APPLICATION OF ANTI-WINDUP TECHNIQUES TO THE CONTROL OF WAVE ENERGY CONVERTERS

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Abstract

This thesis considers control system enhancement for Wave Energy Converters (WECs), of the *point-absorber* type, used for water desalination. The thesis makes several contributions.

Firstly, it is shown that a type of nonlinear control system previously used in the literature provides global stability guarantees for this type of WEC in the absence of input constraints.

Following this, several anti-windup techniques for a certain class of nonlinear systems with input constraints are developed; a nonlinear Internal Model Control (IMC) compensator, a linear reduced-order compensator and a linear sub-optimal performance compensator. It is shown how these anti-windup strategies are natural generalisations of those found elsewhere in the literature and how all of these compensators can be designed such that global exponential stability of the class of systems considered is guaranteed.

Finally, the thesis describes the application of these anti-windup techniques to a nonlinear simulation model of a WEC system where their benefits are clearly demonstrated. It is shown that these compensators improve the performance of the WEC during periods of saturation and, moreover, that the sub-optimal compensator can achieve desirable tracking without causing any damage to the desalination equipment. These results demonstrate the benefit of anti-windup for WEC control and imply potential savings in terms of operation and maintenance costs, thereby contributing to the potential commercialisation of such devices.

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CHAPTER 1

Introduction

1.1 Introduction to wave energy

Initial interest in wave energy developed during the oil crisis of 1973. Due to the high price of oil and the lack thereof, governments begun funding research on harnessing wave energy as an alternative source of energy. The first attempts of constructing devices that could harness wave energy, namely *Wave Energy Converters* (WECs), led to bulky, expensive devices that made electricity generation an expensive procedure, thus turning away governmental funding in the early 80's (Thorpe 1999). The interest in wave energy was reignited in the mid-90's as part of the aim to reduce carbon emissions according to the Kyoto protocol (United Nations 1998) and it was around the same time other forms of renewable energy sources were emerging too.

Wave energy is quite dense in locations that lie between the 30° and 60° latitudes of both hemispheres, due to the characteristic west winds of these areas (Buiges et al. 2006). Looking at Figure 1.1 one can see the west Irish and Scotish coasts, along with Australia, New Zealand and the tip of South America, have some of the best wave climates in the world (Nolan 2006), thus making the UK specifically a very keen player in the research and application of wave energy technology. According to (Esteban et al. 2011), "by the year 2050 the UK offshore renewable sector could produce between 127 and 146 TWh of electricity, which is equivalent to around 57 - 66% of the current energy consumption in the country".

Wave energy is mainly used for two purposes: the production of electricity and



Wave power levels are approximate and given as kW/m of wave front

Figure 1.1: Global wave power levels (Thorpe 2003).

the production of potable water. The majority of the academic community has focused on the electricity production problem, with the production of potable water receiving significantly less attention (Nolan and Ringwood 2006, Sharaf and El-Sayed 2009, Ramudu 2011). Other uses include nutrient rich cold water for mariculture and hydrogen by electrolysis (Buiges et al. 2006), pumping of clean sea water (fish farms, cleaning of contaminated lagoons and other sea areas with insufficient water circulation), heating of sea water (e.g. for fish farms and swimming pools), propulsion of vessels and refrigeration of plants.

1.2 Wave Energy Converter (WEC) technologies

Harnessing wave energy has proven to be a nontrivial task, mainly because of the unpredictability of the waves themselves and the harsh environment the devices must operate in. This is reflected in the research community as well as in industries, where there are many different designs of WECs. Despite the fact that the first design of such a device dates back to the 18^{th} century (Girard 1799), there is not a universally adopted method of capturing wave energy. To date there have been identified more than 100 WECs at various design stages (Falcão 2010).

WECs are categorised in many different ways: usually either with respect to their position in the sea (nearshore or offshore), their size and orientation or with respect to their mode of operation (oscillating bodies, overtopping devices, floating bodies, multi-bodies). An extensive review of all the technologies currently available can be found in (Falcão 2010, Ringwood 2006).

1.2.1 Operation principles of a typical point-absorber WEC

The type of WEC examined in this thesis is a point-absorber heaving¹ buoy thought to be the most cost-efficient technology available to date to extract wave energy (Li and Yu 2012). The heaving buoy is usually an offshore axi-symmetric device, which consists of a single buoy oscillating relative to a fixed reference of the sea bed (Figure 1.2). Its vertical oscillation is the main source of power, however in practice, perfect heave motion is difficult to obtain unless the device is constrained to vertical motion only (Ringwood 2010).



Figure 1.2: Basic heaving buoy configuration.

An ideal wave energy device would capture all the power in the waves that it interacts with (Carbon Trust 2006a). Some devices absorb a lot of energy over a narrow band and very little energy outside this band, while other devices absorb more energy over a broader band, but less energy at particular frequency within this band (Carbon Trust 2006b). The dimensions of point-absorbers are small compared to the wavelength of the sea waves, hence they have a narrow energy capture bandwidth² (Tedeschi and Molinas 2010), however, they have the ability to focus energy onto themselves, by radiating waves that partially cancel the incoming waves (Entec UK Ltd. 2005).

As point-absorbers are characterised by axial symmetry they become omnidirectional³

¹ Heaving motion is the vertical motion of a buoyant body (Entec UK Ltd. 2005).

 $^{^2\,}$ The range of frequencies over which a device captures energy is called its bandwidth (Carbon Trust 2006b).

³ Omnidirectional buoys can absorb energy equally from all directions.

devices, so wave directionality is unimportant with respect to their ability to absorb energy (Entec UK Ltd. 2005). Due to their narrow bandwidth nature, they are designed so as to resonate - i.e. the frequency of the waves is close to the device natural frequency⁴ of oscillation and hence moves with larger amplitudes than the waves themselves. This feature is useful to maximise the amount of power that is available for capture (Carbon Trust 2006c) and can be achieved through some form of active control, otherwise the WEC will not work efficiently. In addition their size can be very small and still generate a lot of power (Entec UK Ltd. 2005).

Since the wave frequency changes over time, the device natural frequency must also change in order for the device to resonate continuously. Changing a device's natural frequency is known as tuning, and may involve adjusting its size, shape, mass, stiffness or damping, or some combination of these (Carbon Trust 2006b).

The body of the WEC (the heaving buoy) is coupled to a Power Take-Off unit (PTO). The function of the PTO system is to convert the energy harnessed by the buoy into fluid energy, which is further converted into electrical energy or it is used to produce potable water. A PTO's primary function is to provide hydraulic damping that tunes or de-tunes the WEC and in addition, it encapsulates the necessary control equipment/machinery to perform the rest of the control objectives depending on the application (e.g. adjust the load-to-controller power extraction, control the quality of the power e.t.c. for electricity generation (Entec UK Ltd. 2005) or maintain a constant pressure or adjust the flow through a valve for water desalination).

PTO systems for point-absorbers are generally either hydraulic or electrical. A hydraulic PTO comprises of a hydraulic circuit that converts the motion of the device into pressurised oil flow by opposing the motion of the oscillating buoy. Electrical PTOs do oppose the motion of the oscillating buoy, however they convert the energy captured by this motion into force. In the WEC studied in this thesis, the PTO used is hydraulic and will be described in more detail in Section 3.1.3.

Lately, there is an increasing interest in developing direct linear generators that will be coupled to the point absorber and will allow for direct conversion of the mechanical energy captured from the waves to electrical. This has the obvious

⁴ The natural frequency or *eigenfrequency* of an oscillating system is the frequency of forcing that gives the highest response from the device (Entec UK Ltd. 2005).

benefits of improved conversion efficiency, as an energy conversion stage is omitted (conversion of mechanical to flow and then to electrical energy), however, there is no energy storage and this can be detrimental to the grid, since wave energy and thus electricity production is intermittent and fluctuating.

1.2.2 Wave energy for producing potable water

As mentioned earlier, most of the research performed and the WEC prototypes developed have focused on producing electricity. However, an alternative application studied in this thesis is the production of potable water, which can prove extremely beneficial in many parts of the world where direct access to potable water is difficult or non-existent and it can be done with relatively low cost (Thorpe 1999).

Wave-powered desalination appears particularly attractive since both of the primary requirements, seawater and energy, are available in abundance at the same location. Moreover, the desalination of seawater can be of great importance in arid areas, in developing countries with coastlines and can also contribute to the drinking water independence of islands around the world. In (Cruz 2008), it is also pointed out that renewable powered desalination plants can act as energy storage devices that allow for more use of the renewable energy power at hand and also overcoming the problem of the weak grid integration of renewable energy as they prevent energy fluctuations.

The system used and studied in this thesis is an autonomous wave-powered desalination system, i.e. the system is powered solely by waves. The production of potable water from sea waves is achieved through the coupling of a buoy with a PTO enclosing a Reverse Osmosis (RO) desalination unit (Nolan 2006). Reverse osmosis is a popular desalination procedure used to produce potable water despite the fact that it is very a energy intensive procedure. In recent years people have incorporated energy recovery technologies in water desalination plants to increase their efficiency, i.e. the water not desalinated the first time round is recycled and used again - this can be up to 60% of the initial water volume (Cruz 2008).

As seen in Figure 1.3, the semi-permeable membrane separates the two water volumes. Normally, water will flow from the low salt concentration side to the

high salt concentration side; this is the process of *osmosis*. However, this flow can be stopped or reversed by applying an external pressure (P_{ro}) on the high salt concentration side (*reverse osmosis*). The pressure that stops the flow of water from the low salt concentration side to the high salt concentration side is called the *osmotic pressure* (P_{osm}) and it has to be exceeded to achieve reverse osmosis (Nolan 2006). The osmotic pressure for sea water is usually 25 - 28 bar (Cruz 2008).



Figure 1.3: The process of osmosis (a) and the process of reverse osmosis (b) (Nolan 2006, page 17).

Applied	Semi-permeable
Pressue	membrane
Salt	Fresh
water	water

Figure 1.4: Desalination through reverse osmosis (Nolan 2006, page 18).

The RO membranes must operate between a minimum and maximum water flow and below a maximum pressure in order to prolong their lifespan, which is typically 2-5 years (Cruz 2008). Manufacturers also suggest to avoid pressure fluctuations to prevent membrane fatigue (Cruz 2008).

1.3 Control of Wave Energy Converters

As mentioned earlier, without some form of active control, WECs are essentially passive devices which would perform inefficiently due to the varying behaviour of the sea. The main aim of an active control system, is to improve the efficiency of the devices and to enable them to function well in different sea states. Active control systems have various objectives, including the maximisation of energy extraction from the waves, the minimisation of damage to the device due to large waves, the optimisation of energy conversion in PTO systems, the sizing and configuration of WECs and the coordination of WEC arrays (Ringwood 2010, Brekken et al. 2011).

When designing a WEC there are some stages established that WECs have to go through before they can be rendered commercially viable (Figure ??).



The problem with this design procedure is the fact that control engineers are not involved in the WEC design until later stages (i.e. stage 2, WEC scale 1:10-25). As a consequence, the engineers involved in the earlier design stages often do not design WECs with "good" control properties in mind (such as stability), which are critical to the device performance, but rather on cost and size considerations. Usually, this results in a quite challenging task for the control engineer to tackle, since nonlinearities and constraints (due to the hydraulic pipes for water regulation, PTO machinery, e.t.c.) appear on the system which needs to be controlled.

Most of the literature concentrates on the energy maximisation problem in WECs, which effectively is an *optimisation* problem of constantly calculating the optimal damping the device needs to receive from the PTO in order to resonate. Although many control designs/approaches have been proposed for electricity production, the majority of them are based on the following two assumptions: firstly, their analysis is based on linear wave theory and secondly, on linear PTOs.

In mathematical descriptions of oscillating systems the term *linear* means that all oscillating variables are sinusoidal and proportional to the wave height, which often implies small motions or amplitudes (Entec UK Ltd. 2005). This is in contrast with the resonance condition necessary for maximum energy absorption, so even though the controllers are designed based on small signal operation, in fact they have to operate under large signal conditions (large amplitudes).

Among the first approaches for maximising power generation is the approach known as *reactive* or *complex conjugate* control (Falnes 2002), which maximises energy capture by creating destructive interference⁵ between the properties of the pointabsorber and the incident waves. This method has a theoretical optimal resonance condition, which results in large amplitudes and large bidirectional energy transfer to and from the PTO mechanism. The method's main drawbacks are the inability to handle physical constraints and its nonapplicability to systems with nonlinear PTO.

An alternative approach, known as *latching* or *phase control*, is a nonlinear control approach in which the point absorber is held⁶ for approximately one quarter of the incoming wave, allowing the force of the WEC to reach high levels, and then it is released (Nolan and Ringwood 2006, Brekken et al. 2011, Ringwood and Butler 2004). Latching controllers do not take into account any constraints; moreover, they result in extreme amplitude oscillations (Nolan 2006, page 176) making their actual implementation impractical.

Another approach is the so-called *passive loading* (Santos et al. 2011) where the energy capture is not maximum, but the power flow is unidirectional. More recently, researchers have begun to use *model predictive control* (MPC) for WEC control, which allows constraints involved with the WEC (position, velocity or force constraints) to be incorporated directly into the controller design (Hals 2010, Brekken 2011, Cretel et al. 2011).

⁵ The combination of two waves such that one of them in some part cancels the other (Entec UK Ltd. 2005).

⁶ One might hold the buoy lower in the water for part of the incoming wave than it would naturally assume under wave action alone (Entec UK Ltd. 2005).

For WEC water desalination however, the nature of the problem differs. The damping in this case is fixed (thus not allowing energy maximisation) and the focus of the control system lies in maintaining a constant pressure on the reverse osmosis unit in order to ensure efficient desalination (higher pressures give better efficiency) and avoid damage to the RO membrane (as a result of excessively high pressures). The analysis in this case is based on nonlinear (hydraulic) PTOs that exhibit some saturation effects during operation (Nolan 2006).

A solution which allows both objectives to be followed involves the use of a recently proposed PTO system (Schlemmer et al. 2011), which effectively couples the RO unit (or electrical power generation) from the PTO input side, where optimum power transfer is effected. While this topic has not been treated extensively in the literature, it has attracted increased attention recently (Nolan 2006, Magagna et al. 2009, Bacelli et al. 2009).

One notable drawback with much of the current literature is that although researchers have been keen to propose new control schemes for WECs, the formal analysis of such schemes is, with a few exceptions, e.g. (Orazov et al. 2012), largely absent. In particular, it is necessary to examine the stability properties of these schemes, because the WECs are often nonlinear and also because some control schemes themselves have nonlinear elements. Although the physical properties of most wave energy devices may allow bounded-input-bounded-output (BIBO) stability to be inferred, internal stability cannot be taken for granted⁷ and may not always be trivial to prove (Orazov et al. 2012). Such analysis also has practical implications because closed-loop internal stability will prevent limit cycles and other strange behaviour occurring in WECs, and thus lead to control schemes which, when implemented on real WECs, bestow safe and reliable operation in varying conditions (Hals 2010).

In recent years, other forms of WEC control have emerged, such as robust control, i.e. the study of survivability properties during extreme weather conditions and fault-tolerant control, i.e. the controller has the ability to operate safely or shut down if necessary (Hals 2010).

⁷ Internal stability implies BIBO stability, but BIBO stability does not necessarily imply internal stability.

1.4 Input-constrained systems

As mentioned above, a formal analysis of WEC control schemes requires one to take account of the input constraints on the actuators. Input saturation is ubiquitous in practice as it usually arises due to restrictions introduced on purpose on actuators to prevent damages to the system or due to physical limitations of actuators. In general, saturation is difficult to handle, because although its small signal behaviour is equal to the identity operation, its large signal behaviour cannot be linearised or inverted (Turner et al. 2007a).



Figure 1.5: Input saturation of a linear system.

The most prevalent problem regarding saturation is that even if a linear plant G is considered with input saturation and a linear controller K designed to perform well, once the saturation becomes active the system can exhibit severely degraded performance and sometimes loss of stability, even in cases where the controller has been designed with large robustness margins (Turner et al. 2007a). This is because when saturation occurs the system behaves open-loop and therefore the open-loop system dynamics have a large effect on the system stabilisation properties.

As mentioned earlier in Section 1.3, most controllers designed for WECs exhibit the following common characteristics/disadvantages: linear wave theory is assumed (i.e. system stays in the small signal vicinity) and no stability properties are examined. In reality though, the real sea waves are irregular and usually WECs are forced to operate outside their linear region, that is, in a large signal area where several linear assumptions made during the design stages of these control schemes no longer hold. For instance, saturation cannot be ignored in a large signal setting and its effects on the system can be detrimental. In addition, these systems might be stable in the linear setting, but their stability properties in a nonlinear setting cannot be guaranteed without some rigorous formal analysis.

When designing a control system that needs to account for saturation there are two routes a designer can follow (Barbu et al. 2005, Tarbouriech and Turner 2009):

1. Saturation handling: The controller is designed so as to accommodate the constraints/saturation problem directly, usually trading some performance for stability guarantees (nonlinear techniques are usually employed in this case). In this particular WEC control problem, one-step saturation handling control laws such as optimal control where the resulting system is bilinear and the cost function is non-convex quadratic have been used (Abraham and Kerrigan 2012, Li et al. 2012). Also, nonlinear Model Predictive Control (MPC) has been used (Richter et al. 2013), which is again a non-convex problem and poses a challenge with respect to the stability properties of the system (i.e. finding a global optimum) and its numerical solution. In addition, nonlinear MPC can be computationally quite expensive.



Figure 1.6: One-step saturation handling architecture

Another hurdle in the use of MPC in the wave energy problem dealt with here is the fact that MPC needs to have information about the future waves ahead of time. Wave prediction is currently a very active area of research (Fusco and Ringwood 2010; 2012).

Other types of controllers that could be used on a switching nonlinear system like the WEC introduced earlier could be *variable structure* controllers (VSC) for example *sliding mode* control and *bang-bang or hysteresis control*. Nevertheless, sliding mode controllers are known for producing excessively chattering control signals, something that would surely wear out the WEC actuators. Similar, bang-bang controllers would again cause actuator wear since the control signal would be switching between the upper and lower saturation limits.

2. Saturation compensation: The controller is designed (usually using linear techniques) to perform as desired and then an additional compensator is

inserted in the unconstrained controller to deal with the saturation present in the system ensuring stability and ideally recovering nominal unconstrained behaviour. This extra compensator is known as *anti-windup* (AW) compensator and it can be linear or nonlinear.



Figure 1.7: Two-step saturation handling architecture.

One should note that in the first case the controller must be completely re-designed to account for saturation, whereas in the second case the nominal controller already exists or it is designed to perform well when no saturation occurs.

Since the control design of WECs is highly model-based, there is a need for an integrated solution to the input-constrained problem. Anti-windup compensators can be a good candidate in this case, as they can be fitted retrospectively without altering the nominal controller. In addition, they are easy to design and implement - at least in the linear setting - and they provide an *a priori* insight regarding the stability guarantees of the closed-loop system. In general, it is common to encounter control problems where many years of experience have gone into the development of a small signal controller (as the ones usually encountered in WECs) and an augmentation of such a controller is desired to handle the effects of input saturation that appear occasionally (Turner et al. 2007a, Grimm et al. 2003a).

The use of AW compensation in the control of WECs could give the possibility to control designers to interact with the WEC designers at the beginning, especially in choosing the appropriate sizing of components, as this is a trade-off between cost and operational efficiency (Nolan 2006, page 244). Handling saturation through AW compensation could lead to a reduction in the size, mass and overall cost of various components (e.g. actuators, sensors, e.t.c.).

1.5 Anti-windup compensation

Designing an AW compensator is a two-step procedure. The first step is to design a linear controller K such that the closed loop is stable and behaves in a desirable manner when there is no saturation (good *small signal* behaviour) (Turner et al. 2007a). The second step involves the design of an extra (usually linear) element Θ such that it becomes active and improves the stability of the system when saturation occurs (improvement of *large signal* behaviour) (Turner et al. 2007a). Consider the block diagram of Figure 1.8 representing a general AW setup



Figure 1.8: Architecture of the anti-windup approach.

where G is the plant, K is the nominal controller and Θ is the anti-windup compensator, which is activated upon saturation occurring.

Anti-windup compensator design has advanced significantly over the past decade or so, with many techniques now available for rigorous analysis and systematic design. The literature is replete with papers on the subject and the interested reader is referred to the surveys (Tarbouriech and Turner 2009, Galeani et al. 2009) or the books (Glattfelder and Schaufelberger 2003, Hippe 2006, Zaccarian and Teel 2011, Tarbouriech et al. 2011), and the references therein for more details. Much of the literature treats the problem of anti-windup design for systems with linear plants and linear controllers. This is appealing because, in the linear setting, anti-windup design and analysis can typically be conducted using tools from convex optimisation (Grimm et al. 2003a, Mulder et al. 2001, Saeki and Wada 2002) or frequency domain absolute stability theory (Wu and Jayasuriya 2001, Kerr et al. 2007; 2011). Moreover, many physical systems can often be approximated as linear systems, at least around a given operating point.

There are of course systems which are not easy to approximate with linear models and, for these systems, anti-windup design becomes more complicated, with accompanying stability and performance properties more difficult to ensure. It seems unlikely that one anti-windup compensation method will be satisfactory for all varieties of nonlinear systems, so researchers have focused their attention on particular classes of nonlinear systems and they have developed dedicated anti-windup procedures for each class.

One of the first papers to rigorously and systematically treat anti-windup synthesis for a class of nonlinear systems was (Morabito et al. 2004) in which the method of (Teel and Kapoor 1997) was adapted for anti-windup design in Euler-Lagrange systems. Work on anti-windup for nonlinear systems under adaptive control was reported in (Kahveci et al. 2008) (and references therein); the work there essentially sought to modify linear anti-windup techniques (Sofrony Esmeral et al. 2010) to adaptive systems.

A differential algebraic representation of nonlinear systems has been proposed by (Coutinho et al. 2004) and applied to the anti-windup problem for nonlinear systems (Oliveira et al. 2011). The attractive feature of (Oliveira et al. 2011) is that various concepts used in linear anti-windup are generalised but the transformation proposed is rather complicated and the anti-windup synthesis conditions are typically non-convex. Furthermore, recent results on constrained control for systems with sector bounded nonlinearities have also been specialised to anti-windup design (da Silva Jr. et al. 2008, da Silva Jr. and Turner 2012).

In addition to the approaches above to nonlinear anti-windup design, a number of papers have appeared on anti-windup design for feedback linearisable systems under Nonlinear Dynamic Inversion (NDI) control (Doyle III 1999, Kapoor and Daoutidis 1999, Herrman et al. 2006; 2010, Valmórbida et al. 2011). The work in (Herrman et al. 2010) showed that, for a particular class of nonlinear systems under NDI control, an anti-windup scheme, exhibiting intuitive parallels to the attractive linear scheme of (Weston and Postlethwaite 2000), could be devised. Although the details of the proof differed, (Herrman et al. 2010) showed that the decoupled scheme appearing in (Weston and Postlethwaite 2000) was effectively preserved, suggesting natural generalisations of the linear scheme's performance properties. Although the results

of (Herrman et al. 2010) are fairly powerful, they only apply to the class of nonlinear systems in which the nonlinearity is Lipschitz, and appears in the *range space* of the system's input distribution matrix. Moreover, in general, anti-windup compensator construction is dependent on the solution of a partial differential inequality.

1.6 Motivation & Objectives

The work in this thesis is motivated by regularly occurring saturation problems (Nolan 2006, page 231) for a wave energy converter used for potable water production. Controlling the WEC was reduced down to maintaining a constant pressure regulation on the RO desalination unit.

Since the nominal unconstrained controller was working efficiently most of the time, a natural approach to tackle any saturation effects would be the application of anti-windup compensation theory. This was in fact also implied by the developer of this control law, as "both the reduction in pressure regulation performance and the possibility of system instability are a direct result of integral windup" (Nolan 2006, page 230).

The main objectives of this thesis are to provide a deeper analysis of the stability properties of the control law designed in (Nolan 2006) and to examine how the controller can be augmented with anti-windup compensation in order to prove stability in the face of actuator saturation. The approach adopted will provide rigorous stability guarantees using Lyapunov's method.

1.7 Contributions

The work carried out in this thesis resulted in the following contributions:

1. The model of the WEC used in this thesis is based on the widely used pointabsorber model found in (Nolan 2006, Falnes 2002, Eidsmoen 1995) (used by many researchers (Nolan 2006, page 77)) and the hydraulic PTO used in the famous McCabe WEC (McCabe 1992) used for water desalination. Global asymptotic stability has been proven for this type of nonlinear WECs.

