially for net portfolio investment. It is admitted that the stringent capital control policies are a good and temporary way to isolate contagion or to reduce the negative impact of a crisis on a local market, as shown by China, which is not highly affected by the crisis. Fourth, an essential measure is to implement consistent domestic macroeconomic and exchange rate policies, improve economic fundamentals, and strengthen financial systems, which reduces the fragility and vulnerability of markets to adverse developments.

Finally, to some extent, the absence of significantly high correlation coefficients during crisis periods implies that the gain from international portfolio diversification on many global markets stocks is not affected by the crisis. However, for the majority of markets, the significantly high volatility of correlation coefficients, which may be attributed to either an unstable covariance, an abnormal variance, or both implies that, certainty of estimated correlation coefficients create some difficulties for portfolio managers and increase the probability that they will make inappropriate decisions. Note that it is difficult to determine the full extent of the gain from the use of wavelets in this context without a true benchmark comparison - hard to really quantify the gain.

Chapter 5

Stock Prices and Liquidity in the U.S. Stock Market: The Response to Economic News across the Business Cycle

5.1 Introduction

Many economic indicators that reflect economic fundamentals have a major impact on financial markets. Therefore, scheduled macroeconomic news announcements, such as the employment report, CPI (consumer price index), and PPI (producer price index), are a natural focus for market participants. How do market participants respond to news announcements? Is public information about the economy immediately reflected in asset prices, or does its effect on volatility persist over the course of several days? How does the impact on financial markets from announcements persist? Market participants' responses to news announcements are interpreted by the way information spreads in the market, which is viewed as information processing. Information processing is an important topic in financial economics. Consequently, much attention has been devoted to its study.

In the early stage, economists found mixed and relatively weak empirical evidence using monthly or daily data. Cornell (1983), Pearce and Roley (1983, 1985), and Hardouvelis (1987) find a negative effect of monetary announcement surprises on stock prices. However, limited evidence supports the view that stock prices significantly react to nonmonetary announcement surprises. Schwert (1981) finds that daily stock prices significantly but weakly react to the announcement of unexpected information in the CPI. A limited impact of inflation (PPI) surprises and no impact of industrial production and unemployment rate surprises on stock prices are found by Pearce and Roley (1985), and only the unemployment rate, trade deficit, and personal income of eleven announcements are found to be significant in Hardouvelis (1987). Edison (1997) only finds a statistically significant response for daily exchange rates and short-term and long-term interest rates for nonfarm payroll information in six U.S. news announcements.

Over time, the sampling frequency of data in the literature has been questioned. Low frequency data, consisting of monthly or daily data, are believed to be unable to capture the impact on financial markets of news announcements. Oberlechner and Hocking (2004) find that the speed of information is the primary characteristic of important foreign exchange market news, the importance of which is actually rated higher than the accuracy and content of news by a questionnaire survey of 321 traders and 63 financial journalists from leading banks and financial news providers in the European foreign exchange market. Market participants' responses to news announcements are so fast that they cause immediate changes in asset prices. The daily or monthly data are unable to reflect these behaviours. Furthermore, several announcements and other news may be released on the same day but at different times. The price behaviour is difficult to attribute to any specific announcement using daily or monthly data. Consequently, higher frequency data have been preferred in the past two decades. A growing number of works in the literature study the impact of macroeconomic news announcements on financial markets using intraday data, such as one-minute or five-minute data. Ederington and Lee (1993) find that most price adjustments to announcements are within one minute of the monthly economic information releases; volatility remains considerably higher than normal for another fifteen minutes or so and slightly higher for several hours. Following this study, Ederington and Lee (1995) study the price changes in interest rate and foreign exchange futures markets to monthly announcements in the short run. They conclude that asset prices adjust to the scheduled announcements within the first 10 seconds. The adjustments are small but rapid, which implies that some trades occur at nonequilibrium prices. Furthermore, it takes 40-50 seconds to complete the major adjustments to the initial releases.

These empirical studies focus on scheduled macroeconomic news announcements and examine their impact on futures markets, including Treasury bond and foreign exchange futures markets, which are highly related to spot markets. However, according to challenges to the efficient market hypothesis, asset prices not only reflect all information in the market but also include market participants' expectations. Market participants constantly discount expectations of the future in their present buying and selling decisions. As a result, the importance of news announcements in financial markets declines. This observation is consistent with the findings of Oberlechner and Hocking (2004): foreign exchange traders regard news that is unanticipated by the foreign exchange market and that contradicts an expectation of the foreign exchange market as more important than the reliability of the news source and the perceived accuracy of the news. This view is also implicitly identified by Ederington and Lee (1993). For a total of 19 monthly announcements, they find that the employment report, PPI, CPI and durable goods orders are most important for the Treasury bond and Eurodollar futures prices. In addition, the monthly announcements with the greatest impact on the dollar-deutsche mark exchange rate are the employment report, merchandise trade deficit, PPI, durable goods orders, GNP, and retail sales: note that they are listed in order of decreasing impact. The employment report that imposes the greatest impact on financial markets is normally the first government release concerning economic activity in a given month. Furthermore, the PPI is released before the CPI. Ederington and Lee (1993) infer that later releases are less important because they are partially predictable based on earlier releases.

Consequently, announcements are worth less as expectations conform more closely to them. The unanticipated components of announcements, but not the announcements themselves, affect asset prices. As a result, the credibility of the findings of previous studies is challenged. Moreover, they allow us to assess the different impacts of several announcements that are released simultaneously. Ederington and Lee (1993) and Fleming and Remolona (1997) study the impact of macroeconomic news announcements on financial markets by using dummy variables to represent announcements as regressors in the model. However, it is impossible to individually evaluate the effects of several announcements that are released at the same time. To study information processing, it is important to estimate expectations. On the one hand, expectations can be produced from extrapolative benchmarks such as ARMA models, a strategy that is adopted by Schwert (1981). On the other hand, expectations can be collected from financial companies, such as the International Money Market Services (MMS) and the Bloomberg Terminal.[41] Compared to the expectations generated by econometric models, the MMS survey data are unbiased and less variable (Andersen *et al.* (2003)). Accordingly, these data are proposed to represent the expectations of market participants on scheduled macroeconomic news. Cornell (1983), Pearce and Roley (1983, 1985), Hardouvelis (1987), Harvey and Huang

[41] Every week since 1977, MMS has conducted a Friday telephone survey of approximately forty money managers, collected forecasts for all figures from news announcements to be released during the next week, and reported the median forecasts from the survey.

(1993), McQueen and Roley (1993), Edison (1997), Balduzzi *et al.* (2001), and Andersen *et al.* (2003) use data from the MMS survey and announced macroeconomic news to generate announcement surprises and then study the response of asset prices to them.

We extend the literature in four directions. First, how does the stock market respond to scheduled news announcements? Previous papers (Ederington and Lee (1993, 1995), Edison (1997), Fleming and Remolona (1997, 1999), Balduzzi *et al.* (2001), Andersen *et al.* (2003), Goldberg and Grisse (2013), Altavilla *et al.* (2014) and Paiardini (2014)) primarily focus on the Treasury bond and foreign exchange markets using low frequency and high frequency data. Although some papers (Schwert (1981), Cornell (1983), Pearce and Roley (1983, 1985), Hardouvelis (1987), and McQueen and Roley (1993)) have studied the impact of news announcements on the stock market using low frequency data−(daily data), to the best of our knowledge, no previous studies have used high frequency data to examine this impact.

Second, do news announcements immediately or eventually affect the stock market in the form of price volatility and trading volume? Ederington and Lee (1993, 1995), Fleming and Remolona (1997, 1999), Balduzzi *et al.* (2001), and Andersen *et al.* (2003) propose that the information contained in news announcements is incorporated in asset prices so immediately that a sharp and instantaneous price change occurs at the news release time. However, the implicit information from a news announcement is not fully learned when it is released. Market participants' responses to the information are based on their initial analyses. They need to adjust their investment decisions to reconcile their different views about the news after the release. The subsequent adjustment of prices induces significant and persistent increases in price volatility and trading volume. Accordingly, the impact of news announcements on the stock market comprises an immediate impact and an eventual impact. Price volatility and trading volume by one-minute intervals and by five-minute intervals, respectively, are used to study these impacts.

Naturally, two questions are raised regarding which announcements immediately or eventually affect the stock price. Based on a questionnaire survey with traders and financial journalists, news that contradicts market expectations is considered far more important than news that confirms these expectations (Oberlechner and Hocking (2004)). To address the first question, we thus regress one-minute price changes on announcement surprises. Regarding the second question, the traditional literature fixes the time before the announcement and shifts the examined time after the announcement. The largest time interval in which the price changes significantly in reaction to the news announcement indicates the time during which the news announcement's impact on the market remains significant. However, the analysis based on these static changes in prices cannot explain why increases in price volatility persist over a longer time, as has been found by Ederington and Lee (1993), Balduzzi *et al.* (2001), and Andersen *et al.* (2003). This issue stems from static changes in prices, which cannot fully reflect the eventual impact of a news announcement on the market. To manage the disadvantages of these static changes, we use wavelets to construct wavelet-scale price changes. These data are then regressed on announcement surprises to answer the second question. Moreover, because wavelets produce an orthogonal decomposition of a data sequence by time scale, wavelet-scale price changes on different time scales are mutually orthogonal, which implies that they are linearly independent. The combination of estimation results from the OLS regression model of static price changes and of wavelet-scale price changes gives us the time-profile for the news announcement's impact on stock prices.

Third, we study the different responses of the stock market to news announcements over various stages of the business cycle. The same type of news is considered a positive signal regarding the economy in some states of the business cycle and a negative signal of the economy in others. Consequently, the market's responses are different. For example, on the one hand, a negative surprise in the announcement of the unemployment rate during a boom reduces asset prices because the market fears that the economy is overheating and policy makers will increase the interest rate to cool it down. On the other hand, the same negative surprise in the unemployment rate during a gloom raises asset prices because the market considers it a signal of economic recovery. Even when the market has the same interpretation of news announcements over different stages of the economy, the response of the market to news announcements is unlikely to be consistent considering the divergent behaviours of market participants conditional on the state of the economy. It is interesting to examine the stability of the market's response over various stages of the business cycle. Due to the significant impact of only monetary announcement surprises on daily stock prices, McQueen and Roley (1993) classify economic states into high, medium, and low based on the industrial production index. They find different responses for daily stock prices from a variety of news announcements conditional on the state of the economy. The low frequency data in this paper motivate us to investigate the market's response to news announcements over different stages of the business cycle using high frequency data.[42]

The fourth direction is to examine a so-called "calm before the storm" effect on the stock market. As the weather is particularly calm before a storm, this effect may also observed in financial markets. Market participants withdraw from the market and stabilise asset prices prior to the arrival of a scheduled news announcement, which is taken as a shock to the market, because they do not want to undergo the news release with heavily held stocks. This strategy implies high uncertainty.

This paper is organised as follows. In section 2, we introduce the tick-by-tick data, news announcements, and corresponding forecast data used in the analysis. In section 3, we document price volatility and trading volume. According to the results, we divide the market's response to news announcements into two stages: the immediate response in the first stage and the subsequent adjustment in the second stage. We infer market participants' behaviours conditional on the stages of the economy. In section 4, we propose a simple "news" model and use the data on news announcements and market surveys to generate announcement surprises. Then, we introduce the wavelet analysis and estimate wavelet-scale price changes. We regress static price changes and wavelet-scale price changes on announcement surprises using an ordinary-least squares (OLS) regression model to identify which announcements immediately and which eventually move the stock market. In section 5, we summarise our findings.

5.2 Data

This section provides a detailed description of the data set used in the empirical analysis: S&P 500 index futures data and data on news announcement release values and the corresponding forecast survey values.

[42]We follow the method of McQueen and Roley (1993) and examine the response of daily closing prices of the S&P 500 index to news announcements conditional on the economic states in the sample, which runs from February 3, 1997 to January 30, 2009. Unlike these authors, our results are quite weak. In our opinion, this difference is due to the emergence of the Internet and other advanced communication technologies that facilitate the spread of information during our sample period. In McQueen and Roley (1993), the sample period covers September 1977 to May 1988. Consequently, low frequency data are not appropriate for examining the impact of news announcements on financial markets.

5.2.1 S&P 500 Index Futures

The sample data collected are tick-by-tick S&P 500 index futures from February 3, 1997 to January 30, 2009.[43] Due to the need for high-frequency data but the lack of high-frequency S&P 500 index data, it is only possible to use tick-by-tick S&P 500 index futures, which are highly related to the spot assets, to study the news announcements' impact on the stock market. The S&P 500 index futures have been listed on the Chicago Mercantile Exchange (CME) since the spring of 1982 and comprise the largest 500 listed stocks. They are traded from 09 : 30 to 16 : 15 Eastern Time (ET) on working days except holidays.[44] The price of the tick-by-tick S&P 500 index futures is recorded when a trading transaction occurs. Therefore, we know the trading prices and the number of ticks in a one-minute interval, but not the second when a trading transaction occurs. The one-minute price change is defined as the difference in the price from the last trade in the previous minute interval to the last trade in the current minute interval by percentage. The two-minute price change is the price of the last trade in the current minute interval subtracted from the price of the last trade in the minute interval two minutes previously in percentage terms, and so on. In addition, the total number of trading transactions in a time period is the proxy of the corresponding trading volume. Following this definition, the one-minute trading volume is the total number of ticks in a one-minute interval, and so on.

5.2.2 News Announcements and Market Survey Data

The data on news announcements and the corresponding market expectations are from the Bloomberg Terminal, which is a global financial market database providing data, business news, and analytics. The forecasts of news announcements' release values are collected from a number of economists in different companies on a variety of days before the news announcement. The last forecast of an news announcement in each month is usually conducted only one day before the announcement. The standard deviation of all forecasts of the same news announcement in each month is small, approximately 0.1% to 0.2%. This standard deviation suggests that the survey data, which reflect the market's expectations on news announcements' release val-

[43]Tarde and Quote (TAQ) database is another source of intraday trades and quotes for all securities listed on the New York Stock Exchange, American Stock Exchange, Nasdaq National Market System and SmallCap issues.

[44]In this paper, the time is based on American Eastern Standard Time.

ues, include all publicly available information one day before the announcements.[45] The median of the survey data for a news announcement represents the market's expectation for it.

The 17 monthly news announcements that we consider are reported in Table [D.1]. Seven of the announcements are released at 8:30, two at 9:15, six at 10:00, one at 14:00, and one at 15:00.[46] The precise release times of these announcements stem from the Bloomberg Terminal. For the nonfarm payrolls, trade balance, consumer confidence, new single-family home sales, PMI (purchasing managers index), federal budget, and consumer credit, we convert the announcement release values into percentage changes from the previous month's announced level.

5.3 Price Volatility and Trading Volume

To examine intraday volatility, the standard deviations of the one-minute price changes at the same time interval across all 3003 trading days are shown in Figure [D.1A].[47] The means of the corresponding trading volumes at the same time interval across all trading days are shown in Figure [D.1B] as well. In these and other figures, the time on the horizontal axis denotes the end of the interval in Eastern Time (e.g., 10:00 for 9:59 to 10:00 price changes). An apparent spike emerges over the 16:15 to 9:30 period in Figure [D.1A]. It is not surprising to find high volatility in price changes at this time interval because of overnight information. Indeed, this price change does not belong to the one-minute price change. The price change over the 9:59 to 10:00 period is also very volatile. The corresponding trading volumes over the 16:15 to 9:30 period and over the 9:59 to 10:00 period are unusually high. The difference between them is that trading volume declines substantially after 9:30 but falls slowly after 10:00. It is observed that 9 of our news announcements occur before 9:30, which is the opening time of the CME; 6 of the news announcements occur at 10:00; 1 news announcement occurs at 14:00; and 1 news announcement occurs at 15:00. To investigate whether the price volatility and trading volume patterns observed in Figures [D.1A] and [D.1B] are attributed to news announcements, we

[45] One could conjecture that the survey data miss some information and cannot precisely reflect the market's expectations because the market updates its expectations on the later released announcement in accordance with the earlier released one. However, the survey data from the Bloomberg Terminal are collected on a variety of days before the announcement. The last forecast is usually conducted one day before the announcement, and the standard deviation of all forecasts of the same news announcement in each month is small, approximately 0.1% to 0.2%, which implies that the survey data do not miss the information before the announcement.

[46] The release times of these announcements change in some months. Here, we classify the times in terms of when they are usually released.

[47] Given the small magnitude of intraday price changes, they are all multiplied by 100.

divide the sample into those days that contain at least one of our seventeen news announcements (1541 days) and those with none (1462 days). As shown in Figure [D.2A], the 9:59 to 10:00 spike in price volatility disappears on nonannouncement days. Figure [D.2B] also indicates that the 9:59 to 10:00 trading volume on announcement days is significantly higher than on nonannouncement days. Moreover, price volatility and trading volume remain considerably higher than normal for a time after the news announcements, particularly for those released at 10:00.

Consequently, the price volatility and trading volume patterns observed in the above figures are due to news releases. Given the price volatility and trading volume over the 16:15 to 9:30 period, it is very difficult to clarify whether news announcements before 9:30 affect the stock market because overnight information dominates. However, the persistent increases in price volatility and trading volume after 9:30 on announcement days are shown in Figures [D.2A] and [D.2B], respectively. It is believed that these patterns are related to news announcements before 9:30.

The above study in price volatility and trading volume only concentrates on the sample period. The findings imply that market participants' response to news is the same in different stages of the economy. However, market participants' behaviours may vary for different macroeconomic environments: there is no reason to believe in a consistent response to news announcements over different stages of the business cycle.[48] An economic expansion denotes an increase in the level of economic activity, whereas an economic contraction represents a decline in the economy and more volatile financial markets. Market participants chase returns in an expansion period, whereas they prefer to decrease their exposure to aggregate risk factors in a contraction period (Cederburg (2008)). McQueen and Roley (1993) utilise an example to show that market participants' behaviours are related to divergent levels of economic activity. A negative surprise in the unemployment rate, which was released on February 4, 1983 after 16 months of recession, increased the Dow Jones Industrial Average because it was viewed as an signal of economic recovery ("The Chairmen of the Council of Economic Advisers, Martin Feldstein, commented that a recovery was either beginning or already here in the *Wall Street Journal*, February 7, 1983"). However, a similar surprise, which was announced on November 4, 1988 after six years of expansion, reduced stock prices because it was considered a signal of tighter Fed Policies that would increase the interest rate, which is called the "policy anticipation effect" by Urich and Wachtel (1981) ("Bond market investors reacted with gloom, sending interest rates higher on fears of tighter Fed Policy. The stock market also fell. They were reported by *Wall Street Journal*, November

[48]It implies that multicollinearity in the data does not exist.

7, 1988"). Consequently, market participants adapt their behaviours to different macroeconomic environments. This observation infers that their response to news announcements will vary for different stages of the business cycle, as McQueen and Roley (1993) argue.

Accordingly, it would be interesting to explore the patterns in price volatility and trading volume over different stages of economic activity. It is necessary to first classify the divergent levels of the economy. Due to the lack of a widely accepted definition, we quote the NBER business cycle as the classification. Regarding the definition of the NBER business cycle, the time periods from February 1997 to March 2001 and from December 2001 to December 2007 are classified as the expansion period, and the rest of the sample is in the contraction period.[49] As a result, the announcement days and nonannouncement days are divided into two groups: the expansion period and the contraction period. There are 1314 announcement trading days in the expansion period and 227 announcement trading days in the contraction period, and there are 1252 nonannouncement trading days in the expansion period and 210 nonannouncement trading days in the contraction period.

On both announcement days and nonannouncement days, Figures [D.5A] and [D.6A] show that price volatility during the contraction period is apparently greater than during the expansion period. As introduced earlier, the spike over the 16:15 to 9:30 period emerges on both announcement days and nonannouncement days, whereas the spike over the 9:59 to 10:00 period only appears on announcement days. Conditional on the business cycle, aside from the greater volatility over the 9:59 to 10:00 period, price volatility remains higher than normal within the minutes following 9:30 or 10:00 in both the expansion period and the contraction period, as shown in Figures [D.3A] and [D.4A]. Moreover, the increases in price volatility persist over a longer period of time in the expansion period compared with the contraction period, especially for 10:00 announcements.

Figures [D.5B] and [D.6B] show that trading volume in the contraction period is generally lower than in the expansion period on both announcement days and nonannouncement days. One of the few time intervals contradicting this finding is from 9:59 to 10:00, when trading volume in the contraction period is higher than in the expansion period on announcement days. As shown in Figures [D.3B] and [D.4B], for the same stage of the business cycle, trading volume over the 9:59 to 10:00 period on announcement trading days is higher than on nonannouncement trading days. Subsequently, trading volume falls less after 10:00 in the expansion period

[49]The NBER indicates that contractions (recessions) start at the peak of a business cycle and end at the trough.

compared with the contraction period. Furthermore, it is found that trading volume remains higher than normal within nearly an hour after 10:00 in the expansion period. However, this persistence in the contraction period is not as obvious, and the time is shorter. In addition, trading volume in the expansion period remains higher than normal within the minutes following the market opening at 9:30, whereas this phenomenon is not observed in the contraction period. It is concluded that persistent increases in price volatility and trading volume emerge after news announcements but vary with different stages of economic activity.

5.3.1 The First Stage: The Immediate Impact of Public Information

In the above section, we briefly introduce the impact of news announcements on price volatility and trading volume. Based on the findings that price volatility and trading volume surge in the news release minute and show persistent increases afterwards, we divide the news announcements' impact on the stock market into two categories: the immediate impact and the eventual impact. To examine the impact of news announcements on the stock market in detail, we form 4 groups of announcements based on their release times: 8:30 & 9:15 announcements, 10:00 announcements, the 14:00 announcement, and the 15:00 announcement.[50] Consequently, the announcement days only include the trading days when a specific-time announcement is released, whereas the nonannouncement days are trading days excluding all days when an announcement is released. Table [D.2] reports price volatility and trading volume by one-minute intervals as well as a comparison between announcement and nonannouncement days using the ratio of price volatility and the difference in trading volume mean. The Brown-Forsythe-modified Levene F-statistic comparing variances for announcement and nonannouncement days and the t-statistic comparing means for announcement and nonannouncement days assuming unequal variances are also included in this table. All one-minute intervals between 9:30 and 9:41 and the time period over 16:15 to 9:30 are examined for the 8:30 & 9:15 announcements. For other announcements, we examine all one-minute intervals from 5 minutes before the announcement to 7 minutes after the announcement.

[50] Because the market is open from 9:30 to 16:15, 8:30 and 9:15 announcements, which are released before the opening time, are put into one group. The 8:30 & 9:15 announcements are CPI, PPI, civilian unemployment, nonfarm payrolls, personal consumption, personal income, the trade balance, capacity utilisation, and IP (industrial production); 10:00 announcements are consumer confidence, durable goods orders, the leading index, manufacturers' new orders, new single-family home sales, and PMI; the 14:00 announcement is federal budget; and the 15:00 announcement is consumer credit.

Panel A of Table [D.2] shows that the ratio of price volatility on announcement days to that on nonannouncement days increases in the news release minute and/or the next minute, reflecting the market's initial reaction to the announcement. The ratio increases in the next minute for the 8:30 & 9:15 announcements and the 14:00 announcement and increases in both the news release minute and the next minute for the 10:00 announcements. In particular, price volatility surges at these time intervals. This surge denotes that price volatility on announcement days is greater than on nonannouncement days. Moreover, the results of the B-F-L test indicate that price volatility between these two different types of days is significantly distinguished in the corresponding time intervals. Meanwhile, trading volume significantly increases for the same time intervals along with an increase in price volatility, particularly for the 10:00 announcements, as shown in Panel B. For the 14:00 announcement, the increase in price volatility is not accompanied by a higher trading volume.

Some important macroeconomic news consisting of civilian unemployment, CPI, and IP fall into the category of 8:30 & 9:15 announcements. Ederington and Lee (1993, 1995), Fleming and Remolona (1997, 1999), Balduzzi *et al.* (2001), and Andersen *et al.* (2003) find a fierce initial response to these types of announcements on other markets including the Treasury bond and foreign exchange markets. However, the market's initial response to 8:30 & 9:15 announcements in terms of stock prices is not as intense as it is for other markets' asset prices and is not as the same as it is for 10:00 announcements. Stock prices significantly change only over the next minute when the market opens because the market participants already know the contents of these news announcements, and the market responds to them based on the performance of other 24-hour markets after these announcements. The stock prices are changed by more homogeneous analyses, whereas other markets' asset prices are affected by more heterogeneous analyses, similar to the change in stock prices from 10:00 announcements.

Tables [D.3] and [D.4] compare announcement with nonannouncement days by one-minute price volatility and trading volume during the expansion and contraction periods, respectively. As shown in Table [D.2] for the entire sample period, price volatility and trading volume rise in the news release minute or the next minute. The difference is that the significant increase in price volatility in the expansion period is larger than in the contraction period, whereas the significant increase in trading volume in the expansion period is smaller than in the contraction period. Accordingly, news announcements induce larger price changes per interval in the expansion period and more price changes in the contraction period. These tables also demonstrate the divergent market's behaviours at different stages of economic activity. In the expansion period, market participants who are relatively calm on normal days are more sensitive to news announcements and prefer to use a lower number of trading transactions to achieve a greater price change. However, in the contraction period, market participants who are relatively susceptible on normal days are less sensitive to news announcements. Their response to announcements is more prudent; they prefer to change stock price on a smaller scale through a higher number of trading transactions.

It is noted that these comparisons are based on the same stage of economic activity. As shown in Figures [D.5A], [D.5B], [D.6A], and [D.6B], Tables [D.3] and [D.4] report higher price volatility and lower trading volume in the contraction period in comparison with the expansion period on both announcement days and nonannouncement days. These figures illustrate that the market is more susceptible and less active during the contraction period compared with the expansion period. It is not surprising to find that the magnitude of price volatility in the news release minute or the next minute over the contraction period is greater than that at the same time intervals over the expansion period, whereas the magnitude of the corresponding trading volume over the contraction period is generally lower than that over the expansion period, with an exception for the trading volume over the 9:59 to 10:00 period.

It is interesting to note that price volatility and trading volume significantly decline 2 or 3 minutes before the 10:00 announcements, as shown in Tables [D.2], [D.3], and [D.4], because market participants reduce trading transactions and stabilise the stock price at those times to prepare for the arrival of announcements. This pattern is called the "calm before the storm" effect by Jones *et al.* (1998), although these authors use daily data to investigate this effect. This effect is consistent with the claim of the financial press that financial markets are particularly quiet prior to scheduled news announcements. However, the effect of news announcements on financial markets lasts only a few hours. Thus, it is doubtful that markets remain silent for the days. We find that the effects only last a few minutes.

In conclusion, the market's initial response to news announcements is strong and induces a sharp and nearly instantaneous price change along with a rise in trading volume. The information contained in news announcements is incorporated into stock prices immediately. There are different market behaviours conditional on the business cycle: a larger initial price change is driven by a lower trading volume in the expansion period, whereas a smaller initial price change is accompanied by a higher trading volume in the contraction period. Moreover, the "calm before the storm" effect arises 2 or 3 minutes before an announcement because market participants withdraw from the market just prior to lower their risk.

5.3.2 The Second Stage: The Eventual Impact of Public Information

After an announcement, market participants need to adjust their initial reactions in accordance with others' behaviour. To obtain profits or avoid losses when an news announcement is released, market participants immediately react based on their initial analyses. After the announcements, they learn others' decisions from changes in stock prices and trading volume. The participants may then realise that their initial behaviours were an over- or underreaction and may then further adjust their response. Consequently, stock prices are still volatile and are still accompanied by high trading volume for a long time before equilibrium is reached.

We characterise the subsequent adjustment to news announcements measured through price volatility and trading volume by five-minute intervals from 9:30 to 10:25, adding the measurements over the closing (16:15) to opening (9:30) time period for 8:30 & 9:15 announcements, and from 5 minutes before to 55 minutes after for other announcements, including the 10:00, 14:00, and 15:00 announcements. Tables [D.5], [D.6], and [D.7] present the comparisons between announcement days and nonannouncement days over the entire sample period, over the expansion period, and over the contraction period, respectively, following the same format as in Tables [D.2], [D.3], and [D.4]. Standard deviations of five-minute price changes across the trading days for the specific economic period are presented in Panel A, and five-minute trading volume means are shown in Panel B.

The sharp initial price change is followed by a significantly prolonged increase in trading volume along with high price volatility for the 8:30 & 9:15 announcements and the 10:00 announcements. This pattern indicates the persistent increases in price volatility and trading volume. Panel A of Table [D.5] and Figure [D.2A] show that price volatility remains significantly higher than normal from 9:45 to 10:20 for 8:30 & 9:15 announcements and from 10:00 to 10:40 for 10:00 announcements, respectively. Panel B of Table [D.5] and Figure [D.2B] indicate significantly higher trading volume across the announcement days from 9:35 to 10:10 for 8:30 & 9:15 announcements and from 10:00 to 10:40 for 10:00 announcements, respectively. We do not find persistent increases in either price volatility or trading volume for the 14:00 or the 15:00 announcement. Because there is only one announcement at these times, the subsequent adjustment is too small to significantly change the stock price or increase the trading volume.

The higher price volatility and trading volume following 8:30 & 9:15 announcements indicate that market participants still need to adjust their initial decisions for more than an hour after the announcements. This length of time implies that market participants cannot accurately absorb the implicit information from news announcements until that time, although they already know the response to these announcements for other markets. Ederington and Lee (1993), Fleming and Remolona (1999), Balduzzi *et al.* (2001), and Andersen *et al.* (2003) find that price volatility remains considerably higher than normal for approximately one hour and up to several hours on the Treasury bond and foreign exchange markets. The persistent increases in price volatility and trading volume on the stock market from 8:30 & 9:15 announcements are accompanied by these continued adjustments in asset prices for other markets.

The above results for the entire sample period vary over the two different stages of the economy. In the expansion period, the eventual effects of announcements on price volatility and trading volume persist over time for as long as they do for the entire sample period. Both price volatility and trading volume remain significantly higher than normal from 9:35 to 10:20. The 10:00 announcements induce higher price volatility and higher trading volume from 10:00 to 10:40. However, in the contraction period, the time encompassing persistent increases in price volatility and trading volume is shorter. For 8:30 & 9:15 announcements, price volatility remains significantly higher than normal only from 10:00 to 10:20, whereas trading volume is significantly higher only over the 10:00 to 10:05 period. For 10:00 announcements, price volatility and trading volume are significantly higher than normal only from 10:00 to 10:05. These results provide evidence for our earlier arguments: in the expansion period, news-sensitive market participants overreact or underreact to news announcements and thus need to spend more time adjusting their initial behaviours. In the contraction period, news-insensitive market participants react to news announcements moderately based on prudent decisions; thus, they spend less time adjusting their initial behaviours.

In conclusion, market participants reconcile the differential views over a prolonged second stage that induces an increase in price volatility and trading volume after the announcements. Because market participants initially overreact or underreact to news announcements, they need to adjust their initial responses according to others' behaviour as seen in the market's performance. This process causes persistent increases in price volatility and trading volume within an hour and over even a longer time. In the expansion period, the patterns in price volatility and trading volume are similar to those for the entire sample period. However, the increases in price volatility and trading volume persist over a shorter time in the contraction period. Compared to nonannouncement days at the same stage of economic activity, news-sensitive market participants make larger subsequent adjustments in stock prices through a higher number of trading transactions across a longer time in the expansion period, whereas news-insensitive market participants are so cautious that they make smaller subsequent adjustments in stock prices through a lower number of trading transactions across a shorter time in the contraction period. These findings imply that there is a more efficient market in the contraction period. Accordingly, news announcements induce substantial and long-term repercussions in the stock market over the expansion period, whereas they have small and short-term repercussions on the stock market over the contraction period.

5.4 Economic News and Stock Prices

The results shown in the above figures and tables demonstrate that scheduled macroeconomic news announcements significantly affect stock prices. Naturally, the next question involves identifying which announcements move the stock market. In this section, we propose a simple "news" model for stock prices to explain why announcement surprises affect them. Then, we study the impact of different news announcements on stock prices, including the immediate impact and the eventual impact.

5.4.1 The Theoretical Framework of the "News" Model

The underlying principle of investment in the stock market is that stock prices are identical to the present discounted values of rationally expected future dividends through infinity, which is called the dividend discount model. This model is expressed as follows:

$$P_t = \sum_{\tau=1}^{\infty} \frac{E_t D_{t+\tau}}{1 + E_t r_{t+\tau}}, \tag{5.1}$$

where P_t is the stock price at time t, $D_{t+\tau}$ is the dividend at time $t+\tau$, $r_{t+\tau}$ is the stochastic discount rate of cash flows at time $t+\tau$, and $E_t[\cdot]$ denotes the mathematical expectation conditional on available information Ω_t at time t. Correspondingly, the stock price at time $t-1$ is P_{t-1}, which is equal to

$$P_{t-1} = \sum_{\tau=1}^{\infty} \frac{E_{t-1}D_{t-1+\tau}}{1+E_{t-1}r_{t-1+\tau}}. \tag{5.2}$$

Consequently, the stock price change from time $t-1$ to t is

$$\begin{aligned} P_t - P_{t-1} &= \sum_{\tau=1}^{\infty} \frac{E_t D_{t+\tau}}{1+E_t r_{t+\tau}} - \sum_{\tau=1}^{\infty} \frac{E_{t-1}D_{t-1+\tau}}{1+E_{t-1}r_{t-1+\tau}} \\ &= -\frac{E_{t-1}D_t}{1+E_{t-1}r_t} + \sum_{\tau=1}^{\infty}\left(\frac{E_t D_{t+\tau}}{1+E_t r_{t+\tau}} - \frac{E_{t-1}D_{t+\tau}}{1+E_{t-1}r_{t+\tau}}\right) \\ &= \frac{D_t}{1+r_t} - \frac{E_{t-1}D_t}{1+E_{t-1}r_t} + \sum_{\tau=1}^{\infty}\left(\frac{E_t D_{t+\tau}}{1+E_t r_{t+\tau}} - \frac{E_{t-1}D_{t+\tau}}{1+E_{t-1}r_{t+\tau}}\right) - \frac{D_t}{1+r_t}. \end{aligned} \tag{5.3}$$

Suppose that the dividend and the discount factor are only determined by the economic fundamentals; we then have

$$\frac{D_t}{1+r_t} = f(z_t), \tag{5.4}$$

where z_t is the vector of fundamental variables, and $f[\cdot]$ is the linear function with the variable z_t. Market participants rationally expect the next period's dividend using Equation (5.4).[51] Specifically, the participants use all publicly available information at time $t-1$ to form their expectation of the dividend at time t:

$$\frac{E_{t-1}D_t}{1+E_{t-1}r_t} = f(E_{t-1}z_t). \tag{5.5}$$

Subtracting Equation (5.5) from Equation (5.4), we have

$$\frac{D_t}{1+r_t} - \frac{E_{t-1}D_t}{1+E_{t-1}r_t} = f(z_t - E_{t-1}z_t), \tag{5.6}$$

where $f(z_t) - f(E_{t-1}z_t) = f(z_t - E_{t-1}z_t)$ because $f[\cdot]$ is the linear function with the variable z_t. $z_t - E_{t-1}z_t$ is the unexpected component of the fundamentals in z_t, which is defined as "news". This component is interpreted as the announcement surprise and shows the deviation of the actual value of the figure on an news announcement from its mathematical expected value. This deviation should be random in the sense that it has an average value of zero and displays no systematic

[51] Here, it is assumed that market participants know the underlying structural model between the endogenous variables, D_t and r_t, and the fundamental variable, z_t. This assumption allows us to infer that the same structural model links the expectations of those variables.

pattern over time. Otherwise, market participants could obtain the potentially predictable element from it to upgrade their expectation at the time. Although the news releases the previous month's data about fundamentals, these data involve the latest reliable information about economic fundamentals. Market participants use these announcements to update their knowledge about fundamentals. The efficient market hypothesis implies that market participants immediately respond to information when it becomes available. As a result, stock prices should react when market participants perceive information about fundamentals, which is at the same time that the institution collects the relative data. However, Schwert (1981) finds the contrary result that the stock market does not respond to unexpected inflation during the period in which CPI data is collected, which is before the release date, but the market significantly reacts to unexpected inflation around the time that it is released. Therefore, $z_t - E_{t-1}z_t$ is viewed as the unexpected component of a news announcement that occurs at time t.

Similarly, the relationship between the expected dividend and the expected fundamentals at time $t+1$ conditional on the information at time t is

$$\frac{E_t D_{t+1}}{1 + E_t r_{t+1}} = f(E_t z_{t+1}). \tag{5.7}$$

The expected dividend and the expected discount factor at time $t+1$ are determined by the expected fundamentals at time $t+1$ based on all available information at time $t-1$:

$$\frac{E_{t-1} D_{t+1}}{1 + E_{t-1} r_{t+1}} = f(E_{t-1} z_{t+1}). \tag{5.8}$$

Consequently, the first term in the bracket on the right-hand side of Equation (5.3) is generated by subtracting Equation (5.8) from Equation (5.7):

$$\frac{E_t D_{t+1}}{1 + E_t r_{t+1}} - \frac{E_{t-1} D_{t+1}}{1 + E_{t-1} r_{t+1}} = f(E_t z_{t+1} - E_{t-1} z_{t+1}). \tag{5.9}$$

The expected fundamentals z_t are generated by using the past information about fundamentals, which is assumed to be

$$E_{t-1} z_t = \sum\nolimits_i \lambda_i z_{t-i}, \tag{5.10}$$

where λ_i is the decreasing weight. At time $t-1$, market participants anticipate the future fundamentals at time $t+1$ using the past fundamentals and the expected future fundamentals at time t based on the information set Ω_{t-1}. Following the same format as Equation (5.10), we have the expected fundamentals at time $t+1$, which are

$$E_{t-1}z_{t+1} = \lambda_1 E_{t-1}z_t + \sum\nolimits_i \lambda_{i+1}z_{t-i}. \tag{5.11}$$

According to Equation (5.10), the expected fundamentals at time $t+1$ conditional on the information set Ω_t, are identical to

$$E_t z_{t+1} = \sum\nolimits_i \lambda_i z_{t+1-i}. \tag{5.12}$$

Subtracting Equation (5.11) from Equation (5.12), we have

$$\begin{aligned} E_t z_{t+1} - E_{t-1}z_{t+1} &= \sum\nolimits_i \lambda_i z_{t+1-i} - (\lambda_1 E_{t-1}z_t + \sum\nolimits_i \lambda_{i+1}z_{t-i}) \\ &= \lambda_1(z_t - E_{t-1}z_t). \end{aligned} \tag{5.13}$$

Therefore, the first term in the bracket on the right-hand side of Equation (5.3) can also be expressed by

$$\frac{E_t D_{t+1}}{1+E_t r_{t+1}} - \frac{E_{t-1}D_{t+1}}{1+E_{t-1}r_{t+1}} = f[\lambda_1(z_t - E_{t-1}z_t)]. \tag{5.14}$$

Based on the same method, it is not difficult to determine that every term in the bracket on the right-hand side of Equation (5.3) can be represented by the function $f[\cdot]$ with the variable $z_t - E_{t-1}z_t$. Consequently, Equation (5.3) is identical to

$$P_t - P_{t-1} = -\frac{D_t}{1+r_t} + \sum_{i=1}^{\infty} f[\mu_i(z_t - E_{t-1}z_t)]. \tag{5.15}$$

The left-hand side of this equation is the stock price change from time $t-1$ to t. The unexpected component of the fundamentals in z_t, which is revealed by the announcement surprise, is the variable in the function $f[\cdot]$ on the right. It is the simple "news" model about stock prices, and it tells us that the announcement surprise affects the stock price change.

5.4.2 "News"

"News" is defined as the difference between the expected and real values for an announcement. The unanticipated component of announcement j is

$$S_{j,t} = \frac{A_{j,t} - E_{j,t}}{\hat{\sigma}_j}, \tag{5.16}$$

where $A_{j,t}$ is the real value for announcement j at time t, $E_{j,t}$ is the median of the market-based survey forecast for announcement j, which is collected from the Bloomberg Terminal, and $\hat{\sigma}_j$ is the sample standard deviation of surprise $A_{j,t} - E_{j,t}$, which is used to facilitate the comparison between stock market responses to different news announcements. Therefore, $S_{j,t}$ is interpreted as the standardised surprise of announcement j. The regression coefficient shows how much a one standard deviation change in the surprise affects the price change when regressing price changes on announcement surprises. Due to the constant standard deviation $\hat{\sigma}_j$ across all observations for announcement j, the standardisation affects neither the significance of the estimates nor the fit of the regression.

Table [D.1] reports the sample standard deviations of announcement surprises and the descriptive statistics for the data on standardised announcement surprises, including the number of observations, the number of zero values for standardised announcement surprises, the means, and the t-statistics testing the zero mean for the entire sample period, for the expansion period, and for the contraction period. The highest ratio for the number of zero values over the number of observations occurs for the leading index (36.17%), with the CPI (32.64%) and civilian unemployment (31.69%) following for the entire sample period. Regarding the different stages of the economy, the highest ratio is for civilian unemployment (35.53%), following the leading index (35%) and CPI (34.96%) during the expansion period. The leading index (42.86%) with the PPI (40%) following for the contraction period. Moreover, the mean of every standardised announcement surprise is close to zero. However, only the standardised surprise of personal income is significantly different from zero for the entire sample period. Regarding the different stages of the business cycle, the means of the standardised surprises are significantly different from zero for civilian unemployment, personal income, and trade balance for the expansion period and PPI and federal budget for the contraction period. These results are consistent with our inference in the "news" model that the average of the deviations for the actual value of an news announcement from its mathematical expectation should be zero.[52] This result is attributed to random deviation; otherwise market participants would be able to improve their expectation according to the potentially predictable information from the deviation. Consequently, this result provides indirect evidence that the survey data from the Bloomberg Terminal are rational forecasts.

[52]The mean of the announcement surprises equals the production of the mean of the standardised surprises and the standard deviation of surprises. Because the standard deviation of surprises is smaller than 1, the mean of announcement surprises is more likely to approximate zero. The t-statistic for the mean of the announcement surprises is identical to that of the standardised surprises.

5.4.3 Which News Announcements Immediately Affect Stock Prices?

To investigate which news announcements immediately affect stock prices, we regress one-minute price changes on the surprises for the 17 economic variables:

$$(P_t - P_{t-1})/P_{t-1} * 100 = C + \sum_{k=1}^{17} \beta_k S'_{k,t} + e_t, \tag{5.17}$$

where

i) P_t is the price of the last trade in the current minute interval;

ii) P_{t-1} is the price of the last trade in the previous minute interval. Because the magnitude of the one-minute price change is small, we multiply it by 100 and interpret it as a change in percentage;

iii) β_k is the response coefficient of the price to the kth announcement;

iv) $S'_{k,t}$ is the kth economic variable. It is equal to the standardised surprise of the kth announcement $S_{k,t}$ when the kth announcement occurs at time t; otherwise, it is identical to zero.

The model takes concurrent news announcements into account. For example, the civilian unemployment and the nonfarm payroll figures are always announced in the same report. The different values for the standardised surprises of concurrent news announcements distinguish them, but the dummy variables that have been used to represent news announcements in previous papers cannot separate them. This difference is one advantage of adopting announcement surprises as regressors. Another advantage is the ability to study whether asset prices increase when news announcements are better than expected by assessing the response coefficients. However, this ability is not available when dummy variables represent news announcements. In the literature, to reduce the computational burden, only data around the news release time are analysed. The results of these two methods are similar, except for the higher adjusted R^2 in the latter method. Because the market is open from 09:30 to 16:15, we take the 8:30 and 9:15 announcements as the 9:30 announcements in the model.

Table [D.8] presents the estimation results of the model, including the response coefficients and the t-statistics.[53] Intercept terms are not listed because they are rarely significant. The results show the significant response of the stock price to 6 announcement surprises regarding PPI, consumer confidence, durable goods orders,

[53] To address the heteroskedasticity issue, standard errors and test statistics use the HAC coefficient covariance matrix. We use the slash line instead of the data if the response coefficient on the announcement surprise is not significant in every time period.

the leading index, PMI, and federal budget. Of these announcements, PPI is released at 8:30, federal budget is released at 14:00, and the other 4 announcements are released at 10:00. The positive signs for the response coefficients on the 10:00 announcement surprises indicate that the stock price rises when these announcements are better than expected, implying that the economy has outperformed market expectation, and drops when these announcements are worse than expected, implying that the economy has underperformed market expectation. Moreover, the negative signs of the response coefficients for the the PPI surprise and the federal budget surprise show a reverse movement in the stock price. An unexpectedly high PPI reduces the stock price, whereas an unexpectedly low PPI increases the stock price. A positive surprise in federal budget reduces the stock price, whereas a negative surprise increases the stock price. The market's response to news announcements is ambiguous. Our results indicate the overall market reaction to these surprises in the sample period.

It would be interesting to examine the stability of the response coefficients over different stages of the business cycle. Seven announcements consisting of CPI, PPI, consumer confidence, the leading index, PMI, federal budget and consumer credit significantly affect stock price in the expansion period, whereas seven different announcements comprising nonfarm payrolls, consumer confidence, durable goods orders, new single-family home sales, PMI, federal budget, and consumer credit significantly affect the stock price in the contraction period. Compared to the results for the entire sample period, more news announcements have a significant impact on stock prices when examining different stages of the economy. This finding implies that the market is only responsive to surprises in some news announcements over a particular economic period but not over the entire sample period. In addition, stock prices react to some announcement surprises over both economic periods, including PMI, consumer confidence, federal budget, and consumer credit. In particular, the signs of the response coefficients for these announcement surprises are quite stable in both stages of the business cycle. The one exceptional announcement is consumer credit, which is negatively related to stock prices in the expansion period and positively related to stock prices in the contraction period. We do not find that the market significantly reacts to consumer credit over the entire sample period because the signs of its response coefficients vary based on the stage of economic activity.

The overall response of the stock market to these news announcements is described as follows: unexpected high figures in the leading index in the expansion period and durable goods orders and new single-family home sales in the contraction period increase the stock price, whereas unexpected low figures in these announcements for the corresponding periods reduce the stock price. Moreover, unexpected CPI and PPI in the expansion period negatively affect the stock price. In both economic periods as well as in the entire sample period, surprises in consumer confidence and PMI have a positive impact on the stock price, whereas in federal budget have a negative impact on the stock price.

In terms of the size of the impact of news announcements on stock prices in the entire sample period, PPI is the most significant. It is noted that the standard deviation of the daily price change for the S&P 500 index futures is 1.37%. A one standard deviation surprise in the PPI, which is related to a 0.27% monthly variation in the index, causes a price change of approximately 11.41% in the normal daily volatility of price changes.[54] Consumer confidence (8.58%), durable goods orders (6.38%), PMI (2.66%), the leading index (2.65%),and federal budget (0.33%) lead to price changes between 0.33% and 8.58% of daily volatility, and their importance decreases by the listing order.[55] According to the effect of news announcements on stock prices in the expansion period, PPI is also the most important announcement with an effect of approximately 11.03% of daily volatility on the price change. In terms of the decreasing size of the news announcements' impact on stock prices, PPI is followed by consumer confidence (8.70%), CPI (7.82%), PMI (2.32%), federal budget (0.30%), and consumer credit (0.30%), where federal budget and consumer credit are of identical importance. In the contraction period, new single-family home sales have the greatest impact on the market with an effect of approximately 171.25% of daily volatility on the price change. Next are nonfarm payrolls with an effect of approximately 65.23% of daily volatility, followed by consumer credit (28.62%), durable goods orders (15.11%), consumer confidence (8.39%), and PMI (3.57%). Federal budget (2.05%) has the smallest effect on stock prices. In conclusion, the magnitudes of the response coefficients on news announcement surprises in the contraction period are generally greater than those in the expansion period. This result suggests that the impact of news announcements on stock prices is more considerable in contraction period. This finding is consistent with Figure [D.5A] and Tables [D.2] and [D.3], which show that price volatility in the contraction period is higher than in the expansion period when news announcements are released.

[54]The seventh column of Table [D.1] reports the sample standard deviations of announcement surprises. The response coefficients in the regression model combined with these standard deviations provide the economic interpretations of the estimation results.

[55]The value in the brackets shows how much the stock price changes given a one standard deviation surprise of an announcement in terms of the normal daily volatility of price changes.

5.4.4 Which News Announcements Eventually Affect Stock Prices?

The results of the regression model of one-minute price changes on announcement surprises identify which announcements move the stock market and confirm the immediate impact of those announcements on the stock price. As discussed earlier, the news announcements' impact on the stock market comprises the immediate and the eventual impact. The eventual impact induces persistent increases in price volatility and trading volume. Regressing price changes from the time before the announcement to the time after the announcement on the surprises in the economic variables or on the dummy variables that represent news announcements is a commonly used method, and this paper also adopts this method to study the immediate impact of announcements on the stock price. To investigate how long news announcements affect asset prices, the time before the announcement is often fixed, and the time after the announcement is moved to obtain the price change for different time intervals. The largest time interval over which the price change is significantly affected by a news announcement tells us the market's response until that time. Because this type of price change is static, the method used to study the impact of news announcements on the price change is defined as static analysis. Ederington and Lee (1993, 1995), Balduzzi *et al.* (2001), and Andersen *et al.* (2003) find that asset prices significantly react to news announcements from between one minute to five minutes after the announcements based on this type of static analysis or price volatility over the short time interval. However, Ederington and Lee (1993), Fleming and Remolona (1999), Balduzzi *et al.* (2001), and Andersen *et al.* (2003) find that price volatility and trading volume remain considerably higher than normal over approximately an hour and up to even several hours.

Consequently, the static analysis cannot explain why increases in price volatility persist over longer than the maximum time discovered by economists. The difference in time between them is caused by temporal aggregation. A simple example illustrates why this question arises. Suppose the asset price is P_1 one minute before an announcement at time t, and it increases considerably to P_2 when the news is released. The price then drops to P_3 one minute after the announcement and rises to P_4 two minutes after the announcement. The relationships between these prices are $P_2 > P_4 > P_1 > P_3$ and $P_1P_4 > P_2P_3$, as illustrated in Figure [D.7A]. By the regression model, the asset price significantly reacts to the news announcement through the one-minute price change $(P_2 - P_1)/P_1$, but it does not react through the two-minute price change $(P_3 - P_1)/P_1$ or the three-minute price change

$(P_4 - P_1)/P_1$ because those magnitudes are smaller than that of one-minute price change $(P_2 - P_1)/P_1$. Thus, the static analysis claims that the effect of the news announcement on the financial market is within one minute. However, the price change $(P_4 - P_3)/P_3$, whose magnitude is greater than that of $(P_2 - P_1)/P_1$, shows that price volatility remains significantly higher than normal within three minutes after the announcement. Consequently, we find that the effects of announcements on price volatility persist longer than the maximum time declared by the static analysis. Because the asset price changes from P_1 to P_2, P_3 and P_4 are attributed to the news announcement, the static analysis cannot properly examine the eventual impact of news announcements on the financial market.

Furthermore, in terms of static price change, two scenarios are indistinguishable. In the first scenario, the asset price is assumed to be P_1 one minute before an announcement. When the announcement is released, the asset price surges to the threshold price P_2, which reflects the significant response of the market to the announcement and remains constant for four minutes. According to the static analysis, we fix the time before the announcement. As a result, the one-minute price change $(P_2 - P_1)/P_1$ is identical to the two-minute price change, and so on. The static analysis shows the impact of the news announcement on the financial market within five minutes, although it occurs within one minute. In the second scenario, the asset price is still assumed to be P_1 one minute before an announcement. The announcement induces a linear increase in the price to P_2 in the fourth minute. As a result, the price changes by different time intervals are distinct. The impact of the news announcement on the financial market is within five minutes. These two different scenarios are shown in Figure [D.7B]. They are deemed to be the same when we regress the five-minute price changes on announcement surprises.

Consequently, these examples motivate us to better aggregate the data to capture the eventual impact of news announcements on stock prices. Consider the first example again. If a mathematical operation called first-difference is applied to the one-minute price change, the price change from the first one minute to the second one minute is $(P_3 - P_2)/P_2 - (P_2 - P_1)/P_1$. Its magnitude is smaller than that of the first one minute ($(P_2 - P_1)/P_1$). However, the price change from the second one minute to the third one minute is $(P_4 - P_3)/P_3 - (P_3 - P_2)/P_2$, which has a magnitude greater than that of the first one minute. Thus, we conclude that it is within three minutes by using the first-difference to investigate the speed of the eventual impact, which is consistent with reality. Moreover, when this type of temporal aggregation is applied to the second example, the two scenarios are easily distinguished.

5.4.4.1 Why Do We Use Wavelets?

The first-difference is a type of filter that performs mathematical operations to rearrange a data structure. In empirical works in economics and finance, the required frequency of observations is commonly not available because it is very expensive or not possible to collect data in the required frequency for particular variables. However, there is no reason to believe that data collected in the required frequency would be able to fully capture the movement of the economy. To solve this issue, a mathematical method referred to as temporal aggregation is required. The implicit assumption of this method is that the underlying stochastic process in continuous time is observed in discrete intervals. When the required frequency of observations is not available, the temporal aggregation is applied to obtain the ideal frequency of data.

Consider a case in which monthly observations of an economic variable are collected from the market. However, quarterly data for this variable are actually required to study an empirical issue. A simple and frequently used way of converting the observations would be to take the sums or the averages of successive sets of three months. This process is equivalent to subjecting the data to a three-point moving sum or average and then subsampling the resulting sequence by picking one in every three points. However, a problem called "aliasing" arises with this procedure. Aliasing refers to an effect that leads to different data sequences that are indistinguishable when sampled. For instance, in the second example, the five-minute price changes in the two scenarios are the same. We cannot use them to distinguish the groups to which they belong.

To avoid the aliasing problem, it is appropriate to use a filter that can better aggregate data without creating this problem and losing any data points in the process. Although several filters fulfil this condition, they cannot make the filtered data linearly independent on different time scales. Linear independence means that when price changes on different time scales are applied in an OLS regression model that studies a linear relationship between price changes and announcement surprises, the results do not affect each other. Otherwise, they are ambiguous. As a result, the time profile for a news announcement's impact on the stock price is revealed. Furthermore, news announcements cause jumps in asset prices (Andersen *et al.* (2003)), which requires the filter to maintain this feature when processing data. Fortunately, wavelet theory provides this type of filter. Wavelets literally mean small waves because they have finite length and are oscillatory. Wavelets on a finite support begin at a point in time and then die out at a later point in time. Their localised nature enables them to be used to analyse episodic variations in the frequency composition of data; thus, they are referred to as a "mathematical microscope". Consequently, wavelets have the ability to isolate jumps at different time scales. The filters, which are based on a Fourier transform, are not appropriate here because they smooth jumps.

There are two different filters in the wavelet theory: the wavelet filter ($\{h_k\}$) and the scaling filter ($\{g_k\}$). Generally, there is a relationship between these filters

$$g_k = (-1)^{k+1} h_{L-k-1}, \tag{5.18}$$

and an inverse relationship

$$h_k = (-1)^k g_{L-k-1}, \tag{5.19}$$

where L, the width of filter, must be even. $\{g_k\}$ is referred to as the "quadrature mirror filter" (QMF), corresponding to $\{h_k\}$. The scaling filter g_k is a lowpass filter that preserves the contents of the signal at a low frequency and discards the contents at a high frequency, whereas the wavelet filter h_k is a highpass filter that retains only the high-frequency components. The wavelet filter and the scaling filter should fulfil three conditions, respectively:

$$\begin{gathered} \sum h_k = 0,\ \sum h_k^2 = 1,\ \sum h_k h_{k+2m} = 0 \quad (m \neq 0), \\ \sum g_k = \sqrt{2},\ \sum g_k^2 = 1,\ \sum g_k g_{k+2m} = 0 \quad (m \neq 0). \end{gathered} \tag{5.20}$$

Furthermore, two more conditions are imposed on the wavelet and scaling filters:

$$\begin{gathered} \sum\nolimits_k g_k h_{k+2m} = 0 \ \ (m \neq 0), \\ \sum\nolimits_k g_k h_k = 0. \end{gathered} \tag{5.21}$$

These conditions guarantee that a data sequence can be decomposed orthogonally into components by time scales via discrete wavelet transform (DWT). Correspondingly, a pyramid algorithm is proposed to implement this transform. Specifically, scaling coefficients from the previous level are used as inputs and processed by wavelet and scaling filters to estimate the current level wavelet and scaling coefficients, respectively. The only exception is at the first level, in which the wavelet and scaling filters are applied to the original data sequence. It is noted that jth level is associated with frequency interval $(\pi/2^j, \pi/2^{j-1}]$. Because frequency ω is related to time horizon T: $\omega = 2\pi/T$, the jth level indicates that the relative data sequence contains information inside the time interval $[2^j, 2^{j+1})$. Wavelet and scaling coefficients can be used to recover the original data sequence or to construct subseries in specific frequency intervals. Here, we use two-channel filter banks to demonstrate wavelet decomposition and synthesis.

