

Memorandum

The accompanying thesis "The effect of power factor on the stability of the doubly fed machine" is based on work conducted by the author in the Engineering Department of the University of Leicester between October 1966 and December 1969.

All work and ideas recorded in this thesis are original unless otherwise acknowledged in the text or by references. None of the work has been submitted for another degree in this or any other university.

A handwritten signature in black ink, appearing to read "I.A. Alston". The signature is written in a cursive style with a large initial 'I' and a long, sweeping underline.

I.A. Alston

The Effect of Power Factor on the Stability
of the Doubly Fed Machine

by

I.A. Alston, B.Sc. Leicester 1966

A thesis submitted in support of an application for the
degree of Doctor of Philosophy in Engineering in the
University of Leicester.

1969

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Summary

The doubly fed machine is similar to the slip ring induction motor except that both the stator and the rotor are simultaneously supplied with polyphase a.c.. The machine develops a synchronous torque at a certain speed (called the 'speed of synchronous operation'), which is proportional to the sum or the difference of the frequencies of the two supplies (depending on the relative phase sequence). When operating synchronously, the machine exhibits several attributes, the most important of which is its ability to act as a variable speed drive (speed variation is caused by variation of one of the supply frequencies) with a speed-independent-of-load characteristic. It is, however, unstable at certain speeds.

This work uses methods similar to that of Prescott and Raju (Proc. I.E.E. 1958) to analyse the doubly fed machine. An equation is produced relating the speed of synchronous operation of the machine to its supply frequencies and the machine is assumed to operate at (or about) this speed. Equations are also produced, using the method of superimposition, relating the various currents, emf's and torques in the machine to the machine parameters and to the supply voltages and frequencies. This is done in such a way that studies of the dynamic behaviour of the machine could be made.

Workers in the past have either defined the ratio of the rotor voltage to the stator voltage in an arbitrary manner or just tried several values. In seeking an equation to define this ratio is it sensible to stipulate the condition that the minimum current (for that load-speed combination) should be drawn from the supplies. This stipulation resolves into two

criteria: one of unity power factor in the stator supply and one of unity power factor in the rotor supply, which can be expressed mathematically as two equations. Only one of these criteria may be used at any one time (the choice of which one to use depends on practical rather than theoretical considerations). The machine is now completely specified. To verify the validity of the various expressions, a series of experiments and computer simulations were executed. (The latter being based on the expressions with the parameters of the test machine, used in the former, as data.) The results of these experiments and computer simulations did not only show good agreement between practice and theory but also showed that several advantages ensued from the use of a unity power factor criterion. These advantages are: an increase in the stable speed range of the machine, an increase in efficiency, a possible increase in the usable torque range (at any particular speed), a reduction in the currents drawn from the supplies. These advantages are suggested by qualitative arguments, which use the similarities between the doubly fed machine and its, more well known, special cases. The analysis mentioned above was required to quantify them. The performance of the doubly fed machine under conditions of unity power factor (both stator and rotor power factor conditions), and the variation ^{OF} ~~fo~~ the rotor voltage (for a fixed stator voltage) to maintain these conditions, form the latter part of the work. Comparisons are also made between unity power factor conditions and non unity power factor conditions.

The final part of the project was the building and testing of a simple feedback device designed to automatically adjust the rotor voltage to maintain unity stator power factor for all speeds and torques. Many practical difficulties were overcome by using this device including those associated with the starting and synchronising of the machine.

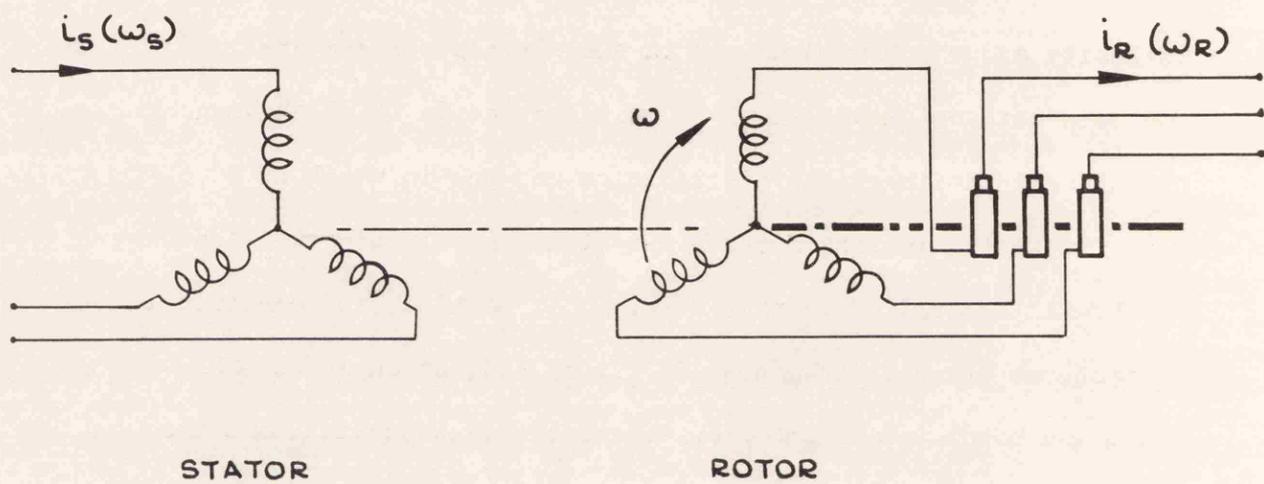


FIG.1.1. REPRESENTATION OF A DOUBLY FED MACHINE

CHAPTER 1

INTRODUCTION

It is well known that all rotating electromagnetic machines depend, for their driving torques, on the interaction of two magnetic fields, one of which is produced by currents in the stator windings and the other of which is produced by currents in the rotor windings. The torque produced is related to the magnetic co-energy (which is an integral function of the field intensity and the flux density) in the air gap. There are two main categories of electric machine: one in which the magnetic circuit affects the field distribution and one in which it does not. It is the latter of these two (the non salient-pole machine) which will be studied in this work. This machine, the electrical connections of which are shown in figure 1.1, strongly resembles the wound rotor induction motor in construction but it forms the basis of a much larger group of machines (as shown in Appendix three).

Figure 1.1 shows that the machine has two separate polyphase a.c. supplies (and consequently it is called 'the doubly fed machine'). The stator field may be assumed to be uniform and rotating at an angular velocity of ω_S^* relative to the stator, where:

$$\omega_S = \frac{2\pi f_S}{\quad}$$

(f_S = stator frequency)

Similarly the rotor field may be assumed to be uniform and rotating at an angular velocity of ω_R^* relative to the rotor where:

$$\omega_R = \frac{2\pi f_R}{\quad}$$

(f_R = rotor frequency)

* in electrical radians

The two fields, whilst inducing further fields in the rotor and stator respectively, are assumed to be independent and are only stationary relative to each other when the mechanical speed of rotation of the machine, $\frac{w}{n}$ ^{*} is such that:

$$w = w_S - w_R$$

and this is termed the 'synchronous speed'. To permit the study of stability, the conditions of non synchronous operation and, ~~to this end~~, the term Δw ^{are} is introduced such that

$$w - \Delta w = w_S - w_R$$

under all conditions. Variation of the synchronous speed of the machine results from the variation of one of the two supply frequencies. In the present work the stator frequency is held constant and the rotor frequency is varied. To facilitate the study of speeds above fundamental^{**} (as well as below) the concept of negative w_R and negative rotor frequency (for reversed phase sequence) is introduced.

The doubly fed machine is analysed by methods similar to that of Prescott and Raju¹ except that more care is taken with regard to the relative phase angles, of various voltages, during superimposition. This is to facilitate the study of the supply phase angle relationships, which is a fundamental part of this work. Equations are developed from first principles which give the machine output torques as a function of the machine parameters, the supply frequencies and voltages and a load angle ' δ ' (which is defined in terms of the apparent phase angle between the stator and rotor supply voltages).

* 'n' is the number of pole pairs

** fundamental speed corresponds to $w_R = 0$ (i.e. $w = w_S$)

The process of deriving these equations requires the machine to be completely specified (regarding currents, induced emf's etc.) except for the ratio of rotor to stator voltage. This ratio can be arbitrarily defined but variations in the ratio can result in large reactive currents being drawn from the supplies⁷. Criteria are, therefore, developed for optimum rotor to stator voltage ratios whereby either the stator or the rotor phase angles are reduced to zero. The performance of a 2 h.p. wound rotor induction motor, operating as a doubly fed machine, is then studied both by experiment and by computer simulation. Comparisons are made between practical and simulated results, and between conditions where stator (or rotor) phase angles are zero and conditions where they are not. The stability of the machine (for small oscillations) is also studied in this manner, the criterion for stability being the Routh-Hurwitz criterion as outlined by Macfarlane¹¹. The machine torques consist of two components, one of which varies with speed and the other of which varies with load angle (roughly). Stability results when the slopes of both these characteristics are positive.

$$\text{(i.e. } \frac{dT_s}{d\delta} > 0 \text{ and } \frac{dT_I}{d\delta} > 0 \text{)}$$

Stability studies then reduce to a study of the speed dependent torques only, as considerations of operation at unstable load angles (where $dT_s/d\delta$ is negative) is unnecessary.

The results of this work show that considerable advantages ensue when reactive currents are reduced to a minimum. The stable speed range available in the doubly fed machine and the available torque range are increased by this method together with the machine efficiency and utilisation. The doubly fed machine becomes a useful component in a variable speed drive system, offering synchronous speed operation, unaffected by load torques, at a wide range of subfundamental ($< \frac{3000}{n}$ rev/m) and superfundamental ($> \frac{3000}{n}$ rev/m) speeds. It is thought to be the ideal load machine for a static inverter or cycloconverter as these are only required to handle a fraction of the machine power (in watts or in volt-amps).

Nomenclature and Symbols

General

Re	=	Real part of -----
Im	=	Imaginary part of -----
w	=	angular velocity of rotor relative to the stator (electrical radians)
w_S	=	angular velocity of stator field relative to the stator (electrical radians)
w_R	=	angular velocity of rotor field relative to the rotor (electrical radians)
Δw	=	$w + w_R - w_S$
ϕ_{S1}	=	phase angle between stator volts and current in stator fed system
ϕ_{S2}	=	phase angle between stator volts and current in rotor fed system
ϕ_{R1}	=	phase angle between rotor volts and current in stator stator fed system
ϕ_{R2}	=	phase angle between rotor volts and current in rotor fed system
δ	=	$\phi_{S1} - \phi_{R2} - \Delta w t$
i_S	=	resultant inward stator current; scalar equivalent I_S
i_R	=	resultant outward rotor current; scalar equivalent I_R
L_S	=	stator self inductance (cyclic)
L_R	=	rotor self inductance (cyclic)
M	=	mutual inductance between rotor and stator (cyclic)
R_S	=	stator resistance
R_R	=	rotor resistance
T	=	Torque/pole-pair/phase

(the turns-ratio is implicit in L_S , L_R and M)

Stator fed terms

<u>Vector</u>		<u>Scalar Equivalent</u>
v_S	= applied stator voltage	V_S
i_{S1}	= stator current	I_{S1}
e_{S1}	= stator e.m.f. induced by i_{S1}	E_{S1}
e_{R1}	= rotor e.m.f. induced by i_{S1}	E_{R1}
i_{R1}	= rotor current	I_{R1}
e_{RR1}	= e.m.f. induced in rotor by i_{R1}	E_{RR1}
e_{SR1}	= e.m.f. induced in stator by i_{R1}	E_{SR1}

Rotor fed terms

<u>Vector</u>		<u>Scalar Equivalent</u>
v_R	= applied rotor voltage	V_R
i_{R2}	= rotor current	I_{R2}
e_{R2}	= e.m.f. induced in rotor by i_{R2}	E_{R2}
e_{S2}	= e.m.f. induced in stator by i_{R2}	E_{S2}
i_{S2}	= stator current	I_{S2}
e_{SS2}	= e.m.f. induced in stator by i_{S2}	E_{SS2}
e_{RS2}	= e.m.f. induced in rotor by i_{S2}	E_{RS2}
e^*	= conjugate vector of e	

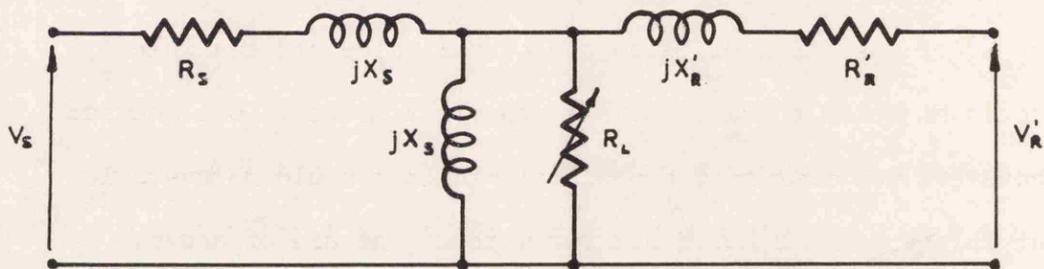


FIG. 2-1. D.F.M. EQUIVALENT CIRCUIT (AFTER ALBERTSON)

CHAPTER 2

PAST WORK ON THE DOUBLY FED
MACHINE

The first 'doubly fed machine', patented in Germany in 1899 (pat. no. DRP 109986), was a polyphase wound rotor induction machine with the rotor and stator supplied with power of the same frequency but opposite phase sequence. In the present notation this results in the condition that

$$\omega_R = -\omega_S \quad \text{and} \quad \omega = 2\omega_S \quad \text{for synchronous operation .}$$

The result was, therefore, a machine which possessed a synchronous speed of operation of twice the normal value and one which could be shown to have several other attributes. One drawback, however, was that most commercial machines would be unstable at this speed and could not run without the aid of a d.c. machine or an induction motor (with gearbox) to stabilise it. The machine also needed to be started and synchronised with the aid of an external motor.

Early analyses of the doubly fed machine were based upon the equivalent circuit of the machine (figure 2.1), where R_L represents the mechanical load of the machine, and were restricted to the steady state. Dynamic analysis was produced by two methods: (1) By using tensorial techniques to produce the equivalent circuit of the doubly fed machine (attributed to Kron) and (2) By re-analysing the machine, under generalised conditions, from first principles (attributed to Prescott and Raju). These three methods of analysis are well represented (in the reverse order) by the work of: (a) Prescott and Raju¹, (b) Albertson et al^{2,3,4}, and (c) (for work in

the steady state) Bird, Chalmers et al^{5,6,7}, which will be discussed below.

The instability of the machine made it of little use and much work was done to produce a stable, efficient machine. The machine was stabilised by various methods, which can be shown to fall into three main classes: (a) alteration of the parameters of the machine to make it naturally stable, (b) the use of feedback, and (c) the superimposition of positive damping such as that produced by an induction motor or a d.c. motor. Although the machine was successfully stabilised by these methods, a sizeable reduction in machine efficiency and utilisation resulted, preventing the machine from becoming a commercial proposition.

The recent production of high power static inverter and frequency-changer circuits (based on the thyristor) has led to the continued study of the doubly fed machine when fed from these devices. Prior to the production of the static convertor, variable speed drives involved the use of commutators which required a great deal of maintenance. With the development of the static convertor the 'variable speed drive' was to comprise an a.c. machine driven from a variable frequency supply (produced by the convertor). Because of the high cost of high power silicon controlled rectifiers, it became desirable that the convertor should only handle part of the machine input power. Reference was made to the changing of the induction motor torque-speed characteristic with increased rotor resistance. The rotor resistance was said to dissipate or remove, power from the machine rotor. The removal of this

power (at slip frequency) could be done by a cycloconverter which would feed the power into the mains. It was also found that, by injecting energy into the rotor, the machine could be made to run at speeds above the fundamental synchronous speed. The actual arrangement used was that of a doubly fed machine. Work has also been conducted recently on the 'asynchronous generator' which used a condition whereby a conventional synchronous generator field was excited with low frequency a.c.. Thus the doubly fed machine is again becoming of interest, as the efficiencies and reliabilities of static converters improve.

2.1 The theory of the doubly fed machine

There have been three types of approach to the analysis of the doubly fed machine and these will be represented by reference to three well-known groups of workers in the field.

Steady State analysis

This method of analysing the doubly fed machine was used by all the early workers in the field and some of the later ones. A good recent example is the work by Chalmers⁷ (1960) which will be outlined below.

The doubly fed machine, in the no load condition, was considered as a transformer with complex energy being fed, say, from the primary to the secondary. As the secondary voltage was raised relative to the primary the flow of energy would be reversed. In particular reactive current would flow 'into' the rotor and stator (or primary and secondary)

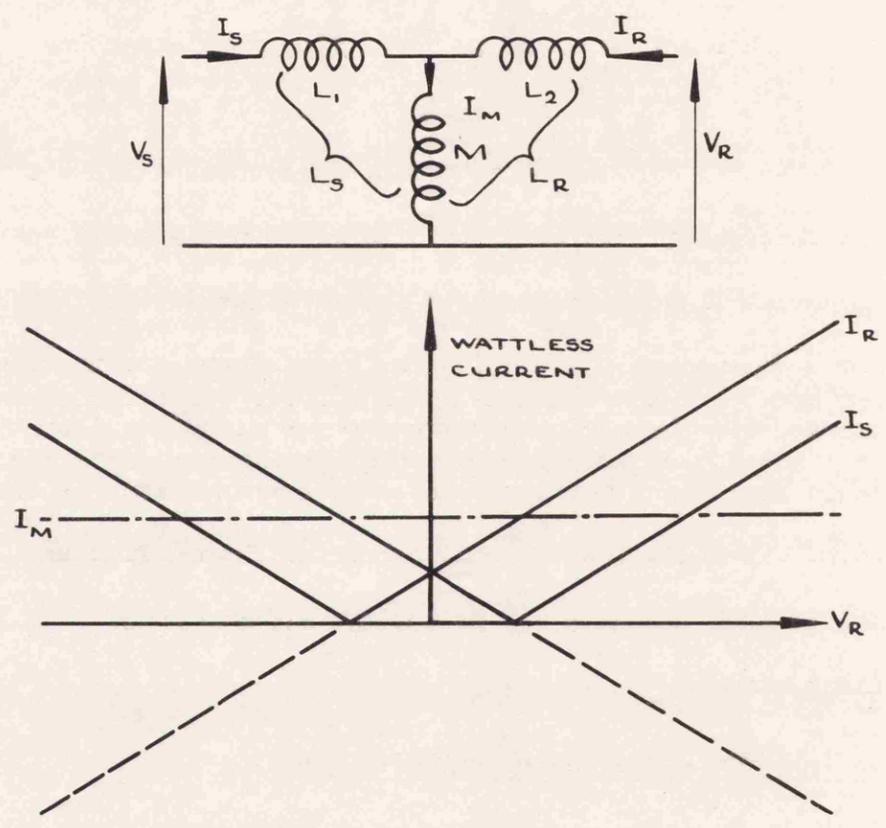


FIG. 2.2.
THE TRANSFORMER EFFECT IN DOUBLY FED MACHINES

such that the total current flowing into the machine was equal to the magnetising current. If the rotor voltage were too low (figure 2.2) the rotor reactive current would be negative and if it were too high the stator reactive current would be negative. In measured^{ing} a.c. currents it is difficult to distinguish in-phase from anti-phase currents and magnitudes appear to be significant. Figure 2.2 clearly shows how high reactive currents could flow in the machine if the ratio of V_R/V_S was far from optimum. It also shows that the area for minimum -net-current-magnitude (which was constant throughout this area) existed between the position where reactive I_R was zero and the position where reactive I_S was zero. To analyse the machine on load, a tertiary winding was inserted (in the equivalent-transformer circuit) with a variable resistance to represent the mechanical load of the machine. This equivalent circuit (shown in figure 2.1) was analysed in much the same way as the synchronous motor equivalent circuit.

Dynamic analysis using equivalent circuits

Most of the work on equivalent circuits to represent the dynamic behaviour of electric machines was pioneered by Gabriel Kron* who applied his expertise to the doubly fed machine, in a paper written jointly with Concordia and Cray⁸, in 1942. A two phase machine (or transformed three phase machine) was analysed in such a way that all the voltages in all the windings were specified including those caused by

 * For an outline of Kron's work see the chapter 'Kron's Generalised Machine' by Alger⁹

mutual inductance with other windings and the currents in those windings. This is the significant feature of this method in that, whereas only one phase in each of the rotor and stator are normally considered, Kron considered all phases in the rotor and stator and the effect of one stator phase on another or one rotor phase on another. The equations were written down as an impedance tensor 'z' such that the applied voltage and the current vectors were related by:

$$\begin{bmatrix} v_{dS} \\ v_{qS} \\ v_{dR} \\ v_{qR} \end{bmatrix} = \begin{bmatrix} z \end{bmatrix} \cdot \begin{bmatrix} i_{dS} \\ i_{qS} \\ i_{dR} \\ i_{qR} \end{bmatrix} \quad \text{or } v = z \cdot i$$

(where d refers to the direct axis, q refers to the quadrature axis, S refers to the stator and R refers to the rotor)

As the rotor axes were moving relative to the stator axes it was necessary to refer the equations to one set of axes.

This was done with a transform matrix 'C' which was such that:

$$C_t^* \cdot C = \text{the unit matrix}$$

$$I = C \cdot i$$

$$V = C_t^* \cdot v$$

$$\text{and } Z = C_t^* \cdot z \cdot C$$

(where I, V and Z are the transformed equivalents of i, v and z.)

Generally followers of Kron (of which Albertson^{2,3} is a good example) chose the new axes to be the positive and negative sequence component axes thus facilitating the study of unbalanced conditions.

The resulting impedance tensor, Z , was used to produce an equivalent circuit which could be set up on the network analyser for simulation of the performance of the machine. Kron et al produced an impedance tensor (and the resultant equivalent circuit) which could represent the performance of the machine during hunting which enabled dynamic studies to be made. The machine torque was represented in terms of the current (I) and induced emf (E) in one of the sets of windings (either rotor or stator) by the relationship^{8,10}:

Torque = real part of $I E^*$ (synchwatt) which could be measured by the suitable use of wattmeters in the equivalent circuit on the network analyser.

Dynamic analysis by superposition

Prescott and Raju¹ introduced a form of analysis of the doubly fed machine which did not involve equivalent circuits. The doubly fed machine was analysed for currents, voltages etc. by considering it as two induction motors: one with the stator supplied and the rotor short circuited and the other with the rotor supplied and the stator short circuited. The final doubly fed machine was formed by the superposition of these two. To do this the voltage-current equations were produced for the two component induction motors and these equations were solved for:

$$I_{S1} \text{ and } I_{R1} \quad \text{for the stator fed induction motor}$$

$$\text{and } I_{S2} \text{ and } I_{R2} \quad \text{for the rotor fed induction motor.}$$

Superimposition was then achieved by taking the stator and rotor currents to be the vectorial sum of the two components thus:

$$I_S = I_{S1} + I_{S2}$$

$$I_R = I_{R1} + I_{R2}$$

It becomes immediately clear that the values of these vector sums depend on the phase angle between the stator and rotor fed components and, by allowing this angle to vary in an arbitrary manner, the dynamic behaviour of the machine can be studied. Prescott and Raju defined this angle (the angle of superposition) as the angle between the rotor 'A phase' and the stator 'A phase' at time, $t = 0$. To study the dynamic behaviour fully they made the mechanical speed of the machine mathematically independent of the supply frequencies. The machine torque was evaluated from an equation, similar to that used by Kron et al, which was written as:

$$T = \text{Real part of } \left[\frac{E_{R1}^*}{(\omega_S - \omega)} + \frac{E_{R2}^*}{\omega_R} \right] \left[I_{R1} + I_{R2} \right]$$

Unfortunately the final expressions for torque contained a component with an erroneous sign in it. This had led to a general mistrust in the work of Prescott and Raju and a perpetuation of the use of Kron's equations (despite their greater complexity).

The present work is similar to that of Prescott and Raju (similar notation and methods being used) but, because of the power-factor aspect of the work, more care is taken in defining the angle of superposition. To study the dynamic behaviour of

the machine either the equations for torque etc. can be manipulated (as Prescott and Raju did for the double speed machine dynamic conditions and Bird and Burbidge⁵ did for steady state conditions of the machine) or the equations can be processed by a digital computer to produce a numerical simulation of the doubly fed machine. No equivalent circuit is used although one could be produced.

2.2 Stabilising the doubly fed machine

The doubly fed machine has long been known for its instability (at double speed) and its good steady-state properties. Thus much work has been done to stabilise the machine for use at double speed. The methods of stabilising the machine fall into three categories (as with most systems) and these are:

- (1) Changing the machine parameters to make it stable
- (2) Imposing a large amount of damping on the machine
- (3) Using a stabilising feed-back loop.

Alteration of the machine parameters

A parallel can be drawn between the stability of the induction motor at various speeds and that of the doubly fed machine at those speeds. The stable speed range of the doubly fed machine can (as can that of the induction motor) be extended by increasing the rotor resistance. Prescott and Raju¹ showed that, by increasing the rotor resistance (or decreasing the leakage reactance) the machine could be stabilised at double speed. This, however, resulted in an increase in copper loss (if the rotor resistance was to be raised sufficiently) which produced a lower machine efficiency.

Application of damping from an external source

External damping can be applied to the machine either by its load or by an auxiliary machine. The auxiliary machine would be required to provide a considerable degree of damping, not only to cancel the negative damping produced by the doubly fed machine but also to provide good stability during load changes. Suitable machines, for this purpose, are the d.c. machine and the induction machine (supplied with an appropriate frequency or coupled through a gearbox). Many of the earlier workers on the doubly fed machine used auxiliary machines to stabilise the doubly fed machine. One of the best examples of the use of this principle however, is that used by Chalmers^{6,7}. Chalmers stabilised his machine by winding an induction motor on the same frame. His early machine had two windings on the stator (a four pole and an eight pole winding) and one winding on the rotor. The rotor winding was such that it presented a short circuit to the eight pole field and an open circuit (open at three slip rings) to the four pole field. Thus, by supplying the rotor slip rings and the stator four pole field with 50Hz, 3 phase (of opposing phase sequence) a double speed, doubly fed machine was produced (with a speed of 3000 rev/m). Also, by supplying the eight pole stator field with 200 Hz an induction machine was produced (with a "synchronous speed" of 3000 rev/m). The induction machine exhibited a high degree of dynamic stability near its "synchronous speed" which could be used to stabilize the doubly fed machine operating at the same speed. Thus Chalmers stabilised his doubly fed machine.

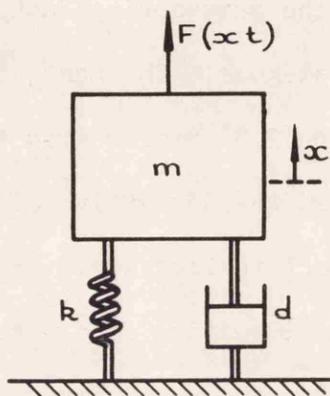
The 2 pole - 4 pole machine (two pole doubly fed machine, four pole induction machine) could not be used as an unbalanced magnetic pull occurred when two such fields, with a pole number differing by two, coexisted. For operation at 6000 rev/m, therefore, he produced a 2 pole - 8 pole machine with an auxiliary (induction winding) supply of 400 Hz.

He also used the induction windings to start and synchronise the machine (as others had done before with their auxiliary stabilising machines), but found difficulties resulting from the Gorges phenomenon associated with his effectively single phase rotor winding.

Stabilisation by feedback

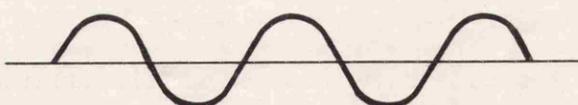
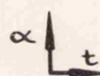
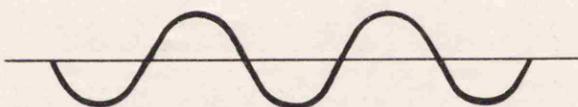
Feedback has long been used to stabilise the doubly fed machine. The universal commutator motor, the Schrage motor, the ~~Scherbius~~ ^{Scherbius} system and numerous other variable speed drives comprise a doubly fed machine and a secondary frequency injector (the output frequency of which is controlled by the machine speed). The doubly fed machine component always operates synchronously (in that $w = w_S - w_R$) not because it is stable but because it governs one of its supply frequencies. Thus, in the equation $w = w_S - w_R$, w_R is the dependent variable with respect to the frequency changer and w is the dependent variable with respect to the doubly fed machine. Thus a very simple form of speed feedback is used to retain stability (this is shown in figure A1 of appendix 3).