- 2. The design of a nonlinear Internal Model Control (IMC) AW compensator for the aforementioned generic class of nonlinear WECs has led to the global exponential stability of the closed-loop system. This type of compensator can be applied on any given system, since IMC compensators are essentially a copy of the plant itself, and requires no optimisation for its implementation.
- 3. A different type of AW compensation has been designed for the same generic class of nonlinear WECs. This compensator has fewer states than the IMC one, it also requires no optimisation and despite the fact it is linear, it guarantees global exponential stability of the nonlinear closed-loop system. In addition, its linear nature makes its implementation easier and cheaper.
- 4. A third AW compensator has been designed similar to the reduced-order linear one, but also containing an extra feedback term. This extra term is making the implementation of the AW compensator a little more complex as a *Linear Matrix Inequality* (LMI) must be solved, but still this is a convex optimisation problem that can be solved trivially nowadays through the use of powerful software tools like Matlab[®] Robust Control Toolbox. This compensator not only guarantees global exponential stability of the nonlinear closed-loop system, but also provides performance improvement over the other compensators proposed.

The work presented in this thesis could act as a blueprint for other researchers when approaching the WEC wave desalination problem. Since the WEC model used consists of widely used subsystems, proving stability properties and designing AW compensation laws for a similar system will also be similar. Even when deploying such WECs in different locations, hence the WEC components' size and ratings would change, the approach to this particular problem could be similar.

1.8 Thesis outline

The thesis is organised as follows:

In Chapter 1, an introduction to wave energy is made and devices that harness this type of energy are introduced. The problem of controlling such devices is stressed and the use of anti-windup compensation to tackle saturation problems associated with the baseline system has been mentioned.

Chapter 2, introduces some background concepts from systems and stability theory. There is a brief introduction to nonlinear systems, followed by some stability theory according to Lyapunov, as well as some tools used to prove this kind of stability for nonlinear systems. An introduction to the basic operation principles of linear AW compensators has also been made.

In Chapter 3, the model of the WEC used in the thesis is presented. The WEC subsystems are analysed and presented in detail and the dynamics of the device are derived. In addition, the controller designed for this specific WEC in (Nolan 2006) is also presented. There are also some simulation results showing that the controller loses track of the reference and even leads the system to instability at times under saturation.

In Chapter 4, the stability properties of the controller mentioned in Chapter 3 are examined and global asymptotic stability of the closed-loop system is proven, despite the fact that it is a switching (nonlinear) controller. This work has been presented in the 2012 IEEE International Conference of Control Applications (Lekka et al. 2012).

Chapter 5 introduces a nonlinear IMC compensator, which complements the controller of Chapter 3. The proposed AW compensator ensures global exponential stability for the nonlinear closed-loop system.

Chapter 6 introduces another type of AW compensation. This compensator has a similar structure to the nonlinear IMC compensator proposed in Chapter 5, however is itself linear and also ensures global exponential stability of the nonlinear closed-loop system.

In Chapter 7, another type of AW compensation is introduced. The proposed compensator has similar structure to the linear one of Chapter 6; it has got an additional term that determines the rate at which the system will return to nominal conditions following saturation. The compensator is itself linear and its synthesis can be cast as a convex optimisation problem solved through an LMI. Simulation results show this compensator achieves the best performance, while ensuring global exponential stability (also proven in this chapter) at the same time.

Finally, Chapter 8 summarizes the main points of the work presented in this thesis, discusses its limitations and presents possible avenues for future work.

1.9 Publications

- Angeliki Lekka, Matthew C. Turner and John V. Ringwood, A class of globally stabilising controllers for the control of wave energy devices for potable water production, 2012 IEEE International Conference on Control Applications (CCA), pp. 1369-1373
- Angeliki Lekka, Matthew C. Turner and Prathyush P. Menon, *Full and reduced order IMC anti-windup compensators for a class of nonlinear systems with application to wave energy converter control*, 2013 American Control Conference (ACC), pp. 4861-4866

CHAPTER 2

Background concepts from systems and stability theory

This chapter introduces some concepts from systems and stability theory that the reader may find useful while reading about the work presented in subsequent chapters. To be specific, this chapter introduces selected elements of nonlinear systems theory, Lyapunov stability, input-output stability and some aspects of anti-windup design for linear systems.

The dynamics of the WEC system, to be introduced in Chapter 3, are inherently nonlinear and not easily linearised; therefore much of the analysis will make a direct appeal to the concepts presented here. In addition, the WEC anti-windup system will be built upon the linear AW approach reviewed here.

2.1 Nonlinear systems

Classical control theory is a mature field in control that has focused mainly on the analysis of linear systems. Recently however, there is an increased interest in studying nonlinear control techniques for application to real world systems, which are often themselves nonlinear.

The importance of studying nonlinear control techniques lies mostly in the analysis of "hard" nonlinearities, such as saturation, deadzone, e.t.c. These nonlinearities are called hard since their non-smooth nature does not allow them to be accurately represented by a linear approximation (which is often what happens when a system is linearised).

These nonlinear functions are often a cause of undesirable system behaviour (performance degradation, limit cycling, e.t.c.) or sometimes even instability, therefore, they need to be studied rigorously, so that their effects can be compensated properly.

A nonlinear time-invariant dynamic system can be represented by a set of first-order nonlinear ordinary differential equations of the form

$$G \sim \begin{cases} \dot{x} = f(x, u) \\ y = h(x, u) \end{cases}$$
(2.1)

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the input vector, f is a vector of general nonlinear functions, $y \in \mathbb{R}^l$ is the output vector and h is a vector of functions that may be nonlinear. A solution of Equation (2.1) is referred to as a state trajectory or system trajectory (Slotine and Li 1991) and gives complete information about the behaviour of the system (Cook 1994). The output vector y usually contains variables that can be physically measured or that require to behave in a particular manner (Khalil 2002).

2.1.1 Linear systems

Before moving further with the theory of nonlinear systems, it is interesting to review briefly the special case of nonlinear systems, widely known as *linear* systems. The study of such systems is interesting and useful for several reasons. Their mathematical analysis is tractable, linear approximations are widely available and used and most of the common control systems design methods are based on linear systems.

Linear time-invariant systems that are finite-dimensional are usually described by a set of first-order differential equations, i.e. their state-space representation

$$G \sim \begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases}$$
(2.2)

where $x \in \mathbb{R}^n$ is the vector containing all the states of the system, $u \in \mathbb{R}^m$

is the system input, $y \in \mathbb{R}^l$ is the system output and A, B, C, D are matrices of appropriate dimensions. Specifically, $A \in \mathbb{R}^{n \times n}$ is the system matrix representing the interconnection among the states, $B \in \mathbb{R}^{n \times m}$ is the input matrix representing the input-to-state direct connection, $C \in \mathbb{R}^{l \times n}$ is the output matrix representing the state-to-output direct connection and $D \in \mathbb{R}^{l \times m}$ is the coupling matrix representing the input-to-output direct connection or input/output coupling. The transfer function of the system described by Equation (2.2) is given by

$$G(s) = C(sI - A)^{-1}B + D$$
(2.3)

Linear systems satisfy the so called *superposition principle*, which is the satisfaction of two properties: the additivity and the homogeneity.

$$\begin{cases} f(x_1) + f(x_2) + \ldots + f(x_n) = f(x_1 + x_2 + \ldots + x_n) & \text{(additivity)} \\ \alpha f(x) = f(\alpha x) & \text{(homogeneity)} \end{cases}$$
(2.4)

Due to the superposition principle there is a large body of mathematical techniques that are applicable to linear systems, such as Fourier and Laplace transforms, linear operator theory, e.t.c.

2.2 Stability of nonlinear systems

In any given system one desires to control, the first and most important issue regarding this system is to explore its stability properties, as an unstable system is useless and potentially dangerous (Slotine and Li 1991), although rarely both. In linear systems the property of *superposition* holds and hence powerful analysis tools can be used. In nonlinear systems however, the superposition principle no longer holds and more advanced tools must be used for analysis.

A common approach in the analysis of nonlinear systems is to linearise the system around an equilibrium point and analyse the resulting linear system. However, with respect to the stability properties of a nonlinear system linearisation cannot give sufficient answers. This is due to the fact that linearisation is essentially an approximation around an operating point and can only give stability guarantees for this local area - it is impossible to ensure global stability throughout the whole state-space. Moreover, some nonlinear phenomena cannot be represented by a linear approximation and the behaviour of systems consisting of such phenomena can vary greatly depending on the nature of the input and/or the initial conditions.

It is thus imperative to study the use of techniques for proving stability of nonlinear systems. This thesis focuses on the use of techniques for proving stability according to Lyapunov; at the same time input-output stability is also presented as it plays a dominant role in the linear anti-windup compensation presented in Section 2.3.

2.2.1 Definitions of stability

As mentioned in Chapter 1, the focus of this thesis is on the study of input-constrained systems. The primary concern around such systems is to examine whether they are stable or not. As these systems are nonlinear, their stability properties are not always trivial to guarantee and any stability conditions given will be either necessary or sufficient or maybe both.

The system of Equation (2.1) is said to be globally asymptotically stable if for $u \equiv 0$

$$\lim_{t \to \infty} x(t) = x_0 \qquad \forall \ x(0) \in \mathbb{R}^n$$
(2.5)

where x(0) denotes initial condition. If possible, the equilibrium is chosen to be the origin of the state-space, i.e. $x_0 = x(0) = 0$, or if that is not the case the system equations can be transformed in such a way that any equilibrium point can be shifted to the origin of the state-space via a change of variables (Slotine and Li 1991, Khalil 2002).

A stronger¹ form of stability is the so called *global exponential* stability where the states decay back to the origin at a certain rate.

¹ Exponential stability is a stronger form of stability when nonlinear systems are concerned. In the case of linear systems the two forms of stability are equivalent.

$$x(t) \leq m e^{-\zeta t} ||x_0||, \quad \text{for some } m, \zeta > 0, \quad \forall x_0 \in \mathbb{R}^n$$

$$(2.6)$$

The constant ζ is called the rate of convergence or decay rate or maximum Lyapunov exponential and it is associated with the speed of response of a system.

2.2.2 Stability according to Lyapunov

A popular way of proving stability of nonlinear systems is to use Lyapunov techniques, their biggest advantage being that they allow the assessment of the stability of equilibrium points of a system without solving the differential equations that describe the system itself.

One of the main methods used to prove global asymptotic and exponential stability of nonlinear systems is the so called second or direct Lyapunov method.

To prove stability according to Lyapunov, the goal is to find an "energy-like" scalar function satisfying the following properties

$$V(x) > 0, \qquad \forall x \in \mathbb{R}^n$$
 (2.7)

$$\dot{V}(x) < 0, \qquad \forall x \in \mathbb{R}^n$$
 (2.8)

$$V(x) \to \infty$$
 as $||x|| \to \infty$ (2.9)

The conditions described above are *sufficient* to prove global asymptotic stability of the origin (Slotine and Li 1991).

A common category of energy-like functions used as Lyapunov function candidates in this thesis are the so called *quadratic* functions. These are a set of scalar functions of the form

$$V(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} P_{ij} x_i x_j = x' P x$$
(2.10)

where x is the state vector and $P \in \mathbb{R}^{n \times n}$ is a real symmetric positive definite matrix. P is said to be positive definite, if all its eigenvalues are strictly positive, i.e. if all its principal minors $(P_{11}, P_{11}P_{22} - P_{21}P_{12}, \dots, det P)$ are positive (Sylvester's theorem (Atherton 1981)). The derivative of Equation (2.10) along the trajectories of the linear system of Equation (2.2) is equal to (Khalil 2002, page 135)

$$\dot{V}(x) = x'P\dot{x} + xPx = x'(PA + A'P)x = -x'Qx$$
 (2.11)

where Q is a symmetric matrix defined by

$$PA + A'P = -Q < 0 \tag{2.12}$$

If Q is positive definite, the equilibrium of the system described by Equation 2.2 is asymptotically stable. Equation (2.12) is known as the *Lyapunov equation*.

In the case of linear systems, the existence of a quadratic Lyapunov function provides a necessary and sufficient guarantee for stability and is also computationally tractable. It is often the case that quadratic Lyapunov functions are chosen to prove stability of a system that has a significant linear part due to their simplicity and tractability, since stability using quadratic Lyapunov functions can be established independently of the nonlinear part.

There are many nonlinear systems which can be represented as a combination of a linear system G and a static nonlinear term $\phi(.) : \mathbb{R}^m \to \mathbb{R}^m$ in the feedback path (Figure 2.1).

The nonlinear term is usually a saturation or deadzone function, i.e. the type of systems studied in this thesis. It is assumed that systems represented in this way are *well-posed*, i.e. there is always a unique solution to the feedback equations.

In addition, systems represented in the way described above allows one to exploit the tools of *absolute stability* in order to guarantee stability of the nonlinear loop based mostly on information about the linear system G and some approximate information about $\phi(.)$. In particular, a standard assumption of absolute stability theory is for



Figure 2.1: System with a memoryless nonlinearity.

the nonlinear term $\phi(.)$ to belong to a certain sector.

A classical tool of absolute stability theory, which leads to tractable anti-windup (AW) synthesis schemes, is the *Circle criterion*. The main requirement for using this tool is for the nonlinearity $\phi(.)$ to be *sector-bounded*.

Definition 2.1. A memoryless nonlinearity $\phi(.) : \mathbb{R}^m \to \mathbb{R}^m$ is said to belong to the sector $[\alpha, \beta]$ where

$$\alpha = diag(\alpha_1, \alpha_2, \dots, \alpha_m) > 0, \quad \beta = diag(\beta_1, \beta_2, \dots, \beta_m) > 0 \tag{2.13}$$

if $\beta - \alpha > 0$ and the following inequality holds

$$[\phi(u) - \alpha u]'[\phi(u) - \beta u] \le 0 \quad \forall u \in \mathbb{R}^m$$
(2.14)

There are many nonlinear functions that satisfy the sector condition of Equation (2.14), but in this thesis the study is focused on the *decentralised* saturation and deadzone functions and the fact that they co-inhabit the same sector bounded by $\alpha = 0$ and $\beta = 1$. In such case, a nonlinear function $\phi(.)$ is said to belong to the sector [0, I] if the following inequality holds:

$$\phi(u)'W[u - \phi(u)] \ge 0 \quad \forall u \in \mathbb{R}^m$$
(2.15)

where W is some positive definite diagonal matrix.



Figure 2.2: Sector-bounded nonlinear function.

According to the *Circle criterion* then (Turner and Bates 2007), the system of Figure 2.1 is globally exponentially stable, if there exists a positive definite function V(x) > 0 such that

$$\dot{V}(x) + \phi(u)W[u - \phi(u)] < 0 \quad \forall \ x \neq 0$$
(2.16)

i.e. V(x) is a quadratic Lyapunov function.

As the system under consideration has a significant linear part to it, quadratic Lyapunov functions and the Circle criterion will be used extensively throughout this thesis to prove stability. However, the stability conditions given in later chapters are sufficient, something which is expected to introduce conservatism to the proposed anti-windup designs.

The WEC system considered in this thesis has a restriction on its control signal (representative of the motion of a valve varying from fully closed (0%) to fully open (100%) - more details on this will be given in Chapter 3). This is represented by a scalar non-symmetric saturation function defined as

$$sat(u) = \begin{cases} u_{max}, & u \ge u_{max} \\ u, & u_{min} < u < u_{max} \\ u_{min}, & u \le u_{min} \end{cases}$$
(2.17)
where u_{min} denotes the lower saturation limit and u_{max} the upper saturation limit.



Figure 2.3: Saturation function.

For the results presented in following chapters the deadzone function must also be defined as it is the signal that activates the AW compensator during saturation:

$$dz(u) = \begin{cases} u - u_{max}, & u \ge u_{max} \\ 0, & u_{min} < u < u_{max} \\ u - u_{min}, & u \le u_{min} \end{cases}$$
(2.18)



Figure 2.4: Deadzone function.

The deadzone function is a complement of the saturation function as

$$sat(u) + dz(u) = u \tag{2.19}$$

Both the saturation and the deadzone functions belong to the sector [0, I], i.e. $sat(.) \in Sector[0, I]$ and $dz(.) \in Sector[0, I]$.

The WEC dynamics contain a nonlinear term represented by the *sign* or *switching* function defined as

$$sgn(u) = \begin{cases} 1, & u > 0 \\ 0, & u = 0 \\ -1, & u < 0 \end{cases}$$
(2.20)



Figure 2.5: Signum function.

2.2.3 Input-Output stability

An alternative way of proving stability of nonlinear systems (when $u \neq 0$) is by the form of *input-output* stability, i.e. the relationship between an input signal $u \in \mathbb{R}^m$ and an output signal $y \in \mathbb{R}^l$. It is common to assume that these signals belong to certain vector spaces (Turner and Bates 2007). As vector spaces are defined by their norms, this type of stability is closely related to the use of the so called \mathcal{L}_p norms, which measure the "size" of a signal and are usually defined as

$$||x||_{p} = \left(\int_{0}^{\infty} \sum_{i=1}^{n} ||x_{i}(t)||^{p} dt\right)^{\frac{1}{p}}$$
(2.21)

where $p \in [1, \infty]$.

The space \mathcal{L}_p is then defined as the space of all signals with finite \mathcal{L}_p norm, that is

$$\mathcal{L}_p := \{ x : \|x\|_p < \infty \}$$
 (2.22)

It follows that if a signal belongs to \mathcal{L}_p it is well-behaved in some sense. The definition for input-output stability then is the following:

Definition 2.2. Consider the system $\mathcal{T}: u \mapsto y$, then the system is

- 1. \mathcal{L}_p stable if $u \in \mathcal{L}_p$ implies that $y \in \mathcal{L}_p$
- 2. Finite gain \mathcal{L}_p stable if $\|y\|_p \leq \gamma \|u\|_p$

A nonlinear system G is said to have an \mathcal{L}_p norm or \mathcal{L}_p gain, if the second item holds; the smallest such γ is the \mathcal{L}_p induced norm or gain of the nonlinear system and it is denoted as $\|\mathcal{T}\|_{i,p}$.

Particular interest appears for the case where p = 2, where the \mathcal{L}_p gain now becomes the \mathcal{L}_2 gain; a quantity associated with the Root-Mean-Square (RMS) energy gain of the system. Usually, the \mathcal{L}_2 gain cannot be explicitly calculated for nonlinear systems, so normally an upper bound for it is obtained.

In (Hu et al. 2005) it is mentioned that the boundedness of the global L_2 gain guarantees global performance of the closed-loop system, but at times it can introduce conservatism in practice.

It is worth mentioning here that when working with energy-like signals/functions, the \mathcal{L}_2 gain approach is complementary to the state-space representation when looking for stability, especially for Multiple-Input-Multiple-Output (MIMO) systems. In fact, Lyapunov functions can help in placing an upper bound on the \mathcal{L}_2 gain problems,

provided one can find a function V(x) such that

$$\underbrace{\dot{V}(x)}_{stability} + \underbrace{\|y\|_2^2 - \gamma^2 \|u\|_2^2}_{performance} < 0$$
(2.23)

The notion of input-output stability is used extensively in the synthesis of most modern anti-windup compensators like the ones presented in Section 2.3.

It will be shown later on that the aim in Chapters 4-6 is to find a Lyapunov function to provide sufficient conditions for the stability of the nonlinear closed-loop antiwindup (AW) system. In addition, in Chapter 7 besides an appropriate Lyapunov function that will guarantee system stability, an L_2 approach is presented also in an attempt to optimise the performance of the proposed globally stabilising AW compensator.

2.3 Anti-windup (AW) compensation

As mentioned briefly in Chapter 1, the basic underlying idea of anti-windup schemes is to introduce a compensator that will be able to impose some control modifications in order to ensure that stability is maintained during and after saturation and to recover the performance lost during this period. The general principle of an antiwindup scheme can be seen in Figure 2.6 where the unconstrained control signal



Figure 2.6: Architecture of the anti-windup approach.

coming from the nominal controller is compared with the constrained signal, u_{sat} , entering the plant. The difference between these two signals activates the so called anti-windup compensator which in turn tries to modify the control effort and to minimise the performance degradation.

More specifically, G represents the linear plant driven by the saturated control signal $u_{sat} \in \mathbb{R}^m$ in the case of saturation. K is the nominal controller driven by the reference signal $r \in \mathbb{R}^r$ and the plant output $y \in \mathbb{R}^l$. The controller is interconnected with a static nonlinear saturation block to the plant. This saturation block represents the limit on the magnitude of the control signal u due to actuator saturation.

To overcome the effect of this nonlinear element the anti-windup compensator Θ is introduced. This compensator can be either a static gain or a transfer function matrix which has the responsibility of "attenuating" the saturation effects on the system (Sofrony Esmeral 2007). The AW compensator Θ is driven by the difference of the unsaturated and the saturated control signals

$$u_{aw} := u - u_{sat} =: Dz(u) \tag{2.24}$$

thus the AW compensator is only active when saturation occurs. During saturation the compensator Θ emits two signals, θ_1 and θ_2 , that affect the closed-loop behaviour. Signal θ_1 is fed into the control signal (i.e. the controller output) and enables the AW controller to have a quick impact on it (the signal does not have to pass through the nominal controller) and θ_2 is fed into the controller input, hence "stabilising" the controller since the controller may contain unstable modes (Sofrony Esmeral 2007, Turner et al. 2007b).

2.3.1 A parametrisation of linear AW schemes

Although the AW architecture of Figure 2.6 is generic and most commonly used to describe AW compensators, because the system examined is nonlinear and this setup does not help in the analysis of AW schemes, an equivalent, more lucid representation based on the so called *M*-parametrisation (Weston and Postlethwaite 2000) will be shown here.

Consider Figure 2.6 where the AW compensator has the following structure:

$$\Theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \sim \begin{bmatrix} M - I \\ GM \end{bmatrix} u_{aw}$$
(2.25)

This parametrisation helps in drawing the closed-loop "decoupled" AW structure shown in Figure 2.7.

The "decoupled" structure now consists of three parts; the nominal linear loop, the nonlinear loop and the disturbance filter. The nominal linear loop represents the behaviour of the nominal system the way it was originally designed to operate and as long as the control signal is small enough not to cause actuator saturation, the system will continue to behave like so. In this structure the output y consists of two signals, the output y_{lin} of the nominal linear plant and the system output θ_2 due to input saturation (this signal only results during saturation). With this structure and under the assumptions that the linear closed-loop is asymptotically stable and the open-loop plant is bounded real² ($G \in \mathcal{R}H_{\infty}$), the problem of stability is reduced to ensuring asymptotic stability of the nonlinear loop. The performance of the AW scheme is determined by the disturbance filter which determines³ the time it takes the system to recover from saturation.

This parametrisation is useful for comparing stability and performance properties of different AW schemes as the majority of (linear) AW designs can be interpreted as different choices of M. The choice of M dictates the stability properties of the system under saturation (Turner et al. 2007a).

The closed-loop performance of the system is closely related to the linear mapping $\mathcal{T}_p : u_{lin} \mapsto \theta_2$, which represents the deviation from nominal linear behaviour in response to a saturation event and can be used as a measure of performance of AW compensators (Turner et al. 2007a;b). If \mathcal{T}_p is "small", it means that the finite \mathcal{L}_2 gain between the linear control signal and the deviation from the linear behaviour is also "small", i.e. the AW compensator successfully keeps the performance close to the desired one (Turner et al. 2007a;b).

² A function $G(j\omega)$ is said to be bounded real, if $\forall \omega \in \mathbb{R} \Rightarrow G(j\omega)G^*(j\omega) < \gamma$, for $\gamma > 0$.

³ The return to linear behaviour is determined by the dynamics of the disturbance filter (i.e. GM).



Figure 2.7: A "decoupled" AW structure.

2.3.1.1 Full-order AW

Full-order anti-windup compensators are defined as those that have the same degree as the plant. They provide maximum freedom in the design of the compensators themselves and they are always feasible for stable plants $G \in \mathcal{R}H_{\infty}$ (Turner et al. 2007a, Grimm et al. 2003a). On the other hand, full-order compensators may contain unnecessary dynamics for certain design specification and generally they are more computationally intensive.

When designing a full-order compensator the general idea is to choose the M(s) as a right coprime factorisation⁴ of G(s) (Weston and Postlethwaite 2000):

$$G(s) = N(s)M(s)^{-1}$$
(2.26)

where $M, N \in \mathcal{RH}_{\infty}$. The choice of M depends on the time requirement the system takes to return to linear behaviour after saturation has occurred (Weston and

 $^{^4}$ M is chosen as a right coprime factorisation of G, so that the poles of G can be cancelled by the zeros of M and consequently, they will not appear in the disturbance filter GM (Weston and Postlethwaite 2000).



Figure 2.8: Architecture of full-order anti-windup compensation (Turner 2009).