5.4.4.2 The Analysis of Two-Channel Filter Banks

A sequence $\{y_t, t = 0, 1, \cdots, T-1\}$, where the tth element of a column vector Y is y_t, goes through a highpass filter H_1 that is constructed by wavelet filters via a downsampling process ($\downarrow 2$) in which the odd-numbered elements of the filtered signal are discarded and the even-numbered elements are preserved. Then, the filtered and downsampled signal, which holds half the information of y_t, is stored and transmitted. Later, this signal goes through an anti-imaging highpass filter C_1, which is constructed by wavelet filters. Prior to this procedure, upsampling ($\uparrow 2$) is performed by inserting zeros between each element of the filtered and downsampled signal. Finally, $\mathbf{w_1}$, involving a half component of the signal y_t in the specific frequency band, is achieved. This process is applied for scaling filters as well, and $\mathbf{v_1}$, which contains the other half part of y_t, is derived. Therefore, the graphic of this flow path is

$$Y \longrightarrow H_1 \longrightarrow \downarrow 2 \longrightarrow \simeq \longrightarrow \uparrow 2 \longrightarrow C_1 \longrightarrow \mathbf{w}_1,$$

$$Y \longrightarrow G_1 \longrightarrow \downarrow 2 \longrightarrow \simeq \longrightarrow \uparrow 2 \longrightarrow D_1 \longrightarrow \mathbf{v}_1 \ ,$$

where the structures of matrices G_1 and D_1 are the same as matrices H_1 and C_1, respectively. Both the lowpass filter G_1 and the anti-imaging lowpass filter D_1 are constructed by scaling filters. Here, filters H_1 and G_1 are called analysis filters, and filters C_1 and D_1 are called synthesis filters. The symbol $\simeq$ represents the storage and transmission of the signal. The output signals formed by the two-channel filter banks are $\mathbf{w}_1$ and $\mathbf{v}_1$, respectively, and their combination is the original signal: $\mathbf{w}_1 + \mathbf{v}_1 = Y$.

Normally, compared to temporal notation, frequency notation is preferred to express this flow path because it can show some properties of these filters. Therefore, the highpass and lowpass flow paths are expressed, respectively, as

$$y(z) \longrightarrow H_1(z) \longrightarrow \downarrow 2 \longrightarrow \simeq \longrightarrow \uparrow 2 \longrightarrow C_1(z) \longrightarrow \mathbf{w}_1(z),$$

$$y(z) \longrightarrow G_1(z) \longrightarrow \downarrow 2 \longrightarrow \simeq \longrightarrow \uparrow 2 \longrightarrow D_1(z) \longrightarrow \mathbf{v}_1(z) \ ,$$

where z could be ω or $e^{-\mathrm{i}\omega}$, and this expression has more generality. It has been proven that the Fourier transforms of $(\downarrow 2)y_t$ and $(\uparrow 2)y_t$ are $[\varepsilon(\omega/2) + \varepsilon(\omega/2 + \pi)]/2$

and $\varepsilon(2\omega)$, respectively.[56] Because $e^{-\mathrm{i}\omega/2} = z^{1/2}$ and $e^{-\mathrm{i}(\omega/2+\pi)} = -z^{1/2}$ where $z = e^{-\mathrm{i}\omega}$, in terms of the z-transform, they can be written as

$$(\downarrow 2)y_t \longleftrightarrow [\varepsilon(z^{1/2}) + \varepsilon(-z^{1/2})]/2 \quad \text{and} \quad (\uparrow 2)[(\downarrow 2)y_t] \longleftrightarrow [\varepsilon(z) + \varepsilon(-z)]/2, \tag{5.22}$$

where "$\longleftrightarrow$" denotes the Fourier transform, and the right term is the Fourier transform coefficient.

In conclusion, in terms of the z-transform, the highpass and lowpass flow paths are expressed by two equations, respectively:

$$\begin{aligned} \mathbf{w}_1(z) &= \frac{1}{2}C_1(z)[H_1(z)y(z) + H_1(-z)y(-z)], \\ \mathbf{v}_1(z) &= \frac{1}{2}D_1(z)[G_1(z)y(z) + G_1(-z)y(-z)]. \end{aligned} \tag{5.23}$$

It is presumed that the synthesis of $\mathbf{w}_1(z)$ and $\mathbf{v}_1(z)$ is $x(z)$; thus,

$$\begin{aligned} x(z) = &\frac{1}{2}[C_1(z)H_1(z) + D_1(z)G_1(z)]y(z) \\ &+ \frac{1}{2}[C_1(z)H_1(-z) + D_1(z)G_1(-z)]y(-z). \end{aligned} \tag{5.24}$$

As $y(-z)$ is caused by aliasing from the downsampling process, it must be eliminated.[57] Here, we set $C_1(z) = -z^{-d}G_1(-z)$, and $D_1(z) = z^{-d}H_1(-z)$, where d is identical to $L-1$, and L is the width of filter. Thus, Equation (5.24) becomes

$$x(z) = \frac{z^{-d}}{2}[H_1(-z)G_1(z) - H_1(z)G_1(-z)]y(z). \tag{5.25}$$

It is noted that the aliasing term $y(-z)$ can be cancelled by any choice of $H_1(z)$ and $G_1(z)$ when the anti-imaging filters $C_1(z)$ and $D_1(z)$ are identical to $-z^{-d}G_1(-z)$ and $z^{-d}H_1(-z)$, respectively. However, a restriction on the choice of $H_1(z)$ and $G_1(z)$ is imposed so that the coefficients of the wavelet and scaling filters are mutually

[56] The details are provided in the appendix: downsampling doubles frequency and upsampling halves frequency.

[57] Aliasing may occur when a temporal sequence is Fourier transformed in terms of its frequency. This occurs because the basis of the Fourier analysis is the cosine and sine functions. Suppose the angular velocity ω exceeds the Nyquist value (π), $\omega \in (\pi, 2\pi]$, and define $\omega_* = 2\pi - \omega$, $\omega_* \in [0, \pi)$. Thus, for all values of $t = 0, \ldots, T-1$, $\cos(\omega t) = \cos(2\pi t - \omega_* t) = \cos(2\pi t)\cos(\omega_* t) + \sin(2\pi t)\sin(\omega_* t) = \cos(\omega_* t)$. Thus, the cosine functions at the frequency ω and ω_* are identical; this problem is called "aliasing". It is not possible to distinguish the angular frequency ω from the value of the cosine (or sine function). To avoid aliasing, the positive frequencies in the spectral analysis of a discrete time process are limited to the interval $[0, \pi]$. Whether an aliasing problem exists depends on the structure of the particular time series. For many econometric time series, the problem does not arise because their positive frequencies are limited to the average $[0, \pi]$.

orthogonal, including sequential orthogonal and lateral orthogonal. To demonstrate this, we assume that the width of the filter is four. Thus,

$$\begin{aligned} G_1(z) &= g_0 + g_1 z + g_2 z^2 + g_3 z^3, \\ H_1(z) &= h_0 + h_1 z + h_2 z^2 + h_3 z^3. \end{aligned} \tag{5.26}$$

Because $\{g_k\}$ is referred to as the "quadrature mirror filter" (QMF) corresponding to $\{h_k\}$, $h_k = (-1)^k g_{L-k-1}$ indicates that Equation (5.26) could be written as

$$\begin{aligned} G_1(z) &= -h_3 + h_2 z - h_1 z^2 + h_0 z^3 = z^3 H_1(-z^{-1}) = D_1(z^{-1}), \\ H_1(z) &= g_3 - g_2 z + g_1 z^2 - g_0 z^3 = -z^3 G_1(-z^{-1}) = C_1(z^{-1}), \end{aligned} \tag{5.27}$$

where

$$\begin{aligned} D_1(z) &= -h_3 + h_2 z^{-1} - h_1 z^{-2} + h_0 z^{-3} = z^{-3} H_1(-z) = G_1(z^{-1}), \\ C_1(z) &= g_3 - g_2 z^{-1} + g_1 z^{-2} - g_0 z^{-3} = -z^{-3} G_1(-z) = H_1(z^{-1}), \end{aligned} \tag{5.28}$$

$C_1(z) = H_1(z^{-1})$ and $D_1(z) = G_1(z^{-1})$ indicate that the synthesis filters are simply the reversed-sequence anti-causal versions of analysis filters. Equations (5.27) and (5.28) tell us that Equation (5.25) can be rendered as

$$\begin{aligned} x(z) &= \frac{1}{2}[H_1(z)H_1(z^{-1}) + G_1(z)G_1(z^{-1})]y(z) \\ &= \frac{1}{2}[D_1(-z)G_1(-z) + D_1(z)G_1(z)]y(z) \\ &= \frac{1}{2}[P(-z) + P(z)]y(z), \end{aligned} \tag{5.29}$$

where

$$\begin{aligned} P(-z) &= D_1(-z)G_1(-z) = H_1(z)H_1(z^{-1}), \\ P(z) &= D_1(z)G_1(z) = G_1(z)G_1(z^{-1}). \end{aligned} \tag{5.30}$$

To achieve the perfect reconstruction in which $x(z)$ is equal to $y(z)$, a condition is imposed:

$$H_1(z)H_1(z^{-1}) + G_1(z)G_1(z^{-1}) = 2. \tag{5.31}$$

This condition guarantees the perfect reconstruction of the original sequence $y(t)$ from outputs by two-channel filter banks. The terms in $H_1(z)H_1(z^{-1})$ and $G_1(z)G_1(z^{-1})$ with an odd power of z are cancelled because of the relationship between the wavelet filter $\{h_l\}$ and the scaling filter $\{g_l\}$ (Equation (5.21)). The orthogonality conditions of $\{h_l\}$ and $\{g_l\}$ (Equation (5.20)) make the terms with an even power of z equal to zero and the terms associated with a zero power of z identical to 2. Consequently, Equation (5.31) is always valid in the wavelet theory. This result is applied to the further decomposition and reconstruction in DWT as well. Thus, the sum of component signals is the original signal: $\sum_{j=1}^{J} \mathbf{w}_j + \mathbf{v}_J = Y$. In conclusion, we briefly introduce the deconstruction and the perfect reconstruction of a time series by two-channel filter banks. These two-channel filter banks offer us the entire architecture for the dyadic wavelet analysis and help us to interpret this analysis more easily.

The above introduction of dyadic wavelet analysis concentrates on frequency notation. Returning to temporal notation, we use the circulant matrix K_T to replace z in Equation (5.26), where $K_T = [e_1, e_2, \cdots, e_{T-1}, e_0]$, which is established by shifting the first column of an identity matrix ($I = [e_0, e_1, \cdots, e_{T-1}]$) to the last column, and T is the length of the original time series. The results are the filter matrices H_1 and G_1. Because $K_T^{-1} = K_T'$, the filter matrices H_1' and G_1' are associated with $H_1(z^{-1})$ and $G_1(z^{-1})$, respectively. Because $C_1(z) = H_1(z^{-1})$ and $D_1(z) = G_1(z^{-1})$ in Equation (5.28), the anti-imaging highpass filter matrix C_1 and lowpass filter matrix D_1 are equal to the transpose of the highpass filter matrix H_1 and the lowpass filter matrix G_1, respectively. The operation "downsampling" can be represented by a matrix V, where $V = \Lambda' = [e_0, e_2, \cdots, e_{T-2}]'$. This comes from the identity matrix ($I = [e_0, e_1, \cdots, e_{T-1}]$), in which the alternate rows are deleted. As introduced earlier by frequency notation, a sequence goes through a highpass filter matrix H_1 or a lowpass filter matrix G_1 and then is downsampled (V). The outputs are wavelet coefficients associated with the highpass filter or scaling coefficients associated with the lowpass filter. The wavelet and scaling coefficients at the first level are referred to as $\alpha_{(1)}$ and $\beta_{(1)}$, respectively, and are identical to

$$\begin{aligned} \alpha_{(1)} &= VH_1Y, \\ \beta_{(1)} &= VG_1Y. \end{aligned} \tag{5.32}$$

To construct the component signals $\mathbf{w}_1$ and $\mathbf{v}_1$, they are first upsampled. Matrix Λ can be used to represent this operation. Second, the upsampled coefficients go through the anti-imaging highpass filter matrix C_1 or the lowpass filter matrix D_1, in which $C_1 = H_1'$ and $D_1 = G_1'$. The results are the component signal $\mathbf{w}_1$ or $\mathbf{v}_1$, which is equal to

$$
\begin{aligned}
\mathbf{w}_1 &= H_1'\Lambda\alpha_{(1)} = H_1'\Lambda V H_1 Y,\\
\mathbf{v}_1 &= G_1'\Lambda\beta_{(1)} = G_1'\Lambda V G_1 Y.
\end{aligned} \tag{5.33}
$$

Because the synthesis of the component signals $\mathbf{w}_1$ and $\mathbf{v}_1$ is the original time series, we have

$$
Y = \mathbf{w}_1 + \mathbf{v}_1 = H_1'\Lambda V H_1 Y + G_1'\Lambda V G_1 Y. \tag{5.34}
$$

This entire process is also applied on the further decomposition and reconstruction. The only difference is that scaling coefficients from the previous level are used as inputs instead. Consequently, the wavelet coefficients associated with the jth-level wavelet filter are

$$
\alpha_{(j)} = V_j H_j V_{j-1} G_{j-1} \cdots V_1 G_1 Y, \tag{5.35}
$$

and the scaling coefficients associated with the jth-level scaling filter are

$$
\beta_{(j)} = V_j G_j V_{j-1} G_{j-1} \cdots V_1 G_1 Y, \tag{5.36}
$$

where $V_j = \Lambda_j' = [e_0, e_2, \cdots, e_{T/2^{j-1}-2}]'$, which is established by deleting the alternate rows of an identity matrix ($I_{T/2^{j-1}} = [e_0, e_1, \cdots, e_{T/2^{j-1}-1}]$).[58] The matrices H_j and G_j, in terms of polynomial expression, can be written as

$$
\begin{aligned}
H_j = H(K_{T/2^{j-1}}) &= h_0 K^0_{T/2^{j-1}} + h_1 K^1_{T/2^{j-1}} + \cdots + h_{L-1} K^{L-1}_{T/2^{j-1}},\\
G_j = G(K_{T/2^{j-1}}) &= g_0 K^0_{T/2^{j-1}} + g_1 K^1_{T/2^{j-1}} + \cdots + g_{L-1} K^{L-1}_{T/2^{j-1}},
\end{aligned} \tag{5.37}
$$

where L is the width of the filter and T is the length of original time series. It is not difficult to find that

$$
\begin{aligned}
V_j H_j V_{j-1} G_{j-1} \cdots V_1 G_1 &= V_j V_{j-1} \cdots V_1 H(K_T^{2j-2}) G(K_T^{2j-4}) \cdots G(K_T),\\
V_j G_j V_{j-1} G_{j-1} \cdots V_1 G_1 &= V_j V_{j-1} \cdots V_1 G(K_T^{2j-2}) G(K_T^{2j-4}) \cdots G(K_T),
\end{aligned} \tag{5.38}
$$

where the jth-level wavelet filter $\{h_{j,k}\}$ forms the matrix:

$$
H(K_T^{2j-2}) G(K_T^{2j-4}) \cdots G(K_T), \tag{5.39}
$$

[58] To unify the matrix expression, the matrices V and Λ in the first decomposed level are rewritten as V_1 and Λ_1, respectively.

and the jth-level scaling filter $\{g_{j,k}\}$ matrix is

$$G(K_T^{2j-2})G(K_T^{2j-4})\cdots G(K_T), \tag{5.40}$$

in which j is no smaller than 2. If $j = 1$, the first-level wavelet and scaling filters correspond to the matrices $H(K_T)$ and $G(K_T)$, respectively. Accordingly, the wavelet and scaling amplitudes can also be expressed by

$$\begin{aligned} \alpha_{(j)} &= V_j V_{j-1} \cdots V_1 H(K_T^{2j-2})G(K_T^{2j-4})\cdots G(K_T)Y, \\ \beta_{(j)} &= V_j V_{j-1} \cdots V_1 G(K_T^{2j-2})G(K_T^{2j-4})\cdots G(K_T)Y. \end{aligned} \tag{5.41}$$

Observing the component signal $\mathbf{w}_1$, we find that it is estimated by multiplying the transpose of the production of matrices V and H_1 by wavelet coefficients $\alpha_{(1)}$, which are the results of multiplying the production itself by the original time series Y. This mathematical operation also works on another component signal $\mathbf{v}_1$ with the scaling filter matrix G_1. Generally, we find that it is always valid for the component signals $\mathbf{w}_j$ and $\mathbf{v}_j$ at the jth level. Consequently, the component signal $\mathbf{w}_j$ is

$$\begin{aligned} \mathbf{w}_j &= [V_j H_j V_{j-1} G_{j-1} \cdots V_1 G_1]' \alpha_{(j)} \qquad (5.42) \\ &= [V_j V_{j-1} \cdots V_1 H(K_T^{2j-2})G(K_T^{2j-4})\cdots G(K_T)]' \alpha_{(j)}, \end{aligned}$$

and the component signal $\mathbf{v}_j$ is

$$\begin{aligned} \mathbf{v}_j &= [V_j G_j V_{j-1} G_{j-1} \cdots V_1 G_1]' \beta_{(j)} \qquad (5.43) \\ &= [V_j V_{j-1} \cdots V_1 G(K_T^{2j-2})G(K_T^{2j-4})\cdots G(K_T)]' \beta_{(j)}. \end{aligned}$$

According to the orthogonality conditions of wavelet and scaling filters on Equations (5.20) and (5.21), we have

$$\begin{bmatrix} V_j H_j \\ V_j G_j \end{bmatrix} \begin{bmatrix} (V_j H_j)' & (V_j G_j)' \end{bmatrix} = \begin{bmatrix} V_j H_j H_j' \Lambda_j & V_j H_j G_j' \Lambda_j \\ V_j G_j H_j' \Lambda_j & V_j G_j G_j' \Lambda_j \end{bmatrix} = \begin{bmatrix} I_{T/2^j} & \mathbf{0}_{T/2^j} \\ \mathbf{0}_{T/2^j} & I_{T/2^j} \end{bmatrix}, \tag{5.44}$$

and

$$\begin{bmatrix} (V_j H_j)' & (V_j G_j)' \end{bmatrix} \begin{bmatrix} V_j H_j \\ V_j G_j \end{bmatrix} = H_j' \Lambda_j V_j H_j + G_j' \Lambda_j V_j G_j = I_{T/2^{j-1}}. \tag{5.45}$$

As a result, we infer that

$$\begin{aligned} &\mathbf{v}_j'\mathbf{w}_j = 0, \\ &\mathbf{w}_j'\mathbf{w}_k = 0 \;\; (j \neq k), \\ &\sum_{j=1}^{J}\mathbf{w}_j + \mathbf{v}_J = Y, \end{aligned} \tag{5.46}$$

which illustrate the lateral orthogonality ($\mathbf{w}_j \perp \mathbf{v}_j$) and sequential orthogonality ($\mathbf{w}_j \perp \mathbf{w}_k$), respectively. A data sequence can be decomposed orthogonally into components by time scales using wavelets. Because orthogonality is a special case of linear independence, the components at different time scales are linearly independent. The component signal $\mathbf{w}_j$ is associated with the frequency interval $(\pi/2^j, \pi/2^{j-1}]$, which implies that it contains information inside the time interval $[2^j, 2^{j+1})$.

In this paper, we use the Daubechies least asymmetric (LA) wavelet filter of width 8 to orthogonally decompose one-minute price changes by ten scales.[59] The tenth scale, which indicates that the time interval of the data sequence is $[2^{10}, 2^{11})$ minutes (approximately one day change), shows the maximum change in time during the ten scales. Because the increases in price volatility persist over approximately an hour in the paper, which is longer than an hour but shorter than the one day suggested by a number of papers, price changes on this scale and smaller scales are sufficient to examine the eventual impact of news announcements on the stock market. Price changes on each scale that are estimated using wavelets are called "wavelet-scale price changes".

5.4.4.3 The Findings of News Announcements' Eventual Impact on Stock Prices

We regress the wavelet-scale price changes on the surprises in the 17 economic variables, as in the static analysis. Table [D.9] reports the estimation results of the model of the wavelet-scale price changes using different time intervals, including response coefficients and t-statistics, following the same format as Table [D.8]. The results show the subsequent adjustment of the stock market to more news announcements. In the entire sample period, there are six announcements that significantly affect the price change at the first scale. In these six announcements, this price change is positively related to unexpected components of consumer confidence, the leading index and PMI, and it is negatively related to unexpected components of

[59]This filter with this width is recommended by Percival and Walden (2000) and is widely used in the literature on wavelet analysis (Kim and In (2003), Gallegati (2008)).

PPI, new single-family home sales and consumer credit. Similarly, three announcements significantly affect the price change at the second scale. In particular, the unexpected announcements of PPI and consumer credit have a positive impact on this price change, whereas the unexpected announcement of new single-family home sales has a negative impact on this price change.

In addition, the price change at the third scale positively reacts to unanticipated changes in capacity utilisation, new single-family home sales and PMI and negatively reacts to unanticipated changes in PPI and nonfarm payrolls. The price change at the fourth scale positively reacts to IP, consumer confidence, and new single-family home sales and negatively reacts to personal consumption. The price change at the fifth scale positively reacts to civilian unemployment and consumer credit and negatively reacts to personal consumption, trade balance, and federal budget. The price change at the sixth scale positively reacts to personal consumption and negatively reacts to capacity utilisation and trade balance. The price change at the seventh scale positively reacts to personal consumption and durable goods orders and negatively reacts to IP and personal income. Finally, the price change at the eighth scale negatively reacts to CPI, personal income, durable goods orders, and consumer credit; the price change at the ninth scale negatively reacts to PPI; and the price change at the tenth scale negatively reacts to consumer confidence.

As shown in the static analysis, the wavelet analysis finds that more announcements affect the stock market conditional on the business cycle, and some have an eventual impact only for a particular type of economic period. To explore this observation further, consumer confidence, the leading index, and federal budget positively affect price change at the first scale, whereas PPI, new single-family home sales, and consumer credit negatively affect this price change in the expansion period. PMI and durable goods orders positively affect the price change and nonfarm payrolls negatively affect the price change by the same time interval in the contraction period. Consumer credit has a positive impact, whereas new single-family home sales have a negative impact on the price change at the second scale in the expansion period. Manufacturers' new orders are positively related, whereas civilian unemployment and personal consumption are negatively related to the price change at the same scale in the contraction period.

Moreover, the price change at the third scale positively reacts to new single-family home sales in the expansion period and capacity utilisation and PMI in the contraction period, and it negatively reacts to CPI, PPI, and personal consumption in the expansion period and civilian unemployment and nonfarm payrolls in the contraction period. The price change at the fourth scale positively reacts to new single-family home sales in the expansion period and PPI, IP, and PMI in the contraction period, and it negatively reacts to federal budget in the contraction period and personal consumption in both economic periods. The price change at the fifth scale positively reacts to consumer credit in the expansion period and civilian unemployment in both economic periods, and it negatively reacts to personal consumption and federal budget in the expansion period, capacity utilisation and new single-family home sales in the contraction period, and trade balance in both economic periods. The price change at the sixth scale positively reacts to federal budget in the expansion period, new single-family home sales in the contraction period, and personal consumption in both economic periods, and it negatively reacts to capacity utilisation in the expansion period and PMI and federal budget in the contraction period. The price change at the seventh scale positively reacts to PPI and manufacturers' new orders in the expansion period, nonfarm payrolls, capacity utilisation, the trade balance, consumer confidence, durable goods orders, and consumer credit in the contraction period, and federal budget in both economic periods. It negatively reacts to personal income in the expansion period and IP and the leading index in the contraction period. The price change at the eighth scale positively reacts to new single-family home sales in the contraction period, and it negatively reacts to CPI, personal income, durable goods orders, and consumer credit in the expansion period and trade balance and federal budget in the contraction period. The price change at the ninth scale positively reacts to PPI and negatively reacts to PMI in the contraction period; and the price change at the tenth scale negatively reacts to consumer confidence in the contraction period.

In sum, more news announcements cause the subsequent adjustments of the stock market when examining different stages of the economy. The eventual impact of news announcements on the stock market is quite stable conditional on the business cycle. Furthermore, the signs of the response coefficients on the same announcement surprise vary over wavelet-scale price changes by different time intervals. This finding is attributed to the market participants' subsequent adjustments to news announcements. Only some announcements significantly affect the stock price based on the static analysis. In comparison, all of the news announcements found by the wavelet analysis impose an eventual impact on the stock market over different time periods, regardless of the size of the impact. The stock market significantly reacts to more announcements through price changes at smaller time scales and reacts to fewer announcements through price changes at bigger time scales.

Kimmel (2004), and Oberlechner and Hocking (2004) find that market participants are concerned about rumours in financial markets. The financial press report these rumours, including the *Wall Street Journal*'s "Heard on the Street" and "Abreast of the Market" columns, *Business Week*'s "Inside Wall Street" column, and *SmartMoney*'s web site.[60] This reporting indicates the importance of rumours and implies that rumours can sometimes affect financial markets as much as scheduled news announcements. Thus, we believe that all reliable news announcements can have a significant impact on the market unless market participants perfectly expect them. This belief contradicts the results of previous papers, which claim that the market responds to only some announcements. As mentioned earlier, previous papers examine the market's response to news announcements based on static changes in prices, which ignores the impact of announcements on the price changes by different time scales, denoting that price change information occurs in different time intervals.

Regarding the size of the eventual impact of news announcements on stock prices, the most important announcements during the entire sample period, the expansion period, and the contraction period are shown as follows, respectively: consumer confidence, PPI, and nonfarm payrolls at the first scale; PPI, new single-family home sales, and nonfarm payrolls at the second scale; capacity utilisation, PPI, and capacity utilisation at the third scale; IP, personal consumption, and federal budget at the fourth scale; civilian unemployment, personal consumption, and new single-family home sales at the fifth scale; capacity utilisation, capacity utilisation, and new single-family home sales at the sixth scale; IP, personal income, and consumer credit at the seventh scale; and personal income, personal income, and new single-family home sales at the eighth scale. In the entire sample period and the contraction period, PPI has a larger eventual impact on the stock price change at the ninth scale, and consumer confidence is the only announcement that still significantly affects the stock price change at the tenth scale.

It is noted that the magnitudes of the response coefficients are considerably reduced over the larger time scales. In particular, the magnitudes for the price change at the sixth scale and the larger scales are smaller than 10^{-6}, whereas almost all magnitudes for the price change at the smaller scales are notably larger than 10^{-6}. These findings imply that the eventual impact of news announcements is very small over longer than an hour after the announcements because the sixth scale indicates a time interval for the data sequence of approximately an hour. Consequently, this finding provides evidence of why price volatility remains considerably higher than normal over approximately an hour but less than a day.

According to the results of the wavelet analysis over the entire sample period,

[60] *SmartMoney*'s content was merged into *MarketWatch* in 2013.

the expansion period and the contraction period, most announcements that impose a significant eventual impact on the stock market are released at 8:30 or 9:15. These announcements belong to the 8:30 & 9:15 announcements category. This result is consistent with the findings in Tables [D.5], [D.6], and [D.7] showing that the increases in price volatility persist over approximately an hour for 8:30 & 9:15 announcements, whereas they persist over a shorter time for other announcements.

5.5 Conclusion

This paper examines the impact of monthly news announcements on the price, trading volume, and price volatility of S&P 500 index futures. The market participants' responses to scheduled news announcements are viewed as information processing in financial markets. In accordance with the participants' initial analyses, they immediately react when a news announcement is released. Then, market participants adjust their investing decisions by observing the market's subsequent performance. Accordingly, the way information spreads in the market is understood by examining market participants' responses to news announcements in two distinct stages.

In sum, the effect of news announcements includes immediate and eventual effects, which are identified by price volatility and trading volume by one-minute and five-minute intervals, respectively. The immediate effect is a sharp and nearly instantaneous price change along with a rise in trading volume, and the eventual effect causes a persistent increase in price volatility and trading volume. Furthermore, the static analysis indicates which announcements immediately affect the stock price, whereas the wavelet analysis shows which announcements eventually affect the stock price. The combination of these results provides us with the time-profile for each type of news announcement's impact on the stock price and demonstrates that the impact is short lived to within a day. Although many announcements do not have an immediate impact on stock price, all announcements impose an eventual impact on stock price over different time periods.

It is important to note that price is more volatile and trading volume is lower over contraction period in comparison with expansion period on both announcement days and nonannouncement days, as shown in Figures [D.5A], [D.5B], [D.6A], and [D.6B], because market participants are more susceptible and less active in the contraction period. This result explains why the magnitudes of response coefficients in the contraction period are always greater than those in the expansion period. On the one hand, market participants react to news announcements more moderately during the contraction period. They are news insensitive and change stock prices by a smaller scale through a higher number of trading transactions. They then make smaller subsequent adjustments to stock prices along with a lower number of trading transactions across a shorter time to reconcile their different views on news announcements. On the other hand, market participants are news sensitive in the expansion period and change stock prices by a larger scale through a lower number of trading transactions. They then make larger subsequent adjustments of stock prices accompanied by a higher number of trading transactions across a longer time to reconcile their different views of news announcements.

Consequently, news announcements create larger immediate price changes per interval in the expansion period and more immediate price changes per interval in the contraction period from the old equilibrium to the approximate new equilibrium. It takes smaller subsequent adjustments of stock prices along with a lower number of trading transactions across a shorter time in the contraction period for the information contained in news announcements to be incorporated fully in stock prices. This finding implies a more efficient market in the contraction period and shows that the market participants' behaviour is conditional on the economic state. The market's response to news announcements varies over different stages of the business cycle, although the signs of the response coefficients are quite consistent which implies no difference of changes in price direction caused by news announcements in terms of upswings and downswings in the economy. The combination of the results from the examination of the two different stages of the business cycle shows that the market significantly reacts to more announcements in comparison with the results from the entire sample period because some news announcements impose a significant impact only in a particular type of economic period.

Because the arrival of scheduled news announcements brings high uncertainty into the market, market participants generally withdraw from the market prior to announcements to avoid the high risk. Price volatility and trading volume thus significantly decline prior to announcements. The "calm before the storm" effect arises 2 or 3 minutes before announcements. The financial press usually claims that this effect is observed over the days prior to the announcements, as supported by Jones *et al.* (1998). However, this finding is questioned because the effect of new announcements on financial markets does not last over a day, a result that has been found by a number of previous papers and this paper. Accordingly, we believe that the "calm before the storm" effect is only observed some minutes prior to the announcements, as found in this paper.

Chapter 6

Conclusion

The previous chapters have introduced wavelet theory in detail and have presented three new applications for wavelets in the economic and financial fields. In the second chapter, wavelet theory, including the discrete wavelet transform (DWT), multiresolution analysis (MRA), orthogonal decomposition and reconstruction of a time series by wavelets, is discussed. A thorough survey of the economic and financial applications of wavelets is presented here as well.

In the third chapter, the wavelet filter is applied in a time series to extract business cycles or trend.[61] As a symmetric filter, the wavelet filter does not cause the phase effect; thus, there are no time differences between the filtered data and the original data. Because the base functions of the wavelet filter are localised in time and in frequency, which implies that it provides a good resolution in the time domain, the wavelet filter is useful for capturing the changing volatility of business cycles. The wavelet filter's performance is compared with the performance of four traditionally used filters. Regarding the orthogonal property of the wavelet filter, its extracted business cycles and trend are linearly independent, which is often a promising result.

Note that the cut-off frequencies are required to be dyadic for the wavelet filter. In addition, we should be cautious in addressing a so-called end-sample problem when detrending a finite time series. If an appropriate approach is not used to resolve this problem, then all of the processed values would be affected. To detrend a data sequence at the beginning or at the end as well as in the middle, we should supply pre-sample and post-sample values for the symmetric filter, which can be either finite or infinite. A so-called transient effect is generated by choosing inappropriate values for the forward or backward pass. There should be no distinguishable disjunction where the beginning and end of the sample are joined. Otherwise, the distortion problem arises. In the third chapter, we interpolate a piece of pseudo-data at that

[61] In this chapter, the wavelet filter is a general notion that describes wavelets.

location to smooth the transition between the end and the beginning for the wavelet filter. These data are estimated by backcasting and forecasting the residuals, which are results in removing the polynomial trend from a data sequence.

The fourth chapter studies the presence of contagion among major world markets based on wavelets, providing new insights regarding short-run relationships among markets. A bivariate VAR-BEKK$(1, 1, 1)$ model and a Granger-causality test are applied to the results of wavelets for 27 representative global markets' daily stock-return data series from 1996.1 to 1997.12 to generate short-run pair-wise contemporaneous correlations and lead-lag relationships, respectively, both of which are involved in short-run relationships. This chapter extends the contagion literature by proposing a more precise definition of contagion and by measuring short-run relationships to distinguish contagion from interdependence.

The fifth chapter uses one-minute and five-minute data to examine the immediate and eventual effects, respectively, of monthly news announcements on the price, trading volume, and price volatility of S&P 500 index futures over various business cycles. Correspondingly, static and wavelet analyses are used to investigate which announcements immediately affect the stock price and which eventually affect the stock price, respectively. This study reveals how the U.S. stock market responds to scheduled news announcements and how the behaviours of market participants vary over the business cycle.

The last three chapters present promising results. In terms of the Monte Carlo simulation, the third chapter indicates that the Baxter-King bandpass filter, the wavelet filter and the digital butterworth filter are dominant in extracting business cycles from annual data, quarterly data and monthly data, respectively. Moreover, from the perspective of estimating a trend, the Baxter-King bandpass filter outperforms the other filters in annual and quarterly data, and the digital butterworth filter generates the best trend from a monthly data sequence. However, the Baxter-King bandpass filter and the digital butterworth filter are not appropriate for the current analysis because the former discards the first and last sample values to avoid a so-called distortion problem and because the latter results in wide deviations at the end of the estimated data sequence. However, this issue does not arise with the wavelet filter, which makes this method more appealing to economists.

Moreover, the base functions of the wavelet filter are localised in time and frequency, and can be stretched and translated with a flexible resolution to capture features that are local in both time and frequency. By contrast, the sine and cosine functions that are the base functions of the Fourier transform are localised only in frequency. Accordingly, compared with the other four filters estimated based on the

Fourier transform, the wavelet filter provides a better resolution in the time domain that is more useful for capturing the changing volatility of business cycles. In addition, the most attractive property of the wavelet filter is that its extracted trend and business cycles are linearly interdependent, which facilitates the investigation of certain issues.

The empirical findings of the fourth chapter illustrate that there is no contagion but only interdependence in the majority of markets, especially for markets in the same region as the shock-hit market. Consequently, the regional view of contagion is not supported. Shocks increase the visibility of the normal transmission mechanism that always exists during tranquil periods. Contagion is merely an illusion of interdependence. This study thus distinguishes between contagion and interdependence, which are believed to be identical in many works (Fratzscher (2003)).

The fifth chapter shows that the effects of news announcements on the stock market comprise both immediate and eventual effects, which are identified based on price volatility and trading volume using one-minute and five-minute intervals, respectively. The immediate effect generates a sharp and nearly instantaneous price change along with a rise in trading volume, and the eventual effect causes a persistent increase in price volatility and trading volume within approximately one hour. The static analysis indicates that only 6 of 17 announcements have a significant immediate effect, whereas the wavelet analysis shows that all announcements have an eventual effect over different time periods. The combination of the results of both analyses provides us with a time profile for the effect of each type of news announcement on stock prices and shows that the effect is significant within approximately one hour but dissipates after a day. The empirical findings also demonstrate that price volatility and trading volume significantly decline prior to announcements. The "calm before the storm" effect arises 2 or 3 minutes before announcements are released.

Furthermore, the market's response to news announcements varies over different stages of the business cycle. News announcements create larger immediate price changes per interval during the expansion period and more immediate price changes per interval during the contraction period from the old equilibrium to the approximate new equilibrium. During the contraction period, smaller subsequent adjustments of stock prices and fewer trading transactions across a shorter time period are needed for the information contained in news announcements to be fully incorporated into stock prices.

In sum, this thesis further extends the application of wavelets to the economics and finance fields. The analyses based on wavelets provide new insights into the study of three interesting issues and show promising results. The ability to work with non-stationary data and to study characteristics, relationships or structures in the time-frequency space makes wavelets a useful tool for exploring economic and financial problems from different perspectives. Wavelets thus have significant potential for empirical economic and financial research.

Although wavelets have been used widely in economics and finance over the last two decades, four categories of their applications summarised by Ramsey (2002) could still be explored further:

1. Exploratory analysis - time scale versus frequency. Many time-domain econometric methodologies are presented for time-varying characteristics of data. However, there is no frequency-domain methodology for frequency-varying characteristics of data. For example, an event causes structural breaks or jumps in frequency components of a time series. Wavelets enable us to detect them.

2. Density estimation and local inhomogeneity. As Lee and Hong (2001), Hong and Kao (2004), and Duchesne (2006a,b) show that wavelet estimators are superior to kernel estimators whenever there are local inhomogeneity, I speculate that many statistical tests could be re-designed using wavelets to replace kernel functions for data that spatial inhomogeneities are embodied in.

3. Time-scale decomposition. It is estimated on a recognition that certain economic issues could be addressed more easily by study relationships between economic and/or financial variables at the disaggregate (scale) level rather than at an aggregate level. Distinguishing financial contagion from interdependence in the fourth chapter is based on this idea. Regarding the complexity of relationships between economic and/or financial variables, wavelets have a promising future in this kind of application.

4. Forecasting by scale. The idea that is decomposing a time series into different time-scale components and then adopting corresponding methodologies to forecast them has already been realised. The results show that wavelets enable to enhance the forecasting. This method is especially useful for data within local inhomogeneity. Moreover, we can use this method to forecast the permanent components of a time series that is contaminated by local noises. For instance, an earthquake temporarily affects a year's GDP data. Wavelets enable to improve the forecast of economic growth trend by removing this effect.

Besides these four categories of economic and financial applications of wavelets, another important application of wavelets is related to wavelet coefficients. Given that wavelets are localised in time and scale (frequency), wavelet coefficients are accordingly concentrated in time and scale. If wavelet coefficients are independent (simulation studies by Whitcher (1998) and Whitcher *et al.* (2000) demonstrate that the decorrelation is good in terms of the test statistic), then the wavelet variance constructed by them will be time independent. This property of wavelet variance is appealing to economics and finance scholars because many econometric models implicitly assume time-independent variance. Wavelets have another advantage in that they provide a zero mean for wavelet coefficients. The zero mean avoids the issue of the bias properties of sample variance, which occurs because the mean is rarely known a priori when estimating the sample variance. In sum, it is reasonable to use wavelet coefficients to construct wavelet variance on a scale-by-scale basis to study the scaling properties or relationships between economic and/or financial variables.

The applications of wavelets in economics and finance introduced above are based on discrete wavelet transform (DWT). In recent years, the economics and financial applications of continuous wavelet transform (CWT) are gradually developed. The CWT avoids one particular problem: in most of the literature from the frequency domain, the cut-off of the frequency band is arbitrary for the analysis. The CWT provides a continuous assessment of relationships or structures, as well as other observations.

Consider the case of spectral analysis, which can identify periodicities in data. The power spectrum is estimated using the Fourier transform; therefore, spectral analysis has the same problems as the Fourier transform. The results based on spectral analysis are misleading when the time series is not stationary. Consequently, spectral analysis is unable to detect transient and irregular cycles and structural breaks in the periodicity of those cycles. Fortunately, wavelet spectral analysis can serve this purpose. Wavelet spectral analysis is analogous to spectral analysis but uses the CWT rather than the Fourier transform. Because wavelets yield frequency and time information simultaneously, the wavelet power spectrum varies over time and across frequencies. Wavelet spectral analysis measures the variance distribution of a time series in the time-frequency space. Changes in periodicity across time may be recorded in the wavelet power spectrum; thus, we can easily capture irregular cycles and identify time periods of different predominant cycles in the time series.

The tools within the CWT used by economists include not only the wavelet power spectrum but also cross-wavelet power, cross-wavelet coherency, the wavelet phase and the wavelet phase-difference. These tools have analogous concepts in Fourier analysis but are based on the CWT rather than the Fourier transform. These tools within wavelet analysis enable us to study the time-frequency dependencies between two time series, which are considered to be important features of economic and financial data.

In conclusion, the potential applications of wavelets in economics and finance are waiting to be explored further. Wavelets lead new insights into economic and financial phenomena.

Appendix A

to Chapter 2

A.1 Sampling Theorem

The sampling theorem establishes a link between a continuous signal and a discrete signal. Under the sampling theorem, without any loss of information, a continuous time signal can be represented by a sequence of values sampled at regular intervals of T time units. However, before formally introducing the sampling theorem, we need to know the impulse function, the train of the impulse function, and the Dirac Delta function.

The unit impulse sequence, also known as the Delta sequence, is the fundamental model of discrete time and a simple signal filter, and it is described as follows:

$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{otherwise} \end{cases} . \tag{A.1}$$

If we delay the sample time by k units, then it becomes

$$\delta(t-k) = \begin{cases} 1 & t = k \\ 0 & \text{otherwise} \end{cases} . \tag{A.2}$$

Moreover, the train of the impulse function is derived from the unit impulse sequence and is as follows:

$$g(t) = \sum_{j=-\infty}^{\infty} \delta(t - jT), \tag{A.3}$$

which is both periodic and discrete. It means that the sum of the impulse disperses along the time axis at intervals of T units of time.

This unit impulse is discrete time, and accordingly, the Dirac Delta function ($\delta(t)$) is the continuous time version of it, which is expressed by

$$\delta(t) = 0 \text{ (for all } t \neq 0) \qquad \text{and} \qquad \int_{-\infty}^{\infty} \delta(t)\, dt = 1. \tag{A.4}$$

These two properties imply that $\delta(t)$ must be infinite at $t = 0$.[62] If we delay the sample time by τ units, then the expression becomes the following:

$$\delta(t-\tau) = 0 \text{ (for all } t \neq \tau) \qquad \text{and} \qquad \int_{-\infty}^{\infty} \delta(t-\tau)\, dt = 1. \tag{A.5}$$

Clearly, $\int_{-\infty}^{\infty} f(t)\delta(t-\tau)\, dt = f(\tau)$, where $f(t)$ is an integrable function. This equation illustrates the *sifting property of Dirac Delta*, which means that the value $f(\tau)$ is selected or sifted out from the continuous time signal $f(t)$.

[62]Because $\int_{-\infty}^{\infty} \delta(t)\, dt = \delta(0) \cdot 0 = 1$, $\delta(0) = \infty$.

The Fourier transform of the Dirac Delta function $\delta(t-\tau)$ is the following:

$$\varepsilon(\omega) = \int_{-\infty}^{\infty} \delta(t-\tau) e^{-\mathrm{i}\omega t}\, dt = e^{-\mathrm{i}\omega\tau}, \tag{A.6}$$

where ω is not the fundamental frequency. If $\tau = 0$, then $\varepsilon(\omega) = 1$. Thus, the Fourier transform of the impulse function, which is localised in time, disperses over the entire line.

Suppose that a function $f(t)$ is the convolution of the unit impulse $\delta(t)$ and a discrete time sequence $y(t)$. Because the transform of $\delta(t-\tau)$ is $\exp(-\mathrm{i}\omega t)$, the signal $f(t)$ and its Fourier transform $\phi(\omega)$ are expressed, respectively, as follows:

$$\begin{aligned} f(t) &= \sum_{\tau=-\infty}^{\infty} \delta(t-\tau) y(\tau), \\ \phi(\omega) &= \sum_{t=-\infty}^{\infty} y(t) e^{-\mathrm{i}\omega t}. \end{aligned} \tag{A.7}$$

The approach to estimate the Fourier transform of $f(t)$ is the discrete temporal sequence $y(t)$ associated with the Fourier transform of the unit impulse $\delta(t-\tau)$, which implies that we have managed to subsume the discrete sequence case under Fourier integral theory.[63]

Here, the impulse functions introduced above are in the time domain, and the corresponding frequency domain impulse functions are quite similar. From a mathematical perspective, these functions are indistinguishable. A frequency domain impulse at $\omega = \omega_0$ with an amplitude 2π is expressed by $\varepsilon(\omega) = 2\pi\delta(\omega - \omega_0)$; thus, its inverse Fourier transform is the following:

$$y(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} 2\pi\delta(\omega - \omega_0) e^{\mathrm{i}\omega t}\, dt = e^{\mathrm{i}\omega_0 t}. \tag{A.8}$$

This expression is the continuous time version of $y(t)$. Suppose that there is a sum of the discrete-frequency impulse $\delta(\omega - j\omega_0)$ that is displaced along the frequency axis at intervals of ω_0 units of frequency and there is its inverse Fourier transform,

[63]The Fourier integral is applied to continuous time functions whose Fourier transform in frequency is continuous as well; Fourier series, which will be mentioned subsequently, are appropriate for continuous time functions with a discrete Fourier transform. It is straightforward to find the difference between these two from the following equations: Fourier integral: $y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \varepsilon(\omega) e^{\mathrm{i}\omega t} d\omega$, and its Fourier transform is $\varepsilon(\omega) = \int_{-\infty}^{\infty} y(t) e^{-\mathrm{i}\omega t}\, dt$; Fourier series: $y(t) = \sum_{j=-\infty}^{\infty} \varepsilon_j e^{\mathrm{i}\omega_j t}$, and its Fourier transform is $\varepsilon_j = \frac{1}{T}\int_0^T y(t) e^{-\mathrm{i}\omega_j t}\, dt$.

they are expressed as follows:

$$
\begin{aligned}
\varepsilon(\omega) &= \sum_{j=-\infty}^{\infty} \varepsilon_j 2\pi\delta(\omega - j\omega_0), \\
y(t) = \sum_{j=-\infty}^{\infty} \varepsilon_j e^{\mathrm{i}\omega_0 jt} &= \sum_{j=-\infty}^{\infty} \varepsilon_j e^{\mathrm{i}\omega_j t}.
\end{aligned}
\tag{A.9}
$$

Equations (A.9) are in exactly the same form of Fourier series representation of a continuous signal. In effect, we successively evaluate the case of the continuous function under the theory of the Fourier integral. It is notable that the relationship between the Fourier integral and Fourier series can be evaluated using the sampling theorem.

Because of the periodic nature of the train of the impulse function $g(t)$, the function is extended to a Fourier series expansion:

$$g(t) = \sum_{j=-\infty}^{\infty} \gamma_j e^{\mathrm{i}\omega_j t}. \tag{A.10}$$

The coefficients of this expansion, γ_j, can be inferred by integrating over merely one cycle.

$$\gamma_j = \frac{1}{T_0}\int_0^{T_0} g(t)e^{-\mathrm{i}\omega_j t}\,dt = \frac{1}{T_0}\int_0^{T_0} \delta(t)e^{-\mathrm{i}\omega_j t}\,dt = \frac{1}{T_0}e^{-\mathrm{i}\omega_j 0} = \frac{1}{T_0}. \tag{A.11}$$

Thus, Equation (A.10) becomes the following:

$$g(t) = \frac{1}{T_0}\sum_{j=-\infty}^{\infty} e^{\mathrm{i}\omega_j t}. \tag{A.12}$$

Because the inverse Fourier transform of the frequency domain impulse $2\pi\delta(\omega - j\omega_0)$ is $\exp(\mathrm{i}\omega_j t)$, the transform of $g(t)$ is as follows:

$$\gamma(\omega) = \frac{1}{T_0}\sum_{j=-\infty}^{\infty} 2\pi\delta(\omega - j\omega_0) = \omega_0 \sum_{j=-\infty}^{\infty} \delta(\omega - j\omega_0), \tag{A.13}$$

where $\omega_0 = 2\pi/T_0$ is the fundamental frequency. This implies a discrete periodic train of the impulse function $g(t)$ in the time domain corresponding to the aperiodic train of the impulse function $\gamma(\omega)$ in the frequency domain.

At this point, we have sufficient materials and information to demonstrate the sampling theorem. As stated previously , the sampling theorem establishes the link between the continuous function and the discrete function. With the assistance of the train of the impulse function $g(t)$, we obtain the following:

$$y_s(t) = y(t)g(t) = \sum_{j=-\infty}^{\infty} y(jT_0)\delta(t - jT_0), \tag{A.14}$$

where $y(t)$ is the continuous function in the time domain, and $y_s(t)$ is the sampled discrete time function.

Observing the last term of Equation (A.14), the Fourier transform $\varepsilon_s(\omega)$ of $y_s(t)$ is the modulation of $\varepsilon(\omega)$ and $\gamma(\omega)$, which are the Fourier transforms of $y(t)$ and $g(t)$, respectively. Thus,

$$\varepsilon_s(\omega) = \gamma(\omega) * \varepsilon(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \gamma(\lambda)\varepsilon(\omega - \lambda)\, d\lambda. \tag{A.15}$$

Using Equation (A.13) to replace $\gamma(\lambda)$, we obtain the following:

$$\begin{aligned}
\varepsilon_s(\omega) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} \omega_0 \sum_{j=-\infty}^{\infty} \delta(\lambda - j\omega_0)\varepsilon(\omega - \lambda)\, d\lambda \\
&= \frac{1}{T_0}\sum_{j=-\infty}^{\infty}\int_{-\infty}^{\infty} \varepsilon(\omega - \lambda)\delta(\lambda - j\omega_0)\, d\lambda \\
&= \frac{1}{T_0}\sum_{j=-\infty}^{\infty} \varepsilon(\omega - j\omega_0).
\end{aligned} \tag{A.16}$$

An alternative approach to obtain the Fourier transform of $y_s(t)$ is directly inferred from the definition. We use Equation (A.12) to substitute $g(t)$ and obtain the Fourier transform of $y_s(t)$:

$$\begin{aligned}
\varepsilon_s(\omega) &= \int_{-\infty}^{\infty} y_s(t)e^{-\mathrm{i}\omega t}\, dt \\
&= \int_{-\infty}^{\infty} [y(t)\frac{1}{T_0}\sum_{j=-\infty}^{\infty} e^{\mathrm{i}\omega_j t}]e^{-\mathrm{i}\omega t}\, dt \\
&= \frac{1}{T_0}\int_{-\infty}^{\infty}\sum_{j=-\infty}^{\infty} y(t)e^{-\mathrm{i}t(\omega-\omega_j)}\, dt \\
&= \frac{1}{T_0}\sum_{j=-\infty}^{\infty} \varepsilon(\omega - \omega_j) \\
&= \frac{1}{T_0}\sum_{j=-\infty}^{\infty} \varepsilon(\omega - j\omega_0),
\end{aligned} \tag{A.17}$$

where $\omega_j = 2\pi j/T_0$, $\omega_0 = 2\pi/T_0$, and $\omega_j = j\omega_0$. This result indicates that the Fourier transform of the sampled function is a periodic function consisting of superimposed copies of the transform of the continuous time signal. That is, sampling in the time domain at intervals of T_0 units replicates the spectrum of the original signal successively through the frequency range $(-\infty, \infty)$ at intervals of ω_0 radians. Caution must be exercised regarding the problem of aliasing. If the original signal $y(t)$ has a frequency band $[0, \omega_c]$, then the sampled frequency ω_0 must be larger than $2\omega_c$ to avoid this problem.

A.2 Downsampling Doubles Frequency

For a sequence $X(t)$, the output of downsampling $X(t)$ is $X(2t)$, which can be expressed by the following:

$$\begin{aligned} X(2t) &= \frac{1}{2\pi}\int_{-\pi}^{\pi} \varepsilon(\omega)e^{\mathrm{i}\omega\cdot 2t}\,d\omega \\ &= \frac{1}{4\pi}\int_{-2\pi}^{2\pi} \varepsilon(\frac{\lambda}{2})e^{\mathrm{i}\lambda t}\,d\lambda \\ &= \frac{1}{4\pi}\int_{-2\pi}^{-\pi} \varepsilon(\frac{\lambda}{2})e^{\mathrm{i}\lambda t}\,d\lambda + \frac{1}{4\pi}\int_{\pi}^{2\pi} \varepsilon(\frac{\lambda}{2})e^{\mathrm{i}\lambda t}\,d\lambda + \frac{1}{4\pi}\int_{-\pi}^{\pi} \varepsilon(\frac{\lambda}{2})e^{\mathrm{i}\lambda t}\,d\lambda \\ &= \frac{1}{4\pi}\int_{0}^{\pi} \varepsilon(\frac{\lambda}{2}-\pi)e^{\mathrm{i}\lambda t}\,d\lambda + \frac{1}{4\pi}\int_{-\pi}^{0} \varepsilon(\frac{\lambda}{2}+\pi)e^{\mathrm{i}\lambda t}\,d\lambda + \frac{1}{4\pi}\int_{-\pi}^{\pi} \varepsilon(\frac{\lambda}{2})e^{\mathrm{i}\lambda t}\,d\lambda, \end{aligned} \tag{A.18}$$

where $\lambda = 2\omega$. Because $\varepsilon(\omega)$ is a function with period 2π, $\varepsilon(\lambda/2-\pi) = \varepsilon(\lambda/2+\pi)$, and

$$\begin{aligned} X(2t) &= \frac{1}{4\pi}\int_{-\pi}^{\pi} \varepsilon(\frac{\lambda}{2}+\pi)e^{\mathrm{i}\lambda t}\,d\lambda + \frac{1}{4\pi}\int_{-\pi}^{\pi} \varepsilon(\frac{\lambda}{2})e^{\mathrm{i}\lambda t}\,d\lambda \\ &= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\varepsilon(\frac{\lambda}{2}+\pi)+\varepsilon(\frac{\lambda}{2})}{2}e^{\mathrm{i}\lambda t}\,d\lambda. \end{aligned} \tag{A.19}$$

It is clearly shown that the frequency (λ) of the downsampled sequence $X(2t)$ is double the original sequence $X(t)$, and the Fourier coefficient is $[\varepsilon(\lambda/2)+\varepsilon(\lambda/2+\pi)]/2$.

A.3 Upsampling Halves Frequency

For a sequence $X(t)$, its upsampled sequence is $v(t)$, which implies that $v(2t) = X(t)$, and $v(2t+1) = 0$. The Fourier transform of $v(t)$ is $v(\omega)$, which can be represented by the following:

$$\begin{aligned} v(\omega) &= \sum\nolimits_t v(t)e^{-\mathrm{i}\omega t} \\ &= \sum\nolimits_t v(2t)e^{-\mathrm{i}\omega\cdot 2t} \\ &= \sum\nolimits_t X(t)e^{-\mathrm{i}\omega\cdot 2t} \\ &= \varepsilon(2\omega). \end{aligned} \tag{A.20}$$

This result indicates that the frequency of the upsampled sequence $v(t)$ is half of the original time series $X(t)$.

A.4 Shannon (Down-) Sampling Theorem

First, a special case is illustrated in which the output $(\downarrow 2)X$ is an impulse $\delta = (\cdots, 0, 1, 0, \cdots)$.[64] The question is how to obtain X from δ. In the band-limited signal ($|\omega| < \pi/2$), the frequency response of input X is

$$\varepsilon(\omega) = \begin{cases} 2 & 0 \leqslant |\omega| < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \leqslant |\omega| < \pi \end{cases}. \tag{A.21}$$

Because downsampling doubles every frequency, $(\downarrow 2)X$ has a full band of frequencies ($[0, \pi)$) in equal amounts.[65] The inverse Fourier transform is the following:

$$X(t) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \varepsilon(\omega)e^{\mathrm{i}\omega t}\, d\omega = \frac{1}{2\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2e^{\mathrm{i}\omega t}\, d\omega = \frac{2}{\pi t}\sin\frac{\pi t}{2}. \tag{A.22}$$

Therefore, the input X before downsampling has been recovered as a sinc function: $X_{\text{sinc}}(t) = \sin(\pi t/2)/(\pi t/2)$, which implies that

$$X(t) = \begin{cases} 0 & t \neq 0 \\ 1 & \text{otherwise} \end{cases}. \tag{A.23}$$

In the Shannon Sampling Theorem, the band-limited frequencies are restricted because downsampling doubles every frequency. Therefore, a signal with full band frequencies is not appropriate. Otherwise, in this case, $X = \delta$ would also be a solution.

Generally, any discrete time sequence $(\downarrow 2)X$ can be regarded as a number of

[64] The precise definition of the discrete time impulse function is $\delta(t) = 1$ (only $t = 0$), otherwise $\delta(t) = 0$.

[65] The proof is provided in the appendix.

discrete values combined with an impulse and can be written as follows:

$$(\downarrow 2)X = (X(0), 0, X(2), 0, \cdots) = X(0)\delta + X(2)\delta^2 + \cdots . \quad ^{66} \tag{A.24}$$

Because the input to yield $(\downarrow 2)X = \delta$ is a sinc function, X can be expressed by the following:

$$\begin{aligned} X(t) &= X(0)X_{\text{sinc}}(t) + X(2)X_{\text{sinc}}(t-2) + \cdots \\ &= \sum_{k=0}^{\infty} X(2k) \frac{\sin((t-2k)\pi/2)}{(t-2k)\pi/2}, \end{aligned} \tag{A.25}$$

for event t, all terms are zero, except $t = 2k$, which yields $X(t)$. When $2k$ is replaced by m, the output of $(\downarrow 2)X$ is the following:

$$X(t) = \sum_{m=0}^{\infty} X(m) \frac{\sin((t-m)\pi/2)}{(t-m)\pi/2}. \tag{A.26}$$

It is more generally applied to all cases. This ordinary Shannon Sampling Theorem indicates that the original time series can be recovered using the downsampled sequence.