When this feedback is used, however, speed control is by voltage instead of frequency and the machine has a torque dependent speed characteristic.



MASS SPRING DAMPER

DOUBLY FED MACHINE



FOR STABILISATION

FOR STABILISATION
(AFTER ALBERTSON)

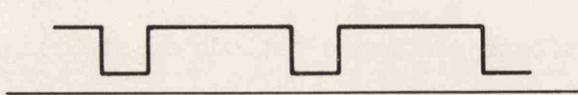
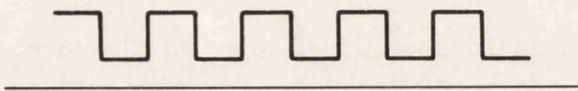
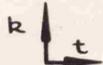
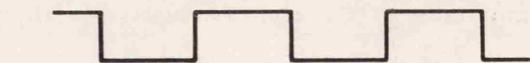
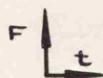


FIG. 2.3. THE STABILISATION OF DOUBLY FED MACHINES USING THE MASS SPRING DASHPOT ANALOGUE

A method of stabilising the machine was developed by Albertson^{2,4} which did not adversely affect the synchronous performance of the machine. Comparison between the dynamic equation of the machine and that of the mass-spring-dashpot system showed the two systems to be analogous. The mass, in the mass spring dashpot system (figure 2.3) represented the machine inertia the spring represented the derivative of the synchronous torque components with respect to load angle and the dashpot represented the derivative of the inductive torque terms with respect to speed. The force 'F' represented fluctuations in load torque or machine output torque (predominantly synchronous).

To represent the machine in its unstable region the mass-spring-dashpot system would have negative damping. In this condition, were it left alone, the system would oscillate with ever increasing amplitude. But if an upwards force, F, were applied to the mass only when it was descending or alternatively if the spring stiffness were temporarily reduced during the times when it was returning energy to the system (during periods of acceleration in the direction of motion of the system) then the oscillations would soon die down. As the synchronous torque components were analogous to both the force F and the spring stiffness both the above methods of stabilising could be achieved by periodically varying these torque components as shown in figure 2.3. To apply this method of stabilisation, however, required the measurement of the variations in the machine speed so that the variations in synchronous torque could be applied at the correct times. If the machine speed were measured, the mean speed would swamp any fluctuations in speed. So the acceleration of the machine was measured. This was done using

a two phase drag-cup servo induction motor with d.c. applied to the direct axis field (say) and the output taken from the quadrature axis field. The output was then roughly proportional to the acceleration of the machine. Albertson had some difficulty due to noise pick-up but, as his machine was only to operate at one speed, this could be filtered out. The signal from this accelerometer was used to time the opening and closing of a gate which introduced an unbalancing resistor into the rotor circuit of the machine (the stator circuit providing the magnetising current). This unbalance reduced the synchronous torque components of the machine and it was stabilised by periodically applying it as shown in figure 2.3.

Albertson assumed however that in the event of instability the machine would oscillate for several cycles before de-synchronising. This would only occur if the damping of the machine (be it positive or negative) was less than critical and the machine stiffness high. His method does not work, therefore, for all machines but it is valuable in that it can be applied to variable speed machines.

2.3 General Comments

Various methods of analysis and various methods of stabilising (for use at double speed) have been mentioned with examples from a few eminent workers in the field, who are representative of most of the workers. A considerable amount of work has been done, however, on the doubly fed machine (under various names) over the past half century and it is impossible to mention all of this here.

As methods of analysis, and applications for the machine, have developed, results on all aspects of the performance of the doubly fed machine, in many situations, have been produced. In all

these studies, however, the ratio of rotor to stator voltage has always been taken to be equal to the ratio of the square roots of the rotor and stator impedances. Little has been done to study the effect of the variation of this ratio on the performance of the machine or the parameters on which an optimum ratio might depend.

STATOR FED

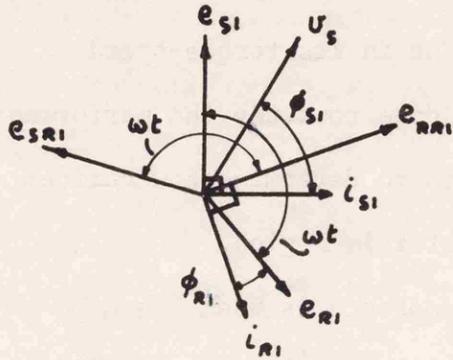


FIG. 3.1

ROTOR FED

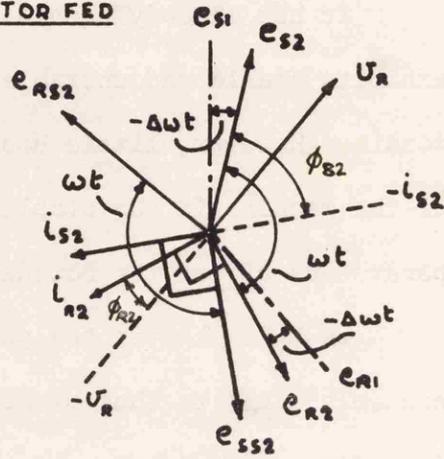


FIG. 3.2

SUPERIMPOSED

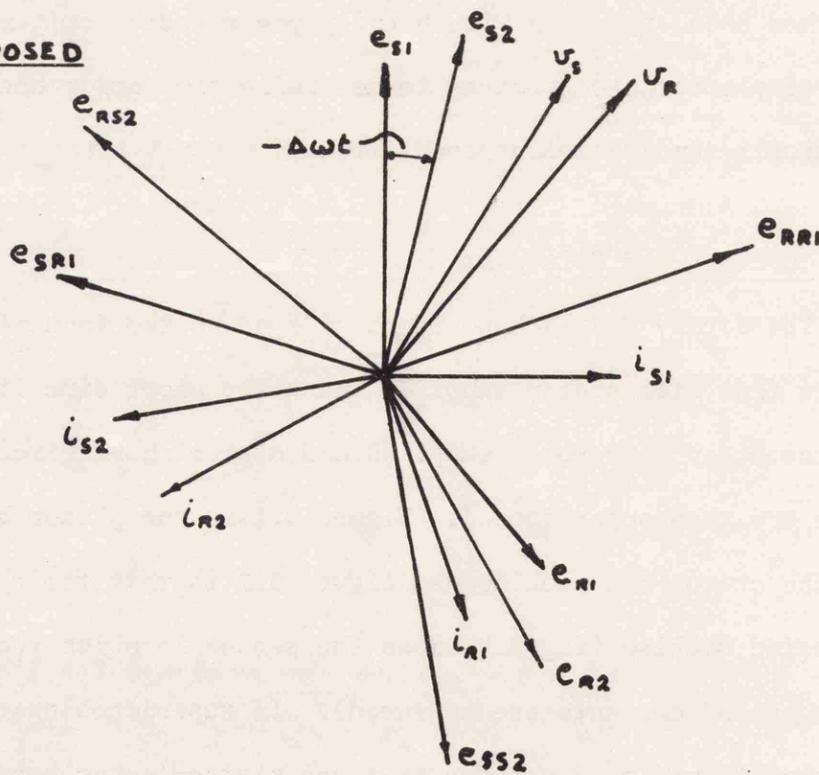


FIG. 3.3

PHASORS IN THE DOUBLY FED MACHINE

CHAPTER 3

THE THEORY OF THE DOUBLY FED MACHINE

It has already been said that the doubly fed machine exhibits stable and unstable regions in its torque-speed domain. However, little has been done to study the performance of the machine in the stable region to determine how various parameters affect the boundaries of this region.

In this chapter the phasor approach is used (in a manner similar to that of Prescott and Raju¹) to determine from first principles, expressions for the currents, voltages and torques in the doubly fed machine. These expressions are then used, together with a unity power factor criterion in a simple computer program to determine the performance of the machine under various conditions.

3.1 Currents and Voltages

The doubly fed machine is considered as two induction motors (one with stator supplied and rotor short circuited and the other with rotor supplied and stator short circuited) which are then superimposed. Figure 3.1 is the phasor diagram for the stator-fed machine and figure 3.2 is that for the rotor-fed machine (fig.3.4 shows the senses in which the voltages and currents are measured). In superimposing these two conditions it is assumed that the stator-fed voltage and current components will not affect the rotor-fed voltage and current components, but will merely add to them (vectorially). It is also assumed that the magnetic circuit of the machine does not saturate.

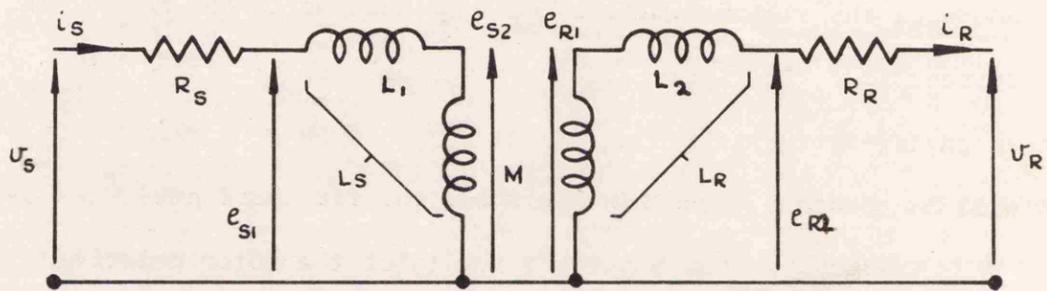


FIG.3-4. DIAGRAM SHOWING SENSE OF VOLTAGES CURRENTS & E.M.F.S.

It will be noted that in both figures 3.1 and 3.2 there are two distinct sets of vectors: (1) those which are associated with the stator (and rotate at an angular velocity of w_S in space) and (2) those associated with the rotor (which rotate at an angular velocity of w_R in space). When these conditions are superimposed, as shown in figure 3.3, care should be taken to choose two reference vectors which are stationary with respect to each other. Two such vectors are e_{S1} , the stator e.m.f. induced by changes in the stator current i_{S1} , and e_{S2} , the stator e.m.f. induced by changes in the rotor current i_{R2} . In figure 3.3 these vectors are displaced by an angle, $-\Delta\omega t$, by which e_{S1} leads e_{S2} . If $\Delta\omega = 0$, this angle will be constant, otherwise $\Delta\omega t^*$ will be time variant.

Taking w as the angular velocity of the rotor relative to the stator (in electrical radians), w_S as the angular velocity of the stator field relative to the stator and w_R as the angular velocity of the rotor field relative to the rotor then the equation:

$$w_S - w_R = w - \Delta\omega$$

will apply to the machine in all conditions. The principal concern is synchronous operation, when $\Delta\omega = 0$. If dynamic instability occurs the machine tries to depart from synchronism (and $\Delta\omega \neq 0$), hence calculations are executed mainly with a finite $\Delta\omega$ which is reduced to zero only in the final stages.

* $\Delta\omega t$ is taken as $\int \Delta\omega dt$ and not $\Delta\omega \cdot t$ as might be assumed.

Currents and voltages with the stator supplied

When the machine is supplied via the stator, with rotor short circuited, the currents i_{S1} and i_{R1} flow in the stator and rotor windings respectively.*

The current i_{S1} induces the e.m.f.'s:

$$e_{S1} = L_S \frac{di_{S1}}{dt} = j\omega_S L_S i_{S1} \text{ in the stator}$$

$$\text{and } e_{R1} = M \frac{d}{dt} (i_{S1} e^{-j\omega t}) = j(\omega_S - \omega) M i_{S1} e^{j\omega t} = j(\omega_R - \Delta\omega) M i_{S1} e^{j\omega t}$$

in the rotor

and the current i_{R1} induces the e.m.f.'s:

$$e_{SR1} = M \frac{d}{dt} (i_{R1} e^{j\omega t}) = j(\omega_R - \Delta\omega + \omega) M i_{R1} e^{j\omega t} = j\omega_S M i_{R1} e^{j\omega t}$$

in the stator

$$\text{and } e_{RR1} = L_R \frac{di_{R1}}{dt} = j(\omega_R - \Delta\omega) L_R i_{R1} \text{ in the rotor}$$

The resultant voltages at the stator and rotor terminals are

$$v_S = R_S i_{S1} + e_{S1} + e_{SR1} \quad (3.1)$$

$$\text{and } 0 = R_R i_{R1} + e_{R1} + e_{RR1} \quad (3.2)$$

Substituting for the e.m.f.'s in equations 3.1 and 3.2 gives:

$$\begin{aligned} v_S &= R_S i_{S1} + j\omega_S L_S i_{S1} + j\omega_S M i_{R1} e^{j\omega t} \\ &= \{R_S + j\omega_S L_S\} i_{S1} + \{j\omega_S M e^{j\omega t}\} i_{R1} \end{aligned} \quad (3.3)$$

$$\begin{aligned} \text{and } 0 &= R_R i_{R1} + j(\omega_R - \Delta\omega) M i_{S1} e^{-j\omega t} + j(\omega_R - \Delta\omega) L_R i_{R1} \\ &= \{R_R + j(\omega_R - \Delta\omega) L_R\} i_{R1} + \{j(\omega_R - \Delta\omega) M e^{-j\omega t}\} i_{S1} \end{aligned} \quad (3.4)$$

* $i_{S1} = (i_{S1})_{t=0} e^{j\omega_S t}$ and $i_{R1} = (i_{R1})_{t=0} e^{j(\omega_R - \Delta\omega)t}$

Rearranging equation 3.4

$$\{J(w_R - \Delta w)Me^{-j\omega t}\}i_{S1} = -\{R_R + j(w_R - \Delta w)L_R\}i_{R1}$$

$$\text{and } i_{S1} = \frac{-\{R_R + j(w_R - \Delta w)L_R\}e^{j\omega t}i_{R1}}{j(w_R - \Delta w)M} \quad (3.5)$$

Substituting equation 3.5 into equation 3.3 gives

$$v = \left[\frac{\{R_S + jw_S L_S\}\{R_R + j(w_R - \Delta w)L_R\}e^{j\omega t}}{-j(w_R - \Delta w)M} + jw_S Me^{j\omega t} \right] i_{R1}$$

$$\begin{aligned} \text{Hence } -j(w_R - \Delta w)Mv_S &= e^{j\omega t} \{R_S R_R - w_S (w_R - \Delta w)L_S L_R + jR_S (w_R - \Delta w)L_R \\ &\quad + jR_R w_S L_S + w_S (w_R - \Delta w)M^2\} i_{R1} \end{aligned}$$

$$\begin{aligned} \text{and } j(w_R - \Delta w)Me^{-j\omega t}v_S &= -\left[\{R_S R_R + w_S (w_R - \Delta w)(M^2 - L_S L_R)\} \right. \\ &\quad \left. + j\{R_S (w_R - \Delta w)L_R + R_R w_S L_S\} \right] i_{R1} \end{aligned}$$

$$\text{and } i_{R1} = \frac{-j(w_R - \Delta w)Me^{-j\omega t}v_S}{\{R_S R_R + w_S (w_R - \Delta w)(M^2 - L_S L_R)\} + j\{R_S (w_R - \Delta w)L_R + R_R w_S L_S\}} \quad (3.6)$$

$$= \frac{-j(w_R - \Delta w)Me^{-j\omega t}v_S}{A + jB} \quad (3.7)$$

Where A is the real part and B is the imaginary part of the denominator of equation (3.6)

Also, substituting equation 3.5 into equation 3.7

$$i_{S1} = \frac{+\{R_R + j(w_R - \Delta w)L_R\}v_S}{A + jB} \quad (3.8)$$

Currents and voltages with rotor supplied

With the stator short circuited and the rotor supplied the currents i_{S2} and i_{R2} flow in the stator and rotor windings respectively.

The current i_{S2} induces the e.m.f's

$$e_{SS2} = j(\omega_S + \Delta\omega)L_S i_{S2} \text{ in the stator}$$

and
$$e_{RS2} = j\omega_R M_{RS} i_{S2} e^{-j\omega t} \text{ in the rotor}$$

and the current i_{R2} induces the e.m.f's

$$e_{S2} = j(\omega_S + \Delta\omega)M_{SR} i_{R2} e^{j\omega t} \text{ in the stator}$$

and
$$e_{R2} = j\omega_R L_R i_{R2} \text{ in the rotor}$$

The resultant voltages at the stator and rotor terminals are

$$0 = R_S i_{S2} + e_{S2} + e_{SS2}$$

$$v_R = R_R i_{R2} + e_{R2} + e_{RS2}$$

or, substituting for the e.m.f's

$$0 = \{R_S + j(\omega_S + \Delta\omega)L_S\} i_{S2} + \{j(\omega_S + \Delta\omega)M\} i_{R2} e^{j\omega t} \quad (3.9)$$

and
$$v_R = \{R_R + j\omega_R L_R\} i_{R2} + \{j\omega_R M e^{-j\omega t}\} i_{S2} \quad (3.10)$$

By a similar method to that used in ~~section 9.1~~ in the stator fed case:

equation 3.9 may be rearranged to give

$$i_{R2} = \frac{\{R_S + j(\omega_S + \Delta\omega)L_S\} i_{S2}}{-j(\omega_S + \Delta\omega)M e^{j\omega t}} \quad (3.11)$$

which, when substituted into equation 3.10 gives:

$$-j(\omega_S + \Delta\omega)Mv_R e^{j\omega t} = \left[\{R_R + j\omega_R L_R\} \{R_S + j(\omega_S + \Delta\omega)L_S\} + \omega_R(\omega_S + \Delta\omega)M^2 \right] i_{S2}$$

$$\text{or } i_{S2} = \frac{-j(\omega_S + \Delta\omega)M e^{j\omega t} v_R}{\{R_S R_R + \omega_R(\omega_S + \Delta\omega)(M^2 - L_S L_R)\} + j\{R_S \omega_R L_R + R_R(\omega_S + \Delta\omega)L_S\}} \quad (3.12)$$

$$= \frac{-j(\omega_S + \Delta\omega)M e^{j\omega t} v_R}{C + jD} \quad (3.13)$$

Where C is the real part and D the imaginary part of the denominator of equation (3.12).

Substituting equation (3.11) into equation (3.13) gives

$$i_{R2} = \frac{\{R_S + j(\omega_S + \Delta\omega)L_S\}v_R}{C + jD} \quad (3.14)$$

Phase angle relationship between v_S and v_R

All the expressions so far have been in terms of vectors rather than scalars and the knowledge of phase-angle relationships has not, as yet, been of use. The final expressions for torque must, however, be in terms of scalar quantities only and to produce these requires a knowledge of the phase angle relationship between the two applied voltages. This phase angle relationship is also used in the determination of the minimum current (or unity power factor) criterion of section 3.3.

With the aid of figures 3.1, 3.2 and 3.3 this relationship can be found as follows:

From fig. 3.1:

$$v_S = -jV_S e^{j\phi_{S1}} \text{ taking } e_{S1} \text{ as reference}$$

From fig. 3.2:

$$e_{S2} = E_{S2} e^{j\Delta\omega t} \text{ with } e_{S1} \text{ as reference}$$

$$\text{and } v_R = jV_R e^{j(\phi_{R2} - \omega t)} \text{ with } e_{S2} \text{ as reference}$$

$$\therefore v_R = jV_R e^{j(\Delta\omega t + \phi_{R2} - \omega t)} \text{ with } e_{S1} \text{ as reference}$$

$$\therefore v_R = -V_R e^{j(\Delta\omega t + \phi_{R2} - \phi_{S1} - \omega t)} \text{ with } v_S \text{ as reference}$$

$$= -V_R e^{-j(\delta + \omega t)} \quad \text{" " " " (3.15)}$$

similarly

$$v_S = -V_S e^{j(\omega t + \phi_{S1} - \phi_{R2} - \Delta\omega t)} \text{ with } v_R \text{ as reference}$$

$$= -V_S e^{j(\delta + \omega t)} \quad \text{" " " " (3.16)}$$

3.2 Torques

Equations for torque have been produced before¹ but the algebraic process executed to obtain them and, indeed, the final results themselves left something to be desired. Rather than criticise previous work, it has been decided to develop expressions for torque from first principles using the M.K.S. system of units.

The basic torque equation

Consider a single, 'loss-free', full pitched, rectangular turn (or 'current loop') in an externally applied uniform magnetic field, B (as shown in fig. 3.5) . Let this loop rotate

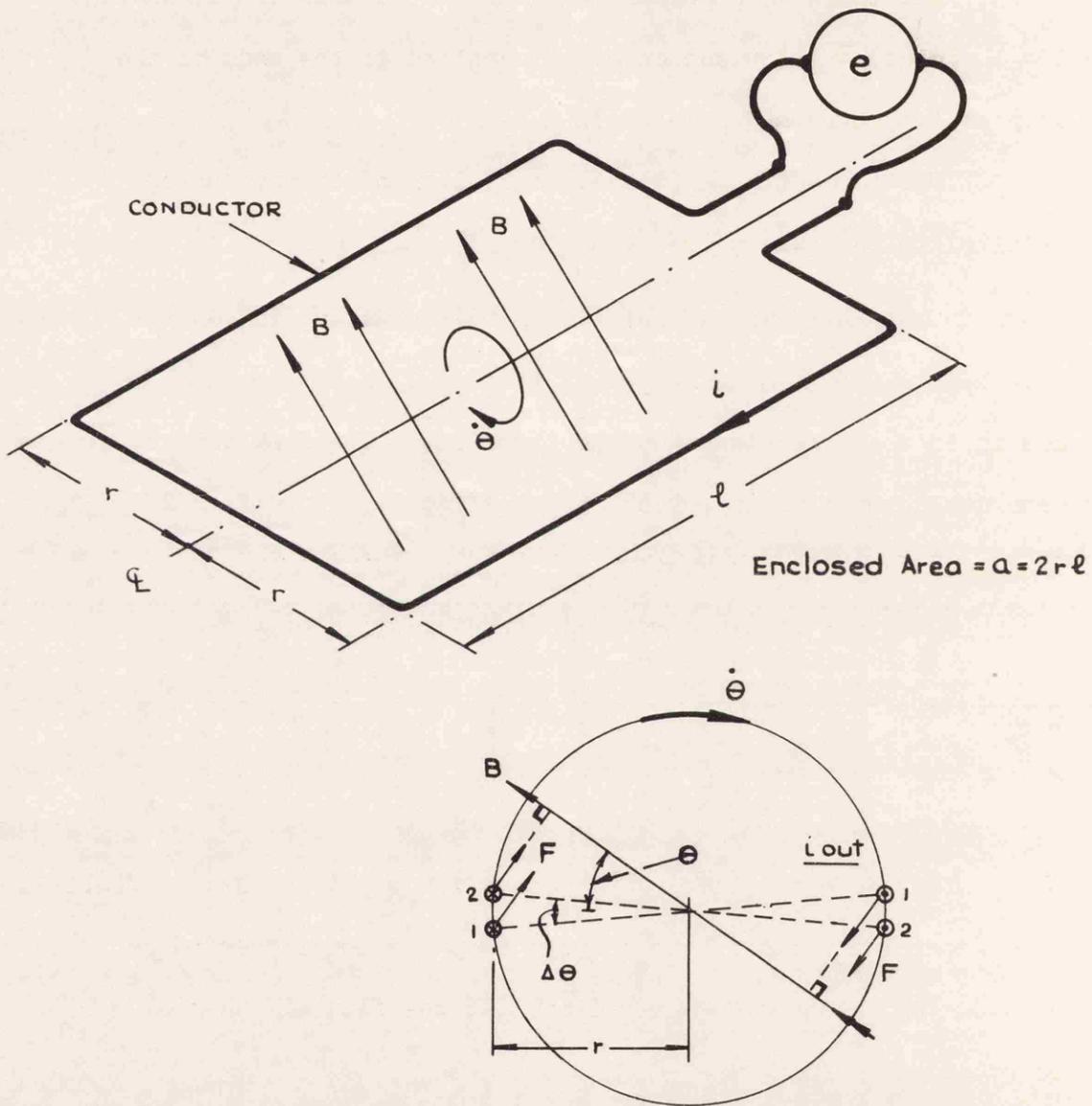


FIG. 3.5. THE FORCE ON A CURRENT LOOP IN A MAGNETIC FIELD

about a centreline (which is equidistant from and parallel to, two of the sides of the loop and which is perpendicular to the field vector) at a constant angular speed of $\dot{\theta}$ relative to the field. Let there be a current flowing in the loop of 'i' where $i = I \sin(\dot{\theta}t) = I \sin\theta$ and an e.m.f. applied to the ends of the loop of 'e' where $e = E \sin(\theta + \alpha)$.

The force (in Newtons) on each limb of the conductor parallel to the axis of rotation is

$$F = B \ell i \quad (\text{by the definition of the Weber, the unit of strength of the field 'B' (M.K.S. system)})$$

and is in the direction shown in fig. 3.5. The work done by each conductor limb in moving from position 1 to position 2 (Figure 3.5) is

$$\begin{aligned} \Delta W &= F \times \text{distance moved in the direction of } F \\ &= F r \Delta\theta \cos\theta \\ &= r B \ell i \Delta\theta \cos\theta \\ &= \frac{a}{2} B i \Delta\theta \cos\theta \end{aligned}$$

$$\text{Therefore the total work done} = 2\Delta W = aBi \Delta\theta \cos\theta \quad (3.17)$$

(the remaining parts of the current loop are considered to be outside the field or do not cut any of the flux of the field 'B' in rotation).

The energy for this work is provided from the voltage source, e, and is $\frac{ei \cos \alpha}{2}$

Assuming time Δt is required for the loop to move through the angle (from position 1 to position 2)

$$\text{Then the work done} = \frac{1}{2}ei \Delta t \cos\alpha$$

$$\text{Therefore } \frac{1}{2}ei \Delta t \cos\alpha = 2\Delta W = aBi \Delta\theta \cos\theta, \text{ from equation}$$

$$(3.17) \text{ and } a B \cos\theta = \frac{(e \cos\alpha) \Delta t}{2 \Delta\theta}$$

$$= \frac{e \cos\alpha}{2 \dot{\theta}}$$

Now the torque on the current loop (in Newton meters) about the axis of rotation is $T = 2Fr \cos\theta$

and, substituting for F $T = 2Bli r \cos\theta$
 $= i aB \cos\theta$

$$= \frac{i e \cos\alpha}{2 \dot{\theta}} \quad \text{or} \quad 2T = \frac{i e \cos\alpha}{\dot{\theta}} \quad (3.18)$$

The e.m.f. 'e' directly opposes the back e.m.f. generated in the loop by the external field 'B', thus the e.m.f. 'e' and the back e.m.f. can be considered to be identical. It will also be noted that the denominator of the expression for torque, $\dot{\theta}$, is the relative speed between the field, 'B', and the loop.

Finally if the current i and the e.m.f. e are both sinusoidal quantities then the expression:

$$i e \cos\alpha$$

can be written in terms of phasors as

The real part of $(i e^*)$

as the angle α is the angle between the two vectors i and e .

Substituting this into equation (3.18) gives the result:

$$2T = \underline{\text{Re}} \left(\frac{e^* i}{\dot{\theta}} \right)$$

If the current i and the e.m.f. e are composed of component parts thus:

$$e = e_1 + e_2$$

$$\text{and} \quad i = i_1 + i_2$$

then the torque expression may be written:

$$\begin{aligned} 2T &= \underline{\text{Re}} \frac{(e_1^* + e_2^*)(i_1 + i_2)}{\dot{\theta}} \\ &= \underline{\text{Re}} \frac{e_1^* i_1}{\dot{\theta}} + \frac{e_1^* i_2}{\dot{\theta}} + \frac{e_2^* i_1}{\dot{\theta}} + \frac{e_2^* i_2}{\dot{\theta}} \end{aligned} \quad (3.19)$$

As the induced e.m.f. e is composed of components so will the inducing field B be composed of components. These components may not be moving at the same angular speed relative to the current loop, thus different angular velocities will be required for each.