Postlethwaite 2000). Given Equation (2.25), the state-space representation of the full-order AW compensator would be

$$\Theta = \begin{bmatrix} M - I \\ GM \end{bmatrix} \sim \begin{bmatrix} \dot{x}_{aw} \\ \theta_1 \\ \theta_2 \end{bmatrix} \begin{bmatrix} A + BF & B \\ F & 0 \\ C + DF & D \end{bmatrix} \begin{bmatrix} x_{aw} \\ u_{aw} \end{bmatrix}$$
(2.27)

where F is chosen so that A + BF is Hurwitz; the eigenvalues of A + BF are the poles of the disturbance filter (Weston and Postlethwaite 2000), hence the disturbance filter is a stable transfer function (Turner et al. 2007a).

The purpose of the full-order compensator is to stabilise the nonlinear loop and optimise the performance of the closed-loop system during saturation via $\|\mathcal{T}\|_{i,2} < \gamma$, i.e. improve the speed and damping of the poles of the disturbance filter (Weston and Postlethwaite 2000). The right coprime factorisation allows the choice of M(s) or equivalently the choice of F so as to satisfy the aforementioned objectives, i.e.

F can be chosen by solving the following Linear Matrix Inequality (LMI), which is a convex optimisation problem (Turner et al. 2007a, Theorem 4):

$$\begin{bmatrix} QA' + AQ + L'B' + BL & BU - L' & 0 & QC' + L'D' \\ \star & -2U & I & UD' \\ \star & \star & -\gamma I & 0 \\ \star & \star & \star & -\gamma I \end{bmatrix} < 0$$
(2.28)

where " \star " denotes the transpose of the off-diagonal terms that make the matrix symmetrical, A, B, C, D are the plant matrices, Q is a positive definite matrix, $L \in \mathbb{R}^{m \times n}$ is an unstructured matrix, U is a positive definite diagonal matrix and γ is a positive scalar. Provided the LMI above is feasible, a suitable F achieving $\|\mathcal{T}\|_{i,2} < \gamma$ is given by $F = LQ^{-1}$, which in turn is used to synthesise the full-order AW compensator.

2.3.1.2 Internal Model Control (IMC) Anti-Windup (AW) compensator

Another AW compensator is the Internal Model Control (IMC) (Figure 2.9) firstly appeared in (Zheng et al. 1993). This is a special case of the full-order AW presented in the previous section since F is chosen equal to 0. Essentially, the IMC AW compensator is a copy of the nominal plant, activated during saturation and driven by the signal u_{aw} . It behaves in such a manner as to ensure that the input which the controller sees during saturation is exactly that of the model of the plant during linear behaviour.

When M = I (F = 0) is chosen, the nonlinear loop becomes the deadzone operator and the disturbance filter becomes the open-loop plant. IMC is a stabilising AW scheme assuming that G is stable, but performance can deteriorate if the plant has slow or lightly damped poles or nonminimum phase zeros (Weston and Postlethwaite 2000, Turner et al. 2007b).



Figure 2.9: IMC anti-windup architecture.

The main attractive characteristics of IMC compensators are i) the fact that it is very easy to design them, i.e. no computations have to be performed, ii) they are very robust especially in the case where uncertainties exist (Turner et al. 2004) and iii) stability is guaranteed unconditionally for open-loop stable linear systems (Herrman et al. 2010). The state-space form of the IMC compensator is given by

$$\Theta = \begin{bmatrix} M - I \\ GM \end{bmatrix} \sim \begin{bmatrix} \dot{x}_{aw} \\ \theta_1 \\ \theta_2 \end{bmatrix} \begin{bmatrix} A & B \\ 0 & 0 \\ C & D \end{bmatrix} \begin{bmatrix} x_{aw} \\ u_{aw} \end{bmatrix}$$
(2.29)

Despite the aforementioned advantages of linear IMC compensators the stability properties of the system conditioned by a nonlinear IMC AW compensator have to be formally analysed. The simplicity of the IMC AW scheme motivates the analysis of the nonlinear IMC compensator described in Chapter 5.

2.4 Conclusions

In this chapter, basic concepts around nonlinear systems have been presented as well as Lyapunov's second method of proving stability for such systems. Also, an introduction of linear AW has been made, especially focused on the M parametrisation (Weston and Postlethwaite 2000); a particular choice of M yields the Internal Model Control (IMC) AW compensator, which will be used later in the thesis.

CHAPTER 3

Wave Energy Converter (WEC) model and performance analysis

In this chapter, the WEC model used throughout the thesis adopted from (Nolan 2006, Falnes 2002, Eidsmoen 1995) is described as well as its nominal controller. Simulation results exhibiting the systems nominal performance and also performance under saturation are included here. Saturation acts in particular as motivation for the work carried out and presented in Chapters 5-7, as different AW techniques are proposed in order to diminish the saturation effects affecting the nominal controller.

3.1 Modelling of Wave Energy Converters

Modelling of WECs has been done traditionally in either the frequency domain or the time domain. Frequency domain design is used to calculate the hydrodynamic parameters of the main body of the WEC, as they are highly frequency and buoy size-dependant. However, in order to design and evaluate control strategies time domain models are used (both transient performance and nonlinear phenomena can be seen in the time domain).

The WEC under consideration is taken from (Nolan 2006) and can be seen in Figure 3.1. The buoy has a diameter of 3.3 m and a height of 5.1 m, of which 3.1 m is submerged when the buoy is at equilibrium (Nolan 2006). The off-shore floating device captures energy from its wave-induced motion via hydraulic rams and a high



pressure power take-off system (Nolan and Ringwood 2006).

Figure 3.1: Conceptual diagram of a heaving WEC (Nolan 2006, page 55), (Ringwood 2010).

The basic operation of the WEC, as described in (Nolan 2006, page 55), is the following: A force inducing motion is exerted on the device from the waves. The relative motion between the buoy and the sea bed activates a hydraulic piston pump sucking sea water and sending it to the Power Take-Off (PTO) unit. The throttle valve controls the pressure and the flow in the hydraulic system and it is due to this pressure that the water particles pass through the semi-permeable membrane in the RO desalination unit in order to produce potable water.

The alternating vertical motion of the buoy causes a series of unidirectional check valves to rectify the water pumped by the device to the hydraulic PTO (Ringwood 2010); this introduces switching expressed through the term $sign(\dot{q})$ and indicates that the PTO subsystem opposes the motion of the buoy, and thus provides damping (Nolan 2006, Ringwood 2010, Bacelli et al. 2009). According to (Nolan 2006), the buoy "drives" the PTO while the latter "loads" the buoy. The interaction (coupling)

between the two subsystems can be seen in Figure 3.2.

Although in general the goal is to achieve energy maximisation by actively adjusting the damping according to the buoy motion (usually when the application of interest is electricity production), the application of water desalination differs. The damping seen by the buoy is effectively constant (for reasons explained in Section 3.3), so there is a trade-off between the energy available for capture and the control goal of the PTO.

In the next sections, the complete WEC model is presented consisting of the mechanical model of the buoy and the model for the PTO. The ocean waves act as an input to the WEC system, so the excitation force model will also be presented.



Figure 3.2: Block diagram of the subsystems consisting the WEC under consideration (Nolan 2006, page 56).

3.1.1 The excitation force model

The excitation force model represents the motion of the ocean waves that exert a force on the buoy, thereby inducing motion. As the ocean waves act as an input to the nonlinear plant (WEC), they affect the system output. At the same time, the excitation force is an exogenous input acting as a disturbance to the system, therefore if one wants to design a controller with good disturbance rejection properties one must have as much knowledge about the disturbance as possible.

Usually, all wave analysis is done based on the assumption that linear wave theory holds and hence the waves are well represented by a sinusoidal form. This relies on the following two assumptions: there are no energy losses due to friction, turbulence, etc. and the wave height, H, is much smaller than the wavelength λ (Ringwood



2010). The basic wave characteristics can be seen in the following figure where κ is

Figure 3.3: Basic wave characteristics.

the amplitude of the wave, H is the height equal to twice the amplitude, λ is the distance between adjacent troughs¹ or crests², T is the period of a wave, i.e. the time it takes for one complete wave to pass a given point and $\eta(t)$ is the surface wave elevation.

As already discussed in Chapter 1, linear wave theory is not very representative of the actual waves leading to misconceptions about the WEC and their controllers' true performance. Instead, a model for irregular waves must be used in order to get more realistic results regarding the system tracking performance, stability, robustness and disturbance rejection properties since irregular waves exhibit a nonlinear nature.

In order to represent real ocean waves, usually a distributed amplitude spectrum with random phases is used (Ringwood 2010). Consequently, real seas can be thought of as the sum of many individual waves that have different wavelengths and different amplitudes (Carbon Trust 2006b). Each sea state is characterised by a specific combination of significant wave height H_s and period T and a discretised spectrum (Figure 3.4). The spectrum is usually a Pierson-Moskowitz (Pierson Jr. and Moskowitz 1964) one or either one of its variants; the Bretschneider (Bretschneider 1952) and JOHNSWAP spectrum (Hasselmann et al. 1973), respectively. In practice, specific sea states last for a few hours (Carbon Trust 2006b).

Here, a model for producing irregular waves, based on the Pierson-Moskowitz (PM)

¹ A trough is a low area between two big waves on the sea.

 $^{^2}$ The crest of a wave is the top of it.

energy spectrum for fully developed seas, has been used and is representative of a large number of ocean locations (typical for locations in the North Atlantic (Carbon Trust 2006b)). The energy spectrum is given by (Michel 1999)

$$E_{\omega}(\omega) = \frac{0.11 H_s^2 T_1}{2\pi} \left(\frac{\omega T_1}{2\pi}\right)^{-5} e^{\left[-0.44 \left(\frac{\omega T_1}{2\pi}\right)^{-4}\right]}$$
(3.1)

where H_s is the significant³ wave height, T_1 is the mean wave period and ω is the wave frequency. The PM spectrum basically consists of a large number of superimposed sine waves with appropriate adjustments of frequency, magnitude and phase.



Figure 3.4: Energy spectrum characterised by $H_s = 6m$ and $T_1 = 10sec$.

The time series corresponding to this spectrum can be calculated by dividing it into 100 equally spaced frequencies at intervals of $\Delta \omega = 0.0325 \ rad/s$ up to a maximum frequency of 3.2459 rad/s (Nolan 2006, page 57). The wave surface elevation $\eta(t)$ is then calculated as

$$\eta(t) = \sum_{i=1}^{100} \kappa(i) \sin\left(\omega(i)t + \psi(i)\right)$$
(3.2)

where

³ The significant wave height is defined as the average height of the highest one-third of the waves and it can also be written as $H_{\frac{1}{2}}$.

$$\kappa(i) = \sqrt{2E_{\omega}(\omega(i))\Delta\omega}$$
(3.3)

where $\kappa(i)$ is the wave amplitude and $\psi(i)$ are random phase angles distributed uniformly from 0 to 2π .

The excitation force (Figure 3.5) corresponding to the energy spectrum of Figure 3.4 (which also acts as an input to the system) is proportional to the wave amplitude and it is calculated as follows (Nolan 2006, page 83):

$$f_e(t) = \eta(t) \cdot \left| F\left(\omega(i)\right) \right| = \sum_{i=1}^n \kappa(i) \left| F\left(\omega(i)\right) \right| \sin\left(\omega(i)t + \psi(i)\right)$$
(3.4)

where $F(\omega(i))$ is the excitation force coefficient that is dependent on the buoy geometry and its value is usually obtained through wave tank experiments (Bacelli et al. 2009).



Figure 3.5: Realisation of the excitation force corresponding to the energy spectrum of Figure 3.4.

3.1.2 The hydrodynamic model of a heaving point-absorber

The mechanical buoy model is derived from its equation of motion and it is based on the oscillation of a cylindrical body in the water. It incorporates only the vertical motion in order to reduce the hydrodynamic complexity of the model⁴. The buoy mathematical model is given by the following integro-differential equation (Nolan 2006, Ringwood 2006, Bacelli et al. 2009):

$$\ddot{q}(t) = \frac{1}{m_b + m_r(\infty)} \left\{ \int_{-\infty}^{\infty} \eta(\tau) f(t-\tau) d\tau - B(t) \dot{q}(t) - \int_{-\infty}^{t} k(t-\tau) \dot{q}(\tau) d\tau - R_f \dot{q}(t) - Sq(t) \right\}$$
(3.5)

where

- \ddot{q} is the vertical acceleration of the buoy,
- \dot{q} is its vertical velocity,
- q is its displacement,
- $\eta(t)$ is the surface elevation mentioned earlier,
- B(t) is a nonlinear damping term representing the PTO system
- R_f is a resistance coefficient representing friction,
- $m_r(\infty)$ is the value of the added mass at infinite frequency,
- m_b is the ballast mass,
- S is the hydrostatic stiffness coefficient
- f(t) is the impulse response of the transfer function relating the wave elevation $\eta(t)$ to the excitation force $f_e(t)$ acting on the buoy
- k(t) is the impulse response of the transfer function relating radiation damping to heave velocity \dot{q}

⁴ This is a common practice in point absorber type WECs as most of them are designed such that they have a dominant response in heave (Brekken et al. 2011).

With the exception of the hydrostatic stiffness S which is static, the rest of the hydrodynamic parameters are wave-frequency and also buoy-shape dependant (Nolan 2006, page 62). Equation (3.5) allows for unconstrained movement of the WEC in heave.

3.1.3 The Power Take-Off (PTO) model

The PTO model represents the mechanism used to convert wave energy into useful products and particularly here the model represents how the wave motion relates to potable water production. This consists of the following subsystems (*Figure 3.2* (p. 39)) described in (Nolan 2006).

- Hydraulic pumps: In any hydraulic system they create a flow of liquid. Here, they are used for energy capture by the WEC.
- Manifold: It collects flow from the hydraulic pump, passes flow in and out of the accumulator and passes flow to the RO unit. A number of hydraulic pumps can be connected to the manifold (Ringwood 2010).
- Accumulator: Stores energy for long or short periods of time and releases the stored energy in the same form it was supplied (Entec UK Ltd. 2005). This particular accumulator stores energy in the form of fluid under pressure and its function is to smooth the delivery from the hydraulic pump so that a relatively constant pressure and flow is experienced in the RO unit.
- Filter: It filters the feed of water into the RO unit and hence protects the RO membrane from wear.
- Reverse Osmosis: This subsystem is responsible for the desalination of sea water. It includes a semi-permeable membrane that is responsible for holding back most of the salt content of the sea water, thereby rendering it potable. The RO pressure P_{ro} must remain constant around a setting point; this is the regulated output of the system.
- Throttle valve: It controls the backpressure⁵ of the hydraulic system as well as

⁵ Backpressure refers to the pressure opposed to the desired flow of a fluid in a confined space such as a pipe.

the hydraulic resistance opposing the motion of the buoy. It can be considered as an orifice, i.e. a restriction of fluid flow passage. Its position, V_{pos} , is the *control variable* u in the system.



Figure 3.6: Basic PTO components for potable water production (Nolan 2006, page 19).

Together, these subsystems can be described by the following differential equation (see Nolan 2006, Section 7.2)

$$\dot{P}_{ro} = -P_{ro} \left[\frac{1}{C_{acc}} \left(\frac{1}{R_{tv}} + \frac{1}{R_{ro}} \right) \right] + \frac{|\dot{q}|A_p}{C_{acc}}$$
(3.6)

where

$$R_{tv} = \frac{200\sqrt{P_{ro}}}{C_r} \tag{3.7}$$

$$R_{ro} = \frac{P_{ro}}{N_{ro}\rho_{ro}(P_{ro} - P_{osm})}$$
(3.8)

 C_{acc} is the accumulator capacitance, C_r is the rated valve flow coefficient, R_{tv} is the resistance introduced by the throttle valve, R_{ro} is the resistance introduced by the RO membrane, ρ_{ro} is the RO permeability coefficient and N_{ro} is the number of the RO units deployed (here $N_{ro} = 1$). A_p is the pump area and is a function of the pump diameter (Nolan 2006, page 139).

The damping experienced by the body of the WEC due to the PTO system (Figure 3.2 (p. 39)) (Nolan 2006) is given by

$$B_{dam}(t) = \frac{A_p P_{ro}}{|\dot{q}|} \tag{3.9}$$

In a water-desalinating WEC the damping experienced will vary with time, it is determined by the pressure control requirements and it cannot be manipulated independently in order to maximise the device's energy absorption. In this thesis, there is no manipulation of the damping provided by the PTO to the buoy; it is a function of the buoy velocity \dot{q} , as seen by Equation (3.9).

For the PTO system to operate efficiently the following conditions must be ensured by the control system (Nolan 2006, Ringwood 2010):

- The pressure P_{ro} must be maintained at $6 \cdot 10^6 P_a$ (or 60 bar)
- Pressure excursions above the setpoint must be avoided as they can damage the RO membrane
- Negative excursions must also be avoided since they result in the loss of efficiency in the RO unit

The positive and negative pressure excursions are limited to 3% (Nolan 2006, page 135); if it is more than this the quality of water produced drops below the required specification, making it useless in commercial terms. In addition, the RO membrane is damaged due to excessive pressure.

3.2 Complete WEC model

Since the control problem investigated in this chapter is a regulation problem, there exists a single operating point ($\overline{P_{ro}} = 6 \cdot 10^6 P_a$) around which the individual subsystems described in Sections 3.1.1 - 3.1.3 are accurate with respect to the full nonlinear WEC model and hence are combined and linearised, in order to design, test and evaluate appropriate control strategies (Nolan 2006). The "linearised" model is a switched model (effectively there is switching between two linear subsystems) with the following generic state-space description:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \hline \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & A_{12} & 0 \\ A_{21} & A_{22} & A_{23}(x_2) \\ \hline 0 & A_{32}(x_2) & A_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \hline x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \hline B_{u3} \end{bmatrix} u + \begin{bmatrix} 0 \\ B_{d2} \\ \hline 0 \end{bmatrix} d \quad (3.10a)$$

$$y = \begin{bmatrix} 0 & 0 & C_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \hline x_3 \end{bmatrix}$$
(3.10b)

where the terms $A_{32}(x_2)$ and $A_{23}(x_2)$ are the cross-coupling terms between the buoy and the PTO and they are dependent upon switching of the x_2 state (the buoy velocity). One can see that the $A_{32}(x_2)$ term introduces disturbance on the x_3 (P_{ro}).

The linearised equations around the RO pressure operating point $\overline{P_{ro}}$, as described in (Nolan 2006, Sec. 7.3) and in (Ringwood 2006, Bacelli et al. 2009) can be seen below:

$$\begin{bmatrix} \dot{x}_1\\ \dot{x}_2\\ \hline{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0\\ -\frac{S}{M} & -\frac{R_f + R_c}{M} & -\frac{A_p^*}{M}\\ \hline{0 & \frac{A_p^*}{C_{acc}}} & -\frac{(\overline{P_{ro}} - P_{osm})\rho_{ro}}{C_{acc}\overline{P_{ro}}} \end{bmatrix} \begin{bmatrix} x_1\\ x_2\\ \hline{x}_3 \end{bmatrix} + \begin{bmatrix} 0\\ 0\\ \hline{-\frac{C_r\sqrt{P_{ro}}}{200C_{acc}}} \end{bmatrix} u + \begin{bmatrix} 0\\ \frac{1}{M}\\ \hline{0} \end{bmatrix} f_e(t)$$
(3.11a)
$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1\\ x_2\\ \hline{x}_3 \end{bmatrix}$$
(3.11b)

where $\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}' = \begin{bmatrix} q & \dot{q} & P_{ro} \end{bmatrix}'$ are the system states, u is the control signal corresponding to a linearised valve characteristic described in (Nolan 2006, Sec. 7.3),(Nolan and Ringwood 2006) and $M = m_b + m_{r(\infty)}$. The magnitude of the control signal is limited between $0 - 100\%^6$. In the model of Equation (3.11a), the kernel k(t) has been approximated by the constant R_c .

The model of Equation (3.11a) is nonlinear due to state-dependent switching, i.e. the switching term $\mathbf{A}_{\mathbf{p}}^*$, which carries the sign of the state \dot{x}_2

$$\mathbf{A}_{\mathbf{p}}^{*} = sign(x_{2})A_{p} = \begin{cases} A_{p}, & \text{if } x_{2} > 0\\ 0, & \text{if } x_{2} = 0\\ -A_{p}, & \text{if } x_{2} < 0 \end{cases}$$
(3.12)

The switching parameter $\mathbf{A}_{\mathbf{p}}^*$ is introduced in order to eliminate the absolute value functions of the velocity \dot{q} in Equations (3.6) and (3.9). One should note here that the

⁶ The control signal corresponds to the position of a throttle valve, which ranges from fully closed to fully open, hence 0-100%.

saturation is a function of a plant parameter, so saturation will be present regardless of the controller.

Looking at the WEC state-space equation carefully one might observe the following:

- The A_{32} term is showing the interaction from the buoy to the PTO
- The A_{23} term is showing the interaction from the PTO to the buoy
- The only control input to the coupled system is the throttle valve position u
- The excitation force $f_e(t)$ acts as a (unmeasurable) disturbance to the system
- The only nonlinearity considered in this work is the switching term $\mathbf{A}^*_\mathbf{p}$

In the next section, the controller designed by (Nolan 2006) is described. Since the WEC model described by Equation (3.11a) is "weakly" nonlinear, i.e. the only nonlinear term is $\mathbf{A}^*_{\mathbf{p}}$ which is a scalar, then a natural control approach would be to use the Nonlinear Dynamical Inversion (NDI) control technique to cancel out all the nonlinear terms (provided the nonlinear terms are invertible) and effectively control a linear system. However, only partial inversion is possible and thus a "partial-NDI" controller is designed.

3.3 Nominal controller

The control approach taken in (Nolan 2006), was to design a state feedback controller in order to control the WEC under consideration. A full state-feedback controller would try to regulate all three states of the system, however the buoy should be able to move freely, which means the states corresponding to WEC position (q) and WEC velocity (\dot{q}) must remain unregulated. As a result, the control design was focused on the regulation of the third system state, i.e. the pressure in the RO desalination unit (P_{ro})

$$\dot{x}_3 = A_{32}(x_2)x_2 + A_{33}x_3 + B_{u3}u \tag{3.13}$$

The cross-coupling term $A_{32}(x_2)$ can be considered as a measurable disturbance that can be cancelled by a feedforward gain in an effort to remove a large part of the disturbance effects on x_3 .

The goal is to design a feedback controller C such that it reduces the effect of the disturbance d and also a feedforward gain K_{ff} so as to track the reference r. It is well known that response time is reduced when using feedforward control, however at the expense of increased control effort. A partial state-feedback controller was designed (Figure 3.7) with the following generic state-space form:

$$C_{wec} \sim \begin{bmatrix} A_c & B_{cr} & B_{cy} \\ \hline C_c & D_{cr} & D_{cy} \end{bmatrix}$$
(3.14)

where the terms B_{cr} and D_{cr} are driven by the reference signal and terms B_{cy} and D_{cy} are driven by the measured plant outputs. The controller designed in (Nolan 2006) is realised as follows:

$$C_{wec} \sim \begin{cases} \dot{x}_c = r - x_3 \\ u = K_i x_c + (K_p + K_{ff})r - K_p x_3 + K_s(x) \end{cases}$$
(3.15)

where $K_i = \frac{\mu_1 \mu_2}{B_{u3}}$, K_p the controller gain equal to $K_p = \frac{A_{33} - (\mu_1 + \mu_2)}{B_{u3}}$, $K_{ff} = -\frac{A_{33}}{B_{u3}}$ and $K_s(x)$ is a nonlinear term chosen to cancel the nonlinear terms appearing in the \dot{x}_2 state equation $K_s(x) = -\frac{1}{B_{u3}}A_{32}(x_2)x_2$, where μ_1, μ_2 are the controller desired closed loop poles placed at 0.12 rad/sec (Nolan and Ringwood 2006). Since the gain $K_s(x)$ is a function of A_{32} , which in turn contains the switching term A_p , then $K_s(x)$ is required to switch according to the sign changes of $\dot{q}(x_2)$, thus making the controller nonlinear. $K_s(x)$ is used on x_2 to enable a large part of the "incident wave" effect on x_3 to be neutralised (Nolan and Ringwood 2006).

The partial state-feedback controller of Equation (3.15) is a pure regulator⁷ as it regulates the system output P_{ro} .

In this controller, the cross-coupling term $A_{23}(x_2)$ is not taken into account, as x_3

⁷ A regulator is a feedback control system in which the reference input is constant for long periods of time, often for the entire time interval during which the system is operational.



Figure 3.7: Block diagram of the state feedback controller proposed in (Nolan and Ringwood 2006).

regulation is of primary importance and the designer accepts that the damping seen by the buoy through the term $A_{23}(x_2)$ is non-optimal (Ringwood 2010), i.e. it cannot be regulated. On the other hand, the interaction term $A_{32}(x_2)$ is taken into account through the feed-forward feedback structure $A_{32}(x_2)x_2$, which acts as a measurable disturbance feedforward term that aims to cancel out the disturbance caused from the waves (Ringwood 2010).