A.5 A Linear Filter and Its Properties

A linear filter modifies a time series by changing the amplitudes of its components in a specific frequency interval and advancing or delaying them in time. The gain and phase functions of the filter yield these effects, respectively. For example, the transfer function of a linear filter $H(L) = \sum_j H_j L^j$ could be derived by replacing L with $e^{-\mathrm{i}\omega}$:

$$H(\omega) = \sum_{j=-\infty}^{\infty} H_j e^{-\mathrm{i}\omega j}, \tag{A.27}$$

whose polar representation is

$$H(\omega) = |H(\omega)| e^{\mathrm{i}\theta(\omega)}. \tag{A.28}$$

According to Euler's formula $e^{\mathrm{i}\omega} = \cos\omega + \mathrm{i}\sin\omega$, $H(\omega)$ is also identical to

$$H(\omega) = H^{\text{re}}(\omega) + \mathrm{i} \cdot H^{\text{im}}(\omega), \tag{A.29}$$

[66] δ^n delays the impulse n units. Fox example, $\delta^2 = (\cdots, 0, 0, 0, 0, 1, \cdots)$.

where $H^{\mathrm{re}}(\omega)$ is a cosine function with a variable ω that is the real part of $H(\omega)$, whereas $H^{\mathrm{im}}(\omega)$ is a sine function with a variable ω that is the imaginary part of $H(\omega)$. $|H(\omega)|$, the modulus of $H(\omega)$ that is identical to $\sqrt{[H^{\mathrm{re}}(\omega)]^2 + [H^{\mathrm{im}}(\omega)]^2}$, is called a gain function; $\theta(\omega)$ is called a phase function, which is equal to $\arctan\{H^{\mathrm{im}}(\omega)/H^{\mathrm{re}}(\omega)\}$. The gain function alters the amplitudes of the original time series, and the phase function imposes phase shifts on the filtered data, which implies time differences between the filtered data and the original data.

In economics, the temporal property of a time series is so important that it should be retained. Thus, a zero phase filter, which is also referred to as a symmetric filter, is preferred. The zero phase filter denotes the symmetry in the filter's weights: $H_j = H_{-j}$. Accordingly, the frequency response of the symmetric filter $H(L)$ for frequency $-\omega$ is as follows:

$$\begin{aligned} H(-\omega) &= \sum\nolimits_j H_j e^{\mathrm{i}\omega j} \\ &= \sum\nolimits_j H_{-j} e^{-\mathrm{i}\omega(-j)} \\ &= \sum\nolimits_k H_k e^{-\mathrm{i}\omega k} \quad (\text{for } k = -j) \\ &= H(\omega). \end{aligned} \tag{A.30}$$

Combining this result with Equation (A.29), we obtain $H^{\mathrm{re}}(\omega) = H^{\mathrm{re}}(-\omega)$ and $H^{\mathrm{im}}(\omega) = H^{\mathrm{im}}(-\omega)$. $H^{\mathrm{im}}(\omega)$ is a sine function that belongs to the odd functions, which implies that $H^{\mathrm{im}}(\omega) = 0$. Consequently, the value of this filter's phase function $\arctan\{H^{\mathrm{im}}(\omega)/H^{\mathrm{re}}(\omega)\}$ is identical to zero, which demonstrates why the symmetric filter is associated with a zero phase. When $e^{-\mathrm{i}\omega}$ is replaced by z in Equation (A.30), the symmetric property of the filter is expressed in terms of the z-transform function: $H(z) = H(z^{-1})$.

A.6 The Method of Daubechies

When designing the filters for a wavelet analysis, the first concern is to ensure the sequential orthogonality of a lowpass filter $G(z)$ and its complementary highpass filter $H(z)$. The focus is typically on the lowpass filter $G(z)$. It is natural to obtain the corresponding highpass filter $H(z)$ via the equation $H(z) = G(-z)$. Thus, the lateral orthogonality of $G(z)$ is automatically satisfied. The following autocorrelation function is a key part of the sequential orthogonality condition:

$$P(z) = G(z)G(z^{-1}) = \sum_{j=0}^{L-1} p_j(z^j + z^{-j}). \tag{A.31}$$

According to the orthogonality properties of the wavelet and scaling filters, a coefficient with even power should be equal to zero: $p_{2j} = 0$, $j = \pm 1, \cdots, \pm(L/2-1)$. The normalisation requirement is $p_0 = 1$. The form of this autocorrelation function could be adjusted as follows:

$$P(z) = G(z)G(z^{-1}) = (\frac{1+z}{2})^{\frac{L}{2}} W(z)(\frac{1+z^{-1}}{2})^{\frac{L}{2}}. \tag{A.32}$$

In terms of the Fourier transform, z is replaced by $e^{-\mathrm{i}\omega}$ in $P(z)$:

$$\begin{aligned} P(\omega) &= (\frac{1+e^{-\mathrm{i}\omega}}{2})^{\frac{L}{2}} W(\omega)(\frac{1+e^{\mathrm{i}\omega}}{2})^{\frac{L}{2}} \\ &= (\cos^2\frac{\omega}{2})^{\frac{L}{2}} W(\omega). \end{aligned} \tag{A.33}$$

The functions $W(z)$ and $W(-z)$ with $z = e^{-\mathrm{i}\omega}$ can be expressed as trigonometrical polynomials:

$$\begin{aligned} W(\omega) &= Q(\sin^2\frac{\omega}{2}), \\ W(\omega+\pi) &= Q(\cos^2\frac{\omega}{2}). \end{aligned} \tag{A.34}$$

Setting $y = \sin^2(\omega/2)$, we can rewrite Equation (A.33) as follows:

$$\begin{aligned} P(\omega) &= (1-y)^{\frac{L}{2}} Q(y), \\ P(\omega+\pi) &= y^{\frac{L}{2}} Q(1-y). \end{aligned} \tag{A.35}$$

According to the sequential orthogonality condition $P(z) + P(-z) = 2$, we obtain the following:

$$P(\omega) + P(\omega+\pi) = (1-y)^{\frac{L}{2}} Q(y) + y^{\frac{L}{2}} Q(1-y) = 2. \tag{A.36}$$

Finally, the function $Q(y)$ is found to be identical to the following:

$$Q(y) = 2\sum_{k=0}^{\frac{L}{2}-1} \binom{\frac{L}{2}+k-1}{k} y^k, \tag{A.37}$$

such that $P(\omega)$ is the following:

$$\begin{aligned} P(\omega) = G(\omega)G(\omega+\pi) &= 2(\cos^2\frac{\omega}{2})^{\frac{L}{2}} \sum_{k=0}^{\frac{L}{2}-1} \binom{\frac{L}{2}+k-1}{k} y^k (\sin^2\frac{\omega}{2})^k \\ &= 2(\cos\frac{\omega}{2})^{L} \sum_{k=0}^{\frac{L}{2}-1} \binom{\frac{L}{2}+k-1}{k} y^k (\sin\frac{\omega}{2})^{2k}. \end{aligned} \tag{A.38}$$

Actually, $G(\omega)G(\omega+\pi)$ is the squared gain function of $G(\omega)$ and denotes that the squared gain function of Daubechies wavelet filters is the following:

$$P(\omega) = 2(\cos\frac{\omega}{2})^{L} \sum_{k=0}^{\frac{L}{2}-1} \binom{\frac{L}{2}+k-1}{k} y^k (\sin\frac{\omega}{2})^{2k}. \tag{A.39}$$

The squared gain function of the complementary Daubechies wavelet filter $H(\omega)$ is the following:

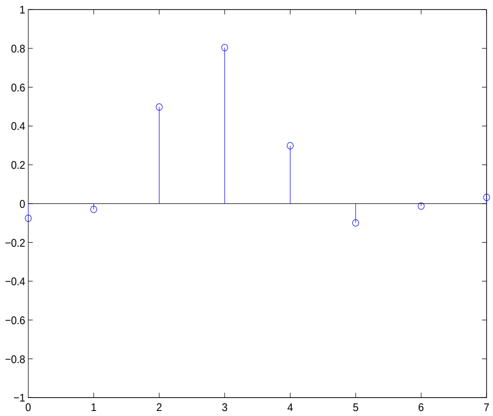
$$P(\omega+\pi) = 2(\sin\frac{\omega}{2})^{L} \sum_{k=0}^{\frac{L}{2}-1} \binom{\frac{L}{2}+k-1}{k} y^k (\cos\frac{\omega}{2})^{2k} = D^{\frac{L}{2}}(\omega)A_L(\omega), \tag{A.40}$$

where

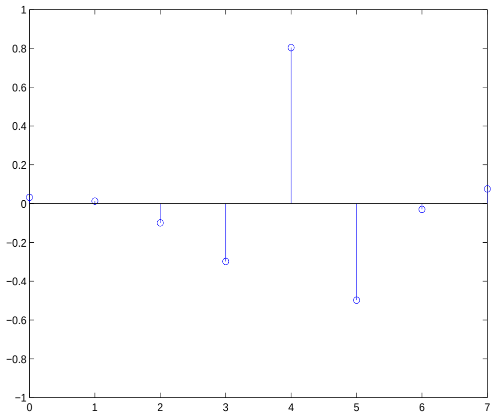
$$\begin{aligned} D(\omega) &= 4\sin^2(\frac{\omega}{2}), \\ A_L(\omega) &= \frac{1}{2^{L-1}} \sum_{k=0}^{\frac{L}{2}-1} \binom{\frac{L}{2}+k-1}{k} y^k (\cos\frac{\omega}{2})^{2k}, \end{aligned} \tag{A.41}$$

$D(\omega)$ is the squared gain function of a first-order backward difference operator $\{a_0 = 1, a_1 = -1\}$. Hence, the Daubechies wavelet filter is regarded as a combination of two filters: the first is an $L/2$th order backward difference filter, and the second is a weighted average filter that is essentially a lowpass filter. $P(\omega)$ converges to the squared gain function of an ideal lowpass filter as L increases.

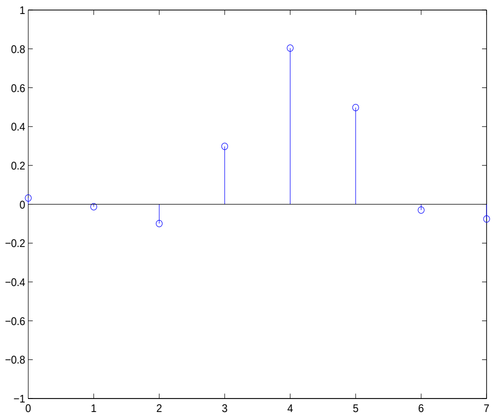
The sequence of the scaling filter $\{g_l\}$ can be estimated by factorising its squared gain function $P(\omega)$, which is known as spectral factorisation. Generally, there are two choices: extremal phase and least asymmetric. The difference in these sequences of scaling filters $\{g_l\}$ is only in their phase functions; their gain functions are the same. Percival and Walden (2000) recommend the LA family of scaling filters, with $L = 8$ in practice, based on the least asymmetric choice, which is denoted by LA(8) or Sym(8). Figure [A.1] shows the Daubechies least asymmetric wavelet filter and its corresponding scaling filter with width 8. In addition, the corresponding synthesis filters are plotted. Furthermore, the squared gain function of the Daubechies wavelet filter from the first level to the fourth level and the squared gain function of Daubechies scaling filter at the fourth level are listed in Figure [A.2]. The dotted lines are associated with the squared gain function of an ideal filter in the specific frequencies.



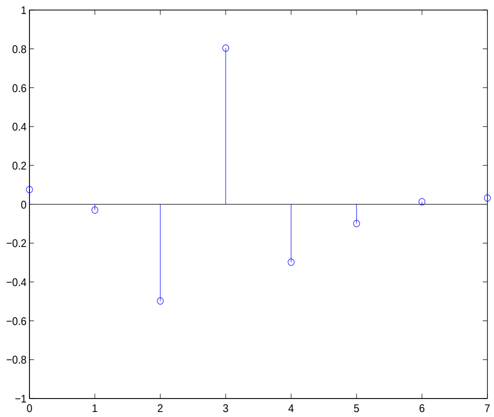
(a) Lowpass analysis filter



(b) Highpass analysis filter

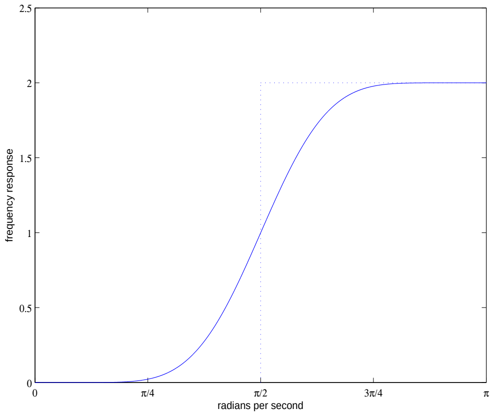


(c) Lowpass synthesis filter

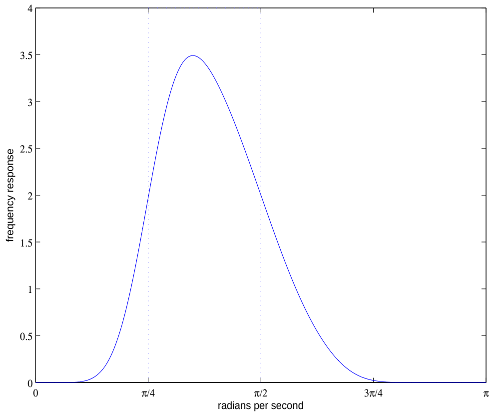


(d) Highpass synthesis filter

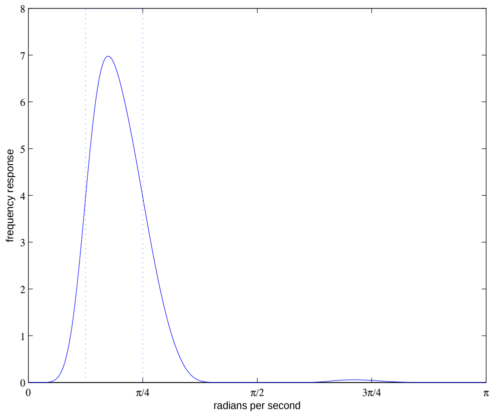
Figure A.1: Impulse response of the analysis and synthesis filters of the LA8



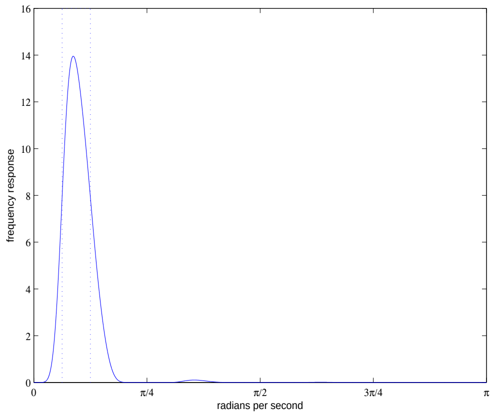
(a) Level 1 detail: 2-4 period cycles



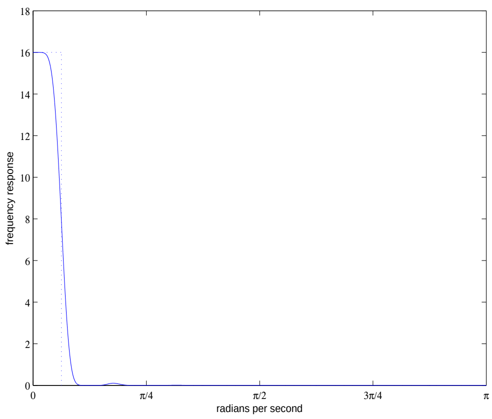
(b) Level 2 detail: 4-8 period cycles



(c) Level 3 detail: 8-16 period cycles



(d) Level 4 detail: 16-32 period cycles



(e) Level 4 approximation: 32+ period cycles

Figure A.2: **The LA8 wavelet and scaling filters in the frequency domain.** Plots (a) to (d) show the squared gain functions of the first-level to the fourth-level wavelet filters, and plot (e) illustrates the squared gain function of the fourth-level scaling filter. The dotted lines exhibit the shape of the squared gain function of an ideal bandpass filter.

Appendix B

to Chapter 3

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The cut-off frequency is $\pi/4$.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	137.0473	137.5130	137.1473	137.7064
BW	143.9370	140.9702	141.2997	142.2430
CF	144.7403	144.5198	144.8704	145.1514
HP	182.5504	184.8813	182.4425	184.5742
Wavelet	151.8003	151.4446	150.1265	151.7139
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	137.0473	137.5130	137.1473	137.7064
BW	143.4780	140.9702	156.3205	146.2913
CF	145.1738	144.5198	145.3381	145.3690
HP	181.6614	184.8813	181.5958	184.0723
Wavelet	153.1769	151.4446	151.8339	152.4812

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The cut-off frequency is $\pi/16$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	61.5497	62.3492	61.8735	62.5741
BW	77.9675	84.2701	87.2960	85.0473
CF	61.7270	63.1383	61.9888	63.1717
HP	63.2365	64.5732	63.5692	64.6400
Wavelet	62.8842	64.3365	63.7720	64.5707
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	61.5497	62.3492	61.8735	62.5741
BW	74.1059	84.2701	137.014	101.4226
CF	61.2053	63.1383	61.3718	62.7532
HP	62.1957	64.5732	62.4322	63.9578
Wavelet	62.7987	64.3365	63.6243	64.4101

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The cut-off frequency is $\pi/48$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	35.1419	35.6295	35.2329	35.8075
BW	22.3592	22.4717	22.5867	22.5849
CF	35.3029	35.9796	35.3333	36.0958
HP	28.4023	28.7733	28.5584	28.8567
Wavelet	30.3554	30.8213	30.7580	31.0149
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	35.1419	35.6295	35.2329	35.8075
BW	22.1608	22.4717	31.6862	25.8047
CF	34.9606	35.9796	34.8492	35.7456
HP	28.0730	28.7733	28.0617	28.5495
Wavelet	30.9117	30.8213	30.0825	30.9121

Table B.1: The filters are adopted to extract the trend from the original time series

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/4$ and the upper cut-off frequency is π.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	172.0446	172.8385	172.3608	173.0487
BW	174.1548	172.8014	172.8837	173.7629
CF	175.5107	175.9319	175.6882	176.3973
HP	220.6110	222.7137	221.3179	222.6406
Wavelet	181.5913	181.5053	180.5388	181.8915
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	172.0446	172.8385	172.3608	173.0487
BW	172.0009	172.8014	186.0572	176.7924
CF	174.9811	175.9319	175.2518	176.1262
HP	219.0572	222.7137	219.7843	221.8141
Wavelet	182.2907	181.5053	181.8747	182.3886

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/16$ and the upper cut-off frequency is $\pi/3$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	58.7263	59.9478	58.7834	60.0909
BW	73.8900	80.6048	83.5477	81.4214
CF	57.5403	59.3390	57.6246	59.3327
HP	71.3867	72.6663	71.4675	72.5733
Wavelet	55.3206	57.1862	55.3235	57.3596
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	58.7263	59.9478	58.7834	60.0909
BW	70.1126	80.6048	134.6369	98.4465
CF	56.8079	59.3390	56.8082	58.7762
HP	70.2407	72.6663	70.2384	71.8527
Wavelet	55.0084	57.1862	54.7001	56.9704

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/48$ and the upper cut-off frequency is $\pi/9$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	32.1790	32.7881	32.2508	33.0897
BW	15.0507	15.3092	15.3405	15.4692
CF	31.5492	32.3288	31.5018	32.5463
HP	30.7232	30.9713	30.7929	31.0590
Wavelet	26.5207	26.8816	26.7734	27.2446
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	32.1790	32.7881	32.2508	33.0897
BW	15.1795	15.3092	27.7699	20.3782
CF	31.1885	32.3288	31.0105	32.1654
HP	30.3973	30.9713	30.3497	30.7815
Wavelet	27.0595	26.8816	25.9653	27.0876

Table B.2: The filters are adopted to extract the business cycles from the original time series

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The cut-off frequency is $\pi/4$.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	137.2574	137.4294	136.8858	137.6588
BW	144.1984	141.0445	141.0390	142.2901
CF	145.0580	144.6829	144.5499	145.2355
HP	182.8203	184.6227	182.4342	184.5261
Wavelet	151.9620	151.4339	149.8745	151.6917
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	137.2574	137.4294	136.8858	137.6588
BW	143.7667	141.0445	156.1861	146.3863
CF	145.5547	144.6829	145.0676	145.4826
HP	181.9920	184.6227	181.6023	184.0514
Wavelet	153.4505	151.4339	151.5894	152.4961

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The cut-off frequency is $\pi/16$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	61.5837	62.2570	61.5634	62.4538
BW	77.4126	83.6315	86.6487	84.3767
CF	61.7293	63.0275	61.6984	63.0367
HP	63.2115	64.5090	63.2200	64.5093
Wavelet	62.9568	64.2726	63.3720	64.4633
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	61.5837	62.2570	61.5634	62.4538
BW	73.5383	83.6315	135.8925	100.6015
CF	61.1152	63.0275	61.0391	62.5799
HP	62.0830	64.5090	62.0405	63.7880
Wavelet	62.7462	64.2726	63.1820	64.2502

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The cut-off frequency is $\pi/48$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	35.0832	35.6501	35.1019	35.7684
BW	22.3269	22.5419	22.5486	22.6025
CF	35.1791	36.0051	35.2063	36.0435
HP	28.3161	28.7999	28.4488	28.8199
Wavelet	30.3455	30.8724	30.6145	31.0021
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	35.0832	35.6501	35.1019	35.7684
BW	22.1407	22.5419	31.6928	25.8354
CF	34.9107	36.0051	34.8311	35.7390
HP	28.0193	28.7999	28.0145	28.5316
Wavelet	30.9118	30.8724	30.0073	30.9129

Table B.3: The filters are adopted to extract the trend from the time series with the drift that is 10 times smaller.

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/4$ and the upper cut-off frequency is π.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	172.2843	172.6708	172.1665	172.9661
BW	174.2696	172.5997	172.7659	173.6481
CF	175.6844	175.6595	175.6596	176.2921
HP	220.9830	222.7256	220.7892	222.6051
Wavelet	181.6650	181.4698	180.4374	181.8580
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	172.2843	172.6708	172.1665	172.9661
BW	172.1472	172.5997	185.6069	176.5947
CF	175.2272	175.6595	175.1803	176.0349
HP	219.5042	222.7256	219.2309	221.7919
Wavelet	182.4991	181.4698	181.6511	182.3612

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/16$ and the upper cut-off frequency is $\pi/3$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	58.9606	59.8268	58.8866	60.1172
BW	74.4137	80.7929	83.3303	81.6260
CF	57.7454	59.3253	57.6735	59.3881
HP	71.5727	72.6169	71.5466	72.6206
Wavelet	55.6203	57.2226	55.5310	57.5055
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	58.9606	59.8268	58.8866	60.1172
BW	70.6112	80.7929	134.0183	98.4062
CF	57.0232	59.3253	56.8473	58.8377
HP	70.4623	72.6169	70.3494	71.9205
Wavelet	55.3236	57.2226	54.8656	57.1199

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/48$ and the upper cut-off frequency is $\pi/9$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	32.1183	32.7157	32.1565	32.9934
BW	15.0804	15.3162	15.4018	15.5000
CF	31.3083	32.2513	31.4437	32.4196
HP	30.6353	30.9496	30.7982	31.0223
Wavelet	26.3257	26.8886	26.8281	27.2052
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	32.1183	32.7157	32.1565	32.9934
BW	15.2020	15.3162	28.2342	20.5823
CF	30.9691	32.2513	30.9954	32.0625
HP	30.3007	30.9496	30.3391	30.7385
Wavelet	26.8350	26.8886	26.0041	27.0313

Table B.4: The filters are adopted to extract the business cycles from the time series with the drift that is 10 times smaller.

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The cut-off frequency is $\pi/4$.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	137.3119	137.7453	137.0283	137.8650
BW	144.0514	141.2767	141.1837	142.4068
CF	144.8859	144.8796	144.8075	145.3582
HP	182.9508	185.0896	182.5759	184.8340
Wavelet	151.9075	151.8566	150.0459	151.9350
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	137.3119	137.7453	137.0283	137.8650
BW	143.6172	141.2767	156.0336	146.4035
CF	145.3662	144.8796	145.2578	145.5776
HP	182.0390	185.0896	181.7255	184.3272
Wavelet	153.3369	151.8566	151.6450	152.6857

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The cut-off frequency is $\pi/16$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	61.7598	62.1322	61.5760	62.4260
BW	77.4196	83.8138	86.7458	84.5168
CF	61.9178	62.9327	61.7563	63.0330
HP	63.4628	64.3639	63.2345	64.4899
Wavelet	63.3121	64.1386	63.3430	64.4566
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	61.7598	62.1322	61.5760	62.4260
BW	73.5364	83.8138	136.5554	100.9493
CF	61.3324	62.9327	61.1209	62.6064
HP	62.3856	64.3639	62.0917	63.8088
Wavelet	63.1015	64.1386	63.2274	64.2865

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The cut-off frequency is $\pi/48$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	35.2612	35.6483	35.2911	35.8677
BW	22.4256	22.5176	22.5417	22.6144
CF	35.3908	35.9106	35.4313	36.1169
HP	28.4972	28.7435	28.5611	28.8703
Wavelet	30.5824	30.8112	30.8202	31.0882
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	35.2612	35.6483	35.2911	35.8677
BW	22.2115	22.5176	31.8048	25.8870
CF	35.0916	35.9106	35.0201	35.8115
HP	28.1787	28.7435	28.1186	28.5877
Wavelet	31.1025	30.8112	30.1714	30.9974

Table B.5: The filters are adopted to extract the trend from the time series with the drift that is 10 times larger.

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/4$ and the upper cut-off frequency is π.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	172.3708	172.8692	172.3536	173.1453
BW	174.4018	172.7698	172.8189	173.7920
CF	175.8701	175.8462	175.7040	176.4537
HP	221.3197	222.6348	221.6098	222.8586
Wavelet	181.9748	181.6795	180.4695	182.0566
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	172.3708	172.8692	172.3536	173.1453
BW	172.3016	172.7698	186.0944	176.8693
CF	175.3462	175.8462	175.3021	176.1981
HP	219.7517	222.6348	220.1392	222.0538
Wavelet	182.6792	181.6795	181.7797	182.5508

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/16$ and the upper cut-off frequency is $\pi/3$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	58.6801	59.8386	58.8619	60.0425
BW	74.2214	80.5388	83.7282	81.5273
CF	57.5565	59.2335	57.7981	59.3169
HP	71.3174	72.6287	71.4693	72.5417
Wavelet	55.2290	57.0618	55.5354	57.3059
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	58.6801	59.8386	58.8619	60.0425
BW	70.4199	80.5388	135.2085	98.6977
CF	56.8196	59.2335	56.9532	58.7661
HP	70.2727	72.6287	70.2896	71.8541
Wavelet	54.9510	57.0618	54.8516	56.9212

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/48$ and the upper cut-off frequency is $\pi/9$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	32.0690	32.6911	32.1784	32.9904
BW	15.1109	15.3472	15.3353	15.5079
CF	31.3739	32.1556	31.4234	32.3995
HP	30.6651	30.9308	30.7249	31.0099
Wavelet	26.2979	26.8633	26.6403	27.1519
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	32.0690	32.6911	32.1784	32.9904
BW	15.2585	15.3472	27.7527	20.4014
CF	31.0363	32.1556	30.9241	32.0241
HP	30.3549	30.9308	30.2588	30.7276
Wavelet	26.8176	26.8633	25.8196	26.9641

Table B.6: The filters are adopted to extract the business cycles from the time series with the drift that is 10 times larger.

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The cut-off frequency is $\pi/4$.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	135.0292	135.5601	134.9868	135.6850
BW	139.3108	138.5507	138.2343	139.1045
CF	140.6430	142.1981	140.8088	141.9393
HP	182.6742	184.9812	182.8827	184.7890
Wavelet	144.1490	143.9664	143.0472	144.2859
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	135.0292	135.5601	134.9868	135.6850
BW	138.6323	138.5507	151.5134	142.5804
CF	140.3886	142.1981	140.5413	141.7708
HP	181.6706	184.9812	181.8762	184.2189
Wavelet	145.1097	143.9664	144.4610	144.8785

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The cut-off frequency is $\pi/16$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	61.6125	62.0767	61.6251	62.3623
BW	77.4868	83.5290	86.7155	84.3363
CF	61.6011	62.7123	61.8316	62.8506
HP	63.2449	64.2985	63.2979	64.4036
Wavelet	62.9906	64.0559	63.4754	64.3628
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	61.6125	62.0767	61.6251	62.3623
BW	73.5400	83.5290	136.6091	100.8577
CF	60.9744	62.7123	61.1610	62.4095
HP	62.0918	64.2985	62.1281	63.6964
Wavelet	62.7147	64.0559	63.3377	64.1668

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The cut-off frequency is $\pi/48$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	35.0480	35.3723	35.0189	35.5942
BW	21.3506	21.3601	21.3862	21.4915
CF	35.0890	35.6588	35.0509	35.7942
HP	27.9202	28.1908	27.9302	28.2822
Wavelet	29.7295	30.0221	29.8655	30.2414
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	35.0480	35.3723	35.0189	35.5942
BW	21.0476	21.3601	30.8920	24.8470
CF	34.6799	35.6588	34.5766	35.4366
HP	27.5001	28.1908	27.4811	27.9697
Wavelet	30.1738	30.0221	29.3467	30.1594

Table B.7: The filters are adopted to extract the trend from the time series with the parameter of cycles that is 10 times smaller.

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/4$ and the upper cut-off frequency is π.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	170.1498	170.9659	170.3334	171.1272
BW	170.1251	170.3184	170.0310	170.8007
CF	171.8992	173.3816	172.0088	173.3000
HP	220.5522	222.5767	221.0939	222.5102
Wavelet	174.8970	174.9555	174.1646	175.4028
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	170.1498	170.9659	170.3334	171.1272
BW	167.8102	170.3184	181.6938	173.3556
CF	170.8140	173.3816	170.9055	172.7089
HP	218.8839	222.5767	219.4442	221.6258
Wavelet	175.2659	174.9555	175.1971	175.7390

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/16$ and the upper cut-off frequency is $\pi/3$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	58.5986	59.7846	58.5775	59.9397
BW	73.4081	80.5057	83.2749	81.1880
CF	57.5490	59.2476	57.4471	59.2472
HP	71.3027	72.6004	71.3153	72.4954
Wavelet	55.2526	57.0462	55.2660	57.2612
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	58.5986	59.7846	58.5775	59.9397
BW	69.5945	80.5057	133.7927	97.9473
CF	56.7021	59.2476	56.7855	58.6974
HP	70.0863	72.6004	70.2327	71.7843
Wavelet	54.8197	57.0462	54.7557	56.8675

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/48$ and the upper cut-off frequency is $\pi/9$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	31.9130	32.5759	31.9766	32.8345
BW	13.3921	13.5037	13.3084	13.7486
CF	31.1028	31.9877	31.1033	32.1542
HP	30.2108	30.4354	30.1819	30.5095
Wavelet	25.6074	25.9388	25.7132	26.3057
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	31.9130	32.5759	31.9766	32.8345
BW	13.3911	13.5037	26.9898	19.1805
CF	30.6502	31.9877	30.6316	31.7484
HP	29.7943	30.4354	29.7528	30.2101
Wavelet	25.9437	25.9388	25.0458	26.1289

Table B.8: The filters are adopted to extract the business cycles from the time series with the parameter of cycles that is 10 times smaller.

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The cut-off frequency is $\pi/4$.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	279.6456	283.2578	279.5625	281.8704
BW	392.6987	315.9696	324.2367	338.8078
CF	372.4385	321.8072	372.3950	348.1383
HP	185.5313	185.4603	185.5501	186.3767
Wavelet	498.6185	504.8087	482.1977	498.0276
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	279.6456	283.2578	279.5625	281.8704
BW	400.3066	315.9696	425.8094	369.3254
CF	399.4302	321.8072	399.5365	363.7631
HP	197.1033	185.4603	197.2038	192.2976
Wavelet	513.6700	504.8087	493.6340	504.4887

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The cut-off frequency is $\pi/16$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	66.1936	67.3305	66.9415	67.5638
BW	85.0901	84.5249	93.3631	88.2582
CF	69.3506	69.4110	69.4960	70.0968
HP	65.3882	66.4168	65.5996	66.6246
Wavelet	67.4850	67.0310	65.3414	67.4190
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	66.1936	67.3305	66.9415	67.5638
BW	85.0780	84.5249	147.9779	107.9028
CF	72.0463	69.4110	70.4408	70.9829
HP	67.4442	66.4168	64.8557	66.8228
Wavelet	67.4442	66.4168	64.8557	66.8228

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The cut-off frequency is $\pi/48$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	46.8677	48.9736	48.4184	48.8816
BW	70.5336	74.8394	75.4303	73.9432
CF	53.0430	51.8844	53.1305	53.1685
HP	59.7766	63.3048	63.9742	62.7872
Wavelet	69.6819	76.3871	75.7406	74.7719
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	46.8677	48.9736	48.4184	48.8816
BW	73.5337	74.8394	76.2136	74.9951
CF	59.4665	51.8844	54.0386	55.4182
HP	63.6939	63.3048	62.6308	63.3320
Wavelet	74.8421	76.3871	70.6200	74.3661

Table B.9: The filters are adopted to extract the trend from the time series with the parameter of cycles that is 10 times larger.

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/4$ and the upper cut-off frequency is π.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	298.4975	301.8130	298.3752	300.5918
BW	405.1267	331.1956	339.3107	353.1147
CF	385.6418	336.8982	385.6457	362.1849
HP	222.8766	223.0432	223.1164	223.8261
Wavelet	508.9333	514.7245	492.7829	508.0429
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	298.4975	301.8130	298.3752	300.5918
BW	411.7537	331.1956	437.4374	382.2795
CF	411.3140	336.8982	411.5293	376.9778
HP	231.8124	223.0432	232.1927	228.3635
Wavelet	523.4256	514.7245	503.8822	514.2875

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/16$ and the upper cut-off frequency is $\pi/3$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	62.9652	64.3396	63.5001	64.5981
BW	81.9767	81.5148	90.5741	85.4795
CF	66.0659	66.3451	66.0179	67.0377
HP	73.4926	74.4104	73.6666	74.5490
Wavelet	61.2012	60.9745	58.4151	61.3738
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	62.9652	64.3396	63.5001	64.5981
BW	82.3057	81.5148	147.2714	106.0397
CF	68.8172	66.3451	67.1485	67.9399
HP	75.1476	74.4104	72.8544	74.6438
Wavelet	65.8416	60.9745	57.7555	62.2526

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/48$ and the upper cut-off frequency is $\pi/9$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	42.2790	44.7530	44.1081	44.7043
BW	68.3513	73.3533	74.1235	72.3306
CF	50.6950	49.8026	51.3946	51.1984
HP	60.8928	64.3254	64.9956	63.8215
Wavelet	67.8722	75.2518	74.5326	73.4681
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	42.2790	44.7530	44.1081	44.7043
BW	71.7387	73.3533	72.3196	72.6755
CF	57.3834	49.8026	52.0287	53.4287
HP	64.7053	64.3254	63.6541	64.3447
Wavelet	73.2131	75.2518	69.3132	73.0392

Table B.10: The filters are adopted to extract the business cycles from the time series with the parameter of cycles that is 10 times larger.

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The cut-off frequency is $\pi/4$.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	124.0619	124.4564	124.1296	124.6764
BW	125.1922	123.3790	123.8339	124.4488
CF	126.8426	126.6347	126.9284	127.3151
HP	178.0898	180.0915	178.7511	180.1579
Wavelet	134.3824	134.2179	133.2739	134.5353
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	124.0619	124.4564	124.1296	124.6764
BW	123.6944	123.3790	138.3874	128.1426
CF	126.9181	126.6347	127.0103	127.3378
HP	176.6740	180.0915	177.3431	179.3993
Wavelet	135.5597	134.2179	134.8513	135.2234

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The cut-off frequency is $\pi/16$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	60.7745	61.7215	61.1733	61.8915
BW	76.8496	83.2711	86.3307	83.9869
CF	60.7701	62.2909	61.1667	62.2980
HP	62.4532	64.0344	62.8299	63.9961
Wavelet	62.0840	63.4669	62.6937	63.6790
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	60.7745	61.7215	61.1733	61.8915
BW	72.7740	83.2711	136.4358	100.5081
CF	59.9904	62.2909	60.4194	61.7841
HP	61.2625	64.0344	61.6237	63.2505
Wavelet	61.7505	63.4669	62.5045	63.4453

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The cut-off frequency is $\pi/48$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	35.1938	35.4259	35.1801	35.7059
BW	22.1620	22.3170	22.3787	22.4060
CF	35.2940	35.7604	35.2281	35.9604
HP	28.3576	28.6055	28.4308	28.7288
Wavelet	30.3168	30.6456	30.5941	30.8800
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	35.1938	35.4259	35.1801	35.7059
BW	21.9510	22.3170	31.6428	25.6842
CF	34.9706	35.7604	34.8196	35.6549
HP	28.0405	28.6055	27.9979	28.4520
Wavelet	30.8915	30.6456	29.9775	30.8063

Table B.11: The filters are adopted to extract the trend from the time series with the parameter of noise variance that is 100 times smaller.

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/4$ and the upper cut-off frequency is π.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	124.5240	124.8613	124.5290	125.0947
BW	125.5859	123.7418	124.2016	124.8222
CF	127.2372	126.9914	127.2841	127.6829
HP	178.5600	180.5036	179.1466	180.5762
Wavelet	134.7466	134.5506	133.6360	134.8829
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	124.5240	124.8613	124.5290	125.0947
BW	124.0737	123.7418	138.7150	128.4990
CF	127.3059	126.9914	127.3532	127.7009
HP	177.1407	180.5036	177.7318	179.8154
Wavelet	135.9159	134.5506	135.2062	135.5675

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/16$ and the upper cut-off frequency is $\pi/3$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	55.8077	57.0630	56.2712	57.3063
BW	71.4956	78.4223	81.4753	79.2839
CF	54.4755	56.3237	54.9023	56.3865
HP	62.5368	64.1201	62.9258	64.0820
Wavelet	53.0511	54.8806	53.5428	55.1963
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	55.8077	57.0630	56.2712	57.3063
BW	67.5084	78.4223	133.4599	96.6754
CF	53.6169	56.3237	54.0966	55.8105
HP	61.3449	64.1201	61.7161	63.3354
Wavelet	52.6644	54.8806	52.9755	54.8239

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/48$ and the upper cut-off frequency is $\pi/9$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	32.0055	32.3814	31.9850	32.7693
BW	14.8396	15.1133	15.1500	15.2667
CF	31.2687	31.8928	31.1941	32.1834
HP	28.3814	28.6297	28.4541	28.7521
Wavelet	26.0936	26.5194	26.4278	26.8744
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	32.0055	32.3814	31.9850	32.7693
BW	14.9673	15.1133	27.8540	20.3009
CF	30.9154	31.8928	30.7438	31.8305
HP	28.0641	28.6297	28.0205	28.4751
Wavelet	26.6368	26.5194	25.6506	26.7198

Table B.12: The filters are adopted to extract the business cycles from the time series with the parameter of noise variance that is 100 times smaller.

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The cut-off frequency is $\pi/4$.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	125.5061	125.8581	125.3886	126.0456
BW	127.1747	125.1818	125.4462	126.2238
CF	128.7573	128.4467	128.5476	129.0807
HP	178.9880	180.5755	178.8115	180.6415
Wavelet	136.3857	136.0350	134.7197	136.2996
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	125.5061	125.8581	125.3886	126.0456
BW	125.7730	125.1818	139.7331	129.8530
CF	128.8975	128.4467	128.6439	129.1223
HP	177.6263	180.5755	177.4115	179.8960
Wavelet	137.5710	136.0350	136.2923	136.9836

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The cut-off frequency is $\pi/16$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	61.1121	61.8322	61.1774	62.0287
BW	77.4083	83.2394	86.2547	84.0540
CF	61.1342	62.4228	61.2170	62.4600
HP	62.7923	64.1274	62.8267	64.1215
Wavelet	62.3212	63.7789	62.5982	63.8684
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	61.1121	61.8322	61.1774	62.0287
BW	73.2704	83.2394	136.4953	100.6148
CF	60.4057	62.4228	60.4257	61.9539
HP	61.6217	64.1274	61.6238	63.3855
Wavelet	62.0949	63.7789	62.4230	63.6479

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The cut-off frequency is $\pi/48$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	35.1975	35.5620	35.1176	35.7587
BW	22.2144	22.3868	22.4163	22.4642
CF	35.2564	35.9114	35.2083	36.0232
HP	28.3775	28.7186	28.4418	28.7955
Wavelet	30.3695	30.7596	30.6528	30.9627
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	35.1975	35.5620	35.1176	35.7587
BW	21.9831	22.3868	31.4484	25.6318
CF	34.9313	35.9114	34.7940	35.6979
HP	28.0494	28.7186	27.9937	28.5008
Wavelet	30.9116	30.7596	30.0160	30.8680

Table B.13: The filters are adopted to extract the trend from the time series with the parameter of noise variance that is 10 times smaller

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/4$ and the upper cut-off frequency is π.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	129.7843	130.1249	129.5391	130.3042
BW	130.9133	129.0531	129.2235	130.0639
CF	132.5197	132.2381	132.2258	132.8603
HP	183.3377	184.8577	182.9713	184.8946
Wavelet	140.0117	139.6458	138.3024	139.9214
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	129.7843	130.1249	129.5391	130.3042
BW	129.3146	129.0531	143.2492	133.5573
CF	132.5329	132.2381	132.2179	132.8408
HP	181.9034	184.8577	181.5278	184.1211
Wavelet	141.1093	139.6458	139.8419	140.5743

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/16$ and the upper cut-off frequency is $\pi/3$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	56.4237	57.4050	56.4472	57.6734
BW	72.2784	78.5308	81.5653	79.5105
CF	55.1066	56.6866	55.1618	56.7806
HP	63.6677	64.9974	63.7359	64.9880
Wavelet	53.4692	55.4024	53.5922	55.5695
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	56.4237	57.4050	56.4472	57.6734
BW	68.2263	78.5308	133.7203	96.9458
CF	54.2846	56.6866	54.2932	56.2044
HP	62.4824	64.9974	62.5193	64.2468
Wavelet	53.1837	55.4024	53.0045	55.1919

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/48$ and the upper cut-off frequency is $\pi/9$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	32.0070	32.5420	31.8867	32.8256
BW	14.8964	15.1894	15.1714	15.3282
CF	31.2234	32.0721	31.1556	32.2552
HP	28.6128	28.9449	28.6657	29.0191
Wavelet	26.1818	26.6778	26.4886	26.9854
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	32.0070	32.5420	31.8867	32.8256
BW	14.9955	15.1894	27.6073	20.2140
CF	30.8714	32.0721	30.7040	31.8806
HP	28.2877	28.9449	28.2187	28.7265
Wavelet	26.6791	26.6778	25.6841	26.8032

Table B.14: The filters are adopted to extract the business cycles from the time series with the parameter of noise variance that is 10 times smaller.

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The cut-off frequency is $\pi/4$.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	221.6521	222.9630	221.2367	223.4942
BW	256.0715	248.7985	246.6210	251.6313
CF	253.3874	255.0373	252.6960	255.5961
HP	220.2442	221.6259	219.6433	222.0490
Wavelet	258.4161	259.3073	255.2111	259.6746
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	221.6521	222.9630	221.2367	223.4942
BW	259.9793	248.7985	266.6231	258.0632
CF	255.5690	255.0373	254.9009	256.6434
HP	222.5999	221.6259	222.0324	223.2136
Wavelet	260.8959	259.3073	257.3441	260.7848

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The cut-off frequency is $\pi/16$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	65.3430	66.0790	65.2942	66.2764
BW	82.6153	89.4878	90.1867	89.6334
CF	66.9117	68.2038	67.0281	68.2690
HP	66.7466	67.8789	66.7064	67.9616
Wavelet	68.3940	69.3936	68.6512	69.6907
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	65.3430	66.0790	65.2942	66.2764
BW	80.4386	89.4878	140.5897	106.2736
CF	67.7574	68.2038	67.8872	68.5743
HP	66.5564	67.8789	66.5084	67.7211
Wavelet	68.8652	69.3936	69.2173	69.8383

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The cut-off frequency is $\pi/48$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	35.4631	35.7111	35.4980	35.9898
BW	23.9633	23.9861	23.9882	24.1029
CF	35.6575	36.1484	35.6890	36.3477
HP	28.9358	29.2077	29.0636	29.3279
Wavelet	31.0582	31.3599	31.3209	31.5926
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	35.4631	35.7111	35.4980	35.9898
BW	24.1550	23.9861	33.1211	27.4130
CF	35.6810	36.1484	35.5702	36.2328
HP	28.6813	29.2077	28.6931	29.0887
Wavelet	31.5828	31.3599	30.7366	31.5136

Table B.15: The filters are adopted to extract the trend from the time series with the parameter of noise variance that is 10 times larger.

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/4$ and the upper cut-off frequency is π.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	397.1558	398.8292	396.8339	399.2572
BW	402.9761	401.6658	400.0938	402.9929
CF	404.1151	406.0848	403.7713	406.4209
HP	448.9787	450.9714	448.2906	451.1223
Wavelet	408.4534	408.7886	406.2623	409.4724
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	397.1558	398.8292	396.8339	399.2572
BW	398.0342	401.6658	414.8510	405.5746
CF	401.7101	406.0848	401.5122	405.1729
HP	446.3498	450.9714	445.7583	449.7482
Wavelet	408.2784	408.7886	407.1955	409.6060

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/16$ and the upper cut-off frequency is $\pi/3$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	79.2066	80.0530	79.1014	80.4314
BW	92.9745	98.6511	100.7533	99.3650
CF	79.2165	80.5561	79.1327	80.7845
HP	124.3310	125.2399	124.5536	125.3036
Wavelet	71.5103	72.9713	71.1994	73.3453
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	79.2066	80.0530	79.1014	80.4314
BW	90.4382	98.6511	150.9323	115.5260
CF	78.9432	80.5561	78.7585	80.4601
HP	123.4695	125.2399	123.5222	124.7202
Wavelet	71.1592	72.9713	70.3570	72.8853

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/48$ and the upper cut-off frequency is $\pi/9$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	33.6729	34.1369	33.7853	34.5087
BW	16.6495	16.9123	16.9051	17.1055
CF	33.0464	33.7754	33.1524	34.0473
HP	46.2803	46.4460	46.3856	46.4990
Wavelet	28.4631	28.8511	28.8419	29.2276
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	33.6729	34.1369	33.7853	34.5087
BW	17.2005	16.9123	30.0116	22.3366
CF	32.9562	33.7754	32.8582	33.8134
HP	45.9768	46.4460	45.9891	46.2629
Wavelet	28.8694	28.8511	27.9860	29.0122

Table B.16: The filters are adopted to extract the business cycles from the time series with the parameter of noise variance that is 10 times larger.

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The cut-off frequency is $\pi/4$.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	590.9072	595.4321	590.9077	598.8825
BW	713.4270	691.0979	684.1064	701.5365
CF	701.0232	708.5082	701.7173	711.6308
HP	438.7163	441.0767	438.3170	446.6595
Wavelet	706.4465	712.2160	699.8049	714.4540
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	590.9072	595.4321	590.9077	598.8825
BW	730.8185	691.0979	733.6463	719.4401
CF	709.6492	708.5082	709.7039	715.5541
HP	455.7741	441.0767	455.0589	455.3796
Wavelet	714.2829	712.2160	705.3526	717.6278

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The cut-off frequency is $\pi/16$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	94.0433	95.7722	93.8341	96.4754
BW	117.5146	127.8873	121.7189	126.9097
CF	103.9468	105.9076	103.7068	106.8199
HP	93.5664	95.1680	93.4144	95.9858
Wavelet	105.9811	107.1221	105.4394	108.3313
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	94.0433	95.7722	93.8341	96.4754
BW	124.8276	127.8873	179.5015	147.1545
CF	112.6609	105.9076	112.3087	111.2148
HP	98.6318	95.1680	98.2869	98.3997
Wavelet	110.0074	107.1221	109.6407	110.2828

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The cut-off frequency is $\pi/48$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	38.0375	38.3459	37.8982	38.6022
BW	35.7969	35.3756	34.8607	35.7271
CF	39.4233	39.7027	39.3388	40.0420
HP	33.5767	33.9243	33.6181	34.0824
Wavelet	36.3898	37.0238	36.4889	37.1323
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	38.0375	38.3459	37.8982	38.6022
BW	38.2492	35.3756	45.5641	40.1205
CF	41.8888	39.7027	41.8932	41.5387
HP	34.1477	33.9243	34.1312	34.3325
Wavelet	36.9906	37.0238	36.6077	37.2361

Table B.17: The filters are adopted to extract the trend from the time series with the parameter of noise variance that is 100 times larger.

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/4$ and the upper cut-off frequency is π.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	1201.3204	1204.5815	1196.6195	1205.9254
BW	1218.9723	1215.4760	1208.0753	1218.7252
CF	1221.7691	1227.4625	1217.3089	1227.7660
HP	1314.8349	1319.0919	1311.6372	1320.0518
Wavelet	1228.1725	1228.5936	1219.5887	1230.4967
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	1201.3204	1204.5815	1196.6195	1205.9254
BW	1204.6999	1215.4760	1243.5876	1224.2025
CF	1214.0783	1227.4625	1209.8132	1223.7003
HP	1307.9552	1319.0919	1304.8300	1316.3985
Wavelet	1226.9495	1228.5936	1221.2581	1230.4082

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/16$ and the upper cut-off frequency is $\pi/3$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	183.7162	186.1239	183.8100	186.8760
BW	195.5582	201.6863	203.5116	202.8000
CF	187.2604	190.1907	187.7166	190.8265
HP	344.0812	345.6181	344.9178	346.1720
Wavelet	157.6630	160.7995	157.2468	161.5500
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	183.7162	186.1239	183.8100	186.8760
BW	196.5656	201.6863	259.5708	219.8112
CF	188.0902	190.1907	188.8064	190.9947
HP	342.6221	345.6181	343.5963	345.2679
Wavelet	156.9503	160.7995	155.4403	160.5614

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/48$ and the upper cut-off frequency is $\pi/9$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	46.4797	47.0580	46.4576	47.4373
BW	28.3346	27.9474	27.7071	28.5381
CF	46.8971	47.4032	46.8235	47.8327
HP	118.5997	118.8379	118.5615	118.8895
Wavelet	44.0604	44.2830	44.2726	44.8566
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	46.4797	47.0580	46.4576	47.4373
BW	30.5889	27.9474	46.7061	36.3282
CF	48.1252	47.4032	48.1181	48.4806
HP	118.3606	118.8379	118.3739	118.7088
Wavelet	43.9710	44.2830	43.4780	44.4941

Table B.18: The filters are adopted to extract the business cycles from the time series with the parameter of noise variance that is 100 times larger.

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The cut-off frequency is $\pi/4$.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	63.7970	64.5222	64.0345	64.7203
BW	79.8477	74.7832	74.4427	76.6011
CF	77.7424	76.5152	78.1528	77.8384
HP	43.8632	43.9874	43.9451	44.6220
Wavelet	84.4420	85.8731	83.2645	85.6296
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	63.7970	64.5222	64.0345	64.7203
BW	81.7403	74.7832	83.0105	79.4514
CF	79.8007	76.5152	80.1821	78.9099
HP	46.0719	43.9874	46.1245	45.7750
Wavelet	85.9602	85.8731	84.3305	86.2493

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The cut-off frequency is $\pi/16$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	9.6941	9.9538	9.7567	10.0099
BW	12.1790	12.9184	12.6085	12.9715
CF	10.7986	11.0317	10.8475	11.1342
HP	9.4851	9.6906	9.5272	9.7681
Wavelet	10.7914	10.9358	10.7045	11.0438
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	9.6941	9.9538	9.7567	10.0099
BW	13.1084	12.9184	18.8858	15.2612
CF	11.8213	11.0317	11.7421	11.6220
HP	10.1591	9.6906	10.0391	10.0610
Wavelet	11.4150	10.9358	11.1207	11.2942

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The cut-off frequency is $\pi/48$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	4.8736	5.0831	5.0171	5.0801
BW	7.5439	7.9351	7.9281	7.8809
CF	5.5440	5.4217	5.5515	5.5694
HP	6.2092	6.5519	6.5915	6.5216
Wavelet	7.2180	7.8737	7.7673	7.7363
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	4.8736	5.0831	5.0171	5.0801
BW	7.9654	7.9351	8.2231	8.0854
CF	6.3325	5.4217	5.8493	5.9141
HP	6.6294	6.5519	6.5089	6.5995
Wavelet	7.7208	7.8737	7.3145	7.7061

Table B.19: The filters are adopted to extract the trend from the time series with the parameter of trend variance that is 100 times smaller.

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/4$ and the upper cut-off frequency is π.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	122.4032	122.7677	122.5583	123.0242
BW	127.1386	124.5515	124.7382	125.6444
CF	126.7979	125.8160	126.9593	126.7413
HP	131.3362	131.7246	131.5498	131.9690
Wavelet	131.6330	131.7750	130.7883	131.8449
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	122.4032	122.7677	122.5583	123.0242
BW	126.0638	124.5515	130.8410	126.9825
CF	126.9289	125.8160	127.0922	126.8091
HP	130.8928	131.7246	131.0948	131.7257
Wavelet	132.1221	131.7750	131.3644	132.0944

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/16$ and the upper cut-off frequency is $\pi/3$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	18.5166	18.8203	18.5425	18.8689
BW	19.8851	20.2730	20.6614	20.4828
CF	19.0122	19.3031	19.0314	19.3584
HP	34.4877	34.6706	34.4602	34.6882
Wavelet	15.9831	16.2428	15.8019	16.3037
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	18.5166	18.8203	18.5425	18.8689
BW	20.1310	20.2730	26.6890	22.3835
CF	19.2227	19.3031	19.1772	19.4167
HP	34.4147	34.6706	34.3428	34.6143
Wavelet	16.1268	16.2428	15.5977	16.2501

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/48$ and the upper cut-off frequency is $\pi/9$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	5.3895	5.5995	5.5330	5.6044
BW	7.2624	7.7196	7.7878	7.6293
CF	6.1473	6.0524	6.1881	6.1930
HP	12.9862	13.1467	13.1749	13.1297
Wavelet	7.6451	8.3101	8.2667	8.1668
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	5.3895	5.5995	5.5330	5.6044
BW	7.6786	7.7196	8.1627	7.8698
CF	6.8074	6.0524	6.3675	6.4514
HP	13.1465	13.1467	13.0850	13.1409
Wavelet	8.1067	8.3101	7.7711	8.1128

Table B.20: The filters are adopted to extract the business cycles from the time series with the parameter of trend variance that is 100 times smaller.

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The cut-off frequency is $\pi/4$.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	73.8251	74.3432	73.8967	74.4889
BW	87.7577	83.2001	82.8701	84.7561
CF	86.3573	85.2243	86.3602	86.2728
HP	69.6458	70.1262	69.7816	70.3091
Wavelet	92.9959	93.9038	91.5451	93.6763
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	73.8251	74.3432	73.8967	74.4889
BW	89.2110	83.2001	92.3639	87.7534
CF	88.0227	85.2243	88.0506	87.1489
HP	70.7100	70.1262	70.8575	70.8452
Wavelet	94.3472	93.9038	92.6735	94.2768

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The cut-off frequency is $\pi/16$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	20.8401	21.0110	20.7816	21.0979
BW	26.2786	28.1830	28.8976	28.4179
CF	21.4348	21.7178	21.3562	21.7764
HP	21.2138	21.4974	21.1389	21.5485
Wavelet	21.7505	22.0032	21.7518	22.1081
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	20.8401	21.0110	20.7816	21.0979
BW	25.6839	28.1830	45.0558	33.8447
CF	21.7682	21.7178	21.6676	21.9045
HP	21.2062	21.4974	21.0855	21.4901
Wavelet	21.9878	22.0032	21.9327	22.1802

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The cut-off frequency is $\pi/48$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	11.5915	11.7272	11.6356	11.8087
BW	9.8896	10.1704	10.2132	10.1475
CF	11.9088	11.9452	11.8670	12.0788
HP	10.4169	10.6605	10.6532	10.6792
Wavelet	11.4656	11.9411	11.9184	11.9199
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	11.5915	11.7272	11.6356	11.8087
BW	10.1399	10.1704	12.4660	10.9892
CF	12.1793	11.9452	11.9108	12.1507
HP	10.5777	10.6605	10.5218	10.6569
Wavelet	11.8983	11.9411	11.4988	11.8775

Table B.21: The filters are adopted to extract the trend from the time series with the parameter of trend variance that is 10 times smaller.

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/4$ and the upper cut-off frequency is π.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	127.8355	128.3504	127.6614	128.4580
BW	132.0475	129.9027	129.6107	130.7801
CF	131.9956	131.3604	131.8573	132.0590
HP	141.9695	142.6578	141.7368	142.6614
Wavelet	136.8952	137.2735	135.8055	137.1869
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	127.8355	128.3504	127.6614	128.4580
BW	130.8472	129.9027	136.6179	132.3276
CF	132.0049	131.3604	131.9054	132.0666
HP	141.3353	142.6578	141.1225	142.3230
Wavelet	137.3842	137.2735	136.4565	137.4503

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/16$ and the upper cut-off frequency is $\pi/3$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	25.1773	25.4072	25.0586	25.5269
BW	29.5787	31.1519	32.1977	31.5218
CF	25.2888	25.6206	25.1482	25.7098
HP	39.4580	39.6309	39.3268	39.6566
Wavelet	22.7637	23.1169	22.5777	23.2652
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	25.1773	25.4072	25.0586	25.5269
BW	28.8413	31.1519	48.3326	36.7871
CF	25.2191	25.6206	25.1212	25.6366
HP	39.1781	39.6309	39.0421	39.4828
Wavelet	22.7299	23.1169	22.3312	23.1419

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/48$ and the upper cut-off frequency is $\pi/9$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	10.9659	11.1610	11.0190	11.2685
BW	8.2989	8.6978	8.7736	8.6429
CF	11.1420	11.2435	11.1265	11.4015
HP	15.4812	15.6258	15.6178	15.6257
Wavelet	10.7811	11.3321	11.3050	11.3134
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	10.9659	11.1610	11.0190	11.2685
BW	8.6605	8.6978	11.6280	9.7766
CF	11.3977	11.2435	11.1250	11.4443
HP	15.5397	15.6258	15.4826	15.5871
Wavelet	11.2093	11.3321	10.8098	11.2379

Table B.22: The filters are adopted to extract the business cycles from the time series with the parameter of trend variance that is 10 times smaller.