Thus if $B = B_1 + B_2$
 then B_1 moves at an angular velocity of $\dot{\theta}_1$ relative to the loop,
 whilst B_2 moves at an angular velocity of $\dot{\theta}_2$ relative to the
 loop. In this case equation (3.19) must be rewritten as

$$2T = \underline{\text{Re}} \frac{e_1^* i_1}{\dot{\theta}_1} + \frac{e_1^* i_2}{\dot{\theta}_1} + \frac{e_2^* i_1}{\dot{\theta}_2} + \frac{e_2^* i_2}{\dot{\theta}_2} \quad (3.20)$$

Although this expression for torque was derived for a single turn winding it follows that, because both torque and induced e.m.f. are proportional to the number of turns, it holds good for any full-pitch, non-spread, non-~~corded~~^{chorded} single layer winding. For any other winding type the usual factors should be applied.

The full torque equation

It is not specified whether the winding (or current loop) under consideration is part of the rotor or the stator windings because the torque resulting from e.m.f.'s and currents in the rotor is of the same magnitude (but opposite sign) to that resulting from stator e.m.f.'s and currents. This results from Newtons law of action and reaction and can be used as a test for the validity of the expression. The expression for motoring torque for the case of the stator will be developed below, that for the rotor is shown in Appendix 1 as it serves only as a check.

Equation 3.20, restated in terms of stator e.m.f.'s and currents is:

$$2T = \underline{\text{Re}} \left(\frac{e_{SR1}^* i_{S1}}{w_S} + \frac{e_{SR1}^* i_{S2}}{w_S} \right) + \underline{\text{Re}} \left(\frac{e_{S2}^* i_{S1}}{w_S + \Delta w} + \frac{e_{S2}^* i_{S2}}{w_S + \Delta w} \right)$$

$$= 2(T_1 + T_2) + 2(T_3 + T_4)$$

The units of torque here are $\frac{\text{Newton metres}}{\text{Pole pair-phase}}$

Torque can now be evaluated in terms of the machine parameters and applied frequencies and voltages. But in doing so care must be taken to use a consistent set of reference vectors which can be either the current or the e.m.f. in each component of torque. Thus the expression for torque becomes

$$2T = \underline{\text{Re}} \left(\frac{e_{SR1}^* i_{S1}}{w_S} + \frac{e_{SR1}^* i_{S2}}{w_S} \right) \left. \begin{array}{l} \text{w.r.t. } e_{SR1} \\ \\ \\ \text{w.r.t. } e_{S2} \end{array} \right\} (3.21)$$

$$+ \underline{\text{Re}} \left(\frac{e_{S2}^* i_{S1}}{w_S + \Delta w} + \frac{e_{S2}^* i_{S2}}{w_S + \Delta w} \right)$$

which will be evaluated in its four component parts:

T_1, T_2, T_3 and T_4 as follows:

First term T_1

$$2T_1 = \underline{\text{Re}} \left(\frac{e_{SR1}^* i_{S1}}{w_S} \right) \quad \text{from equation (3.21)}$$

$$= \underline{\text{Re}} \left(\frac{-jw_S M_{R1}^* e^{-j\omega t} i_{S1}}{w_S} \right)$$

$$= \underline{\text{Re}} \left(-jM_e^{-j\omega t} i_{R1}^* i_{S1} \right) \quad (3.22)$$

Substituting for i_{R1}^* and i_{S1} in (3.22) gives,

$$2T_1 = \underline{\text{Re}} \left[\frac{-jMe^{-j\omega t} \{j(\omega_R - \Delta\omega)Me^{j\omega t}\} v_S^* \{R_R + j(\omega_R - \Delta\omega)L_R\} v_S}{(A+jB)(A-jB)} \right]$$

$$= \underline{\text{Re}} \left[\frac{M^2(\omega_R - \Delta\omega) \{R_R + j(\omega_R - \Delta\omega)L_R\} v_S v_S^*}{A^2 + B^2} \right]_{\text{w.r.t. } e_{SR1}}$$

$$v_S v_S^* = V_S e^{j\theta} V_S e^{-j\theta} = V_S^2$$

$$\therefore 2T_1 = \underline{\text{Re}} \left[\frac{M^2(\omega_R - \Delta\omega) \{R_R + j(\omega_R - \Delta\omega)L_R\} V_S^2}{A^2 + B^2} \right]$$

$$\text{and } 2T_1 = \frac{M^2 R_R V_S^2 (\omega_R - \Delta\omega)}{A^2 + B^2} \quad (3.23)$$

Second term T_2

$$2T_2 = \underline{\text{Re}} \left(\frac{e_{SR1}^* i_{S2}}{w_S} \right) \quad \text{from equation (3.21)}$$

w.r.t. e_{SR1}

$$= \underline{\text{Re}} \left(\frac{-jw_S M i_{R1}^* e^{-j\omega t} i_{S2}}{w_S} \right) = \underline{\text{Re}} (-jMe^{-j\omega t} i_{R1}^* i_{S2})$$

$$\therefore 2T_2 = \underline{\text{Re}} \left[\frac{-jMe^{-j\omega t} \{j(\omega_R - \Delta\omega)Me^{j\omega t}\} v_S^* \{-j(\omega_S + \Delta\omega)Me^{j\omega t}\} v_R}{(A-jB)(C+jD)} \right]$$

$$= \underline{\text{Re}} \left[\frac{-jM^3(\omega_R - \Delta\omega)(\omega_S + \Delta\omega)e^{j\omega t} v_S^* v_R}{(A-jB)(C+jD)} \right]$$

w.r.t. e_{SR1} (3.24)

Now $e_{SR1} = jE_{SR1} e^{-j\phi_{R1}}$ when e_{S1} is taken as reference

and $v_S = -jV_S e^{j\phi_{S1}}$ " " " " " "

$\therefore v_S = -V_S e^{j(\phi_{S1} + \phi_{R1})}$ " " " " "

$\therefore v_S^* = -V_S e^{-j(\phi_{S1} + \phi_{R1})}$ " " " " "

and $v_R = jV_R e^{j(\Delta\omega t - \omega t + \phi_{R2})}$ with e_{S1} taken as a reference

$\therefore v_R = V_R e^{j(\Delta\omega t - \omega t + \phi_{R1} + \phi_{R2})}$ " " " " "

$\therefore e^{j\omega t} v_S^* v_R = -V_S V_R e^{j(\Delta\omega t + \phi_{R2} - \phi_{S1})}$

$$= -V_S V_R e^{-j\delta} \text{ with } e_{SR1} \text{ taken as a reference} \quad (3.25)$$

Hence substituting (3.25) in (3.24) gives,

$$2T_2 = \underline{\text{Re}} \left[\frac{jM^3 (w_R - \Delta\omega)(w_S + \Delta\omega) V_S V_R e^{-j\delta}}{(A - jB)(C + jD)} \right]$$

$$= \underline{\text{Re}} \left[\frac{(A + jB)(C - jD)(jM^3 V_S V_R)(w_R - \Delta\omega)(w_S + \Delta\omega) e^{-j\delta}}{(A^2 + B^2)(C^2 + D^2)} \right]$$

$$= \frac{M^3 V_S V_R (w_S + \Delta\omega)(w_R - \Delta\omega)}{(A^2 + B^2)(C^2 + D^2)} \underline{\text{Re}} \left[j(A + jB)(C - jD) e^{-j\delta} \right]$$

$$\therefore 2T_2 = \frac{M^3 V_S V_R (w_S + \Delta\omega)(w_R - \Delta\omega)}{(A^2 + B^2)(C^2 + D^2)} \left[(AC + BD)\sin\delta + (AD - BC)\cos\delta \right]$$

(3.26)

Third Term T_3

$$2T_3 = \underline{\text{Re}} \left(\frac{e_{S2}^* i_{S1}}{w_S + \Delta\omega} \right) \text{ from equation (3.21)}$$

w.r.t. e_{S2}

$$\begin{aligned}
&= \underline{\text{Re}} \left(\frac{-j(\omega_S + \Delta\omega) M i_{R2}^* e^{-j\omega t} i_{S1}}{\omega_S + \Delta\omega} \right) \\
&= \underline{\text{Re}} \left(-j M e^{-j\omega t} i_{R2}^* i_{S1} \right) \\
&= \underline{\text{Re}} \left[\frac{-j M e^{-j\omega t} \{R_S - j(\omega_S + \Delta\omega)L_S\} v_R^* \{R_R + j(\omega_R - \Delta\omega)L_R\} v_S}{(C - jD)(A + jB)} \right] \\
&= \underline{\text{Re}} \left[\frac{-j M \{R_S - j(\omega_S + \Delta\omega)L_S\} \{R_R + j(\omega_R - \Delta\omega)L_R\} e^{-j\omega t} v_R^* v_S}{(C - jD)(A + jB)} \right]
\end{aligned}$$

The real part of a complex number and its conjugate are the same. Hence, taking the conjugate:

$$2T_3 = \underline{\text{Re}} \left[\frac{j M \{R_S + j(\omega_S + \Delta\omega)L_S\} \{R_R - j(\omega_R - \Delta\omega)L_R\} e^{j\omega t} v_R v_S^*}{(C + jD)(A - jB)} \right] \quad (3.27)$$

Using the process for the evaluation of T_2 ,

$$(e^{j\omega t} v_S^* v_R)_{\text{w.r.t. } e_{S2}} = -v_S v_R e^{-j\delta}$$

equation 3.27 becomes,

$$2T_3 = \underline{\text{Re}} \left[\frac{-j M \{R_S + j(\omega_S + \Delta\omega)L_S\} \{R_R - j(\omega_R - \Delta\omega)L_R\} v_S v_R e^{-j\delta}}{(C + jD)(A - jB)} \right]$$

Hence,

$$2T_3 = \frac{-M v_S v_R}{(A^2 + B^2)(C^2 + D^2)} \underline{\text{Re}} \left[j(C - jD)(A + jB) \{R_S + j(\omega_S + \Delta\omega)L_S\} \{R_R - j(\omega_R - \Delta\omega)L_R\} e^{-j\delta} \right]$$

$$\text{and } 2T_3 = \frac{-M v_S v_R}{(A^2 + B^2)(C^2 + D^2)} \underline{\text{Re}} \left[\{R_S R_R + (\omega_S + \Delta\omega)(\omega_R - \Delta\omega)L_S L_R\} + j \{R_R(\omega_S + \Delta\omega)L_S - R_S(\omega_R - \Delta\omega)L_R\} \right]$$

$$\left(\{ (AC + BD)\sin\delta + (AD - BC)\cos\delta \} + j \{ (BC - AD)\sin\delta + (AC + BD)\cos\delta \} \right)$$

Hence,

$$2T_3 = \frac{-MV_S V_R}{(A^2 + B^2)(C^2 + D^2)} \left[\{R_S R_R + (w_S + \Delta w)(w_R - \Delta w)L_S L_R\} \{(AC+BD)\sin\delta + (AD-BC)\cos\delta\} + \{R_S (w_R - \Delta w)L_R - R_R (w_S + \Delta w)L_S\} \{(BC-AD)\sin\delta + (AC+BD)\cos\delta\} \right] \quad (3.28)$$

Fourth Term T_4

$$\begin{aligned} 2T_4 &= \underline{\text{Re}} \left(\frac{e_{S2}^* i_{S2}}{w_S + \Delta w} \right) \text{taking } e_{S2} \text{ as reference} \\ &\quad \text{from equation (3.21)} \\ &= \underline{\text{Re}} \left(\frac{-j(w_S + \Delta w)M i_{R2}^* e^{-j\omega t} i_{S2}}{w_S + \Delta w} \right) \\ &= \underline{\text{Re}} \left(-jM e^{-j\omega t} i_{R2}^* i_{S2} \right) \\ &= \underline{\text{Re}} \left[\frac{-jM e^{-j\omega t} \{R_S - j(w_S + \Delta w)L_S\} V_R^* \{-j(w_S + \Delta w)M e^{j\omega t}\} V_R}{(C-jD)(C+jD)} \right] \\ &= \underline{\text{Re}} \left[\frac{-M^2 (w_S + \Delta w) \{R_S - j(w_S + \Delta w)L_S\} V_R^* V_R}{C^2 + D^2} \right] \\ &= \underline{\text{Re}} \left[\frac{-M^2 (w_S + \Delta w) \{R_S - j(w_S + \Delta w)L_S\} V_R^2}{C^2 + D^2} \right] \\ \text{Hence } 2T_4 &= \frac{-M^2 V_R^2 R_S (w_S + \Delta w)}{C^2 + D^2} \quad (3.29) \end{aligned}$$

The Total Torque

$T = T_1 + T_2 + T_3 + T_4$ and so equations 3.23, 3.26, 3.28 and 3.29 combine to give

$$\begin{aligned}
 2T = & \frac{M^2 R_S V_S^2 (w_R - \Delta w)}{A^2 + B^2} - \frac{M^2 R_S V_R^2 (w_S + \Delta w)}{C^2 + D^2} \\
 & + \frac{M V_S V_R}{(A^2 + B^2)(C^2 + D^2)} \left[\{(w_S + \Delta w)(w_R - \Delta w)(M^2 - L_S L_R) - R_S R_R\} \right. \\
 & \qquad \qquad \qquad \left. \{(AD - BC)\cos\delta + (AC + BD)\sin\delta\} \right. \\
 & \qquad \qquad \qquad \left. + \{R_R (w_S + \Delta w)L_S - R_S (w_R - \Delta w)L_R\} \{(AC + BD)\cos\delta - (AD - BC)\sin\delta\} \right]
 \end{aligned}$$

(3.30)

It can be seen that equation 3.30 can be derived irrespective of the reference vector that is used, providing it is consistently used. This is because in every case a vector is multiplied by the conjugate of a vector in order to derive a torque expression. Equation 3.21 expressed in rotor terms will lead to an equation similar to 3.30 (but of opposite sign) as shown in Appendix 1.

Finally it should be noted that if the quantities V_S and V_R are R.M.S. and not peak values then equation 3.30 becomes the expression for motoring torque 'T' instead of '2T'.

3.3 Minimum Current Criterion

It has already been said⁷ that a transformer effect exists whereby, if the rotor and stator voltages are not matched, large, reactive, currents circulate through the machine and its supplies reducing the machine's efficiency, This is verified by practical experience and there exists, for each value of speed and load, an optimum ratio of V_S/V_R whereby either the stator or the rotor current is minimised. Experiment has further shown the condition of current minimum to coincide with that of unity power factor or zero reactive current. With this as a basis the required relationship between V_S and V_R can be determined as follows:

Minimum Rotor Current (or $\phi_R = 0$)

$$i_R = i_{R1} + i_{R2}$$

If $\phi_R = 0$, $\underline{\text{Im}} i_R = 0$ taking v_R as a reference vector

$$\therefore \underline{\text{Im}} (i_{R1} + i_{R2}) = 0$$

Section 3.1 shows that

$$i_{R1} = \frac{-j(\omega_R - \Delta\omega) M e^{-j\omega t} v_S}{A + jB} \quad (3.6)$$

where $A = R_S R_R + \omega_S (\omega_R - \Delta\omega) (M^2 - L_S L_R)$

and $B = R_S (\omega_R - \Delta\omega) L_R + R_R \omega_S L_S$.

Substituting for v_S into 3.6 gives

$$i_{R1} = \frac{-j(\omega_R + \Delta\omega) M e^{-j\omega t} - V_S e^{j(\phi_{S1} - \Delta\omega t + \omega t - \phi_{R2})}}{A + jB} \quad (3.31)$$

Similarly section 3.1 shows that

$$i_{R2} = \frac{[R_S + j(\omega_S + \Delta\omega)L_S] V_R}{C + jD} \quad (3.14)$$

where $C = R_S R_R + \omega_R (\omega_S + \Delta\omega) (M^2 - L_S L_R)$

and $D = R_S \omega_R L_R + R_R (\omega_S + \Delta\omega) L_S$

The machine is operating synchronously with $\Delta\omega = 0$ and so $C = A$ and $D = B$. Equations (3.31) and (3.14) now become

$$i_{R1} = \frac{[j\omega_R M V_S e^{j(\phi_{S1} - \Delta\omega t - \phi_{R2})}] (A - jB)}{A^2 + B^2}$$

and $i_{R2} = \frac{[R_S + j\omega_S L_S] V_R (A - jB)}{A^2 + B^2}$

For minimum rotor current $\underline{\text{Im}} i_R = 0$

$$\therefore \underline{\text{Im}} (i_{R1} + i_{R2}) = 0$$

and $0 = A [j\omega_S L_S V_R + j\omega_R M V_S \cos(\phi_{S1} - \Delta\omega t - \phi_{R2})]$

$$-jB [R_S V_R - \omega_R M V_S \sin(\phi_{S1} - \Delta\omega t - \phi_{R2})] \quad (3.32)$$

Equation (3.32) rearranged gives

$$0 = V_R [\omega_S L_S A - R_S B] + V_S M \omega_R [A \cos(\phi_{S1} - \Delta\omega t - \phi_{R2}) + B \sin(\phi_{S1} - \Delta\omega t - \phi_{R2})]$$

$$\text{or } 0 = V_R [w_S L_S A - R_S B] + V_S M w_R [A \cos \delta + B \sin \delta]$$

$$\text{where } \delta = \phi_{S1} - \Delta \omega t - \phi_{R2}$$

Hence for minimum rotor current

$$\frac{V_S}{V_R} = \frac{B R_S - A L_S w_S}{M w_R [A \cos \delta + B \sin \delta]} \quad (3.33)$$

Minimum Stator Current (or $\phi_S = 0$)

Similarly, stator current can be minimised from the conditions that $\phi_S = 0$, $i_S = i_{S1} + i_{S2}$ and $\text{Im } i_S = 0$ with reference to V_S . This gives the relationship

$$\frac{V_S}{V_R} = \frac{M w_S [A \cos \delta - B \sin \delta]}{B R_R - A w_R L_R} \quad (3.34)$$

The expression, (3.33) or (3.34), for the optimum ratio of V_S/V_R is of little practical importance as there are many well developed methods for measuring phase angle (in a supply line) for use in a feedback loop. In theoretical studies of the machine performance under unity power factor conditions, however, they are of importance. They are used in the next section in conjunction with the expression for torque (equation (3.30)).

3.4 The performance of the machine and the limits of its stable region

The equations (3.30), (3.33) and (3.34) of sections 3.2 and 3.3 have been written into a short computer program which evaluates the optimum rotor voltage (if required), the motoring

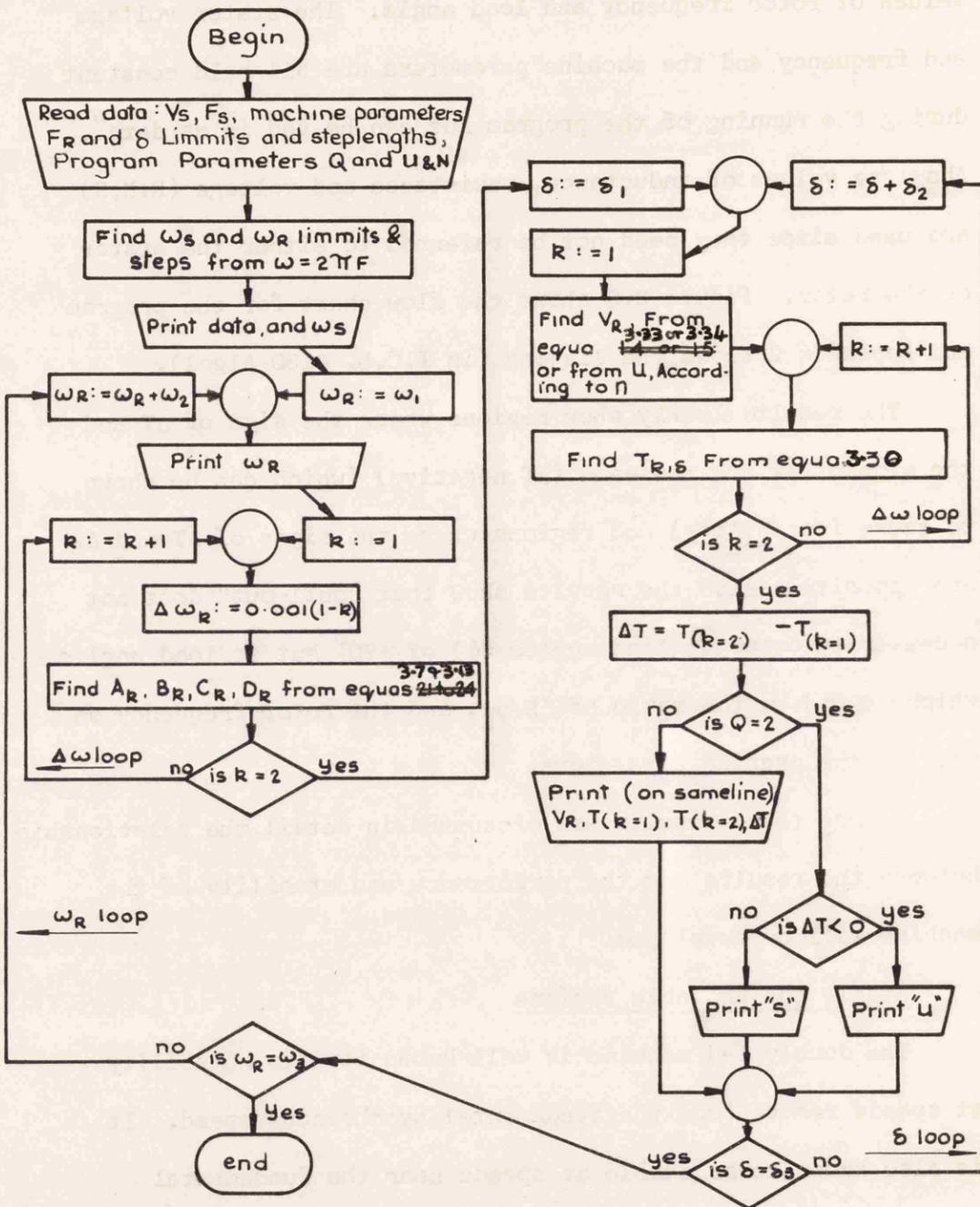


FIG.3.6. D.F.M TORQUES PROGRAM FLOW CHART

torque of the machine (for $\Delta\omega = 0$), and the variation of motoring torque (ΔT) with a small change of $\Delta\omega$ (to $\Delta\omega = -10^{-3}$) for various values of rotor frequency and load angle. The stator voltage and frequency and the machine parameters are all held constant during the running of the program but can be fed in as data*. Absolute values of inductance, resistance and voltage (R.M.S) are used since they need not be referred to either the stator or the rotor. Figure 3.6 shows the flow chart for the program and Appendix 2 shows the listing (in I.C.L. 4130 Algol).

The results clearly show regions where the sign of ΔT and the sign of $\Delta\omega$ are the same (ΔT negative) (which can be shown to cause instability) and regions where the signs of ΔT and $\Delta\omega$ are opposite. Also the results show that "pull-out" does not necessarily occur at load angles (δ) of $+90^\circ$ but at load angles which depend on the ratio of V_S/V_R , and the rotor frequency as well as the machine parameters.

Before these results are discussed in detail the relationship between the results and the performance and stability of the machine will be developed.

Stable and unstable regions

The doubly fed machine is well known for its instability at speeds remote from the fundamental synchronous speed. It is also known to be stable at speeds near the fundamental synchronous speed. It is, however, not fully understood where the limits of the stable region are or what affects the position of these limits and the degree of stability. It is therefore useful to develop a criterion for stability which can easily be applied.

*in all the runs discussed here this data is that of the machine and conditions used in the experimental trials (see table 4.1).

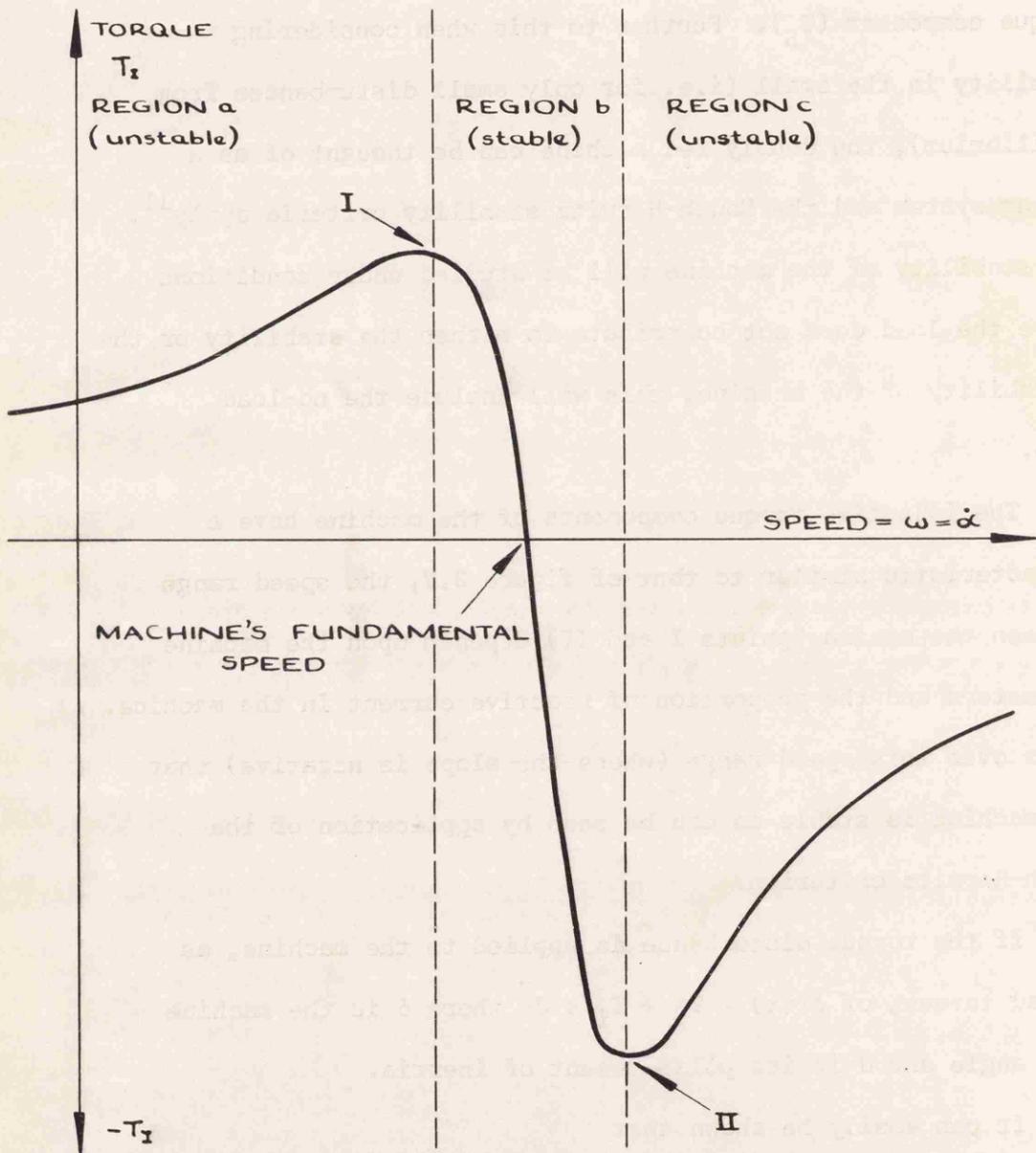


FIG.3.7. TYPICAL INDUCTIVE TORQUE CHARACTERISTIC

The torque of the doubly fed machine can be considered to be composed of two parts, an inductive or speed dependent torque component (T_i) and a synchronous or angular dependent torque component (T_S). Further to this when considering stability in the small (i.e. for only small disturbances from equilibrium), the doubly fed machine can be thought of as a linear system and the Routh-Herwitz stability criteria apply¹¹. The stability of the machine will be studied under conditions where the load does not contribute to either the stability or the instability of the machine, this will include the no-load case.

The inductive torque components of the machine have a characteristic similar to that of figure 3.7, the speed range between the maxima (points I and II) depends upon the machine parameters and the proportion of reactive current in the machine. It is over this speed range (where the slope is negative) that the machine is stable as can be seen by application of the Routh-Herwitz criterion:

If the torque disturbance is applied to the machine, as a load torque, of $\Delta T(t) = T_S + T_i + J\ddot{\delta}$ where δ is the machine load angle and J is its polar moment of inertia.