The reference input is the pressure that the system needs to maintain. The control variable u, representing the throttle valve position in the system, is one of the inputs to the plant. The other input to the plant is the disturbance wave excitation force, whose realisation is based on the description of irregular waves given in Section 3.1.1. The regulated output is the pressure, which is fed back to the controller along with the value and sign of the buoy's vertical velocity \dot{q} .

According to (Nolan 2006), this controller provides regulation around the operating point $(6 \cdot 10^6 P_a)$, suitable transient performance and integral action to ensure zero steady-state error. In addition, the poles of the controller are placed at the same frequency as is the peak of the wave spectrum in order to achieve optimal disturbance rejection (Ringwood 2010).

In order to achieve this disturbance rejection the poles of the controller are placed as

far into the left-half plane as possible, however to implement such a fast controller in practice the required control signal would be very large in magnitude, something which is not feasible in practice resulting in actuator saturation. According to (Nolan 2006), actuator saturation occurs regularly and poses a significant challenge to the operation of WECs, as their performance deteriorates rapidly after saturation occurs.



Figure 3.8: Block diagram of the system implemented in this thesis.

3.4 Performance of the constrained system

This section presents some simulation results of the closed-loop system, consisting of the WEC model presented in Section 3.2 and the controller presented in Section 3.3, without saturation compared to the performance of the closed-loop system under saturation.

Before moving on to describing the simulation environment, it should be mentioned that the results presented in this section are based solely on computer simulations and not on an actual system. As mentioned in Chapter 1 the second step in the WEC development process, after the initial system design, is testing a scale of that system in a wave tank. This was impossible to do while developing the work presented in this thesis for reasons such as the following: wave tanks as well as appropriate sensing and measurement equipment are available at certain UK and Irish Universities (like University of Maynooth and University of Cork in Ireland where devoted research groups and facilities exist), hence could not be performed at the University of Leicester. Access to such facilities around the UK and Europe could be granted with all expenses covered for up to a week through the Marine Renewables Infrastructure Network (MARINET) (Marine Renewable Infrastructure Network), however the lack of a scaled WEC prototype, the inability to construct a scaled PTO system to test the control algorithms designed and the lack of supporting staff in designing and performing such experiments while using such as the aforementioned facilities, forbade for wave tank experiments to take place. In addition, access to such facilities could be granted even a year after applying for it, a time frame which is very tight with respect to the typical duration of a PhD.

Moving on to the description of the simulation setup, for the reference signal r a step-type signal has been used and also, the saturation limits correspond to the throttle valve being fully closed (0%) or fully open (100%). The simulations are performed for certain wave heights ($H_s = 0.5 \text{ m}, 0.8 \text{ m}$ and 1.2 m) and for a wave period of $T_1 = 10 \text{ s}$.

It is worth noting here that for higher wave heights $(H_s > 1.5 \text{ m})$ both the unconstrained controller as well as the AW compensators developed in this thesis cannot track the reference at all (the output pressure reaches destructive values). This is probably due to the fact that the WEC model of Section 3.2 is a good representation of the full nonlinear WEC model, however no one can guarantee its validity away from this operating point (Nolan 2006, p. 214). Hence, results for wave heights higher than 1.5 m will not be shown in this thesis.

Figure 3.9 shows how the reference signal is tracked perfectly when the WEC is allowed to move in an unconstrained manner, however this can only be done at the expense of a large and hence practically infeasible control signal.

In Figure 3.10 one can see that the system under saturation experiences difficulties in tracking the system reference as the output goes to infinity, hence it is unstable. This can compromise the structural integrity of the WEC, which is something highly undesirable considering its capital cost as well as its maintainance/repair cost.

In Figure 3.11 the magnitude of the control signal of the nominal closed-loop system



Figure 3.9: Step response of the nominal unconstrained closed-loop system for wave height $H_s = 0.5$ m and wave period $T_1 = 10$ s



Figure 3.10: Step response of the closed-loop system under saturation for $H_s = 0.5$ m and $T_1 = 10$ s.

is quite high due to the fast controller poles, as pointed out in Section 3.3, which is practically infeasible.



Figure 3.11: Unconstrained WEC control signal for $H_s = 0.5$ m and $T_1 = 10$ s.

When actuator constraints are introduced (Figure 3.12) the system operates mostly on the lower saturation limit, due to the irregular WEC input (Nolan 2006, p.231).



Figure 3.12: WEC control signal under saturation for $H_s = 0.5$ m and $T_1 = 10$ s.

In higher wave amplitudes (Figure 3.13), the saturated system oscillates around the reference, however there is a high overshoot that will destroy the desalination membrane.



Figure 3.13: Output for $H_s = 1.2 \text{ m}$ and $T_1 = 10 \text{ s}$.



Figure 3.14: Constrained control signal for $H_s = 1.2$ m and $T_1 = 10$ s.

One can see from the simulation results that the nominal system achieves perfect regulation at $6 \cdot 10^6 P_a$, however the control signal/activity achieves very high values that cannot be realised in practice by an actuator. Therefore, when the actuator limits are imposed, it is clear that the constrained system loses its tracking ability and poses a hazard for the RO unit structural integrity (the pressure reaches very high values, a lot bigger than the permitted 3%). In other cases, the constrained system also loses its stability properties, which is highly undesirable.

3.5 Conclusions

In this chapter, the WEC under consideration has been presented; its main operation principles have been analysed and its dynamics have been given, as formulated in (Nolan 2006). The WEC dynamics proved to be nonlinear due to the device's switching. As a result, a partial-NDI switching state-feedback controller was designed by (Nolan 2006). The benefit of this control approach is that the wave-induced disturbance was eliminated by cancelling part of the nonlinear switching WEC dynamics.

Simulation results showed that when no saturation occurs the controller decouples the PTO subsystem from the mechanical subsystem and allows excellent pressure regulation to be achieved. However, due to large control activity of the nominal unconstrained system, when actuator limits are imposed the system performance deteriorates significantly (there is no tracking and the RO membrane is destroyed). Nevertheless, noting the good performance of the baseline control system, it seems natural to tackle these saturation issues using anti-windup compensation. It is well-known that AW compensators are not effective in systems under constant saturation, so it is expected that AW compensation will only be effective for those wave amplitudes that cause the closed-loop system to get in and out of saturation. This can be also verified in later chapters based on simulation results.

CHAPTER 4

Control system stability analysis for a class of point-absorber WECs

This chapter focuses on the stability properties of a generic class of pseudo-Nonlinear Dynamic Inversion (NDI) controllers applicable to point-absorber type heaving Wave Energy Converters (WECs). A special case of these controllers is considered in (Nolan 2006, Nolan and Ringwood 2006) and has been presented in Section 3.3, where a partial state feedback controller with feedforward and integral gains was proposed. There it was shown that the controller performed well in simulations, although a formal stability analysis was not carried out. In (Nolan 2006), it was shown that the system was open-loop stable, but the proof was dependent on the numerical solution of a number of Lyapunov equations and did not exploit the structure of the WEC equations in the analysis. In addition, this analysis was open-loop and not closed-loop. In this chapter, it is shown that the pseudo-NDI controller proposed in (Nolan 2006, Nolan and Ringwood 2006) belongs to a class of control systems which indeed can provide global closed-loop stability for the WEC system under examination. The work in this chapter has been presented in the 2012 IEEE International Conference on Control Applications (Lekka et al. 2012).

4.1 Plant

Recalling Equations (3.10a) - (3.11b) the following state-space notation is adopted, which will prove useful for the derivation of results for the remainder of the thesis.

The following system partition allows for the nonlinearities to become distinct and hence be dealt with individually.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \mathcal{A}_{11} & \mathcal{A}_{12}(x_1) \\ \mathcal{A}_{21}(x_1) & \mathcal{A}_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \mathcal{B}_1 \\ 0 \end{bmatrix} d + \begin{bmatrix} 0 \\ \mathcal{B}_2 \end{bmatrix} u \qquad (4.1a)$$
$$y = \begin{bmatrix} 0 & \mathcal{C}_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad (4.1b)$$

where

$$\begin{aligned} \boldsymbol{x}_{1} &= [x_{1} \ x_{2}]' & \boldsymbol{x}_{2} = x_{3} & \mathcal{A}_{11} = \begin{bmatrix} 0 & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \\ \mathcal{A}_{12}(\boldsymbol{x}_{1}) &= \begin{bmatrix} 0 \\ A_{23}(\boldsymbol{x}_{2}) \end{bmatrix} & \mathcal{A}_{21}(\boldsymbol{x}_{1}) = \begin{bmatrix} 0 & A_{32}(\boldsymbol{x}_{2}) \end{bmatrix} & \mathcal{A}_{22} = A_{33} \\ \mathcal{B}_{1} &= \begin{bmatrix} 0 \\ B_{d2} \end{bmatrix} & \mathcal{B}_{2} = B_{u3} & \mathcal{C}_{2} = C_{3} \end{aligned}$$

Equations (4.1a) and (4.1b) can be written more compactly as

$$\dot{\boldsymbol{x}} = A\boldsymbol{x} + A_s(\boldsymbol{x})\boldsymbol{x} + \mathcal{B}_1 d + \mathcal{B}_2 u \tag{4.2a}$$

$$y = \mathcal{C}_2 x \tag{4.2b}$$

where matrix A contains all the linear parts and matrix $A_s(x)$ contains the two switching nonlinear terms.

$$A = \begin{bmatrix} \mathcal{A}_{11} & 0 \\ 0 & \mathcal{A}_{22} \end{bmatrix} \qquad \qquad A_s(\boldsymbol{x}) = \begin{bmatrix} 0 & \mathcal{A}_{12}(\boldsymbol{x}_1) \\ \mathcal{A}_{21}(\boldsymbol{x}_1) & 0 \end{bmatrix}$$

It can be seen from Equation (4.1a) that $f_e(t)$ is not in the range space of \mathcal{B}_2 , so it will be impossible to cancel the nonlinear term $\mathcal{A}_{12}(x_1)$ without some additional control effort. Also note that, disregarding the nonlinear terms, the system can be decoupled into two *independent* subsystems: an x_1 subsystem driven by the disturbance $f_e(t)$ and an x_2 subsystem driven by the control input u(t). Moreover, subsystem x_1 is unobservable from the output y(t), hence it does not affect it.

4.2 Controller

With the above observations in mind, a natural control strategy is one of decoupling the two subsystems. While complete decoupling is not possible, it is possible to use a pseudo-NDI type of control strategy to cancel the influence of the x_1 dynamics on the x_2 subsystem. Therefore, consider the following controller

$$\mathcal{K} \sim \begin{cases} \dot{x}_c = \mathcal{A}_c x_c + \mathcal{B}_{cr} r + \mathcal{B}_{cy} \mathcal{Y} \\ u = \mathcal{C}_c x_c + \mathcal{D}_{cr} r + \mathcal{D}_{cy} \mathcal{Y} + K_s(x) \end{cases}$$
(4.3)

where the nonlinear term $K_s(x)$ is chosen to "cancel" the nonlinear term appearing in the x_2 subsystem, viz:

$$K_s(\boldsymbol{x}) = -\frac{1}{\mathcal{B}_2} \mathcal{A}_{21}(\boldsymbol{x}_1) \boldsymbol{x}_1 \tag{4.4}$$

Since the controller cannot cancel the nonlinearity in the x_1 subsystem, the problem is different than the standard NDI. In standard NDI problems the goal is to cancel "all" the nonlinear dynamics of the plant when designing the controller. This implies that the nonlinear dynamics are invertible in order to be cancelled; in this case not all of the nonlinear terms can be cancelled and therefore more analysis is needed in order to establish stability.

Using the controller \mathcal{K} and substituting u from Equation (4.3) to the x_2 subsystem yields:

$$\dot{x}_2 = \mathcal{A}_{21}(x_1)x_1 + \mathcal{A}_{22}x_2 + \mathcal{B}_2 u \tag{4.5}$$

$$= \mathcal{A}_{21}(\boldsymbol{x}_1)\boldsymbol{x}_1 + \mathcal{A}_{22}\boldsymbol{x}_2 + \mathcal{B}_2\Big[\mathcal{C}_c\boldsymbol{x}_c + \mathcal{D}_{cr}\boldsymbol{r} + \mathcal{D}_{cy}\boldsymbol{y} + K_s(\boldsymbol{x})\Big]$$
(4.6)

$$= \mathcal{A}_{21}(\boldsymbol{x}_1)\boldsymbol{x}_1 + \mathcal{A}_{22}\boldsymbol{x}_2 + \mathcal{B}_2\mathcal{C}_c\boldsymbol{x}_c + \mathcal{B}_2\mathcal{D}_{cr}r + \mathcal{B}_2\mathcal{D}_{cy}\boldsymbol{y} + \mathcal{B}_2\left[-\frac{1}{\mathcal{B}_2}\mathcal{A}_{21}(\boldsymbol{x}_1)\boldsymbol{x}_1\right]$$

$$(4.7)$$

and using Equation (4.2b) the expression for the dynamics of the (x_2, x_c) subsystem,

expressed in state-space form, is the following

$$\begin{bmatrix} \dot{x}_2 \\ \dot{x}_c \end{bmatrix} = \underbrace{\begin{bmatrix} \mathcal{A}_{22} + \mathcal{B}_2 \mathcal{D}_{cy} \mathcal{C}_2 & \mathcal{B}_2 \mathcal{C}_c \\ \mathcal{B}_{cy} \mathcal{C}_2 & \mathcal{A}_c \end{bmatrix}}_{\mathcal{A}_{2c}} \begin{bmatrix} x_2 \\ x_c \end{bmatrix} + \underbrace{\begin{bmatrix} \mathcal{B}_2 \mathcal{D}_{cr} \\ \mathcal{B}_{cr} \end{bmatrix}}_{\mathcal{B}_{2c}} r \qquad (4.8)$$

Note that these dynamics are completely independent from the x_1 subsystem; they only depend on the controller state and the reference at any given time.

For this subsystem to be stable it is necessary to design the controller \mathcal{K} so as to stabilise the (x_2, x_c) dynamics. This can be done using any standard linear method provided $K_s(x)$ is chosen as indicated in Equation (4.4), the (x_2, x_c) dynamics become completely linear and that the number of nonlinearities is reduced down to one.

Similarly, the dynamics of the x_1 subsystem read

$$\dot{x}_1 = \mathcal{A}_{11}x_1 + \mathcal{A}_{12}(x_1)x_2 + \mathcal{B}_1f_e(t)$$
(4.9)

The dynamics of the x_1 subsystem are still nonlinear, since they are dependent on the sign of x_1 and the output x_2 . Stacking the states x_2, x_c in another state defined as $x_{2c} := [x_2 \ x_c]'$ the complete closed-loop system dynamics can be written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_{2c} \end{bmatrix} = \begin{bmatrix} \mathcal{A}_{11} & \mathcal{A}_{12}(x_1) \\ 0 & \mathcal{A}_{2c} \end{bmatrix} \begin{bmatrix} x_1 \\ x_{2c} \end{bmatrix} + \begin{bmatrix} \mathcal{B}_1 \\ 0 \end{bmatrix} f_e(t) + \begin{bmatrix} 0 \\ \mathcal{B}_{2c} \end{bmatrix} r \quad (4.10)$$

4.3 Closed-loop stability analysis

From the state-space realisation of Equation (4.10), it is clear that the x_{2c} dynamics are excited by the reference r and not the x_1 dynamics. The x_1 dynamics are dependent on the x_{2c} dynamics, however it is useful to observe for the WEC system considered here that

• A_{11} is Hurwitz (this is known from the physical properties of the WEC)

- \mathcal{A}_{2c} is Hurwitz (assuming the controller has been designed appropriately)
- The discontinuous term $\mathcal{A}_{12}(x_1)$ is bounded, i.e. $\|\mathcal{A}_{12}(x_1)\| \leq \beta := A_p/M$ (again due to the WEC physical characteristics)

Thus, the controller \mathcal{K} provides a cascade system in which the two sub-systems in isolation are themselves asymptotically stable. Under the third observation (boundedness), one can prove that if the two subsystems are combined together they are stable (which does not necessarily hold for general nonlinear systems).

To prove *internal stability* of the system of Equation (4.10) let $f_e(t) = r(t) = 0$ and choose the Lyapunov function candidate as the sum of two quadratic Lyapunov functions associated with each sub-system, viz:

$$V(x) = x'_{1}P_{1}x_{1} + \alpha x'_{2c}P_{2c}x_{2c}$$
(4.11)

where the matrices P_1 and P_{2c} are positive definite and $\alpha > 0$ is a free scalar design parameter. The time derivative of this Lyapunov function is

$$\dot{V}(x) = 2x'_{1}P_{1}[\mathcal{A}_{11}x_{1} + \mathcal{A}_{12}(x_{1})x_{2c}] + 2\alpha x'_{2c}P_{2c}\mathcal{A}_{2c}x_{2c}$$

$$= x'_{1}[P_{1}\mathcal{A}_{11} + A'_{11}P_{1}]x_{1} + 2x'_{1}P_{1}\mathcal{A}_{12}(x_{1})x_{2c} + \alpha x'_{2c}[P_{2c}\mathcal{A}_{2c} + A'_{2c}P_{2c}]x_{2c}$$

$$\leq -x'_{1}Q_{1}x_{1} - \alpha x'_{2c}Q_{2c}x_{2c} + 2||x_{1}|||P_{1}|||\mathcal{A}_{12}(x_{1})|||x_{2c}|| \qquad (4.12)$$

where, because \mathcal{A}_{11} and \mathcal{A}_{2c} are Hurwitz, P_1 and P_{2c} can be chosen to satisfy the following Lyapunov equations for $Q_1 > 0$ and $Q_{2c} > 0$

$$P_1 \mathcal{A}_1 + A_1' P_1 = -Q_1 < 0 \tag{4.13}$$

$$P_{2c}\mathcal{A}_{2c} + A'_{2c}P_{2c} = -Q_{2c} < 0 \tag{4.14}$$

Also, noting that $\|\mathcal{A}_{12}(x_1)\| \leq \beta$, enables one to write

$$\dot{V}(x) \leq -x'_{1}Q_{1}x_{1} - \alpha x'_{2c}Q_{2c}x_{2c} + 2\|x_{1}\|\|x_{2c}\|\beta\|P_{1}\| \\
\leq -\lambda_{min}(Q_{1})\|x_{1}\|^{2} - \alpha \lambda_{min}(Q_{2c})\|x_{2c}\|^{2} + 2\|x_{1}\|\|x_{2c}\|\beta\|P_{1}\| \\
= \begin{bmatrix} \|x_{1}\| \\ \|x_{2c}\| \end{bmatrix}' \begin{bmatrix} -\lambda_{min}(Q_{1}) & \beta\|P_{1}\| \\ \star & -\alpha \lambda_{min}(Q_{2c}) \end{bmatrix} \begin{bmatrix} \|x_{1}\| \\ \|x_{2c}\| \end{bmatrix}$$
(4.15)

where α is a free design parameter. It is always possible to choose a large enough α so that this inequality is negative definite, which thus implies the system is *globally* asymptotically stable.

Note that this analysis is independent of the *precise* form of the controller \mathcal{K} : all that is required is that it has the structure given in Equation (4.3) and be such that the matrix \mathcal{A}_{2c} be Hurwitz (which can be done using standard linear design techniques). It is also interesting to observe that the unconstrained controller \mathcal{C}_{wec} given in Section 3.3 (Equation 3.15 (p. 49)) is a special form of this controller and thus it is globally asymptotically stable too.

4.4 Conclusions

In (Nolan 2006) a stability analysis based on writing the system as a switched linear system and then searching for a common Lyapunov function was used. While this technique was able to prove *open-loop* stability (Nolan 2006, p. 220), no conclusions about *closed-loop* stability could be made. The analysis presented in this chapter was based on a more classical approach, where the system was divided into two subsystems that were proven to be stable independently and then subsequently when combined together, closed-loop asymptotic stability was also proven for the complete system dynamics.

Despite the fact that global asymptotic stability was proven for the class of WECs under consideration, based on the specific type of controller, as mentioned in (Nolan 2006) and shown in Section 3.4, the performance of the closed-loop system is still suffering from saturation. Under such conditions, the stability of the closed-loop nonlinear system cannot be guaranteed without some rigorous analysis. In Chapters 5-7, anti-windup techniques will be used for the development of compensators that will augment the existing type of controllers and will ensure that the closed-loop system remains stable even under saturation (Chapters 5-7) and ideally recover some performance as well (Chapter 7).
CHAPTER 5

Nonlinear full-order Internal Model Control (IMC) Anti-Windup (AW) compensator

In this chapter, the partial Nonlinear Dynamic Inversion controller of Chapter 3 is augmented with an Internal Model Control (IMC) anti-windup compensator. The proposed AW compensator is a natural generalisation of a linear IMC compensator, as one of the attractive properties of the IMC AW techniques is that stability is guaranteed unconditionally for open-loop stable linear systems (Herrman et al. 2010). The analysis performed in this chapter will show that this is also the case for the class of nonlinear WEC dynamics considered here. The work in this chapter constitutes the first part of the work presented at the 2013 American Control Conference (Lekka et al. 2013).

5.1 Nominal system

5.1.1 Plant

Recalling the generic class of nonlinear systems under consideration (presented in Chapter 4) represented by the following state-space equations

$$G \sim \begin{cases} \dot{x}_{1} = \mathcal{A}_{11}x_{1} + \mathcal{A}_{12}(x_{1})x_{2} + \mathcal{B}_{1}d \\ \dot{x}_{2} = \mathcal{A}_{21}(x_{1}) + \mathcal{A}_{22}x_{2} + \mathcal{B}_{2}u_{sat} \\ y = \mathcal{C}_{2}x_{2} \end{cases}$$
(5.1)

where $x_1 \in \mathbb{R}^{n_1}$, $x_2 \in \mathbb{R}^{n_2}$ and $n = n_1 + n_2$ is the total state dimension of the plant, $d \in \mathbb{R}^{n_d}$ represents a disturbance vector, $u_{sat} = \operatorname{sat}(u) \in \mathbb{R}^m$ represents the control input to the plant and $y \in \mathbb{R}^p$ represents the output of the system available to the linear part of the controller.

The term $\mathcal{A}_{12}(.) : \mathbb{R}^{n_1} \to \mathbb{R}^{n_1 \times n_2}$ represents an "unmatched" nonlinearity which is not in the range space of the input distribution matrix \mathcal{B}_2 . The term $\mathcal{A}_{21}(.) : \mathbb{R}^{n_1} \to \mathbb{R}^{n_2}$ is the "matched" nonlinearity, since it enters the system through the input distribution matrix \mathcal{B}_2 , and is formalised below.

For the analysis presented next several assumptions are made:

Assumption 5.1. Consider the plant of Equation (5.1). It is assumed that when $d \equiv 0, u_{sat} \equiv 0, G$ is quadratically stable, i.e. there exists a positive definite matrix P > 0 and a scalar $\alpha > 0$ such that with $V(x) = \alpha x' P x$ there exists a scalar $\epsilon > 0$ such that $\dot{V}(x) < -\alpha \epsilon ||x||^2$.

Quadratic stability implies exponential stability which, of course, implies asymptotic stability. Assumption (5.1) stems from the baseline system presented in Section 3.2 and it is similar to that used in linear anti-windup when seeking global results: the open-loop plant should be asymptotically stable (Turner et al. 2007a).

Assumption 5.2. The matrices \mathcal{A}_{11} and \mathcal{A}_{22} are Hurwitz.

The requirement that \mathcal{A}_{11} and \mathcal{A}_{22} be Hurwitz is not strictly necessary in general, but it is compatible with the form of partial-NDI controller introduced next (presented in Chapter 3).

Assumption 5.3. $\mathcal{A}_{21}(z) = \mathcal{B}_2 f_1(z)$ where $f_1(.) : \mathbb{R}^{n_1} \to \mathbb{R}^m$ is Lipschitz with constant k_1 .

Assumption (5.3) effectively requires the nonlinearity in the second state equation to be in the range space of the input distribution matrix and is a structural requirement for the results derived next. Assumption 5.4. $\|\mathcal{A}_{12}(z)\| \leq \beta \quad \forall z \in \mathbb{R}^{n_1}.$

Assumption (5.4) effectively limits the boundedness of the nonlinear term in the first state equation and again is a structural requirement.