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The cut-off frequency is $\pi/4$.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	389.2604	390.7984	390.0717	391.4511
BW	386.5665	386.4315	387.5156	388.2675
CF	393.1554	396.6983	394.0607	396.8238
HP	562.9296	570.8244	565.2315	570.2979
Wavelet	405.8384	404.7268	404.2807	406.7177
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	389.2604	390.7984	390.0717	391.4511
BW	381.2191	386.4315	427.7873	398.0710
CF	391.2870	396.6983	392.1166	395.7608
HP	558.3963	570.8244	560.6518	567.8233
Wavelet	408.4597	404.7268	408.5220	408.4468

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The cut-off frequency is $\pi/16$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	193.6910	195.4098	192.6015	195.9711
BW	244.3351	263.5520	271.9014	265.6318
CF	193.6801	197.3513	192.5133	197.3285
HP	199.1445	202.6740	198.1200	202.6826
Wavelet	198.0775	201.3278	197.7310	201.9454
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	193.6910	195.4098	192.6015	195.9711
BW	231.2769	263.5520	429.5330	317.3868
CF	191.2381	197.3513	190.3467	195.7358
HP	195.3581	202.6740	194.4854	200.3774
Wavelet	197.0774	201.3278	197.3569	201.2562

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The cut-off frequency is $\pi/48$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	110.1286	111.4813	110.7824	112.2156
BW	67.2460	67.2398	67.4151	67.6923
CF	110.0795	112.2589	110.8239	112.7307
HP	87.9368	88.8828	88.4000	89.2471
Wavelet	93.7223	94.5911	94.5230	95.4098
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	110.1286	111.4813	110.7824	112.2156
BW	66.1972	67.2398	97.2740	78.2137
CF	108.7822	112.2589	109.3090	111.5947
HP	86.6614	88.8828	86.9153	88.2607
Wavelet	95.1381	94.5911	92.7852	95.1454

Table B.23: The filters are adopted to extract the trend from the time series with the parameter of trend variance that is 10 times larger.

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/4$ and the upper cut-off frequency is π.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	402.9229	404.6037	403.7479	405.2448
BW	398.7721	399.0436	399.9096	400.8102
CF	405.4075	409.0240	406.2390	409.1749
HP	576.4564	584.3954	578.8860	583.8078
Wavelet	417.9111	416.9088	416.2051	418.8800
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	402.9229	404.6037	403.7479	405.2448
BW	392.8562	399.0436	439.3595	410.1991
CF	403.2294	409.0240	403.9294	407.9430
HP	571.7728	584.3954	574.0559	581.2362
Wavelet	420.3046	416.9088	420.2706	420.5018

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/16$ and the upper cut-off frequency is $\pi/3$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	178.6937	181.3818	177.7728	182.1688
BW	227.8963	248.6502	257.0861	251.2092
CF	174.3920	179.1926	173.2535	179.2934
HP	201.7571	205.4156	200.9213	205.3629
Wavelet	170.1558	174.7660	169.2630	175.6871
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	178.6937	181.3818	177.7728	182.1688
BW	215.1177	248.6502	420.5221	305.6465
CF	171.7001	179.1926	170.7887	177.4606
HP	198.0249	205.4156	197.2533	203.0636
Wavelet	169.0078	174.7660	167.5467	174.5030

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/48$ and the upper cut-off frequency is $\pi/9$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	99.9575	101.9248	100.7480	102.9596
BW	42.1875	42.5406	42.4071	43.3716
CF	97.1028	100.0319	98.0231	100.7110
HP	88.6923	89.6370	89.1479	89.9861
Wavelet	80.0512	81.1829	80.9553	82.4343
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	99.9575	101.9248	100.7480	102.9596
BW	42.0648	42.5406	84.6155	60.1247
CF	95.6653	100.0319	96.3329	99.3843
HP	87.4230	89.6370	87.6634	89.0030
Wavelet	81.2464	81.1829	78.7051	81.8879

Table B.24: The filters are adopted to extract the business cycles from the time series with the parameter of trend variance that is 10 times larger.

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The cut-off frequency is $\pi/4$.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	1218.2514	1222.0215	1216.0693	1223.4778
BW	1199.0401	1203.0040	1201.9216	1206.7343
CF	1222.7693	1234.7701	1221.1837	1233.8158
HP	1786.3305	1802.8886	1776.5875	1801.1820
Wavelet	1261.3193	1254.4062	1249.1478	1260.8260
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	1218.2514	1222.0215	1216.0693	1223.4778
BW	1180.3196	1203.0040	1322.4402	1235.3043
CF	1215.2430	1234.7701	1213.5305	1229.6490
HP	1771.4708	1802.8886	1761.4996	1793.1032
Wavelet	1268.5304	1254.4062	1261.9104	1265.8704

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The cut-off frequency is $\pi/16$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	609.6496	618.3494	607.5198	618.9619
BW	772.4767	832.0477	861.4746	840.0234
CF	610.3015	622.8136	606.2472	622.2969
HP	626.6585	641.2351	624.3331	639.9555
Wavelet	623.6939	636.8545	622.9598	637.6937
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	609.6496	618.3494	607.5198	618.9619
BW	731.0110	832.0477	1358.4283	1004.0937
CF	602.3323	622.8136	598.5350	616.9928
HP	614.7454	641.2351	612.0291	632.3645
Wavelet	620.5130	636.8545	620.9108	635.1820

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The cut-off frequency is $\pi/48$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	350.4753	354.0003	350.0668	356.0514
BW	211.4117	211.4934	211.4482	212.7415
CF	350.3722	356.8549	350.3043	357.8886
HP	279.0803	281.3946	278.6710	282.3920
Wavelet	297.6313	299.5347	298.2006	302.0470
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	350.4753	354.0003	350.0668	356.0514
BW	208.0847	211.4934	308.1085	246.9251
CF	345.5887	356.8549	345.3003	354.0026
HP	274.4010	281.3946	273.9705	279.0822
Wavelet	301.4179	299.5347	292.6799	301.0200

Table B.25: The filters are adopted to extract the trend from the time series with the parameter of trend variance that is 100 times larger.

The sample size of annual data is 197. The first 3 and last 3 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/4$ and the upper cut-off frequency is π.

	beginning (4 : 50)	middle (51 : 147)	end (148 : 194)	entire (4 : 194)
BK	1223.0341	1226.4768	1220.4360	1228.0040
BW	1203.4029	1207.1277	1205.9695	1210.9234
CF	1227.1525	1238.8105	1225.1612	1237.9510
HP	1790.9574	1807.1113	1780.8454	1805.4882
Wavelet	1265.7207	1258.2327	1253.1781	1264.8573
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	1223.0341	1226.4768	1220.4360	1228.0040
BW	1184.4927	1207.1277	1326.1686	1239.3396
CF	1219.5268	1238.8105	1217.4238	1233.7399
HP	1775.9975	1807.1113	1765.7532	1797.3846
Wavelet	1272.7777	1258.2327	1265.8947	1269.8556

The sample size of quarterly data is 197. The first 12 and last 12 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/16$ and the upper cut-off frequency is $\pi/3$.

	beginning (13 : 50)	middle (51 : 147)	end (148 : 185)	entire (13 : 185)
BK	559.8732	572.1359	557.8039	573.1561
BW	718.8015	783.6578	812.8447	793.0585
CF	547.1867	563.3659	542.8592	563.1446
HP	627.5725	642.1545	625.2499	640.8539
Wavelet	533.3701	551.8859	531.5168	553.4064
	beginning (1 : 50)	middle (51 : 147)	end (148 : 197)	entire (1 : 197)
BK	559.8732	572.1359	557.8039	573.1561
BW	678.3311	783.6578	1328.5771	965.7863
CF	538.5527	563.3659	534.4799	557.1983
HP	615.6203	642.1545	612.9684	633.2591
Wavelet	529.6694	551.8859	525.4698	549.3793

The sample size of monthly data is 405. The first 36 and last 36 processed data are excluded in the top of table. The lower cut-off frequency is $\pi/48$ and the upper cut-off frequency is $\pi/9$.

	beginning (37 : 120)	middle (121 : 285)	end (286 : 369)	entire (37 : 369)
BK	318.6158	323.8301	318.2406	326.8319
BW	131.9465	132.4225	131.4061	135.0955
CF	309.7218	318.0531	309.6065	319.8163
HP	279.3037	281.6223	278.8883	282.6112
Wavelet	254.4389	256.8345	255.0871	260.8053
	beginning (1 : 120)	middle (121 : 285)	end (286 : 405)	entire (1 : 405)
BK	318.6158	323.8301	318.2406	326.8319
BW	131.4008	132.4225	267.6639	189.3563
CF	304.3884	318.0531	304.1121	315.3498
HP	274.6235	281.6223	274.2009	279.3044
Wavelet	257.3252	256.8345	247.9769	258.8657

Table B.26: The filters are adopted to extract the business cycles from the time series with the parameter of trend variance that is 100 times larger.

Appendix C

to Chapter 4

Table C.1: Developing countries, countries in transition, and newly industrialised economies: net capital flows[1] *(In billions of U.S. dollars).*

	1984 – 89[2]	1990 – 96[2]	1992	1993	1994	1995	1996	1997
Total								
Net private capital flows[3]	15.0	151.1	124.9	162.4	147.2	191.5	259.3	181.5
Net direct investment	13.1	61.7	37.4	56.2	77.9	93.6	115.9	125.6
Net portfolio investment	3.6	54.9	58.6	104.6	95.5	29.3	39.6	18.2
Other net investment	~	~	~	~	~	~	~	~
Developing countries								
Net private capital flows[3]	18.8	130.6	119.7	142.0	116.2	149.4	216.3	144.6
Net direct investment	12.1	54.6	33.8	49.5	71.9	78.8	101.6	106.2
Net portfolio investment	4.1	47.7	51.6	88.9	84.1	15.6	39.2	28.1
Other net investment	2.6	28.2	34.3	3.6	−40.0	54.6	75.1	10.1
Africa								
Net private capital flows[3]	4.5	5.3	~	2.8	9.0	10.9	12.9	6.8
Net direct investment	1.1	2.8	2.0	2.0	3.5	3.3	5.0	5.2
Net portfolio investment	−0.8	0.0	−0.7	0.8	0.4	1.9	0.6	0.2
Other net investment	4.2	2.5	−1.2	~	5.1	5.8	7.3	1.4
Asia								
Net private capital flows[3]	13.0	55.3	21.0	53.4	62.4	89.2	101.2	34.2
Net direct investment	4.5	32.2	17.6	34.1	43.4	49.6	58.9	51.1
Net portfolio investment	1.5	5.8	1.0	11.7	10.0	9.3	7.9	0.2
Other net investment	7.0	17.2	2.4	7.6	8.9	30.3	34.4	−17.0
Middle East and Europe								
Net private capital flows[3]	2.0	23.9	42.8	22.6	−1.0	12.2	19.1	15.7
Net direct investment	1.1	1.5	1.3	1.8	1.8	1.4	1.2	2.2
Net portfolio investment	4.4	13.0	21.0	15.3	12.5	11.6	5.6	4.1
Other net investment	−3.5	9.5	20.5	5.5	−15.3	−0.9	12.3	9.4
Western Hemisphere								
Net private capital flows[3]	−0.8	46.1	55.9	63.3	45.8	37.1	83.1	87.9
Net direct investment	5.4	18.1	12.9	11.6	23.2	24.6	36.6	47.7
Net portfolio investment	−1.0	28.9	30.4	61.1	61.1	−7.2	25.0	23.7
Other net investment	−5.2	−1.0	12.6	−9.4	−38.7	19.4	21.2	16.4
Countries in transition								
Net private capital flows[3]	−1.0	14.0	7.7	12.1	17.3	29.5	28.6	30.0
Net direct investment	−0.1	6.3	4.2	6.0	5.4	13.1	13.0	15.6
Net portfolio investment	~	2.3	−0.8	3.6	2.9	3.8	5.5	7.5
Other net investment	~	~	~	~	~	~	~	~
Newly industrialised economies[4]								
Net private capital flows[3]	−2.7	6.5	−2.5	8.3	13.6	12.6	14.3	6.8
Net direct investment	1.1	0.9	−0.6	0.8	0.6	1.7	1.3	3.8
Net portfolio investment	−0.4	4.9	7.8	12.1	8.5	9.8	−5.2	−17.4
Other net investment	−3.4	−1.1	−8.4	−7.1	2.8	−1.1	10.7	12.2

[1]Net capital flows comprise net direct investment, net portfolio investment, and other long- and short-term net investment flows, including official and private borrowing.
[2]Annual averages.
[3]Because of data limitations, other net investment may include some official flows.
[4] Hong Kong, Korea, Singapore, Taiwan, and Israel.
Source: IMF, "World Economic Outlook: Interim Assessment", December 1997 table 6.

Table C.2: Selected Asian economies: capital flows[1] *(In percent of GDP)*

	1983 – 88[2]	1989 – 95[2]	1991	1992	1993	1994	1995	1996	1997
China									
Net private capital flows[3]	1.2	2.5	1.7	−0.9	4.5	5.6	5.2	4.7	3.7
Net direct investment	0.4	2.9	0.9	1.7	5.3	5.9	4.8	4.6	4.3
Net portfolio investment	0.2	0.2	0.1	~	0.7	0.7	0.1	0.3	0.2
Other net investment	0.5	−0.6	0.7	−2.6	−1.5	−0.9	0.2	−0.3	−0.8
India									
Net private capital flows[3]	1.5	1.2	1.0	0.3	1.4	1.7	1.5	2.0	2.9
Net direct investment	0.1	0.2	0.1	0.1	0.2	0.4	0.6	0.6	0.7
Net portfolio investment	~	0.5	~	0.1	1.1	1.2	0.8	0.8	0.8
Other net investment	1.5	0.6	0.9	0.2	0.1	0.1	0.1	0.6	1.4
Indonesia									
Net private capital flows[3]	1.5	4.2	4.6	2.5	3.1	3.9	6.2	6.3	1.6
Net direct investment	0.4	1.3	1.2	1.2	1.2	1.4	2.3	2.8	2.0
Net portfolio investment	0.1	0.4	~	~	1.1	0.6	0.7	0.8	−0.4
Other net investment	1.0	2.6	3.5	1.4	0.7	1.9	3.1	2.7	0.1
Korea									
Net private capital flows[3]	−1.1	2.1	2.2	2.4	1.6	3.1	3.9	4.9	2.8
Net direct investment	0.2	−0.1	−0.1	−0.2	−0.2	−0.3	−0.4	−0.4	−0.2
Net portfolio investment	0.3	1.4	1.1	1.9	3.2	1.8	1.9	2.3	−0.3
Other net investment	−1.6	0.8	1.3	0.7	−1.5	1.7	2.5	3.0	3.4
Malaysia									
Net private capital flows[3]	3.1	8.8	11.2	15.1	17.4	1.5	8.8	9.6	4.7
Net direct investment	2.3	6.5	8.3	8.9	7.8	5.7	4.8	5.1	5.3
Net portfolio investment	~	~	~	~	~	~	~	~	~
Other net investment	0.8	2.3	2.9	6.2	9.7	−4.2	4.1	4.5	−0.6
Philippines									
Net private capital flows[3]	−2.0	2.7	1.6	2.0	2.6	5.0	4.6	9.8	0.5
Net direct investment	0.7	1.6	1.2	1.3	1.6	2.0	1.8	1.6	1.4
Net portfolio investment	~	0.2	0.3	0.1	−0.1	0.4	0.3	−0.2	−5.3
Other net investment	−2.7	0.9	0.2	0.6	1.1	2.5	2.4	8.5	4.5
Singapore									
Net private capital flows[3]	5.0	3.8	1.7	−2.7	9.4	2.5	1.3	−10.1	−5.5
Net direct investment	8.7	6.0	8.8	2.1	5.5	4.8	4.9	4.3	5.3
Net portfolio investment	−0.5	0.1	−2.1	3.3	0.5	1.1	0.9	−16.2	−14.4
Other net investment	−3.2	−2.4	−5.1	−8.0	3.4	−3.4	−4.6	1.8	3.6
Taiwan									
Net private capital flows[3]	0.2	−4.0	−1.2	−3.2	−2.1	−0.6	−3.6	−3.2	−3.8
Net direct investment	−0.2	−1.2	−0.3	−0.5	−0.7	−0.5	−0.4	−0.7	−0.6
Net portfolio investment	−0.3	~	~	0.2	0.5	0.4	0.2	−0.4	−0.6
Other net investment	0.7	−2.8	−0.9	−3.0	−1.9	−0.5	−3.3	−2.1	−2.6
Thailand									
Net private capital flows[3]	3.1	10.2	10.7	8.7	8.4	8.6	12.7	9.3	−10.9
Net direct investment	0.8	1.5	1.5	1.4	1.1	0.7	0.7	0.9	1.3
Net portfolio investment	0.7	1.3	~	0.5	3.2	0.9	1.9	0.6	0.4
Other net investment	1.5	7.4	9.2	6.8	4.1	7.0	10.0	7.7	−12.6

[1]Net capital flows comprise net direct investment, net portfolio investment, and other long- and short-term net investment flows, including official and private borrowing.
[2]Annual averages.
[3]Because of data limitations, other net investment may include some official flows.
Source: IMF, "World Economic Outlook: Interim Assessment", December 1997 table 1.

Table C.3: Descriptive statistics on stock returns (01/01/1996 – 12/31/1997)

Market[1]	Stock Market Index[2]	Stock Returns[3]			
		Mean	St.Dev.	Skewness	Kurtosis
Argentina	Merval Buenos Aires (MERVAL)	0.0005	0.0178	−1.7041	13.8805
Australia	S&P/ASX 200	0.0003	0.0087	−0.8673	14.9229
Austria	ATX	0.0006	0.0104	−1.2192	14.3054
Brazil	MSCI Brazil	0.0017	0.0229	−1.0593	11.5980
Canada	S&P/TSX	0.0007	0.0069	−1.8782	18.2237
Chile	Chile Santiago Se General (IGPA)	−0.0003	0.0057	0.0953	5.0487
China	Shanghai Se Composite (SEE)	0.0015	0.0241	−0.5387	6.5611
France	CAC 40	0.0009	0.0111	−0.1351	5.7664
Germany	DAX 30	0.0012	0.0120	−1.0228	10.0196
Hong Kong	Hang Seng Index	0.0001	0.0189	−0.0874	25.8897
India	India BSE 100	0.0002	0.0137	0.1053	6.9039
Indonesia	IDX Composite	−0.0005	0.0158	−0.3300	12.9872
Italy	Milan MIBTEL	0.0011	0.0118	0.0399	5.4234
Japan	NIKKEI 225	−0.0005	0.0137	−0.0054	6.4259
Korea	Korea Se Composite (KOSPI)	−0.0016	0.0193	−0.3584	9.3217
Malaysia	FTSE Bursa Malaysia KLCI	−0.0010	0.0174	0.1676	16.0818
Mexico	Mexico IPC	0.0012	0.0148	−0.9113	25.3799
Netherlands	AEX	0.0012	0.0117	−0.3269	5.6694
Peru	Lima Se General (IGBL)	0.0007	0.0110	−0.4152	7.9143
Philippines	Philippines Se I (PSEi)	−0.0006	0.0145	−0.4508	9.6063
Singapore	Straits Times Index (STI)	−0.0008	0.0104	−0.3528	14.2087
Spain	Madrid Se General (IGBM)	0.0013	0.0103	−0.3055	6.8540
Sweden	OMX Stockholm 30	0.0011	0.0117	−0.0418	5.9429
Taiwan	Taiwan Se Weighted TAIEX	0.0009	0.0145	−0.5482	6.2611
Thailand	Bangkok S.E.T.	−0.0024	0.0183	0.5038	5.6854
UK	FTSE 100	0.0006	0.0079	−0.1919	4.5710
USA	S&P 500	0.0009	0.0095	−0.6837	10.4409

[1] In order to save space, the names of markets in the paper are abbreviated as follows: Argentina (ARG); Australia (AUS2); Austria (AUS1); Brazil (BRA); Canada (CAN); Chile (CHI1); China (CHI2); France (FRA); Germany (GER); Hong Kong (HK); India (IND1); Indonesia (IND2); Italy (ITA); Japan (JAP); Korea (KOR); Malaysia (MAL); Mexico (MEX); Netherlands (NET); Peru (PER); Philippines (PHI); Singapore (SIN); Spain (SPA); Sweden (SWE); Taiwan (TAI); Thailand (THA); UK (UK); USA (USA); Singapore (SIN).
[2]The indexes are daily adjusted closing prices from 01/01/1996 – 12/31/1997. The number of data points in each market is not consistent, because of differences in the holidays for each market. In order to maintain consistency, the prices in holidays are assumed to be the same as the previous trading data. In sum, there are 519 observations for each time series in the sample, including 389 observations in the tranquil period and 130 observations in the crisis period.
[3] Daily log-difference of stock market closing prices.
Data source: Datastream.

Table C.4: Estimation results of the bivariate VAR-BEKK(1,1,1) model at the first level

	Variance equations							
	$c_{1,1}$	$c_{1,2}$	$c_{2,2}$	$a_{1,1}$	$a_{2,2}$	$g_{1,1}$	$g_{2,2}$	H_0
ARG-THA	0.0014∗	$4.67E-06$	0.0008∗	1.0538∗	0.7717∗	−0.1497 ∗ ∗	0.6979∗	3413.631∗
	(18.7605)	(0.0268)	(7.4278)	(14.1844)	(11.4979)	(−1.6522)	(21.5311)	0.0000
BRA-THA	0.0018∗	$-2.71E-05$	0.0010∗	1.0491∗	0.8162∗	−0.1388 ∗ ∗	0.6219∗	1954.153∗
	(13.7155)	(−0.1637)	(10.0059)	(16.6182)	(12.2843)	(−1.8847)	(16.0981)	0.0000
CHI1-THA	0.0005∗	0.0002 ∗ ∗	0.0011∗	0.9327∗	0.8102∗	0.0640	0.5871∗	1017.115∗
	(20.6073)	(1.7830)	(8.8728)	(13.2222)	(12.6733)	(0.6787)	(13.7333)	0.0000
MEX-THA	0.0010∗	0.0005∗	0.0010∗	1.1908∗	0.8442∗	−0.0160	0.5700∗	1713.118∗
	(12.6342)	(2.6009)	(10.2275)	(15.1198)	(11.4235)	(−0.1880)	(16.0023)	0.0000
PER-THA	0.0011∗	−0.0001	0.0014∗	0.8904∗	1.0176∗	0.1242	−0.1460	243.079∗
	(19.9459)	(−1.0282)	(14.8183)	(12.4490)	(13.4130)	(0.8616)	(−1.3487)	0.0000
AUS1-THA	0.0007∗	0.0002	0.0015∗	0.9580∗	0.9076∗	0.3817∗	−0.2686∗	665.204∗
	(11.1775)	(1.0776)	(15.4852)	(13.5404)	(12.3134)	(6.7342)	(−3.9779)	0.0000
AUS2-THA	0.0007∗	0.0002	0.0012∗	1.0438∗	0.8291∗	0.1360∗	−0.5675∗	1170.619∗
	(14.5091)	(1.1300)	(8.1818)	(13.7045)	(11.3157)	(2.3268)	(−10.5764)	0.0000
CAN-THA	0.0005∗	0.0001	0.0010∗	0.9711∗	0.8652∗	−0.0951	0.5447∗	1206.563∗
	(15.7426)	(0.8713)	(10.4231)	(14.6611)	(12.0502)	(−0.9936)	(11.0361)	0.0000
FRA-THA	0.0009∗	0.0004∗	0.0008∗	0.9265∗	0.8798∗	−0.3010	0.5640∗	1789.139∗
	(13.9167)	(2.5853)	(6.6074)	(12.4723)	(12.5394)	(−5.4058)	(15.4836)	0.0000
GER-THA	0.0012∗	0.0003∗	0.0016∗	1.0169∗	0.9377∗	0.0913	−0.1295	247.417∗
	(23.8766)	(2.1310)	(19.1443)	(13.5254)	(12.5481)	(1.0621)	(−1.2004)	0.0000
ITA-THA	0.0012∗	−0.0001	0.0009∗	0.8976∗	0.8328∗	−0.2154∗	0.5850∗	1243.796∗
	(15.4276)	(−0.3849)	(8.4020)	(11.8977)	(12.8012)	(−3.1165)	(13.6048)	0.0000
JAP-THA	0.0005∗	0.0001	0.0016∗	0.7510∗	0.9072∗	0.7348∗	−0.1185	2933.477∗
	(3.9472)	(0.2967)	(18.2910)	(13.4787)	(13.7735)	(22.9797)	(−1.2358)	0.0000
NET-THA	0.0010∗	0.0001	0.0016∗	0.8297∗	0.8950∗	0.4854∗	−0.1452	531.724∗
	(10.6530)	(0.5435)	(21.6550)	(11.6013)	(12.7691)	(7.4165)	(−1.4450)	0.0000
SPA-THA	0.0009∗	0.0002	0.0009∗	0.8424∗	0.8677∗	0.3333∗	−0.5546∗	1345.004∗
	(11.9351)	(1.5314)	(8.8338)	(11.2825)	(12.2589)	(4.1717)	(−14.4274)	0.0000
SWE-THA	0.0011∗	0.0003∗	0.0008∗	0.8657∗	0.8619∗	−0.2682∗	0.5803∗	1706.163∗
	(14.5835)	(2.1081)	(6.9805)	(12.4357)	(12.3946)	(−3.6996)	(14.9622)	0.0000
UK-THA	0.0008∗	0.0001	0.0011∗	0.9088∗	0.8408∗	−0.1878∗	0.5630∗	1338.908∗
	(17.8905)	(0.7822)	(11.4844)	(13.0719)	(12.5417)	(−2.7075)	(13.7267)	0.0000
USA-THA	0.0011∗	0.0002	0.0013∗	0.9476∗	0.9733∗	−0.2050 ∗ ∗	0.3012∗	344.240∗
	(12.1640)	(1.1579)	(13.3738)	(11.9101)	(12.0257)	(−1.7675)	(3.7822)	0.0000
CHI2-THA	0.0023∗	0.0001	0.0009∗	1.0308∗	0.7800∗	−0.0225	0.6541∗	1909.955∗
	(20.4075)	(1.1842)	(8.2836)	(15.1786)	(12.7389)	(−0.2944)	(19.7226)	0.0000
KOR-THA	0.0008∗	−0.0001	0.0016∗	0.8157∗	0.9327∗	0.6254∗	−0.0190	1203.239∗
	(8.0745)	(−0.5134)	(18.0088)	(11.8907)	(12.7915)	(15.6108)	(−0.1749)	0.0000
SIN-THA	0.0008∗	0.0006∗	0.0009∗	0.8625∗	0.7498∗	−0.3957∗	0.6216∗	1794.698∗
	(16.4412)	(2.7990)	(4.9068)	(11.5702)	(11.1476)	(−6.0700)	(14.4751)	0.0000
MAL-THA	0.0009∗	0.0001	0.0010∗	1.0955∗	0.7658∗	0.0697∗	0.6667∗	1955.425∗
	(17.1912)	(0.4235)	(9.5904)	(18.2230)	(12.8157)	(2.0347)	(19.5131)	0.0000
HK-THA	0.0007∗	0.0007∗	0.0016∗	0.8489∗	0.8318∗	0.6215∗	−0.1044	41787.228∗
	(8.6826)	(2.9861)	(12.1614)	(13.9526)	(12.6184)	(16.3311)	(−1.1067)	0.0000
IND1-THA	0.0014∗	−0.0001	0.0019∗	0.9100∗	0.8442∗	−0.0013	0.0031	194.101∗
	(18.8117)	(−0.4644)	(19.6765)	(12.0834)	(11.5692)	(−0.0001)	(0.0004)	0.0000
IND2-THA	0.0006∗	0.0003	0.0014∗	0.7740∗	0.9955∗	0.6365∗	−0.1586∗	1754.243∗
	(9.1689)	(1.0168)	(16.8615)	(11.7522)	(14.0789)	(14.1820)	(−2.8748)	0.0000
PHI-THA	0.0009∗	−0.0001	−0.0010∗	1.0021∗	0.7977∗	−0.1179	0.6081∗	1861.945∗
	(16.6573)	(−0.9071)	(−10.3120)	(13.9617)	(10.5914)	(−1.5828)	(16.4668)	0.0000
TAI-THA	0.0011∗	−0.0003	0.0008∗	1.0852∗	0.7929∗	−0.1142 ∗ ∗	0.6616∗	2650.311∗
	(12.4111)	(−1.6258)	(9.3825)	(14.9058)	(12.3914)	(−1.8054)	(22.4018)	0.0000

The parameters and Z−statistic values (in parentheses) are estimated on Equation (4.33) in the text. The lag length p in Equation (4.36) is determined by the AIC and LM criteria. Because the optimal lag length p in every case is different, the parameters and Z−statistic values of variables in the mean equation are not attached here. The null hypothesis H_0 is that all of the parameters of variables in the variance equation are equal to zero. Accordingly, the χ^2−values and the corresponding p−values are listed here as well. ∗ and ∗∗ represent statistical significance at the 5% and 10% levels, respectively.

Table C.5: Estimation results of the bivariate VAR-BEKK(1,1,1) model at the second level

	Variance equations							
	$c_{1,1}$	$c_{1,2}$	$c_{2,2}$	$a_{1,1}$	$a_{2,2}$	$g_{1,1}$	$g_{2,2}$	H_0
ARG-THA	0.0007*	0.0001	0.0007*	0.6753*	0.5958*	0.7484*	0.7975*	5032.088*
	(8.4044)	(0.8865)	(7.1266)	(12.9932)	(12.2053)	(25.1820)	(30.5330)	0.0000
BRA-THA	0.0002*	0.0001	0.0003*	0.6029*	0.4878*	0.8308*	0.8775*	18619.440*
	(5.1731)	(1.1815)	(7.7414)	(12.1207)	(11.9472)	(36.3975)	(53.8448)	0.0000
CHI1-THA	0.0003*	$-1.12E-05$	-0.0007*	0.6261*	0.6024*	0.7648*	0.7850*	5746.925*
	(7.2691)	(-0.1433)	(-7.5783)	(11.2478)	(10.9569)	(21.1308)	(27.0023)	0.0000
MEX-THA	0.0007*	$-1.02E-05$	0.0007*	0.6863*	0.6235*	0.6985*	0.7749*	3832.052*
	(9.5044)	(-0.1307)	(7.0488)	(10.5205)	(10.7630)	(17.2341)	(24.1439)	0.0000
PER-THA	0.0006*	0.0001	0.0008*	0.5896*	0.6379*	0.7588*	0.7474*	4053.847*
	(7.9509)	(1.2464)	(7.9541)	(10.5063)	(10.0161)	(20.4267)	(20.0694)	0.0000
AUS1-THA	0.0004*	0.0002**	0.0008*	0.6926*	0.6046*	0.7345*	0.7648*	4035.971*
	(8.9560)	(1.9135)	(7.8787)	(13.3921)	(11.1379)	(23.8287)	(24.1666)	0.0000
AUS2-THA	0.0004*	0.0001	0.0008*	0.6316*	0.6052*	0.7750*	0.7716*	5795.607*
	(8.0192)	(1.0434)	(7.8730)	(11.2014)	(10.8820)	(23.9741)	(23.8237)	0.0000
CAN-THA	0.0002*	-0.0001	0.0010*	0.5899*	0.6910*	0.8287*	0.6881*	17254.248*
	(5.8173)	(-0.7988)	(8.6933)	(12.9309)	(11.4113)	(35.4806)	(15.3456)	0.0000
FRA-THA	0.0002*	$-2.01E-05$	0.0004*	0.5963*	0.7031*	0.8301*	0.7522*	6790.609*
	(5.8583)	(-0.3375)	(6.9108)	(11.2025)	(11.5645)	(32.3848)	(20.8743)	0.0000
GER-THA	0.0003*	$-3.67E-05$	0.0008*	0.5837*	0.6225*	0.8292*	0.7529*	12196.640*
	(6.3133)	(-0.2854)	(9.4674)	(15.7732)	(11.7003)	(57.8845)	(23.2427)	0.0000
ITA-THA	0.0005*	0.0003*	0.0007*	0.6035*	0.6427*	0.7766*	0.7620*	4512.278*
	(7.2570)	(3.6304)	(7.7799)	(10.0851)	(12.4669)	(21.5241)	(30.7871)	0.0000
JAP-THA	0.0005*	0.0001	0.0008*	0.6383*	0.5593*	0.7763*	0.7934*	6376.603*
	(7.4944)	(0.5198)	(7.2936)	(9.0775)	(9.8986)	(21.7143)	(22.8767)	0.0000
NET-THA	0.0002*	$2.85E-05$	0.0004*	0.6038*	0.6645*	0.8248*	0.7811*	8445.232*
	(7.2459)	(0.5886)	(7.0790)	(12.2674)	(13.1285)	(37.0202)	(26.4112)	0.0000
SPA-THA	0.0004*	0.0002**	0.0009*	0.5645*	0.6986*	0.8185*	0.6945*	4976.217*
	(6.0390)	(1.9188)	(8.5487)	(13.4768)	(11.7116)	(30.3315)	(17.0357)	0.0000
SWE-THA	0.0005*	0.0001	0.0008*	0.5692*	0.6191*	0.8073*	0.7455*	4013.517*
	(6.2112)	(1.1146)	(8.6936)	(9.5565)	(10.7707)	(24.1601)	(19.7424)	0.0000
UK-THA	0.0002*	0.0001	0.0011*	0.6128*	0.7457*	0.8117*	0.5809*	5734.087*
	(5.5288)	(0.2861)	(14.0017)	(12.0020)	(12.3969)	(35.0178)	(12.5218)	0.0000
USA-THA	0.0005*	$-4.92E-05$	0.0008*	0.6496*	0.6290*	0.7465*	0.7467*	3041.043*
	(6.7673)	(-0.5514)	(6.9697)	(10.3551)	(9.4932)	(18.2081)	(16.5921)	0.0000
CHI2-THA	0.0009*	$-4.25E-05$	0.0007*	0.6994*	0.5957*	0.7270*	0.7918*	4917.172*
	(8.0928)	(-0.4653)	(6.4772)	(12.2220)	(10.6163)	(19.7823)	(23.4944)	0.0000
KOR-THA	0.0007*	0.0001	0.0006*	0.7087*	0.5841*	0.7225*	0.8145*	6722.523*
	(11.0926)	(1.1307)	(5.5877)	(12.5020)	(13.0122)	(23.9244)	(32.7477)	0.0000
SIN-THA	0.0003*	0.0003*	0.0003*	0.6728*	0.5746*	0.7807*	0.8536*	16634.240*
	(9.4141)	(5.3981)	(3.4193)	(14.4835)	(16.0867)	(34.9732)	(64.6940)	0.0000
MAL-THA	0.0004*	0.0001	0.0007*	0.6578*	0.6809*	0.7819*	0.7425*	5623.079*
	(7.5435)	(1.3891)	(8.4585)	(13.0248)	(12.1473)	(28.8814)	(22.8040)	0.0000
HK-THA	0.0006*	0.0002*	0.0006*	0.7136*	0.5420*	0.7085*	0.8292*	6939.115*
	(9.3531)	(3.2383)	(6.2060)	(12.0275)	(12.4923)	(20.9659)	(36.2389)	0.0000
IND1-THA	0.0007*	$6.21E-06$	0.0006*	0.6493*	0.6343*	0.7545*	0.7765*	4846.642*
	(7.3768)	(0.0712)	(6.5954)	(11.7733)	(10.8979)	(22.2067)	(24.5793)	0.0000
IND2-THA	0.0005*	0.0001	0.0007*	0.6915*	0.6889*	0.7626*	0.7415*	5702.456*
	(9.3173)	(1.3536)	(7.4285)	(15.5759)	(12.5657)	(39.5097)	(22.1980)	0.0000
PHI-THA	0.0002*	0.0001*	0.0002*	0.5474*	0.5517*	0.8567*	0.8597*	15517.400*
	(5.9900)	(2.7353)	(5.2983)	(12.6034)	(11.7309)	(46.2015)	(42.1025)	0.0000
TAI-THA	0.0008*	$1.24E-05$	0.0008*	0.6336*	0.6395*	0.7392*	0.7512*	3640.078*
	(7.1339)	(0.1760)	(7.8829)	(9.0122)	(10.9533)	(14.9954)	(23.5803)	0.0000

The parameters and Z−statistic values (in parentheses) are estimated on Equation (4.33) in the text. The lag length p in Equation (4.36) is determined by the AIC and LM criteria. Because the optimal lag length p in every case is different, the parameters and Z−statistic values of variables in the mean equation are not attached here. The null hypothesis H_0 is that all of the parameters of variables in variance equation equal zero. Accordingly, the χ^2-values and the corresponding $p-$values are listed here as well. $*$ and $**$ represent statistical significance at the 5% and 10% levels, respectively.

Table C.6: Estimation results of the bivariate VAR-BEKK(1,1,1) model at the third level

	Variance equations							
	$c_{1,1}$	$c_{1,2}$	$c_{2,2}$	$a_{1,1}$	$a_{2,2}$	$g_{1,1}$	$g_{2,2}$	H_0
ARG-THA	$3.75E-05$*	$-4.59E-05$**	0.0001*	0.6498*	0.6948*	0.8300*	0.7659*	9798.123*
	(3.3813)	(−1.9594)	(4.5563)	(14.0318)	(12.2305)	(46.6986)	(26.4065)	0.0000
BRA-THA	0.0001*	$-3.43E-05$	0.0001*	0.6904*	0.6350*	0.7805*	0.7868*	6213.569*
	(3.7136)	(−1.5492)	(6.1299)	(12.4081)	(10.2588)	(29.9398)	(24.5035)	0.0000
CHI1-THA	$4.76E-05$*	0.0001*	$7.24E-06$	0.4983*	0.5209*	0.8337*	0.8560*	21137.190*
	(7.5768)	(6.9885)	($4.99E-05$)	(13.6683)	(14.5496)	(39.7679)	(55.0452)	0.0000
MEX-THA	$4.59E-05$*	$-1.44E-05$	0.0001*	0.6540*	0.7375*	0.8013*	0.7688*	6084.660*
	(4.8992)	(−0.7540)	(4.9981)	(12.6578)	(12.7163)	(33.9708)	(26.4923)	0.0000
PER-THA	0.0001*	$9.36E-06$	0.0001*	0.7873*	0.6716*	0.6924*	0.7944*	5843.239*
	(7.7755)	(0.4607)	(4.5290)	(13.2794)	(11.6098)	(19.9903)	(32.5092)	0.0000
AUS1-THA	$2.16E-05$*	−0.0001*	$-5.06E-07$	0.6138*	0.6904*	0.8385*	0.7943*	11676.180*
	(4.5217)	(−4.4346)	(−0.0006)	(13.3475)	(14.2229)	(42.3666)	(39.3959)	0.0000
AUS2-THA	$1.98E-05$*	0.0001*	$3.72E-06$	0.5420*	0.4300*	0.8541*	0.9010*	26411.250*
	(5.1828)	(5.0264)	($1.59E-05$)	(16.3000)	(11.0432)	(54.6619)	(64.4772)	0.0000
CAN-THA	$1.81E-05$*	0.0001*	$5.21E-06$	0.4088*	0.4688*	0.9109*	0.8798*	29461.900*
	(4.1213)	(4.3407)	($1.93E-05$)	(12.7873)	(11.1192)	(62.4348)	(48.2637)	0.0000
FRA-THA	0.0001*	$2.32E-05$**	0.0001*	0.6994*	0.5465*	0.7048*	0.8600*	8798.825*
	(9.1098)	(1.8946)	(3.0941)	(11.4088)	(10.2087)	(18.4215)	(40.6018)	0.0000
GER-THA	$1.42E-05$*	0.0001*	$4.87E-06$	0.4464*	0.4417*	0.9040*	0.8851*	29794.490*
	(2.9773)	(3.9391)	($1.21E-05$)	(13.5312)	(10.7802)	(74.9732)	(51.3429)	0.0000
ITA-THA	0.0001*	$1.86E-05$	0.0001*	0.7163*	0.5960*	0.7051*	0.8030*	4439.383*
	(7.7190)	(1.1441)	(5.1747)	(11.5214)	(10.5864)	(17.1030)	(27.3574)	0.0000
JAP-THA	$3.31E-05$*	$-1.42E-06$	0.0001*	0.5840*	0.5973*	0.8434*	0.8245*	9933.670*
	(4.1534)	(−0.0959)	(4.7192)	(11.1783)	(11.3042)	(37.2113)	(36.1729)	0.0000
NET-THA	$2.25E-05$*	0.0001*	$1.86E-05$	0.5026*	0.4174*	0.8794*	0.9005*	42252.220*
	(4.2390)	(6.2414)	(0.0003)	(14.0832)	(11.0056)	(62.5991)	(57.6940)	0.0000
SPA-THA	$1.74E-05$*	0.0001*	$7.24E-06$	0.5022*	0.4654*	0.8810*	0.8764*	22402.010*
	(2.8087)	(4.6391)	($2.84E-05$)	(13.1432)	(11.7276)	(57.0393)	(52.4007)	0.0000
SWE-THA	$3.37E-05$*	$-3.39E-05$*	0.0001*	0.6836*	0.6231*	0.7918*	0.8233*	9997.279*
	(3.9011)	(−2.0635)	(3.3973)	(12.2096)	(11.6907)	(31.7616)	(37.3923)	0.0000
UK-THA	$1.88E-05$*	0.0001*	$5.82E-06$	0.4153*	0.4595*	0.9117*	0.8852*	38967.050*
	(4.3597)	(4.9498)	(0.0001)	(12.6284)	(12.1782)	(74.6149)	(58.5057)	0.0000
USA-THA	$3.85E-05$*	0.0001*	0.0001*	0.6391*	0.6664*	0.8025*	0.7798*	6159.888*
	(6.2662)	(3.6309)	(6.5347)	(12.0459)	(12.3761)	(32.5910)	(28.7101)	0.0000
CHI2-THA	$3.77E-05$*	0.0001*	0.0001*	0.6391*	0.6664*	0.8025*	0.7798*	6385.954*
	(6.2662)	(3.6309)	(6.5347)	(12.0459)	(12.3761)	(32.5910)	(28.7101)	0.0000
KOR-THA	0.0001*	$2.82E-05$	0.0001*	0.6335*	0.6016*	0.8121*	0.8194*	9629.555*
	(5.5279)	(1.4863)	(4.6253)	(12.6397)	(11.6282)	(37.8931)	(36.2558)	0.0000
SIN-THA	$4.47E-05$*	$4.84E-05$*	0.0001*	0.6532*	0.7048*	0.7997*	0.7848*	8990.161*
	(6.6133)	(3.4231)	(3.6281)	(13.5475)	(13.8541)	(38.8710)	(37.7445)	0.0000
MAL-THA	$3.28E-05$*	0.0001*	$3.07E-05$	0.4445*	0.4301*	0.9004*	0.8984*	34719.040*
	(4.9178)	(6.2750)	(0.0010)	(13.6122)	(13.1866)	(72.3048)	(69.1353)	0.0000
HK-THA	$4.02E-05$*	$1.73E-05$	$2.80E-05$*	0.6553*	0.6373*	0.8150*	0.8362*	16514.980*
	(4.9853)	(1.2001)	(3.0280)	(14.4320)	(14.3926)	(41.5267)	(48.1755)	0.0000
IND1-THA	0.0001*	$5.20E-06$	0.0001*	0.8498*	0.7741*	0.6941*	0.7667*	6575.631*
	(7.5170)	(0.3419)	(3.7765)	(14.2362)	(14.0294)	(23.5579)	(34.6770)	0.0000
IND2-THA	0.0001*	0.0001*	0.0001*	0.8170*	0.6479*	0.6671*	0.8106*	8238.811*
	(7.7259)	(3.7347)	(4.6819)	(14.1133)	(12.1610)	(22.7031)	(38.3236)	0.0000
PHI-THA	$4.05E-05$*	$2.68E-05$	0.0001*	0.6597*	0.5648*	0.8067*	0.8624*	17092.420*
	(4.7023)	(1.3948)	(3.5733)	(13.3499)	(12.2488)	(40.4305)	(48.8490)	0.0000
TAI-THA	0.0001*	$3.61E-05$*	0.0001*	1.5490*	1.5023*	0.2471*	0.3257*	969.200*
	(12.8154)	(2.4026)	(9.0109)	(17.6563)	(17.6759)	(13.1938)	(15.3455)	0.0000

The parameters and Z−statistic values (in parentheses) are estimated on Equation (4.33) in the text. The lag length p in Equation (4.36) is determined by the AIC and LM criteria. Because the optimal lag length p in every case is different, the parameters and Z−statistic values of variables in the mean equation are not attached here. The null hypothesis H_0 is that all of the parameters of the variables in the variance equation equal zero. Accordingly, the χ^2−values and the corresponding p−values are listed here as well. * and ** represent statistical significance at the 5% and 10% levels, respectively.

Table C.7: Estimation results of the bivariate VAR-BEKK(1,1,1) model at the fourth level

	Variance equations							
	$c_{1,1}$	$c_{1,2}$	$c_{2,2}$	$a_{1,1}$	$a_{2,2}$	$g_{1,1}$	$g_{2,2}$	H_0
ARG-THA	1.41E-05*	1.17E-05*	3.02E-06	0.4471*	0.4954*	0.8947*	0.8840*	24074.430*
	(4.6837)	(5.0469)	(0.0003)	(11.4229)	(13.3421)	(52.6091)	(59.2230)	0.0000
BRA-THA	1.99E-06**	1.81E-06	1.55E-06	0.4150*	0.3604*	0.9308*	0.9438*	130998.700*
	(1.6985)	(0.4993)	(0.4420)	(11.9054)	(14.3174)	(89.8390)	(139.1314)	0.0000
CHI1-THA	2.04E-05*	1.23E-05*	4.33E-07	0.3873*	0.5087*	0.7746*	0.8763*	20113.890*
	(12.3695)	(6.3167)	(1.18E-05)	(13.5391)	(12.5037)	(32.5906)	(52.7267)	0.0000
MEX-THA	2.98E-06*	1.89E-06	2.59E-06**	0.4359*	0.3921*	0.9210*	0.9351*	88325.940*
	(2.3262)	(1.0239)	(1.7181)	(11.3390)	(14.5945)	(76.3852)	(131.3863)	0.0000
PER-THA	5.90E-06*	1.31E-05*	1.03E-07	0.3946*	0.4973*	0.9215*	0.8801*	31808.140*
	(4.1730)	(3.9190)	(1.73E-06)	(14.0943)	(13.6324)	(87.1790)	(66.9674)	0.0000
AUS1-THA	1.41E-06*	7.54E-07	3.31E-06*	0.4584*	0.3871*	0.9103*	0.9346*	76797.150*
	(2.4922)	(0.3949)	(2.9471)	(11.9645)	(13.0407)	(77.6348)	(108.2390)	0.0000
AUS2-THA	1.18E-05*	3.66E-06*	−1.45E-07	0.3873*	0.2446*	0.7746*	0.9711*	171008.000*
	(8.9318)	(3.6498)	(−2.11E-06)	(5.9124)	(8.9373)	(17.4870)	(170.4483)	0.0000
CAN-THA	9.18E-07	1.59E-06	5.14E-06*	0.6455*	0.6523*	0.8452*	0.8403*	22597.020*
	(1.5932)	(0.5998)	(2.9964)	(15.4504)	(15.1532)	(65.0053)	(62.8954)	0.0000
FRA-THA	9.80E-06*	1.18E-05*	2.77E-07	0.3876*	0.4959*	0.9163*	0.8834*	26048.770*
	(5.2231)	(4.7857)	(5.62E-06)	(11.9738)	(13.0319)	(55.0107)	(57.0499)	0.0000
GER-THA	7.47E-06*	1.23E-05*	7.69E-07	0.4338*	0.4690*	0.9030*	0.8940*	24710.250*
	(5.2528)	(4.5679)	(1.57E-05)	(12.4252)	(13.4446)	(55.0675)	(65.2539)	0.0000
ITA-THA	−1.93E-06*	6.37E-07	2.12E-06*	0.4769*	0.4353*	0.9022*	0.9208*	82790.490*
	(−4.3838)	(0.6574)	(2.1064)	(14.7057)	(14.2664)	(95.6779)	(104.3618)	0.0000
JAP-THA	7.10E-06*	1.06E-05*	2.55E-06	0.4294*	0.5110*	0.9064*	0.8781*	35900.140*
	(4.5454)	(4.8950)	(0.0003)	(14.1503)	(14.5676)	(71.7881)	(63.8982)	0.0000
NET-THA	2.86E-06*	3.76E-06	−4.35E-06*	0.6812*	0.6799*	0.8327*	0.8201*	15479.140*
	(3.0934)	(1.2841)	(−2.9222)	(16.2804)	(16.0390)	(59.0323)	(58.1633)	0.0000
SPA-THA	1.07E-05*	1.24E-05*	5.55E-07	0.4352*	0.5036*	0.8938*	0.8784*	29234.810*
	(6.7404)	(6.0670)	(2.51E-05)	(11.8649)	(12.2604)	(50.4591)	(50.7278)	0.0000
SWE-THA	6.03E-06*	1.16E-05*	2.16E-07	0.4436*	0.5043*	0.9049*	0.8792*	32150.600*
	(3.6180)	(3.5798)	(4.81E-06)	(12.3703)	(12.9971)	(62.0274)	(61.5771)	0.0000
UK-THA	−1.33E-06	5.61E-06	3.32E-06	0.6191*	0.6131*	0.8515*	0.8474*	23593.970*
	(−0.9206)	(1.3652)	(0.7301)	(14.4067)	(13.2458)	(66.6922)	(61.4660)	0.0000
USA-THA	1.01E-06	−2.01E-06	−1.58E-08	0.4767*	0.4102*	0.9125*	0.9304*	119853.800*
	(1.2499)	(−0.7238)	(−0.0001)	(13.2869)	(15.0353)	(83.0383)	(130.5689)	0.0000
CHI2-THA	3.03E-05*	2.81E-05*	4.12E-06	0.3873*	0.3873*	0.7746*	0.7746*	330035.500*
	(20.2055)	(20.6100)	(0.0017)	(38.1583)	(40.4551)	(128.3997)	(100.8617)	0.0000
KOR-THA	4.23E-06*	1.08E-05*	3.56E-06	0.4389*	0.5028*	0.9128*	0.8816*	42811.900*
	(2.7704)	(2.8930)	(0.0002)	(16.7390)	(15.5878)	(107.3187)	(64.3556)	0.0000
SIN-THA	1.44E-05*	3.93E-06*	1.38E-10	0.3873*	0.2194*	0.7746*	0.9759*	176505.800*
	(10.3914)	(3.6605)	(3.84E-09)	(6.8328)	(7.5415)	(21.0047)	(171.1253)	0.0000
MAL-THA	2.01E-06*	8.70E-07	3.66E-06*	0.3339*	0.3747*	0.9499*	0.9383*	103146.500*
	(3.5058)	(0.5060)	(3.3152)	(10.9385)	(12.3156)	(108.0987)	(101.7455)	0.0000
HK-THA	7.61E-07	−3.38E-07	3.16E-06*	0.4513*	0.3632*	0.9204*	0.9413*	114411.600*
	(0.3982)	(−0.0844)	(2.4814)	(13.4672)	(12.9652)	(90.9940)	(121.7725)	0.0000
IND1-THA	1.12E-05*	1.36E-05*	7.44E-07	0.4709*	0.4836*	0.8882*	0.8853*	25844.000*
	(4.2629)	(4.6256)	(2.47E-05)	(11.7457)	(13.1231)	(50.8842)	(55.1409)	0.0000
IND2-THA	1.48E-05*	2.81E-05*	2.77E-05	0.3873*	0.3873*	0.7746*	0.7747*	200267.400*
	(26.7952)	(27.3224)	(0.0023)	(45.0238)	(45.3322)	(157.6261)	(148.7455)	0.0000
PHI-THA	1.46E-05*	1.35E-05*	1.70E-06	0.3877*	0.5009*	0.9142*	0.8772*	23452.350*
	(5.2472)	(5.7321)	(0.0001)	(12.5415)	(13.2018)	(64.9900)	(56.0077)	0.0000
TAI-THA	6.58E-06*	1.55E-05*	4.55E-07	0.4451*	0.5100*	0.9023*	0.8687*	28807.990*
	(5.3920)	(4.2304)	(8.22E-06)	(15.2539)	(13.0559)	(79.7514)	(52.4853)	0.0000

The parameters and Z−statistic values (in parentheses) are estimated on Equation (4.33) in the text. The lag length p in Equation (4.36) is determined by the AIC and LM criteria. Because the optimal lag length p in every case is different, the parameters and Z−statistic values of variables in the mean equation are not attached here. The null hypothesis H_0 is that all of the the parameters of the variables in the variance equation equal zero. Accordingly, the χ^2−values and the corresponding p−values are listed here as well. $*$ and $**$ represent statistical significance at the 5% and 10% levels, respectively.