It can easily be shown that

$$\Delta T(t) = T_S + T_i + J\ddot{\delta} = \frac{dT_S}{d\delta}\delta + \frac{dT_i}{d\delta}\dot{\delta} + J\ddot{\delta}$$

or, taking Laplace transforms:

$$\Delta T(t) = \delta \left[\frac{dT_S}{d\delta} + \frac{pdT_i}{d\delta} + p^2J \right] \quad (3.35)$$

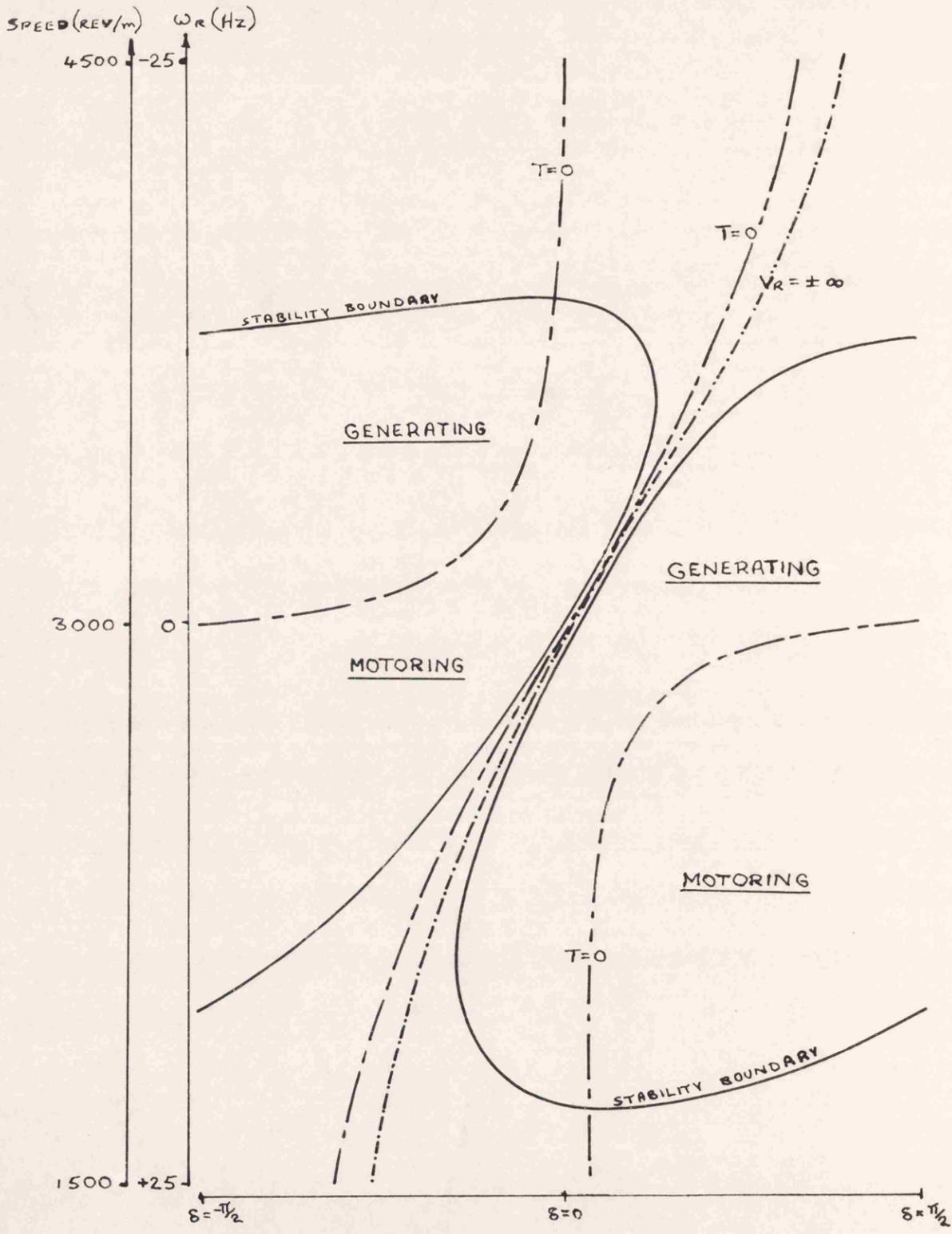


FIG.3.B. 'D.F.M. TORQUES' PRINT OUT MAP FOR MINIMUM I_s

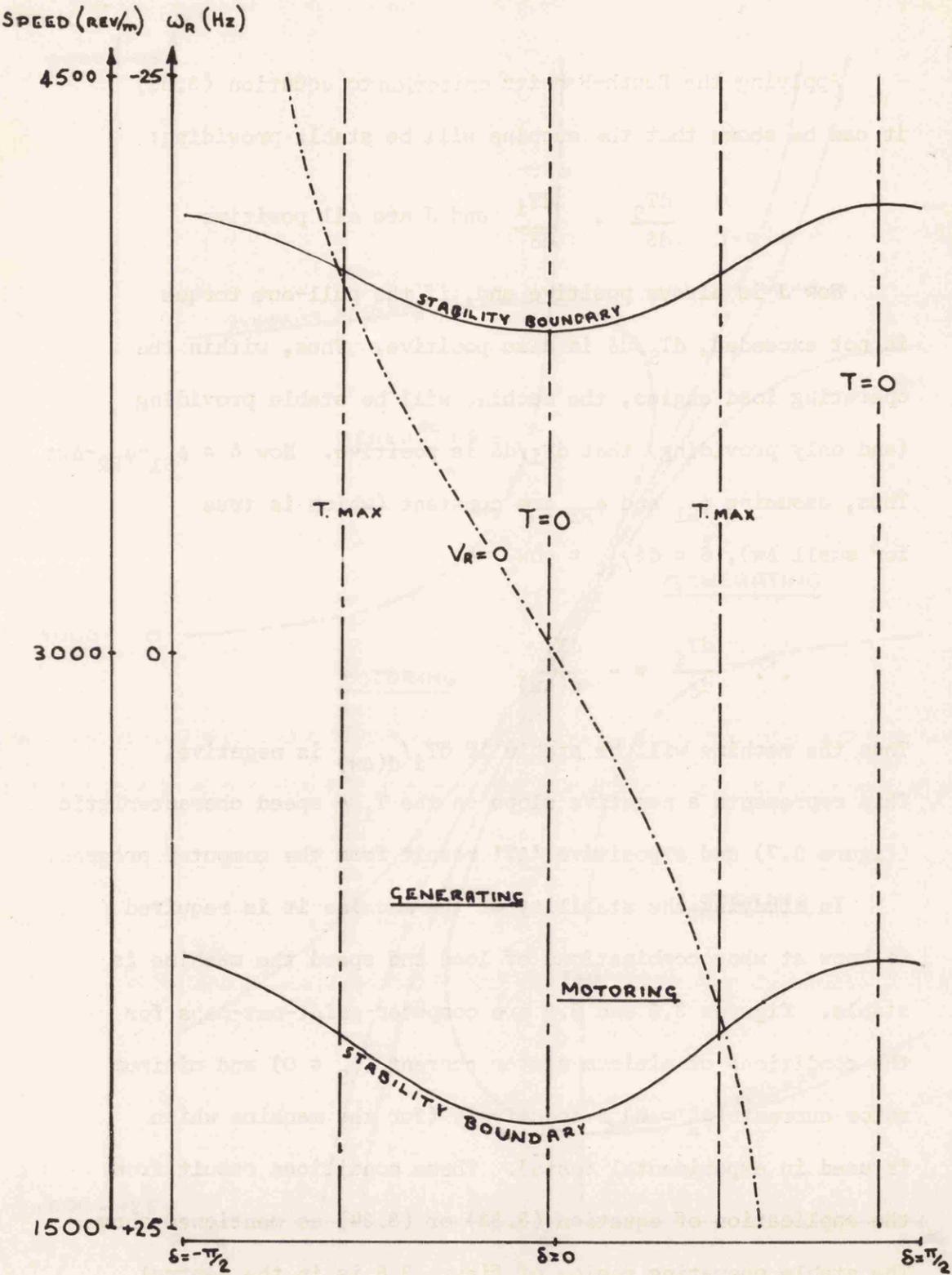


FIG 3-9 DFM TORQUES PRINT OUT MAP FOR MINIMUM I_R

Applying the Routh-Hurwitz criterion to equation (3.35) it can be shown that the machine will be stable providing:

$$\frac{dT_S}{d\delta}, \quad \frac{dT_i}{d\delta} \quad \text{and } J \text{ are all positive}$$

Now J is always positive and, if the pull-out torque is not exceeded, $dT_S/d\delta$ is also positive. Thus, within the operating load angles, the machine will be stable providing (and only providing) that $dT_i/d\delta$ is positive. Now $\delta = \phi_{S1} - \phi_{R2} - \Delta\omega t$. Thus, assuming ϕ_{S1} and ϕ_{R2} are constant (which is true for small $\Delta\omega$), $\dot{\delta} = d\delta/dt = -\Delta\omega$

$$\therefore \frac{dT_i}{d\dot{\delta}} = - \frac{dT_i}{d(\Delta\omega)}$$

Thus the machine will be stable if $dT_i/d(\Delta\omega)$ is negative. This represents a negative slope on the T_i - speed characteristic (figure 3.7) and a positive ' ΔT ' result from the computer program.

In studying the stability of the machine it is required to know at what combinations of load and speed the machine is stable. Figures 3.8 and 3.9 are computer-print-out-maps for the conditions of minimum stator current ($\phi_S = 0$) and minimum rotor current ($\phi_R = 0$) respectively (for the machine which is used in experimental tests). These conditions result from the application of equation (3.33) or (3.34) as mentioned above. The stable operating region of figure 3.8 is in the central portion where the machine is stable. It will be noted that there is an unstable 'channel' about the line where optimum V_R (for $\phi_S = 0$) is $\pm \infty$ (+ve to the left, -ve to the right); it should also be noted, although it is not shown, that the

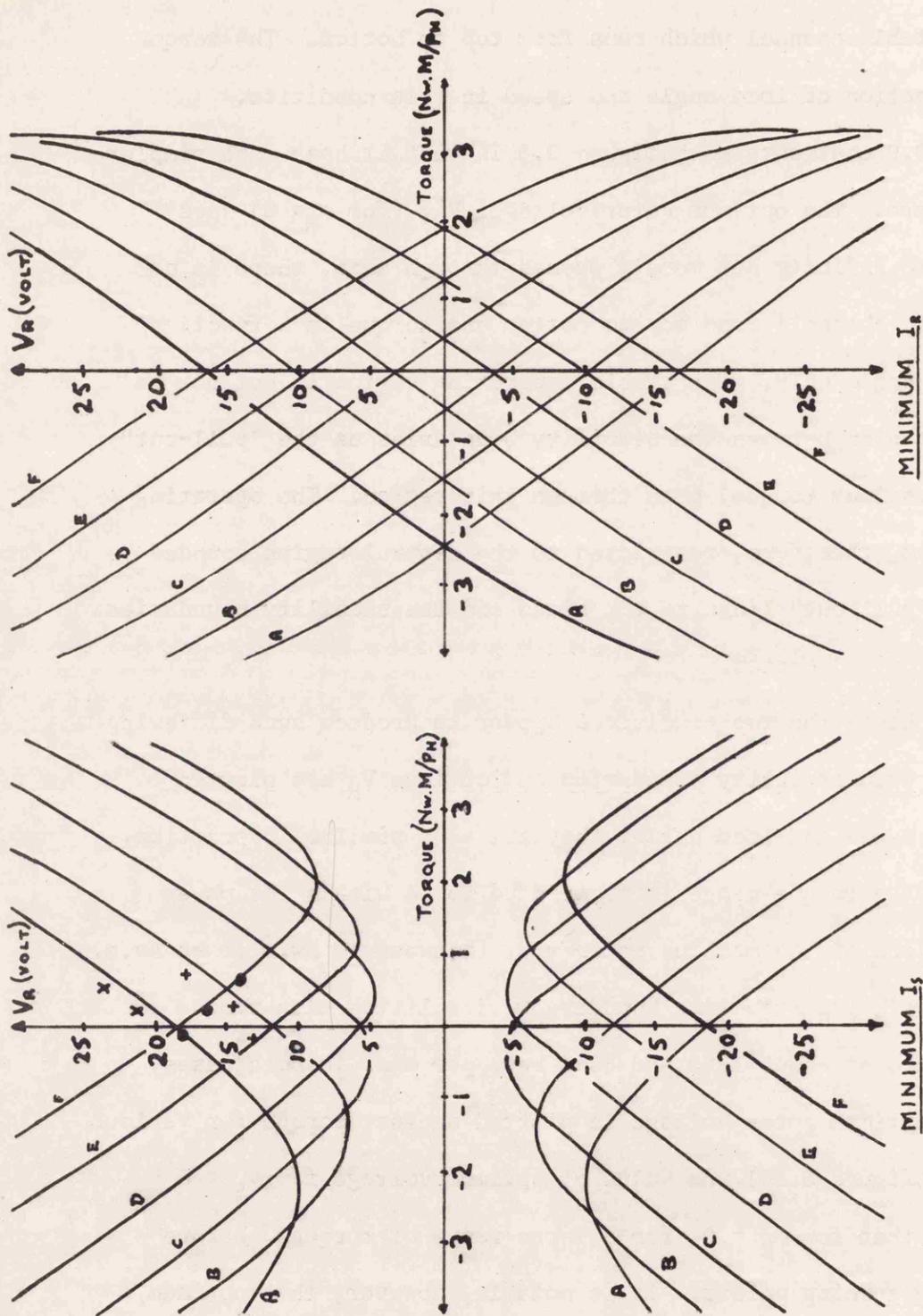


FIG. 3-10. PLOT OF OPTIMUM ROTOR VOLTAGE AGAINST TORQUE (PER PHASE)

FOR VARIOUS SPEEDS (AND STATOR VOLTAGE OF 240 V.)

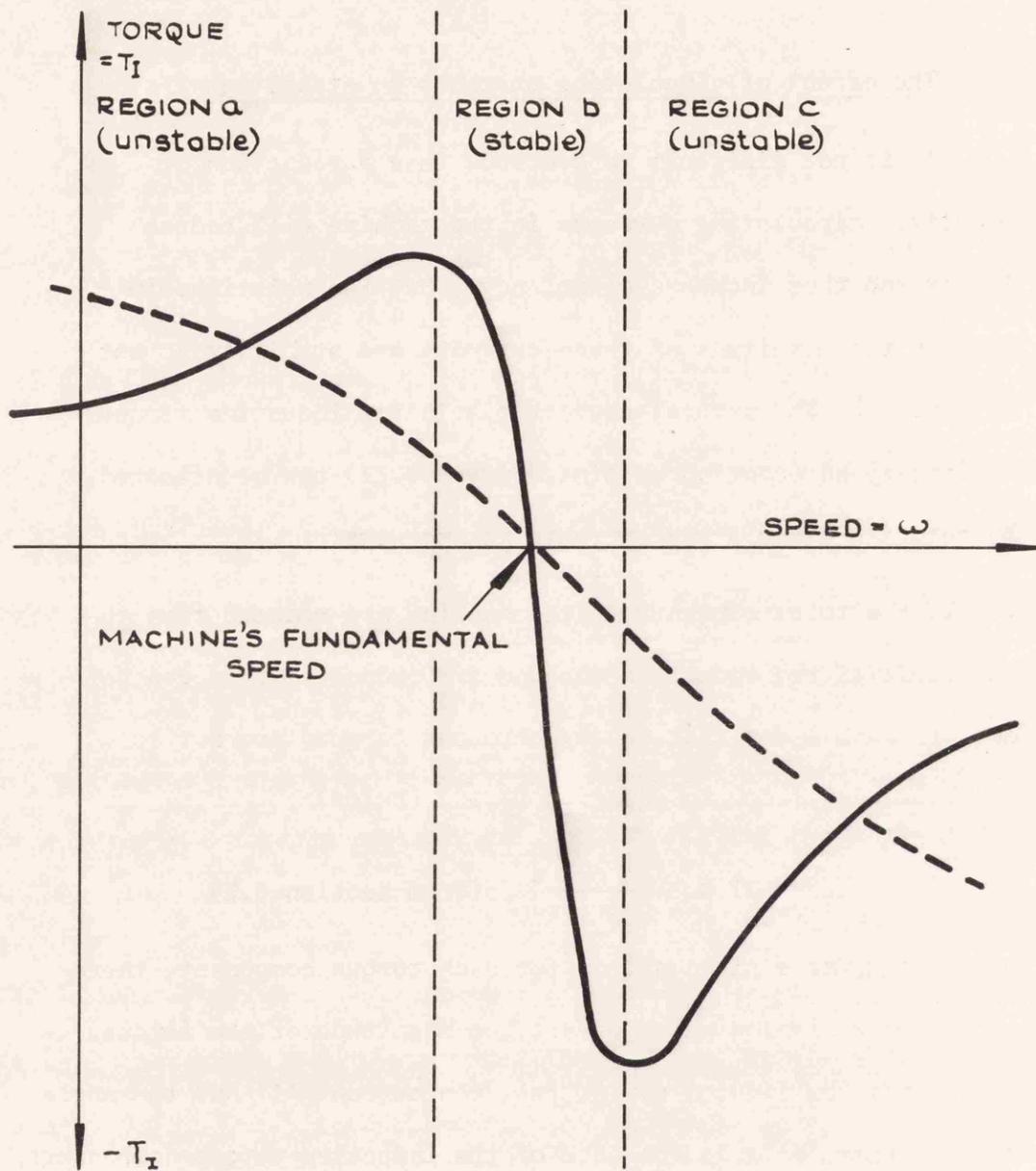
\times = A = 3300 r.p.m.; $+$ = B = 3180 r.p.m.; \circ = C = 3060 r.p.m.; \square = D = 2940 r.p.m.; \triangle = E = 2820 r.p.m.; \bullet = F = 2700 r.p.m..

pull-out load angles (i.e. maximum torque lines) also lie in the unstable channel which runs from top to bottom. The torque is a function of load angle and speed in this condition.

Figure 3.9 contrasts with figure 3.8 in that it has a much simpler appearance. The optimum rotor voltage, V_R , (for $\phi_R = 0$) does not go to infinity but merely passes through zero, there is no unstable 'channel' from top to bottom and torque is a function of load-angle only. The, stable, operating region is not now the entire region between the stability boundaries as the "pull-out" lines (maximum torque) pass through this region. The operating region is, therefore, restricted to the central region bounded by the "pull-out" lines to the sides and the stability boundaries at the top and bottom.

Although the two conditions appear to produce such differing results when stability boundaries and optimum V_R are plotted against speed and load angle, they are very similar in practice.

If the torque range is restricted to be within the power limitations of the machine under test (between ± 9 Nw.m or ± 3 Nw.m. per phase), the stability boundary varies little with torque and occurs at about 1750 and 3800 rev. per min. in both cases. If the optimum rotor voltage is plotted against torque for various speeds (figure 3.10) the value of optimum voltage for $\phi_S = 0$ follows that for $\phi_R = 0$, for a large range of torques, before reaching turning points. It is notable, however, that optimum V_R for $\phi_R = 0$ passes through zero whilst that for $\phi_S = 0$ does not.



——— NATURAL CHARACTERISTIC
 - - - CHARACTERISTIC WHEN CURRENT MINIMISATION IS USED.

FIG 3.11. INDUCTIVE TORQUE CHARACTERISTIC FOR THE D.F.M.

The effect of circulating currents on stability

It is not difficult to conclude that a reduction of reactive, circulating currents in the machine will reduce losses and thus increase efficiency. But the relationship between the magnitude of these currents and stability is not so obvious. The natural characteristic for inductive torques-against-speed (continuous line, figure 3.11) can be affected by reactive circulating currents in two ways:

(1) If the total currents in the machine are reduced then the magnitude of the inductive torques are reduced. This can be done in such a way that the synchronous torques are not so reduced. Reference to equation (3.18) (below)

$$2T = \frac{i e \cos \alpha}{\dot{\theta}} \quad (\text{from section 3.2})$$

shows that, at a given speed, for each torque component, there are three variables which affect the magnitude of the torque. These are: the induced e.m.f. 'e', the current 'i' and the angle between them 'α'. In the case of the inductive torque components the e.m.f. 'e' is directly or indirectly induced by the current 'i'. The e.m.f. 'e' and the angle 'α' are, therefore, both dependent on the current. In the case of the synchronous torque components, however, the e.m.f. 'e' is induced by an external current (e.g. 'e' is induced in the stator by a rotor current or vice versa). Thus the angle 'α' is an independent variable and, by reducing it simultaneously with the reduction in overall current, the synchronous torque components can be maintained.

The inductive torque components can amount to a considerable portion of the machines net output torque. This can cause the

range of operating torques to be restricted to torques of too large a magnitude (either in the motoring or the generating mode) for the machine to be usable - at low torques (or zero torque) the pull-out torque can be exceeded merely because of the presence of the inductive torque components. Thus a reduction in the overall magnitude of the inductive torque components (relative to the synchronous torque components) is desirable and current minimisation can achieve this end.

(2) The overall magnitude of the inductive torque components affects the range of torques over which the machine will operate but does not directly affect the stability of the machine. The slope of the inductive torque - speed characteristic (fig. 3.11) affects the degree of stability (if it is negative) or instability (if it is positive) of the machine whilst the position of the maxima determines the stable speed range. Thus, if the maxima can be moved to speeds more remote from each other (the dashed line of fig. 3.11) then the stable speed range can be increased at the cost of the degree of stability within that range. This is desirable if the machine is to be operated over a wide range of speeds.

Prescott and Raju³ inserted resistance into the rotor to modify the form of the inductive characteristics to this high resistance rotor form. Similar results are produced by 'tuning-out' some of the inductive effect of the machine thus greatly reducing the electrical time constant of the machine and increasing the R/L ratio. This can be achieved by reducing reactive currents in the machine to a minimum or to zero.

Computer results show a fall in the slope of the inductive torque-speed characteristic (shown as a fall in the magnitude of ΔT) and a widening of the stable speed range of the machine. The latter is also clearly demonstrated in practice. These results are accompanied by a rise in machine efficiency, rather than the fall in efficiency which Prescott and Raju experienced on inserting additional resistance into the rotor circuit. The extent to which the stable region can be extended is restricted, however, as it is found that the maxima of figure 3.11 cannot be set much further apart than their position for the machine operating as a simple (stator fed) induction motor. It may, therefore, be necessary to insert additional rotor resistance to overcome this effect if the desired speed range is very large. If current minimisation is used, however, in conjunction with the addition of resistance, then the amount of resistance to be added for a given stable speed range will be reduced and the efficiency increased accordingly.

Thus the technique of current minimisation (whereby reactive, circulating currents are reduced to a minimum, or zero, by suitable choice of the V_S/V_R ratio) has four main advantages:

- (1) Magnetic and other losses are reduced as the current is reduced, thus increasing efficiency.
- (2) The torque-load angle characteristics become such that the machine is usable over the whole of the stable speed range.
- (3) The stable speed range is increased, and
- (4) In the regions where the machine is unstable the degree of instability is less (as the slope of the inductive

torque-speed characteristic is less). This will render the machine more readily stabilisable by external means.^{2,7,...}

The computer results

The print-out consists of sets of values of:

- (1) rotor voltage
- (2) T_A = machine torque for $\Delta\omega = 0$
- (3) T_B = machine torque for $\Delta\omega = -10^{-3}$
- (4) $\Delta T = T_B - T_A$, (which is roughly proportional to $\frac{dT_i}{d\delta}$ as has already been shown).

Each of these four results is printed for each combination of rotor frequency (taken in steps of 2 Hz. from -25Hz. to +25 Hz.) and load angle (18° steps from -180° to $+180^\circ$). The input data is comprised of the stator phase voltage (240V), the stator supply frequency (50Hz), the machine parameters (as listed in table 4.1) and some control data which dictates whether the stator current should be minimised, the rotor current should be minimised or the rotor voltage should be set to some input value.

Figures 3.8 and 3.9 have already been discussed regarding their apparent dissimilarity. It has been pointed out that, for a given torque, the results shown in these two figures (figure 3.8 refers to $\phi_S = 0$ condition and figure 3.9 refers to $\phi_R = 0$ condition) are similar - at least regarding stability. The plot of optimum voltage against torque (figure 3.10) shows that the optimum voltage is roughly the same for $\phi_S = 0$ as it is for $\phi_R = 0$ over certain ranges of torque but, whilst the

optimum voltage for $\phi_R = 0$ passes through zero, that for $\phi_S = 0$ turns at a minimum and goes to infinity. Figure 3.8 reflects this in the $V_R = \pm \infty$ line (running top to bottom). This line is surrounded by an unstable region inside which the pull-out load angles (for torque maxima) also lie. This unstable band is ineffective as (over most of the speed range) the rotor voltage and the torque are too high for operation within that region to be considered. A full working range of torques can be reached without crossing the unstable band at any speed regardless of whether the load angle is to the left or the right of it. The range of torques available (for a reasonable range of rotor voltages) when stator current is minimised (i.e. $\phi_S = 0$) is more than that available between the pull-out torques for rotor current minimisation ($\phi_R = 0$) (figure 3.9). Thus, though the plot for $\phi_S = 0$ appears more complex than that for $\phi_R = 0$, it is practically preferable to use stator current minimisation. This is because torque, speed, stability and voltage are of prime consideration, functions such as load angle (δ) serve merely for analytical purposes.

The degree of reactive current in the machine is low for any voltage ratio (V_S/V_R) near the optimum for minimum stator or minimum rotor current. It is therefore more relevant to compare the performance and stability of the machine for optimum voltage ratio (be it for $\phi_S = 0$ or $\phi_R = 0$) with that for non-optimum voltage ratio (where the voltage ratio is more than, say 10%, above or below optimum).

It is assumed that, for mechanical or thermal reasons study of the performance of the machine should be restricted to the torque range - 9Nw.m. to + 9Nw.m. for this particular

Table 3.1

Operating torque limits for near-optimum rotor voltage conditions

$$V_R = U(w_R + 6.0)$$

Speed (rev/min)	Possible operating torque range (Nw.m/phase)				
	U = -.4	U = -.5	U = -.6	U = -.7	U = -.8
3780	Below -0.4	Below 0.5	-2.0* to 1.4	0* to 2.2	Above +1.6*
3660	Below -0.7	Below 0.1	Below 0.9	Below 1.7	Below 2.4
3540	Below -0.9	Below -0.2	Below 0.5	Below 1.6	Below 1.8
3420	Below -0.9	Below -0.4	Below 0.2	Below .7	Below 1.2
3300	Below -0.8	Below -0.5	Below -0.1	Below .3	Below 0.6
3180	-2.3 to -0.7	-2.5 to -0.5	-2.7 to -0.3	-2.9 to -1.25	Below 0.06
3060	-0.5 to -0.46	-0.5 to -0.45	-0.51 to -0.45	-.5 to -.45	-.51 to -.45
2940	-0.2 to 1.1	-0.4 to 1.3	-0.6 to 1.5	-.75 to 1.7	-.9 to 1.8
2820	0 to .25	-0.4 to 2.8	Above -0.7	Above -1.0	Above -1.4
2700	Above 0.13	Above -0.4	Above -0.9	Above -1.4	Above -1.8
2580	Above 0.2	Above -0.4	Above -1.1	Above -1.7	Above -2.4
2460	Above 0.2	Above -0.6	Above -1.3	Above -2.1	Above -2.9
2340	Above 0.1	Above -1.0	Above -1.6	Above -2.5	Full Range
2220	Above -0.1	Above -1.3	Above -2.0	Full Range	Full Range

* Due to instability

machine. This torque range is equivalent to $-3Nw.m.$ to $+3Nw.m.$ per phase. It has already been predicted that the full range of torques (between these limits) can be reached, stably, for speeds ranging between 1750 and 3800 rev. per min. from computer results for both minimum stator current and minimum rotor current. This is not the case under other circumstances. Computer runs were made in which the rotor voltage was varied with rotor frequency but not with torque. Thus only a crude, rule-of-thumb, optimisation was applied whereby the current(s) were only minimised for certain torques (within the operating range of -3 to $+3nw.m.$ per phase). Table 3.1 indicates the possible torque ranges which may be achieved for various conditions at various speeds. The parameter 'u' is shown at the top of each column,

$$\text{where } V_R = U(w_R + 6.0)^*$$

In table 3.1 the terms: "Full Range" indicates that all torques between $-3NW.m./\text{phase}$ and $+3Nw.m./\text{phase}$ are obtainable (as is the case universally with current minimisation); "Below" indicates that torques between $-3Nw.m./\text{phase}$ and the stated figure are available and "Above ..." indicates that torques between the stated figure and $+3Nw.m./\text{phase}$ are available .