5.1.2 Controller

Under nominal conditions, the system above (Equation (5.1)) is assumed to be controlled using a partial-NDI controller. The dynamics of the stabilising partial-NDI *unconstrained* controller are given below:

$$C \sim \begin{cases} \dot{x}_c = \mathcal{A}_c x_c + \mathcal{B}_{cy} (y - \theta_2) + \mathcal{B}_{cr} r \\ u = \mathcal{C}_c x_c + \mathcal{D}_{cy} (y - \theta_2) + \mathcal{D}_{cr} r - f_1(x_1) + \theta_1 \end{cases}$$
(5.2)

where $x_c \in \mathbb{R}^{n_c}$ is the controller's state and $r \in \mathbb{R}^{n_r}$ is the reference input. Assuming the nominal controller has been designed without the saturation effects in mind, the controller is augmented with the output signals $\theta_1 \in \mathbb{R}^m$ and $\theta_2 \in \mathbb{R}^p$ generated by the anti-windup compensator. In the absence of saturation, $\theta_1 \equiv 0$ and $\theta_2 \equiv 0$.

In addition to the measurement of the system output y, the controller is also assumed to have access to either the state x_1 or measurement of the function $f_1(x_1)$. This allows the controller to partially¹ (and only partially) "cancel" the nonlinear terms present.

5.1.3 Closed-loop system

Without anti-windup $(\theta_1 \equiv 0 \text{ and } \theta_2 \equiv 0)$ and without saturation $(u_{sat} = u)$ and with the reference, r, and disturbance, d, set to zero, the *unconstrained* closed-loop dynamics are given by

¹ Nonlinear Dynamic Inversion controllers need to have access to all the states in order to cancel all the system nonlinearities.

$$\begin{cases} \dot{x}_1 = \mathcal{A}_{11}x_1 + \mathcal{A}_{12}(x_1)x_2 \\ \begin{bmatrix} \dot{x}_2 \\ \dot{x}_c \end{bmatrix} = \begin{bmatrix} \mathcal{A}_{22} + \mathcal{B}_2\mathcal{D}_{cy}\mathcal{C}_2 & \mathcal{B}_2\mathcal{C}_c \\ \mathcal{B}_{cy}\mathcal{C}_2 & \mathcal{A}_c \end{bmatrix} \begin{bmatrix} x_2 \\ x_c \end{bmatrix}$$
(5.3)

Similar to Chapter 4, Equation (5.3) can be written more concisely as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_{2c} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathcal{A}_{11} & 0 \\ 0 & \mathcal{A}_{2c} \end{bmatrix}}_{stable} \begin{bmatrix} x_1 \\ x_{2c} \end{bmatrix} + \underbrace{\begin{bmatrix} \mathcal{A}_{12}(x_1)x_2 \\ 0 \end{bmatrix}}_{disturbance}$$
(5.4)

This motivates the assumption given below.

Assumption 5.5. The matrix \mathcal{A}_{2c} is Hurwitz

This is obviously a natural necessary condition for closed-loop stability, and through a quadratic Lyapunov argument (Lekka et al. 2012) and under Assumption (5.1), can be shown to be sufficient for ensuring global asymptotic (exponential) stability of the nominal closed-loop system (as shown in Chapter 4).

5.2 Proposed nonlinear Internal Model Control (IMC) anti-windup (AW) compensator

The anti-windup compensator, similar to the nonlinear IMC compensator found in (Herrman et al. 2010), is basically a copy of the plant augmented with a state-feedback term (Figure 5.1), viz:

$$\mathcal{AW}_{F} \sim \begin{cases} \dot{x}_{a1} = \mathcal{A}_{11}x_{a1} + \mathcal{A}_{12}(x_{a1})x_{a2} \\ \dot{x}_{a2} = \mathcal{A}_{21}(x_{a1}) + \mathcal{A}_{22}x_{a2} - \mathcal{B}_{2}\mathrm{Dz}(u) \\ \theta_{1} = f_{1}(x_{a1}) \\ \theta_{2} = \mathcal{C}_{2}x_{a2} \end{cases}$$
(5.5)

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Chapter 5. Nonlinear full-order Internal Model Control (IMC) Anti-Windup (AW) compensator



Figure 5.1: System block diagram of the proposed nonlinear IMC anti-windup compensator.

where $x_a = \begin{bmatrix} x'_{a1} & x'_{a2} \end{bmatrix}'$ is the anti-windup state vector, with $x_{a1} \in \mathbb{R}^{n_1}$ and $x_{a2} \in \mathbb{R}^{n_2}$. As with standard linear anti-windup compensators, the above nonlinear IMC compensator is driven by the deadzone function $u_{aw} := Dz = u - u_{sat}$, only when $u \neq u_{sat}$ meaning that it is activated upon saturation occurring.

Before stating the main result, it is useful to perform a coordinate transformation² similar to (Kapoor and Daoutidis 1999, Herrman et al. 2010, Valmórbida et al. 2011).

Let $r \equiv 0$ and $d \equiv 0$ and $u_{sat} = \operatorname{sat}(u)$ and define the error coordinates $e_1 := x_1 - x_{a1}$, $e_2 := x_2 - x_{a2}$ and $e_{2c} := [e'_2 \quad x'_c]'$. The closed-loop interconnection of the plant (Equation (5.1)), controller (Equation (5.2)) and anti-windup compensator (Equation (5.5)) can be described in the $(e_1, e_{2c}, x_{a1}, x_{a2})$ coordinates as follows:

² Changing the coordinates of a system is a change of its internal representation, not a change of the system itself. Since the state-space representation of a system is not unique, if something is proven in one set of coordinates (i.e. stability), it will hold for another set of coordinates for the same system.

$$\begin{cases} \dot{e}_{1} = \mathcal{A}_{11}e_{1} + \mathcal{A}_{12}(x_{1})x_{2} - \mathcal{A}_{12}(x_{a1})x_{a2} \\ \dot{e}_{2c} = \mathcal{A}_{2c}e_{2c} \\ \dot{x}_{a1} = \mathcal{A}_{11}x_{a1} + \mathcal{A}_{12}(x_{a1})x_{a2} \\ \dot{x}_{a2} = \mathcal{A}_{21}(x_{a1}) + \mathcal{A}_{22}x_{a2} - \mathcal{B}_{2}\mathrm{Dz}(u) \\ u = \mathcal{C}_{2c}e_{2c} - f_{1}(x_{1}) + f_{1}(x_{a1}) \end{cases}$$

$$(5.6)$$

where $C_{2c} = \begin{bmatrix} C_2 D_{cy} & C_c \end{bmatrix}$. The derivative of e_{2c} is derived as follows:

$$\dot{e}_{2c} = \begin{bmatrix} \dot{x}_2 - \dot{x}_{a2} \\ \dot{x}_c \end{bmatrix} = \begin{bmatrix} \mathcal{A}_{21}(\boldsymbol{x}_1) + \mathcal{A}_{22}\boldsymbol{x}_2 + \mathcal{B}_2\boldsymbol{u}_{sat} - \mathcal{A}_{21}(\boldsymbol{x}_{a1}) - \mathcal{A}_{22}\boldsymbol{x}_{a2} + \mathcal{B}_2\boldsymbol{D}\boldsymbol{z}(\boldsymbol{u}) \\ \mathcal{A}_c\boldsymbol{x}_c + \mathcal{B}_{cy}(\boldsymbol{y} - \boldsymbol{\theta}_2) \end{bmatrix}$$
(5.7)

Rearranging Equation 2.19 (p. 28) and substituting for Dz(u), the previous equation becomes:

$$\dot{e}_{2c} = \begin{bmatrix} \mathcal{A}_{21}(\boldsymbol{x}_1) + \mathcal{A}_{22}(\boldsymbol{x}_2 - \boldsymbol{x}_{a2}) - \mathcal{A}_{21}(\boldsymbol{x}_{a1}) + \mathcal{B}_2\left(\boldsymbol{u} - D\boldsymbol{z}(\boldsymbol{u}) + D\boldsymbol{z}(\boldsymbol{u})\right) \\ \mathcal{A}_c \boldsymbol{x}_c + \mathcal{B}_{cy} \mathcal{C}_2(\boldsymbol{x}_2 - \boldsymbol{x}_{a2}) \end{bmatrix}$$
(5.8)
$$\begin{bmatrix} \mathcal{A}_{21}(\boldsymbol{x}_1) - \mathcal{A}_{21}(\boldsymbol{x}_{a1}) + \mathcal{A}_{22}\boldsymbol{e}_2 + \mathcal{B}_2\boldsymbol{u} \end{bmatrix}$$
(5.9)

$$= \begin{bmatrix} \mathcal{A}_{21}(x_1) - \mathcal{A}_{21}(x_{a1}) + \mathcal{A}_{22}e_2 + \mathcal{B}_2 u\\ \mathcal{A}_c x_c + \mathcal{B}_{cy} \mathcal{C}_2 e_2 \end{bmatrix}$$
(5.9)

Now, u is equal to:

$$u = \mathcal{C}_c \boldsymbol{x}_c + \mathcal{D}_{cy}(\boldsymbol{y} - \boldsymbol{\theta}_2) - f_1(\boldsymbol{x}_1) + \boldsymbol{\theta}_1$$
(5.10)

$$= C_c x_c + D_{cy} C_2 e_2 - f_1(x_1) + f_1(x_{a1})$$
(5.11)

and substituting in Equation (5.9) yields:

$$\dot{e}_{2c} = \begin{bmatrix} \mathcal{A}_{21}(x_1) - \mathcal{A}_{21}(x_{a1}) + \mathcal{A}_{22}e_2 + \mathcal{B}_2\left(\mathcal{C}_c x_c + \mathcal{D}_{cy}\mathcal{C}_2 e_2 - f_1(x_1) + f_1(x_{a1})\right) \\ \mathcal{A}_c x_c + \mathcal{B}_{cy}\mathcal{C}_2 e_2 \end{bmatrix}$$

$$= \begin{bmatrix} \mathcal{A}_{22}e_2 + \mathcal{B}_2\mathcal{C}_c x_c + \mathcal{B}_2\mathcal{D}_{cy}\mathcal{C}_2 e_2 + \mathcal{B}_2\left(f_1(x_{a1}) - f_1(x_1)\right) + \mathcal{A}_{21}(x_1) - \mathcal{A}_{21}(x_{a1}) \\ \mathcal{A}_c x_c + \mathcal{B}_{cy}\mathcal{C}_2 e_2 \end{bmatrix}$$
(5.13)

which from Assumption 5.3 (p. 64) becomes:

$$\dot{e}_{2c} = \begin{bmatrix} \mathcal{A}_{22}e_2 + \mathcal{B}_2\mathcal{C}_c x_c + \mathcal{B}_2\mathcal{D}_{cy}\mathcal{C}_2 e_2 + \mathcal{B}_2\left(f_1(x_{a1}) - f_1(x_1) + f_1(x_1) - f_1(x_{a1})\right) \\ \mathcal{A}_c x_c + \mathcal{B}_{cy}\mathcal{C}_2 e_2 \end{bmatrix}$$

$$=\underbrace{\begin{bmatrix} \mathcal{A}_{22} + \mathcal{B}_2 \mathcal{D}_{cy} \mathcal{C}_2 & \mathcal{B}_2 \mathcal{C}_c \\ \mathcal{B}_{cy} \mathcal{C}_2 & \mathcal{A}_c \end{bmatrix}}_{\mathcal{A}_{2c}} \underbrace{\begin{bmatrix} e_2 \\ x_c \end{bmatrix}}_{e_{2c}}$$
(5.15)

Proposition 5.1. Consider the interconnection of Equations (5.1), (5.2) and (5.5) through $u_{sat} = sat(u)$. Let $r \equiv 0$, $d \equiv 0$ and let Assumptions (5.1) and (5.5) be satisfied. Then the origin of the interconnected system is globally exponentially stable.

Proof 5.1. As indicated above, instead of working in the (x_1, x_{2c}, x_a) coordinates, it is more convenient to work in the (e_1, e_{2c}, x_a) coordinates. Thus consider the closed-loop interconnection represented by Equation (5.6). The approach taken in the proof is to define quadratic Lyapunov function candidates for each "subsystem" and then to sum together these functions, and their time derivatives, in order to obtain a positive definite (radially unbounded) Lyapunov function with a negative definite derivative.

Lyapunov function candidate 5.1. Let $V_a(x_a) = \alpha x'_a P_a x_a > 0$ be a quadratic Lyapunov function candidate. Then the time derivative of this function along the trajectories of the x_a subsystem is

$$\dot{V}_a(x_a, t) = \frac{\partial V(x_a, t)}{\partial x_a} \frac{dx_a}{dt} = 2\alpha x'_a P_a \dot{x}_a$$
(5.16)

Substituting $\dot{x}_{a1}, \dot{x}_{a2}$ from Equation (5.6) yields:

$$\dot{V}_{a}(x_{a}) = 2\alpha x_{a}' P_{a} \begin{bmatrix} \mathcal{A}_{11}x_{a1} + \mathcal{A}_{12}(x_{a1})x_{a2} \\ \mathcal{A}_{21}(x_{a1}) + \mathcal{A}_{22}x_{a2} - \mathcal{B}_{2}Dz(u) \end{bmatrix}$$
(5.17)

where $\phi := \mathrm{Dz}(u)$.

$$\dot{V}_{a}(x_{a}) = 2\alpha x_{a}' P_{a} \left(\begin{bmatrix} \mathcal{A}_{11}x_{a1} + \mathcal{A}_{12}(x_{a1})x_{a2} \\ \mathcal{A}_{21}(x_{a1}) + \mathcal{A}_{22}x_{a2} \end{bmatrix} - \begin{bmatrix} 0 & \mathcal{B}_{2}' \end{bmatrix}' \phi \right)$$
(5.18)

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$$= 2\alpha x'_{a} P_{a} \begin{bmatrix} \mathcal{A}_{11} x_{a1} + \mathcal{A}_{12} (x_{a1}) x_{a2} \\ \mathcal{A}_{21} (x_{a1}) + \mathcal{A}_{22} x_{a2} \end{bmatrix} - 2\alpha x'_{a} P_{a} [0 \quad \mathcal{B}'_{2}]' \phi$$
(5.19)

Due to Assumption 5.1 (p. 64), it thus follows that there exists an $\epsilon > 0$ such that

$$\dot{V}_a(x_a) \le -\alpha \epsilon \|x_a\|^2 - 2\alpha x'_a P_a[0 \quad \mathcal{B}'_2]'\phi$$
(5.20)

Then using the sector condition (Equation 2.15 (p. 25)) yields

$$\dot{V}_{a}(x_{a}) \leq -\alpha \epsilon \|x_{a}\|^{2} - 2\alpha x_{a}' P_{a}[0 \quad \mathcal{B}_{2}']' \phi \qquad (5.21)$$

$$+ 2\phi' W[\underbrace{\mathcal{C}_{2c}e_{2c} - f_{1}(x_{1}) + f_{1}(x_{a1})}_{u \ from \ Eq. \ (5.6)} - \phi] \qquad (5.21)$$

$$\leq \begin{bmatrix} x_{a} \\ \phi \end{bmatrix}' \begin{bmatrix} -\alpha \epsilon I_{n} \quad \alpha P_{a} \begin{bmatrix} 0 \\ \mathcal{B}_{2} \\ \star \quad -2W \end{bmatrix} \begin{bmatrix} x_{a} \\ \phi \end{bmatrix} \qquad (5.22)$$

$$+ 2\|\phi\|\|W\| \left(\|\mathcal{C}_{2c}e_{2c}\| + \|f_{1}(x_{1}) - f_{1}(x_{a1})\|\right)$$

where W is some positive definite diagonal matrix to be decided by the designer. Thus, using Young's inequality (Khalil 2002)

$$2x'y \le \gamma \|x\|^2 + \frac{1}{\gamma} \|y\|^2 \qquad \forall x \in \mathbb{R}^p, y \in \mathbb{R}^q, \gamma > 0$$
(5.23)

it follows that for any $\delta_a > 0$,

$$\dot{V}(x_a) \leq \begin{bmatrix} x_a \\ \phi \end{bmatrix}' \begin{bmatrix} -\alpha \epsilon & \alpha P_a \begin{bmatrix} 0 \\ \mathcal{B}_2 \end{bmatrix} \\ \star & -2W \end{bmatrix} \begin{bmatrix} x_a \\ \phi \end{bmatrix}$$

$$+ \delta_a \|\phi\|^2 \|W\|^2 + \frac{2}{\delta_a} (\|\mathcal{C}_{2c}\|^2 \|e_{2c}\|^2 + k_1^2 \|e_1\|^2)$$
(5.24)

Lyapunov function candidate 5.2. Let $V_1(e_1) = e'_1 P_1 e_1 > 0$ be a quadratic Lyapunov function candidate. The time derivative of this function along the trajectories

of the e_1 subsystem is then

$$\dot{V}_1(e_1, t) = \frac{\partial V(e_1, t)}{\partial e_1} \frac{de_1}{dt} = e'_1 P_1 \dot{e}_1$$
(5.25)

Substituting \dot{e}_1 from Equation 5.6 (p. 68) yields:

$$\dot{V}_1(e_1) = 2e'_1 P_1[\mathcal{A}_{11}e_1 + \mathcal{A}_{12}(x_1)x_2 - \mathcal{A}_{12}(x_{a1})x_{a2}]$$
(5.26)

$$= 2e'_{1}P_{1}\mathcal{A}_{11}e_{1} + 2e'_{1}P_{1}[\mathcal{A}_{12}(x_{1})x_{2} - \mathcal{A}_{12}(x_{a1})x_{a2}]$$
(5.27)

$$\leq e_1' P_1 \mathcal{A}_{11} e_1 + e_1' P_1 \mathcal{A}_{11} e_1 + 2e_1' P_1 [\mathcal{A}_{12}(x_1) x_2 - \mathcal{A}_{12}(x_{a1}) x_{a2}]$$
(5.28)

$$\leq e_1'(P_1\mathcal{A}_{11} + \mathcal{A}_{11}'P_1)e_1 + 2\|e_1\|\|\mathcal{A}_{12}(x_1)x_2 - \mathcal{A}_{12}(x_{a1})x_{a2}\|$$
(5.29)

Now, by adding and subtracting the term $\mathcal{A}_{12}(x_{a1})x_2$ from $\mathcal{A}_{12}(x_1)x_2 - \mathcal{A}_{12}(x_{a1})x_{a2}$ yields

$$\mathcal{A}_{12}(x_1)x_2 - \mathcal{A}_{12}(x_{a1})x_{a2} \tag{5.30}$$

$$= \mathcal{A}_{12}(x_1)x_2 - \mathcal{A}_{12}(x_{a1})x_2 + \mathcal{A}_{12}(x_{a1})x_2 - \mathcal{A}_{12}(x_{a1})x_{a2}$$
(5.31)

$$= [\mathcal{A}_{12}(x_1) - \mathcal{A}_{12}(x_{a1})]x_2 + \mathcal{A}_{12}(x_{a1})e_2$$
(5.32)

So,

$$\dot{V}_{1}(e_{1}) \leq e_{1}'(P_{1}\mathcal{A}_{11} + \mathcal{A}_{11}'P_{1})e_{1}$$

$$+ 2\|e_{1}\|\|P_{1}\|\left[\|\mathcal{A}_{12}(\boldsymbol{x}_{1}) - \mathcal{A}_{12}(\boldsymbol{x}_{a1})\|\|\boldsymbol{x}_{2}\| + \|\mathcal{A}_{12}(\boldsymbol{x}_{a1})\|\|e_{2}\|\right]$$

$$(5.33)$$

After the application of Assumption 5.4 (p. 64)

$$\dot{V}_{1}(e_{1}) \leq e_{1}'(P_{1}\mathcal{A}_{11} + \mathcal{A}_{11}'P_{1}')e_{1}$$

$$+ 2\|e_{1}\|\|P_{1}\|\Big[\big(\|\mathcal{A}_{12}(\boldsymbol{x}_{1})\| + \|\mathcal{A}_{12}(\boldsymbol{x}_{a1})\|\big)\|\boldsymbol{x}_{2}\| + \beta\|e_{2}\|\Big]$$

$$\leq e_{1}'(P_{1}\mathcal{A}_{12} + \mathcal{A}_{12}'P_{1}') + 2\|P_{1}\|[2P_{1}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|P_{2}\|$$

$$\leq e_1'(P_1\mathcal{A}_{11} + \mathcal{A}_{11}'P_1')e_1 + 2\|e_1\|\|P_1\|\left[2\beta\|x_2\| + \beta\|e_2\|\right]$$
(5.35)

$$\leq e_1'(P_1\mathcal{A}_{11} + \mathcal{A}_{11}'P_1')e_1 + 2\|e_1\|\|P_1\|\left[3\beta\|e_2\| + 2\beta\|x_{a2}\|\right]$$
(5.36)

$$\leq e_1'(P_1\mathcal{A}_{11} + \mathcal{A}_{11}'P_1')e_1 + 2\|e_1\|\|P_1\|\left[3\beta\|e_{2c}\| + 2\beta\|x_a\|\right]$$
(5.37)

So, $3\beta ||e_2|| < 3\beta ||e_{2c}||$ and similarly, $2\beta ||x_{a2}|| < 2\beta ||x_a||$, where the terms involving β arise from repeated application of Assumption (5.4). This implies, for any δ_1 , that

$$\dot{V}_1(e_1) \le e_1'(P_1\mathcal{A}_{11} + \mathcal{A}_{11}'P_1)e_1 + 2\|e_1\|\|P_1\|\beta(3\|e_{2c}\| + 2\|x_a\|)$$
(5.38)

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$$\leq e_1'(P_1\mathcal{A}_{11} + \mathcal{A}_{11}'P_1 + \delta_1 \|P_1\|^2 I)e_1 + \frac{2\beta^2}{\delta_1}(9\|e_{2c}\|^2 + 4\|x_a\|^2)$$
(5.39)

Lyapunov function candidate 5.3. Let $V_2(e_{2c}) = e'_{2c}P_2e_{2c} > 0$ be a quadratic Lyapunov function candidate. The time derivative of this function along the trajectories of the e_{2c} subsystem is then

$$\dot{V}_{2c}(e_{2c},t) = \frac{\partial V_2(e_{2c},t)}{\partial e_{2c}} \frac{de_{2c}}{dt}$$
(5.40)

$$= 2e'_{2c}P_1\dot{e}_{2c} \tag{5.41}$$

$$= e_{2c}'(\mathcal{A}_{2c}'P_2 + P_2\mathcal{A}_{2c})e_{2c} \tag{5.42}$$

Sum of Lyapunov function candidates 5.1. Define the state-vector of the interconnection as $\xi := [e'_1 \ e'_{2c} \ x'_a]'$, then a Lyapunov function candidate for the entire system can be formed as

$$V_{tot}(\xi) := V_a(x_a) + V_1(e_1) + V_{2c}(e_{2c}) > 0$$

Obviously V_{tot} is positive definite and radially unbounded. Using the expressions above, its derivative is bounded by

$$\begin{split} \dot{V}_{tot}(\xi) &= \dot{V}_{a}(x_{a}) + \dot{V}_{1}(e_{1}) + \dot{V}_{2c}(e_{2c}) \\ &\leq \begin{bmatrix} x_{a} \\ \phi \end{bmatrix}' \begin{bmatrix} -a\epsilon & \alpha P_{a} \begin{bmatrix} 0 \\ \mathcal{B}_{2} \\ \star & -2W \end{bmatrix} \begin{bmatrix} x_{a} \\ \phi \end{bmatrix} + \delta_{a} ||W||^{2} ||\phi||^{2} \\ &+ \frac{1}{\delta_{a}} \left(||\mathcal{C}_{2c}||^{2} ||e_{2c}||^{2} + k_{1}^{2} ||e_{1}||^{2} \right) \\ &+ e_{1}' \left(P_{1}\mathcal{A}_{11} + \mathcal{A}_{11}'P_{1} + \delta_{1} ||P_{1}||^{2}I \right) e_{1} + \frac{2\beta^{2}}{\delta_{1}} (9||e_{2c}||^{2} + 4||x_{a}||^{2}) \\ &+ e_{2c}' \left(\mathcal{A}_{2c}'P_{2} + P_{2}\mathcal{A}_{2c} \right) e_{2c} \\ &\leq \begin{bmatrix} x_{a} \\ \phi \end{bmatrix}' \begin{bmatrix} -a\epsilon & \alpha P_{a} \begin{bmatrix} 0 \\ \mathcal{B}_{2} \\ \star & -2W \end{bmatrix} \end{bmatrix} \begin{bmatrix} x_{a} \\ \phi \end{bmatrix} + \delta_{a} ||W||^{2} ||\phi||^{2} \\ &+ \frac{1}{\delta_{a}} ||\mathcal{C}_{2c}||^{2} ||e_{2c}||^{2} + \frac{1}{\delta_{a}} k_{1}^{2} ||e_{1}||^{2} \\ &+ e_{1}' \left(P_{1}\mathcal{A}_{11} + \mathcal{A}_{11}'P_{1} + \delta_{1} ||P_{1}||^{2}I \right) e_{1} + \frac{18\beta^{2}}{\delta_{1}} ||e_{2c}||^{2} + \frac{8\beta^{2}}{\delta_{1}} ||x_{a}||^{2} \end{split}$$
(5.44)

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$$+ e_{2c}'(\mathcal{A}_{2c}'P_{2} + P_{2}\mathcal{A}_{2c})e_{2c}$$

$$\leq \begin{bmatrix} x_{a} \\ \phi \end{bmatrix}' \begin{bmatrix} (-\alpha\epsilon + \frac{4\beta^{2}}{\delta_{1}})I_{n} & \alpha P_{a} \begin{bmatrix} 0 \\ \mathcal{B}_{2} \end{bmatrix} \\ \star & -2W + \delta_{a} ||W||^{2}I_{m} \end{bmatrix} \begin{bmatrix} x_{a} \\ \phi \end{bmatrix}$$

$$+ e_{1}'(P_{1}\mathcal{A}_{11} + \mathcal{A}_{11}'P_{1} + \delta_{1} ||P_{1}||^{2}I_{n_{1}} + \frac{2k_{1}^{2}}{\delta_{a}}I_{n_{1}})e_{1}$$

$$+ e_{2c}'(\mathcal{A}_{2c}'P_{2} + P_{2}\mathcal{A}_{2c} + \left(\frac{18\beta^{2}}{\delta_{1}} + \frac{2||\mathcal{C}_{2c}||^{2}}{\delta_{a}}\right)I_{n_{2}+n_{c}})e_{2c}$$
(5.45)

For stability it is sufficient for each of the three terms in the above inequality to themselves be negative definite. Consider each term individually.