Table C.8: Estimation results of the bivariate VAR-BEKK(1,1,1) model at the fifth level

	Variance equations							
	$c_{1,1}$	$c_{1,2}$	$c_{2,2}$	$a_{1,1}$	$a_{2,2}$	$g_{1,1}$	$g_{2,2}$	H_0
ARG-THA	$2.07E-06*$	$3.14E-06*$	$2.39E-06$	0.3873*	0.3873*	0.7746*	0.7746*	123308.800*
	(19.6178)	(21.0015)	(0.0023)	(38.2056)	(37.1974)	(92.0072)	(119.9974)	0.0000
BRA-THA	$2.87E-06*$	$3.05E-06*$	$1.56E-05$	0.3873*	0.3873*	0.7746*	0.7746*	99765.410*
	(19.7850)	(18.3726)	(0.0174)	(44.6975)	(37.0928)	(115.3288)	(92.1710)	0.0000
CHI1-THA	$1.57E-06*$	$3.39E-06*$	$3.28E-06$	0.3873*	0.3873*	0.7746*	0.7746*	86299.300*
	(22.6675)	(23.4899)	(0.0002)	(46.6992)	(44.4631)	(134.2086)	(124.8247)	0.0000
MEX-THA	$2.40E-06*$	$3.23E-06*$	$3.77E-06$	0.3873*	0.3873*	0.7746*	0.7746*	134811.700*
	(22.9454)	(20.3831)	(0.0024)	(53.0790)	(37.8994)	(149.4166)	(97.9754)	0.0000
PER-THA	$2.54E-06*$	$3.21E-06*$	$2.08E-06$	0.3873*	0.3873*	0.7746*	0.7746*	143000.500*
	(25.4687)	(24.9269)	(0.0030)	(54.6929)	(56.3808)	(182.7606)	(172.9094)	0.0000
AUS1-THA	$1.66E-06*$	$2.92E-06*$	$8.00E-06$	0.3873*	0.3873*	0.7746*	0.7746*	95559.990*
	(21.1015)	(18.0296)	(0.0015)	(36.8328)	(44.6981)	(100.4933)	(121.7015)	0.0000
AUS2-THA	$1.46E-06*$	$2.77E-06*$	$5.89E-06$	0.3873*	0.3873*	0.7746*	0.7746*	103580.900*
	(19.0710)	(17.9813)	(0.0004)	(36.3690)	(38.4579)	(99.1873)	(100.0788)	0.0000
CAN-THA	$1.09E-06*$	$3.14E-06*$	$1.84E-06$	0.3873*	0.3873*	0.7746*	0.7746*	136983.000*
	(28.8844)	(27.8518)	(0.0005)	(47.6034)	(41.0487)	(126.4934)	(120.2028)	0.0000
FRA-THA	$1.69E-06*$	$2.85E-06*$	$8.88E-06$	0.3873*	0.3873*	0.7746*	0.7746*	90557.040*
	(21.2172)	(18.9138)	(0.0014)	(42.4734)	(29.4057)	(102.2710)	(83.0021)	0.0000
GER-THA	$1.43E-06*$	$2.96E-06*$	$7.43E-06$	0.3873*	0.3873*	0.7746*	0.7746*	97253.030*
	(23.7554)	(20.5622)	(0.0014)	(45.9311)	(30.2755)	(127.7858)	(88.5824)	0.0000
ITA-THA	$2.80E-07*$	$3.99E-08*$	$1.28E-07$	0.4237*	0.4691*	0.9225*	0.9100*	119109.600*
	(4.4326)	(0.3740)	(1.1322)	(13.0081)	(12.3778)	(108.3116)	(86.6369)	0.0000
JAP-THA	$1.54E-06*$	$2.55E-06*$	$9.59E-06$	0.3873*	0.3873*	0.7746*	0.7746*	81496.650*
	(15.1225)	(15.3876)	(0.0016)	(41.4438)	(37.3514)	(102.0058)	(96.6542)	0.0000
NET-THA	$1.81E-06*$	$3.01E-06*$	$1.40E-05$	0.3873*	0.3873*	0.7746*	0.7746*	104812.200*
	(24.9067)	(21.9066)	(0.0059)	(45.7183)	(41.9595)	(120.1525)	(106.7851)	0.0000
SPA-THA	$1.57E-06*$	$2.79E-06*$	$6.44E-06$	0.3873*	0.3873*	0.7746*	0.7746*	94174.780*
	(20.5156)	(18.7604)	(0.0011)	(32.3725)	(38.7203)	(87.1399)	(97.6448)	0.0000
SWE-THA	$1.88E-06*$	$3.02E-06*$	$7.29E-06$	0.3873*	0.3873*	0.7746*	0.7746*	91660.010*
	(22.2430)	(18.9483)	(0.0013)	(40.7060)	(37.5749)	(123.5265)	(103.5310)	0.0000
UK-THA	$1.25E-06*$	$2.65E-06*$	$6.85E-06$	0.3873*	0.3873*	0.7746*	0.7746*	94663.690*
	(17.7169)	(16.2074)	(0.0012)	(30.9728)	(37.6189)	(86.1559)	(114.7183)	0.0000
USA-THA	$1.95E-06*$	$2.87E-06*$	$1.13E-05$	0.3873*	0.3873*	0.7746*	0.7746*	70256.690*
	(16.9316)	(16.1131)	(0.0014)	(35.3031)	(33.1681)	(89.2878)	(90.9220)	0.0000
CHI2-THA	$4.30E-06*$	$2.98E-06*$	$4.10E-06$	0.3873*	0.3873*	0.7746*	0.7746*	67500.460*
	(18.6048)	(17.7708)	(0.0009)	(28.5654)	(33.0710)	(77.2868)	(89.6074)	0.0000
KOR-THA	$1.73E-06*$	$2.85E-06*$	$1.71E-06$	0.3873*	0.3873*	0.7746*	0.7746*	154303.500*
	(23.1252)	(22.1388)	(0.0009)	(41.7596)	(36.3790)	(111.9661)	(93.4963)	0.0000
SIN-THA	$1.36E-06*$	$3.18E-06*$	$3.66E-06$	0.3873*	0.3873*	0.7746*	0.7746*	159161.900*
	(28.0801)	(28.7069)	(0.0037)	(57.4469)	(42.1051)	(153.4686)	(115.1208)	0.0000
MAL-THA	$1.73E-07*$	$5.41E-08*$	$1.16E-07$	0.4140*	0.4233*	0.9196*	0.9245*	118466.900*
	(6.4469)	(0.5101)	(1.1080)	(12.9884)	(12.9678)	(98.4455)	(108.2768)	0.0000
HK-THA	$2.46E-06*$	$3.22E-06*$	$5.16E-06$	0.3873*	0.3873*	0.7746*	0.7746*	125760.100*
	(25.5114)	(21.3931)	(0.0045)	(47.2829)	(34.0268)	(140.8639)	(88.5907)	0.0000
IND1-THA	$4.55E-06*$	$4.75E-06*$	$1.70E-05$	0.3873*	0.3873*	0.7746*	0.7746*	215486.700*
	(19.8473)	(19.2079)	(0.0316)	(39.9219)	(38.0357)	(106.8361)	(103.4827)	0.0000
IND2-THA	$3.13E-07*$	$1.01E-07*$	$4.77E-09$	0.4256*	0.4363*	0.9191*	0.9200*	130145.800*
	(6.6939)	(1.2203)	(0.0028)	(10.8281)	(13.4732)	(80.3668)	(108.4662)	0.0000
PHI-THA	$1.59E-06*$	$3.19E-06*$	$7.00E-06$	0.3873*	0.3873*	0.7746*	0.7746*	143241.900*
	(22.0721)	(24.3542)	(0.0022)	(41.7529)	(37.9060)	(99.7180)	(105.3203)	0.0000
TAI-THA	$1.82E-06*$	$3.16E-06*$	$1.11E-05$	0.3873*	0.3873*	0.7746*	0.7746*	84195.010*
	(20.0446)	(18.9617)	(0.0013)	(45.2985)	(36.3502)	(120.2650)	(95.6740)	0.0000

The parameters and Z−statistic values (in parentheses) are estimated on Equation (4.33) in the text. The lag length p in Equation (4.36) is determined by the AIC and LM criteria. Because the optimal lag length p in every case is different, the parameters and Z−statistic values of the variables in the mean equation are not attached here. The null hypothesis H_0 is that all of the parameters of the variables in the variance equation equal zero. Accordingly, the χ^2-values and the corresponding $p-$values are listed here as well. $*$ and $**$ represent statistical significance at the 5% and 10% levels, respectively.

Table C.9: Tests of changes in pair-wise conditional correlations between Thailand and other 26 markets stock returns at the first level during two different phases of the 1997 Asian crisis (1/1/1996-12/31/1997)

	ARG	BRA	CHI1	MEX	PER	AUS1	AUS2	CAN	FRA	GER	ITA	JAP	NET
Mean equation													
Constant	0.0094	−0.0263	0.0053	0.0621∗	−0.0467 ∗ ∗	0.0276	0.0442∗	0.0073	0.0496∗	0.0194	−0.0165	0.0015	0.0086
	(0.5581)	(−1.2596)	(0.3115)	(3.1070)	(−2.5395)	(1.3494)	(2.2167)	(0.3582)	(2.2821)	(1.0580)	(−0.8109)	(0.0856)	(0.4639)
$DM_{1,t}$	0.0152	0.0386	0.0118	−0.0451	−0.0072	−0.0252	0.0184	−0.0581	0.0526	−0.0029	0.0181	0.0290	0.0129
	(0.3482)	(0.8111)	(0.3592)	(−1.0363)	(−0.1825)	(−0.5873)	(0.3818)	(−1.4051)	(1.0516)	(−0.0626)	(0.3648)	(0.7917)	(0.2671)
Variance equation													
Constant	0.2983∗	0.2013∗	0.1364∗	0.1101∗	0.2852∗	0.3954∗	0.2781∗	0.2965∗	0.2656∗	0.3093∗	0.1891∗	0.2510∗	0.1973∗
	(11.9445)	(4.5841)	(4.1109)	(2.4825)	(13.1304)	(18.4259)	(11.1407)	(11.8741)	(9.1059)	(11.2130)	(6.7992)	(12.1048)	(6.5689)
ε^2_{t-1}	0.0079∗	−0.0914∗	−0.1200∗	−0.1314∗	−0.0266 ∗ ∗	0.0089	−0.0707∗	−0.0296	−0.1053∗	−0.0296∗	−0.1204∗	−0.0196	−0.0977∗
	(97.3579)	(−2.7296)	(−11.9559)	(−5.6528)	(−2.4767)	(1.2393)	(−2.5411)	(−1.6368)	(−3.5777)	(−2.8792)	(−4.3800)	(−1.5865)	(−2.1973)
h_{t-1}	−1.0104∗	−0.3330	0.0547	0.3985	−0.9667 ∗ ∗	−1.0070∗	−0.7681∗	−0.9591∗	−0.6376∗	−0.9620∗	−0.1845	−0.9761∗	−0.5457∗
	(−359.6911)	(−1.1892)	(0.2116)	(1.3765)	(−60.2050)	(−174.2163)	(−7.2655)	(−36.5172)	(−4.6528)	(−61.7083)	(−1.2449)	(−56.1402)	(−2.1725)
$DM_{1,t}$	0.0805	0.1074∗	0.0159	0.0115	0.1023∗	0.0287	0.1470∗	0.0441	0.1014∗	0.1766∗	0.0832∗	0.0484	0.1547∗
	(1.4294)	(2.0497)	(1.5755)	(0.9017)	(2.2423)	(0.7546)	(2.3565)	(1.1485)	(2.1825)	(3.7660)	(2.5414)	(1.3635)	(3.7384)

	SPA	SWE	UK	USA	CHI2	KOR	SIN	MAL	HK	IND1	IND2	PHI	TAI
Mean equation													
Constant	0.0075	0.0110	−0.0165	−0.0017	0.0607∗	−0.0167	0.1211∗	0.0100	0.0726∗	0.0141	0.0205	−0.0006	−0.0188
	(0.4140)	(0.5179)	(−0.9264)	(−0.0803)	(3.4302)	(−1.0969)	(6.0444)	(0.6068)	(4.1133)	(0.7955)	(1.0746)	(−0.0305)	(−0.9464)
$DM_{1,t}$	−0.0141	−0.0113	0.0340	−0.0798 ∗ ∗	0.0190	0.0503	0.0160	−0.0207	−0.0201	0.0005	0.0187	0.0058	−0.0166
	(−0.3213)	(−0.3112)	(0.7669)	(−1.9339)	(0.5983)	(1.4506)	(0.3936)	(−0.5532)	(−0.5303)	(0.0147)	(0.4242)	(0.1102)	(−0.5976)
Variance equation													
Constant	0.2558∗	0.2834∗	0.2160∗	0.1626∗	0.2729∗	0.2207∗	0.0161	0.1603∗	0.2334∗	0.2509∗	0.2469∗	0.2269∗	0.2356∗
	(12.0188)	(11.2290)	(7.6402)	(2.4598)	(12.4862)	(15.9038)	(1.4321)	(5.4923)	(12.4078)	(12.1296)	(8.8578)	(10.4327)	(9.7291)
ε^2_{t-1}	−0.0316∗	−0.0830∗	−0.1062∗	−0.1148∗	−0.0225∗	0.0041	0.1314 ∗ ∗	−0.1257∗	−0.0174	−0.0278∗	−0.0885∗	−0.0842∗	−0.1156∗
	(−2.1347)	(−4.6560)	(−3.4986)	(−9.6437)	(−2.2195)	(0.7317)	(1.9164)	(−10.3852)	(−1.3637)	(−2.0001)	(−2.6388)	(−2.9140)	(−4.4691)
h_{t-1}	−0.9531∗	−0.7700∗	−0.5115∗	0.0334	−0.9715∗	−1.0066∗	0.7197∗	−0.3332	−0.9774∗	−0.9610∗	−0.6605∗	−0.7117∗	−0.5759∗
	(−36.4992)	(−8.0526)	(−2.4054)	(0.0778)	(−66.7069)	(−122.0455)	(4.8049)	(−1.3152)	(−48.9806)	(−41.1300)	(−3.9222)	(−5.4120)	(−3.3574)
$DM_{1,t}$	0.1550∗	0.0073	0.0740∗	0.0145	−0.0271	0.0316	0.0125	0.0872∗	0.0882∗	0.0600	0.1326∗	0.2218∗	0.0756∗
	(2.9792)	(0.1918)	(2.5052)	(0.8153)	(−1.1964)	(1.1721)	(1.1338)	(3.1813)	(2.1336)	(1.5670)	(2.4519)	(4.1612)	(2.1377)

The parameters and Z−statistic values (in parentheses) are estimated on Equations (4.40) and (4.41) in the text. The lag length p in Equation (4.40) is determined by the AIC and LM criteria. Because the optimal lag length for each time series $\{\rho_{ij,t}\}$ is different, the parameters and Z−statistic values of lagged terms of $\rho_{ij,t}$ are not attached to the table. $DM_{1,t}$ is a dummy variable to distinguish tranquil period and crisis period. ∗ and ∗∗ represent statistical significance at the 5% and 10% levels, respectively. Because the first level is associated with a time interval of $[2, 4)$ days, the relationship between Thailand and another market is related to the time interval of $[2, 4)$ days as well.

Table C.10: Tests of changes in pair-wise conditional correlations between Thailand and other 26 markets stock returns at the second level during two different phases of the 1997 Asian crisis (1/1/1996-12/31/1997)

	ARG	BRA	CHI1	MEX	PER	AUS1	AUS2	CAN	FRA	GER	ITA	JAP	NET
Mean equation													
Constant	−0.0012	0.0057	0.0064	−0.0026	0.0152	0.0185	0.0075	−0.0056	−0.0010	0.0040	0.0367*	0.0049	0.0209 **
	(−0.1050)	(0.4818)	(0.6359)	(−0.2049)	(1.3069)	(1.5241)	(0.6251)	(−0.5209)	(−0.1150)	(0.4700)	(3.3104)	(0.5520)	(1.8591)
$DM_{1,t}$	0.0277	0.0126	0.0103	0.0085	0.0492*	0.0363	0.0252	0.0041	0.0216	0.0127	−0.0082	0.0175	0.0107
	(1.1859)	(0.6095)	(0.4660)	(0.3583)	(2.3106)	(1.5747)	(1.2055)	(0.2002)	(1.2801)	(0.7911)	(−0.3500)	(1.0423)	(0.5279)
Variance equation													
Constant	0.0178 **	0.0200	0.0904*	0.0196	0.0283 **	0.0043	0.0327	0.1024*	0.0692*	0.0694*	0.0017*	0.0790*	0.0757*
	(1.9207)	(1.5476)	(17.3961)	(0.9576)	(1.7022)	(1.5843)	(1.4635)	(14.8267)	(18.1552)	(17.9521)	(2.3944)	(36.1388)	(14.7140)
ε^2_{t-1}	−0.0707*	−0.0415*	−0.0341*	−0.0360	−0.0626*	−0.0064	−0.0529*	−0.0034	−0.0125*	−0.0068	−0.0024	0.0100*	−0.0448*
	(−6.1084)	(−2.3768)	(−7.1893)	(−1.2771)	(−3.8082)	(−0.5510)	(−10.3028)	(−0.7595)	(−3.7464)	(−1.1088)	(−0.3004)	(2.4152)	(−4.3813)
h_{t-1}	0.6722*	0.4626	−0.9334*	0.6855*	0.4092	0.9218*	0.3106	−0.9970*	−0.9839*	−0.9929*	0.9523*	−1.0108*	−0.8275*
	(3.3510)	(1.2485)	(−79.2292)	(1.9706)	(1.0756)	(16.8647)	(0.6103)	(−200.2285)	(−190.7788)	(−111.1678)	(43.5461)	(−252.5039)	(−12.2112)
$DM_{1,t}$	0.0009	0.0008	0.0017	−0.0024	−0.0039	−0.0002	−0.0002	0.0042	0.0187*	0.0048	0.0012*	0.0020	−0.0026
	(0.5210)	(0.3807)	(0.1929)	(−0.8163)	(−1.2679)	(−0.3010)	(−0.0733)	(0.7276)	(5.2214)	(1.1258)	(2.6252)	(0.3436)	(−0.3088)

	SPA	SWE	UK	USA	CHI2	KOR	SIN	MAL	HK	IND1	IND2	PHI	TAI
Mean equation													
Constant	0.0200*	0.0043	−0.0012	−0.0070	−0.0007	0.0048	0.0211*	−0.0043	0.0240*	−0.0035	0.0035	0.0007	−0.0108
	(1.8961)	(0.3805)	(−0.1131)	(−0.5862)	(−0.0587)	(0.4232)	(2.1619)	(−0.4679)	(2.0122)	(−0.3836)	(0.3712)	(0.0999)	(−1.0431)
$DM_{1,t}$	−0.0044	0.0145	0.0181	0.0192	−0.0103	0.0500*	−0.0192	0.0170	−0.0091	0.0276	0.0226	0.0043	0.0033
	(−0.2230)	(0.6971)	(0.7830)	(0.7668)	(−0.4337)	(2.0294)	(−1.1447)	(0.9487)	(−0.4566)	(1.4470)	(1.0459)	(0.2825)	(0.1604)
Variance equation													
Constant	0.0927*	0.0137*	0.1168*	0.0135*	0.0406	0.0280*	0.0814*	0.0846*	0.0803*	0.0799*	0.1079*	0.0513*	0.0728*
	(14.0284)	(2.5178)	(33.5283)	(2.2785)	(0.4589)	(2.0007)	(15.8288)	(35.7274)	(16.5867)	(22.2130)	(14.6500)	(17.7574)	(7.7261)
ε^2_{t-1}	0.0022	−0.0780*	0.0065*	−0.0658*	−0.0199	−0.0514*	−0.0005	0.0044	−0.0140*	0.0057	−0.0442*	−0.0105*	−0.0613*
	(0.8191)	(−4.8482)	(11.1393)	(−3.8945)	(−0.5041)	(−5.1468)	(−0.2469)	(1.1964)	(−4.7535)	(0.8865)	(−6.8249)	(−8.1007)	(−8.3022)
h_{t-1}	−1.0013*	0.7337*	−1.0085*	0.8018*	0.2400	0.4560	−0.9986*	−1.0091*	−0.9800*	−1.0082*	−0.9503*	−0.9812*	−0.5602*
	(−533.5875)	(5.7421)	(−7851.3160)	(7.7198)	(0.1424)	(1.5126)	(−707.3479)	(−155.8998)	(−166.5029)	(−126.6241)	(−92.5846)	(−309.8824)	(−2.4993)
$DM_{1,t}$	−0.0007	−0.0005	0.0073	0.0020	−0.0013	−0.0015	−0.0019	0.0006	0.0086	0.0111	0.0120	0.0059*	0.0002
	(−0.0771)	(−0.3744)	(0.7933)	(1.1770)	(−0.2168)	(−0.6002)	(−0.4920)	(0.1125)	(1.2582)	(1.5739)	(1.1524)	(2.0975)	(0.0250)

The parameters and Z−statistic values (in parentheses) are estimated on Equations (4.40) and (4.41) in the text. The lag length p in Equation (4.40) is determined by the AIC and LM criteria. Because the optimal lag length for each time series $\{\rho_{ij,t}\}$ is different, the parameters and Z−statistic values of lagged terms of $\rho_{ij,t}$ are not attached to the table. $DM_{1,t}$ is a dummy variable to distinguish the tranquil period and crisis period. * and ** represent statistical significance at the 5% and 10% levels, respectively. Because the second level is associated with a time interval of $[4, 8)$ days, the relationship between Thailand and another market is related to the time interval of $[4, 8)$ days as well.

Table C.11: Tests of changes in pair-wise conditional correlations between Thailand and other 26 markets stock returns at the third level during two different phases of the 1997 Asian crisis (1/1/1996-12/31/1997)

	ARG	BRA	CHI1	MEX	PER	AUS1	AUS2	CAN	FRA	GER	ITA	JAP	NET
Mean equation													
Constant	−0.0018	−0.0077	0.0270∗	0.0069	0.0253∗	−0.0064	0.0348∗	0.0297∗	0.0270∗	0.0152∗	0.0269∗	0.0014	0.0285∗
	(−0.1765)	(−0.7601)	(2.6968)	(0.6261)	(2.2632)	(−0.5818)	(4.3762)	(4.4293)	(2.5119)	(2.2727)	(2.2137)	(0.1683)	(3.3963)
$DM_{1,t}$	0.0024	−0.0067	−0.0133	−0.0274	−0.0205	0.0404 ∗ ∗	−0.0001	−0.0211 ∗ ∗	−0.0258	0.0101	−0.0248	0.0285 ∗ ∗	−0.0251 ∗ ∗
	(0.1175)	(−0.3306)	(−0.7689)	(−1.0807)	(−0.8154)	(1.8894)	(−0.0103)	(−1.6651)	(−1.0733)	(0.8033)	(−1.1444)	(1.9178)	(−1.6549)
Variance equation													
Constant	0.0138	0.0088∗	0.0176∗	0.0108 ∗ ∗	0.0139 ∗ ∗	0.0529∗	0.0254∗	0.0195∗	0.0249	0.0023	0.0013∗	0.0461∗	0.0084
	(1.5207)	(2.8837)	(3.6476)	(1.8735)	(1.7771)	(3.9606)	(6.8400)	(7.2897)	(0.9684)	(1.0049)	(7.0213)	(7.4370)	(0.3774)
ε^2_{t-1}	0.0848 ∗ ∗	0.1206∗	0.1858∗	0.0638∗	0.1075 ∗ ∗	0.0841	0.1504∗	0.1095∗	−0.0399	−0.0180	−0.0427∗	0.1114∗	−0.0169
	(1.6994)	(2.7893)	(2.8816)	(1.9493)	(1.8364)	(1.5005)	(2.5732)	(2.8315)	(−1.4494)	(−1.0542)	(−5.7346)	(2.8367)	(−0.4799)
h_{t-1}	0.5855∗	0.6877∗	0.0312	0.6977∗	0.6046∗	−0.4118	−0.2298 ∗ ∗	−0.4086∗	0.3768	0.8649∗	1.0102∗	−0.5778∗	0.5335
	(2.3827)	(7.4978)	(0.1538)	(4.8249)	(3.0487)	(−1.4913)	(−1.7945)	(−2.9965)	(0.5572)	(5.9727)	(202.4092)	(−3.3064)	(0.4226)
$DM_{1,t}$	−0.0002	−0.0003	0.0079∗	0.0048	0.0022	0.0024	−0.0046 ∗ ∗	0.0014	0.0125	−0.0004	0.0006∗	−0.0130∗	0.0019
	(−0.0996)	(−0.1726)	(2.4484)	(1.6166)	(0.8486)	(0.3320)	(−1.6949)	(0.5106)	(0.9320)	(−0.9725)	(4.6666)	(−2.8625)	(0.3609)

	SPA	SWE	UK	USA	CHI2	KOR	SIN	MAL	HK	IND1	IND2	PHI	TAI
Mean equation													
Constant	0.0274∗	0.0037	0.0276∗	0.0504∗	0.0110	−0.0014	0.0546∗	0.0589∗	0.0016	0.0034	0.0676∗	0.0215∗	0.0138
	(3.4358)	(0.3568)	(3.4164)	(4.5766)	(1.2132)	(−0.1723)	(4.5707)	(5.8299)	(0.3839)	(0.2756)	(5.0996)	(2.2820)	(0.6578)
$DM_{1,t}$	−0.0160	−0.0015	−0.0238 ∗ ∗	−0.0591∗	−0.0210	0.0088	−0.0406 ∗ ∗	−0.0213	0.0255∗	−0.0030	−0.0170	0.0073	0.1020∗
	(−1.0708)	(−0.0678)	(−1.6656)	(−2.7730)	(−0.8004)	(0.4578)	(−1.7700)	(−1.5730)	(2.1170)	(−0.1151)	(−0.7340)	(0.3635)	(2.5077)
Variance equation													
Constant	0.0091 ∗ ∗	0.0482∗	0.0006∗	0.0434∗	0.0113∗	0.0019∗	0.0001∗	0.0001∗	0.0000	0.0094∗	0.0108∗	0.0096∗	0.0160∗
	(1.7264)	(4.0440)	(2.4921)	(3.4772)	(5.3925)	(2.0076)	(4.7603)	(2.7073)	(0.8391)	(3.2239)	(4.5012)	(3.0525)	(2.8296)
ε^2_{t-1}	0.0741	0.0685	0.0789∗	0.0886 ∗ ∗	0.4795∗	0.0817∗	0.0415∗	0.1135∗	1.2331∗	0.1261∗	0.2781∗	0.1155∗	0.0883∗
	(1.6166)	(1.4934)	(3.3201)	(1.6879)	(5.8313)	(3.1471)	(5.7256)	(6.0422)	(10.2736)	(3.2619)	(4.6008)	(2.5980)	(3.1065)
h_{t-1}	0.4649	−0.4827	0.8772∗	−0.3127	0.4241∗	0.8609∗	0.9603∗	0.8863∗	0.4908∗	0.7227∗	0.4623∗	0.5777∗	0.8458∗
	(1.6034)	(−1.5294)	(24.5935)	(−1.0473)	(6.4459)	(19.8686)	(133.9797)	(58.2570)	(16.1278)	(10.7297)	(5.0031)	(4.4571)	(20.1872)
$DM_{1,t}$	0.0007	0.0244∗	0.0005	0.0122 ∗ ∗	0.0059∗	0.0011	0.0002	0.0005∗	0.0014∗	0.0020	0.0129∗	0.0026 ∗ ∗	−0.0092∗
	(0.4141)	(2.5033)	(1.5028)	(1.7225)	(2.2734)	(1.5190)	(0.6561)	(2.5205)	(1.9768)	(1.0250)	(3.5207)	(1.8449)	(−2.3742)

The parameters and Z−statistic values (in parentheses) are estimated on Equations (4.40) and (4.41) in the text. The lag length p in Equation (4.40) is determined by the AIC and LM criteria. Because the optimal lag length for each time series $\{\rho_{ij,t}\}$ is different, the parameters and Z−statistic values of lagged terms of $\rho_{ij,t}$ are not attached to the table. $DM_{1,t}$ is a dummy variable to distinguish the tranquil period and crisis period. ∗ and ∗∗ represent statistical significance at the 5% and 10% levels, respectively. Because the third level is associated with a time interval of $[8, 16)$ days, the relationship between Thailand and another market is related to the time interval of $[8, 16)$ days as well.

Table C.12: Tests of changes in pair-wise conditional correlations between Thailand and other 26 markets stock returns at the fourth level during two different phases of the 1997 Asian crisis (1/1/1996-12/31/1997)

	ARG	BRA	CHI1	MEX	PER	AUS1	AUS2	CAN	FRA	GER	ITA	JAP	NET
Mean equation													
Constant	0.0005	−0.0007	0.0113	0.0055	0.0020	0.0043	0.0181∗	0.0024	0.0002	−0.0010	−0.0043	0.0045	0.0009
	(0.0871)	(−0.1419)	(1.5059)	(1.1665)	(0.5789)	(0.6000)	(2.9557)	(0.5921)	(0.0363)	(−0.2261)	(−1.5551)	(1.0142)	(0.4822)
$DM_{1,t}$	0.0081	0.0086	0.0146 ∗ ∗	0.0182 ∗ ∗	0.0050	0.0089	0.0087	−0.0006	0.0097	0.0129	0.0174 ∗ ∗	−0.0139	0.0013
	(0.8894)	(0.6834)	(1.6986)	(1.6649)	(0.8474)	(0.7504)	(1.1172)	(−0.1080)	(1.3634)	(1.5644)	(1.9306)	(−1.1923)	(0.7131)
Variance equation													
Constant	0.0010∗	0.0071	0.0003∗	0.0051	0.0002∗	0.0130∗	0.0018∗	0.0004∗	0.0003∗	0.0005∗	0.0002∗	0.0001∗	$4.29E-05$ ∗ ∗
	(2.4694)	(1.2825)	(2.4559)	(4.1794)	(2.3554)	(2.0035)	(23.9889)	(8.8336)	(2.5539)	(2.8848)	(8.3545)	(2.9631)	(1.9569)
ε^2_{t-1}	0.1246∗	−0.0294∗	0.1243∗	−0.0212∗	0.2577∗	−0.0436∗	−0.0390∗	0.7849∗	0.3177∗	0.2247∗	0.2707∗	0.5278∗	1.2777∗
	(2.7933)	(−6.4600)	(4.8135)	(−2.0341)	(4.9918)	(−13.0411)	(−5.5136)	(8.0540)	(5.4487)	(4.9180)	(8.8943)	(7.7599)	(10.7463)
h_{t-1}	0.7915∗	0.2107	0.8550∗	0.4803∗	0.7372∗	0.1873	0.7066∗	0.5480∗	0.7110∗	0.7452∗	0.7601∗	0.6374∗	0.4593∗
	(11.2037)	(0.3312)	(30.5900)	(3.5991)	(16.4155)	(0.4462)	(98.9309)	(20.1053)	(20.1827)	(17.3300)	(41.1981)	(19.1189)	(18.2136)
$DM_{1,t}$	−0.0004 ∗ ∗	−0.0016	−0.0002	−0.0014∗	−0.0001	−0.0069 ∗ ∗	−0.0004∗	−0.0004∗	−0.0001	−0.0001	0.0005∗	0.0008∗	−0.0001 ∗ ∗
	(−1.8110)	(−1.2224)	(−1.2379)	(−3.4607)	(−1.2686)	(−1.8979)	(−3.5795)	(−8.8560)	(−0.5131)	(−0.4255)	(3.0027)	(2.2507)	(−1.7390)

	SPA	SWE	UK	USA	CHI2	KOR	SIN	MAL	HK	IND1	IND2	PHI	TAI
Mean equation													
Constant	0.0033	−0.0032	0.0036	0.0013	0.0242	0.0005	0.0197∗	0.0030	−0.0002	−0.0058 ∗ ∗	0.0315	0.0100∗	0.0009
	(1.1711)	(−0.7026)	(1.1776)	(0.2040)	(0.8021)	(0.0820)	(6.0545)	(0.5201)	(−0.0334)	(−1.6473)	(0.9550)	(2.2898)	(0.2382)
$DM_{1,t}$	0.0078	0.0161∗	−0.0095	0.0197	−0.0023	0.0111	0.0037	0.0133	0.0135	0.0046	0.0011	−0.0028	0.0034
	(0.6559)	(2.1842)	(−1.5193)	(1.4493)	(−0.1276)	(0.9988)	(0.7617)	(1.3498)	(1.3963)	(0.6106)	(0.1462)	(−0.6778)	(0.4811)
Variance equation													
Constant	0.0001∗	0.0008∗	0.0001∗	0.0118	0.0005∗	0.0011∗	0.0072∗	0.0046	0.0090 ∗ ∗	0.0001	0.0002∗	$3.95E-05$∗	0.0003∗
	(3.2500)	(5.5147)	(4.8176)	(1.0194)	(3.2231)	(4.0651)	(15.1232)	(0.9827)	(1.7563)	(1.6395)	(4.2091)	(3.0288)	(3.8485)
ε^2_{t-1}	0.4217∗	0.2451∗	0.8310∗	−0.0228	0.0564∗	0.2608∗	−0.0008 ∗ ∗	−0.0211∗	−0.0358∗	0.3014∗	0.1144∗	0.2791∗	0.1942∗
	(6.4076)	(5.5564)	(6.2698)	(−0.7315)	(4.7072)	(4.8889)	(−1.6684)	(−3.6632)	(−10.5366)	(6.1558)	(5.5082)	(6.8107)	(4.7506)
h_{t-1}	0.6567∗	0.7228∗	0.5609∗	0.1979	0.9309∗	0.7005∗	−0.9984∗	0.4679	0.2846	0.7582∗	0.8631∗	0.7892∗	0.8006∗
	(15.3096)	(18.4106)	(12.7517)	(0.2463)	(74.3544)	(14.9085)	(−2059.1350)	(0.8338)	(0.6689)	(28.1228)	(34.5258)	(33.6901)	(23.7042)
$DM_{1,t}$	0.0016∗	−0.0006∗	−0.0001	−0.0042	−0.0003∗	−0.0002	0.0058∗	0.0001	−0.0033	0.0001	0.0000	0.0000	−0.0003∗
	(3.8653)	(−4.9012)	(−1.6130)	(−1.0275)	(−2.2052)	(−1.0039)	(4.5714)	(0.1977)	(−1.7112)	(0.5888)	(0.4974)	(−0.0202)	(−3.6733)

The parameters and Z−statistic values (in parentheses) are estimated on Equations (4.40) and (4.41) in the text. The lag length p in Equation (4.40) is determined by the AIC and LM criteria. Because the optimal lag length for each time series $\{\rho_{ij,t}\}$ is different, the parameters and Z−statistic values of lagged terms of $\rho_{ij,t}$ are not attached to the table. $DM_{1,t}$ is a dummy variable to distinguish the tranquil period and crisis period. ∗ and ∗∗ represent statistical significance at the 5% and 10% levels, respectively. Because the fourth level is associated with a time interval of $[16, 32)$ days, the relationship between Thailand and another market is related to the time interval of $[16, 32)$ days as well.

Table C.13: Tests of changes in pair-wise conditional correlations between Thailand and other 26 markets stock returns at the fifth level during two different phases of the 1997 Asian crisis (1/1/1996-12/31/1997)

	ARG	BRA	CHI1	MEX	PER	AUS1	AUS2	CAN	FRA	GER	ITA	JAP	NET
Mean equation													
Constant	0.0725*	0.0218*	0.0540*	0.0874*	0.0589*	0.0226 **	0.0172	0.0307	0.0231*	0.0581*	0.0049	0.0105*	0.0438*
	(3.7939)	(3.3138)	(2.9086)	(5.6547)	(2.9705)	(1.8762)	(1.6423)	(1.3675)	(2.3867)	(4.7981)	(0.8836)	(1.6782)	(6.6585)
$DM_{1,t}$	−0.0595*	−0.0107*	−0.0034	−0.0225	−0.0046	−0.0040	0.0182 **	−0.0111	−0.0066	−0.0230*	0.0020	0.0119	−0.0051
	(−2.6657)	(−2.0690)	(−0.4207)	(−1.3227)	(−0.2942)	(−0.4761)	(1.9547)	(−0.6739)	(−1.0884)	(−2.9722)	(0.1772)	(1.2970)	(−1.0518)
Variance equation													
Constant	0.0068	$3.28E-06$*	0.0038*	0.0107*	0.0012*	0.0031*	0.0081*	$2.23E-05$*	$2.95E-06$ **	$9.96E-07$	0.0184*	0.0109*	0.0041*
	(1.2800)	(3.6045)	(18.0983)	(19.7836)	(2.0321)	(3.2950)	(4.0844)	(5.8850)	(1.7640)	(0.2754)	(23.0788)	(19.4843)	(22.2284)
ε^2_{t-1}	−0.0390*	0.0552*	−0.0735*	−0.0344*	−0.0271*	−0.0595*	−0.0420*	0.1285*	0.0424*	0.0489*	−0.0262*	−0.0328*	−0.0281*
	(−9.4427)	(8.3586)	(−5.2107)	(−3.5860)	(−6.2838)	(−9.4095)	(−4.6898)	(7.8832)	(4.9270)	(5.4060)	(−412.7987)	(−6.9050)	(−21.6742)
h_{t-1}	0.1034	0.9541*	0.3895*	−0.7766*	0.2722	0.5000*	0.1968	0.8910*	0.9663*	0.9613*	−0.8339*	−0.7697*	−0.8323*
	(0.1422)	(185.4493)	(12.4616)	(−8.3480)	(0.7480)	(2.9764)	(1.0226)	(82.4411)	(117.3522)	(107.7587)	(−23.9895)	(−9.9317)	(−20.2982)
$DM_{1,t}$	0.0237	$-4.89E-05$*	−0.0005	0.0204*	0.0073*	−0.0005	−0.0033*	0.0003	−0.0001*	−0.0001*	0.0028 **	−0.0048*	−0.0010*
	(1.2489)	(−6.7096)	(−1.3098)	(5.6008)	(2.0674)	(−1.5438)	(−3.4161)	(1.3306)	(−5.8411)	(−3.2315)	(1.8481)	(−6.5010)	(−3.1165)

	SPA	SWE	UK	USA	CHI2	KOR	SIN	MAL	HK	IND1	IND2	PHI	TAI
Mean equation													
Constant	0.0304*	0.0426*	0.0527*	0.0184*	0.0019	0.0321*	0.1569*	0.0054	0.1093*	0.0227*	0.0034	0.0428*	0.0274*
	(3.2798)	(4.2789)	(6.9723)	(2.1549)	(0.1385)	(3.3298)	(18.5909)	(0.9055)	(14.9287)	(2.8572)	(0.6695)	(2.6931)	(3.0717)
$DM_{1,t}$	0.0137	0.0055	−0.0005	−0.0038	0.0068	0.0067	−0.0294*	0.0041	−0.0316*	0.0059	0.0052	−0.0038	−0.0096
	(1.3368)	(0.6179)	(−0.0535)	(−0.8465)	(0.3661)	(0.3071)	(−3.2114)	(0.3517)	(−2.7770)	(1.4840)	(0.7651)	(−0.7315)	(−1.2756)
Variance equation													
Constant	0.0134*	0.0113*	0.0160*	0.0000	0.0142	0.0223*	0.0089*	0.0064*	0.0137*	0.0004*	0.0004*	0.0000	0.0016 **
	(9.5933)	(13.9311)	(37.0596)	(1.2522)	(1.4967)	(18.7811)	(19.2659)	(2.0683)	(18.6518)	(6.7711)	(6.2536)	(1.1886)	(1.7684)
ε^2_{t-1}	−0.0304*	−0.0436*	−0.0225*	0.0686*	−0.0359*	−0.0323*	0.0020*	−0.0307*	0.0022*	0.1606*	0.1125*	0.1059*	−0.0372*
	(−56.9463)	(−13.0725)	(−4.3429)	(6.1457)	(−2.5609)	(−13.9885)	(9.9251)	(−1.9943)	(7.7946)	(5.5753)	(5.9416)	(5.8143)	(−3.1885)
h_{t-1}	−0.6231*	−0.7230*	−0.9243*	0.9443*	0.0980	−0.8662*	−1.0000*	0.4754 **	−1.0003*	0.7759*	0.8686*	0.9116*	0.4848
	(−3.1823)	(−5.4179)	(−42.4901)	(107.2757)	(0.1582)	(−48.6044)	(−71214.7400)	(1.7962)	(−13202.8300)	(35.9530)	(51.4801)	(63.9558)	(1.6139)
$DM_{1,t}$	−0.0020 **	−0.0010	−0.0046*	−0.0001*	0.0025	0.0143*	0.0182*	0.0011 **	0.0190*	−0.0004*	−0.0003*	0.0000	−0.0004
	(−1.8156)	(−0.8550)	(−4.2883)	(−4.5249)	(1.2328)	(3.9176)	(4.0852)	(1.6743)	(3.5979)	(−6.7739)	(−6.2376)	(−0.7309)	(−1.1572)

The parameters and Z−statistic values (in parentheses) are estimated on Equations (4.40) and (4.41) in the text. The lag length p in Equation (4.40) is determined by the AIC and LM criteria. Because the optimal lag length for each time series $\{\rho_{ij,t}\}$ is different, the parameters and Z−statistic values of lagged terms of $\rho_{ij,t}$ are not attached to the table. $DM_{1,t}$ is a dummy variable to distinguish the tranquil period and crisis period. * and ** represent statistical significance at the 5% and 10% levels, respectively. Because the fifth level is associated with a time interval of $[32, 64)$ days, the relationship between Thailand and another market is related to the time interval of $[32, 64)$ days as well.

Table C.14: Granger Causality Test at the first level

H_0: A⇏B	Tranquil Period			Crisis Period		
H_1: A⇒B	χ^2 value	p value	Causality	χ^2 value	p value	Causality
THA⇒ARG	53.2421	0.0000	Yes∗	4.0741	0.5388	No
ARG⇒THA	13.0202	0.0232	Yes∗	2.6504	0.7537	No
THA⇒BRA	2.0364	0.8441	No	25.4035	0.0001	Yes∗
BRA⇒THA	16.3463	0.0059	Yes∗	37.0080	0.0000	Yes∗
THA⇒CHI1	16.6149	0.0053	Yes∗	3.4833	0.6259	No
CHI1⇒THA	2.3836	0.7939	No	2.2478	0.8139	No
THA⇒MEX	7.0571	0.4230	No	92.3563	0.0000	Yes∗
MEX⇒THA	31.9789	0.0000	Yes∗	16.6674	0.0052	Yes∗
THA⇒PER	6.2058	0.5159	No	35.1427	0.0000	Yes∗
PER⇒THA	74.9207	0.0000	Yes∗	11.8169	0.0374	Yes∗
THA⇒AUS1	7.0474	0.4240	No	4.6126	0.4650	No
AUS1 ⇒THA	10.6696	0.1537	No	15.5721	0.0082	Yes∗
THA⇒AUS2	35.3452	0.0000	Yes∗	27.4943	0.0000	Yes∗
AUS2⇒THA	21.6103	0.0006	Yes∗	68.8211	0.0000	Yes∗
THA⇒CAN	23.3989	0.0007	Yes∗	4.3581	0.4991	No
CAN ⇒THA	18.6155	0.0049	Yes∗	2.4009	0.7913	No
THA⇒FRA	37.0738	0.0000	Yes∗	5.6251	0.3444	No
FRA⇒THA	11.1842	0.1308	No	134.2011	0.0000	Yes∗
THA⇒GER	2.5154	0.7742	No	9.4922	0.0910	Yes∗∗
GER⇒THA	2.8446	0.7239	No	26.2865	0.0001	Yes∗
THA⇒ITA	17.5070	0.0076	Yes∗	14.4133	0.0132	Yes∗
ITA⇒THA	43.8591	0.0000	Yes∗	16.2175	0.0062	Yes∗
THA⇒JAP	31.0183	0.0001	Yes∗	3.3226	0.6504	No
JAP⇒THA	15.3628	0.0316	Yes∗	57.2069	0.0000	Yes∗
THA⇒NET	17.6701	0.0136	Yes∗	1.5520	0.9070	No
NET⇒THA	30.5820	0.0001	Yes∗	10.8377	0.0547	Yes∗∗
THA⇒SPA	14.8998	0.0108	Yes∗	11.2107	0.0474	Yes∗
SPA⇒THA	7.0829	0.2145	No	10.8520	0.0544	Yes∗∗
THA⇒SWE	1.4094	0.9653	No	3.5583	0.6146	No
SWE⇒THA	4.7019	0.5826	No	75.6129	0.0000	Yes∗
THA⇒UK	8.2583	0.3104	No	10.3164	0.0667	Yes∗∗
UK⇒THA	9.0232	0.2510	No	10.8301	0.0549	Yes∗∗
THA⇒USA	30.9352	0.0001	Yes∗	6.8745	0.2301	No
USA⇒THA	111.8163	0.0000	Yes∗	5.7240	0.3340	No
THA⇒CHI2	12.2708	0.0313	Yes∗	2.7903	0.7323	No
CHI2⇒THA	14.0905	0.0150	Yes∗	0.5609	0.9897	No
THA⇒KOR	7.4426	0.1897	No	7.8049	0.1673	No
KOR⇒THA	6.6940	0.2444	No	35.7908	0.0000	Yes∗
THA⇒SIN	4.8419	0.4355	No	6.9114	0.2273	No
SIN⇒THA	29.1283	0.0000	Yes∗	36.1568	0.0000	Yes∗
THA⇒MAL	11.0243	0.0509	Yes∗∗	17.6810	0.0034	Yes∗
MAL⇒THA	1.3632	0.9283	No	5.2804	0.3826	No
THA⇒HK	19.5487	0.0066	Yes∗	3.7489	0.5861	No
HK⇒THA	24.3701	0.0010	Yes∗	137.0034	0.0000	Yes∗
THA⇒IND1	0.7249	0.9816	No	12.5908	0.0275	Yes∗
IND1⇒THA	16.7667	0.0050	Yes∗	32.6856	0.0000	Yes∗
THA⇒IND2	29.2535	0.0000	Yes∗	11.9097	0.0360	Yes∗
IND2⇒THA	28.8133	0.0000	Yes∗	41.5384	0.0000	Yes∗
THA⇒PHI	13.2213	0.0669	Yes∗∗	15.9496	0.0070	Yes∗
PHI⇒THA	123.9025	0.0000	Yes∗	2.3454	0.7996	No
THA⇒TAI	24.1019	0.0011	Yes∗	37.4920	0.0000	Yes∗
TAI⇒THA	16.6009	0.0202	Yes∗	43.8210	0.0000	Yes∗

∗ and ∗∗ represent statistical significance at the 5% and 10% levels, respectively. Because the first level is associated with a time interval of $[2, 4)$ days, the relationship between Thailand and another market is related to the time interval of $[2, 4)$ days as well.

Table C.15: Granger Causality Test at the second level

H_0: A⇏B	Tranquil Period			Crisis Period		
H_1: A⇒B	χ^2 value	p value	Causality	χ^2 value	p value	Causality
THA⇒ARG	6.0149	0.1980	No	9.9061	0.0420	Yes∗
ARG⇒THA	1.4624	0.8333	No	20.7396	0.0004	Yes∗
THA⇒BRA	5.2979	0.2581	No	11.5858	0.0719	Yes∗∗
BRA⇒THA	16.7593	0.0022	Yes∗	29.6612	0.0000	Yes∗
THA⇒CHI1	21.2106	0.0003	Yes∗	4.6387	0.7954	No
CHI1⇒THA	12.2271	0.0157	Yes∗	16.0644	0.0415	Yes∗
THA⇒MEX	10.6767	0.0989	Yes∗∗	14.8526	0.0214	Yes∗
MEX⇒THA	44.4203	0.0000	Yes∗	4.7276	0.5792	No
THA⇒PER	100.4227	0.0000	Yes∗	4.5794	0.5988	No
PER⇒THA	24.2648	0.0021	Yes∗	0.1069	1.0000	No
THA⇒AUS1	26.0826	0.0000	Yes∗	5.2540	0.5117	No
AUS1 ⇒THA	0.5274	0.9708	No	17.8241	0.0067	Yes∗
THA⇒AUS2	8.3797	0.0786	Yes∗∗	74.3693	0.0000	Yes∗
AUS2⇒THA	14.2222	0.0066	Yes∗	51.6653	0.0000	Yes∗
THA⇒CAN	15.7486	0.0461	Yes∗	13.3106	0.0384	Yes∗
CAN ⇒THA	20.8726	0.0075	Yes∗	6.4310	0.3767	No
THA⇒FRA	21.6431	0.0056	Yes∗	28.2088	0.0001	Yes∗
FRA⇒THA	33.9946	0.0000	Yes∗	6.5414	0.3653	No
THA⇒GER	9.6367	0.2915	No	6.8442	0.3355	No
GER⇒THA	4.8814	0.7702	No	3.4572	0.7497	No
THA⇒ITA	4.8550	0.3025	No	95.9746	0.0000	Yes∗
ITA⇒THA	7.9574	0.0932	Yes∗∗	5.3774	0.4964	No
THA⇒JAP	86.4466	0.0000	Yes∗	8.2588	0.2197	No
JAP⇒THA	16.2680	0.0387	Yes∗	2.5414	0.8638	No
THA⇒NET	38.4533	0.0000	Yes∗	30.9235	0.0000	Yes∗
NET⇒THA	10.9458	0.2048	No	9.7386	0.1361	No
THA⇒SPA	10.1039	0.0387	Yes∗	31.1769	0.0000	Yes∗
SPA⇒THA	8.6256	0.0712	Yes∗∗	4.1887	0.6512	No
THA⇒SWE	3.0666	0.5467	No	5.1588	0.5236	No
SWE⇒THA	10.1511	0.0380	Yes∗	1.9518	0.9241	No
THA⇒UK	27.6967	0.0005	Yes∗	39.0049	0.0000	Yes∗
UK⇒THA	10.6719	0.2210	No	21.1606	0.0017	Yes∗
THA⇒USA	78.8937	0.0000	Yes∗	9.8869	0.1295	No
USA⇒THA	8.9467	0.3468	No	4.3170	0.6339	No
THA⇒CHI2	10.6607	0.0307	Yes∗	33.2600	0.0000	Yes∗
CHI2⇒THA	2.3013	0.6805	No	18.7580	0.0046	Yes∗
THA⇒KOR	10.6501	0.0308	Yes∗	38.2276	0.0000	Yes∗
KOR⇒THA	1.3390	0.8547	No	40.9109	0.0000	Yes∗
THA⇒SIN	1.7714	0.7777	No	35.0495	0.0000	Yes∗
SIN⇒THA	4.3808	0.3569	No	17.8604	0.0223	Yes∗
THA⇒MAL	8.2056	0.4136	No	7.3820	0.4960	No
MAL⇒THA	42.1047	0.0000	Yes∗	0.0002	1.0000	No
THA⇒HK	39.7953	0.0000	Yes∗	17.7445	0.0014	Yes∗
HK⇒THA	20.0883	0.0100	Yes∗	23.3187	0.0001	Yes∗
THA⇒IND1	8.1615	0.0858	Yes∗∗	12.0827	0.0601	Yes∗∗
IND1⇒THA	1.1199	0.8911	No	14.7606	0.0222	Yes∗
THA⇒IND2	0.9154	0.9223	No	1.0245	0.9061	No
IND2⇒THA	3.9055	0.4190	No	27.2560	0.0000	Yes∗
THA⇒PHI	19.8554	0.0005	Yes∗	26.6793	0.0002	Yes∗
PHI⇒THA	15.6929	0.0035	Yes∗	10.5764	0.1024	No
THA⇒TAI	0.4805	0.9754	No	84.2997	0.0000	Yes∗
TAI⇒THA	11.4296	0.0221	Yes∗	16.2563	0.0389	Yes∗

∗ and ∗∗ represent statistical significance at the 5% and 10% levels, respectively. Because the second level is associated with a time interval of $[4, 8)$ days, the relationship between Thailand and another market is related to the time interval of $[4, 8)$ days as well.

Table C.16: Granger Causality Test at the third level

H_0: A$\nRightarrow$B	Tranquil Period			Crisis Period		
H_1: A$\Rightarrow$B	χ^2 value	p value	Causality	χ^2 value	p value	Causality
THA⇒ARG	38.5550	0.0000	Yes∗	16.1316	0.0405	Yes∗
ARG⇒THA	64.5048	0.0000	Yes∗	1537.5510	0.0000	Yes∗
THA⇒BRA	45.3247	0.0000	Yes∗	23.1051	0.0032	Yes∗
BRA⇒THA	145.3844	0.0000	Yes∗	78.3395	0.0000	Yes∗
THA⇒CHI1	6.2181	0.5145	No	7.5386	0.4798	No
CHI1⇒THA	22.7813	0.0019	Yes∗	269.9656	0.0000	Yes∗
THA⇒MEX	23.5717	0.0027	Yes∗	3.6922	0.8838	No
MEX⇒THA	20.9139	0.0074	Yes∗	496.6629	0.0000	Yes∗
THA⇒PER	40.2970	0.0000	Yes∗	25.8201	0.0011	Yes∗
PER⇒THA	22.7513	0.0019	Yes∗	90.5100	0.0000	Yes∗
THA⇒AUS1	53.1646	0.0000	Yes∗	21.4473	0.0060	Yes∗
AUS1 ⇒THA	30.1635	0.0002	Yes∗	44.2529	0.0000	Yes∗
THA⇒AUS2	31.4955	0.0001	Yes∗	38.4411	0.0000	Yes∗
AUS2⇒THA	13.9471	0.0832	Yes∗∗	379.3201	0.0000	Yes∗
THA⇒CAN	83.3122	0.0000	Yes∗	27.3080	0.0006	Yes∗
CAN ⇒THA	55.9003	0.0000	Yes∗	195.3355	0.0000	Yes∗
THA⇒FRA	36.9100	0.0000	Yes∗	25.2872	0.0014	Yes∗
FRA⇒THA	64.4105	0.0000	Yes∗	524.3319	0.0000	Yes∗
THA⇒GER	102.6478	0.0000	Yes∗	31.2945	0.0001	Yes∗
GER⇒THA	38.2739	0.0000	Yes∗	381.8075	0.0000	Yes∗
THA⇒ITA	102.6290	0.0000	Yes∗	35.7107	0.0000	Yes∗
ITA⇒THA	24.1030	0.0022	Yes∗	77.5642	0.0000	Yes∗
THA⇒JAP	28.4399	0.0004	Yes∗	318.5542	0.0000	Yes∗
JAP⇒THA	132.7427	0.0000	Yes∗	474.5543	0.0000	Yes∗
THA⇒NET	46.2769	0.0000	Yes∗	19.3755	0.0130	Yes∗
NET⇒THA	18.2300	0.0196	Yes∗	33.1825	0.0001	Yes∗
THA⇒SPA	34.2853	0.0000	Yes∗	34.8713	0.0000	Yes∗
SPA⇒THA	12.6452	0.1247	No	371.6014	0.0000	Yes∗
THA⇒SWE	19.5589	0.0033	Yes∗	56.4999	0.0000	Yes∗
SWE⇒THA	17.4004	0.0079	Yes∗	763.7298	0.0000	Yes∗
THA⇒UK	42.5432	0.0000	Yes∗	6.7408	0.5648	No
UK⇒THA	55.7307	0.0000	Yes∗	207.5835	0.0000	Yes∗
THA⇒USA	30.7880	0.0002	Yes∗	8.9973	0.3425	No
USA⇒THA	25.5292	0.0013	Yes∗	1193.3340	0.0000	Yes∗
THA⇒CHI2	15.4811	0.0168	Yes∗	35.4941	0.0000	Yes∗
CHI2⇒THA	28.2366	0.0001	Yes∗	14.5998	0.0674	Yes∗∗
THA⇒KOR	6.5888	0.0371	Yes∗	6.7731	0.5613	No
KOR⇒THA	3.5007	0.1737	No	151.0501	0.0000	Yes∗
THA⇒SIN	127.8665	0.0000	Yes∗	12.9131	0.1149	No
SIN⇒THA	6.3789	0.3821	No	9.0477	0.3383	No
THA⇒MAL	20.5171	0.0085	Yes∗	13.1565	0.1066	No
MAL⇒THA	34.3658	0.0000	Yes∗	30.6140	0.0002	Yes∗
THA⇒HK	68.5327	0.0000	Yes∗	4.4903	0.8104	No
HK⇒THA	58.3491	0.0000	Yes∗	136.6980	0.0000	Yes∗
THA⇒IND1	90.9311	0.0000	Yes∗	24.1845	0.0021	Yes∗
IND1⇒THA	22.6271	0.0020	Yes∗	11.7307	0.1636	No
THA⇒IND2	44.9289	0.0000	Yes∗	16.8048	0.0322	Yes∗
IND2⇒THA	19.6152	0.0032	Yes∗	21.1478	0.0068	Yes∗
THA⇒PHI	29.3527	0.0003	Yes∗	2.4568	0.9637	No
PHI⇒THA	19.0619	0.0145	Yes∗	0.2946	1.0000	No
THA⇒TAI	31.2692	0.0000	Yes∗	23.1452	0.0016	Yes∗
TAI⇒THA	6.6178	0.3576	No	68.4112	0.0000	Yes∗

∗ and ∗∗ represent statistical significance at the 5% and 10% levels, respectively. Because the third level is associated with a time interval of $[8, 16)$ days, the relationship between Thailand and another market is related to the time interval of $[8, 16)$ days as well.

Table C.17: Granger Causality Test at the fourth level

H_0: A⇏B	Tranquil Period			Crisis Period		
H_1: A⇒B	χ^2 value	p value	Causality	χ^2 value	p value	Causality
THA⇒ARG	1.3864	0.8466	No	30.3671	0.0002	Yes∗
ARG⇒THA	2.7161	0.6064	No	23.4649	0.0028	Yes∗
THA⇒BRA	34.9917	0.0000	Yes∗	18.0963	0.0205	Yes∗
BRA⇒THA	87.8369	0.0000	Yes∗	14.2839	0.0747	Yes∗∗
THA⇒CHI1	4.8334	0.3048	No	9.3899	0.0521	Yes∗∗
CHI1⇒THA	1.8898	0.7560	No	15.6997	0.0034	Yes∗
THA⇒MEX	40.1227	0.0000	Yes∗	5.7308	0.6774	No
MEX⇒THA	44.0177	0.0000	Yes∗	7.1861	0.5167	No
THA⇒PER	4.8458	0.3035	No	54.8706	0.0000	Yes∗
PER⇒THA	7.2968	0.1210	No	41.4151	0.0000	Yes∗
THA⇒AUS1	0.9398	0.9188	No	26.3661	0.0009	Yes∗
AUS1 ⇒THA	1.2672	0.8669	No	29.8714	0.0002	Yes∗
THA⇒AUS2	2.0587	0.7250	No	49.6910	0.0000	Yes∗
AUS2⇒THA	1.0194	0.9068	No	33.3905	0.0001	Yes∗
THA⇒CAN	38.9962	0.0000	Yes∗	14.1906	0.0769	Yes∗∗
CAN ⇒THA	6.1913	0.6258	No	11.8595	0.1576	No
THA⇒FRA	9.1300	0.1040	No	4.2291	0.3759	No
FRA⇒THA	3.4790	0.6266	No	17.4683	0.0016	Yes∗
THA⇒GER	70.0920	0.0000	Yes∗	11.9271	0.0179	Yes∗
GER⇒THA	12.5359	0.1288	No	13.0883	0.0109	Yes∗
THA⇒ITA	1.6442	0.8008	No	27.2774	0.0001	Yes∗
ITA⇒THA	3.7854	0.4358	No	86.5583	0.0000	Yes∗
THA⇒JAP	2.1842	0.8231	No	23.8871	0.0024	Yes∗
JAP⇒THA	17.1469	0.0042	Yes∗	116.8539	0.0000	Yes∗
THA⇒NET	47.7525	0.0000	Yes∗	5.9538	0.2026	No
NET⇒THA	41.3133	0.0000	Yes∗	11.7064	0.0197	Yes∗
THA⇒SPA	25.2864	0.0001	Yes∗	4.5939	0.8000	No
SPA⇒THA	28.9341	0.0000	Yes∗	10.7773	0.2146	No
THA⇒SWE	0.7120	0.9498	No	13.7642	0.0081	Yes∗
SWE⇒THA	3.9162	0.4175	No	21.7905	0.0002	Yes∗
THA⇒UK	11.4507	0.0219	Yes∗	82.9299	0.0000	Yes∗
UK⇒THA	7.5422	0.1099	No	58.2164	0.0000	Yes∗
THA⇒USA	27.2543	0.0006	Yes∗	18.1943	0.0011	Yes∗
USA⇒THA	8.0119	0.4323	No	23.6894	0.0001	Yes∗
THA⇒CHI2	22.9398	0.0034	Yes∗	18.3516	0.0187	Yes∗
CHI2⇒THA	1.7495	0.9878	No	18.1357	0.0202	Yes∗
THA⇒KOR	2.0377	0.7288	No	27.2689	0.0001	Yes∗
KOR⇒THA	1.5311	0.8211	No	41.0970	0.0000	Yes∗
THA⇒SIN	25.7934	0.0011	Yes∗	50.5612	0.0000	Yes∗
SIN⇒THA	8.2053	0.4137	No	28.5986	0.0001	Yes∗
THA⇒MAL	10.5505	0.2285	No	20.1368	0.0026	Yes∗
MAL⇒THA	8.1604	0.4180	No	34.1449	0.0000	Yes∗
THA⇒HK	27.1039	0.0007	Yes∗	18.5993	0.0049	Yes∗
HK⇒THA	23.3310	0.0030	Yes∗	38.7831	0.0000	Yes∗
THA⇒IND1	1.1285	0.8897	No	16.3816	0.0118	Yes∗
IND1⇒THA	3.6215	0.4596	No	68.7475	0.0000	Yes∗
THA⇒IND2	39.7109	0.0000	Yes∗	43.1889	0.0000	Yes∗
IND2⇒THA	9.8270	0.2774	No	125.3331	0.0000	Yes∗
THA⇒PHI	17.1607	0.0285	Yes∗	6.1508	0.1882	No
PHI⇒THA	24.6936	0.0018	Yes∗	10.1302	0.0383	Yes∗
THA⇒TAI	16.2091	0.0028	Yes∗	9.5415	0.0489	Yes∗
TAI⇒THA	6.3093	0.1772	No	14.5413	0.0058	Yes∗

∗ and ∗∗ represent statistical significance at the 5% and 10% levels, respectively. Because the fourth level is associated with a time interval of $[16, 32)$ days, the relationship between Thailand and another market is related to the time interval of $[16, 32)$ days as well.