Table 3.1 shows that only restricted torque ranges are available for most conditions when the rotor voltage is not varied with torque. Most of these restrictions are due to pull-out torque (maximum, or minimum, torque) being reached and in most cases the figures indicated are pull-out torques.

*The value of u for $V_S/V_R - n w_S/w_R$ (n turns ratio) is $u = -.56$

It will be noted that the torque limits are not symmetrical about zero, as might at first be expected. Indeed, some torque ranges are so displaced that, whilst one limit is small in magnitude, the other limit is very large (and at a value of torque which is quite unusable). This displacement of the torque range is the most significant feature of these results and is caused by large, inductive torque components as mentioned earlier.

The usefulness of the machine can be retained by raising the rotor voltage (as indicated in the $u = -.7$ and $u = -.8$ columns) as this, naturally, broadens the torque range. Raising the voltage in this manner leads to a narrowing of the stable speed range, however, which is not advantageous.

One more result accrues from this series of runs. This result is the one regarding load angle. When V_R is not varied with torque pull-out torques exist at load angles which are 180° apart for all speeds, the position of the pull-out load angles and the value of the corresponding torques, however, varies with speed. Torque variations occur round the full 360° of load angle and do not repeat themselves. This is not true when current minimisation is applied, however. With the rotor current minimised pull-out load angles are separated by 90° and their positions, and the values of the corresponding torques (-4.76 Nw.m./phase and $+3.26$ Nw.m./phase) are independent of speed. With stator current minimisation pull-out torque is not reached before the machine becomes unstable (the values of torques reached however exceed the range -5 Nw.m./phase to $+5$ Nw.m./phase, for all speeds from 1760 to 3780 rev./min.,

which is far in excess of what is normal for the machine).

In both the cases of current minimisation the torque-load angle distribution for load angles of 0 to $+180^\circ$ is repeated (with rotor voltages of the same magnitude but opposite sign) over the range 180° to 360° (or -180° to 0).

3.5 Comments on Chapter 3

Expressions for currents, voltages and torques for the doubly fed machine have been developed in a manner similar to that of Prescott and Raju. These expressions completely describe the machine in all aspects except that of the ratio of rotor to stator voltage. Criteria regarding this ratio have been developed for two conditions: that of minimum stator current (or $\phi_S = 0$) and that of minimum rotor current (or $\phi_R = 0$). A computer program has been used to find the response of the machine, at various speeds and loads, to various voltage ratios (including those for minimum current). This program predicted that the machine responds best under conditions of minimum current where efficiency and availability of a wide range of speeds and torques are concerned. The degree of damping (shown by the magnitude of the ' ΔT ' result) is expected to be reduced under conditions of minimum current - this will result in (prolonged) oscillations occurring either after a sudden torque change or during speed changes. In both these points it appeared to be of no practical importance whether the stator or the rotor current was reduced, as the two conditions result in almost identical performances in all aspects except

that of load angle. All numerical studies were carried out for the machine which was used for the experimental tests (and described in the next chapter) and it is noteworthy that available torques from the machine in the doubly fed mode far exceed the rated torque of the machine as an induction motor/generator.

The stability studies used in this chapter are based on the variations of torque with load angle (and its derivative) and on the Routh-Hurwitz criterion for the stability of linear systems (or for studies of "stability in the small"). The load angle is somewhat arbitrarily defined to represent the phase angle between the rotor voltage and the stator voltage (if allowances are made for their differing rotational speed), rather than the angle between rotor and stator m.m.f's. The study of stability was reduced to a study of the sign of $\frac{dT_i}{d\delta}$ (positive for stable operation) which was finally represented as the sign of a parameter ' ΔT ' in the computer program print-out. Figures 3.7 and 3.11 have been used to describe the variation of $\frac{dT_i}{d\delta}$ and to this extent they are useful. Figures 3.7 and 3.11 do not, however, represent the variation of torque with speed as variations of w_R do not have the same effect as variations in $(-\Delta w)$, thus these figures should not be taken at face value.

No chapter on the theory of the doubly fed machine would be complete without reference to the wider range of electrical machines which are special cases of the doubly fed machine. A brief discussion of these 'special cases' is given in Appendix 3.

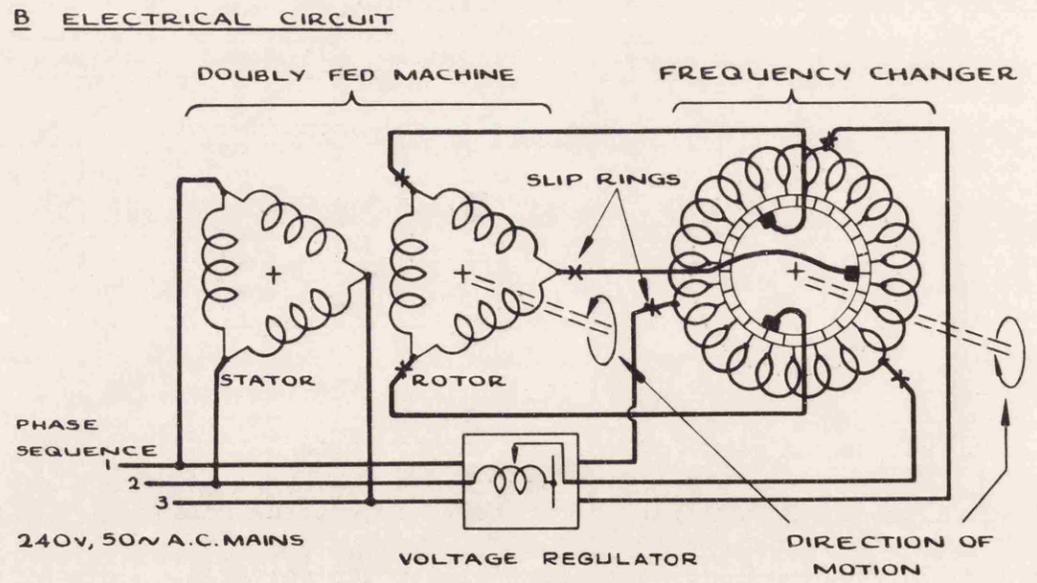
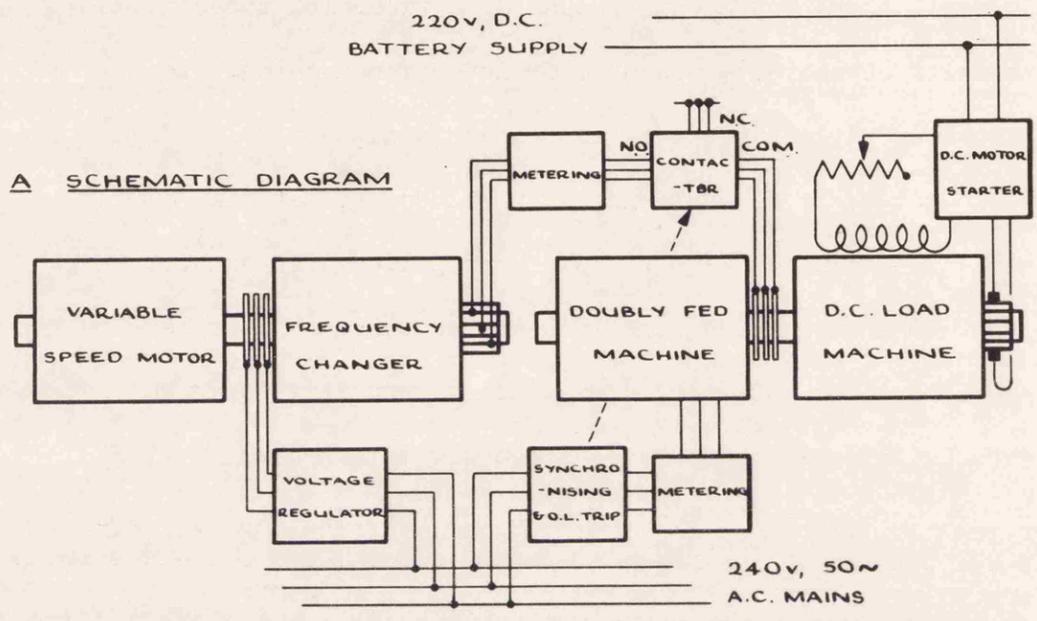


FIG. 4.1. EXPERIMENTAL LAYOUT USING ELECTRO-MECHANICAL FREQUENCY CHANGER

CHAPTER 4

EXPERIMENTAL WORK

The machine used as the experimental doubly fed machine was a universal laboratory machine wound as a two-pole, three-phase (delta-delta connected), wound rotor induction machine. The electrical supply arrangements are shown in figures 4.1. Figure 4.1A also shows, in schematic form, the loading, metering and protection equipment. With the stator supplied with an a.c. supply of 50Hz (and 240V), the fundamental synchronous speed of the machine was 3000 rev/m. Variation from this speed was achieved by injecting a suitable frequency into the rotor slip rings and, to retain the conditions assumed during the analysis, this frequency had to be independent of any variation caused by loading the test machine. A suitable source of the rotor supply is shown in figure 4.1A and is referred to as 'the rotor supply injection circuit'. This circuit consists of two components which modify the 3 phase 50Hz mains, in voltage and in frequency, to a form suitable for injection into the rotor windings of the test machine. The voltage regulator was a 3 phase 7 amp 'variac' variable auto-transformer. This regulator was used to match the rotor supply voltage to the desired value, for synchronisation, and to adjust the rotor-to-stator voltage ratio (to 'optimise' the rotor voltage) thus ^a affecting the power factors in the stator and rotor supplies. The frequency changer* was a four-pole wound rotor machine with slip rings and commutator. The speed of rotation of the machine was related to the output frequency (f_R) by the equation:

* For further details see Appendix 4.

$$f_R = \frac{1500-n}{30} \text{ Hz}$$

(where n is the rotational speed in rev/m)

This relationship was required so that the speed of the frequency changer was related to that of the test machine (when synchronised) enabling a stroboscope to be used to measure load angles in the latter. In practice the frequency changer was a converted Schrage motor driven at the desired speeds by a universal commutator motor (the speed of which was varied by the use of a phase-shift transformer in the armature supply).

The test machine was loaded by a d.c. load machine coupled to a common shaft and floated on the laboratory battery supply. This load machine stator was mounted in a swinging frame so that torque measurements could be made using a spring balance and torque-arm arrangement attached to the stator. The mechanical speed of the test machine was measured using an a.c. tachogenerator - or, in some cases, a stroboscope. The stroboscope was also used, in conjunction with a photocell pick-up, to measure the load angle of the machine (the photocell being triggered from a black-and-white band on the shaft of the frequency convertor).

Rotational vibrations in the machine were measured with a rotational vibration transducer* These 'vibrations' were related to the acceleration of, or changes of load angle in, the test machine.

For electrical measurements* a wattmeter, a voltmeter and an ammeter were provided for each of the stator and rotor supply circuits. Three phase measurements were made using a plugboard arrangement. R.m.s. measurements were required so all meters were of the moving-iron or dynamometer type. To retain accuracy at low frequencies, no transformers could be used in the rotor

*For further details see Appendix 4.

circuit and a cathode-ray-oscilloscope was used to measure rotor voltages. A number of problems arose from the meter loadings in the supplies. A discussion of these problems and their solution is given in Appendix 4.

4.1 The test machine

The test machine was a 'Mawdsleys student demonstration set' wound as a two-pole, three phase, wound rotor induction motor operating on 240V, 50Hz stator supply and with both windings delta connected.

The parameters of the machine (used in the computer program of section 3.4, and shown in table 4.1) were measured using the meters which were used in the remainder of the experimental work.

The stator was supplied with three phase voltage (V_S) whilst the rotor was open circuit and the machine at standstill. The stator current (I_1) and the induced rotor e.m.f. (E_R) were measured. The rotor was then supplied (with the voltage, V_R) whilst the rotor current (I_2) and the e.m.f. (E_S) induced in the, open circuit, stator were measured.

Typical results were

$$V_S = 100.5V \text{ (at 50Hz)}$$

$$I_1 = .586 \text{ A (line)}$$

$$E_R = 71.56V$$

and $V_R = 50.3V \text{ (at 50Hz)}$

$$I_2 = .562 \text{ A (line)}$$

$$E_S = 65.54V$$

From these readings, and assuming that the effects of resistance are negligible at this frequency, the machine self inductances were determined as

$$L_S = \frac{V_S}{100\pi I_1} = .9455 \text{ H}$$

and
$$L_R = \frac{V_R}{100\pi I_2} = .4934 \text{ H}$$

The value of mutual inductance from the stator fed readings differed slightly from that from the rotor fed readings. This is thought to result from the differing magnetic paths (and iron geometry) of the rotor and the stator. As the term 'M²' frequently appears in the torque equations the geometric mean of these was taken.

$$\begin{aligned} \text{Thus } M &= \sqrt{\frac{E_R}{100\pi I_1} \times \frac{E_S}{100\pi I_2}} \\ &= \sqrt{.6732 \times .6429} \text{ H} = .6579 \text{ H} \end{aligned}$$

To determine stator and rotor resistance both stator and rotor were isolated and the deltas were broken at one corner enabling the resistance of individual phases to be measured. A 'Wheatstone Bridge' was set up with a phase winding as the unknown, and three standard resistance boxes in the other limbs of the bridge. The supply was from a d.c. powerpack and the detector was an avometer set to 50μA d.c.

Typical results were:

	Phase 1	Phase 2	Phase 3	Average
Stator, R _S	4.35	4.36	4.36	4.357 Ω
Rotor, R _R	3.78	3.78	3.765	3.775 Ω

TABLE 4.1 : Test Machine Parameters

$$R_R = 3.775 \text{ Ohms}$$

$$R_S = 4.357 \text{ "}$$

$$L_R = 0.4934 \text{ Henries}$$

$$L_S = 0.9455 \text{ "}$$

$$M = 0.6579 \text{ "}$$

Table 4.1 lists the values of resistance and inductance of the test machine (as found in these tests). These parameters were used in the computer ~~assimilation~~ simulation of the machine (section 3.4) which provided a useful basis for comparison between practical and theoretical results.

Other parameters of interest are the turns ratio, magnetic coupling and rated power capabilities of the machine. The turns ratio and coupling were determined from the a.c. tests (see above) and are:

$$\text{turns ratio: } n = \frac{\text{number of stator turns}}{\text{number of rotor turns}}$$

$$= \sqrt{\frac{V_S E_S}{E_R V_R}} = 1.353$$

$$\text{magnetic coupling coefficient: } k = \sqrt{\frac{E_S E_R}{V_S V_R}} = \sqrt{0.9278}$$

$$= 0.9632 = \sqrt{L_S L_R / M^2}$$

The machine was designed primarily as an induction motor (although it is a generalised machine) with a power rating of 2 b.h.p. at 2750 rev/min (and 240V stator phase voltage). This represents a torque of +5.2 Nw.m. or +1.7 Nw.m. per phase. It is assumed that this is the rated torque for all speeds (as torque depends primarily on the currents - which can be assumed to have the same heating effects at all speeds). It will be noted from the results of chapter 3 that the predicted maximum torques are 2 or 3 times this value and rise as the speed varies from fundamental.

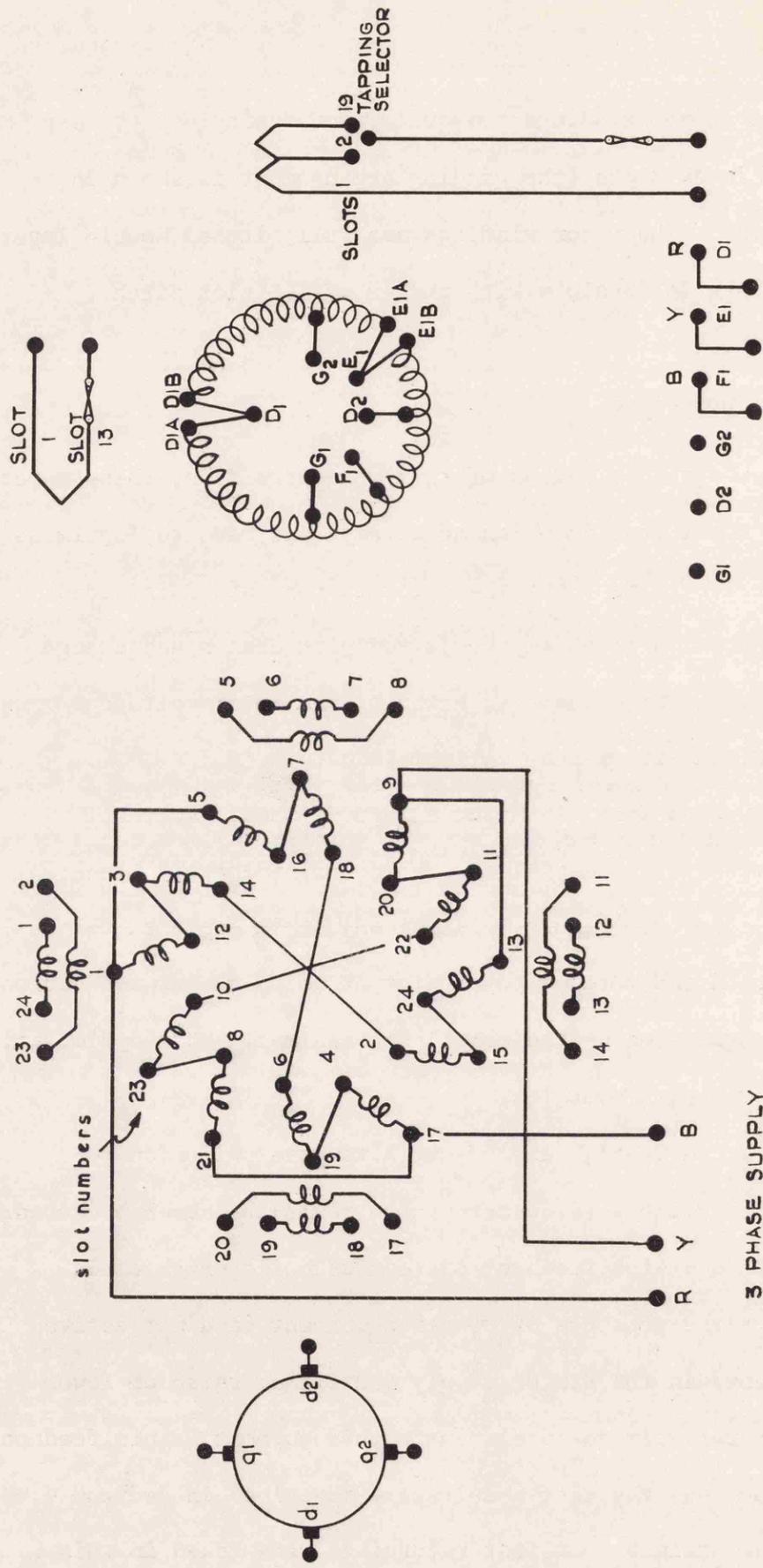


FIG 4.2. MACHINE WINDING CONNECTIONS

The stator windings are full pitched single layer lap windings in 24 slots (the winding arrangement is shown in figure 4.2). The rotor windings are full pitched double layer lap windings in 36 slots with a skew of 1.5 slot pitch.

4.2 Test procedure

Three distinct series of test runs were made, these were:

- (1) The 'First Test' in which no attempt was made to 'optimise' the rotor voltage but the rotor current was maintained at approximately 5 amp (adjusted with each speed change only). At high torques, however, the rotor voltage coincided with a near optimum value.
- (2) The 'Second Test' in which a form of manual voltage optimisation was applied. The rotor circuit 'voltage' regulator (see figure 4.1) was adjusted, for each change in speed and torque, to a point at which the stator current (as registered on the meter) was minimal. This optimisation improved with practice.
- (3) The 'Third Test', carried out after the criterion for optimum voltage (i.e. zero phase angle) had been produced. For this test a feedback system was built which could detect the presence of reactive current (and not active current) in the stator supply and either raise or lower the rotor voltage to eliminate this current. This feedback system, and the test results are described in section 4.4 below. Mainly transient recordings were taken in this test as the second test fully illustrated the steady state performance of the machine under these conditions.

To commence each test run the machine was run-up and synchronised in the following manner: (1) The 'doubly fed machine' (figure 4.1) was run-up to speed by the d.c. load machine (which had proper starting and control equipment associated with it). (2) The 'frequency changer' was driven at a similar speed by its driving motor and the voltage supplied to it was raised to an 'appropriate' value. (3) The rotor supply was fed onto the slip rings of the doubly fed machine causing it to generate a stator voltage which could be compared (for phase, frequency and voltage) with the mains (by the use of a three-lamp synchroscope). (4) Final adjustments were made to the frequency changer speed (which controls the rotor frequency) and to the rotor supply voltage to match the stator frequency and voltage to those of the mains. (5) The stator was 'closed onto' the mains when the correct phase relationship was indicated by the synchroscope. The machine was now operating under conditions of double supply and load or speed variations could be applied. Load variations were achieved by varying the field voltage of the load machine. Speed variations were achieved by varying the speed of the 'rotor frequency-changer' (and consequently the rotor supply frequency). Care had to be taken to match the load-machine speed to that of the doubly fed machine to prevent large load torques being applied and it was preferred to disconnect the load machine for these tests. (Thus most tests were conducted either at constant speed or at zero torque.)

Variations of torque (and, at higher speeds, variations of speed) often caused sufficient disturbance to the machine to cause it to oscillate and fall-out of synchronism. Without

optimisation (the first test) certain torques could not be reached at some speeds and, indeed, the reaching of those which are recorded could be thought of as a 'matter of luck'. In the second tests also great care had to be taken to 'optimise' the rotor voltage at every step and not to cause too large a disturbance on the system. Only in the third test, when the rotor voltage was automatically optimised, could speed and load variations be made with ease (it was not until the third test that the speed was varied whilst the machine was on load or that the load machine could be 'tripped-out' without first slowly reducing the load torque to near zero).

The load machine supplied a considerable degree of damping to the test machine when it was running. This was undesirable for simple studies of the extent of the stable region and, for these studies, it was necessary to disconnect the load machine and conduct tests at 'no-load'. These tests provided information on the stable speed range of the machine at no load only. This information, however, was of significant use despite its restricted nature.

The remaining information was based on measurements (of voltage, current, power, torque, speed etc.) which were made with the machine on load. For these tests the machine was synchronised to the desired speed and loaded to various torques (the speed remaining constant) whilst readings were taken. The machine was then disconnected and resynchronised to a different speed for the process to be repeated.

Thus the majority of tests were conducted either at no-load or at constant speed (as mentioned earlier).

Steady-state readings were taken for all three tests but results were computed only for the first and second tests (as those of the third test appeared to provide no additional information). Traces of the dynamic response of the machine to disturbances only became possible in the second and third tests. These traces, taken on an ultra-violet-recorder, are of: speed (in rev/m), variations in load angle or acceleration (taken from the torsional vibration transducer) and (where relevant) torque (taken from the strain gauge unit).

4.3 Experimental results - the first two tests

A stroboscope (triggered by a light cell directed at a black and white band on the rotor frequency changer shaft) was directed at a calibrated disc on the test machine shaft. This arrangement could be used to demonstrate synchronism (by an apparent stationary appearance of the test machine shaft) and to measure load angle and speed. The doubly fed machine was shown to run 'synchronously' with load angles which varied with load, speed and the rotor-to-stator voltage ratio. These load angles agreed with the predicted values. If oscillations occurred they were not noticeable as they either decayed rapidly (the machine being stable) or soon built up to an amplitude whereby the machine was desynchronised (maximum torque having been exceeded and the machine being unstable).

The range of speeds attainable, with stable operation, was found by no load tests. Speeds ranging from 2400 rev/m to 3200 rev/m were obtained during the first test whilst speeds attained in the second test (with manual voltage optimisation) ranged from 1800 rev/m to 3500 rev/m. During