1.

$$M_1 := \begin{bmatrix} (-\alpha \epsilon + \frac{4\beta^2}{\delta_1})I_n & \alpha P_a \begin{bmatrix} 0\\ \mathcal{B}_2 \end{bmatrix} \\ \star & -2W + \delta_a \|W\|^2 I_m \end{bmatrix}$$

For any (arbitrarily small) δ_1 , it is always possible to choose α sufficiently large such that $-\alpha \epsilon + \frac{4\beta^2}{\delta_1} < 0$. Next, for any W it is possible to choose, δ_a sufficiently small such that $-2W + \delta_a ||W||^2 < 0$ for any W > 0. Finally, choosing $W = \alpha_W \tilde{W}$, it is always possible to choose $\alpha_W > 0$ sufficiently large such that $M_1 < 0$.

2.

$$M_2 := P_1 \mathcal{A}_{11} + \mathcal{A}'_{11} P_1 + \delta_1 \|P_1\|^2 I_{n_1} + \frac{k_1^2}{\delta_a} I_{n_1}$$

By Assumption 5.1 (p. 64) there exists a \tilde{P}_1 such that $\tilde{P}_1 \mathcal{A}_{11} + \mathcal{A}'_{11}\tilde{P}_1 = -\tilde{Q}_1 < 0$. Then with $P_1 = \alpha_1 \tilde{P}_1$, choosing δ_1 sufficiently small ensures that $\alpha_1(\tilde{P}_1 \mathcal{A}_{11} + \mathcal{A}'_{11}\tilde{P}_1) + \alpha_1^2 \delta_1 \|\tilde{P}_1\|^2 I_{n_1} < 0$ for any \tilde{P}_1 . Next choosing α_1 sufficiently large ensures that $M_2 < 0$.

3.

$$M_3 := \mathcal{A}'_{2c} P_2 + P_2 \mathcal{A}_{2c} + \frac{9\beta^2}{\delta_1} I_{n_2+n_c} + \frac{\|\mathcal{C}_{2c}\|^2}{\delta_a} I_{n_2+n_c}$$

By Assumption 5.5 (p. 66) there exists a $P_2 = \alpha_2 \tilde{P}_2$ such that $\alpha_2 (\mathcal{A}'_{2c} \tilde{P}_2 + \tilde{P}_2 \mathcal{A}_{2c}) = -\alpha_2 Q_2 < 0$. Choosing α_2 sufficiently large then implies that M_3 is negative definite for any given $\beta, \delta_1, \delta_a$ and \mathcal{C}_{2c} .

Thus by judicious choice of the free parameters W, δ_a , δ_1 , α , α_1 and α_2 , it is always

possible to ensure $\dot{V}_{tot}(\xi) \leq -\eta \|\xi\|^2$ and, thus, the system is globally *exponentially* stable.

5.3 Simulation results

The following table shows the values of every term used in the models presented in this thesis and in simulations (Nolan 2006, Eidsmoen 1995):

m_b	mass of the buoy	9700 Kg
$m_{r(\infty)}$	added mass at infinite frequency	8700 Kg
S	hydrostatic stiffness of the buoy	$86.4 \ kN/m$
R_f	friction resistance coefficient	200 Kg
A_p	pump area	$0.1m^2$
p_o	accumulator pre-charge pressure	$2.5 \cdot 10^{6} P_{a}$
γ	ratio of specific heats of gas at constant	1.4
	temperature and volume	
C_{acc}	accumulator capacitance	$2.464 \cdot 10^{-7} \ m^3/P_a$
Posm	RO osmotic pressure	$27 \cdot 10^5 P_a$
ρ_{ro}	RO permeability coefficient (single unit)	$1.2121 \cdot 10^{-10}$
		$m^3/s/P_a$
C_r	Rated valve flow coefficient	$1.1397 \cdot 10^{-5} \ m^3 P_a^{0.5}/s$
R_c	Static approximation of kernel $k(t)$	722.1

Table 5.1: Parameters used for design and simulation

As shown in Chapter 3, the nominal unconstrained system tracks the reference signal perfectly at the expense of an infeasible control signal, hence results regarding the unconstrained system are omitted here since they are similar to those of Chapter 3.

Figure 5.2 shows that the nominal system under saturation loses its stability properties leading to a possible WEC structural damage; on the other hand, the proposed nonlinear IMC compensator stabilises the system, thus maintaining the WEC structural integrity, however the reference is not tracked, i.e. the water produced will not be of sufficient quality to be rendered potable.

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Figure 5.2: Output of the closed-loop system under saturation and the AW compensator for $H_s = 0.5$ m and $T_1 = 10$ s.

Looking at the control signal of the nominal system under saturation (Figure 5.3) one can see that the saturated control signal spends most of the time at the lower saturation limit. It is well known that an AW compensator can be effective when there is occasional saturation, i.e. when the system falls in and out of saturation intermittently. This could explain the fact that the proposed compensator cannot fully recover the performance desired (in terms of the reference tracking, since stability is ensured).



Figure 5.3: Control signal of the closed-loop system under saturation and the AW compensator for $H_s = 0.5$ m and $T_1 = 10$ s.

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Figure 5.4: Output of the closed-loop system under saturation for $H_s = 0.8 \text{ m}$ and $T_1 = 10 \text{ s}$.

In Figure 5.4 and for a wave amplitude of 0.8 m, one can see that the output of the saturated system reaches very high values of pressure, which will most certain destroy the RO membrane, while the proposed IMC AW compensator maintains system stability (the output is bounded) and despite not achieving enough tracking to render the water extracted of adequate quality³, it preserves the membrane intact.

As mentioned earlier, as the system exits saturation only for a brief time (Figure 5.5) it is very difficult for the IMC (or any other AW compensator) to have an effect on the system's tracking ability.

 $^{^3}$ It is reminded here that the system output should vary between $\pm 3\%$ of the reference.

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Figure 5.5: Control signal of the closed-loop system under saturation for $H_s = 0.8$ m and $T_1 = 10$ s.

Similarly, in Figure 5.6 one can see that both the saturated system and the AW closed-loop system stay within the allowable tracking limits, however the saturated system exhibits an overshoot that will indeed compromise the integrity of the RO membrane, while this can be avoided by augmenting the system with the proposed IMC compensator.



Figure 5.6: Output of the closed-loop system under saturation for $H_s = 1.3$ m and $T_1 = 10$ s.

In Figure 5.7 one can see that the saturated control signal varies between the two

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saturation limits, i.e. the system is not constantly in saturation and therefore the proposed AW compensator has proven effective in tracking the reference during saturation, besides ensuring stability.



Figure 5.7: Control signal of the closed-loop system under saturation for $H_s = 1.3 \text{ m}$ and $T_1 = 10 \text{ s}$.

5.4 Conclusions

In this chapter, a nonlinear IMC AW compensator has been proposed; the compensator is a copy of the nonlinear switched plant falling into the class of systems presented at the beginning of this chapter. This IMC compensator ensures global exponential stability of the overall closed-loop nonlinear system and its synthesis requires no optimisation. Simulation results have verified that the proposed AW compensator stabilises the saturated closed-loop system in all cases, however tracking can only be achieved when the nominal system is not constantly under saturation. The fact that it features the same number of states as present in the plant may cast it as computationally expensive to implement.

CHAPTER 6

Linear reduced-order AW compensator

In this chapter, a new anti-windup (AW) compensator is proposed. Its structure is similar to the one proposed in Section 5.2, but its order is reduced as the analysis is based on the x_2 subsystem dynamics. It is proven that this reduced-order AW compensator ensures global stability for the class of systems studied in the thesis.

One of the main reasons for looking at the design of a lower order AW compensator is the fact that the dynamics of full-order AW compensators may contain unnecessary complex dynamics that result in poor time domain behaviour (Brieger et al. 2008). In (Brieger et al. 2008), it was shown that low-order AW compensators that retain the advantages of full-order compensators can perform better and are more easily implementable and hence practical.

Before moving on to presenting the work done in this chapter one should take into consideration that the WEC dynamics presented and used throughout the thesis are of low order already (only 3 states in the system), so the advantages of using a low-order compensator may not be so obvious. Nevertheless, the nonlinear AW theory developed in this thesis is valid for a more general set of nonlinear systems whose dynamics may be of higher order; in such case a reduced-order compensator similar to the one presented in this chapter could provide the advantages of low-order compensators mentioned earlier.

6.1 Proposed linear reduced-order AW compensator

The partial linear controller (Equation 5.2 (p. 65)) works, under nominal conditions, by cancelling the nonlinear terms in the x_2 state equation and so the linear part of the controller is chosen so as to bestow desirable properties on the x_2 dynamics. Due to the stability of the x_1 subsystem, this part of the plant will remain well-behaved and not interfere with the system's operation (Lekka et al. 2012) (as shown in Chapter 4).

Complete decoupling of the two sub-systems becomes impossible when saturation occurs, but it does suggest that focusing anti-windup design only on the x_2 subsystem could be beneficial.



Figure 6.1: System block diagram of the proposed linear reduced-order anti-windup compensator.

With the x_{a1} state equation omitted and x_{a1} , and functions thereof, set to zero wherever else they appear, the proposed compensator dynamics are described by the

following state-space description:

$$\mathcal{AW}_{RO} \sim \begin{cases} \dot{x}_{a2} = \mathcal{A}_{22} x_{a2} - \mathcal{B}_2 \mathrm{Dz}(u) \\ \theta_1 = 0 \\ \theta_2 = \mathcal{C}_2 x_{a2} \end{cases}$$
(6.1)

Note that this compensator is of order n_2 and is itself linear. It looks very much like a reduced-order IMC compensator (and if the x_1 dynamics were indeed absent, would operate exactly like a linear IMC compensator) and it transpires that it is able to guarantee stability of the full nonlinear system.

Proposition 6.1. Consider the interconnection of the plant (Equation 5.1 (p. 64)), the controller (Equation 5.2 (p. 65)) and the proposed AW compensator (Equation (6.1)) through $u_{sat} = sat(u)$. Let $r \equiv 0$, $d \equiv 0$ and Assumptions 5.1 (p. 64) and 5.5 (p. 66) be satisfied. Then the origin of the interconnected system is globally exponentially stable.

Proof 6.1. Note that with $x_{a1} \equiv 0$, and recalling the notation from the proof of Proposition 5.2 (p. 69), $e_1 \equiv x_1$. The approach taken in the proof is similar to that in Proposition (5.2) and involves one proving that the system in the coordinates (e_1, e_2, x_c, x_{a2}) is exponentially stable. This is achieved by defining quadratic Lyapunov function candidates for each subsystem and then summing together these functions, and their time derivatives, in order to obtain a positive definite (radially unbounded) Lyapunov function with a negative definite derivative. It proceeds in the same way as in Section 5.2, i.e. the closed-loop interconnection of Equations (5.1), (5.2) and (6.1) can be described in the $(e_1, e_{2c}, x_{a1}, x_{a2})$ coordinates as follows:

$$\begin{pmatrix}
\dot{e}_{1} = \mathcal{A}_{11}e_{1} + \mathcal{A}_{12}(x_{1})x_{2} \\
\dot{e}_{2c} = \mathcal{A}_{2c}e_{2c} \\
\dot{x}_{a2} = \mathcal{A}_{22}x_{a2} - \mathcal{B}_{2}\mathrm{Dz}(u) \\
u = \mathcal{C}_{2c}e_{2c} - f_{1}(x_{1})
\end{cases}$$
(6.2)

where $C_{2c} = [C_2 D_{cy} \quad C_c]$. Similar to Section 5.2, the derivative of e_{2c} is derived as follows:

$$\dot{e}_{2c} = \begin{bmatrix} \dot{x}_2 - \dot{x}_{a2} \\ \dot{x}_c \end{bmatrix} = \begin{bmatrix} \mathcal{A}_{21}(\boldsymbol{x}_1) + \mathcal{A}_{22}\boldsymbol{x}_2 + \mathcal{B}_2\boldsymbol{u}_{sat} - \mathcal{A}_{22}\boldsymbol{x}_{a2} + \mathcal{B}_2\boldsymbol{D}\boldsymbol{z}(\boldsymbol{u}) \\ \mathcal{A}_c\boldsymbol{x}_c + \mathcal{B}_{cy}(\boldsymbol{y} - \theta_2) \end{bmatrix}$$
(6.3)

Using Equation 2.19 (p. 28) the previous one becomes:

$$\dot{e}_{2c} = \begin{bmatrix} \mathcal{A}_{21}(x_1) + \mathcal{A}_{22}(x_2 - x_{a2}) + \mathcal{B}_2(u - Dz(u) + Dz(u)) \\ \mathcal{A}_c x_c + \mathcal{B}_{cy} \mathcal{C}_2(x_2 - x_{a2}) \end{bmatrix}$$
(6.4)

$$= \begin{bmatrix} \mathcal{A}_{21}(\boldsymbol{x}_1) + \mathcal{A}_{22}\boldsymbol{e}_2 + \mathcal{B}_2\boldsymbol{u} \\ \mathcal{A}_c\boldsymbol{x}_c + \mathcal{B}_{cy}\mathcal{C}_2\boldsymbol{e}_2 \end{bmatrix}$$
(6.5)

Now, u is equal to (from Equation 5.2 (p. 65)):

$$u = \mathcal{C}_c x_c + \mathcal{D}_{cy}(y - \theta_2) - f_1(x_1) + \theta_1$$
(6.6)

$$= \mathcal{C}_c x_c + \mathcal{D}_{cy} \mathcal{C}_2 e_2 - f_1(x_1) \tag{6.7}$$

and substituting in Equation (6.5) yields:

$$\dot{e}_{2c} = \begin{bmatrix} \mathcal{A}_{21}(x_1) - \mathcal{A}_{21}(x_{a1}) + \mathcal{A}_{22}e_2 + \mathcal{B}_2\left(\mathcal{C}_c x_c + \mathcal{D}_{cy}\mathcal{C}_2 e_2 - f_1(x_1)\right) \\ \mathcal{A}_c x_c + \mathcal{B}_{cy}\mathcal{C}_2 e_2 \end{bmatrix}$$
(6.8)

which from Assumption 5.3 (p. 64) becomes:

$$\dot{e}_{2c} = \begin{bmatrix} \mathcal{A}_{21}(x_1) + \mathcal{A}_{22}e_2 + \mathcal{B}_2\mathcal{C}_c x_c + \mathcal{B}_2\mathcal{D}_{cy}\mathcal{C}_2 e_2 - \mathcal{B}_2 f_1(x_1) \\ \mathcal{A}_c x_c + \mathcal{B}_{cy}\mathcal{C}_2 e_2 \end{bmatrix}$$
(6.9)

and collecting terms in state-space form

$$\dot{e}_{2c} = \underbrace{\begin{bmatrix} \mathcal{A}_{22} + \mathcal{B}_2 \mathcal{D}_{cy} \mathcal{C}_2 & \mathcal{B}_2 \mathcal{C}_c \\ \mathcal{B}_{cy} \mathcal{C}_2 & \mathcal{A}_c \end{bmatrix}}_{\mathcal{A}_{2c}} \underbrace{\begin{bmatrix} e_2 \\ x_c \end{bmatrix}}_{e_{2c}}$$
(6.10)

Lyapunov function candidate 6.1. Let $V_{a2}(x_{a2}) = x'_{a2}P_{a2}x_{a2} > 0$ be a quadratic Lyapunov function candidate. Then, similar to the proof of Proposition (5.2), and using Assumption 5.1 (p. 64), it follows that the time derivative of this function along the trajectories of the x_{a2} subsystem is equal to:

$$\dot{V}_{a2}(x_{a2},t) = \frac{\partial V_{a2}(x_{a2},t)}{\partial x_{a2}} \frac{dx_{a2}}{dt} = 2x'_{a2}P_{a2}\dot{x}_{a2}$$
(6.11)

Substituting x_{a2} from Equation (6.2) yields:

$$\dot{V}_{a2} = 2x'_{a2}P_{a2}\Big(\mathcal{A}_{22}x_{a2} - \mathcal{B}_2\mathrm{Dz}(u)\Big)$$
(6.12)

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Using the sector condition (Equation 2.15 (p. 25)) yields:

$$\dot{V}_{a2} \le 2x'_{a2}P_{a2}\mathcal{A}_{22}x_{a2} - 2x'_{a2}P_{a2}\mathcal{B}_2\mathrm{Dz}(u)$$
 (6.13)

$$\leq x'_{a2}(P_{a2}\mathcal{A}_{22} + \mathcal{A}'_{22}P_{a2})x_{a2} + 2x'_{a2}P_{a2}\mathcal{B}_{2}\phi + 2\phi'W[\mathcal{C}_{2c}e_{2c} - f_{1}(x_{1}) - \phi]$$
(6.14)

$$\underbrace{\left[\underbrace{e_{2c}e_{2c}}_{2c}-\underbrace{f_{1}(x_{1})}_{u\ from\ Eq.\ (6.2)}-\phi\right]}_{u\ from\ Eq.\ (6.2)}$$

$$\leq x'_{a2}(P_{a2}\mathcal{A}_{22} + \mathcal{A}'_{22}P_{a2})x_{a2} + 2x'_{a2}P_{a2}\mathcal{B}_{2}\phi + 2\phi'W\mathcal{C}_{2c}e_{2c} - 2\phi'Wf_{1}(x_{1}) - 2\phi'W\phi$$
(6.15)

Collecting terms in state-space form gives:

$$\dot{V}_{a2} \leq \begin{bmatrix} x_{a2} \\ \phi \end{bmatrix}' \begin{bmatrix} P_{a2}\mathcal{A}_{22} + \mathcal{A}'_{22}P_{a2} & P_{a2}\mathcal{B}_2 \\ \star & -2W \end{bmatrix} \begin{bmatrix} x_{a2} \\ \phi \end{bmatrix}$$
(6.16)

$$+ 2\phi' W \Big(\mathcal{C}_{2c} e_{2c} - f_1(x_1) \Big)$$
(6.17)

Then, applying the triangular inequality yields:

$$\dot{V}_{a2} \leq \begin{bmatrix} x_{a2} \\ \phi \end{bmatrix}' \begin{bmatrix} P_{a2}\mathcal{A}_{22} + \mathcal{A}'_{22}P_{a2} & P_{a2}\mathcal{B}_2 \\ \star & -2W \end{bmatrix} \begin{bmatrix} x_{a2} \\ \phi \end{bmatrix}$$
(6.18)

$$+ 2\|\phi'\|\|W\|\Big(\|\mathcal{C}_{2c}e_{2c}\| + \|f_1(x_1)\|\Big)$$
(6.19)

which from Assumption 5.3 (p. 64) becomes:

$$\dot{V}_{a2} \leq \begin{bmatrix} x_{a2} \\ \phi \end{bmatrix}' \begin{bmatrix} P_{a2}\mathcal{A}_{22} + \mathcal{A}'_{22}P_{a2} & P_{a2}\mathcal{B}_2 \\ \star & -2W \end{bmatrix} \begin{bmatrix} x_{a2} \\ \phi \end{bmatrix}$$
(6.20)

$$+ 2\|\phi\|\|W\|(\|\mathcal{C}_{2c}e_{2c}\| + k_1\|e_1\|)$$
(6.21)

and from Young's inequality (Equation 5.23 (p. 70))

$$\dot{V}_{a2} \leq \begin{bmatrix} x_{a2} \\ \phi \end{bmatrix}' \begin{bmatrix} P_{a2}\mathcal{A}_{22} + \mathcal{A}'_{22}P_{a2} & P_{a2}\mathcal{B}_{2} \\ \star & -2W \end{bmatrix} \begin{bmatrix} x_{a2} \\ \phi \end{bmatrix} \\ + \delta_{a} \|W\|^{2} \|\phi\|^{2} + \frac{2}{\delta_{a}} (\|\mathcal{C}_{2c}\|^{2} \|e_{2c}\|^{2} + k_{1}^{2} \|e_{1}\|^{2})$$
(6.22)

where again W > 0 is some positive definite diagonal matrix and $\delta_a > 0$ is some positive scalar.

Lyapunov function candidate 6.2. Let $V_1(e_1) = e'_1 P_1 e_1 > 0$ be a quadratic Lyapunov function candidate. Then, using Assumption 5.1 (p. 64), its time derivative along the trajectories of the e_1 subsystem is:

$$\dot{V}_1(e_1, t) = \frac{\partial V(e_1, t)}{\partial e_1} \frac{de_1}{dt}$$
(6.23)

$$=2e_1'P_1\dot{e}_1\tag{6.24}$$

Substituting \dot{e}_1 from Equation 6.2 (p. 81) yields:

$$\dot{V}(e_1) \le e_1'(P_1\mathcal{A}_{11} + \mathcal{A}_{11}'P_1)e_1 + 2\|e_1\|\|P_1\|\|\mathcal{A}_{12}(e_1)x_2\|$$
(6.25)

and using Young's inequality (Equation 5.23 (p. 70))

$$\dot{V}(e_1) \le e_1'(P_1\mathcal{A}_{11} + \mathcal{A}_{11}'P_1)e_1 + \delta_1 \|P_1\|^2 \|e_1\|^2 + \frac{1}{\delta_1} \|\mathcal{A}_{12}(e_1)x_2\|^2$$
(6.26)

and Assumption 5.4 (p. 64)

$$\dot{V}(e_1) \le e_1'(P_1\mathcal{A}_{11} + \mathcal{A}_{11}'P_1)e_1 + \delta_1 \|P_1\|^2 \|e_1\|^2 + \frac{\beta^2}{\delta_1} \|x_2\|^2$$
(6.27)

$$\dot{V}(e_1) \le e_1'(P_1\mathcal{A}_{11} + \mathcal{A}_{11}'P_1)e_1 + \delta_1 \|P_1\|^2 \|e_1\|^2 + \frac{2\beta^2}{\delta_1}(\|x_{2c}\|^2 + \|x_{a2}\|^2)$$
(6.28)

where $\delta_1 > 0$ is some positive scalar.