Table C.18: Granger Causality Test at the fifth level

H_0: A$\nRightarrow$B	Tranquil Period			Crisis Period		
H_1: A$\Rightarrow$B	χ^2 value	p value	Causality	χ^2 value	p value	Causality
THA⇒ARG	573.0763	0.0000	Yes∗	26.8082	0.0002	Yes∗
ARG⇒THA	195.2163	0.0000	Yes∗	32.5374	0.0000	Yes∗
THA⇒BRA	42.4251	0.0000	Yes∗	135.8198	0.0000	Yes∗
BRA⇒THA	36.4852	0.0000	Yes∗	40.3831	0.0000	Yes∗
THA⇒CHI1	169.1729	0.0000	Yes∗	32.4224	0.0001	Yes∗
CHI1⇒THA	671.0209	0.0000	Yes∗	17.6178	0.0243	Yes∗
THA⇒MEX	130.2006	0.0000	Yes∗	106.7552	0.0000	Yes∗
MEX⇒THA	693.2796	0.0000	Yes∗	59.6874	0.0000	Yes∗
THA⇒PER	818.5763	0.0000	Yes∗	38.2643	0.0000	Yes∗
PER⇒THA	1826.1110	0.0000	Yes∗	42.3725	0.0000	Yes∗
THA⇒AUS1	48.7968	0.0000	Yes∗	20.2614	0.0011	Yes∗
AUS1 ⇒THA	196.7827	0.0000	Yes∗	20.7238	0.0009	Yes∗
THA⇒AUS2	56.3279	0.0000	Yes∗	2060.7870	0.0000	Yes∗
AUS2⇒THA	120.1372	0.0000	Yes∗	529.4557	0.0000	Yes∗
THA⇒CAN	37.8283	0.0000	Yes∗	203.7735	0.0000	Yes∗
CAN ⇒THA	61.6278	0.0000	Yes∗	76.5512	0.0000	Yes∗
THA⇒FRA	77.4852	0.0000	Yes∗	104.5377	0.0000	Yes∗
FRA⇒THA	28.7319	0.0004	Yes∗	37.2463	0.0000	Yes∗
THA⇒GER	15.2896	0.0538	Yes∗∗	10.0237	0.0746	Yes∗∗
GER⇒THA	53.4492	0.0000	Yes∗	35.8421	0.0000	Yes∗
THA⇒ITA	820.4581	0.0000	Yes∗	102.4740	0.0000	Yes∗
ITA⇒THA	1089.5910	0.0000	Yes∗	65.3976	0.0000	Yes∗
THA⇒JAP	30.6549	0.0002	Yes∗	95.4223	0.0000	Yes∗
JAP⇒THA	22.7810	0.0037	Yes∗	39.3317	0.0000	Yes∗
THA⇒NET	68.1279	0.0000	Yes∗	10.1164	0.0720	Yes∗∗
NET⇒THA	234.8626	0.0000	Yes∗	33.4311	0.0000	Yes∗
THA⇒SPA	22.4703	0.0041	Yes∗	66.5360	0.0000	Yes∗
SPA⇒THA	23.6164	0.0027	Yes∗	62.5048	0.0000	Yes∗
THA⇒SWE	9.2733	0.3198	No	6.0730	0.2992	No
SWE⇒THA	183.9887	0.0000	Yes∗	35.3768	0.0000	Yes∗
THA⇒UK	38.3176	0.0000	Yes∗	15.9114	0.0071	Yes∗
UK⇒THA	75.4174	0.0000	Yes∗	30.1577	0.0000	Yes∗
THA⇒USA	21.8480	0.0052	Yes∗	13.3182	0.0206	Yes∗
USA⇒THA	33.3751	0.0001	Yes∗	31.6011	0.0000	Yes∗
THA⇒CHI2	26.0827	0.0010	Yes∗	10.0892	0.0727	Yes∗∗
CHI2⇒THA	99.5972	0.0000	Yes∗	31.4390	0.0000	Yes∗
THA⇒KOR	13482.9000	0.0000	Yes∗	7.3703	0.3914	No
KOR⇒THA	42396913.0000	0.0000	Yes∗	23.5234	0.0014	Yes∗
THA⇒SIN	318.6406	0.0000	Yes∗	120.4053	0.0000	Yes∗
SIN⇒THA	193.8153	0.0000	Yes∗	25.3869	0.0013	Yes∗
THA⇒MAL	386.8816	0.0000	Yes∗	8.8718	0.3532	No
MAL⇒THA	1023.6460	0.0000	Yes∗	1.1479	0.9971	No
THA⇒HK	140.1609	0.0000	Yes∗	9.8438	0.0798	Yes∗∗
HK⇒THA	721.8621	0.0000	Yes∗	37.8274	0.0000	Yes∗
THA⇒IND1	47.6346	0.0000	Yes∗	256.0565	0.0000	Yes∗
IND1⇒THA	232.6314	0.0000	Yes∗	173.9112	0.0000	Yes∗
THA⇒IND2	79.0141	0.0000	Yes∗	464.7581	0.0000	Yes∗
IND2⇒THA	17.3184	0.0270	Yes∗	469.0665	0.0000	Yes∗
THA⇒PHI	636.6009	0.0000	Yes∗	675.7232	0.0000	Yes∗
PHI⇒THA	340.5465	0.0000	Yes∗	274.2892	0.0000	Yes∗
THA⇒TAI	193.1572	0.0000	Yes∗	9944.6950	0.0000	Yes∗
TAI⇒THA	317.1534	0.0000	Yes∗	58.4895	0.0000	Yes∗

∗ and ∗∗ represent statistical significance at the 5% and 10% levels, respectively. Because the fifth level is associated with a time interval of $[32, 64)$ days, the relationship between Thailand and another market is related to the time interval of $[32, 64)$ days as well.

Table C.19: The Baht currency crisis triggers a series of negative events in the global markets.

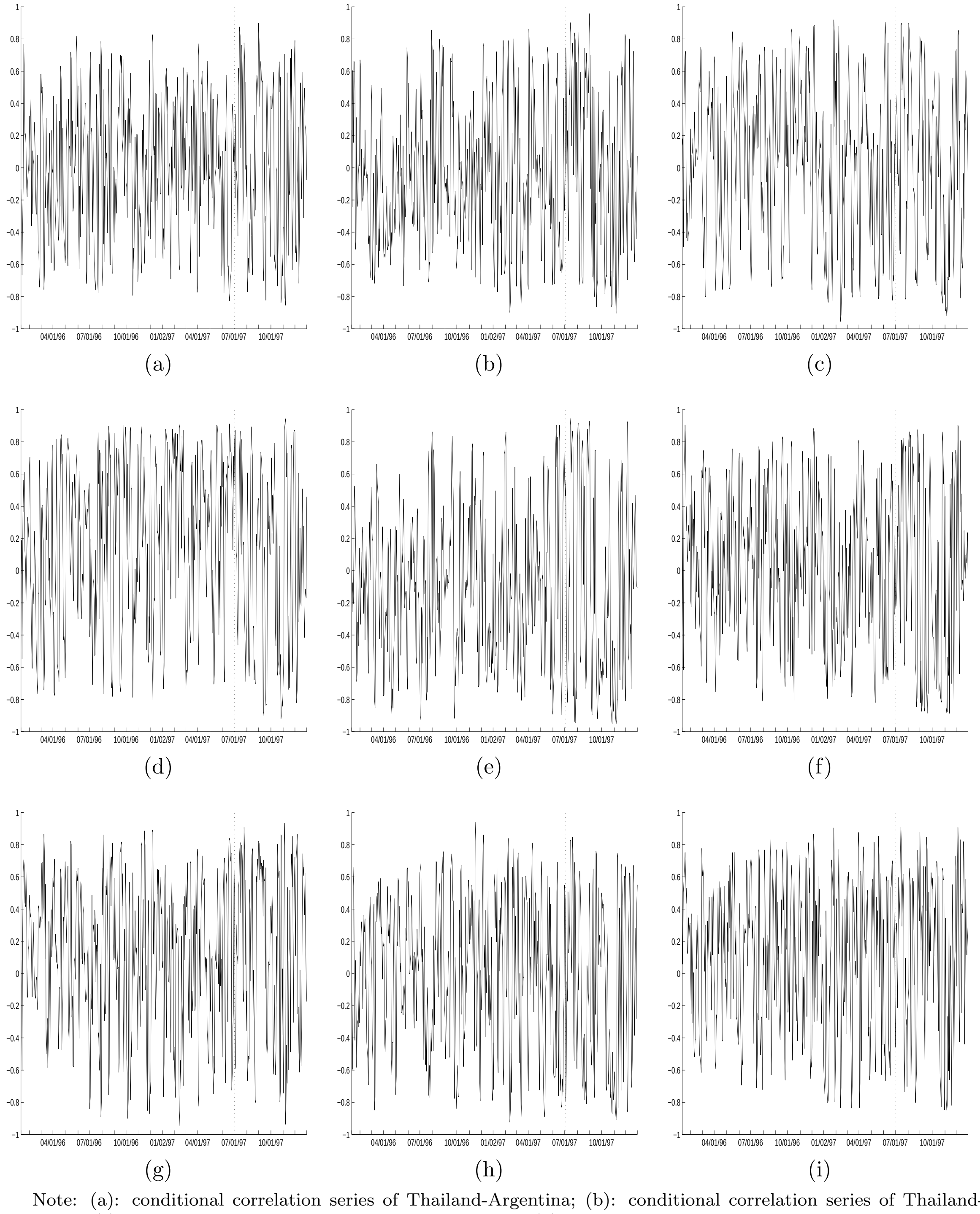
Date	Country	Description of what happened
July 2, 1997	Thailand	After four months of defending the weakening Baht, the Bank of Thailand announces the free float of Baht. Baht loses 10% of its pre-float value.
July 8	Malaysia	Malaysia's central bank intervenes to defend its currency, the ringgit.
July 11	Philippine	The Philippine peso is devalued.
	Indonesia	Indonesia widens its trading band for the rupiah in a move to discourage speculators.
July 20	Philippines	IMF grants US $1000 million as emergency grant after Peso falls outside widened a band to defend the basket peg.
July (undated)	Thailand	IMF warns Thailand to cut its spending, requests that it take a loan from the IMF.
July 24	Malaysia	Malaysian Ringgit comes under speculative attack. Asian currencies fall dramatically.
	Singapore	The Singapore dollar starts a gradual decline.
August 5	Thailand	Thailand agrees to adopt tough economic measures proposed by the IMF in return for a US $17 billion loan from the international lender and Asian nations. The Thai government closes 42 ailing finance companies and imposes tax hikes as part of the IMF's insistence on austerity.
August 11	Thailand	IMF led by Japan's pressure pledges US $16 billion to Thailand as rescue package.
August 13-14	Indonesia	The Indonesian rupiah comes under severe pressure. Indonesia abolishes its system of managing its exchange rate through the use of a band.
August 28	Asia	Asian stock markets plunge in unison: 9.3% in Manila; 4.5% in Jakarta, etc.
September 4	Philippines	Philippine Peso falls to the lowest level before central bank intervenes to maintain basket peg.
	Malaysia	Malaysian ringgit continues to fall. Malaysia spends US $20 billion to prop up the share markets.
October 8	Indonesia	Rupiah hits a low. Indonesia considers asking IMF for an emergency bailout after the rupiah falls more than 30% in two months, despite interventions by the country's central bank to prop up the currency.
October 23	Korea	The South Korean won begins to weaken.
October 27	U.S.A	New York share market loses 7.2% in value.
October 28	Korea	The value of the Korean won drops as investors sell Korean stocks.
October 23-28	Hong Kong	Hong Kong share market declines by nearly 25% in value.
October 31	Indonesia	The IMF agrees to a loan package for Indonesia that eventually swells to US $40 billion. In return, the government closes 16 financially insolvent banks and promises other wide-ranging reforms.
	Russia	The IMF announces that it will delay a US $700 million quarterly disbursement to Russia due to the country's lax tax collection.
November 3	Japan	Japan's Sanyo Securities files for bankruptcy with liabilities of more than US $3 billion.
	Korea	Korea Won loses 7%, largest one-day loss.
		Korea begins talk with IMF for tens of billions in emergency aid.
November 5	Indonesia	The IMF announces a stabilisation package of approximately US $40 billion for Indonesia. The United States pledges a standby credit of US $3 billion.

Table 19 (continued)

Date	Country	Description of what happened
November 8	Japan	Japan's third financial house to apply for closure: the seventh-largest Yamaichi Securities.
November 17	Japan	Hokkaido Takushoku Bank Ltd., one of Japan's top 10 banks, collapses under a pile of bad loans.
	Korea	The Bank of Korea abandons its effort to prop up the value of the won, allowing it to fall below 1000 against the dollar, a record low.
November 20	Korea	Korean Stock Market plunges with a loss of 7.2%.
November 24	Japan	Tokyo City Bank, regional bank, closes.
November 25	Korea	Korea agrees to IMF conditions for restructuring US $55 billion.
December 3	Malaysia	Malaysia imposes tough reforms to reduce its balance of payments deficit.
	Korea	Korea and IMF agree on US $57 billion support package, the largest in history.
December 8	Thailand	The Thai government announces that it will close 56 insolvent finance companies as part of the IMF's economic restructuring plan. 30,000 white-collar workers lose their jobs.
December 12	Russia	The IMF restarts its loan disbursement to Russia. The pact releases US $700 million delayed in October. In the accord, the IMF urges Russia to boost revenues and cut spending.
December 22	Korea	Korean Won plunges further.
December 23	Korea	In an unprecedented move, the World Bank releases an emergency loan of US $3 billion, part of a US $10 billion support package, to South Korea to help salvage its economy.
December 25	Korea	IMF and lender nations move to finance US $10 billion loan to Korea.

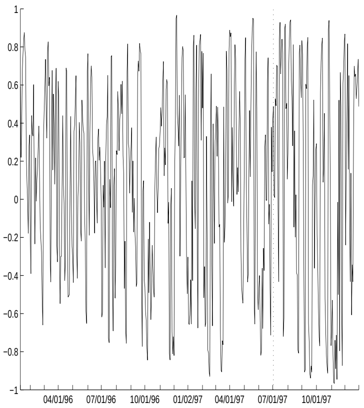
Source: Internet; congressional research service report: the 1997-98 Asian financial crisis; Khalid and Kawai (2003), Was financial market contagion the source of economic crisis in Asia? Evidence using a multivariate VAR model, table 1.

Figure C.1: Pair-wise conditional correlation series between Thailand and other 26 markets at the first level



Note: (a): conditional correlation series of Thailand-Argentina; (b): conditional correlation series of Thailand-Brazil; (c): conditional correlation series of Thailand-Chile; (d): conditional correlation series of Thailand-Mexico; (e): conditional correlation series of Thailand-Peru; (f): conditional correlation series of Thailand-Austria; (g): conditional correlation series of Thailand-Australia; (h): conditional correlation series of Thailand-Canada; (i): conditional correlation series of Thailand-France; The dotted line, which is associated with the day 07/02/1997, divides the entire sample time into the tranquil period and crisis period.

Figure C.1 (continued)



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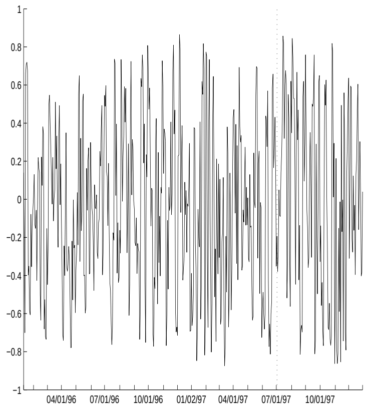

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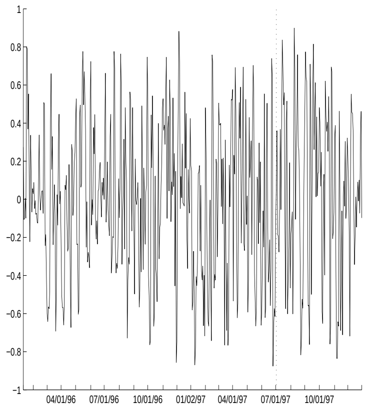

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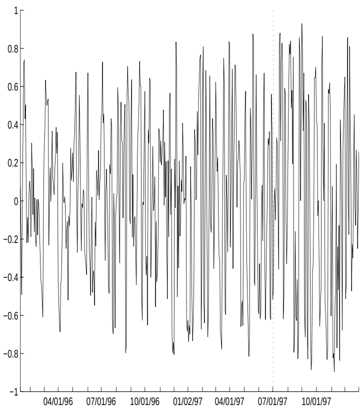

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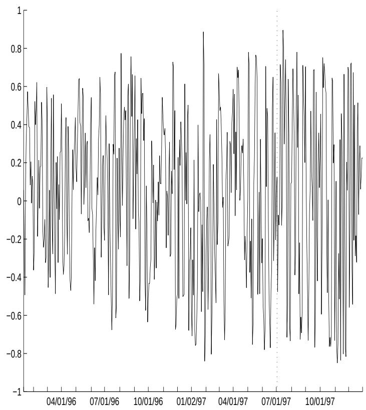

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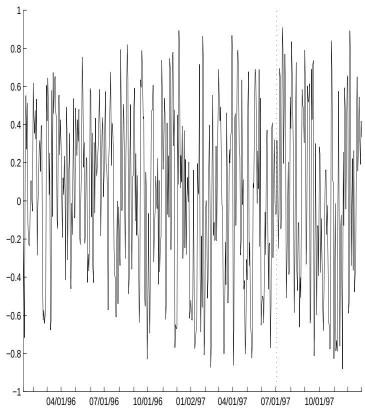

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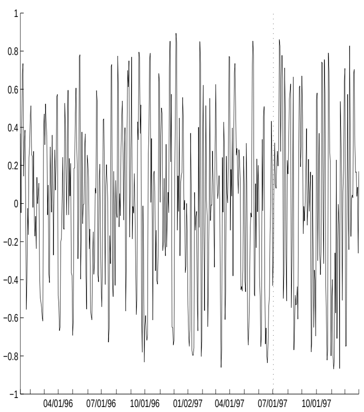
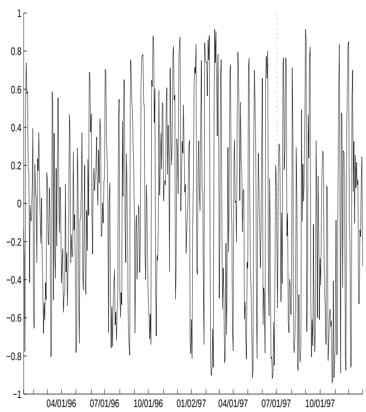
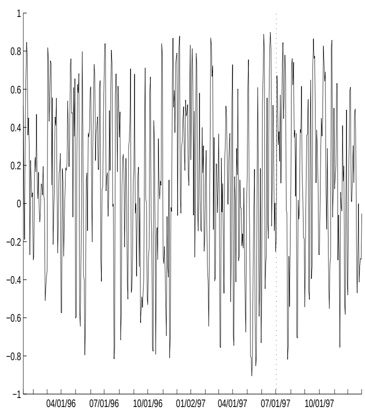


(g)



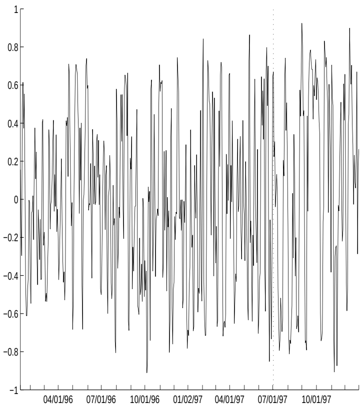
(h)



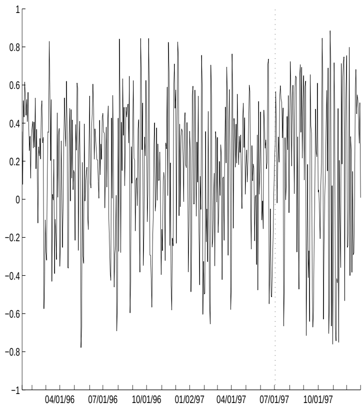
(i)

Note: (a): conditional correlation series of Thailand-Germany; (b): conditional correlation series of Thailand-Italy; (c): conditional correlation series of Thailand-Japan; (d): conditional correlation series of Thailand-Netherlands; (e): conditional correlation series of Thailand-Spain; (f): conditional correlation series of Thailand-Sweden; (g): conditional correlation series of Thailand-UK; (h): conditional correlation series of Thailand-USA; (i): conditional correlation series of Thailand-China; The dotted line, which is associated with the day 07/02/1997, divides the entire sample time into the tranquil period and crisis period.

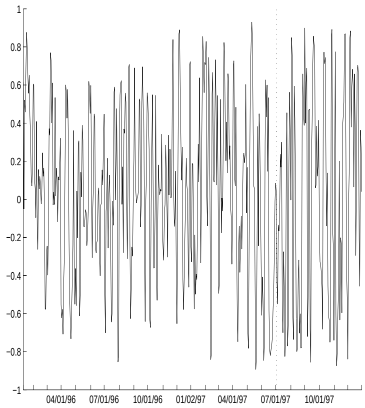
Figure C.1 (continued)



(a)



(b)



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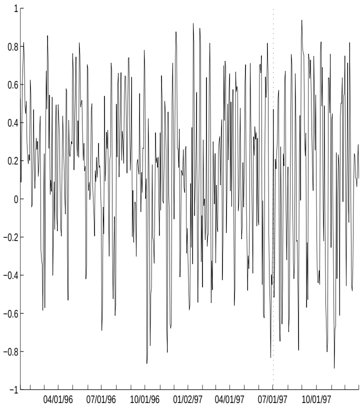

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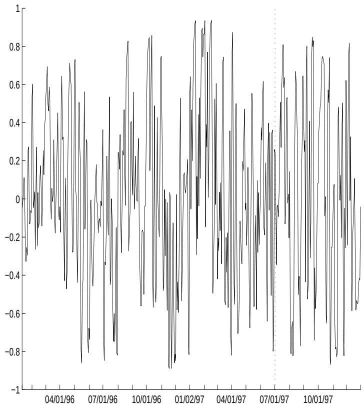

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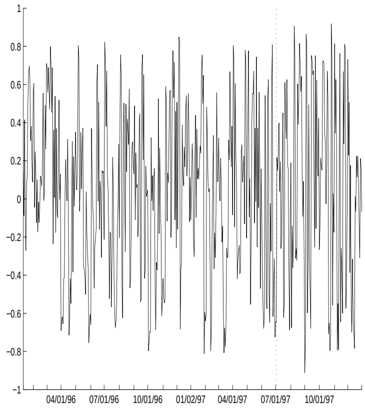

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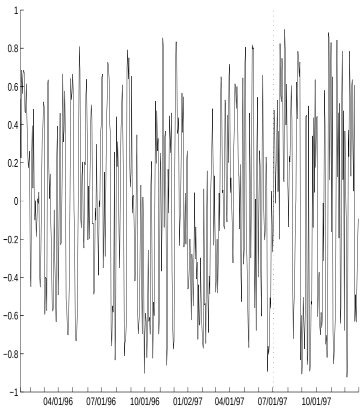

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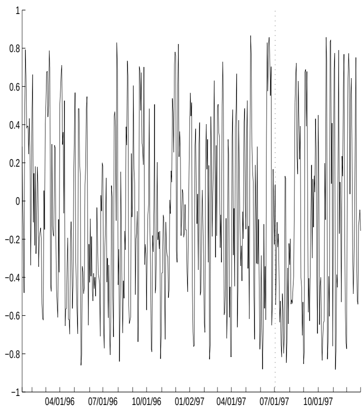
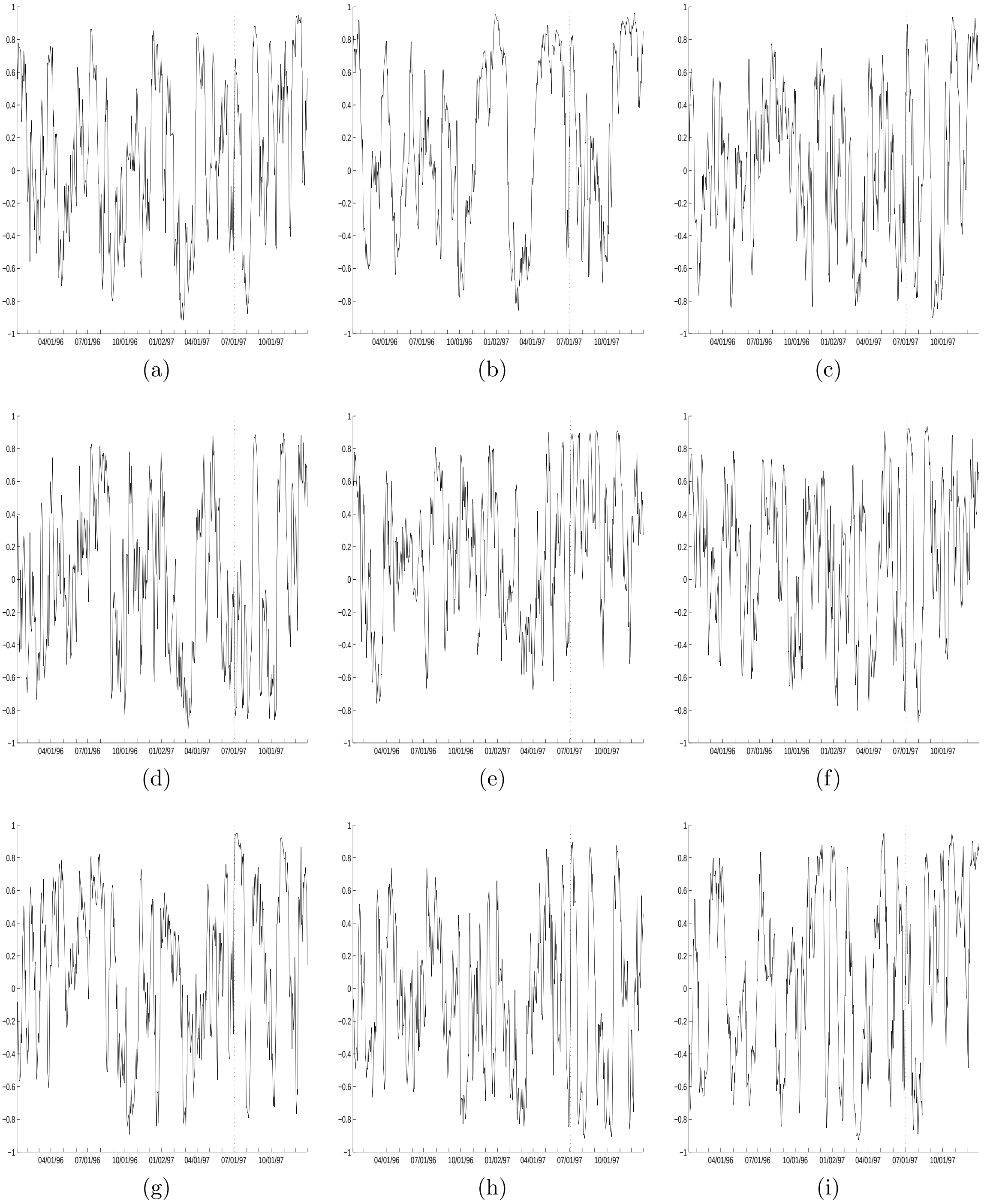


(h)

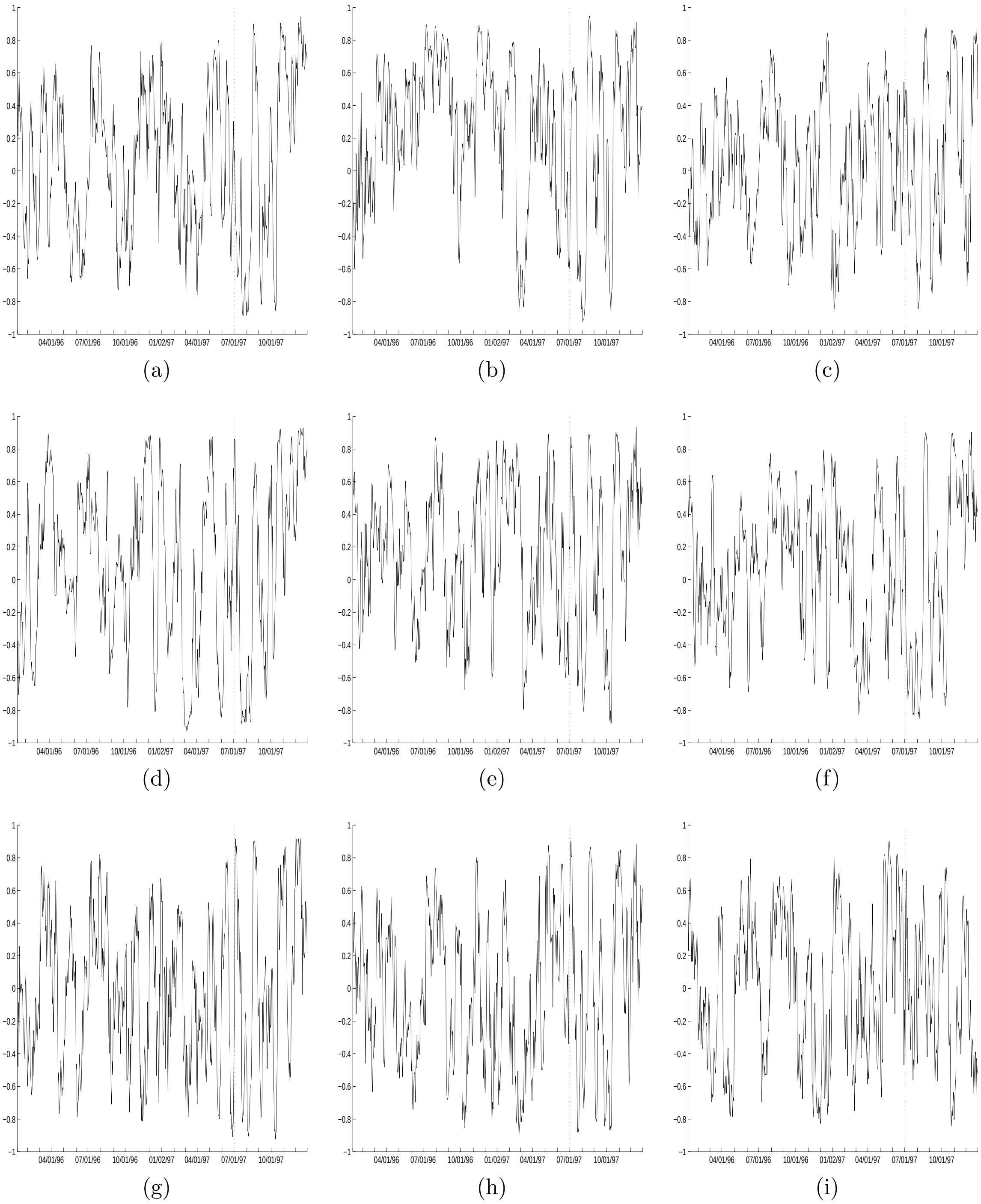
Note: (a): conditional correlation series of Thailand-Korea; (b): conditional correlation series of Thailand-Singapore; (c): conditional correlation series of Thailand-Malaysia; (d): conditional correlation series of Thailand-Hong Kong; (e): conditional correlation series of Thailand-India; (f): conditional correlation series of Thailand-Indonesia; (g): conditional correlation series of Thailand-Philippines; (h): conditional correlation series of Thailand-Taiwan; The dotted line, which is associated with the day 07/02/1997, divides the entire sample time into the tranquil period and crisis period.

Figure C.2: Pair-wise conditional correlation series between Thailand and other 26 markets at the second level



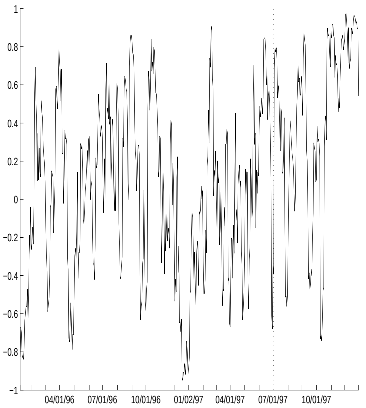
Note: (a): conditional correlation series of Thailand-Argentina; (b): conditional correlation series of Thailand-Brazil; (c): conditional correlation series of Thailand-Chile; (d): conditional correlation series of Thailand-Mexico; (e): conditional correlation series of Thailand-Peru; (f): conditional correlation series of Thailand-Austria; (g): conditional correlation series of Thailand-Australia; (h): conditional correlation series of Thailand-Canada; (i): conditional correlation series of Thailand-France; The dotted line, which is associated with the day 07/02/1997, divides the entire sample time into the tranquil period and crisis period.

Figure C.2 (continued)

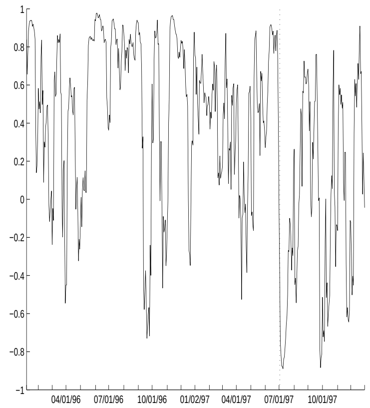


Note: (a): conditional correlation series of Thailand-Germany; (b): conditional correlation series of Thailand-Italy; (c): conditional correlation series of Thailand-Japan; (d): conditional correlation series of Thailand-Netherlands; (e): conditional correlation series of Thailand-Spain; (f): conditional correlation series of Thailand-Sweden; (g): conditional correlation series of Thailand-UK; (h): conditional correlation series of Thailand-USA; (i): conditional correlation series of Thailand-China; The dotted line, which is associated with the day 07/02/1997, divides the entire sample time into the tranquil period and crisis period.

Figure C.2 (continued)



(a)



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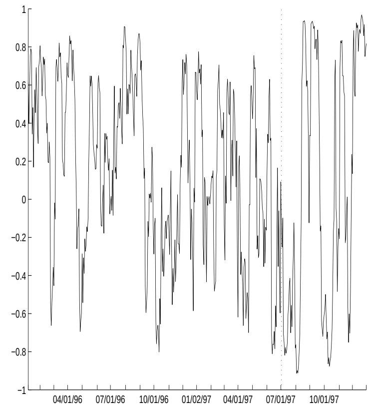

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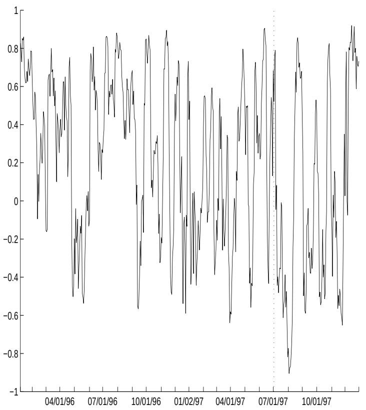

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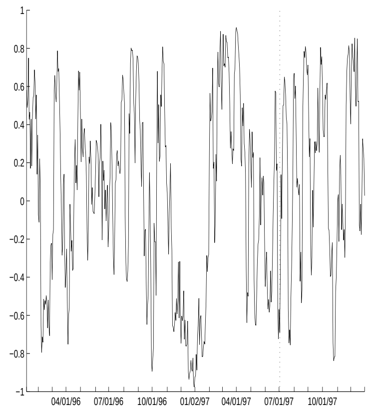

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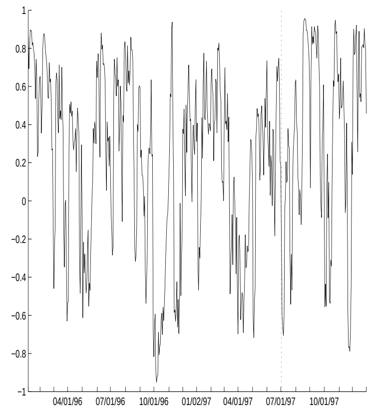

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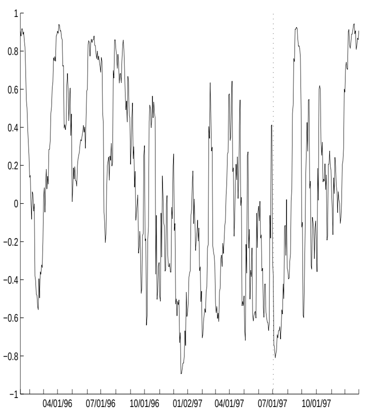

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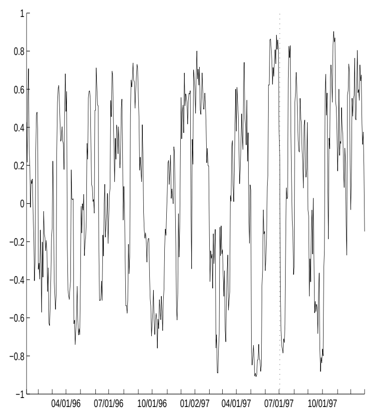
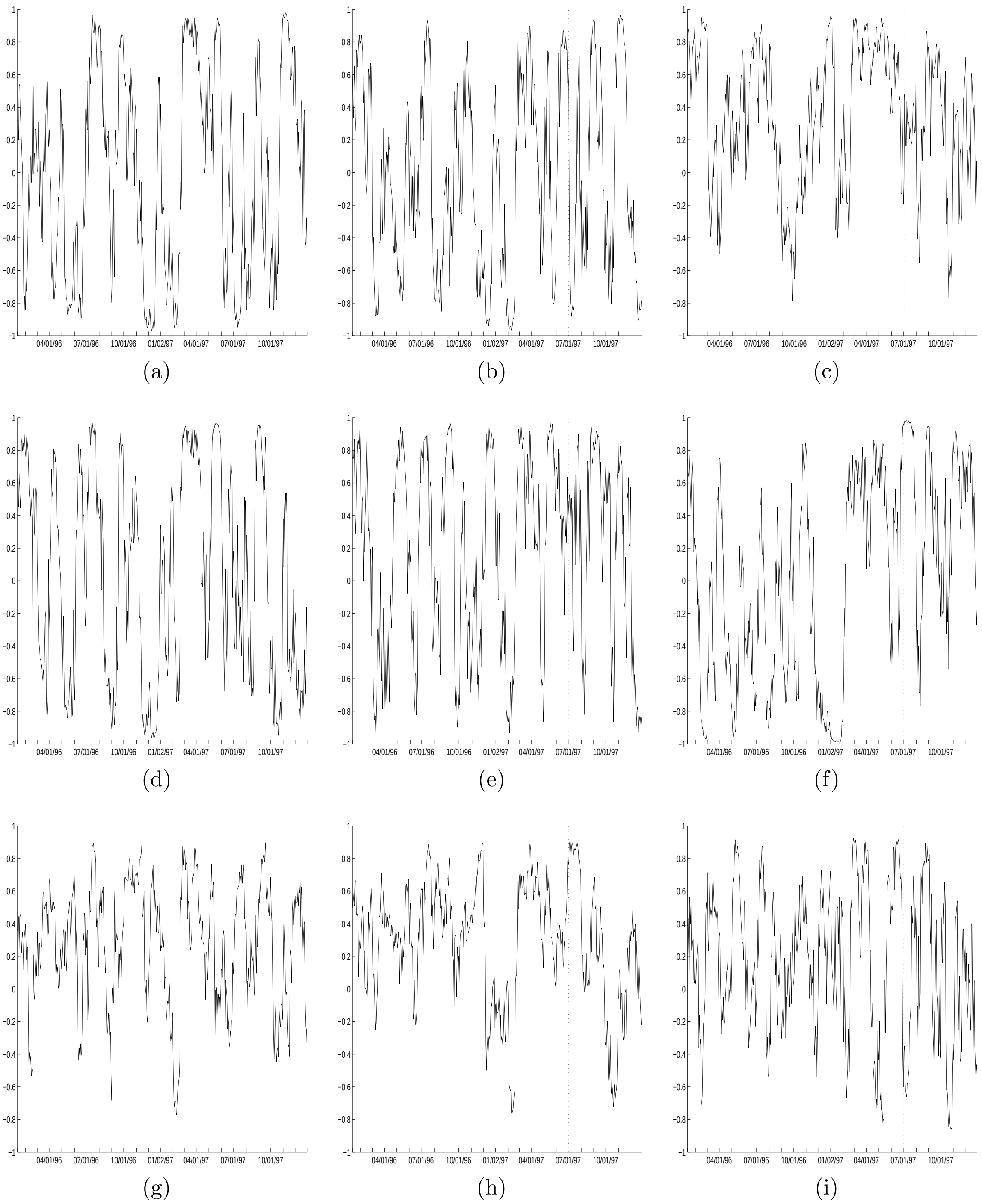


(h)

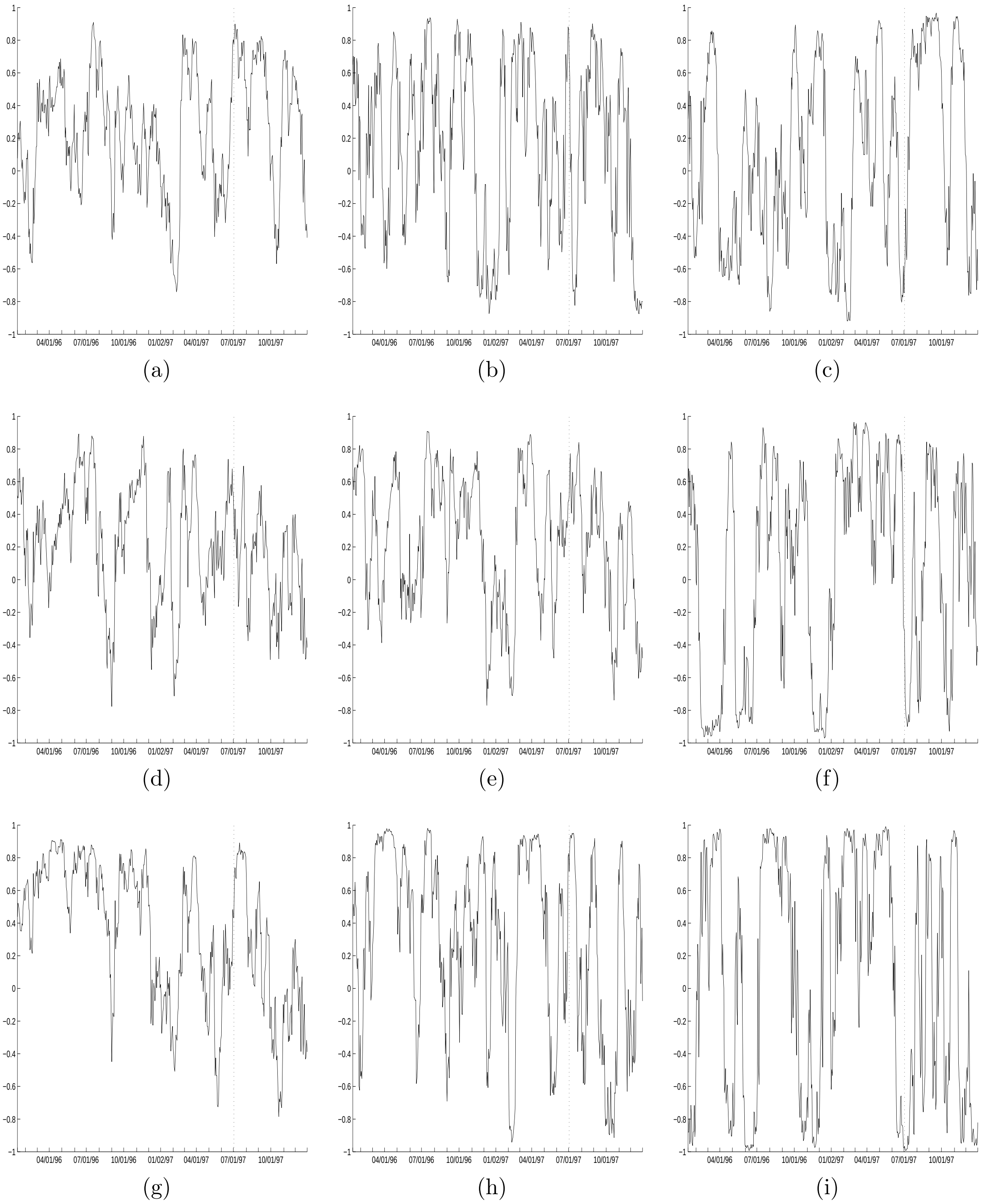
Note: (a): conditional correlation series of Thailand-Korea; (b): conditional correlation series of Thailand-Singapore; (c): conditional correlation series of Thailand-Malaysia; (d): conditional correlation series of Thailand-Hong Kong; (e): conditional correlation series of Thailand-India; (f): conditional correlation series of Thailand-Indonesia; (g): conditional correlation series of Thailand-Philippines; (h): conditional correlation series of Thailand-Taiwan; The dotted line, which is associated with the day 07/02/1997, divides the entire sample time into the tranquil period and crisis period.

Figure C.3: Pair-wise conditional correlation series between Thailand and other 26 markets at the third level



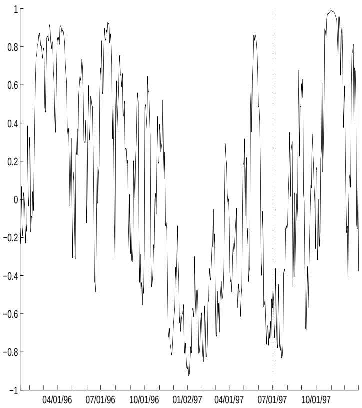
Note: (a): conditional correlation series of Thailand-Argentina; (b): conditional correlation series of Thailand-Brazil; (c): conditional correlation series of Thailand-Chile; (d): conditional correlation series of Thailand-Mexico; (e): conditional correlation series of Thailand-Peru; (f): conditional correlation series of Thailand-Austria; (g): conditional correlation series of Thailand-Australia; (h): conditional correlation series of Thailand-Canada; (i): conditional correlation series of Thailand-France; The dotted line, which is associated with the day 07/02/1997, divides the entire sample time into the tranquil period and crisis period.

Figure C.3 (continued)

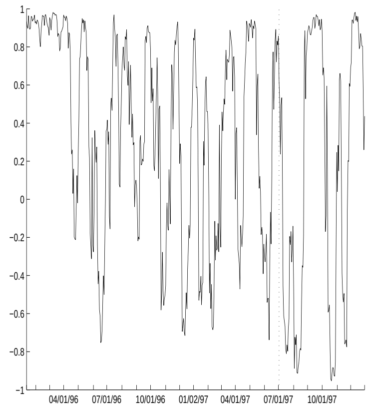


Note: (a): conditional correlation series of Thailand-Germany; (b): conditional correlation series of Thailand-Italy; (c): conditional correlation series of Thailand-Japan; (d): conditional correlation series of Thailand-Netherlands; (e): conditional correlation series of Thailand-Spain; (f): conditional correlation series of Thailand-Sweden; (g): conditional correlation series of Thailand-UK; (h): conditional correlation series of Thailand-USA; (i): conditional correlation series of Thailand-China; The dotted line, which is associated with the day 07/02/1997, divides the entire sample time into the tranquil period and crisis period.

Figure C.3 (continued)



(a)



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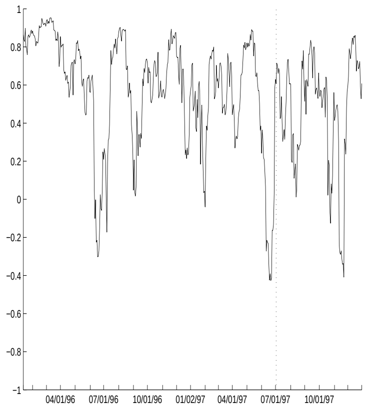

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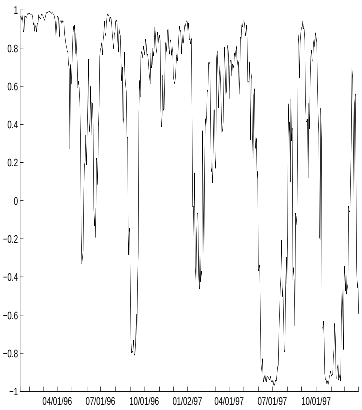

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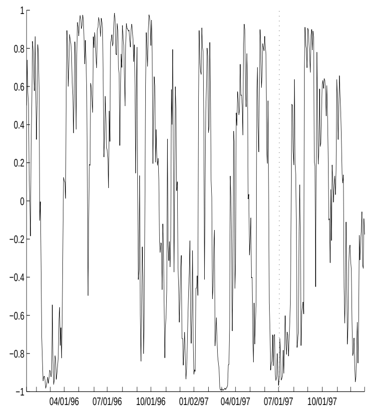

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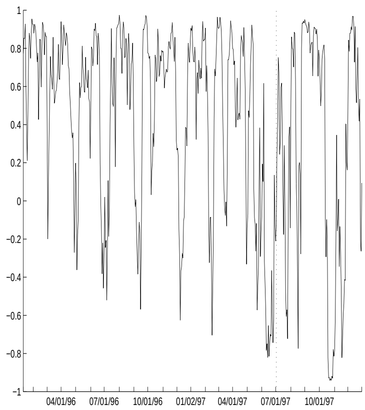

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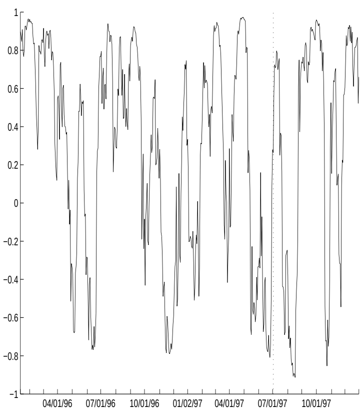

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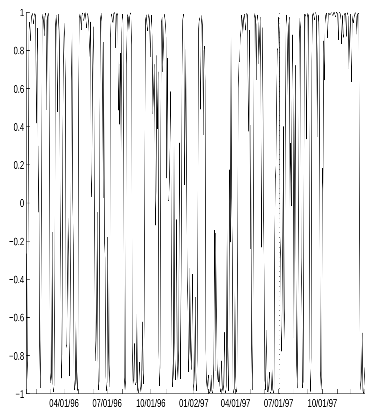
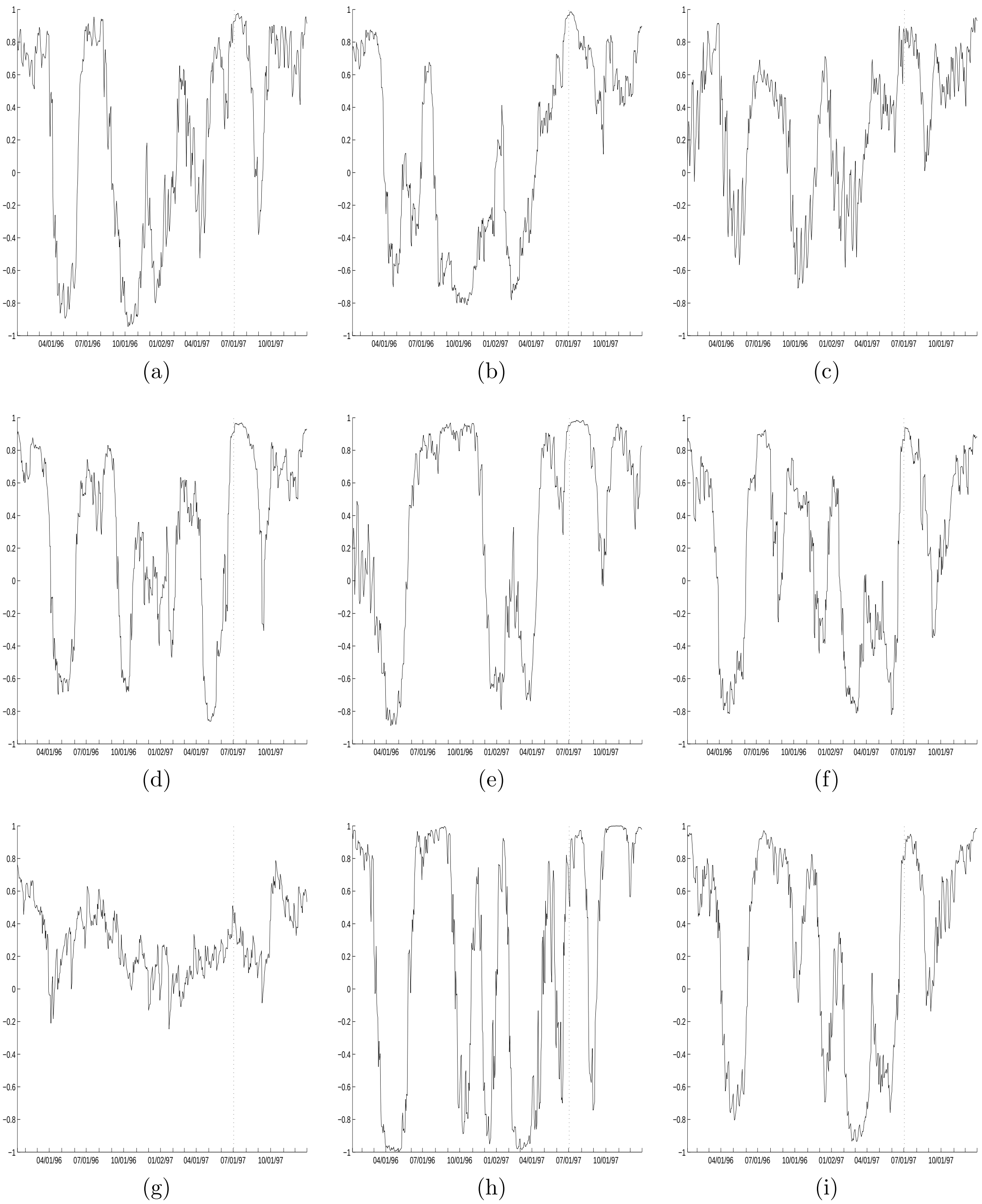


(h)

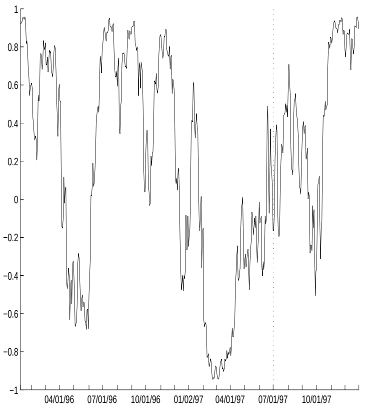
Note: (a): conditional correlation series of Thailand-Korea; (b): conditional correlation series of Thailand-Singapore; (c): conditional correlation series of Thailand-Malaysia; (d): conditional correlation series of Thailand-Hong Kong; (e): conditional correlation series of Thailand-India; (f): conditional correlation series of Thailand-Indonesia; (g): conditional correlation series of Thailand-Philippines; (h): conditional correlation series of Thailand-Taiwan; The dotted line, which is associated with the day 07/02/1997, divides the entire sample time into the tranquil period and crisis period.

Figure C.4: Pair-wise conditional correlation series between Thailand and other 26 markets at the fourth level

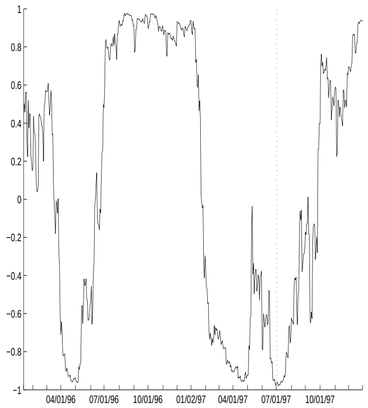


Note: (a): conditional correlation series of Thailand-Argentina; (b): conditional correlation series of Thailand-Brazil; (c): conditional correlation series of Thailand-Chile; (d): conditional correlation series of Thailand-Mexico; (e): conditional correlation series of Thailand-Peru; (f): conditional correlation series of Thailand-Austria; (g): conditional correlation series of Thailand-Australia; (h): conditional correlation series of Thailand-Canada; (i): conditional correlation series of Thailand-France; The dotted line, which is associated with the day 07/02/1997, divides the entire sample time into the tranquil period and crisis period.