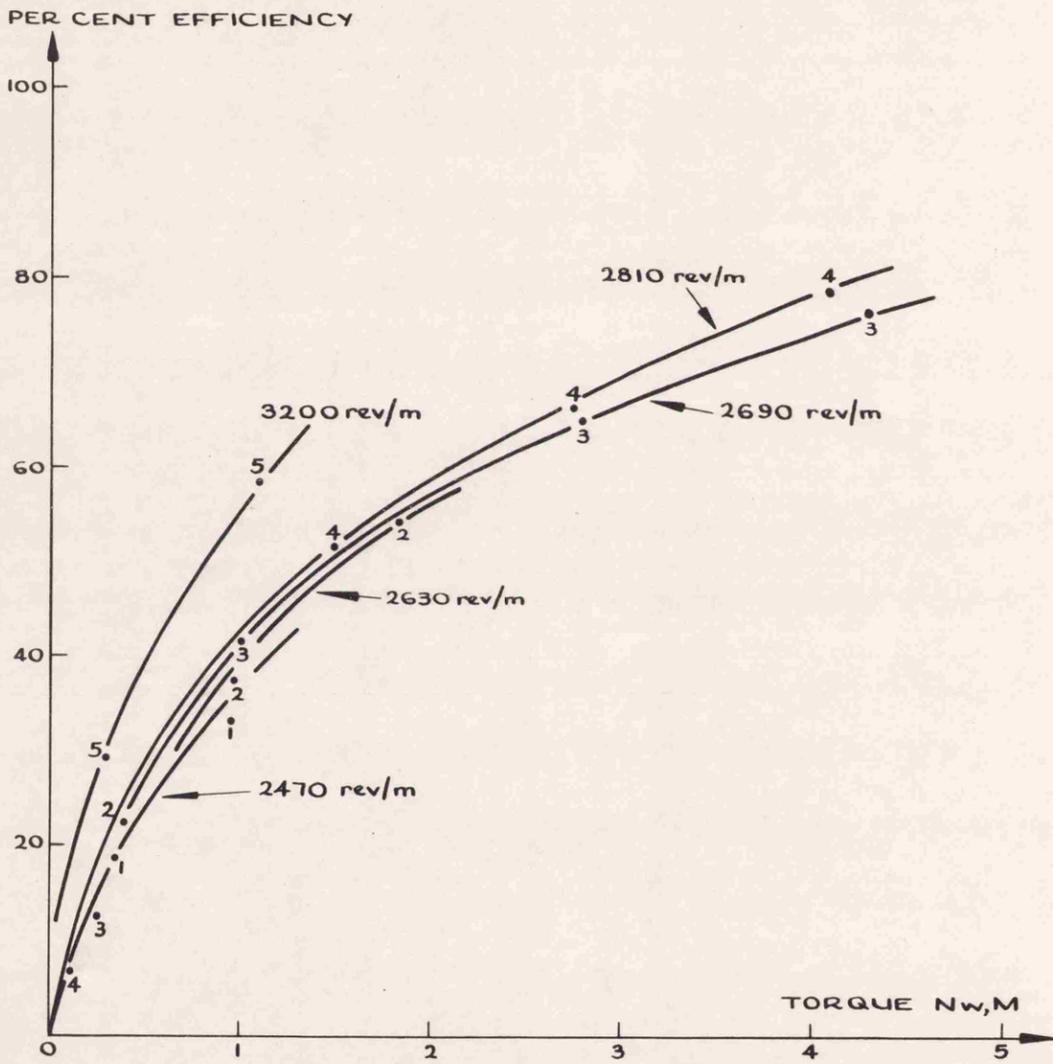


FIG. 4.3 a. EFFICIENCY vs TORQUE FOR VARIOUS SPEEDS
FIRST TEST

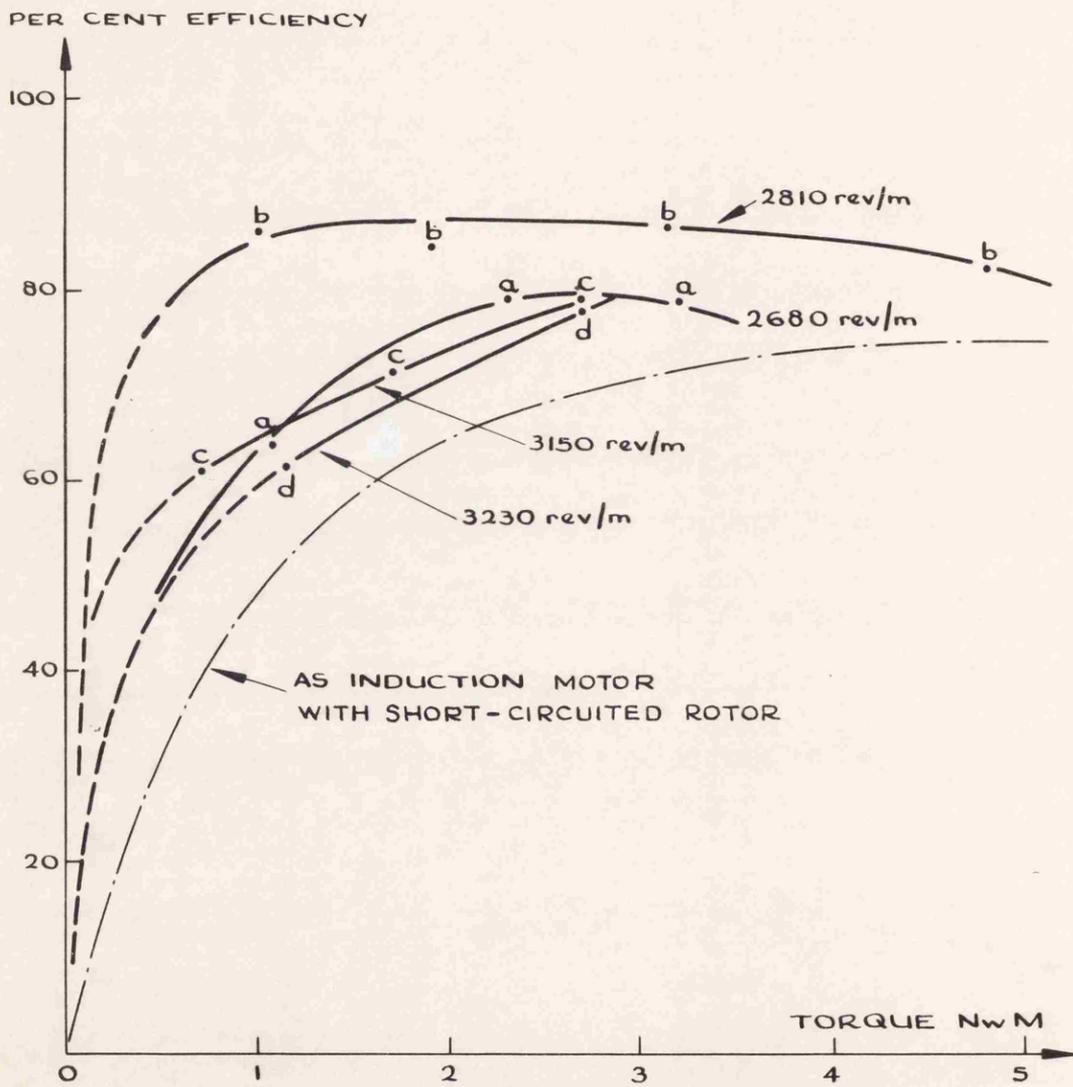


FIG 4.3b. EFFICIENCY vs TORQUE FOR VARIOUS SPEEDS
SECOND TEST

RATIO OF COMPLEX POWER
 FED THROUGH ROTOR TO
 TOTAL COMPLEX POWER

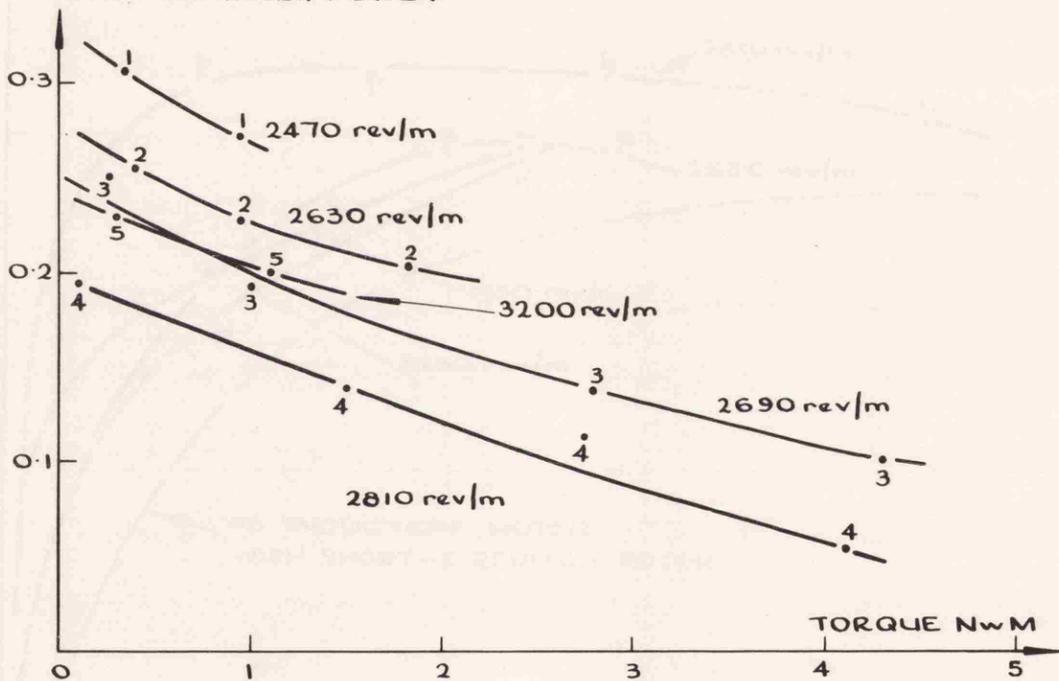


FIG 4a. THE RATIO OF COMPLEX POWER FED THROUGH ROTOR TO TOTAL COMPLEX POWER PLOTTED AGAINST TORQUE FOR VARIOUS SPEEDS - FIRST TEST

RATIO OF COMPLEX POWER
 FED THROUGH ROTOR TO
 TOTAL COMPLEX POWER

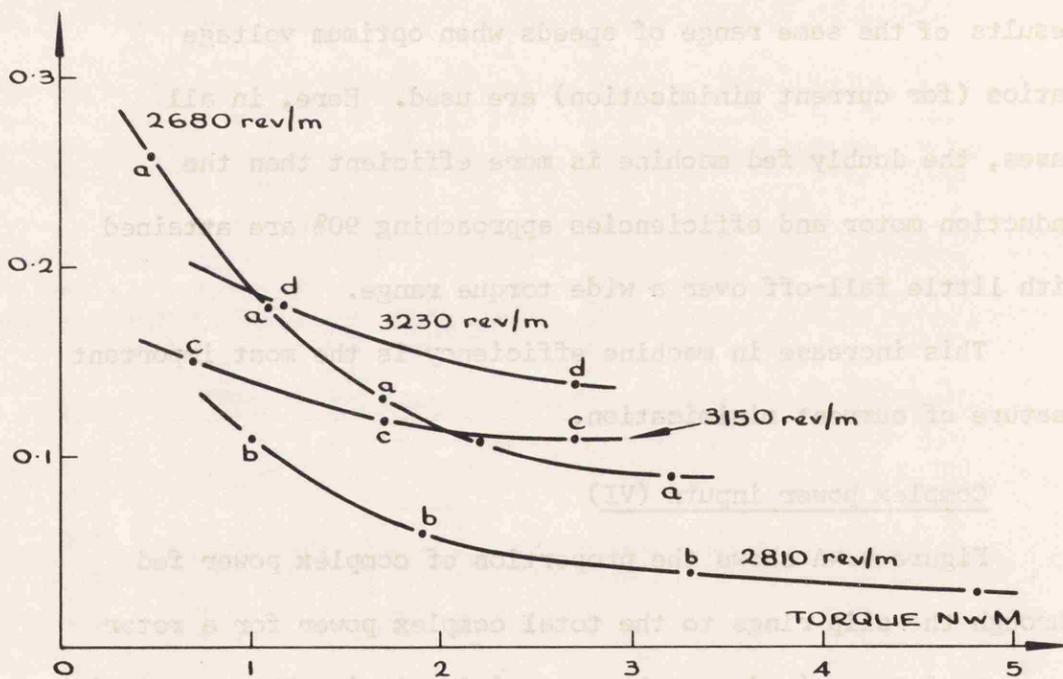


FIG 4.4b. THE RATIO OF COMPLEX POWER FED THROUGH ROTOR TO TOTAL COMPLEX POWER PLOTTED AGAINST TORQUE FOR VARIOUS SPEEDS - SECOND TEST

Efficiency

Two sets of results are presented. Only in the second set of results is the rotor voltage optimised (although, at high torques, the voltages used in obtaining the first set of results coincide with near optimal values). Figure 4.3A shows efficiencies for non optimum voltage conditions. Here efficiencies of 80% are only possible near to fundamental speed. In practice they were only attainable below this speed and fell off badly with reduced torque (as rotor voltages tended to differ from optimum). Figure 4.3B shows efficiency results of the same range of speeds when optimum voltage ratios (for current minimisation) are used. Here, in all cases, the doubly fed machine is more efficient than the induction motor and efficiencies approaching 90% are attained with little fall-off over a wide torque range.

This increase in machine efficiency is the most important feature of current minimisation.

Complex power inputs (VI)

Figure 4.4A shows the proportion of complex power fed through the slip rings to the total complex power for a rotor current of 5 amp (and no voltage optimisation). This proportion is reduced by up to a factor of three when voltage optimisation is used as shown in figure 4.4B. The power capability required in the rotor injector is reduced accordingly.

Optimum rotor voltage V_R

Figure 4.5 shows the variation of optimum rotor voltage (as used in the second test) for current minimisation. The stator voltage was 240 volt throughout. These rotor voltages, which can be seen to vary with torque and with speed, were

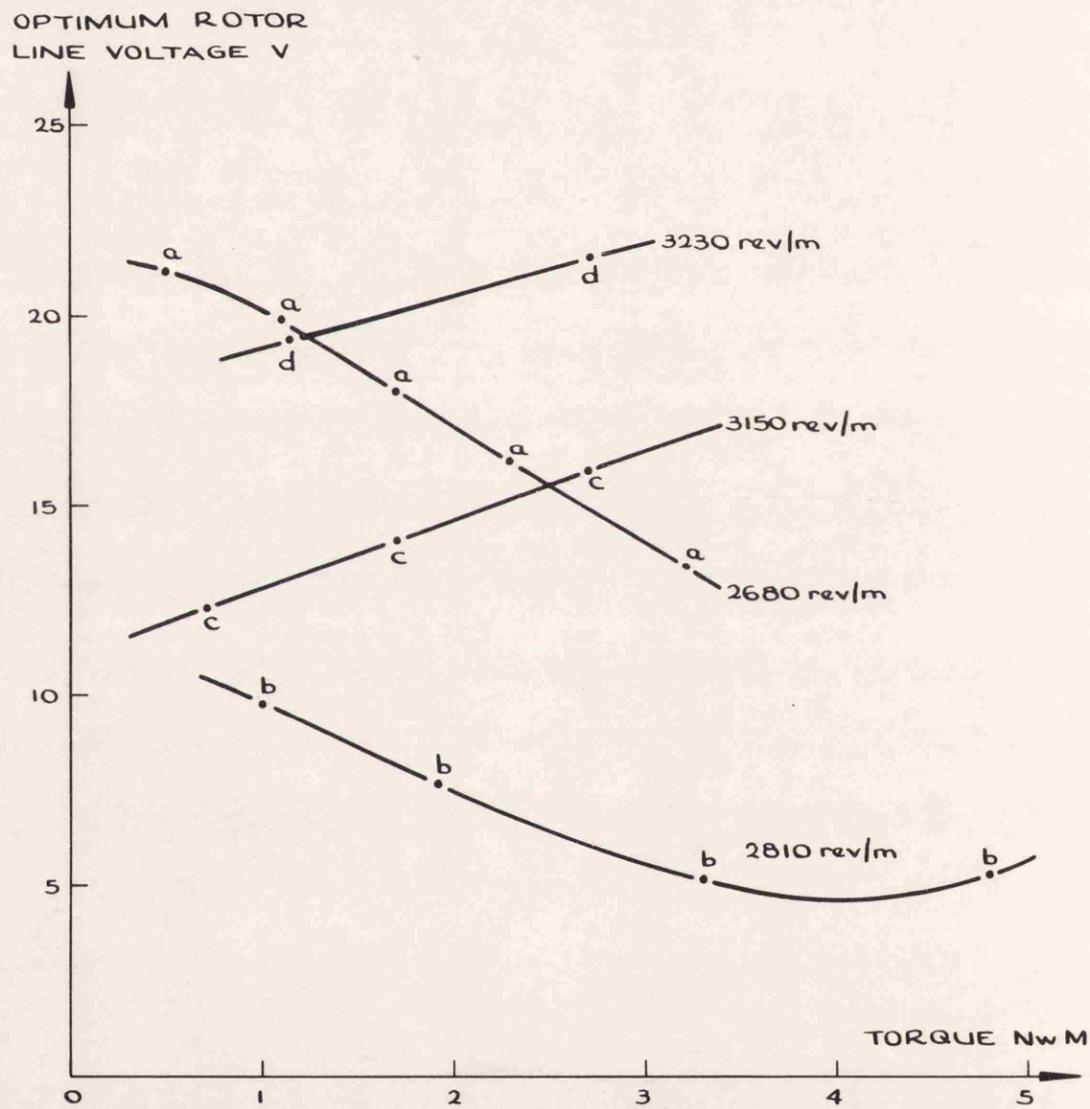


FIG 4.5. OPTIMUM ROTOR LINE VOLTAGE PLOTTED AGAINST TORQUE FOR VARIOUS SPEEDS - SECOND TEST

(Stator line voltage is held at 240v. and stator current is minimised)

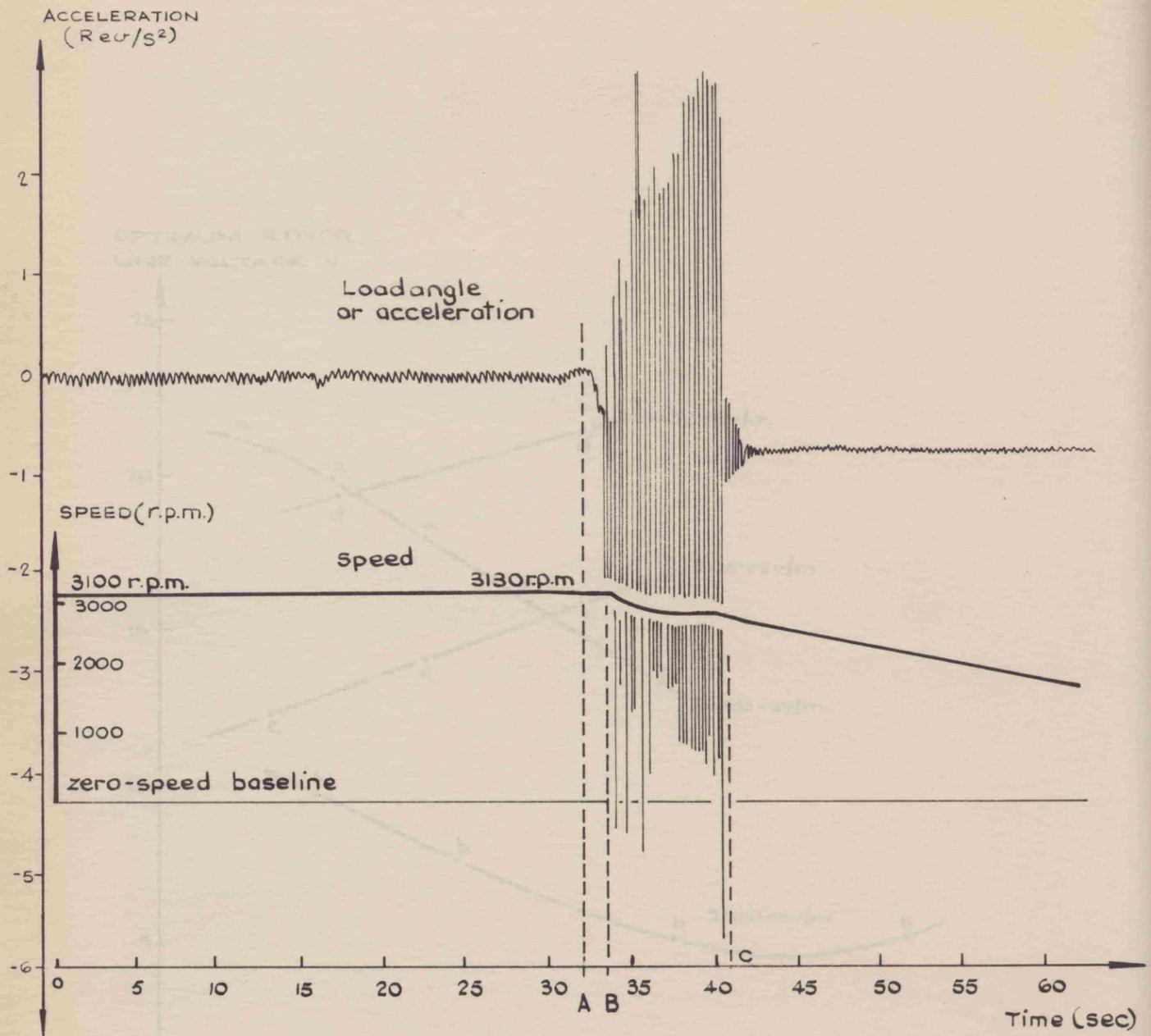


FIG.4.6a. LOSS OF SYNCHRONISM DURING AN ATTEMPTED SPEED VARIATION
 (from 3100 r.p.m. to 3200 r.p.m.) DUE TO POOR V_R OPTIMISATION

- A = Loss of stability
- B = Loss of synchronism
- C = Machine tripped off the supplies

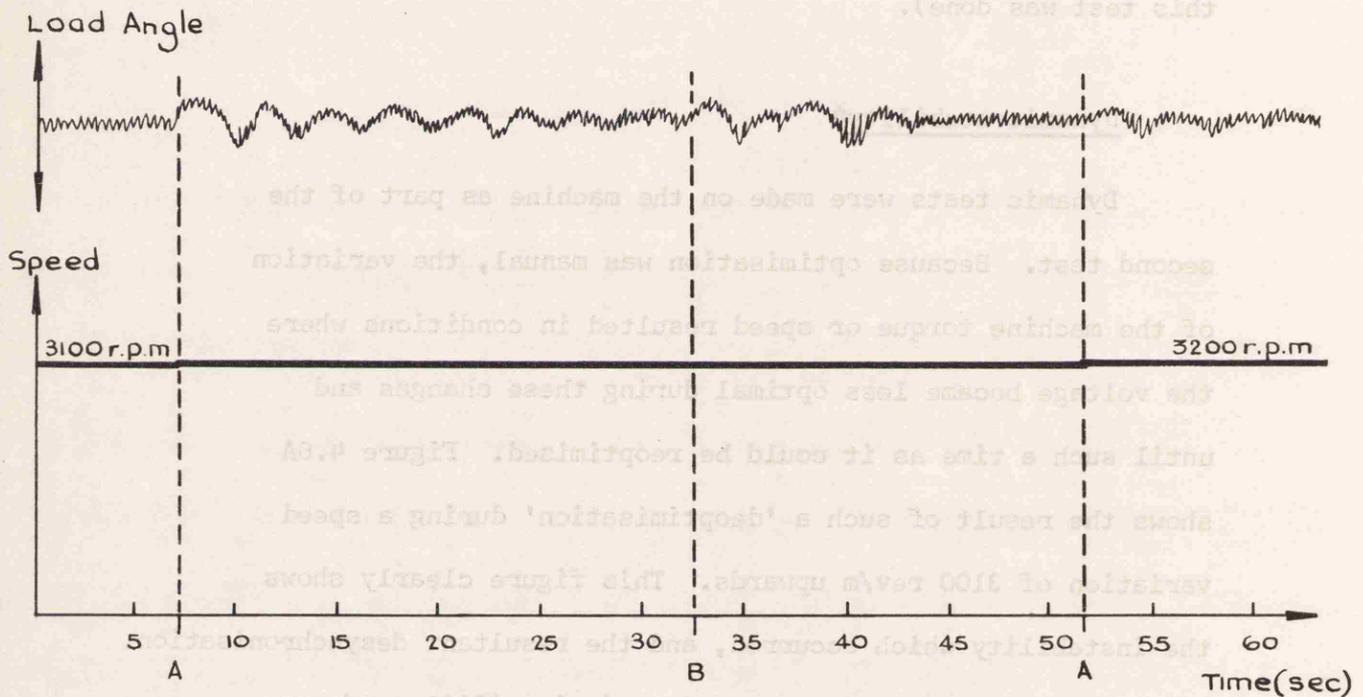


FIG 4.6b THE SUCCESSFUL VARIATION OF SPEED (from 3100r.p.m to 3200r.p.m) IN TWO STEPS WITH INTERMEDIATE V_R OPTIMISATION

A = change in rotor frequency

B = V_R Optimised

not perfect optimum as they were selected manually. The curves, however, follow the lines of the predicted voltages (figure 3.10) fairly well (or, in the cases of curve 'b' and 'c', perfectly). This gives a fair indication of the degree of optimisation attained in the second test (as the results from which figure 3.10 was plotted were not available when this test was done).

Dynamic stability*

Dynamic tests were made on the machine as part of the second test. Because optimisation was manual, the variation of the machine torque or speed resulted in conditions where the voltage became less optimal during these changes and until such a time as it could be reoptimised. Figure 4.6A shows the result of such a 'deoptimisation' during a speed variation of 3100 rev/m upwards. This figure clearly shows the instability which occurred, and the resultant desynchronisation. Figure 4.6B, shows the same speed variation (3100 rev/m to 3200 rev/m) with intermediate optimisation. Here the machine remains synchronised and stable.

It has already been said that without optimisation, speeds outside the range 2400 to 3200 rev/m were unattainable. The attainability of these speeds with optimisation is demonstrated by figures 4.7 and 4.8. Figure 4.7 shows the machine response to a 'step change' in rotor frequency of 5Hz (or a 'step change' in speed command from 2400 rev/m to 2100 rev/m). It can be seen that the machine takes up the

*In all dynamic traces the attenuation of 'speed' and 'load angle' (or 'acceleration') traces is constant.

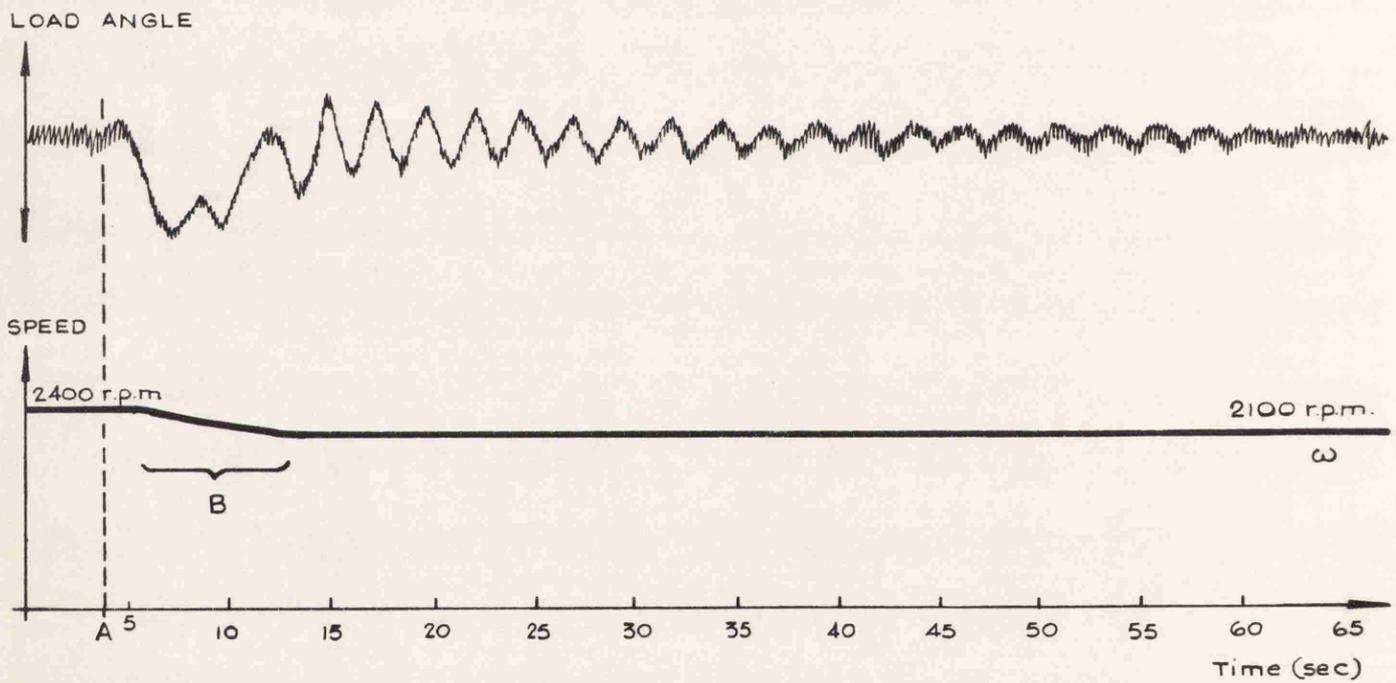


FIG. 4-7. DYNAMIC RESPONSE TO A STEP CHANGE IN ROTOR FREQUENCY

A = Change in rotor frequency.

B = Machine speed change.