Lyapunov function candidate 6.3. Let $V_{2c}(e_{2c}) = x'_{2c}P_2x_{2c} > 0$ be a quadratic Lyapunov function candidate. Then it follows that the time derivative of this function along the trajectories of the e_{2c} subsystem is equal to:

$$\dot{V}_{2c}(e_{2c},t) = \frac{\partial V(e_{2c},t)}{\partial e_{2c}} \frac{de_{2c}}{dt}$$
(6.29)

$$=2e'_{2c}P_2\dot{e}_{2c} \tag{6.30}$$

$$= e_{2c}'(\mathcal{A}_{2c}'P_2 + P_2\mathcal{A}_{2c})e_{2c} \tag{6.31}$$

Sum of Lyapunov function candidates 6.1. Collecting terms together as in the proof of Proposition (5.2), and defining the state-vector $\xi_r := [e'_1 \ e'_{2c} \ x'_{a2}]'$, then allows one to write

$$\dot{V}_r(\xi_r) = \dot{V}_{a2}(x_{a2}) + \dot{V}_1(e_1) + \dot{V}_{2c}(e_{2c})$$

$$\begin{aligned}
&\leq \left[\begin{array}{c} x_{a2} \\ \phi \end{array} \right]' \left[\begin{array}{c} P_{a2}\mathcal{A}_{22} + \mathcal{A}'_{22}P_{a2} & P_{a2}\mathcal{B}_{2} \\ \star & -2W \end{array} \right] \left[\begin{array}{c} x_{a2} \\ \phi \end{array} \right] \\
&+ \delta_{a} \|W\|^{2} \|\phi\|^{2} + \frac{1}{\delta_{a}} \left(\|\mathcal{C}_{2c}\|^{2} \|e_{2c}\|^{2} + k_{1}^{2} \|e_{1}\|^{2} \right) \\
&+ \epsilon_{1}'(P_{1}\mathcal{A}_{11} + \mathcal{A}'_{11}P_{1})e_{1} + \delta_{1} \|P_{1}\|^{2} \|e_{1}\|^{2} + \frac{2\beta^{2}}{\delta_{1}} (\|x_{2c}\|^{2} + \|x_{a2}\|^{2}) \\
&+ \epsilon_{2c}'(\mathcal{A}'_{2c}P_{2} + P_{2}\mathcal{A}_{2c})e_{2c} \\
&\leq \left[\begin{array}{c} x_{a2} \\ \phi \end{array} \right]' \left[\begin{array}{c} P_{a2}\mathcal{A}_{22} + \mathcal{A}'_{22}P_{a2} & P_{a2}\mathcal{B}_{2} \\ \star & -2W \end{array} \right] \left[\begin{array}{c} x_{a2} \\ \phi \end{array} \right] \\
&+ \delta_{a} \|W\|^{2} \|\phi\|^{2} + \frac{1}{\delta_{a}} \|\mathcal{C}_{2c}e_{2c}\|^{2} + \frac{1}{\delta_{a}}k_{1}^{2}\|e_{1}\|^{2} \\
&+ \epsilon_{1}'(P_{1}\mathcal{A}_{11} + \mathcal{A}'_{11}P_{1})e_{1} + \delta_{1}\|P_{1}\|^{2}\|e_{1}\|^{2} + \frac{2\beta^{2}}{\delta_{1}}\|x_{2c}\|^{2} + \frac{2\beta^{2}}{\delta_{1}}\|x_{a2}\|^{2} \\
&\leq \left[\begin{array}{c} x_{a2} \\ \phi \end{array} \right]' \left[\begin{array}{c} P_{a2}\mathcal{A}_{22} + \mathcal{A}'_{22}P_{a2} + \frac{\beta^{2}}{\delta_{1}}I_{a_{2}} \\ &\times & -2W + \delta_{a}\|W\|^{2}I_{m} \end{array} \right] \left[\begin{array}{c} x_{a2} \\ \phi \end{array} \right] \\
&+ \epsilon_{2c}'(\mathcal{A}'_{2c}P_{2} + P_{2}\mathcal{A}_{2c})e_{2c} \\
&\leq \left[\begin{array}{c} x_{a2} \\ \phi \end{array} \right]' \left[\begin{array}{c} P_{a2}\mathcal{A}_{22} + \mathcal{A}'_{22}P_{a2} + \frac{\beta^{2}}{\delta_{1}}I_{a_{2}} & P_{a2}\mathcal{B}_{2} \\ &\times & -2W + \delta_{a}\|W\|^{2}I_{m} \end{array} \right] \left[\begin{array}{c} x_{a2} \\ \phi \end{array} \right] \\
&+ \epsilon_{1}'\left(P_{1}\mathcal{A}_{11} + \mathcal{A}'_{11}P_{1} + \delta_{1}\|P_{1}\|^{2}I_{a_{1}} + \frac{k_{1}^{2}}{\delta_{a}}I_{a_{1}} \right)e_{1} \\
&+ \epsilon_{2c}'\left(\mathcal{A}'_{2c}P_{2} + P_{2}\mathcal{A}_{2c} + \left(\frac{\beta^{2}}{\delta_{1}} + \frac{\|\mathcal{C}_{2c}\|^{2}}{\delta_{a}} \right)I_{a_{2}+a_{c}} \right)e_{2c} \end{aligned}$$

$$(6.34)$$

For stability it is sufficient for each of the three terms in the above inequality to themselves be negative definite. Consider each term individually.

1.

$$M_{1r} := \begin{bmatrix} P_{a2}\mathcal{A}_{22} + \mathcal{A}'_{22}P_{a2} + \frac{\beta^2}{\delta_1}I_{n_2} & P_{a2}\mathcal{B}_2\\ \star & -2W + \delta_a \|W\|^2 I_m \end{bmatrix}$$

For any (arbitrarily small) δ_1 , $P_{a2}\mathcal{A}_{22} + \mathcal{A}'_{22}P_{a2} + \frac{\beta^2}{\delta_1} < 0$. Next, for any W it is possible to choose, δ_a sufficiently small such that $-2W + \delta_a ||W||^2 < 0$ for any W > 0. Finally, choosing $W = \alpha_W \tilde{W}$, it is always possible to choose $\alpha_W > 0$ sufficiently large such that $M_{1r} < 0$.

2.

$$M_{2r} := P_1 \mathcal{A}_{11} + \mathcal{A}'_{11} P_1 + \delta_1 \|P_1\|^2 I_{n_1} + \frac{k_1^2}{\delta_a} I_{n_1}$$

By Assumption 5.1 (p. 64) there exists a \tilde{P}_1 such that $\tilde{P}_1 \mathcal{A}_{11} + \mathcal{A}'_{11}\tilde{P}_1 =$

 $-\tilde{Q}_1 < 0$. Then with $P_1 = \alpha_1 \tilde{P}_1$, choosing δ_1 sufficiently small ensures that $\alpha_1(\tilde{P}_1 \mathcal{A}_{11} + \mathcal{A}'_{11}\tilde{P}_1) + \alpha_1^2 \delta_1 \|\tilde{P}_1\|^2 I_{n_1} < 0$ for any \tilde{P}_1 . Next choosing α_1 sufficiently large ensures that $M_{2r} < 0$.

3.

$$M_{3r} := \mathcal{A}'_{2c} P_2 + P_2 \mathcal{A}_{2c} + \frac{\beta^2}{\delta_1} I_{n_2 + n_c} + \frac{\|\mathcal{C}_{2c}\|^2}{\delta_a} I_{n_2 + n_c}$$

By Assumption 5.5 (p. 66) there exists a $P_2 = \alpha_2 \tilde{P}_2$ such that $\alpha_2 (\mathcal{A}'_{2c} \tilde{P}_2 + \tilde{P}_2 \mathcal{A}_{2c}) = -\alpha_2 Q_2 < 0$. Choosing α_2 sufficiently large then implies that M_{3r} is negative definite for any given $\beta, \delta_1, \delta_a$ and \mathcal{C}_{2c} .

Thus by judicious choice of the scalars $\delta_1, \delta_a, \alpha_1, \alpha_2$ and the matrices W, P_{a2}, P_1 and P_2 , it follows that $\dot{V}_r(\xi_r) \leq -\eta \|\xi_r\|^2$ and, thus, the system is exponentially stable.

6.2 Simulation results

As seen in this chapter, the proposed reduced-order linear AW compensator guarantees global exponential stability of the closed-loop system. Its implementation can be done in two different ways. In the first case, one can set $\mathcal{A}_{11} = \mathcal{A}_{12}(x_1) = \mathcal{A}_{21}(x_1) = 0$ and design the AW on the basis of \mathcal{A}_{22} and \mathcal{B}_2 , i.e. the nonlinear model. In the second case, one can set $\mathbf{A}^*_{\mathbf{p}}$ to either A_p or $-A_p$ and use the whole (linear) model.

In both cases the results are identical and in fact, the results for the reduced-order linear compensator are identical to the ones presented in Chapter 5 for the nonlinear IMC compensator, hence they are omitted here. This is expected if one considers that the dynamics of the reduced-order compensator contain only the state (x_2) that the control signal regulates (see Chapter 4).

6.3 Conclusions

In this chapter, a linear AW compensator has been proposed; the compensator is a linear first-order compensator its dynamics being part of the nonlinear IMC compensator proposed in Chapter 5. It is important to note that the compensator is not equivalent to a compensator calculated from the Jacobian linearisation of the model, but it is obtained by simply setting the nonlinear terms to zero.

This reduced-order linear compensator ensures global exponential stability of the overall closed-loop nonlinear system. Simulation results have verified that the system is indeed stable, however tracking was not achieved in any case (in reality the response was identical to the nonlinear IMC of Chapter 5). The benefits of the reduced-order compensator, besides its linear nature, is the ease in its implementation (only one state) and the possible reduction in cost compared to the full-order IMC one.

While this is not very clear in the current WEC application due to the system's inherent low state nature, in a more generic nonlinear system with a higher number of states, a reduction in the order of states of the AW compensator can result in fewer computational resources for its implementation.

CHAPTER 7

A sub-optimal performance linear anti-windup (AW) compensator

The AW compensators proposed in Chapters 5 and 6 provide global exponential stability to the nonlinear class of systems described by Equation 5.1 (p. 64); a category of systems the WEC under consideration falls into. Nevertheless, simulation results (Sections 5.3 and 6.2) have shown that tracking is infeasible by either of the aforementioned compensators. As highlighted in (Valmórbida et al. 2011), IMC compensators yield well-known robustness and performance properties for linear systems, however their properties for nonlinear systems are less clear.

In this chapter, an extra "state-feedback-like" term \mathbf{F} is added to the x_{a2} dynamics of the AW compensator of Equation 6.1 (p. 81), inspired by similar techniques for linear systems (Zaccarian and Teel 2011, Turner et al. 2007b). The addition of such a term provides some freedom to the design of the AW problem and leads to the solution of a simple LMI in order to synthesise this compensator. It is thought that choosing the state-feedback matrix \mathbf{F} different from zero is likely to lead to improved anti-windup performance (Valmórbida et al. 2011). The work in this chapter is part of a journal paper submitted in the IEEE Transactions on Control Systems & Technology.

7.1 Linear Matrix Inequalities for Anti-windup compensator synthesis

For linear systems the Lyapunov stability problem can be posed as a convex optimisation problem which can be solved easily using appropriate tools (like the Robust Control toolbox from Mathworks (Balas et al. 2010)) and in a numerically tractable manner. In proving the stability of a system, there are usually a number of inequalities of various variables that need to be satisfied at the same time, so there is a system of *Linear Matrix Inequalities* (LMIs).

Generally speaking, in order to solve LMIs, all the constraints are cast as inequalities and if one can find the variables that satisfy these constraints, then the LMI problem has a feasible solution, otherwise it is not feasible. Sometimes the variables can be matrices, i.e. the Lyapunov equation A'P + PA < 0, where $A \in \mathbb{R}^{n \times n}$ is a known matrix and $P = P' \in \mathbb{R}^{n \times n}$ is the matrix variable.

The stability analysis and anti-windup synthesis problems can also be formed as an LMI (or set of LMIs). In Section 7.2 a new linear AW compensator is proposed with a structure similar to the one proposed in Chapter 6. The stability proof of the closed-loop AW system will be similar to the proofs of Chapters 5 and 6 using quadratic Lyapunov function candidates, however an LMI will need to be solved, which will give sufficient stability conditions.

Quadratic Lyapunov functions can be also used to determine some level of performance by placing a lower bound on the system decay rate, i.e. the maximum Lyapunov exponential that specifies the speed at which the states will return to equilibrium. The problem of finding this optimal response can be cast as a *Generalised Eigenvalue Problem* (GEVP), which can be solved with the Robust Control toolbox in Matlab[®].

7.2 Proposed AW compensator

In section 6.1, it was proved that, despite omitting the x_{a1} dynamics, the system is globally exponentially stable. This compensator could be interpreted as reduced-order IMC compensator because of its structure. However, IMC compensators of this type are well known to suffer performance problems (Grimm et al. 2003b) and can lead to unsatisfactory transient performance, despite their global stabilising properties.

Here, an extra term is added to the x_{a2} dynamics of the AW compensator of Equation 6.1 (p. 81), equal to \mathcal{B}_2Fx_{a2} , where $F \in \mathbb{R}^{m \times n}$. The addition of this term gives design freedom to the anti-windup problem and, as will be shown below leads to an LMI-based condition for compensator design.

Similar to Section 6.1, the x_{a1} state equation is omitted and x_{a1} , and functions thereof, are set to zero wherever else they appear. In this case, the augmented anti-windup compensator state-space description becomes

$$\mathcal{AW}_{I} \sim \begin{cases} \dot{x}_{a2} = \mathcal{A}_{22}x_{a2} - \mathcal{B}_{2}\mathrm{Dz}(u) + \mathcal{B}_{2}Fx_{a2} \\ \theta_{1} = Fx_{a2} \\ \theta_{2} = \mathcal{C}_{2}x_{a2} \end{cases}$$
(7.1)

Note that this compensator is of order n_2 and is entirely linear.

Similarly to section 6.1, the following Proposition shows that the proposed suboptimal compensator (shown in Figure 7.1) is able to guarantee stability of the nonlinear system providing a certain LMI is satisfied.

Proposition 7.1. Consider the interconnection of the plant (Equation 5.1 (p. 64)), the controller (Equation 5.2 (p. 65)) and the proposed AW compensator (Equation (7.1)). Let $r \equiv 0, d \equiv 0$ and $u_{sat} = \operatorname{sat}(u)$. Let Assumption 5.1 (p. 64) and 5.5 (p. 66) be satisfied. Then there exists an anti-windup compensator of the form of Equation (7.1) which ensures the origin of the interconnected system is globally exponentially stable, provided there exist a symmetric positive definite matrix $\tilde{Q} > 0$, a diagonal matrix $\tilde{U} > 0$ and a matrix \tilde{L} such that the following LMI is satisfied.



Figure 7.1: System block diagram of the proposed sub-optimal performance linear anti-windup compensator.

$$\begin{bmatrix} \mathcal{A}_{22}\tilde{Q} + \tilde{Q}\mathcal{A}_{22}' + \mathcal{B}_{2}\tilde{L} + \tilde{L}'\mathcal{B}_{2}' & -\mathcal{B}_{2}\tilde{U} + \tilde{L} \\ \star & -2\tilde{U} \end{bmatrix} < 0$$
(7.2)

F will then be equal to $\tilde{L}\tilde{Q}^{-1}$ and will be used in the synthesis of the proposed sub-optimal AW compensator.

Proof 7.1. Note that with $x_{a1} \equiv 0$, and recalling the notation from the proof of Propositions (5.2) and (6.1), $e_1 = x_1$. As indicated earlier, instead of working in the (x_1, x_{2c}, x_a) coordinates, it is more convenient to work in the (e_1, e_2, x_c, x_{a2}) coordinates. The approach taken in the proof is similar to that in Propositions (5.2) and (6.1); to define quadratic Lyapunov function candidates for each "subsystem" and then to sum together these functions, and their time derivatives, in order to obtain a positive definite (radially unbounded) Lyapunov function with a negative definite derivative, i.e. proving that the system in the coordinates (e_1, e_2, x_c, x_{a2}) is exponentially stable.

The closed-loop interconnection (Figure 7.1) of the plant (Eq. 5.1 (p. 64)), the controller (Eq. 5.2 (p. 65)) and the anti-windup compensator (7.1) can be described

in the (e_1, e_2, x_c, x_{a2}) coordinates as follows:

$$\begin{cases} \dot{e}_{1} = \mathcal{A}_{11}x_{1} + \mathcal{A}_{12}(x_{1})x_{2} \\ \dot{e}_{2c} = \mathcal{A}_{2c}e_{2c} \\ \dot{x}_{a2} = (\mathcal{A}_{22} + \mathcal{B}_{2}F)x_{a2} - \mathcal{B}_{2}\mathrm{Dz}(u) \\ u = \mathcal{C}_{2c}e_{2c} - f_{1}(x_{1}) + Fx_{a2} \end{cases}$$
(7.3)

where $C_{2c} = [C_2 D_{cy} \quad C_c]$. Similar to Sections 5.2 and 6.1 the derivative of e_{2c} is derived as follows:

$$\dot{e}_{2c} = \begin{bmatrix} \dot{x}_2 - \dot{x}_{a2} \\ \dot{x}_c \end{bmatrix}$$
(7.4)

$$= \begin{bmatrix} \mathcal{A}_{21}(\boldsymbol{x}_1) + \mathcal{A}_{22}\boldsymbol{x}_2 + \mathcal{B}_2\boldsymbol{u}_{sat} - (\mathcal{A}_{22} + \mathcal{B}_2F)\boldsymbol{x}_{a2} + \mathcal{B}_2D\boldsymbol{z}(\boldsymbol{u}) \\ \mathcal{A}_c\boldsymbol{x}_c + \mathcal{B}_{cy}(\boldsymbol{y} - \theta_2) \end{bmatrix}$$
(7.5)

Again, using Equation 2.19 (p. 28) the previous one becomes:

$$\dot{e}_{2c} = \begin{bmatrix} \mathcal{A}_{21}(x_1) + \mathcal{A}_{22}(x_2 - x_{a2}) - \mathcal{B}_2 F x_{a2} + \mathcal{B}_2 \left(u - Dz(u) + Dz(u) \right) \\ \mathcal{A}_c x_c + \mathcal{B}_{cy} \mathcal{C}_2 (x_2 - x_{a2}) \end{bmatrix}$$
(7.6)

$$= \begin{bmatrix} \mathcal{A}_{21}(\boldsymbol{x}_1) + \mathcal{A}_{22}\boldsymbol{e}_2 - \mathcal{B}_2F\boldsymbol{x}_{a2} + \mathcal{B}_2\boldsymbol{u} \\ \mathcal{A}_c\boldsymbol{x}_c + \mathcal{B}_{cy}\mathcal{C}_2\boldsymbol{e}_2 \end{bmatrix}$$
(7.7)

Now, u is equal to (from Eq. 5.2 (p. 65)):

$$u = \mathcal{C}_c \boldsymbol{x}_c + \mathcal{D}_{cy}(\boldsymbol{y} - \theta_2) - f_1(\boldsymbol{x}_1) + \theta_1$$
(7.8)

$$= \mathcal{C}_c x_c + \mathcal{D}_{cy} \mathcal{C}_2 e_2 - f_1(x_1) + F x_{a2}$$

$$\tag{7.9}$$

and substituting in Equation (7.7) yields:

$$\dot{e}_{2c} = \begin{bmatrix} \mathcal{A}_{21}(x_1) + \mathcal{A}_{22}e_2 - \mathcal{B}_2Fx_{a2} + \mathcal{B}_2(\mathcal{C}_cx_c + \mathcal{D}_{cy}\mathcal{C}_2e_2 - f_1(x_1) + Fx_{a2}) \\ \mathcal{A}_cx_c + \mathcal{B}_{cy}\mathcal{C}_2e_2 \end{bmatrix}$$
(7.10)

$$= \begin{bmatrix} \mathcal{A}_{21}(\boldsymbol{x}_1) + \mathcal{A}_{22}\boldsymbol{e}_2 + \mathcal{B}_2\mathcal{C}_c\boldsymbol{x}_c + \mathcal{B}_2\mathcal{D}_{cy}\mathcal{C}_2\boldsymbol{e}_2 - \mathcal{B}_2f_1(\boldsymbol{x}_1) \\ \mathcal{A}_c\boldsymbol{x}_c + \mathcal{B}_{cy}\mathcal{C}_2\boldsymbol{e}_2 \end{bmatrix}$$
(7.11)

which from Assumption 5.3 (p. 64) becomes:

$$\dot{e}_{2c} = \begin{bmatrix} \mathcal{A}_{22}e_2 + \mathcal{B}_2\mathcal{C}_c x_c + \mathcal{B}_2\mathcal{D}_{cy}\mathcal{C}_2 e_2 + \mathcal{B}_2\left(-f_1(x_1) + f_1(x_1)\right) \\ \mathcal{A}_c x_c + \mathcal{B}_{cy}\mathcal{C}_2 e_2 \end{bmatrix}$$
(7.12)

$$=\underbrace{\begin{bmatrix} \mathcal{A}_{22} + \mathcal{B}_2 \mathcal{D}_{cy} \mathcal{C}_2 & \mathcal{B}_2 \mathcal{C}_c \\ \mathcal{B}_{cy} \mathcal{C}_2 & \mathcal{A}_c \end{bmatrix}}_{\mathcal{A}_{2c}} \underbrace{\begin{bmatrix} e_2 \\ x_c \end{bmatrix}}_{e_{2c}}$$
(7.13)

Lyapunov function candidate 7.1. Let $V_{a2}(x_{a2}) = x'_{a2}P_{a2}x_{a2} > 0$ be a quadratic Lyapunov function candidate. Then, similar to the proof of Propositions (5.2) and (6.1) and using Assumption 5.1 (p. 64), it follows that

$$\dot{V}_{a2} = 2x'_{a2}P_{a2}\dot{x}_{a2}$$

= $2x'_{a2}P_{a2}[(\mathcal{A}_{22} + \mathcal{B}_2F)x_{a2} - \mathcal{B}_2\phi]$ (7.14)

and using the sector condition of Equation 2.15 (p. 25)

$$\dot{V}_{a2} \leq x'_{a2}[P_{a2}(\mathcal{A}_{22} + \mathcal{B}_{2}F) + (\mathcal{A}_{22} + \mathcal{B}_{2}F)'P_{a2}]x_{a2} - 2x'_{a2}P_{a2}\mathcal{B}_{2}\phi
+ 2\phi'W\Big(\underbrace{\mathcal{C}_{2c}e_{2c} - f_{1}(x_{1}) + Fx_{a2}}_{u \ from \ Eq.(7.3)} - \phi\Big)$$

$$\leq x'_{a2}[P'_{a2}(\mathcal{A}_{22} + \mathcal{B}_{2}F) + (\mathcal{A}_{22} + \mathcal{B}_{2}F)'P_{a2}]x_{a2} - 2x'_{a2}P_{a2}\mathcal{B}_{2}\phi
+ 2\|\phi'\|\|W\|\|\mathcal{C}_{2c}e_{2c}\| - 2\|\phi'\|\|W\|\|f_{1}(x_{1})\|
+ 2\|\phi'\|\|W\|\|Fx_{a2}\| - 2\|\phi'\|\|W\|\|\phi\|$$
(7.16)

Collecting terms in state-space form yields:

$$\dot{V}_{a2} \leq \begin{bmatrix} x_{a2} \\ \phi \end{bmatrix}' \begin{bmatrix} P_{a2}(\mathcal{A}_{22} + \mathcal{B}_{2}F) + (\mathcal{A}_{22} + \mathcal{B}_{2}F)'P_{a2} & -P_{a2}\mathcal{B}_{2} + F'W \\ \star & -2W \end{bmatrix} \begin{bmatrix} x_{a2} \\ \phi \end{bmatrix} + 2\|\phi\|\|W\|(\|\mathcal{C}_{2c}e_{2c} - f(x_{1})\|)$$
(7.17)

Using Young's inequality (Equation 5.23 (p. 70)) it follows, for any $\delta_a > 0$, that

$$\dot{V}(x_{a}) \leq \begin{bmatrix} x_{a2} \\ \phi \end{bmatrix}' \begin{bmatrix} P_{a2}(\mathcal{A}_{22} + \mathcal{B}_{2}F) + (\mathcal{A}_{22} + \mathcal{B}_{2}F)'P_{a2} & -P_{a2}\mathcal{B}_{2} + F'W \\ \star & -2W \end{bmatrix} \begin{bmatrix} x_{a2} \\ \phi \end{bmatrix} + \delta_{a} \|\phi\|^{2} \|W\|^{2} + \frac{1}{\delta_{a}} \|\mathcal{C}_{2c}e_{2c} - f(x_{1})\|^{2}$$
(7.18)

which from Assumption 5.3 (p. 64) becomes:

$$\dot{V}(x_a) \leq \begin{bmatrix} x_{a2} \\ \phi \end{bmatrix}' \begin{bmatrix} P_{a2}(\mathcal{A}_{22} + \mathcal{B}_2 F) + (\mathcal{A}_{22} + \mathcal{B}_2 F)' P_{a2} & -P_{a2}\mathcal{B}_2 + F'W \\ \star & -2W \end{bmatrix} \begin{bmatrix} x_{a2} \\ \phi \end{bmatrix} + \delta_a \|\phi\|^2 \|W\|^2 + \frac{1}{\delta_a} \|\mathcal{C}_{2c}\|^2 \|e_{2c}\|^2 + \frac{1}{\delta_a} k_1^2 \|e_1\|^2$$
(7.19)

where again W > 0 is some positive definite diagonal matrix and $\delta_a > 0$ is some positive scalar.