Figure C.4 (continued)



(a)



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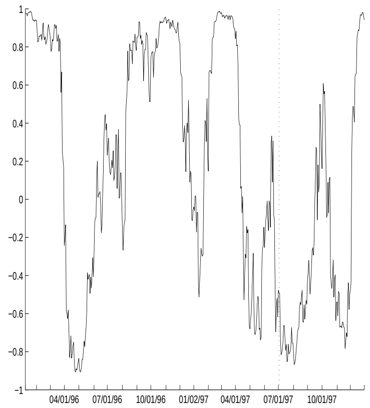

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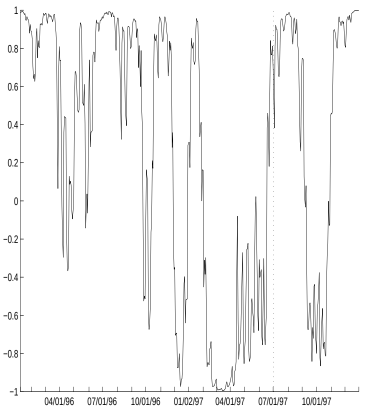

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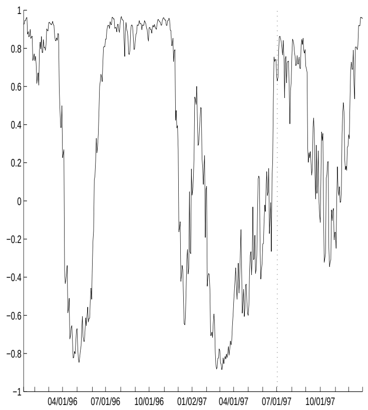

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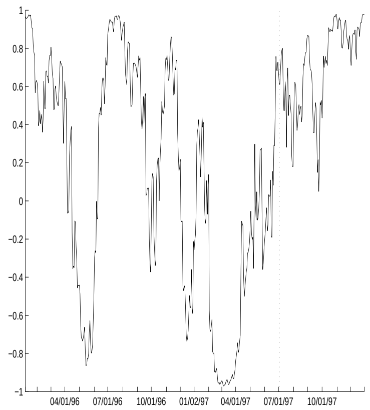

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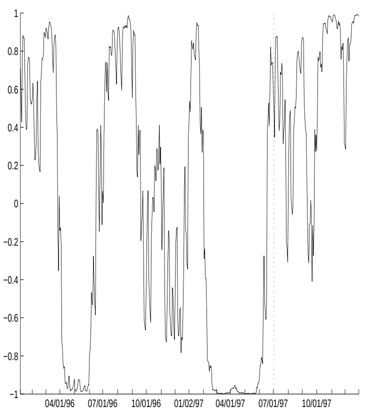
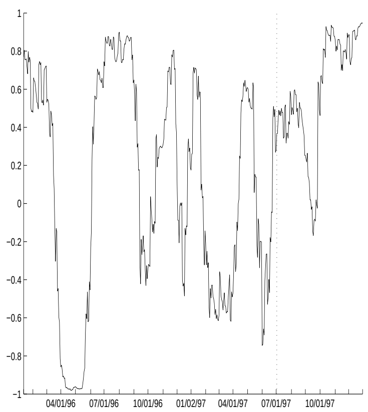


(g)



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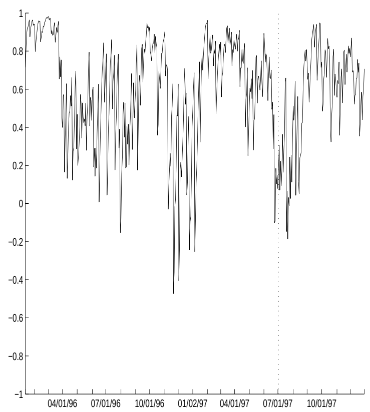
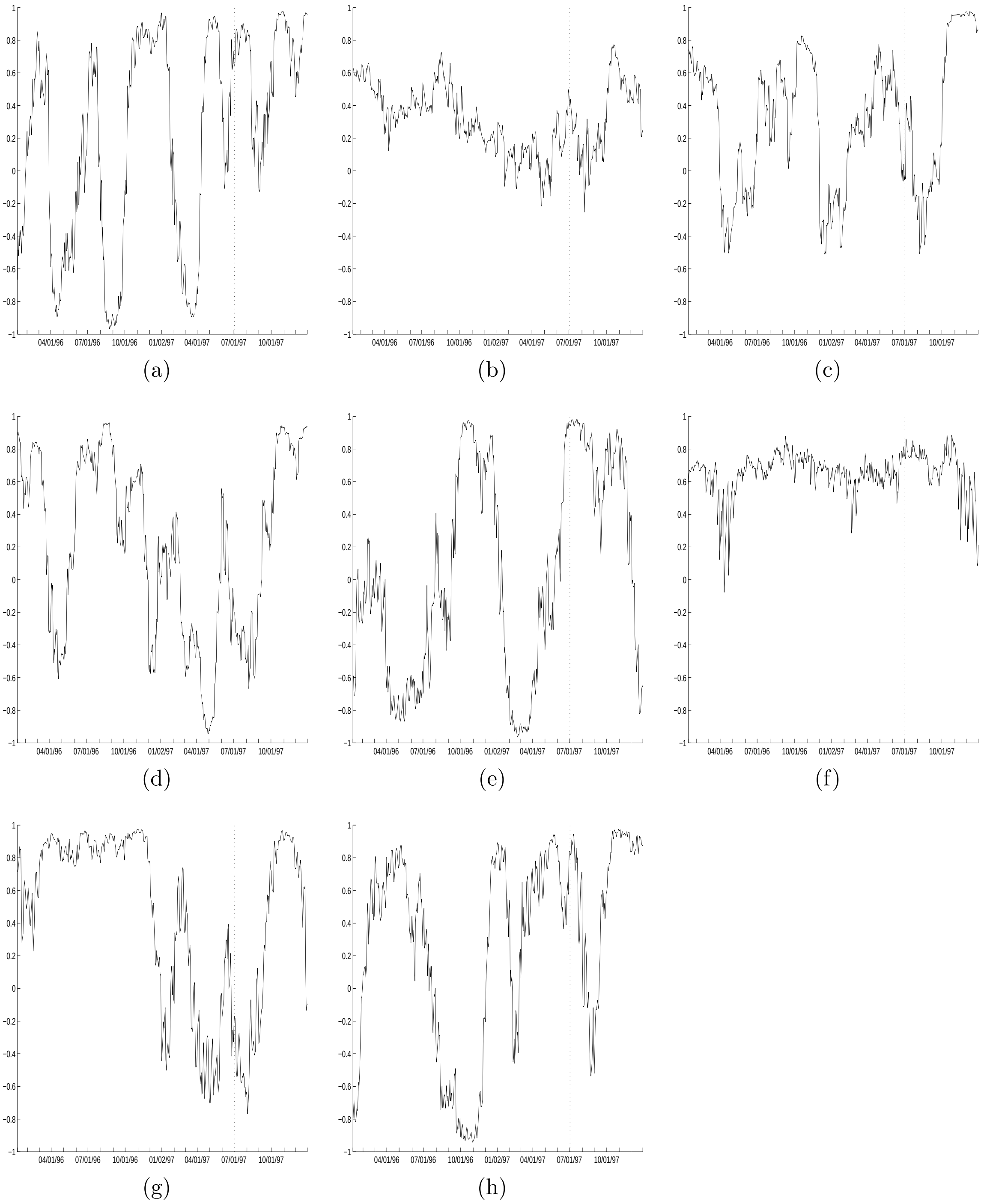


(i)

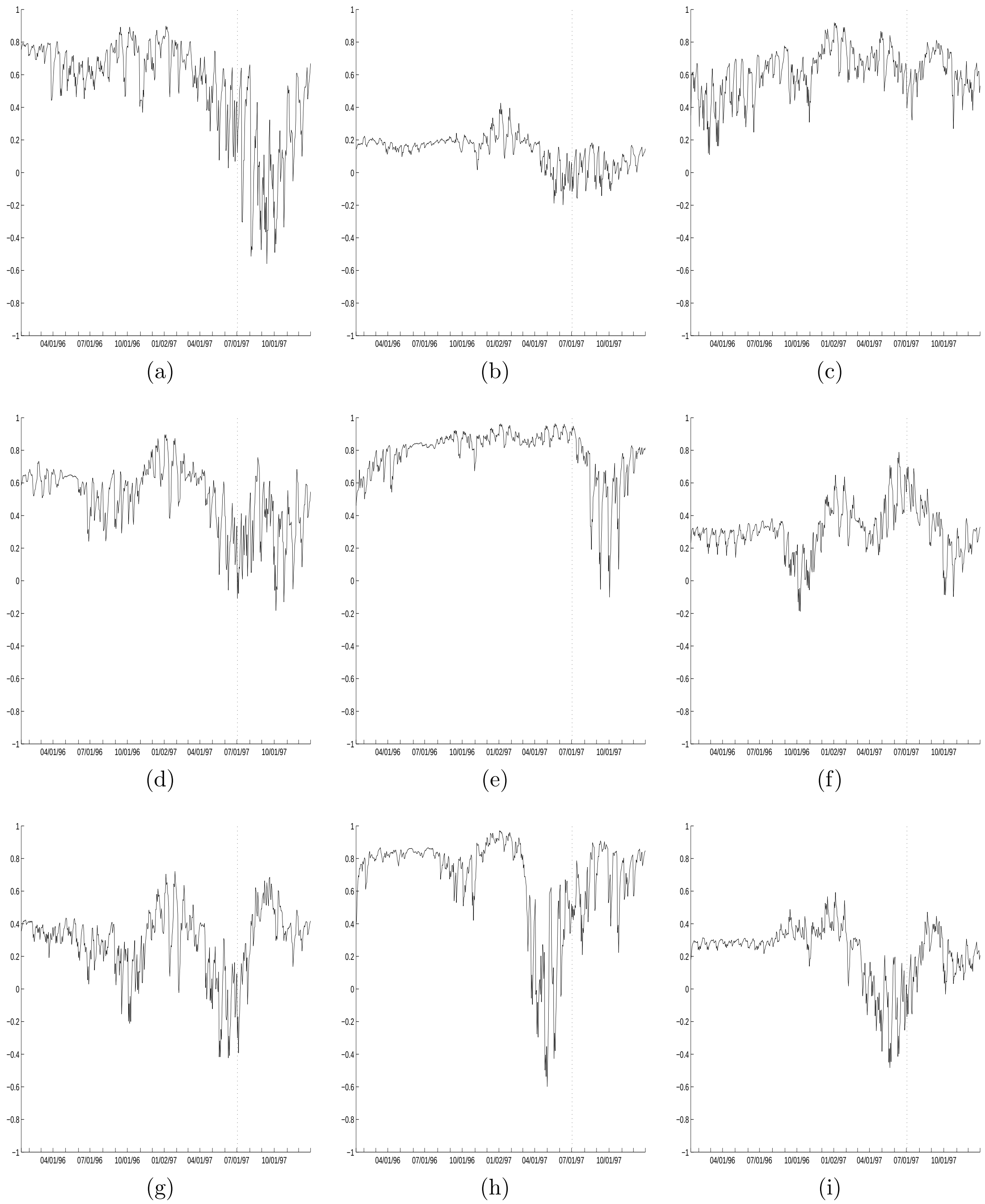
Note: (a): conditional correlation series of Thailand-Germany; (b): conditional correlation series of Thailand-Italy; (c): conditional correlation series of Thailand-Japan; (d): conditional correlation series of Thailand-Netherlands; (e): conditional correlation series of Thailand-Spain; (f): conditional correlation series of Thailand-Sweden; (g): conditional correlation series of Thailand-UK; (h): conditional correlation series of Thailand-USA; (i): conditional correlation series of Thailand-China; The dotted line, which is associated with the day 07/02/1997, divides the entire sample time into the tranquil period and crisis period.

Figure C.4 (continued)



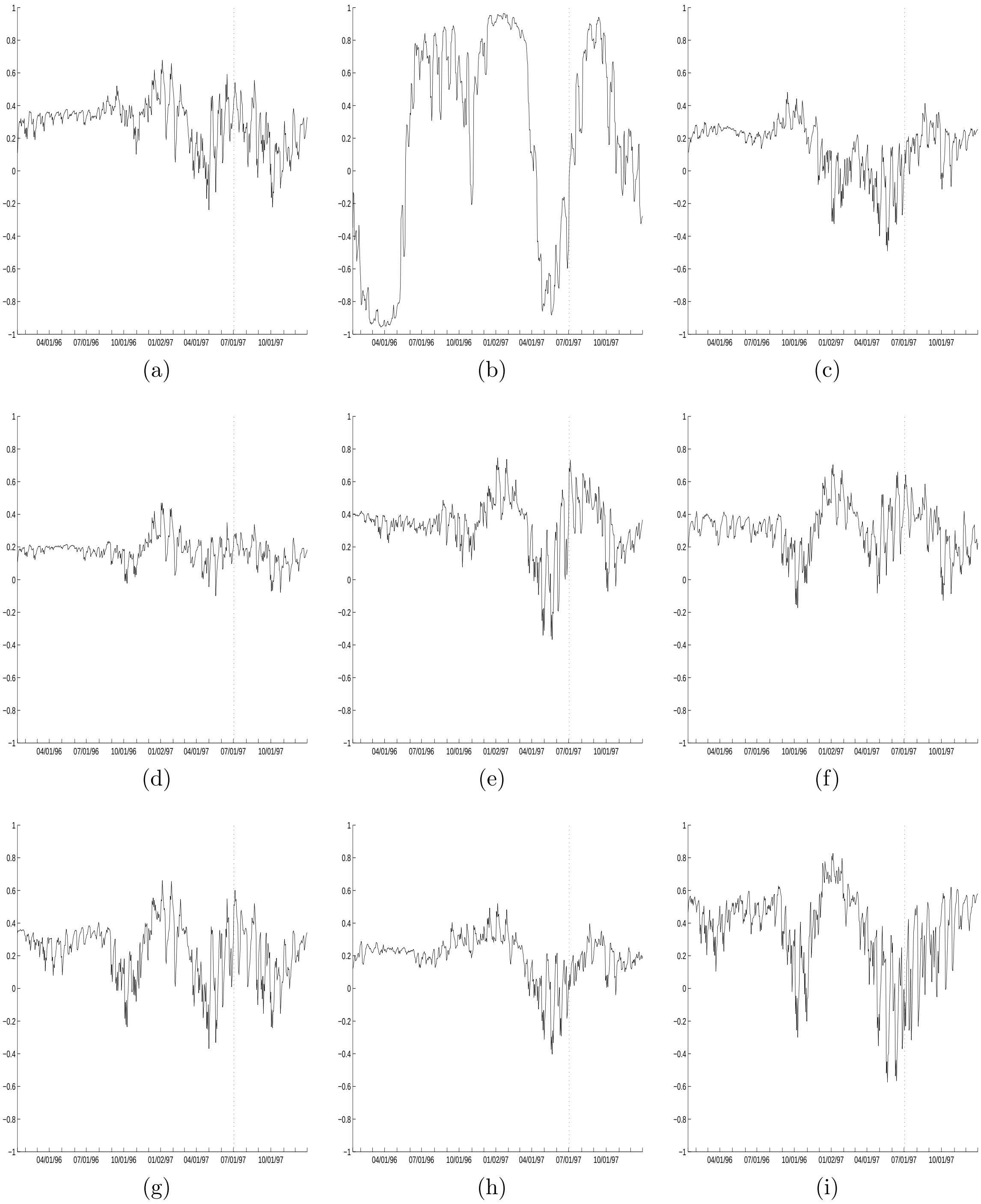
Note: (a): conditional correlation series of Thailand-Korea; (b): conditional correlation series of Thailand-Singapore; (c): conditional correlation series of Thailand-Malaysia; (d): conditional correlation series of Thailand-Hong Kong; (e): conditional correlation series of Thailand-India; (f): conditional correlation series of Thailand-Indonesia; (g): conditional correlation series of Thailand-Philippines; (h): conditional correlation series of Thailand-Taiwan; The dotted line, which is associated with the day 07/02/1997, divides the entire sample time into the tranquil period and crisis period.

Figure C.5: Pair-wise conditional correlation series between Thailand and other 26 markets at the fifth level



Note: (a): conditional correlation series of Thailand-Argentina; (b): conditional correlation series of Thailand-Brazil; (c): conditional correlation series of Thailand-Chile; (d): conditional correlation series of Thailand-Mexico; (e): conditional correlation series of Thailand-Peru; (f): conditional correlation series of Thailand-Austria; (g): conditional correlation series of Thailand-Australia; (h): conditional correlation series of Thailand-Canada; (i): conditional correlation series of Thailand-France; The dotted line, which is associated with the day 07/02/1997, divides the entire sample time into the tranquil period and crisis period.

Figure C.5 (continued)



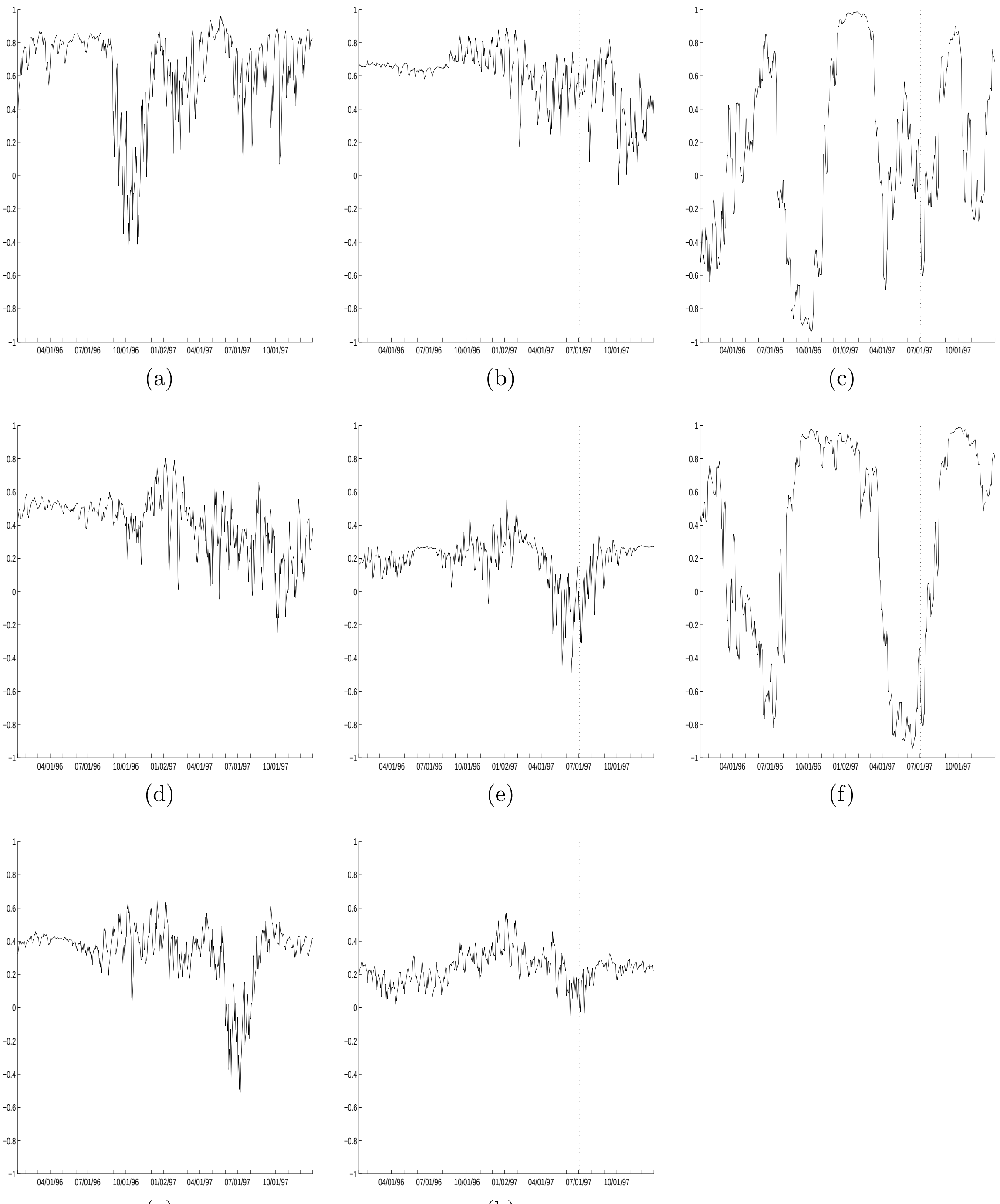
(a) (b) (c)

(d) (e) (f)

(g) (h) (i)

Note: (a): conditional correlation series of Thailand-Germany; (b): conditional correlation series of Thailand-Italy; (c): conditional correlation series of Thailand-Japan; (d): conditional correlation series of Thailand-Netherlands; (e): conditional correlation series of Thailand-Spain; (f): conditional correlation series of Thailand-Sweden; (g): conditional correlation series of Thailand-UK; (h): conditional correlation series of Thailand-USA; (i): conditional correlation series of Thailand-China; The dotted line, which is associated with the day 07/02/1997, divides the entire sample time into the tranquil period and crisis period.

Figure C.5 (continued)



(a) (b) (c) (d) (e) (f) (g) (h)

Note: (a): conditional correlation series of Thailand-Korea; (b): conditional correlation series of Thailand-Singapore; (c): conditional correlation series of Thailand-Malaysia; (d): conditional correlation series of Thailand-Hong Kong; (e): conditional correlation series of Thailand-India; (f): conditional correlation series of Thailand-Indonesia; (g): conditional correlation series of Thailand-Philippines; (h): conditional correlation series of Thailand-Taiwan; The dotted line, which is associated with the day 07/02/1997, divides the entire sample time into the tranquil period and crisis period.

Appendix D

to Chapter 5

Table D.1

News announcements and descriptive statistics of data on standardised announcement surprises

News Announcements[1]	Number of Observations[2]			Number of Zeros[3]			$\hat{\sigma}_j^4$	Means[5]			T-statistics		
8:30 Announcements	W[6]	E[6]	C[6]	W[6]	E[6]	C[6]	W[6]	W[6]	E[6]	C[6]	W[6]	E[6]	C[6]
1. CPI (Consumer Price Index)	144	123	21	47	43	4	0.0014	-0.0689	-0.0634	-0.1013	-0.8269	-0.7906	-0.3043
2. PPI (Producer Price Index)	142	122	20	37	29	8	0.0027	-0.0079	-0.0643	0.3361∗	-0.0940	-0.6875	2.1628
3. Nonfarm Payrolls	142	121	21	2	2	0	5.0445	-0.0606	-0.0832	0.0701	-0.7217	-0.8540	0.8865
4. Civilian Unemployment	142	121	21	45	43	2	0.0014	-0.1222	-0.2150∗∗	0.4130	-1.4556	-2.6338	1.3884
5. Personal Consumption	139	123	16	38	36	2	0.0017	0.0292	0.0283	0.0362	0.3442	0.3271	0.1104
6. Personal Income	140	123	17	36	33	3	0.0023	0.2073∗	0.1874∗	0.3515	2.4530	2.2720	0.9593
7. Trade Balance	144	123	21	7	6	1	0.0843	0.1056	0.1764∗	-0.3092	1.2667	1.9808	-1.4235
9:15 Announcements													
8. Capacity Utilisation	141	121	20	13	13	0	0.0034	-0.0597	-0.0216	-0.2905	-0.7095	-0.2724	-0.8223
9. IP (Industrial Production)	141	121	20	22	21	1	0.0035	-0.0807	-0.0306	-0.3842	-0.9587	-0.4153	-0.9736
10:00 Announcements													
10. Consumer Confidence	141	120	21	0	0	0	0.0555	0.0076	0.0976	-0.5065	0.0900	1.2891	-1.4438
11. Durable Goods Orders	134	113	21	2	1	1	0.0277	-0.0032	-0.0227	0.1015	-0.0374	-0.2526	0.3762
12. Leading Index	141	120	21	51	42	9	0.0015	-0.0327	-0.0659	0.1568	-0.3883	-0.7638	0.5604
13. Manufacturers' New Orders	143	122	21	10	9	1	0.0104	-0.0603	-0.0526	-0.1050	-0.7213	-0.5753	-0.5019
14. New Single-Family Home Sales	142	121	21	2	1	1	1.1187	-0.0485	-0.0552	-0.0101	-0.5779	-0.5600	-0.8609
15. PMI (Purchasing Managers Index)	142	121	21	4	3	1	0.0392	0.0056	0.0253	-0.1083	0.0662	0.3011	-0.3596
14:00 Announcement													
16. Federal Budget	143	122	21	5	5	0	1.6684	0.1261	0.1225	0.1468∗	1.5080	1.2586	2.1046
15:00 Announcement													
17. Consumer Credit	143	122	21	2	2	0	9.8418	0.1022	0.1131	0.0387	1.2219	1.1552	1.2814

1. Sample period: February 1997-January 2009. The release times of news announcements change in some months. Here, we classify the times in terms of when they are usually released.
2. A total of 144 months are in the entire sample period, 123 months are in the expansion period, and 21 months are in the contraction period. For almost all types of news announcements, some observations are missed.
3. A zero value for the announcement surprise indicates that survey participants accurately forecast the announced value of news in the month.
4. $\hat{\sigma}_j$ represents the sample standard deviation of the surprise of announcement j, which is used in Equation (5.16).
5. ∗ and ∗∗ denote statistical significance at the 5% and 1% levels, respectively.
6. The entire sample period is abbreviated as "W", the expansion period as "E", and the contraction period as "C".

Table D.2

Price volatility and trading volume by one-minute intervals in the entire sample period

One-minute price change standard deviations and trading volume means are reported and compared for announcement (at a specific time) and nonannouncement days for the U.S. S&P 500 index futures for the entire sample period. Announcement days are defined as those with announcements released at the specific time, including 8:30 & 9:30, 10:00, 14:00 or 15:00 announcements. Nonannouncement days are those with no 8:30 & 9:30, 10:00, 14:00 and 15:00 announcements. We list the Brown-Forsythe-modified Levene F-statistic comparing variances for announcement and nonannouncement days and the t-statistic comparing means for announcement and nonannouncement days assuming unequal variances. All one-minute intervals between 9:30 and 9:41 and the time period over 16:15 to 9:30 are examined for 8:30 & 9:15 announcements. All one-minute intervals from 9:55 to 10:07 are examined for 10:00 announcements. All one-minute intervals from 13:55 to 14:07 are examined for the 14:00 announcement. All one-minute intervals from 14:55 to 15:07 are examined for the 15:00 announcement. The data period is February 03, 1997, to January 30, 2009.

Panel A: Price Volatility												
8 : 30 & 9 : 15 announcements	16:15-9:30	9:30-9:31	9:31-9:32	9:32-9:33	9:33-9:34	9:34-9:35	9:35-9:36	9:36-9:37	9:37-9:38	9:38-9:39	9:39-9:40	9:40-9:41
Announcement day	0.644	0.075	0.071	0.078	0.075	0.077	0.100	0.070	0.094	0.074	0.077	0.069
Nonannouncement day	0.697	0.074	0.079	0.077	0.162	0.071	0.072	0.074	0.069	0.069	0.071	0.072
Standard deviation ratio	0.925	1.014	0.895	1.014	0.462	1.072	1.396	0.938	1.364	1.070	1.078	0.959
F-ratio	1.070	2.746∗	0.103	1.574	0.468	0.147	2.436	0.658	2.824∗	2.127	1.886	0.498
10 : 00 announcements	9:55-9:56	9:56-9:57	9:57-9:58	9:58-9:59	9:59-10:00	10:00-10:01	10:01-10:02	10:02-10:03	10:03-10:04	10:04-10:05	10:05-10:06	10:06-10:07
Announcement day	0.068	0.059	0.058	0.056	0.142	0.113	0.100	0.095	0.083	0.083	0.073	0.071
Nonannouncement day	0.067	0.066	0.068	0.070	0.084	0.074	0.076	0.070	0.070	0.074	0.067	0.067
Standard deviation ratio	1.011	0.885	0.857	0.803	1.684	1.521	1.318	1.341	1.184	1.120	1.088	1.066
F-ratio	0.111	5.312∗∗	3.031∗	8.334∗∗∗	109.734∗∗∗	98.197∗∗∗	25.730∗∗∗	38.533∗∗∗	24.231∗∗∗	5.955∗∗	7.190∗∗∗	6.933∗∗∗
14 : 00 announcement	13:55-13:56	13:56-13:57	13:57-13:58	13:58-13:59	13:59-14:00	14:00-14:01	14:01-14:02	14:02-14:03	14:03-14:04	14:04-14:05	14:05-14:06	14:06-14:07
Announcement day	0.061	0.044	0.058	0.050	0.063	0.069	0.056	0.064	0.050	0.048	0.055	0.066
Nonannouncement day	0.051	0.048	0.055	0.050	0.062	0.056	0.055	0.052	0.053	0.054	0.054	0.051
Standard deviation ratio	1.198	0.913	1.058	0.982	1.013	1.223	1.013	1.236	0.938	0.888	1.026	1.283
F-ratio	4.602∗∗	0.361	0.006	0.133	0.319	4.024∗∗	0.429	5.668∗∗	0.297	0.525	0.003	2.991∗
15 : 00 announcement	14:55-14:56	14:56-14:57	14:57-14:58	14:58-14:59	14:59-15:00	15:00-15:01	15:01-15:02	15:02-15:03	15:03-15:04	15:04-15:05	15:05-15:06	15:06-15:07
Announcement day	0.073	0.065	0.071	0.069	0.096	0.072	0.066	0.069	0.072	0.062	0.064	0.058
Nonannouncement day	0.055	0.058	0.066	0.064	0.069	0.067	0.063	0.066	0.064	0.062	0.067	0.063
Standard deviation ratio	1.323	1.127	1.089	1.068	1.392	1.064	1.038	1.054	1.131	0.992	0.947	0.930
F-ratio	4.297∗∗	0.089	0.976	0.318	10.297∗	0.406	2.407	0.127	1.209	0.001	0.016	0.056
Panel B: Trading Volume												
8 : 30 & 9 : 15 announcements	16:15-9:30	9:30-9:31	9:31-9:32	9:32-9:33	9:33-9:34	9:34-9:35	9:35-9:36	9:36-9:37	9:37-9:38	9:38-9:39	9:39-9:40	9:40-9:41
Announcement day	10.600	8.786	8.624	9.040	9.023	9.596	9.226	9.172	9.370	9.421	9.354	8.983
Nonannouncement day	10.951	8.687	8.604	9.057	8.913	9.341	8.947	9.180	8.995	9.062	9.079	9.028
Difference in means	-0.351	0.099	0.020	-0.017	0.110	0.255	0.279	-0.008	0.376	0.359	0.275	-0.045
t-statistic value	-0.693	0.667	0.131	-0.108	0.657	1.523	1.704 ∗	-0.047	2.218∗∗	2.121∗∗	1.633	-0.261
10 : 00 announcements	9:55-9:56	9:56-9:57	9:57-9:58	9:58-9:59	9:59-10:00	10:00-10:01	10:01-10:02	10:02-10:03	10:03-10:04	10:04-10:05	10:05-10:06	10:06-10:07
Announcement day	8.316	8.254	8.056	8.054	11.322	10.597	10.248	9.911	9.791	9.886	9.692	9.394
Nonannouncement day	8.681	8.692	8.733	8.622	9.960	9.564	9.327	9.244	9.030	9.249	9.006	8.910
Difference in means	-0.365	-0.438	-0.678	-0.568	1.362	1.033	0.921	0.668	0.761	0.637	0.686	0.484
t-statistic value	-1.998∗∗	-2.410∗∗	-3.588∗∗∗	-3.192∗∗∗	6.702∗∗∗	5.263∗∗∗	4.917∗∗∗	3.591∗∗∗	4.060∗∗∗	3.328∗∗∗	3.839∗∗∗	2.598∗∗∗
14 : 00 announcement	13:55-13:56	13:56-13:57	13:57-13:58	13:58-13:59	13:59-14:00	14:00-14:01	14:01-14:02	14:02-14:03	14:03-14:04	14:04-14:05	14:05-14:06	14:06-14:07
Announcement day	6.322	5.909	5.441	6.182	6.958	6.818	6.517	6.189	5.979	6.287	6.399	6.140
Nonannouncement day	5.600	5.747	5.876	6.068	6.735	6.404	6.308	6.243	6.133	6.207	6.105	6.088
Difference in means	0.722	0.162	-0.436	0.113	0.223	0.414	0.210	-0.054	-0.154	0.080	0.294	0.052
t-statistic value	2.194∗∗	0.502	-1.424	0.345	0.741	1.303	0.639	-0.174	-0.542	0.254	0.892	0.157
15 : 00 announcement	14:55-14:56	14:56-14:57	14:57-14:58	14:58-14:59	14:59-15:00	15:00-15:01	15:01-15:02	15:02-15:03	15:03-15:04	15:04-15:05	15:05-15:06	15:06-15:07
Announcement day	6.741	6.958	6.294	6.748	7.699	7.832	7.259	6.748	7.469	6.790	7.084	6.874
Nonannouncement day	6.525	6.516	6.755	6.687	7.311	6.930	6.782	6.702	6.731	6.811	6.788	6.731
Difference in means	0.217	0.442	-0.461	0.061	0.389	0.902	0.477	0.046	0.737	-0.021	0.296	0.144
t-statistic value	0.655	1.291	-1.388	0.162	1.134	2.570∗∗	1.353	0.138	2.082∗∗	-0.067	0.874	0.454

1. ∗, ∗∗ and ∗∗∗ represent statistical significance at the 10%, 5% and 1% levels, respectively.
2. Please note that the price change over the 16:15 to 9:30 period is not the one-minute price change.

Table D.3

Price volatility and trading volume by one-minute intervals in the expansion period

One-minute price change standard deviations and trading volume means are reported and compared for announcement (at a specific time) and nonannouncement days for the U.S. S&P 500 index future in the expansion period. Announcement days are defined as those with announcements released at the specific time, including 8:30 & 9:30, 10:00, 14:00 or 15:00 announcements. Nonannouncement days are those with no 8:30 & 9:30, 10:00, 14:00 and 15:00 announcements. We list the Brown-Forsythe-modified Levene F-statistic comparing variances for announcement and nonannouncement days and the t-statistic comparing means for announcement and nonannouncement days assuming unequal variances. All one-minute intervals between 9:30 and 9:41 and the time period over 16:15 to 9:30 are examined for 8:30 & 9:15 announcements. All one-minute intervals from 9:55 to 10:07 are examined for 10:00 announcements. All one-minute intervals from 13:55 to 14:07 are examined for the 14:00 announcement. All one-minute intervals from 14:55 to 15:07 are examined for the 15:00 announcement. The data period is February 03, 1997, to January 30, 2009.

Panel A: Price Volatility												
8 : 30 & 9 : 15 announcements	16:15-9:30	9:30-9:31	9:31-9:32	9:32-9:33	9:33-9:34	9:34-9:35	9:35-9:36	9:36-9:37	9:37-9:38	9:38-9:39	9:39-9:40	9:40-9:41
Announcement day	0.569	0.064	0.064	0.066	0.063	0.061	0.061	0.062	0.063	0.066	0.059	0.059
Nonannouncement day	0.532	0.066	0.074	0.061	0.066	0.065	0.060	0.064	0.061	0.061	0.058	0.064
Standard deviation ratio	1.070	0.970	0.854	1.085	0.952	0.951	1.022	0.960	1.027	1.079	1.016	0.922
F-ratio	4.707**	1.268	0.094	3.491*	0.204	0.631	0.881	0.568	0.881	2.488	0.944	0.346
10 : 00 announcements	9:55-9:56	9:56-9:57	9:57-9:58	9:58-9:59	9:59-10:00	10:00-10:01	10:01-10:02	10:02-10:03	10:03-10:04	10:04-10:05	10:05-10:06	10:06-10:07
Announcement day	0.055	0.052	0.055	0.052	0.125	0.105	0.092	0.083	0.079	0.076	0.070	0.066
Nonannouncement day	0.058	0.062	0.058	0.060	0.072	0.069	0.068	0.066	0.061	0.066	0.062	0.057
Standard deviation ratio	0.945	0.836	0.939	0.870	1.736	1.532	1.358	1.263	1.299	1.147	1.125	1.154
F-ratio	0.142	7.519***	0.746	4.259**	99.709***	83.357***	22.471***	26.940***	35.891***	11.366***	10.108***	11.893***
14 : 00 announcement	13:55-13:56	13:56-13:57	13:57-13:58	13:58-13:59	13:59-14:00	14:00-14:01	14:01-14:02	14:02-14:03	14:03-14:04	14:04-14:05	14:05-14:06	14:06-14:07
Announcement day	0.047	0.041	0.045	0.044	0.061	0.059	0.049	0.060	0.040	0.040	0.042	0.048
Nonannouncement day	0.046	0.042	0.045	0.045	0.054	0.050	0.048	0.045	0.047	0.046	0.047	0.046
Standard deviation ratio	1.011	0.967	1.014	0.973	1.132	1.184	1.007	1.325	0.849	0.866	0.890	1.049
F-ratio	0.557	0.000	0.010	0.703	1.415	3.225**	0.005	6.036***	0.469	0.717	0.584	0.799
15 : 00 announcement	14:55-14:56	14:56-14:57	14:57-14:58	14:58-14:59	14:59-15:00	15:00-15:01	15:01-15:02	15:02-15:03	15:03-15:04	15:04-15:05	15:05-15:06	15:06-15:07
Announcement day	0.070	0.064	0.059	0.050	0.073	0.061	0.058	0.056	0.056	0.051	0.053	0.051
Nonannouncement day	0.051	0.052	0.053	0.053	0.060	0.058	0.054	0.056	0.056	0.056	0.056	0.053
Standard deviation ratio	1.374	1.247	1.119	0.943	1.225	1.060	1.080	0.996	0.994	0.910	0.942	0.961
F-ratio	4.907**	0.335	0.828	0.799	3.367*	0.087	2.507	0.160	0.008	0.013	0.003	0.045
Panel B: Trading Volume												
8 : 30 & 9 : 15 announcements	16:15-9:30	9:30-9:31	9:31-9:32	9:32-9:33	9:33-9:34	9:34-9:35	9:35-9:36	9:36-9:37	9:37-9:38	9:38-9:39	9:39-9:40	9:40-9:41
Announcement day	10.727	8.791	8.590	9.005	8.882	9.564	9.162	9.195	9.323	9.396	9.287	8.911
Nonannouncement day	11.142	8.712	8.591	9.096	8.847	9.248	8.849	9.089	8.938	9.069	9.001	8.985
Difference in means	-0.415	0.079	-0.002	-0.090	0.035	0.316	0.312	0.106	0.385	0.326	0.286	-0.074
t-statistic value	-0.704	0.488	-0.009	-0.515	0.192	1.709*	1.756*	0.581	2.073**	1.748*	1.546	-0.384
10 : 00 announcements	9:55-9:56	9:56-9:57	9:57-9:58	9:58-9:59	9:59-10:00	10:00-10:01	10:01-10:02	10:02-10:03	10:03-10:04	10:04-10:05	10:05-10:06	10:06-10:07
Announcement day	8.327	8.295	8.085	8.243	11.124	10.636	10.291	10.092	9.915	9.989	9.787	9.520
Nonannouncement day	8.707	8.749	8.720	8.640	9.843	9.606	9.413	9.355	9.090	9.264	9.003	8.931
Difference in means	-0.380	-0.454	-0.635	-0.396	1.281	1.030	0.878	0.738	0.824	0.725	0.784	0.590
t-statistic value	-1.904*	-2.262**	-3.063***	-2.036**	5.792***	4.751***	4.268***	3.594***	3.982***	3.484***	3.984***	2.872***
14 : 00 announcement	13:55-13:56	13:56-13:57	13:57-13:58	13:58-13:59	13:59-14:00	14:00-14:01	14:01-14:02	14:02-14:03	14:03-14:04	14:04-14:05	14:05-14:06	14:06-14:07
Announcement day	6.434	6.016	5.492	6.270	7.123	7.098	6.615	6.303	6.033	6.361	6.516	6.123
Nonannouncement day	5.719	5.879	5.987	6.147	6.830	6.557	6.440	6.359	6.256	6.323	6.261	6.173
Difference in means	0.716	0.137	-0.495	0.124	0.293	0.542	0.175	-0.055	-0.223	0.038	0.255	-0.050
t-statistic value	2.012**	0.380	-1.474	0.337	0.893	1.550	0.474	-0.159	-0.696	0.107	0.709	-0.133
15 : 00 announcement	14:55-14:56	14:56-14:57	14:57-14:58	14:58-14:59	14:59-15:00	15:00-15:01	15:01-15:02	15:02-15:03	15:03-15:04	15:04-15:05	15:05-15:06	15:06-15:07
Announcement day	6.705	7.066	6.213	6.377	7.549	7.787	7.180	6.713	7.525	6.869	6.934	6.836
Nonannouncement day	6.632	6.616	6.833	6.773	7.326	7.036	6.872	6.813	6.843	6.911	6.827	6.728
Difference in means	0.073	0.450	-0.620	-0.396	0.223	0.751	0.308	-0.100	0.682	-0.042	0.108	0.108
t-statistic value	0.203	1.182	-1.684*	-1.091	0.588	1.989**	0.837	-0.270	1.738*	-0.123	0.298	0.306

1. *, **, and *** represent statistical significance at the 10%, 5%, and 1% levels, respectively.
2. Please note that the price change over the 16:15 to 9:30 period is not the one-minute price change.

Table D.4

Price volatility and trading volume by one-minute intervals in the contraction period

One-minute price change standard deviations and trading volume means are reported and compared for announcement (at a specific time) and nonannouncement days for the U.S. S&P 500 index future in the contraction period. Announcement days are defined as those with announcements released at the specific time, including 8:30 & 9:30, 10:00, 14:00 or 15:00 announcements. Nonannouncement days are those with no 8:30 & 9:30, 10:00, 14:00 and 15:00 announcements. We list the Brown-Forsythe-modified Levene F-statistic comparing variances for announcement and nonannouncement days and the t-statistic comparing means for announcement and nonannouncement days assuming unequal variances. All one-minute intervals between 9:30 and 9:41 and the time period over 16:15 to 9:30 are examined for 8:30 & 9:15 announcements. All one-minute intervals from 9:55 to 10:07 are examined for 10:00 announcements. All one-minute intervals from 13:55 to 14:07 are examined for the 14:00 announcement. All one-minute intervals from 14:55 to 15:07 are examined for the 15:00 announcement. The data period is February 03, 1997, to January 30, 2009.

Panel A: Price Volatility												
8 : 30 & 9 : 15 announcements	16:15-9:30	9:30-9:31	9:31-9:32	9:32-9:33	9:33-9:34	9:34-9:35	9:35-9:36	9:36-9:37	9:37-9:38	9:38-9:39	9:39-9:40	9:40-9:41
Announcement day	0.910	0.119	0.103	0.127	0.124	0.134	0.216	0.106	0.191	0.112	0.141	0.111
Nonannouncement day	1.303	0.109	0.104	0.139	0.396	0.104	0.121	0.119	0.101	0.108	0.123	0.108
Standard deviation ratio	0.699	1.097	0.992	0.916	0.313	1.293	1.786	0.892	1.892	1.044	1.145	1.022
F-ratio	1.950	1.562	0.001	0.014	0.373	2.282	1.467	0.220	1.673	0.086	0.858	0.272
10 : 00 announcements	9:55-9:56	9:56-9:57	9:57-9:58	9:58-9:59	9:59-10:00	10:00-10:01	10:01-10:02	10:02-10:03	10:03-10:04	10:04-10:05	10:05-10:06	10:06-10:07
Announcement day	0.114	0.087	0.076	0.076	0.213	0.146	0.137	0.140	0.103	0.115	0.092	0.096
Nonannouncement day	0.105	0.088	0.110	0.114	0.136	0.099	0.113	0.093	0.111	0.110	0.094	0.108
Standard deviation ratio	1.084	0.992	0.688	0.669	1.563	1.473	1.210	1.504	0.927	1.044	0.970	0.889
F-ratio	0.612	0.027	4.094**	5.206**	14.132***	15.335***	3.852**	11.347***	0.002	0.546	0.044	0.091
14 : 00 announcement	13:55-13:56	13:56-13:57	13:57-13:58	13:58-13:59	13:59-14:00	14:00-14:01	14:01-14:02	14:02-14:03	14:03-14:04	14:04-14:05	14:05-14:06	14:06-14:07
Announcement day	0.110	0.060	0.104	0.077	0.068	0.111	0.089	0.088	0.082	0.082	0.105	0.129
Nonannouncement day	0.072	0.074	0.095	0.076	0.096	0.085	0.086	0.082	0.078	0.089	0.084	0.077
Standard deviation ratio	1.522	0.804	1.103	1.014	0.706	1.305	1.037	1.069	1.063	0.930	1.257	1.673
F-ratio	7.278*	1.067	0.025	0.103	0.399	0.918	0.759	0.555	0.001	0.042	0.974	2.869***
15 : 00 announcement	14:55-14:56	14:56-14:57	14:57-14:58	14:58-14:59	14:59-15:00	15:00-15:01	15:01-15:02	15:02-15:03	15:03-15:04	15:04-15:05	15:05-15:06	15:06-15:07
Announcement day	0.094	0.071	0.121	0.136	0.179	0.111	0.100	0.121	0.133	0.107	0.107	0.090
Nonannouncement day	0.078	0.086	0.115	0.111	0.108	0.107	0.104	0.106	0.098	0.093	0.111	0.101
Standard deviation ratio	1.204	0.823	1.048	1.229	1.666	1.033	0.960	1.139	1.352	1.157	0.969	0.887
F-ratio	0.059	0.111	0.190	2.705	8.247***	0.111	0.182	0.942	2.513	0.027	0.052	0.025
Panel B: Trading Volume												
8 : 30 & 9 : 15 announcements	16:15-9:30	9:30-9:31	9:31-9:32	9:32-9:33	9:33-9:34	9:34-9:35	9:35-9:36	9:36-9:37	9:37-9:38	9:38-9:39	9:39-9:40	9:40-9:41
Announcement day	9.868	8.752	8.822	9.240	9.837	9.783	9.597	9.039	9.643	9.566	9.744	9.395
Nonannouncement day	9.810	8.533	8.681	8.829	9.310	9.895	9.529	9.724	9.333	9.014	9.548	9.286
Difference in means	0.059	0.219	0.141	0.412	0.528	-0.112	0.068	-0.685	0.310	0.552	0.197	0.110
t-statistic value	0.176	0.593	0.389	1.032	1.271	-0.291	0.164	-1.688*	0.764	1.397	0.490	0.281
10 : 00 announcements	9:55-9:56	9:56-9:57	9:57-9:58	9:58-9:59	9:59-10:00	10:00-10:01	10:01-10:02	10:02-10:03	10:03-10:04	10:04-10:05	10:05-10:06	10:06-10:07
Announcement day	8.255	8.029	7.892	7.010	12.412	10.382	10.010	8.912	9.108	9.314	9.167	8.696
Nonannouncement day	8.529	8.352	8.810	8.514	10.652	9.314	8.814	8.581	8.671	9.157	9.024	8.790
Difference in means	-0.274	-0.323	-0.917	-1.504	1.759	1.068	1.196	0.331	0.436	0.157	0.143	-0.094
t-statistic value	-0.598	-0.771	-2.018**	-3.573***	3.497***	2.358**	2.673***	0.799	1.026	0.324	0.342	-0.221
14 : 00 announcement	13:55-13:56	13:56-13:57	13:57-13:58	13:58-13:59	13:59-14:00	14:00-14:01	14:01-14:02	14:02-14:03	14:03-14:04	14:04-14:05	14:05-14:06	14:06-14:07
Announcement day	5.667	5.286	5.143	5.667	6.000	5.190	5.952	5.524	5.667	5.857	5.714	6.238
Nonannouncement day	4.890	4.957	5.214	5.600	6.167	5.495	5.519	5.552	5.405	5.514	5.171	5.581
Difference in means	0.776	0.329	-0.071	0.067	-0.167	-0.305	0.433	-0.029	0.262	0.343	0.543	0.657
t-statistic value	0.893	0.487	-0.097	0.095	-0.226	-0.469	0.674	-0.046	0.478	0.543	0.671	0.904
15 : 00 announcement	14:55-14:56	14:56-14:57	14:57-14:58	14:58-14:59	14:59-15:00	15:00-15:01	15:01-15:02	15:02-15:03	15:03-15:04	15:04-15:05	15:05-15:06	15:06-15:07
Announcement day	6.952	6.333	6.762	8.905	8.571	8.095	7.714	6.952	7.143	6.333	7.952	7.095
Nonannouncement day	5.886	5.924	6.290	6.176	7.219	6.300	6.243	6.038	6.067	6.214	6.557	6.748
Difference in means	1.067	0.410	0.471	2.729	1.352	1.795	1.471	0.914	1.076	0.119	1.395	0.348
t-statistic value	1.241	0.556	0.628	1.979*	1.802*	1.856*	1.330	1.121	1.343	0.153	1.483	0.538

1. *, **, and *** represent statistical significance at the 10%, 5%, and 1% levels, respectively.
2. Please note that the price change over the 16:15 to 9:30 period is not the one-minute price change.

Table D.5

Price volatility and trading volume by five-minute intervals in the entire sample period

Five-minute price change standard deviations and trading volume means are reported and compared for announcement (at a specific time) and nonannouncement days for the U.S. S&P 500 index future in the entire sample period. Announcement days are defined as those with announcements released at the specific time, including 8:30 & 9:30, 10:00, 14:00 or 15:00 announcements. Nonannouncement days are those with no 8:30 & 9:30, 10:00, 14:00 and 15:00 announcements. We list the Brown-Forsythe-modified Levene F-statistic comparing variances for announcement and nonannouncement days and the t-statistic comparing means for announcement and nonannouncement days assuming unequal variances. All five-minute intervals between 9:30 and 10:25 and the time period over 16:15 to 9:30 are examined for 8:30 & 9:15 announcements. All five-minute intervals from 9:55 to 10:55 are examined for 10:00 announcements. All five-minute intervals from 13:55 to 14:55 are examined for the 14:00 announcement. All five-minute intervals from 14:55 to 15:55 are examined for the 15:00 announcement. The data period is February 03, 1997, to January 30, 2009.

Panel A: Price Volatility												
8 : 30 & 9 : 15 announcements	16:15-9:30	9:30-9:35	9:35-9:40	9:40-9:45	9:45-9:50	9:50-9:55	9:55-10:00	10:00-10:05	10:05-10:10	10:10-10:15	10:15-10:20	10:20-10:25
Announcement day	0.644	0.185	0.174	0.183	0.171	0.169	0.184	0.182	0.166	0.153	0.153	0.134
Nonannouncement day	0.697	0.210	0.156	0.161	0.166	0.147	0.162	0.160	0.139	0.147	0.137	0.134
Standard deviation ratio	0.925	0.879	1.114	1.138	1.031	1.149	1.141	1.137	1.197	1.038	1.121	0.996
F-ratio	1.070	0.058	1.206	1.849	2.942*	5.309**	4.620**	6.101**	16.401***	0.764	7.185***	1.357
10 : 00 announcements	9:55-10:00	10:00-10:05	10:05-10:10	10:10-10:15	10:15-10:20	10:20-10:25	10:25-10:30	10:30-10:35	10:35-10:40	10:40-10:45	10:45-10:50	10:50-10:55
Announcement day	0.193	0.219	0.165	0.155	0.155	0.142	0.147	0.137	0.133	0.162	0.137	0.149
Nonannouncement day	0.162	0.160	0.139	0.147	0.137	0.134	0.142	0.132	0.120	0.140	0.129	0.115
Standard deviation ratio	1.197	1.369	1.189	1.052	1.129	1.056	1.041	1.045	1.106	1.160	1.069	1.291
F-ratio	12.736***	50.564***	26.126***	1.540	4.241**	2.734*	1.119	1.433	4.353**	0.478	0.930	2.399
14 : 00 announcement	13:55-14:00	14:00-14:05	14:05-14:10	14:10-14:15	14:15-14:20	14:20-14:25	14:25-14:30	14:30-14:35	14:35-14:40	14:40-14:45	14:45-14:50	14:50-14:55
Announcement day	0.128	0.117	0.145	0.145	0.153	0.124	0.172	0.129	0.160	0.134	0.149	0.144
Nonannouncement day	0.119	0.123	0.112	0.122	0.141	0.125	0.130	0.124	0.130	0.130	0.123	0.126
Standard deviation ratio	1.078	0.955	1.292	1.184	1.086	0.990	1.321	1.035	1.232	1.032	1.214	1.137
F-ratio	0.789	0.569	2.130	0.250	0.094	0.001	4.326**	0.226	0.462	0.240	1.797	1.159
15 : 00 announcement	14:55-15:00	15:00-15:05	15:05-15:10	15:10-15:15	15:15-15:20	15:20-15:25	15:25-15:30	15:30-15:35	15:35-15:40	15:40-15:45	15:45-15:50	15:50-15:55
Announcement day	0.172	0.154	0.148	0.135	0.141	0.134	0.127	0.163	0.139	0.131	0.127	0.121
Nonannouncement day	0.141	0.145	0.140	0.148	0.148	0.142	0.148	0.162	0.160	0.168	0.140	0.148
Standard deviation ratio	1.219	1.059	1.059	0.912	0.949	0.949	0.862	1.003	0.868	0.780	0.905	0.821
F-ratio	2.636	0.003	0.648	0.001	0.055	0.044	1.158	0.000	0.713	0.518	0.398	0.036
Panel B: Trading Volume												
8 : 30 & 9 : 15 announcements	16:15-9:30	9:30-9:35	9:35-9:40	9:40-9:45	9:45-9:50	9:50-9:55	9:55-10:00	10:00-10:05	10:05-10:10	10:10-10:15	10:15-10:20	10:20-10:25
Announcement day	10.600	45.069	46.544	46.132	46.085	44.913	45.315	47.734	45.942	44.596	43.857	42.202
Nonannouncement day	10.951	44.603	45.262	45.326	45.021	44.198	44.688	46.414	44.648	43.813	43.016	41.369
Difference in means	-0.351	0.466	1.282	0.806	1.064	0.715	0.627	1.320	1.294	0.783	0.840	0.832
t-statistic value	-0.693	0.902	2.336**	1.457	1.932*	1.320	1.173	2.334**	2.318**	1.388	1.500	1.504
10 : 00 announcements	9:55-10:00	10:00-10:05	10:05-10:10	10:10-10:15	10:15-10:20	10:20-10:25	10:25-10:30	10:30-10:35	10:35-10:40	10:40-10:45	10:45-10:50	10:50-10:55
Announcement day	44.002	50.433	47.090	45.418	44.066	42.244	42.720	42.221	41.036	40.447	39.027	37.349
Nonannouncement day	44.688	46.414	44.648	43.813	43.016	41.369	41.397	41.456	39.622	39.568	38.700	37.410
Difference in means	-0.687	4.019	2.442	1.605	1.050	0.874	1.324	0.765	1.414	0.878	0.327	-0.061
t-statistic value	-1.144	6.386***	4.041***	2.605***	1.699*	1.431	2.218**	1.255	2.246**	1.417	0.517	-0.096
14 : 00 announcement	13:55-14:00	14:00-14:05	14:05-14:10	14:10-14:15	14:15-14:20	14:20-14:25	14:25-14:30	14:30-14:35	14:35-14:40	14:40-14:45	14:45-14:50	14:50-14:55
Announcement day	30.811	31.790	31.392	30.937	32.014	32.028	32.979	33.385	32.545	32.874	33.392	33.021
Nonannouncement day	30.026	31.295	30.438	31.108	32.036	31.834	32.716	33.062	32.081	32.108	32.111	32.504
Difference in means	0.785	0.495	0.953	-0.171	-0.022	0.194	0.263	0.322	0.464	0.766	1.281	0.517
t-statistic value	0.692	0.441	0.754	-0.140	-0.017	0.163	0.228	0.248	0.367	0.623	1.044	0.439
15 : 00 announcement	14:55-15:00	15:00-15:05	15:05-15:10	15:10-15:15	15:15-15:20	15:20-15:25	15:25-15:30	15:30-15:35	15:35-15:40	15:40-15:45	15:45-15:50	15:50-15:55
Announcement day	34.441	36.098	34.350	34.371	34.420	34.378	35.538	36.385	35.301	35.196	35.853	36.818
Nonannouncement day	33.794	33.956	33.475	33.415	33.042	33.375	34.044	34.553	34.205	34.634	34.192	34.156
Difference in means	0.646	2.142	0.875	0.955	1.377	1.003	1.494	1.832	1.096	0.562	1.662	2.662
t-statistic value	0.525	1.790*	0.688	0.750	1.051	0.811	1.145	1.364	0.875	0.431	1.351	2.204**

1. *, **, and *** represent statistical significance at the 10%, 5%, and 1% levels, respectively.
2. Please note that the price change over the 16:15 to 9:30 period is not the five-minute price change.

Table D.6

Price volatility and trading volume by five-minute intervals in the expansion period

Five-minute price change standard deviations and trading volume means are reported and compared for announcement (at a specific time) and nonannouncement days for the U.S. S&P 500 index future in the expansion period. Announcement days are defined as those with announcements released at the specific time, including 8:30 & 9:30, 10:00, 14:00, or 15:00 announcements. Nonannouncement days are those with no 8:30 & 9:30, 10:00, 14:00, and 15:00 announcements. We list the Brown-Forsythe-modified Levene F-statistic comparing variances for announcement and nonannouncement days and the t-statistic comparing means for announcement and nonannouncement days assuming unequal variances. All five-minute intervals between 9:30 and 10:25 and the time period over 16:15 to 9:30 are examined for 8:30 & 9:15 announcements. All five-minute intervals from 9:55 to 10:55 are examined for 10:00 announcements. All five-minute intervals from 13:55 to 14:55 are examined for the 14:00 announcement. All five-minute intervals from 14:55 to 15:55 are examined for the 15:00 announcement. The data period is February 03, 1997, to January 30, 2009.