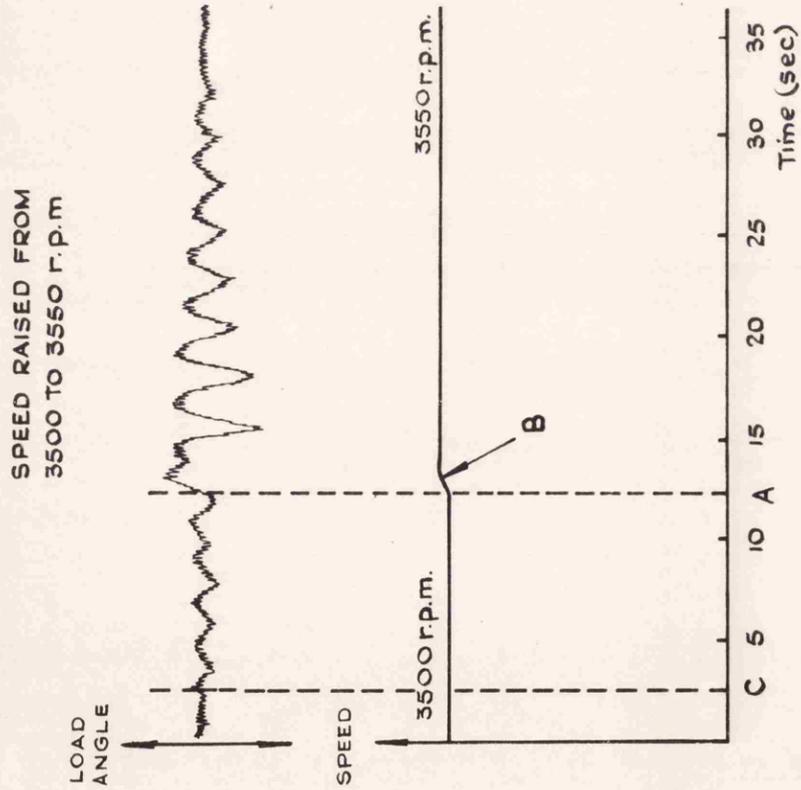
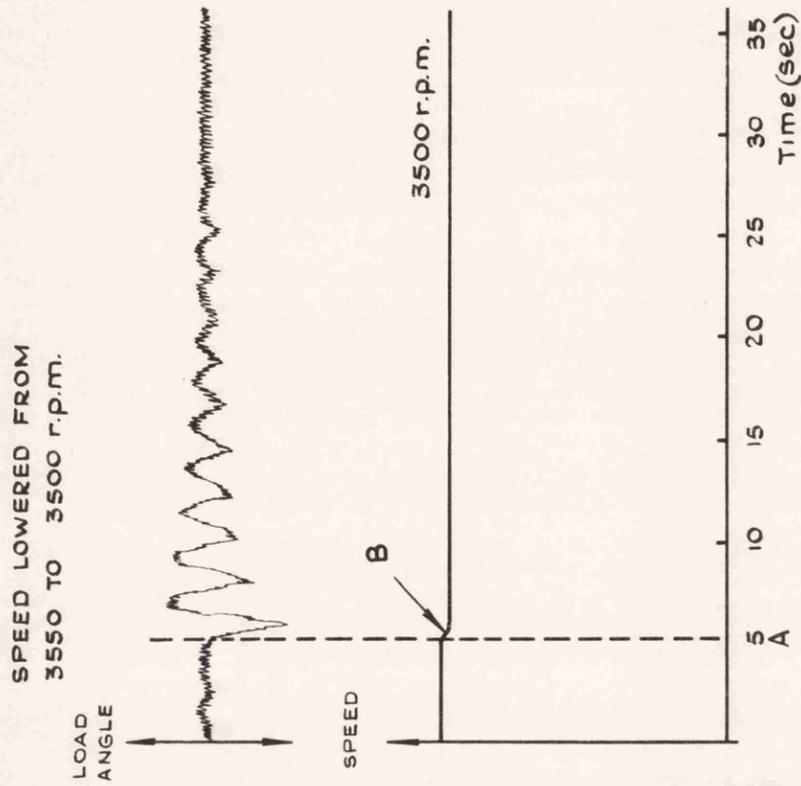


FIG. 4.8. DYNAMIC RESPONSE TO A STEP CHANGE IN ROTOR FREQUENCY

- A = Change in rotor frequency
- B = Machine speed change
- C = V_R adjusted to be optimum for the new speed

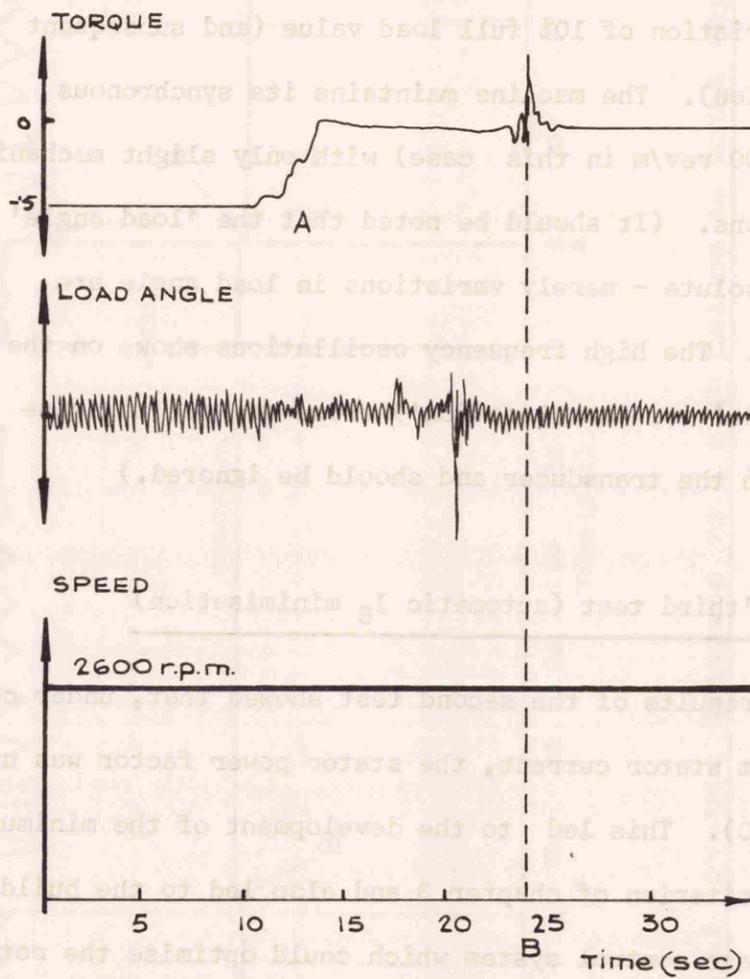


FIG.4.9. DYNAMIC RESPONSE TO A SUDDEN CHANGE IN LOAD TORQUE AT 2600 R.P.M.

- A = Change in load torque (-5 N.m to zero)
- B = V_R optimisation

new speed within 7 sec. and that oscillations die away completely within 60 sec.

Figure 4.8 shows a similar response to such changes between 3500 rev/m and 3550 rev/m (in both directions).

Finally, figure 4.9 shows a typical response to a torque variation of 10% full load value (and subsequent optimisation). The machine maintains its synchronous speed (2600 rev/m in this case) with only slight mechanical oscillations. (It should be noted that the 'load angle' trace is not absolute - merely variations in load angle are indicated. The high frequency oscillations shown on the 'load angle' (or 'acceleration') trace are due to noise pick-up in the transducer and should be ignored.)

4.4 The 'third test (automatic I_g minimisation)

The results of the second test showed that, under conditions of minimum stator current, the stator power factor was unity (or $\phi_S = 0$). This led to the development of the minimum current criterion of chapter 3 and also led to the building of a voltage control system which could optimise the rotor voltage from a knowledge of ϕ_S . The results of this test (with 'automatic stator current minimisation') for the steady-state performance of the machine were similar to those of the second test whilst those for transient performance showed some differences. The control system used and the results obtained are described below.

The control system (for voltage optimisation)

The rotor injection circuit used in the first two tests comprised a 'variac' voltage regulator and a rotating frequency changer (figure 4.1) both of which were electro-mechanical in nature. For practical reasons it was decided to retain this injection circuit (rather than build an electronic one).

It was desired that a phase angle dependent signal should be derived (from the stator supply lines) which could be used to control the rotor voltage (raising it for inductive power factors and lowering it for capacitive power factors). To do this two units were required, these were:

- (1) A unit capable of deriving a phase angle dependent signal from the stator supply lines. The inputs to this unit were to be a voltage and a current and the output was to be a d.c. voltage related to the phase angle between this voltage and current. The output was to be positive for inductive stator phase angles negative for capacitive stator phase angles and zero for zero stator phase angles.
- (2) A unit which would raise the rotor voltage for positive control voltage (the 'control' voltage being the output from the first unit), and lower it for a negative control voltage. This unit was to be composed of the, already existing 'variac', rotor voltage regulator and an electromechanical device to move the control shaft of this 'variac'.

The second of these will be described first.

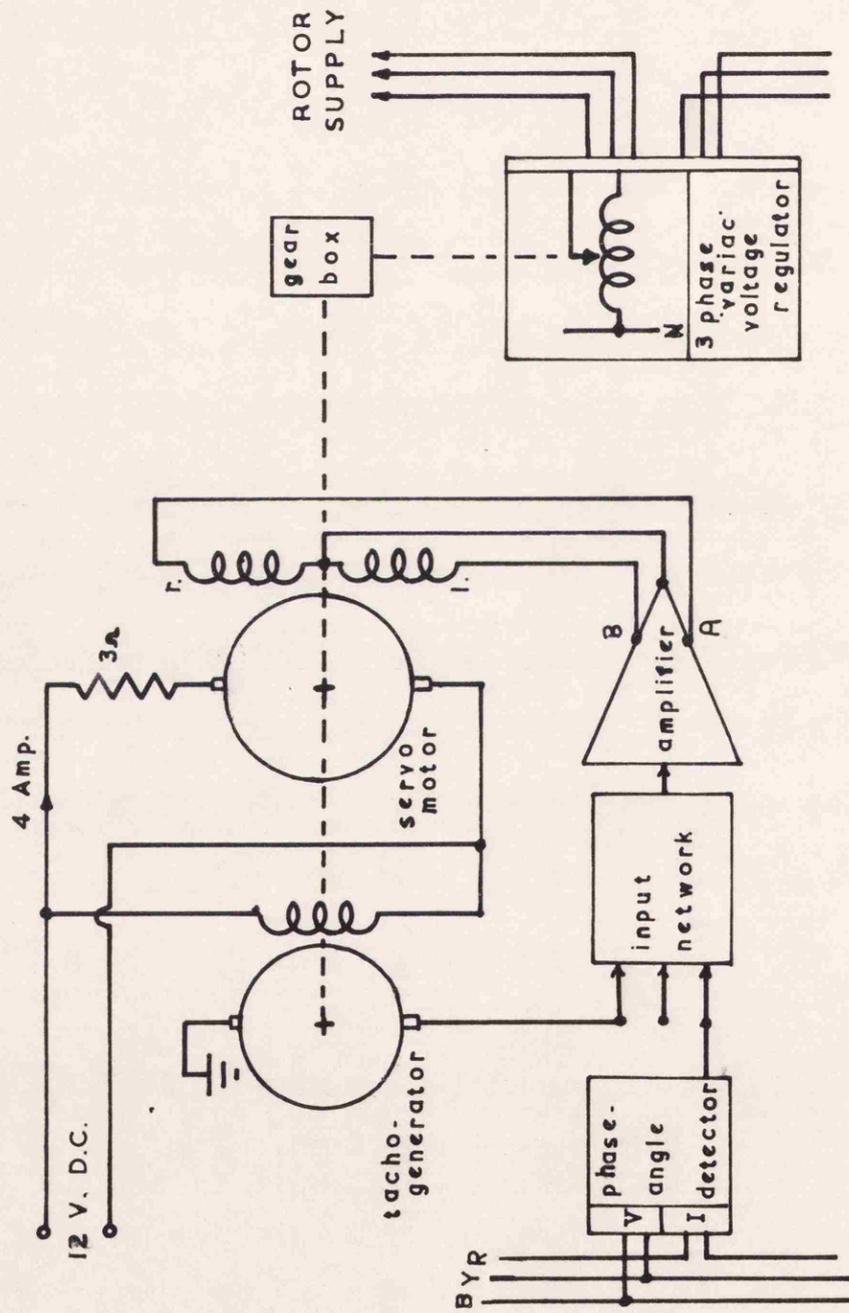


FIG. 4.10 a. VOLTAGE OPTIMISATION FEEDBACK SYSTEM - BLOCK DIAGRAM

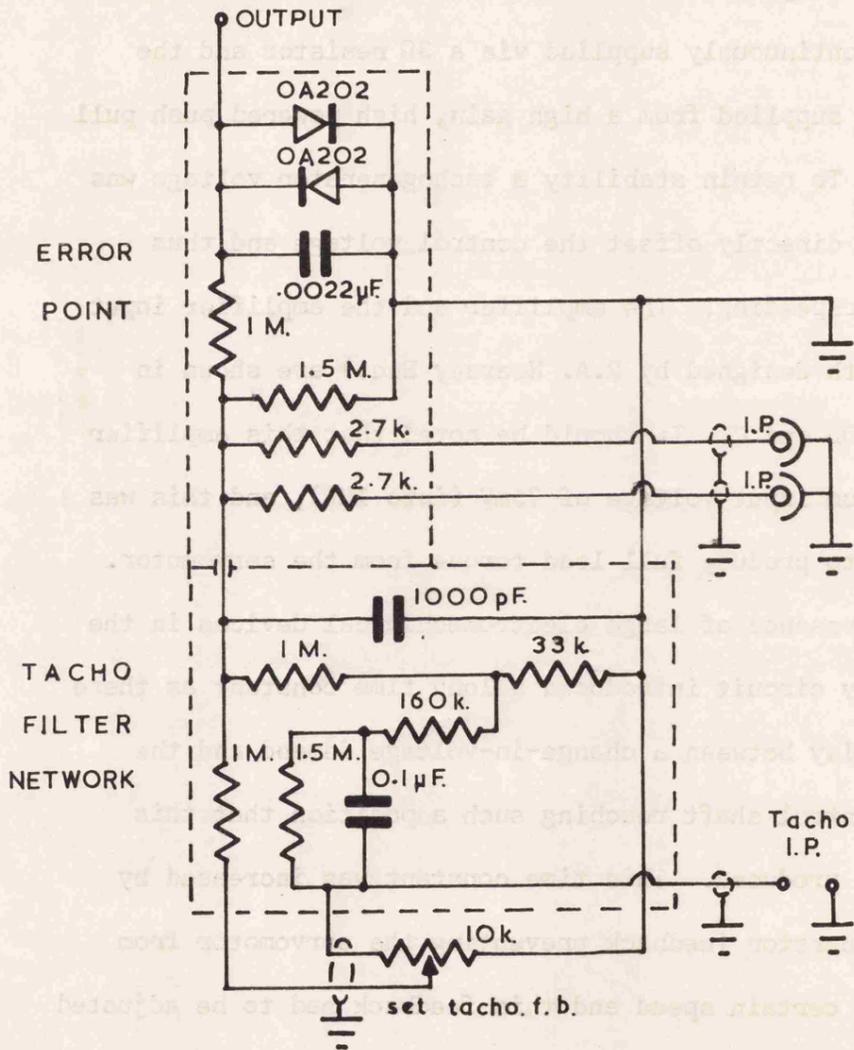


FIG.4.10c. AMPLIFIER INPUT NETWORK FOR VOLTAGE

OPTIMISATION SYSTEM

Figure 4.10A shows, in schematic form, the whole of the rotor voltage control system. With the exception of the phase angle detector (which is unit(1)) this was all contained in unit (2). The variac control shaft was driven (via a gearbox) by a split field servomotor. The armature of this motor was continuously supplied via a 3Ω resistor and the fields were supplied from a high gain, high powered push pull amplifier. To retain stability a tachogenerator voltage was fed back to directly offset the control voltage and thus prevent overspeeding. The amplifier and the amplifier input network (both designed by R.A. Hearsey Esq.) are shown in figures 4.10B and C. It should be noted that this amplifier had a maximum input voltage of 75mV (into $1M\Omega$), and this was sufficient to produce full load torque from the servomotor.

The presence of large electromechanical devices in the rotor supply circuit introduced a long time constant as there was some delay between a change-in-voltage demand and the 'variac' control shaft reaching such a position that this voltage was produced. This time constant was increased by the tachogenerator feedback preventing the servomotor from exceeding a certain speed and this feedback had to be adjusted so that maximum speed of operation was achieved without overshoot occurring (i.e. the servo system had to be 'critically damped'). It should be noted, however, that if the rotor injection circuit were replaced by a reliable cycloconvertor then the control voltage could be used to directly control the output voltage of this cycloconvertor and the electro-mechanical devices (including unit (2) described above) would

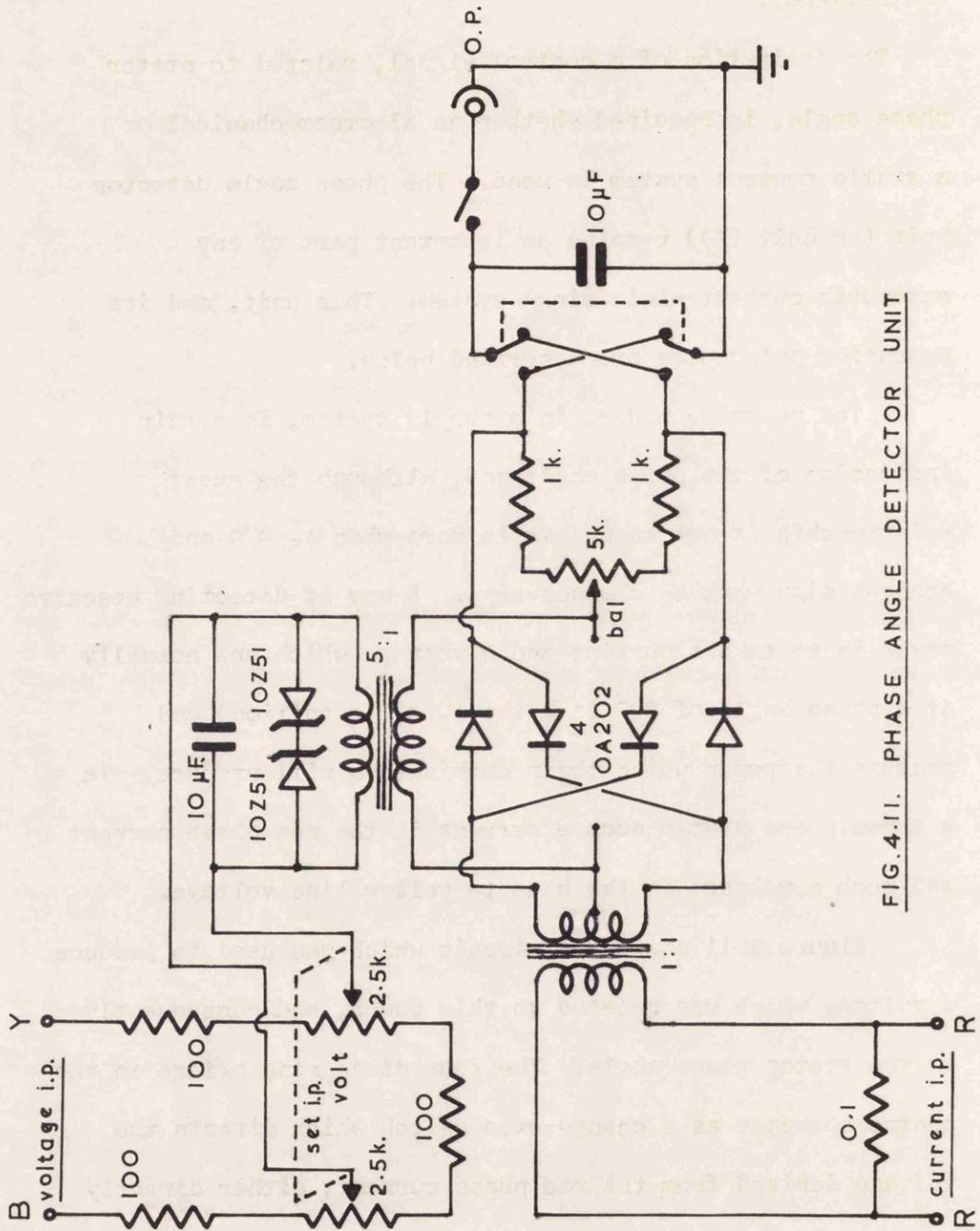


FIG. 4.11. PHASE ANGLE DETECTOR UNIT

be dispensed with. This would produce a shorter response time (as is usually associated with static convertors and controllers).

The derivation of a control signal, related to stator phase angle, is required whether an electromechanical or a static control system is used. The phase angle detector unit (or unit (1)) remains an important part of any automatic current minimising* system. This unit, and its operating principles are described below.

The reactive power, in a supply system, is a fair indication of the phase angle and, although the exact relationship is not known, it is zero when $\phi_S = 0$ and changes sign when ϕ_S changes sign. A way of detecting reactive power is to take a current and a voltage which are normally at a phase angle of 90° (for the $\phi_S = 0$ condition) and measure the power which their combination will produce. In a three phase system such a current is the red phase current and such a voltage is the blue to yellow line voltage.

Figure 4.11 shows the circuit which was used to produce a voltage which was related to this power, and consequently to the stator phase angle. The four diode ring bridge in the centre operates as a change-over switch which directs the voltage derived from the red phase current, either directly or in an inverted form, to the output where it is smoothed by the $10\mu\text{F}$ capacitor and appears as a 'd.c. voltage level'. This change-over switch is operated by the blue to yellow line voltage which is suitably attenuated and clipped (to

*otherwise called 'voltage optimising' or 'unity power factor'

form a square wave). The various transformers and the balancing resistors serve merely to ensure that the voltage only operates the 'change-over switch' (or diode bridge) and that the various inputs are isolated from each other and from the output. To describe operation of this circuit the following series of events should be considered. Assume that the blue phase voltage is positive relative to the yellow phase voltage. A current will flow through the top and bottom diodes shown in the bridge turning them on (the other two diodes being reverse biased and thus turned off). These two diodes then connect the voltage derived from the red phase current directly to the output capacitor. (If the 5K resistor is balanced correctly the voltage input magnitude will not affect the output voltage.) Assume now that the blue to yellow voltage changes sign. The two diodes which were on will be turned off and those which were off will be turned on. The voltage derived from the red phase current will now be connected to the output in inverted form.

If the voltage and current waveforms are in phase the output voltage will be positive (say), if they are 180° out of phase it will be negative and if they are 90° out of phase it will (after smoothing by the capacitor) be zero. The condition of 90° phase angle between red phase current and the blue to yellow line voltage is that of unity power factor or $\phi_S = 0$. Thus we have a voltage which is related (in magnitude and in sign) to the stator phase angle (ϕ_S) and which is suitable for feeding into the control system amplifier. Two points on the operation of this circuit should be noted

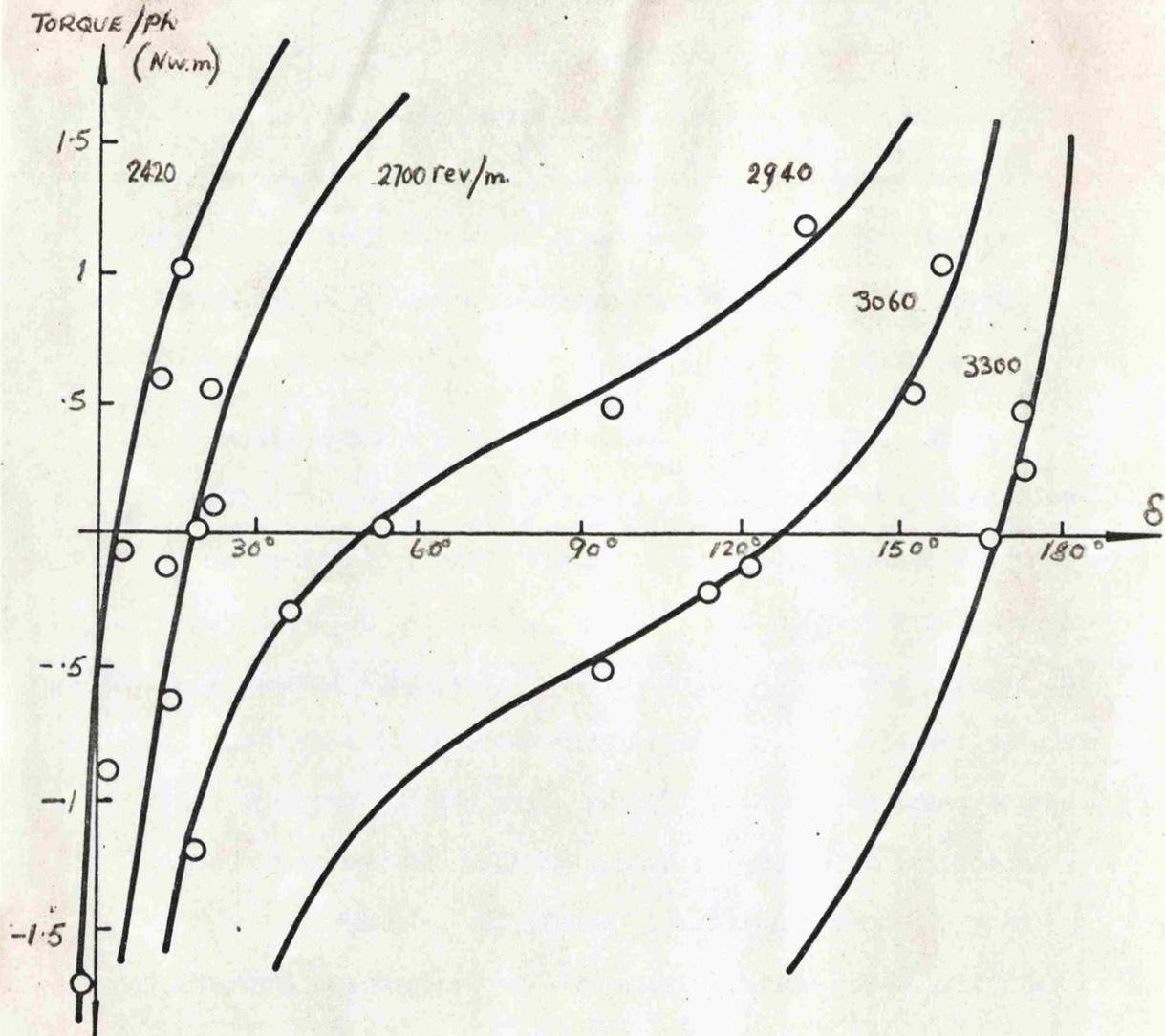
before passing on, however, these are:

- (1) The circuit depends, for its working, on the forward turn on voltage of the diodes in the bridge and diodes with a high turn on voltage should be used.
- (2) The current input sees the smoothing capacitor on the output whereas the voltage input does not. An additional capacitor has, therefore, to be inserted into the voltage input circuit to prevent the introduction of undesirable phase shifts.

The entire control system was tested using a three phase switchable resistor bank to load the lines from which the feedback signal was derived. It was found to be slow but showed the correct responses to various phase angles. When the system was placed in the doubly fed machine network (detecting stator phase angles), however, it was found to have a much better response. Sensitivity was high and the response fast (the inductance of the frequency changer and the test machine causing the most significant lags in the system). To retain stability of the control system the tacho feedback from the servomotor had to be increased.

The results of the third test

It has already been stated (section 4.3) that a restricted speed range (of 2400 to 3200 rev/m) was available during the first test. This speed range was widened (to that of 1800 to 3500 rev/m) by manual voltage optimisation in the second test but this required a considerable degree of skill.



Plot of Torque Against Load Angle (δ) for Various
Speeds with Automatic Stator Current Minimisation

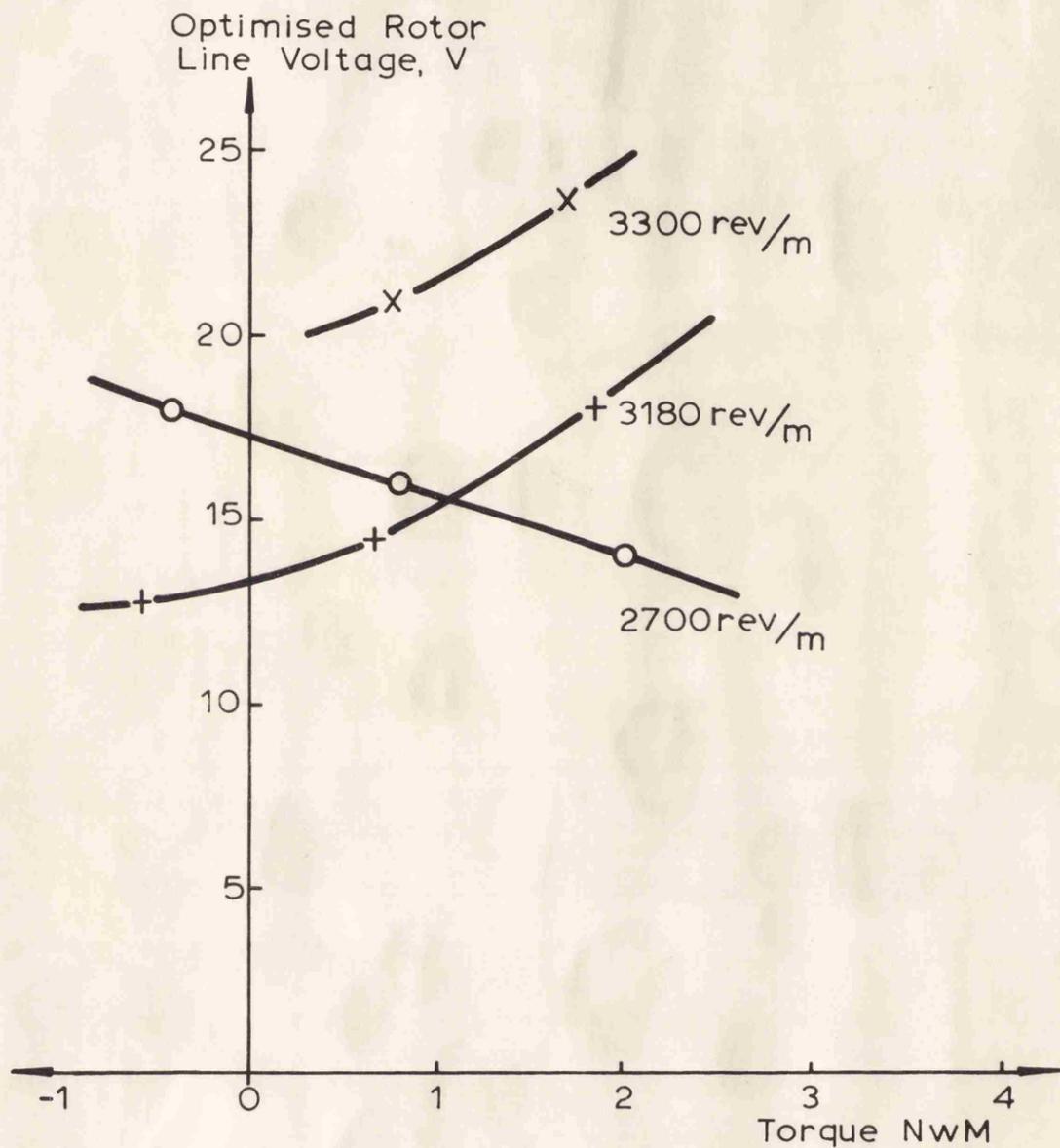
———— THEORETICAL (COMP. PRINT-OUT)
 ○ ○ EXPERIMENTAL (THIRD TEST)

The speed range available in the third tests (of 1700 to 3600 rev/m), whilst not being much greater than that of the second tests, was approaching the predicted speed range (computer simulation of Chapter 3). As the voltage optimisation was done automatically by the control system the running of the machine was no longer a highly skilled operation. Providing the servo system could 'keep-up' with the voltages required of it any variations of torque (within the capabilities of the load machine) and speed (within the range 1700 to 3600 rev/m) could be made. In the second test only small variations in torque or speed could be made without intermediate voltage optimisation (c.f. figures 4.6A and B). In the first test only a restricted speed range (2400 to 3200 rev/m) was available and all the desired torques could not be reached due to a high torque offset resulting from inductive torque components. None of these problems arose in the third test and the limiting conditions mainly resulted from the limitations of the auxiliary equipment.