Lyapunov function candidate 7.2. Let $V_1(e_1) = e'_1 P_1 e_1 > 0$. Then, using Assumption 5.1 (p. 64)

$$\dot{V}(e_{1}) = 2e'_{1}P_{1}\left(\mathcal{A}_{11}e_{1} + \mathcal{A}_{12}(e_{1})x_{2}\right)$$

$$= 2e'_{1}P_{1}\mathcal{A}_{11}e_{1} + 2e'_{1}P_{1}\mathcal{A}_{12}(e_{1})x_{2}$$

$$\leq e'_{1}(P_{1}\mathcal{A}_{11} + \mathcal{A}'_{11}P_{1})e_{1} + 2||e_{1}||||P_{1}|||\mathcal{A}_{12}(e_{1})x_{2}||$$

$$\leq e'_{1}(P_{1}\mathcal{A}_{11} + \mathcal{A}'_{11}P_{1})e_{1} + 2||e_{1}|||P_{1}||\underbrace{||\mathcal{A}_{12}(e_{1})||}_{\leq\beta Ass. (5.4)}||x_{2}||$$
(7.20)

Using Young's inequality (Equation 5.23 (p. 70))

$$\dot{V}(e_1) \le e_1'(P_1\mathcal{A}_{11} + \mathcal{A}_{11}'P_1)e_1 + \delta_1 \|P_1\|^2 \|e_1\|^2 + \frac{\beta^2}{\delta_1} \|x_2\|^2$$
(7.21)

where $\delta_1 > 0$ is some positive scalar. Since $x_2 = e_2 - x_{a2}$, hence

$$\dot{V}_1(e_1) \le e_1'(P_1\mathcal{A}_{11} + \mathcal{A}_{11}'P_1)e_1 + \delta_1 \|P_1\|^2 \|e_1\|^2 + \frac{2\beta^2}{\delta_1}(\|e_2\|^2 + \|x_{a2}\|^2)$$
(7.22)

Lyapunov function candidate 7.3. As before $V_2(e_{2c}) = e'_{2c}P_2e_{2c} > 0$. Then, using Assumption 5.5 (p. 66),

$$\dot{V}(e_{2c}) = e'_{2c}(\mathcal{A}'_{2c}P_2 + P_2\mathcal{A}_{2c})e_{2c}$$
(7.23)

Sum of Lyapunov function candidates 7.1. Collecting terms together and defining the state-vector $\xi_{ro} := [e'_1, e'_{2c}, x_{2a}]'$, then allows one to write

$$V_{tot}(\xi_{ro}) = V_{a2}(x_{a2}) + V_1(e_1) + V_2(e_{2c})$$

$$\dot{V}_{tot}(\xi_{ro}) = \begin{bmatrix} x_{a2} \\ \phi \end{bmatrix}' \begin{bmatrix} P_{a2}(\mathcal{A}_{22} + \mathcal{B}_{2}F) + (\mathcal{A}_{22} + \mathcal{B}_{2}F)'P_{a2} & -P_{a2}\mathcal{B}_{2} + F'W \\ \star & -2W + \delta_{a} ||W||^{2} \end{bmatrix} \begin{bmatrix} x_{a2} \\ \phi \end{bmatrix} \\ + \frac{2}{\delta_{a}} ||\mathcal{C}_{2c}||^{2} ||e_{2c}||^{2} + \frac{2}{\delta_{a}}k_{1}||e_{1}||^{2} \\ + e_{1}'(P_{1}\mathcal{A}_{11} + \mathcal{A}_{11}'P_{1} + \delta_{1}||P_{1}||^{2}I_{n1})e_{1} + \frac{2\beta^{2}}{\delta_{1}}(||e_{2c}||^{2} + ||x_{a2}||^{2}) \\ + e_{2c}'(P_{2}\mathcal{A}_{2c} + \mathcal{A}_{2c}'P_{2})e_{2c} \\ < \begin{bmatrix} x_{a2} \\ \phi \end{bmatrix}' \begin{bmatrix} P_{a2}(\mathcal{A}_{22} + \mathcal{B}_{2}F) + (\mathcal{A}_{22} + \mathcal{B}_{2}F)'P_{a2} + \frac{2\beta^{2}}{\delta_{1}}I & -P_{a2}\mathcal{B}_{2} + F'W \\ -2W + \delta_{a}||W||^{2} \end{bmatrix} \begin{bmatrix} x_{a2} \\ \phi \end{bmatrix} \\ + e_{1}'(P_{1}\mathcal{A}_{11} + \mathcal{A}_{11}'P_{1} + \delta_{1}||P_{1}||^{2} + \frac{2}{\delta_{a}}k_{1}^{2}I_{n1})e_{1} \\ + e_{2c}'(P_{2}\mathcal{A}_{2c} + \mathcal{A}_{2c}'P_{2} + \frac{2}{\delta_{a}}||\mathcal{C}_{2c}||^{2}I_{n2+c} + \frac{2\beta^{2}}{\delta_{1}}I_{n2+c})e_{2c}$$

$$(7.24)$$

For stability it is sufficient for each of the three terms in the above inequality to themselves be negative definite. Consider each term individually.

1.

$$M_{1ro} := \begin{bmatrix} P_{a2}(\mathcal{A}_{22} + \mathcal{B}_2 F) + (\mathcal{A}_{22} + \mathcal{B}_2 F)' P_{a2} + \frac{2\beta^2}{\delta_1} I_{n2} & -P_{a2}\mathcal{B}_2 + F'W \\ \star & -2W + \delta_a \|W\|^2 \end{bmatrix}$$
(7.25)

Choosing $P_{a2} = \alpha \tilde{P}_a$ and $W = \alpha \tilde{W}$ yields

$$M_{1ro} = \alpha \begin{bmatrix} \tilde{P}_a(\mathcal{A}_{22} + \mathcal{B}_2 F) + (\mathcal{A}_{22} + \mathcal{B}_2 F)' \tilde{P}_a & -\tilde{P}_a \mathcal{B}_2 + F' \tilde{W} \\ \star & -2 \tilde{W} \end{bmatrix} + \begin{bmatrix} \frac{2\beta^2}{\delta_1} I_{n2} & 0 \\ 0 & \delta_a \alpha^2 \|\tilde{W}\| I_m \end{bmatrix}$$
(7.26)

Now for large enough α and small enough δ_a this holds for any (arbitrarily small) δ_1 provided

$$\begin{bmatrix} \tilde{P}_a(\mathcal{A}_{22} + \mathcal{B}_2 F) + (\mathcal{A}_{22} + \mathcal{B}_2 F)'\tilde{P}_a & -\tilde{P}_a\mathcal{B}_2 + F'\tilde{W} \\ \star & -2\tilde{W} \end{bmatrix} < 0$$
(7.27)

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which is a matrix inequality in the variables \tilde{P}_a and F. Note that Equation (7.27) is a Bilinear Matrix Inequality (BMI), since there are products of variables ($\tilde{P}_a F$ and $F'\tilde{W}$). To overcome this, the congruence transformation in conjunction with a change of variables can be used.

The basic idea of congruence transformation is to pre- and post-multiply the matrix of inequality (7.27) by a full rank matrix, so that its definiteness remains the same. It is common to choose the aforementioned matrix diagonal, so here choosing $diag(\tilde{P}_a^{-1}, \tilde{W}^{-1}) = diag(\tilde{Q}, \tilde{U})$ yields

$$\begin{bmatrix} \mathcal{A}_{22}\tilde{Q} + \tilde{Q}\mathcal{A}_{22}' + \mathcal{B}_2F\tilde{Q} + \tilde{Q}F'\mathcal{B}_2' & -\mathcal{B}_2\tilde{U} + \tilde{Q}'F \\ \star & -2\tilde{U} \end{bmatrix} < 0$$
(7.28)

which is a new matrix inequality in the variables \tilde{Q} and F, although it is still nonlinear due to the product $\tilde{Q}F$. By defining new variables this nonlinear problem can now be posed as a linear one which can be easily solved, so defining $\tilde{L} = F\tilde{Q}$

$$\begin{bmatrix} \mathcal{A}_{22}\tilde{Q} + \tilde{Q}\mathcal{A}'_{22} + \mathcal{B}_{2}\tilde{L} + \tilde{L}'\mathcal{B}'_{2} & -\mathcal{B}_{2}\tilde{U} + \tilde{L} \\ \star & -2\tilde{U} \end{bmatrix} < 0$$
(7.29)

which is an LMI in \tilde{Q} and \tilde{L} .

2.

$$M_{2ro} := P_1 \mathcal{A}_{11} + \mathcal{A}'_{11} P_1 + \delta_1 \|P_1\|^2 + \frac{2}{\delta_a} k_1^2 I_{n1}$$
(7.30)

Choosing $P_1 = \alpha_1 \tilde{P}_1$ yields

$$M_{2ro} = \alpha_1 (\tilde{P}_1 \mathcal{A}_{11} + \mathcal{A}'_{11} \tilde{P}_1) + \alpha_1^2 \delta_1 \|\tilde{P}_1\|^2 + \frac{2}{\delta_a} k_1^2 I_{n1}$$
(7.31)

Now by choosing α_1 sufficiently large and δ_1 sufficiently small M_{2ro} is negative definite for any (arbitrarily small) δ_a .

3.

$$M_{3ro} := P_2 \mathcal{A}_{2c} + \mathcal{A}'_{2c} P_2 + \left(\frac{2}{\delta_a} \|\mathcal{C}_{2c}\|^2 + \frac{2\beta^2}{\delta_1}\right) I_{n2+c}$$
(7.32)

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Choosing $P_2 = \alpha_2 \tilde{P}_2$ yields

$$M_{3ro} = \alpha_2 (\tilde{P}_2 \mathcal{A}_{2c} + \mathcal{A}'_{2c} \tilde{P}_2) + \left(\frac{2}{\delta_a} \|\mathcal{C}_{2c}\|^2 + \frac{2\beta^2}{\delta_1}\right) I_{n2+c}$$
(7.33)

So for any arbitrarily small δ_a and δ_1 , then α_2 can always be chosen so that M_{3ro} is negative definite.

Thus by judicious choice of the free parameters W, δ_a , δ_1 , α , α_1 and α_2 , it is always possible to ensure that $\dot{V}_{tot}(\xi_{ro}) \leq -\eta \|\xi_{ro}\|^2$ and, thus, the system is exponentially stable.

It is noteworthy that the improved AW compensator is entirely linear (as with the case of the reduced IMC compensator of Chapter 6) and that it can be synthesised by simply solving an LMI problem. This is in contrast to the equivalent technique proposed in (Herrman et al. 2010) where a matrix partial differential inequality needed to be solved. Note also, that when $F \equiv 0$ (and thus $\tilde{L} \equiv 0$), satisfaction of the LMI in Proposition 7.1 is guaranteed and hence the reduced order IMC compensator of Chapter 6 appears as a special case of Proposition 7.1, as one might expect.

Note that the LMI (7.2) could be combined with other approaches in order to guarantee attractive solutions. One simple manner to adapt it would be to solve the LMI associated with linear anti-windup design appearing in (Turner et al. 2007b), but for the reduced-order dynamics associated with the compensator (7.1). The solution of the aforementioned LMI (Turner et al. 2007b) guarantees stability, performance and robustness (with some trade-off between the last two) of a linear mapping. The upper left block of this LMI is identical to the LMI (7.2), hence the motivation for testing this approach (Sec 7.3.2), however there is no such linear mapping in the WEC dynamics/class of systems examined here, so there is no robust analysis that the solution of this LMI indeed minimises an L_2 gain of such a mapping. Nonetheless, simulation results indicate that this is probably the case (Section 7.3.2), however, rigorous analysis to prove that is still needed. This approach is called *the* L_2 *approach* since the linear LMIs deal with minimising a given L_2 gain.

An alternative construction would replace the LMI (7.2) with the generalised eigen-

value problem (GEVP) involving the positive scalar λ

$$\begin{bmatrix} \mathcal{A}_{22}\tilde{Q} + \tilde{Q}\mathcal{A}'_{22} + \mathcal{B}_{2}\tilde{L} + \tilde{L}'\mathcal{B}'_{2} + \lambda\tilde{Q} & -\mathcal{B}_{2}\tilde{U} + \tilde{L} \\ \star & -2\tilde{U} \end{bmatrix} < 0$$
(7.34)

Maximising λ then leads to a compensator with a maximum decay rate, which might lead to an improvement of the anti-windup compensator performance. This approach is called *the decay rate* approach and is presented in Section 7.3.1.

7.3 Simulation results

As mentioned earlier in Chapter 2, the primary aim of AW compensation is to improve system performance during saturation, however this cannot be guaranteed just by solving the LMI (7.2). Therefore, the two slightly different approaches mentioned earlier will be used for simulation that include stability and performance together (the *decay rate* approach) or stability, performance and robustness together (the L_2 approach).

7.3.1 The decay rate approach

As already mentioned in Chapter 5, the nominal unconstrained system tracks the reference signal perfectly (as shown in Section 3.4), so again the results for the unconstrained system are also omitted here as they are identical to those of Chapter 3.


Figure 7.2: Output of the closed-loop system under saturation and the AW compensator for $H_s = 0.5$ m and $T_1 = 10$ s.

One can see in Figure 7.2 that while the saturated system becomes unstable (output signal goes to infinity) the proposed AW compensator stabilises the system and recovers nominal performance and tracking.



Figure 7.3: Control signal of the closed-loop system under saturation and the AW compensator for $H_s = 0.5$ m and $T_1 = 10$ s.

Nevertheless, this is achieved at the price of highly switching control activity (Figure 7.3), which may indicate poor system robustness.

For a wave amplitude of 0.8 m, the response of the saturated system indicates clearly that the RO membrane will be destroyed, whereas again the proposed compensator recovers the closed-loop system from saturation and achieves both stability and tracking of the reference.



Figure 7.4: Output of the closed-loop system under saturation for $H_s=0.8\,{\rm m}~{\rm and}~T_1=10\,{\rm s}.$



Figure 7.5: Control signal of the closed-loop system under saturation for $H_s = 0.8 \text{ m}$ and $T_1 = 10 \text{ s}$.



Figure 7.6: Output of the closed-loop system under saturation for $H_s = 1.2 \text{ m}$ and $T_1 = 10 \text{ s}$.



Figure 7.7: Control signal of the closed-loop system under saturation for $H_s = 1.2 \text{ m}$ and $T_1 = 10 \text{ s}$.



Figure 7.8: Output of the closed-loop system under saturation for $H_s = 1.3$ m and $T_1 = 10$ s.

In Figures 7.6 and 7.8 it can be seen that the proposed AW compensator oscillates between the allowable limits, while the saturated system exceeds them and has a slightly more oscillatory response.



Figure 7.9: Control signal of the closed-loop system under saturation for $H_s = 1.3$ m and $T_1 = 10$ s.

Observing the AW compensator control signals (Figures 7.7 and 7.9) it is obvious that they remain very oscillatory, something that will certainly cause wear and tear of the WEC actuators.

7.3.2 The L_2 approach

Here simulation results for the L_2 approach are given using the same inputs as in the decay rate approach for the sake of comparison. One can observe that the output responses are identical between the two approaches, however the L_2 approach achieves the same results with much less and significantly smoother control activity. Such kind of control will be easier to implement in practice.



Figure 7.10: Output of the closed-loop system under saturation and the AW compensator for $H_s = 0.5$ m and $T_1 = 10$ s.

One can see in Figure 7.10 that while the saturated system becomes unstable (output signal goes to infinity) the proposed AW compensator stabilises the system and recovers nominal performance and tracking.



Figure 7.11: Control signal of the closed-loop system under saturation and the AW compensator for $H_s = 0.5$ m and $T_1 = 10$ s.

The control activity (Figure 7.11) is significantly less than that of Figure 7.3.

For a wave amplitude of 0.8 m, the response of the saturated system indicates clearly that the RO membrane will be destroyed, whereas again the proposed compensator recovers the closed-loop system from saturation and achieves both stability and tracking of the reference.



Figure 7.12: Output of the closed-loop system under saturation for $H_s = 0.8 \text{ m}$ and $T_1 = 10 \text{ s}$.





Figure 7.13: Control signal of the closed-loop system under saturation for $H_s = 0.8 \,\mathrm{m}$ and $T_1 = 10 \,\mathrm{s}$.



Figure 7.14: Output of the closed-loop system under saturation for $H_s = 1.2$ m and $T_1 = 10$ s.



Figure 7.15: Control signal of the closed-loop system under saturation for $H_s = 1.2 \,\mathrm{m}$ and $T_1 = 10 \,\mathrm{s}$.



Figure 7.16: Output of the closed-loop system under saturation for $H_s = 1.3$ m and $T_1 = 10$ s.



Figure 7.17: Control signal of the closed-loop system under saturation for $H_s = 1.3$ m and $T_1 = 10$ s.

7.4 Conclusions

It is noteworthy that the proposed sub-optimal performance AW compensator is linear and it is able to ensure global exponential stability of the overall closed-loop nonlinear system. This compensator seems to force the system to return to nominal behaviour fast after saturation occurs; it has a similar structure to that in (Herrman et al. 2010), however its construction requires only the solution of an LMI rather than a nonlinear partial differential equation (PDE) as in (Herrman et al. 2010).

It has been also shown that this compensator can be combined with other approaches in order to guarantee attractive solutions. The first one, the *decay rate* approach, posed as a GEVP problem, ensured global exponential stability and sharp performance improvement, however at the cost of very high control activity.

The second approach, the L_2 approach, involved the solution of the LMI (23) proposed in (Turner et al. 2007b), but for the reduced-order dynamics of the proposed AW compensator. This LMI dealt with minimising a given L_2 gain, which although there was no rigorous analysis in this thesis regarding such a gain, simulation results showed that the performance improvement was identical to the decay rate approach, however with much less control activity.

CHAPTER 8

Conclusions

The research described in this thesis originally began as an attempt to understand some of the nonlinearities associated with a WEC system coupled to a hydraulic Power Take-Off unit. Indeed the thesis has studied the nonlinear control system proposed in earlier work and has also looked at how control signal constraints affect the performance of the system. However, the initial study on control signal constraints revealed that some of the existing anti-windup techniques which can be found in the literature are not entirely appropriate for the class of WEC systems studied. This led to the development of several new variants of anti-windup compensator for a class of systems to which the WEC system belongs. It is emphasised that the techniques developed have broader application and are not limited to the WEC system on which they are applied in this thesis. Therefore, the thesis makes several contributions, some specifically related to wave power systems and some more directed towards the growing literature on nonlinear anti-windup synthesis. The particular contributions are summarised in the next section.

8.1 Conclusions

In this thesis, a WEC (point-absorber coupled to a hydraulic PTO) has been studied for the production of potable water from waves, together with a pseudo-NDI controller for this type of WEC. The work carried out in this thesis applies to a whole class of nonlinear input-constrained systems (the WEC dynamics fall into this category) and a family of pseudo-NDI controllers (again, the dynamics of the WEC controller fall into this category). In specific:

- Chapter 4 studied the WEC system and the pseudo-NDI controller proposed in earlier work (Nolan 2006, Nolan and Ringwood 2006). It was proved that the controller developed by Nolan belonged to a family of such controllers which bestow global asymptotic stability on the (unconstrained) closed-loop system. This observation was not made in earlier work and thus the effectiveness of the controller was, hitherto, only guaranteed by performing extensive simulation. This chapter therefore provides a useful guarantee which should be of some use in practice.
- In Chapter 5, a nonlinear IMC anti-windup compensator has been proposed, which proved to guarantee global exponential stability of the overall closed-loop system for this class of input-constrained systems and the respective type of controllers. The compensator is a natural counterpart to the IMC compensator derived in (Herrman et al. 2010); however, construction of the proposed compensator requires no optimisation, in contrast to the one in (Herrman et al. 2010), whose synthesis required the solution of a non-convex partial differential inequality.
- In Chapter 6, a second AW compensator has been proposed. This is a reducedorder compensator featuring only the linear part of the dynamics of the aforementioned IMC compensator. This compensator also proved to guarantee global exponential stability of the overall nonlinear closed-loop system and again its synthesis requires no optimisation. This compensator can be constructed in two ways: one is by using the nonlinear dynamics of the IMC compensator of Chapter 5, but only using part of these dynamics (the linear part) and the other one is by choosing one of the switching values for the switching term in the plant, thereby forming a linear subsystem and using this to synthesise the AW compensator. Simulation results have shown that the compensator responses are identical whether synthesised using one way or the other.
- In Chapter 7, a third AW compensator has been proposed. This compensator is itself linear of the same order as the reduced-order compensator of Chapter 6 and it also proved to guarantee global exponential stability of the overall nonlinear closed-loop system. While the other two proposed compensators

required no optimisation, this compensator involves an extra feedback term whose optimisation can lead to performance improvement. Its synthesis is based on the solution of a simple convex 2×2 Linear Matrix Inequality. This compensator can also be synthesised using two different ways: one is to include an extra term in the aforementioned inequality that corresponds to the decay rate and solving this augmented inequality as a GEVP can lead to the synthesis of a compensator with optimal performance. The second way, is more intuitive; the LMI forms part of the LMI (23) in (Turner et al. 2007b) which was used to minimise a given L_2 gain. As the class of input-constrained systems studied in this thesis are nonlinear, a more rigorous analysis would be needed to guarantee such an L_2 gain minimisation, however it has not been performed. Simulation results have shown that constructing the proposed compensator using either way provides a large performance improvement in tracking the desired setpoint, however the control activities differ significantly: the compensator with the decay rate approach requires significant control activity to achieve good performance which translates in to a very oscillatory control signal. On the other hand, the L_2 synthesis approach can provide the same high level of performance, but with much less control activity (the control signal is more feasible in practice).

The results derived here have been applied to, and indeed were motivated by, the problem of a WEC pressure regulation. Simulation results have shown that all three proposed compensators stabilise the system, however only the third one can address the pressure regulation problem by providing tracking. The tracking window achieved (waves of height 0.5 - 1.3m) can be considered quite narrow, which is due to a very slow pole in the specific WEC dynamics under consideration. Nevertheless, the success of the proposed compensators leads one to believe that a more sophisticated nonlinear anti-windup compensator (akin to that proposed in (Herrman et al. 2010)) could yield further improvements in the WEC example and perhaps enable a wider variety of wave heights to be catered for without tracking performance breakdown.

8.2 Future work

- 1. In the particular type of WEC studied in this thesis for the production of potable water, the constraint studied involved the control signal magnitude saturation (the control signal represents the position of a valve varying from fully closed to fully open). As mentioned in (Nolan 2006, Bacelli et al. 2009) though, the control signal also exhibits rate saturation. In addition, there is also a constraint on the vertical buoy oscillation ensuring WEC structural integrity, which can be compromised during large waves. It would be interesting to perform a similar analysis including both of these constraints which have not been studied here and perhaps develop AW techniques that are even more applicable to the actual WEC system.
- 2. As mentioned in Chapter 3, the particular type of WEC considered in this thesis is the very popular heaving point-absorber. According to its principle of operation, the PTO coupled to the buoy must always oppose this heaving buoy motion in order to provide damping and maximise wave energy absorbtion. As a consequence, the author believes that the techniques used here to prove stability of the closed-loop system and to the design and synthesis of stabilising and performance improving AW compensators can be performed similarly on any WEC system consisting of a heaving buoy coupled to a PTO. This could prove particularly useful in the use of WECs for electricity production, since up to this point researchers have focused on maximising wave energy absorbtion considering a very simplified linear version of a generic PTO for electricity production. However, to advance in the race for WEC device commercialisation, research community should begin to focus more on the stability and robustness of WEC control schemes (Hals 2010). The lack of such a systematic analysis could be one of the reasons why commercially available WECs are still small in numbers.
- 3. In Chapter 7, the LMI (23) found in (Turner et al. 2007b) has been used in order to design a linear AW compensator that indeed provides good tracking performance with a control signal whose nature makes it more easily implementable in practice than the one produced by using the decay rate approach. The solution of this LMI though resulted in the minimisation of a linear mapping so as to minimise an L_2 gain. As highlighted in Chapter 7, there is no

such linear mapping in the system under examination, so there is a need to perform some rigorous analysis to try and minimise a nonlinear mapping for such a system. Although it will not be trivial to do, the results of Section 7.3.2 do suggest that if a nonlinear mapping could be minimised, perhaps a compensator with better tracking performance and smoother control signal could be synthesised and hence be more useful in a more practical manner.

4. Another significant factor needed is to test the work proposed in this thesis in the environment of a wave tank in order to test the robustness of the proposed AW compensators in practice and also test their stabilising properties as well as the desired performance recovery claims made in Chapter 7 in particular. Wave tank tests should be performed i) to the WEC model, to validate both the heaving buoy dynamic behaviour as well as the PTO, ii) test the effectiveness and robustness of the baseline control system proposed by (Nolan 2006) and used throughout this thesis and iii) test each of the proposed AW compensators individually; test their stabilising properties within the operating window and for those proposed in Chapter 7 test also their impact on the system's performance and report any improvements in tracking, implementation gain and simplicity, e.t.c. This would be a significant step towards the practical application of the proposed anti-windup techniques in wave energy as well as an opportunity to provide useful insights for the people involved in every design stage (the WEC design stages as seen in Figure ?? (p. ??)) with the hope of taking the race for WEC commercialisation one step further.

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