Panel A: Price Volatility												
8 : 30 & 9 : 15 announcements	16:15-9:30	9:30-9:35	9:35-9:40	9:40-9:45	9:45-9:50	9:50-9:55	9:55-10:00	10:00-10:05	10:05-10:10	10:10-10:15	10:15-10:20	10:20-10:25
Announcement day	0.569	0.140	0.145	0.138	0.146	0.146	0.157	0.164	0.150	0.138	0.133	0.128
Nonannouncement day	0.532	0.141	0.134	0.141	0.138	0.131	0.142	0.150	0.124	0.135	0.124	0.122
Standard deviation ratio	1.070	0.988	1.088	0.978	1.062	1.119	1.110	1.096	1.213	1.021	1.072	1.045
F-ratio	4.707**	0.056	2.942*	0.660	4.204**	3.213*	4.094**	2.920*	13.990***	0.271	3.986**	1.488
10 : 00 announcements	9:55-10:00	10:00-10:05	10:05-10:10	10:10-10:15	10:15-10:20	10:20-10:25	10:25-10:30	10:30-10:35	10:35-10:40	10:40-10:45	10:45-10:50	10:50-10:55
Announcement day	0.166	0.206	0.153	0.149	0.151	0.136	0.142	0.131	0.123	0.126	0.130	0.118
Nonannouncement day	0.142	0.150	0.124	0.135	0.124	0.122	0.129	0.120	0.108	0.119	0.119	0.108
Standard deviation ratio	1.173	1.376	1.239	1.097	1.222	1.107	1.099	1.086	1.137	1.058	1.088	1.091
F-ratio	6.657***	47.126***	29.799***	2.800*	6.094**	3.119*	2.516	2.081	4.205**	0.804	1.879	0.285
14 : 00 announcement	13:55-14:00	14:00-14:05	14:05-14:10	14:10-14:15	14:15-14:20	14:20-14:25	14:25-14:30	14:30-14:35	14:35-14:40	14:40-14:45	14:45-14:50	14:50-14:55
Announcement day	0.107	0.114	0.123	0.147	0.128	0.094	0.144	0.098	0.122	0.101	0.113	0.129
Nonannouncement day	0.103	0.104	0.104	0.109	0.118	0.112	0.117	0.111	0.112	0.119	0.105	0.112
Standard deviation ratio	1.037	1.097	1.189	1.352	1.080	0.839	1.229	0.882	1.090	0.844	1.078	1.151
F-ratio	1.593	1.753	1.878	1.644	0.002	0.549	3.611*	0.097	0.379	0.586	0.844	1.952
15 : 00 announcement	14:55-15:00	15:00-15:05	15:05-15:10	15:10-15:15	15:15-15:20	15:20-15:25	15:25-15:30	15:30-15:35	15:35-15:40	15:40-15:45	15:45-15:50	15:50-15:55
Announcement day	0.145	0.111	0.131	0.110	0.121	0.121	0.129	0.137	0.129	0.121	0.126	0.117
Nonannouncement day	0.121	0.124	0.125	0.128	0.130	0.126	0.136	0.140	0.136	0.137	0.115	0.113
Standard deviation ratio	1.203	0.897	1.047	0.856	0.930	0.963	0.946	0.980	0.950	0.886	1.102	1.035
F-ratio	0.529	0.493	0.048	0.323	0.077	0.005	0.378	0.357	0.207	0.547	3.912**	1.773
Panel B: Trading Volume												
8 : 30 & 9 : 15 announcements	16:15-9:30	9:30-9:35	9:35-9:40	9:40-9:45	9:45-9:50	9:50-9:55	9:55-10:00	10:00-10:05	10:05-10:10	10:10-10:15	10:15-10:20	10:20-10:25
Announcement day	10.727	44.832	46.362	45.871	45.926	45.030	45.139	47.830	46.102	44.716	44.110	42.269
Nonannouncement day	11.142	44.494	44.946	45.151	45.027	44.234	44.660	46.728	44.678	44.011	43.154	41.575
Difference in means	-0.415	0.337	1.416	0.720	0.899	0.796	0.479	1.102	1.424	0.705	0.956	0.694
t-statistic value	-0.704	0.594	2.348**	1.196	1.497	1.348	0.824	1.785*	2.346**	1.141	1.564	1.137
10 : 00 announcements	9:55-10:00	10:00-10:05	10:05-10:10	10:10-10:15	10:15-10:20	10:20-10:25	10:25-10:30	10:30-10:35	10:35-10:40	10:40-10:45	10:45-10:50	10:50-10:55
Announcement day	44.075	50.924	47.544	45.977	44.776	42.826	43.185	42.783	41.417	40.941	39.208	37.801
Nonannouncement day	44.660	46.728	44.678	44.011	43.154	41.575	41.659	41.880	39.942	39.866	38.931	37.593
Difference in means	-0.585	4.195	2.865	1.966	1.622	1.251	1.526	0.903	1.475	1.076	0.277	0.208
t-statistic value	-0.886	6.041***	4.315***	2.908***	2.401**	1.873*	2.347**	1.363	2.146**	1.578	0.394	0.297
14 : 00 announcement	13:55-14:00	14:00-14:05	14:05-14:10	14:10-14:15	14:15-14:20	14:20-14:25	14:25-14:30	14:30-14:35	14:35-14:40	14:40-14:45	14:45-14:50	14:50-14:55
Announcement day	31.336	32.410	31.951	32.049	32.951	32.738	33.451	33.943	33.459	33.770	33.779	33.762
Nonannouncement day	30.562	31.934	31.020	31.716	32.387	32.280	33.296	33.683	32.755	32.746	32.748	33.120
Difference in means	0.774	0.476	0.931	0.334	0.563	0.458	0.154	0.260	0.704	1.024	1.030	0.642
t-statistic value	0.626	0.379	0.663	0.246	0.391	0.343	0.118	0.179	0.504	0.755	0.752	0.492
15 : 00 announcement	14:55-15:00	15:00-15:05	15:05-15:10	15:10-15:15	15:15-15:20	15:20-15:25	15:25-15:30	15:30-15:35	15:35-15:40	15:40-15:45	15:45-15:50	15:50-15:55
Announcement day	33.910	36.074	33.877	33.926	34.254	34.057	35.352	35.992	35.246	35.148	35.516	36.770
Nonannouncement day	34.180	34.475	33.799	33.974	33.463	33.749	34.341	34.761	34.207	34.699	34.223	34.100
Difference in means	-0.270	1.599	0.078	-0.048	0.791	0.308	1.011	1.231	1.039	0.449	1.294	2.671
t-statistic value	-0.203	1.210	0.056	-0.035	0.550	0.233	0.728	0.843	0.749	0.310	0.960	2.030**

1. *, **, and *** represent statistical significance at the 10%, 5%, and 1% levels, respectively.
2. Please note that the price change over the 16:15 to 9:30 period is not the five-minute price change.

Table D.7

Price volatility and trading volume by five-minute intervals in the contraction period

Five-minute price change standard deviations and trading volume means are reported and compared for announcement (at a specific time) and nonannouncement days for the U.S. S&P 500 index future in the contraction period. Announcement days are defined as those with announcements released at the specific time, including 8:30 & 9:30, 10:00, 14:00, or 15:00 announcements. Nonannouncement days are those with no 8:30 & 9:30, 10:00, 14:00, and 15:00 announcements. We list the Brown-Forsythe-modified Levene F-statistic comparing variances for announcement and nonannouncement days and the t-statistic comparing means for announcement and nonannouncement days assuming unequal variances. All five-minute intervals between 9:30 and 10:25, and the time period over 16:15 to 9:30, are examined for 8:30 & 9:15 announcements. All five-minute intervals from 9:55 to 10:55 are examined for 10:00 announcements. All five-minute intervals from 13:55 to 14:55 are examined for the 14:00 announcement. All five-minute intervals from 14:55 to 15:55 are examined for the 15:00 announcement. The data period is February 03, 1997, to January 30, 2009.

Panel A: Price Volatility												
8 : 30 & 9 : 15 announcements	16:15-9:30	9:30-9:35	9:35-9:40	9:40-9:45	9:45-9:50	9:50-9:55	9:55-10:00	10:00-10:05	10:05-10:10	10:10-10:15	10:15-10:20	10:20-10:25
Announcement day	0.910	0.345	0.288	0.343	0.270	0.267	0.296	0.262	0.240	0.219	0.241	0.163
Nonannouncement day	1.303	0.435	0.251	0.248	0.280	0.222	0.249	0.211	0.207	0.203	0.198	0.190
Standard deviation ratio	0.699	0.792	1.146	1.379	0.963	1.202	1.189	1.241	1.158	1.076	1.215	0.861
F-ratio	1.950	0.002	0.112	1.144	0.055	2.060	0.700	3.842**	2.858*	0.550	3.077*	0.019
10 : 00 announcements	9:55-10:00	10:00-10:05	10:05-10:10	10:10-10:15	10:15-10:20	10:20-10:25	10:25-10:30	10:30-10:35	10:35-10:40	10:40-10:45	10:45-10:50	10:50-10:55
Announcement day	0.300	0.281	0.220	0.185	0.171	0.171	0.174	0.172	0.180	0.291	0.175	0.260
Nonannouncement day	0.249	0.211	0.207	0.203	0.198	0.190	0.200	0.185	0.177	0.228	0.175	0.152
Standard deviation ratio	1.208	1.332	1.063	0.912	0.862	0.905	0.871	0.925	1.017	1.278	0.998	1.716
F-ratio	5.245**	4.745**	0.837	0.344	0.217	0.027	0.638	0.057	0.335	0.089	0.438	2.867*
14 : 00 announcement	13:55-14:00	14:00-14:05	14:05-14:10	14:10-14:15	14:15-14:20	14:20-14:25	14:25-14:30	14:30-14:35	14:35-14:40	14:40-14:45	14:45-14:50	14:50-14:55
Announcement day	0.200	0.134	0.235	0.127	0.252	0.233	0.279	0.244	0.288	0.255	0.280	0.210
Nonannouncement day	0.187	0.200	0.152	0.183	0.233	0.185	0.189	0.185	0.206	0.181	0.199	0.190
Standard deviation ratio	1.068	0.669	1.545	0.694	1.083	1.259	1.473	1.319	1.399	1.408	1.407	1.105
F-ratio	0.052	0.279	0.207	1.291	0.115	0.913	0.867	1.222	0.053	0.036	0.589	0.012
15 : 00 announcement	14:55-15:00	15:00-15:05	15:05-15:10	15:10-15:15	15:15-15:20	15:20-15:25	15:25-15:30	15:30-15:35	15:35-15:40	15:40-15:45	15:45-15:50	15:50-15:55
Announcement day	0.287	0.283	0.228	0.231	0.223	0.197	0.112	0.270	0.191	0.184	0.128	0.144
Nonannouncement day	0.228	0.234	0.206	0.234	0.229	0.212	0.202	0.257	0.262	0.294	0.242	0.275
Standard deviation ratio	1.261	1.211	1.107	0.986	0.973	0.930	0.553	1.049	0.727	0.625	0.528	0.526
F-ratio	2.724*	0.591	1.601	0.518	0.004	0.213	1.386	0.433	0.791	0.166	1.553	0.953
Panel B: Trading Volume												
8 : 30 & 9 : 15 announcements	16:15-9:30	9:30-9:35	9:35-9:40	9:40-9:45	9:45-9:50	9:50-9:55	9:55-10:00	10:00-10:05	10:05-10:10	10:10-10:15	10:15-10:20	10:20-10:25
Announcement day	9.868	46.434	47.589	47.636	47.000	44.240	46.333	47.178	45.016	43.907	42.395	41.814
Nonannouncement day	9.810	45.248	47.148	46.371	44.986	43.981	44.857	44.538	44.467	42.633	42.195	40.143
Difference in means	0.059	1.186	0.442	1.264	2.014	0.259	1.476	2.640	0.549	1.274	0.200	1.671
t-statistic value	0.176	0.968	0.343	0.908	1.460	0.189	1.078	1.884*	0.387	0.930	0.145	1.311
10 : 00 announcements	9:55-10:00	10:00-10:05	10:05-10:10	10:10-10:15	10:15-10:20	10:20-10:25	10:25-10:30	10:30-10:35	10:35-10:40	10:40-10:45	10:45-10:50	10:50-10:55
Announcement day	43.598	47.725	44.588	42.333	40.147	39.029	40.157	39.118	38.931	37.716	38.029	34.853
Nonannouncement day	44.857	44.538	44.467	42.633	42.195	40.143	39.833	38.929	37.710	37.795	37.324	36.319
Difference in means	-1.259	3.187	0.122	-0.300	-2.048	-1.113	0.324	0.189	1.222	-0.080	0.706	-1.466
t-statistic value	-0.887	2.220**	0.085	-0.207	-1.403	-0.756	0.218	0.125	0.785	-0.055	0.499	-1.032
14 : 00 announcement	13:55-14:00	14:00-14:05	14:05-14:10	14:10-14:15	14:15-14:20	14:20-14:25	14:25-14:30	14:30-14:35	14:35-14:40	14:40-14:45	14:45-14:50	14:50-14:55
Announcement day	27.762	28.190	28.143	24.476	26.571	27.905	30.238	30.143	27.238	27.667	31.143	28.714
Nonannouncement day	26.829	27.486	26.971	27.486	29.943	29.176	29.257	29.362	28.067	28.305	28.310	28.833
Difference in means	0.933	0.705	1.171	-3.010	-3.371	-1.271	0.981	0.781	-0.829	-0.638	2.833	-0.119
t-statistic value	0.331	0.330	0.436	-1.343	-1.260	-0.580	0.496	0.308	-0.311	-0.249	1.127	-0.049
15 : 00 announcement	14:55-15:00	15:00-15:05	15:05-15:10	15:10-15:15	15:15-15:20	15:20-15:25	15:25-15:30	15:30-15:35	15:35-15:40	15:40-15:45	15:45-15:50	15:50-15:55
Announcement day	37.524	36.238	37.095	36.952	35.381	36.238	36.619	38.667	35.619	35.476	37.810	37.095
Nonannouncement day	31.495	30.862	31.543	30.081	30.533	31.143	32.276	33.310	34.195	34.248	34.005	34.490
Difference in means	6.029	5.376	5.552	6.871	4.848	5.095	4.343	5.357	1.424	1.229	3.805	2.605
t-statistic value	1.848*	1.930*	1.811*	2.092**	1.511	1.466	1.141	1.556	0.501	0.429	1.268	0.839

1. *, **, and * * * represent statistical significance at the 10%, 5%, and 1% levels, respectively.
2. Please note that the price change over the 16:15 to 9:30 period is not the five-minute price change.

Table D.8

The immediate effect of announcement surprises on the stock price

News Announcements	Response Coefficients		
	W[1]	E[1]	C[1]
1. CPI	-0.0868	-0.1072*	-0.0446
	(-1.6079)	(-1.6783)	(-0.4511)
2. PPI	-0.1563***	-0.1511***	-0.2152
	(-2.9859)	(-2.9489)	(-0.7837)
3. Nonfarm Payrolls	-0.0674	-0.0511	-0.8937***
	(-1.4417)	(-1.3553)	(-2.7471)
4. Civilian Unemployment	/	/	/
	/	/	/
5. Personal Consumption	/	/	/
	/	/	/
6. Personal Income	/	/	/
	/	/	/
7. Trade Balance	/	/	/
	/	/	/
8. Capacity Utilisation	/	/	/
	/	/	/
9. IP	/	/	/
	/	/	/
10. Consumer Confidence	0.1175***	0.1192***	0.1150***
	(7.5844)	(5.6781)	(5.1795)
11. Durable Goods Orders	0.0874 **	0.0514	0.2070*
	(2.2164)	(1.3484)	(1.8437)
12. Leading Index	0.0363**	0.0205	0.0856
	(2.0183)	(2.4443)	(1.4310)
13. Manufacturers' New Orders	/	/	/
	/	/	/
14. New Single-Family Home Sales	0.0048	0.0021	2.3461***
	(1.5368)	(1.0949)	(4.4629)
15. PMI	0.0364 ***	0.0318***	0.0489*
	(3.0906)	(2.6721)	(1.7067)
16. Federal Budget	-0.0045***	-0.0041***	-0.0281*
	(-6.7236)	(-5.2428)	(-1.6455)
17. Consumer Credit	-0.0029	-0.0041***	0.3921*
	(-1.1115)	(-2.7033)	(1.7694)

1. The entire sample period is abbreviated as "W", the expansion period as "E", and the contraction period as "C".
2. *, ** and *** represent statistical significance at the 10%, 5% and 1% levels, respectively.
3. If the response coefficient on the announcement surprise is not significant in every time period, we use the slash line instead of the data.

Table D.9

The eventual effect of announcement surprises on the stock price

News Announcements	Response Coefficients								
	First Scale			Second Scale			Third Scale		
	W^1	E^1	C^1	W^1	E^1	C^1	W^1	E^1	C^1
1. CPI	/	/	/	/	/	/	-2.13E-05	-2.38E-04**	3.21E-04
	/	/	/	/	/	/	(-0.1706)	(-1.9765)	(1.0393)
2. PPI	-0.0035***	-0.0037***	-0.0005	9.71E-05*	6.90E-05	4.17E-04	-2.78E-04*	-3.34E-04**	1.17E-04
	(-2.8321)	(-3.0170)	(-0.0836)	(1.7625)	(1.4349)	(1.0463)	(-1.7375)	(-2.1421)	(0.1222)
3. Nonfarm Payrolls	-0.0010	-0.0005	-0.0261**	-7.71E-05*	-4.48E-05	-2.55E-03***	/	/	/
	(-0.7519)	(-0.4844)	(-2.0152)	(-1.6464)	(-1.4465)	(-4.7115)	/	/	/
4. Civilian Unemployment	/	/	/	-7.86E-05	-2.01E-05	-2.28E-04*	2.32E-05	1.65E-04	-3.45E-04**
	/	/	/	(-1.1208)	(-0.2487)	(-1.7210)	(0.1747)	(1.0068)	(-2.0940)
5. Personal Consumption	/	/	/	9.48E-07	5.74E-05	-2.46E-04**	-2.02E-04	-2.85E-04**	1.48E-04
	/	/	/	(0.0199)	(1.1643)	(-2.2992)	(-1.5803)	(-2.4728)	(0.5529)
6. Personal Income	/	/	/	/	/	/	/	/	/
	/	/	/	/	/	/	/	/	/
7. Trade Balance	/	/	/	/	/	/	/	/	/
	/	/	/	/	/	/	/	/	/
8. Capacity Utilisation	/	/	/	/	/	/	4.97E-04*	4.66E-05	1.28E-03**
	/	/	/	/	/	/	(1.8400)	(0.3278)	(2.3492)
9. IP	/	/	/	/	/	/	/	/	/
	/	/	/	/	/	/	/	/	/
10. Consumer Confidence	0.0036*	0.0019*	0.0062	/	/	/	/	/	/
	(1.8504)	(1.7060)	(1.3550)	/	/	/	/	/	/
11. Durable Goods Orders	0.0016	0.0008	0.0043*	/	/	/	/	/	/
	(1.4797)	(0.6485)	(1.6820)	/	/	/	/	/	/
12. Leading Index	0.0019*	0.0013*	0.0035	/	/	/	/	/	/
	(1.8513)	(1.8383)	(1.0311)	/	/	/	/	/	/
13. Manufacturers' New Orders	/	/	/	-6.92E-06	-2.79E-05	1.33E-04*	/	/	/
	/	/	/	(-0.0998)	(-0.4218)	(1.6886)	/	/	/
14. New Single-Family Home Sales	-0.0006***	-0.0007***	0.0744	-6.05E-05***	-5.96E-05***	-2.16E-03	6.94E-05***	6.55E-05***	9.21E-03
	(-2.7675)	(-3.1606)	(1.5670)	(-4.5690)	(-4.5522)	(-1.2550)	(3.7691)	(4.2221)	(1.1585)
15. PMI	0.0017*	0.0011	0.0033*	/	/	/	2.46E-04**	3.84E-05	8.02E-04***
	(1.7601)	(0.9435)	(1.7532)	/	/	/	(2.2355)	(0.4689)	(3.0410)
16. Federal Budget	3.10E-05	0.0002***	-0.0095	/	/	/	/	/	/
	(0.0980)	(3.4465)	(-0.6311)	/	/	/	/	/	/
17. Consumer Credit	-0.0003**	-0.0002***	-0.0154	3.22E-05**	3.04E-05***	6.63E-04	/	/	/
	(-2.2970)	(-2.8307)	(-1.2351)	(2.3562)	(2.6450)	(0.3655)	/	/	/

Table D.9 (Continued)

News Announcements	Response Coefficients								
	Fourth Scale			Fifth Scale			Sixth Scale		
	W^1	E^1	C^1	W^1	E^1	C^1	W^1	E^1	C^1
1. CPI	/	/	/	/	/	/	/	/	/
	/	/	/	/	/	/	/	/	/
2. PPI	1.42E-05	3.32E-07	1.31E-04**	/	/	/	/	/	/
	(1.3125)	(0.0348)	(2.4386)	/	/	/	/	/	/
3. Nonfarm Payrolls	/	/	/	/	/	/	/	/	/
	/	/	/	/	/	/	/	/	/
4. Civilian Unemployment	/	/	/	1.49E-05**	1.06E-05*	2.56E-05*	/	/	/
	/	/	/	(2.3062)	(1.6667)	(1.6817)	/	/	/
5. Personal Consumption	-2.47E-05***	-2.60E-05***	-1.94E-05*	-9.10E-06**	-1.08E-05**	-1.74E-06	3.91E-07***	2.76E-07***	8.92E-07***
	(-3.1118)	(-2.7142)	(-1.9039)	(-2.3012)	(-2.2658)	(-0.3965)	(3.7518)	(2.9917)	(3.2181)
6. Personal Income	/	/	/	/	/	/	/	/	/
	/	/	/	/	/	/	/	/	/
7. Trade Balance	/	/	/	-1.26E-05***	-9.38E-06**	-2.93E-05*	-2.94E-07**	-1.89E-07	-8.80E-07**
	/	/	/	(-2.7034)	(-2.4570)	(-1.6930)	(-2.1868)	(-1.4934)	(-2.1712)
8. Capacity Utilisation	/	/	/	-6.86E-06	9.20E-06	-3.66E-05*	-4.19E-07**	-3.53E-07***	-5.39E-07
	/	/	/	(-0.5744)	(1.2301)	(-1.7568)	(-2.0145)	(-2.8625)	(-1.0471)
9. IP	4.98E-05**	-6.96E-07	1.12E-04**	/	/	/	/	/	/
	(2.4836)	(-0.0592)	(2.5344)	/	/	/	/	/	/
10. Consumer Confidence	2.49E-05*	1.35E-05	4.15E-05	/	/	/	/	/	/
	(1.7098)	(1.0472)	(1.5597)	/	/	/	/	/	/
11. Durable Goods Orders	/	/	/	/	/	/	/	/	/
	/	/	/	/	/	/	/	/	/
12. Leading Index	/	/	/	/	/	/	/	/	/
	/	/	/	/	/	/	/	/	/
13. Manufacturers' New Orders	/	/	/	/	/	/	/	/	/
	/	/	/	/	/	/	/	/	/
14. New Single-Family Home Sales	1.29E-06*	1.21E-06*	2.83E-04	-1.92E-07	-1.49E-09	-4.52E-04*	2.42E-08	1.96E-08	1.11E-05**
	(1.7447)	(1.7907)	(1.0516)	(-0.1549)	(-0.0011)	(-1.8276)	(1.5366)	(1.6056)	(2.3982)
15. PMI	1.41E-05	-1.23E-06	5.50E-05*	/	/	/	-1.08E-07	1.08E-07	-6.89E-07*
	(0.9024)	(-0.0777)	(1.7134)	/	/	/	(-0.7670)	(0.9190)	(-1.9203)
16. Federal Budget	-4.16E-06	-1.06E-06	-1.81E-04***	-1.11E-06**	-1.04E-06***	-4.95E-06	2.43E-08	2.27E-08***	1.18E-07
	(-1.1029)	(-1.3262)	(-4.3519)	(-2.1580)	(-6.1818)	(-0.1744)	(1.5582)	(2.9715)	(0.1478)
17. Consumer Credit	/	/	/	3.05E-06***	2.90E-06***	5.40E-05	/	/	/
	/	/	/	(6.4837)	(5.3754)	(0.7267)	/	/	/

Table D.9 (Continued)

News Announcements	Response Coefficients											
	Seventh Scale			Eighth Scale			Ninth Scale			Tenth Scale		
	W[1]	E[1]	C[1]	W[1]	E[1]	C[1]	W[1]	E[1]	C[1]	W[1]	E[1]	C[1]
1. CPI	/	/	/	-2.26E-09*	-1.79E-09**	-3.33E-09	/	/	/	/	/	/
	/	/	/	(-1.8123)	(-2.0844)	(-0.9861)	/	/	/	/	/	/
2. PPI	1.47E-08	2.16E-08**	-2.27E-08	/	/	/	5.65E-10*	4.22E-10	2.26E-09**	/	/	/
	(1.4587)	(2.1539)	(-0.4325)	/	/	/	(1.7615)	(1.2619)	(2.0040)	/	/	/
3. Nonfarm Payrolls	-8.00E-10	-5.96E-09	2.61E-07*	/	/	/	/	/	/	/	/	/
	(-0.1754)	(-1.5083)	(1.6775)	/	/	/	/	/	/	/	/	/
4. Civilian Unemployment	/	/	/	/	/	/	/	/	/	/	/	/
	/	/	/	/	/	/	/	/	/	/	/	/
5. Personal Consumption	1.53E-08*	6.83E-09	5.17E-08	/	/	/	/	/	/	/	/	/
	(1.7034)	(0.8390)	(1.2338)	/	/	/	/	/	/	/	/	/
6. Personal Income	-2.80E-08**	-3.00E-08***	-2.58E-08	-3.10E-09**	-3.87E-09***	-8.61E-10	/	/	/	/	/	/
	(-2.5264)	(-3.2505)	(-0.8625)	(-2.3840)	(-2.9365)	(-0.3601)	/	/	/	/	/	/
7. Trade Balance	8.44E-09	-2.45E-09	6.81E-08**	-1.20E-09	-2.44E-10	-6.72E-09**	/	/	/	/	/	/
	(0.6228)	(-0.1771)	(2.3270)	(-1.2210)	(-0.3066)	(-2.1092)	/	/	/	/	/	/
8. Capacity Utilisation	3.48E-08	3.79E-09	6.14E-08**	/	/	/	/	/	/	/	/	/
	(1.4677)	(0.2000)	(2.3572)	/	/	/	/	/	/	/	/	/
9. IP	-6.20E-08**	3.40E-09	-1.29E-07***	/	/	/	/	/	/	/	/	/
	(-2.1579)	(0.1445)	(-4.7581)	/	/	/	/	/	/	/	/	/
10. Consumer Confidence	5.80E-10	-3.19E-08	4.77E-08**	/	/	/	/	/	/	-1.01E-10***	-9.26E-12	-2.18E-10***
	(0.0296)	(-1.2505)	(2.5741)	/	/	/	/	/	/	(-4.3198)	(-0.2302)	(-4.4954)
11. Durable Goods Orders	2.60E-08**	1.76E-08	5.31E-08**	-2.22E-09**	-3.69E-09***	2.80E-09	/	/	/	/	/	/
	(2.2885)	(1.3758)	(2.3293)	(-2.0459)	(-3.4538)	(0.9799)	/	/	/	/	/	/
12. Leading Index	-1.63E-08	5.06E-09	-8.46E-08***	/	/	/	/	/	/	/	/	/
	(-1.5589)	(0.5426)	(-2.9798)	/	/	/	/	/	/	/	/	/
13. Manufacturers' New Orders	1.27E-08	1.30E-08**	1.10E-08	/	/	/	/	/	/	/	/	/
	(1.2738)	(2.0472)	(0.1719)	/	/	/	/	/	/	/	/	/
14. New Single-Family Home Sales	/	/	/	2.60E-11	-5.55E-11	1.94E-07**	/	/	/	/	/	/
	/	/	/	(0.0812)	(-0.1830)	(2.2287)	/	/	/	/	/	/
15. PMI	/	/	/	/	/	/	-2.22E-11	3.81E-10	-1.12E-09 *	/	/	/
	/	/	/	/	/	/	(-0.0693)	(1.0145)	(-1.8274)	/	/	/
16. Federal Budget	1.64E-08	1.14E-08**	3.01E-07 ***	-1.26E-09	-1.40E-10	-6.59E-08**	/	/	/	/	/	/
	(1.5394)	(2.0258)	(3.0614)	(-0.9309)	(-1.2599)	(-2.0534)	/	/	/	/	/	/
17. Consumer Credit	7.41E-10	-1.61E-09	8.13E-07***	-1.12E-09***	-1.16E-09***	1.15E-08	/	/	/	/	/	/
	(0.1531)	(-0.5544)	(2.6083)	(-2.7872)	(-3.3079)	(0.2289)	/	/	/	/	/	/

1. The entire sample period is abbreviated as "W", the expansion period as "E", and the contraction period as "C".
2. *, ** and *** represent statistical significance at the 10%, 5% and 1% levels, respectively.
3. If the response coefficient on the announcement surprise is not significant in every time period, we use the slash line instead of the data.

Figure D.1A

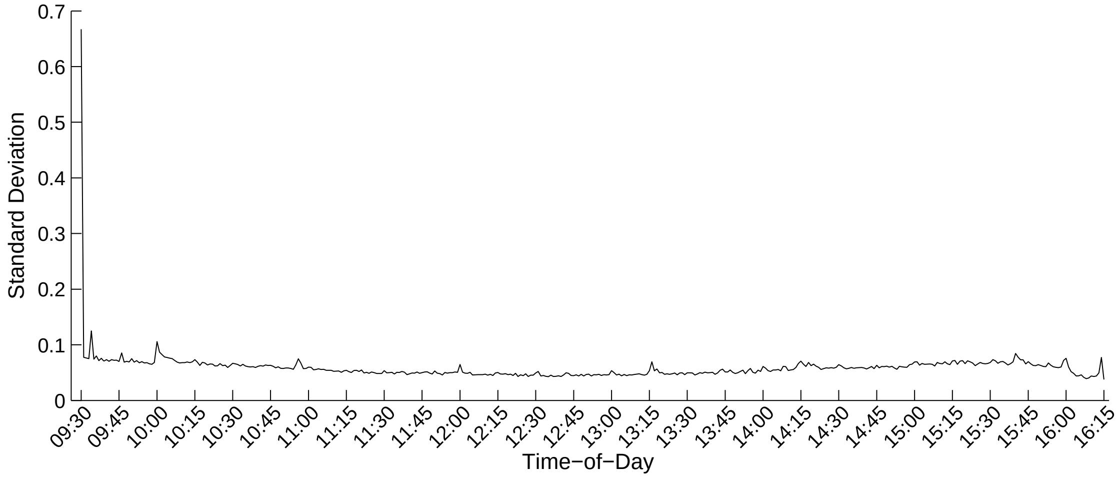


Figure D.1B

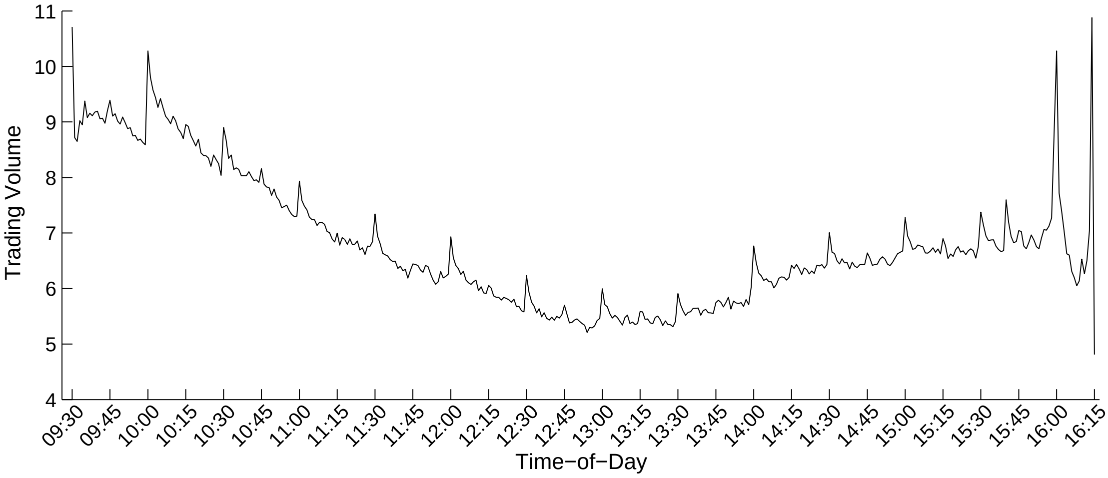


Figure D.1: **Price volatility and trading volume.** The standard deviations of one-minute price changes and the means of one-minute trading volumes, which are proxied by the number of ticks in one minute intervals, at the same time interval across all 3003 trading days from February 3, 1997 to January 30, 2009 are shown in Figures [D.1A] and [D.1B], respectively. The one-minute price changes are the calculated values times 10^2, and the times shown on the horizontal line are the interval ending times.

Figure D.2A

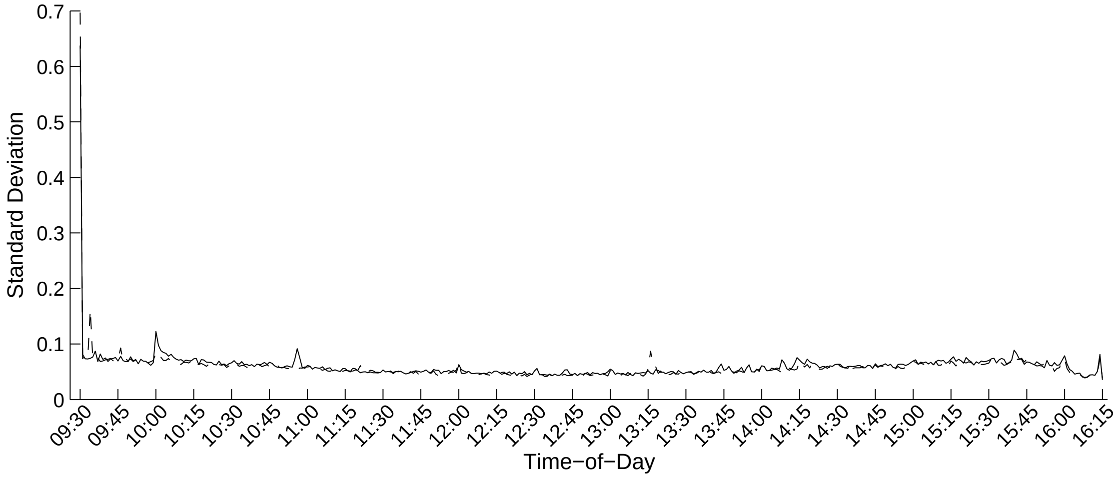


Figure D.2B

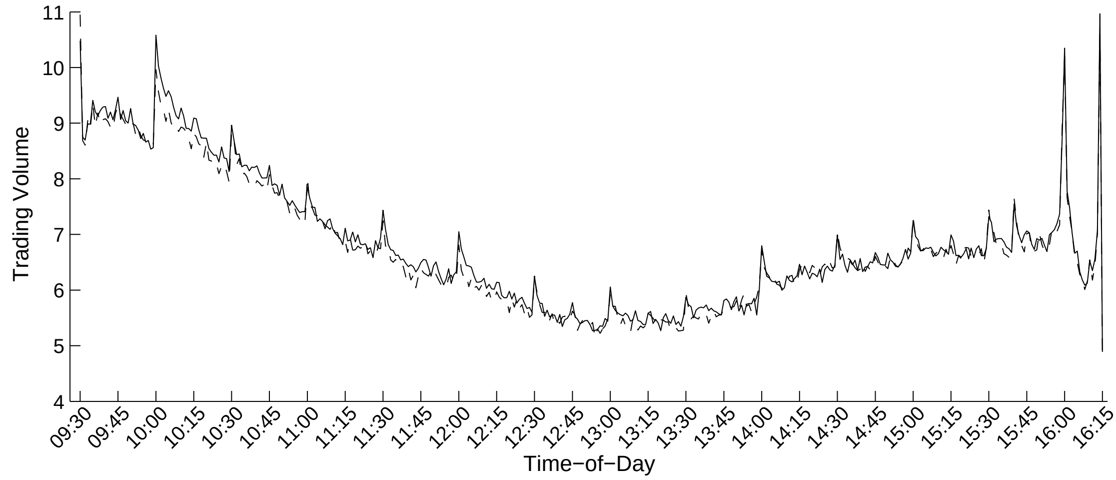


Figure D.2: **Price volatility and trading volume on announcement and nonannouncement days.** The standard deviations of one-minute price changes and the means of one-minute trading volumes, which are proxied by the number of ticks in one minute intervals, are reported for days with at least one of the seventeen announcements (solid line) and days with none of these announcements (dashed line) in Figures [D.2A] and [D.2B], respectively. The one-minute price changes are the calculated values times 10^2, and the times shown on the horizontal line are the interval ending times.

Figure D.3A

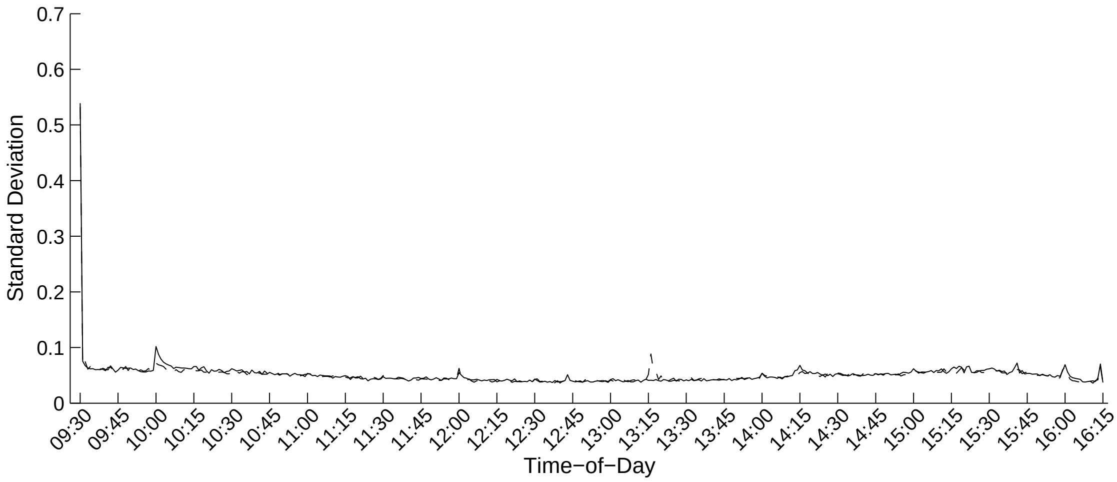


Figure D.3B

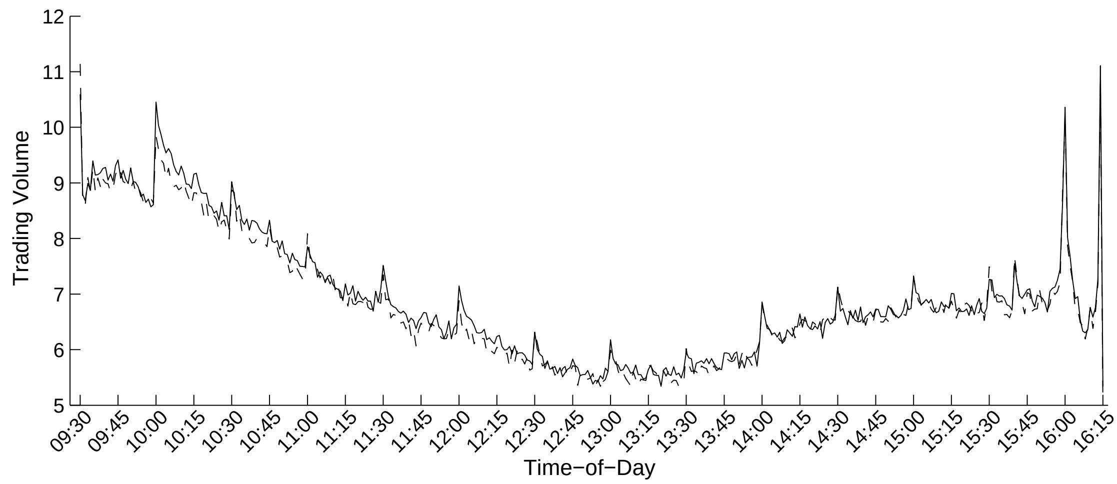


Figure D.3: **Price volatility and trading volume on announcement and nonannouncement days in the expansion period.** According to the NBER business cycle, trading days from February 3, 1997 to March 30, 2001 and from December 3, 2001 to December 31, 2007 are the expansion period. The standard deviations of one-minute price changes and the means of one-minute trading volumes, which are proxied by the number of ticks in one minute intervals, are reported for days in the expansion period with at least one of the seventeen announcements (solid line) and days in the expansion period with none of these announcements (dashed line) in Figures [D.3A] and [D.3B], respectively. The one-minute price changes are the calculated values times 10^2, and the times shown on the horizontal line are the interval ending times.

Figure D.4A

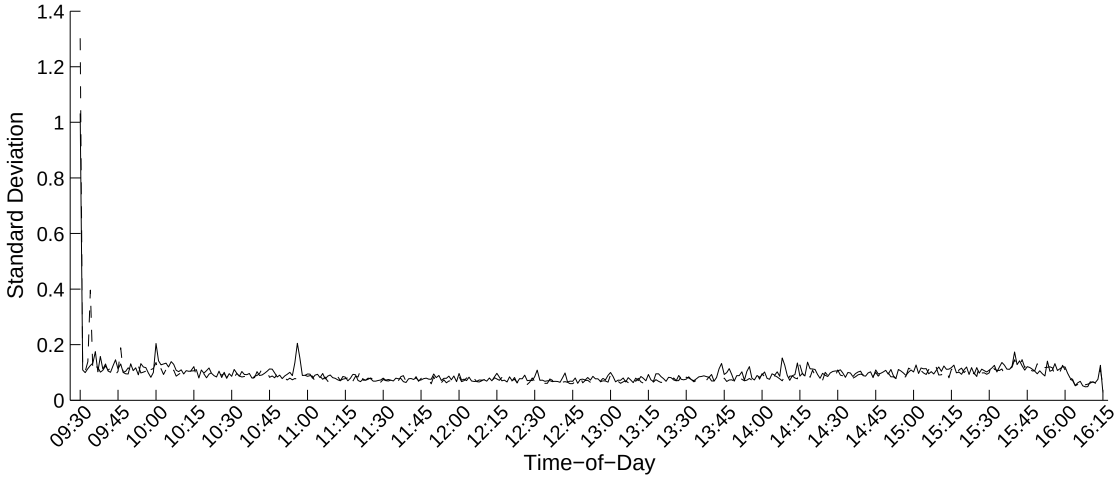


Figure D.4B

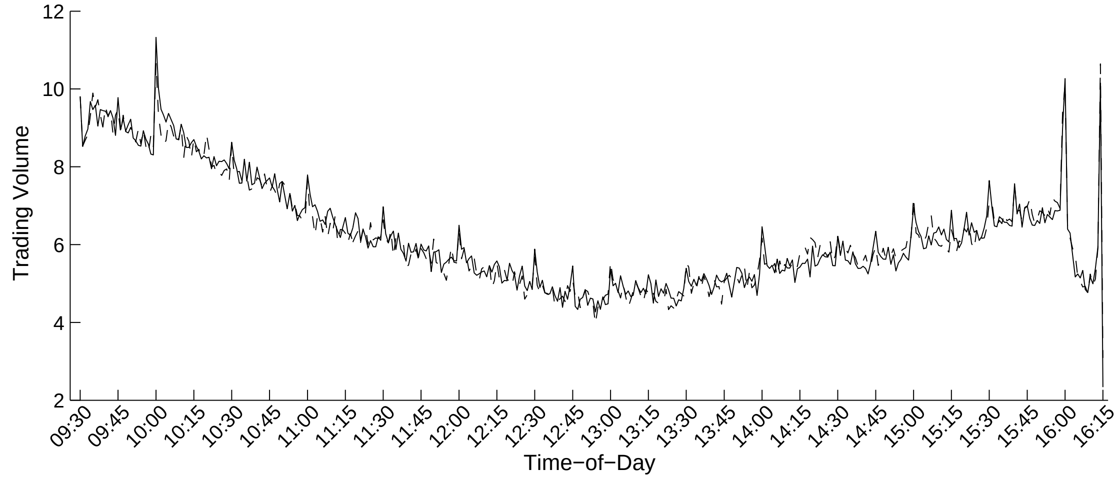


Figure D.4: **Price volatility and trading volume on announcement and nonannouncement days in the contraction period.** According to the NBER business cycle, trading days from April 2, 2001 to November 30, 2001 and from January 2, 2008 to January 30, 2009 are the contraction period. The standard deviations of one-minute price changes and the means of one-minute trading volumes, which are proxied by the number of ticks in one minute intervals, are reported for days in the contraction period with at least one of the seventeen announcements (solid line) and days in the contraction period with none of these announcements (dashed line) in Figures [D.4A] and [D.4B], respectively. The one-minute price changes are the calculated values times 10^2, and the times shown on the horizontal line are the interval ending times.

Figure D.5A

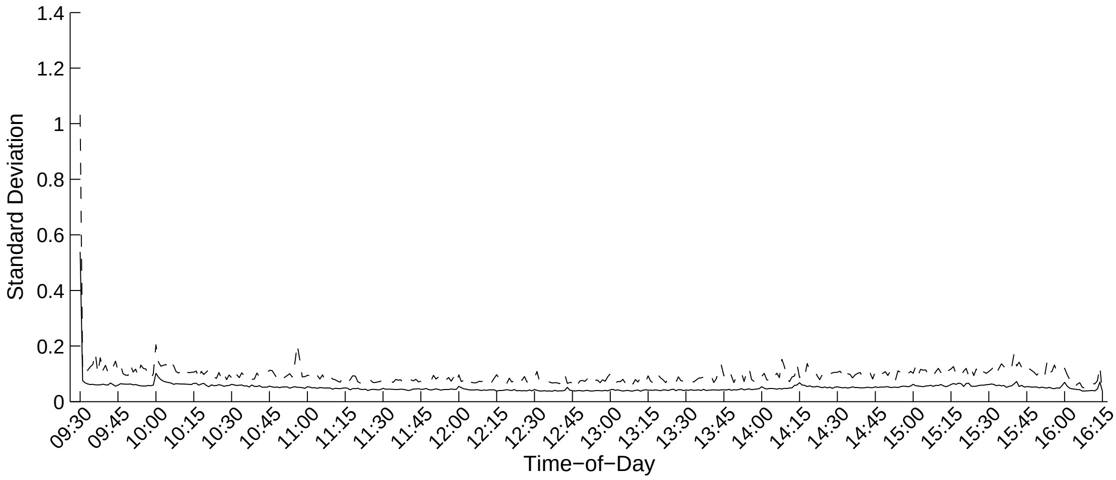


Figure D.5B

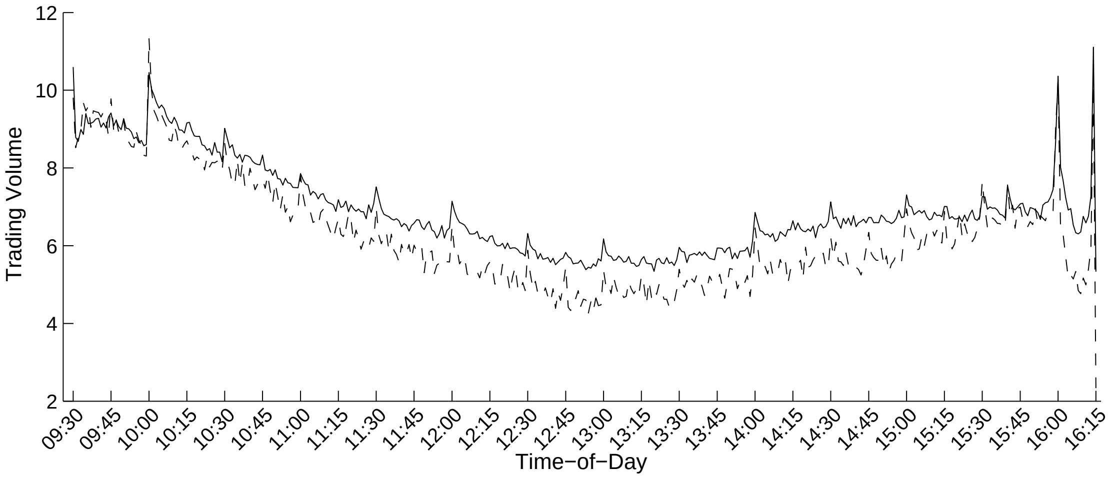


Figure D.5: **Price volatility and trading volume on announcement days in the expansion period and in the contraction period.** According to the NBER business cycle, trading days from February 3, 1997, to March 30, 2001, and from December 03, 2001, to December 31, 2007, are the expansion period, and trading days from April 2, 2001, to November 30, 2001, and from January 2, 2008, to January 30, 2009, are the contraction period. The standard deviations of one-minute price changes and the means of one-minute trading volumes, which are proxied by the number of ticks in one minute intervals, are reported for days in the expansion period with at least one of the seventeen announcements (solid line) and days in the contraction period with at least one of the seventeen announcements (dashed line) in Figures [D.5A] and [D.5B], respectively. The one-minute price changes are the calculated values times 10^2, and the times shown on the horizontal line are the interval ending times.

Figure D.6A

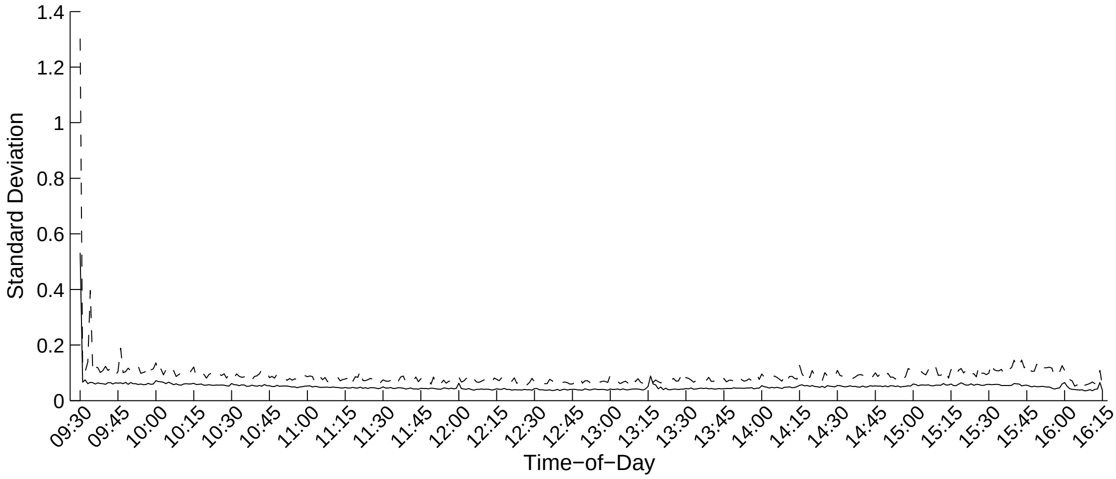


Figure D.6B

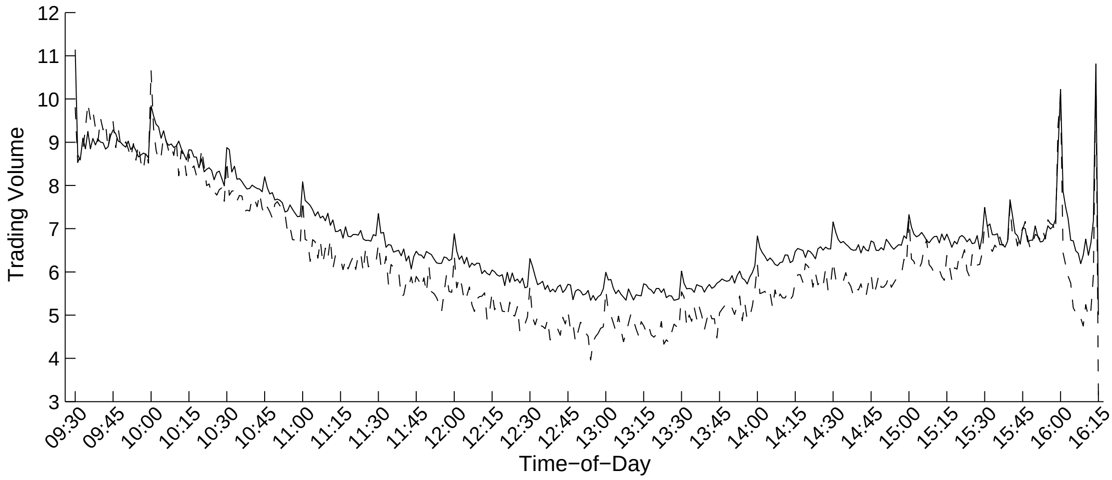


Figure D.6: **Price volatility and trading volume on nonannouncement days in the expansion period and in the contraction period.** According to the NBER business cycle, trading days from February 3, 1997 to March 30, 2001 and from December 3, 2001 to December 31, 2007 are the expansion period, and trading days from April 2, 2001 to November 30, 2001 and from January 2, 2008 to January 30, 2009 are the contraction period. The standard deviations of the one-minute price changes and the means of the one-minute trading volumes, which are proxied by the number of ticks in one minute intervals, are reported for days in the expansion period with none of seventeen announcements (solid line) and days in the contraction period with none of seventeen announcements (dashed line) in Figures [D.6A] and [D.6B], respectively. The one-minute price changes are the calculated values times 10^2, and the times shown on the horizontal line are the interval ending times.

Figure D.7A

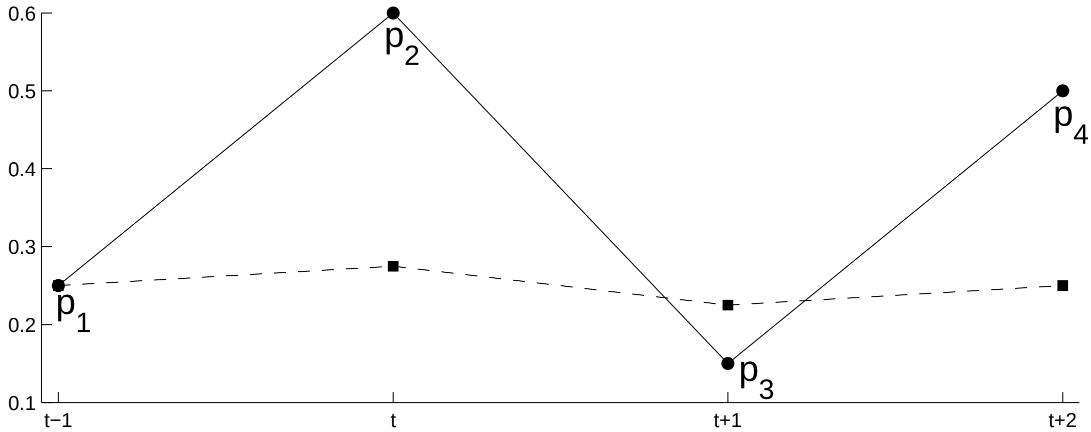


Figure D.7B

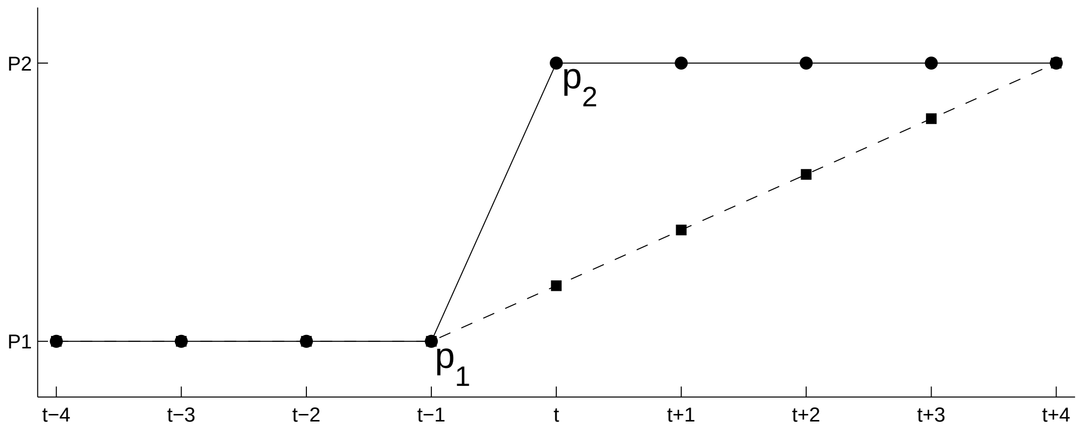


Figure D.7: **The difference between static price changes.** The times shown on the horizontal line are the interval ending times. A news announcement is released at time t. Price changes with and without the effect of a news announcement are shown by the solid and dashed lines, respectively, in Figure [D.7A]. Under the impact of a news announcement at time t on the market, price changes in the first scenario and in the second scenario are drawn with a solid line and a dashed line, respectively, in Figure [D.7B].

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