The optimum voltage rises as the speeds progress further from the fundamental and as torque rises. It therefore became necessary to reduce the stator voltage (thus allowing the optimum rotor-to-stator voltage ratio to be reached) at speeds below 2000 rev/m and consequently few tests were done at these speeds.

The majority of the steady state results, from the third tests, provided little more information than those of the second tests (being identical) and are, therefore, not included here. Steady state results which were of interest,

DERIVATION OF EXPERIMENTAL POINTS OF FIG 3-10



AUTOMATICALLY OPTIMISED ROTOR LINE VOLTAGE PLOTTED
AGAINST TORQUE FOR VARIOUS SPEEDS-THIRD TEST

(stator line voltage is held at 240v. and stator current is minimised)

however, were those relating to the optimum rotor voltage. These are included in figure 3.10 where a comparison can be made between them and the predicted values (for I_s Min.). Although these voltages do not exactly coincide with the predicted values, it was found that the load angle-torque-speed characteristics of the machine operating with these voltages coincided exactly with the predicted values from the computer simulation of Chapter 3 (see fig. facing page 70).

The load angle of the machine varied with rotor voltage and, whilst any oscillations were occurring in the rotor voltage, oscillations would occur in load angle. Oscillations in the machine caused oscillations in the supply currents and phase angles and, with the optimising feedback system in use, this would cause oscillations in the rotor voltage. To prevent the complete system becoming unstable a large amount of damping was introduced by employing a high servo-motor-tachogenerator feedback. The system was stable in the large, if not in the small, even without this damping and a stable limit-cycle often resulted from low tacho feedback. At speeds of about 3400 changes in load torque could not be made in small steps (due to the loading arrangements) and such changes often resulted in loss of synchronism. After loss of synchronism, however, the machine operates entirely as an induction motor, with inductive power factors. The effect of an inductive power factor in the stator supply on the optimising control system causes the rotor voltage to be progressively raised and often the increased voltage would cause the machine to be resynchronised even against

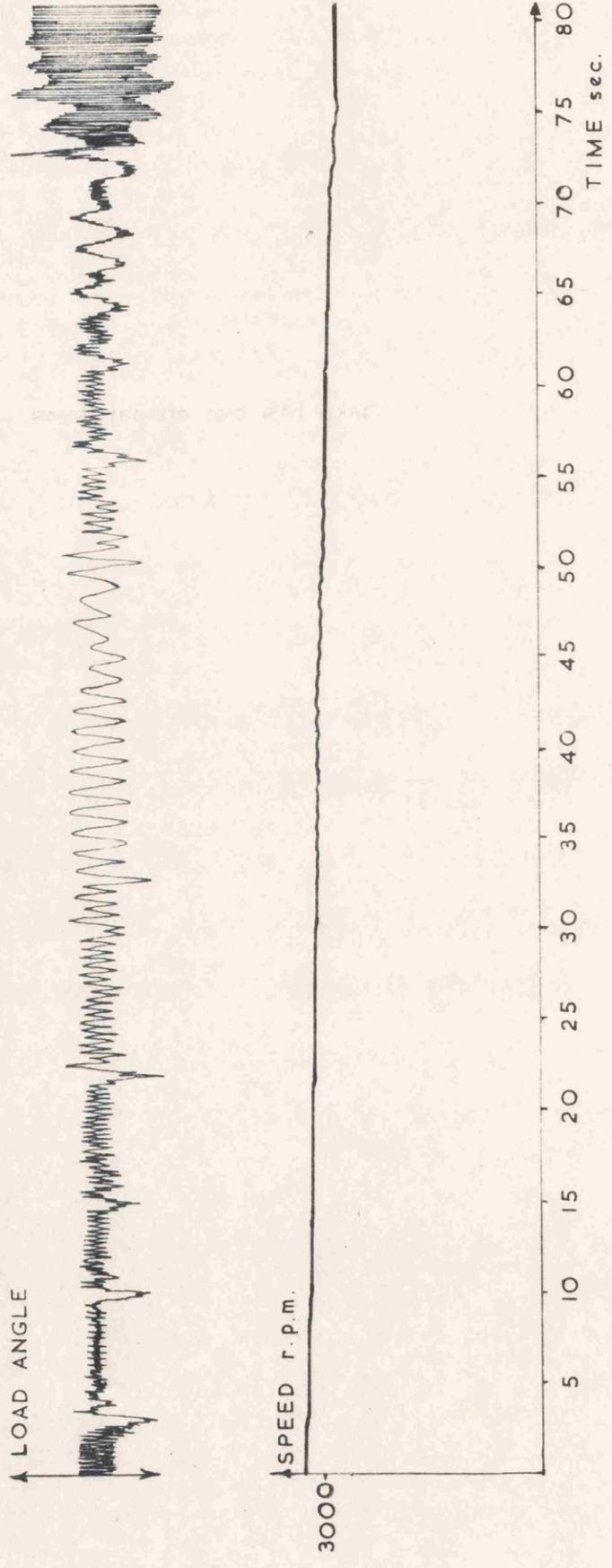


FIG. 4.12. DYNAMIC RESPONSE DURING SPEED REDUCTION (3180 to 2700 r.p.m.). THIRD TEST

load torques of 60% rated value. This property was found to be useful in synchronising the machine in many instances and towards the end of the tests the long synchronising procedure given in section 4.2 was dispensed with.

The dynamic traces which were taken in this tests were all taken under conditions of maximum servomotor tachogenerator feedback and, whilst general comments will be made on them, the reader is asked to make his own comparisons between the performance (and degree of stability) shown in these traces and that shown in the traces from the second test. The attenuation of the 'load-angle' (or 'acceleration') traces and the speed traces is kept constant (as for the second test) so that comparisons can be made.

The response of the machine to speed variations is very similar to that of the second test and a typical one is shown in figure 4.12 (3180 to 2700 rev/m in 80 sec). The high frequency oscillations on the 'load-angle' trace should be ignored as these are noise (picked-up from the frequency convertor drive motor commutator), only the spikes which accompany speed changes and the oscillations at the 2700 rev/m end are relevant. Good stability is shown on this trace and speed oscillations only occur around 3000 rev/m (fundamental speed) where the optimisation of the rotor voltage is more critical. The speed variation was made in ten steps of about 50 rev/m.

In the second test only torque steps of 10% full load value could be made stably and these produced similar responses at all speeds. (An example of the response is shown in figure 4.9.) A change in load torque, however, required a change

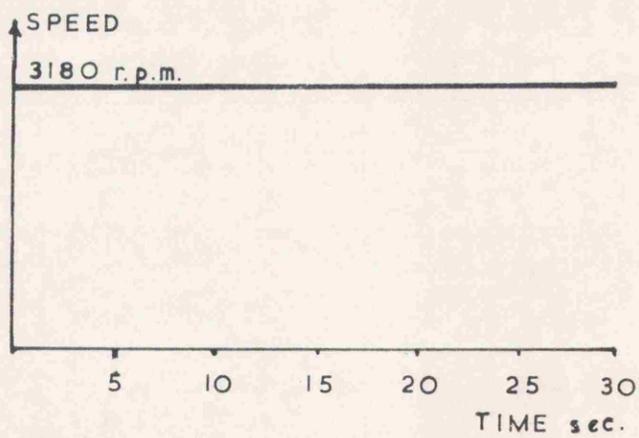
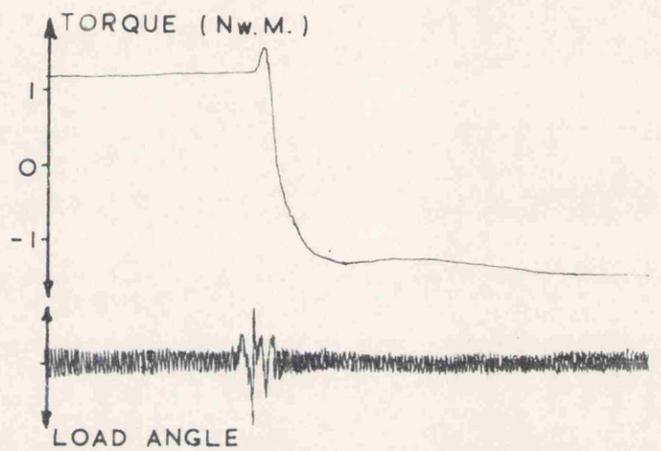


FIG. 4.13a. DYNAMIC RESPONSE TO A TORQUE
STEP (1.2 to -1.5 Nw.M. at 3180 r.p.m.), THIRD TEST

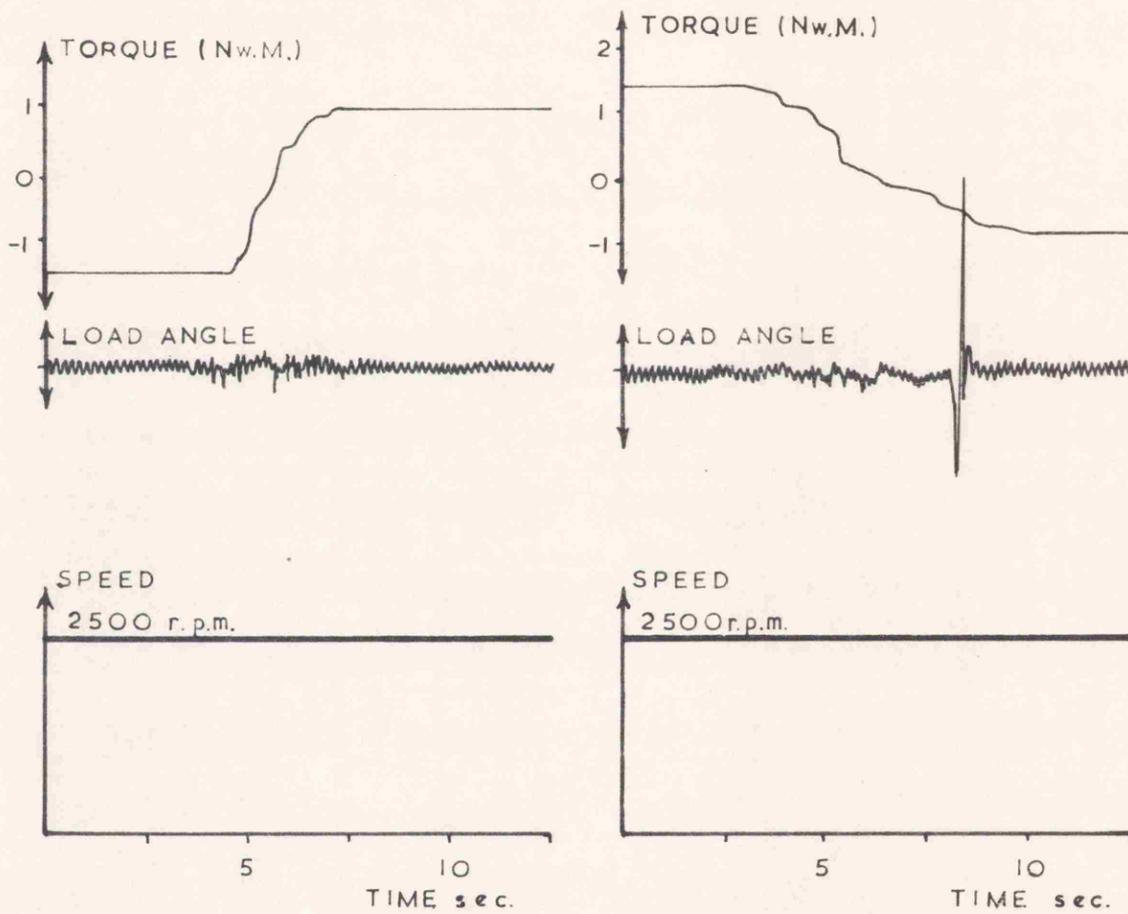


FIG. 4-13b. DYNAMIC RESPONSE TO TORQUE STEPS AT 2500 r.p.m.
THIRD TEST

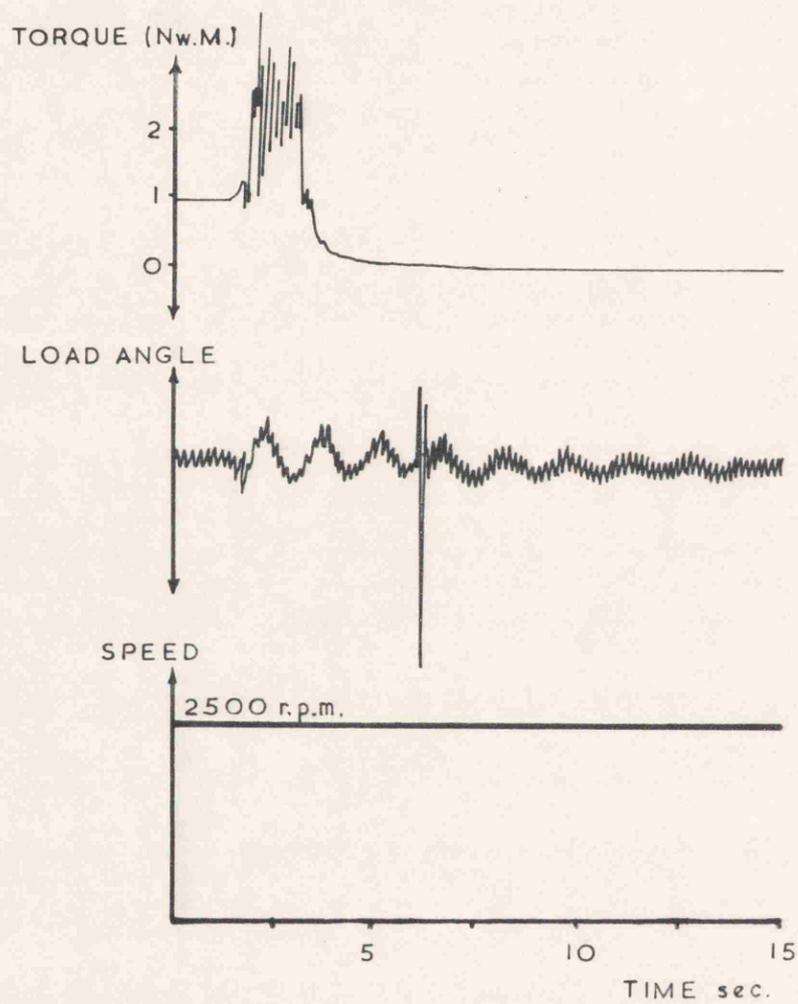


FIG. 4.14a. DYNAMIC RESPONSE TO A TORQUE
IMPULSE AT 2500 r.p.m., THIRD TEST

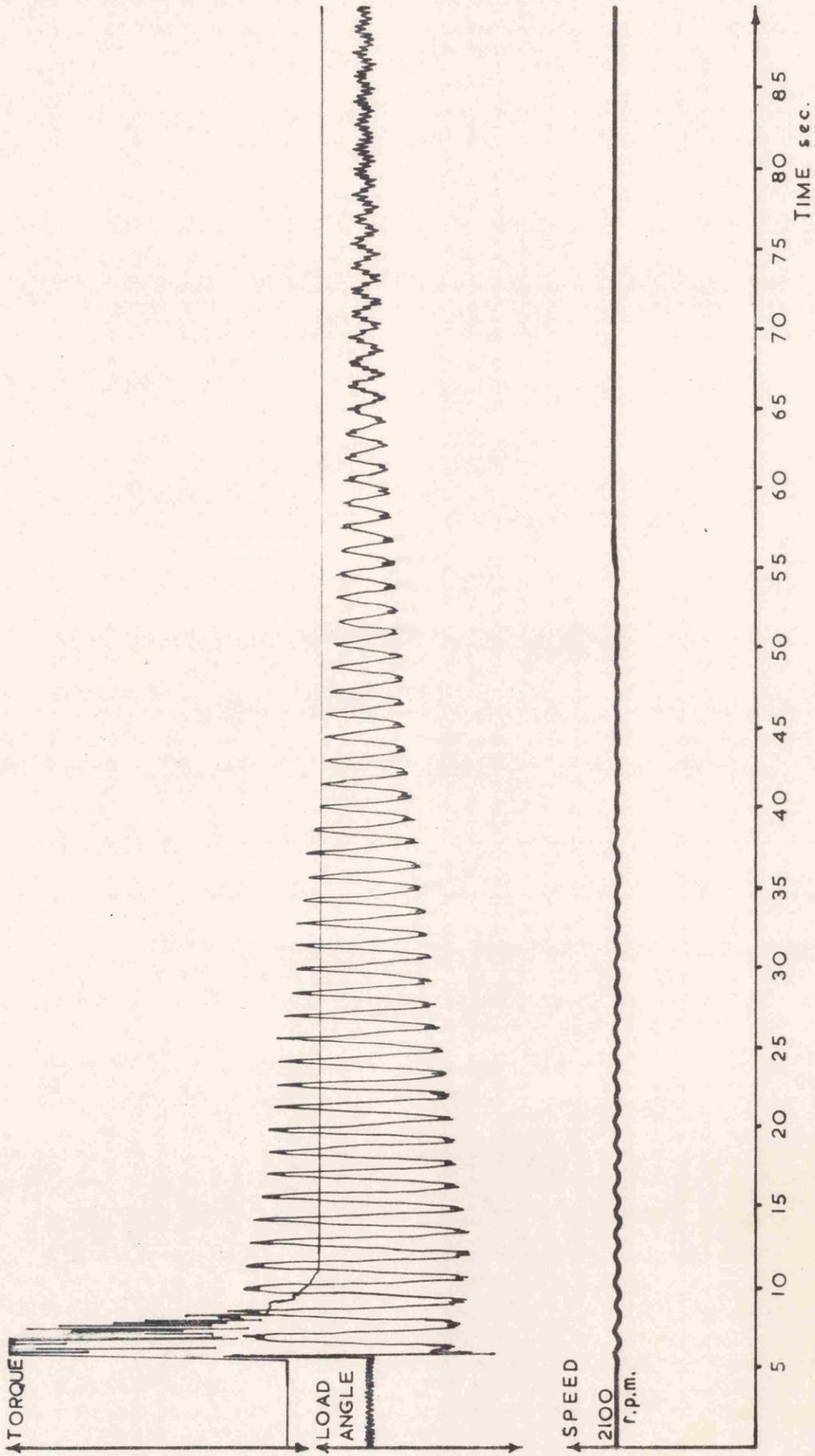


FIG. 4.14b. DYNAMIC RESPONSE TO A -4.5 N.w.m. TORQUE IMPULSE AT 2100 r.p.m. THIRD TEST

in optimum rotor voltage. The voltage optimising system oscillated at many speeds in an attempt to regain optimum voltage conditions after a torque change. This voltage oscillation produced the 'load-angle' oscillation which appears on some of the traces now to be discussed.

Figures 4.13A and B show the machine responses to load changes of 40% full load value at 3180 and 2500 rev/m respectively. The 'load angle' trace here is not absolute but merely shows variations in load angle (being derived from a torsional vibration transducer). The large spike which appears on one trace is 'noise' picked up from the load machine field and should be ignored together with the high frequency oscillations. These traces show a high degree of stability but it should be noted that the loading machine is providing a large amount of damping to the machine. At these speeds (and at speeds more remote from fundamental) the machine (when working with true optimum voltage) operates at virtually constant load-angle and no variations of load-angle (or speed) should be observed during a load change.

Figures 4.14A and B show the response of the machine to torque impulses at 2500 and 2100 rev/m respectively. Figure 4.14A shows (by comparison with figure 4.13B) the deceptive nature of tests made with the load machine connected. After the torque impulses (of figures 4.14A and B) were applied the load machine was de-energised and, therefore, could not provide any damping. In figure 4.14A the machine oscillates as the automatic voltage optimising control oscillates and the decay of these oscillations depends on the damping of the control servomotor. Figure 4.14B shows the

response of the machine to a load impulse of 90% rated value. Here oscillations in load angle are so great that their derivative appears as a variation in speed. These oscillations, after being built up to a maximum (at time = 10 sec), die away exponentially and virtually disappear after 90 seconds.

4.5 Summary of, and conclusions from Chapter 4

A two b.h.p. wound rotor induction motor was connected as a doubly fed machine with a stator supply (of 240V, 50Hz) from the three phase mains and a rotor supply (of varying voltage and frequency) produced by electromechanical means. Facilities were provided for loading the machine and for taking all the necessary readings. The performance (efficiency etc.) and stability of the machine (and not the machine system as a whole) were evaluated in three tests. The first of these tests was conducted without any attempt at current minimisation (or voltage optimisation) and machine currents were kept at about 5 amp for all torques and speeds. The second test followed the realisation that these high currents were not essential and manual attempts were made to minimise them by varying the rotor voltage (minimum current condition was observed by merely looking at the ammeters). Initially minimisation of both currents was tried but later minimisation of the stator current was concentrated on. This was for two reasons: (1) it was easier - the stator supply frequency and voltage being constant throughout, and (2) it did not appear to make any difference which current was minimised (at these torques). One result produced in the second test was that of unity stator power

factor throughout. This led to the minimum current criteria of Chapter 3 and the production of a device which used this result to automatically minimise the stator current (at $\phi_S = 0$). The testing of the machine with this device controlling the rotor voltage constituted the third (and final) test.

The machine responded best under conditions of minimum (stator) current where efficiency and a wide range of speeds and torques were concerned. The degree of damping (shown by the rate of decay of oscillations after a disturbance) was thought to be reduced with minimum current but the extent of this reduction was difficult to determine. High load torques were not imposed upon the machine due to the inadequacy of the loading system and only in the first test (where high inductive torque components were present) was the maximum available machine torque (or pull-out torque) reached. In all cases the machine performed as predicted in the computer simulation of Chapter 3.

CHAPTER 5

GENERAL CONCLUSIONS AND FUTURE WORK

5.1 Conclusions

The doubly fed machine is a versatile machine having a power handling capability comparable to that of the synchronous machine (which is one of several special-cases of the doubly fed machine). As a variable speed drive the machine has several advantages over its counterparts:

- (1) The speed of the machine can be controlled externally, without the use of feedback loops which, together with its synchronous torque characteristics, gives it a speed-independent-of-load characteristic.
- (2) As an a.c. variable speed drive, the machine speed depends on the difference (or sum) of the two supply frequencies. Thus one supply may be of fixed (mains) frequency whilst the other is of a variable frequency. This results in a situation whereby only a fraction of the machine power need be handled by the control equipment.
- (3) The speed range of the machine, for a given frequency range, is increased as, for each pair of supply frequencies, there exist two speeds: one with rotor and stator phase sequences in the same direction (related to the difference of the frequencies) and one with the rotor and stator phase sequences in opposition (related to the sum of the frequencies).

These points recommend the machine for use in many applications, including one in which several drives were required to remain in step with each other. The speed range, however, is affected

by the stability of the machine as most machines are only stable over a restricted speed range about the machines fundamental synchronous speed. To increase this speed range generally requires alteration of the machine parameters (which makes it inefficient as more copper losses are introduced) or the use of external stabilization. Analysis has shown, however, that a certain amount of increase in the stable speed range can be achieved by correctly proportioning the rotor-to-stator voltage ratio so as to reduce the stator or rotor currents to a minimum. This is accompanied by an increase in efficiency.

The analysis used was similar to that of Prescott and Raju¹ except that, to enable supply power factors to be studied, more care was taken in defining the angle of superimposition. This angle, $\Delta\omega t$, was defined as the angle between two stator emf's, one induced in the stator fed condition and one induced in the rotor fed condition. This angle was related to the machine speed by the equation

$$w - \Delta\omega = w_S - w_R$$

where $\Delta\omega t$ was the integral of $\Delta\omega$ with respect to time. The variations of various parameters with $\Delta\omega$ permitted dynamic studies to be made. The machine would be operating synchronously if the term $\Delta\omega$ was zero (which was the steady state condition) and small variations from this condition were used, together with the Routh-Hurwitz criteria, to test for stability. To overcome the problems which some authors have found in studying super-fundamental* and subfundamental speeds (where the phase

*The term 'fundamental speed' was introduced, defining the speed where $w = w_S$ (normally called the synchronous speed), to avoid confusion with the speed of synchronous operation of the machine (where $\Delta\omega = 0$).

sequence of the rotor supply is reversed) the rotor frequency and the term ' w_R ' were permitted to become negative. Equations were developed which defined all aspects of the machine performance (as a balanced, nonsalient pole machine) in terms of the machine parameters and the supply frequencies and voltages. No equation was produced to define the relationship between the rotor and stator supply voltages, so criteria were produced which related these two voltages for unity stator supply power factor and for unity rotor supply power factor. The effects of applying these conditions were found (both theoretically and practically) to be:

- (1) As the rotor to stator voltage ratio is optimised (or adjusted for unity stator or rotor power factor), the supply currents are reduced. This reduces the machine copper losses which causes an increased efficiency.
- (2) If the voltage ratio differs slightly from optimum the magnitude of the inductive torque component becomes so large, compared to the maximum value of the synchronous torque component that, at some speeds, the torque range of the machine becomes restricted or so displaced that the machine cannot operate except at quite ^{un}practical loads. The optimization of the voltage ratio ensures that the full load torque range is available at all speeds.
- (3) The extent of the stable speed range can be greatly increased by optimizing the voltage ratio.

Thus the use of an optimum rotor-to-stator voltage ratio, which varies with speed and with load; increases the machine efficiency (and reduces the rating of the auxiliary equipment

to be used with the machine) and widens the range of speeds and torques over which the machine may be used.

The load angle-torque variations of the machine also differ under optimum voltage conditions in that, whereas normally the torque distribution is around the full 360° of load angle, under these conditions the torque distribution is completed over 180° of load angle and repeated for the remaining 180° of load angle. This can be explained in terms of the fact that the optimum rotor voltage (for a given stator voltage) is a complex sine function of the load angle ' δ ' and the synchronous torque terms (i.e. those which vary with δ) are linear functions of the rotor voltage and complex sine functions of the load angle. Thus in the torque expression terms of ' $\sin^2\delta$ ', ' $\cos^2\delta$ ', and ' $\cos\delta \sin\delta$ ' are produced which then simplify into double angle ($\sin 2\delta$ etc.) terms.

The conditions of unity stator power factor and of unity rotor power factor produced results which showed a number of differences as listed below:

- (1) For a given stator voltage, the rotor voltage which produces unity rotor power factor passes through zero whilst that which produces unity stator power factor turns at a minimum value. The latter of these two voltages also goes-to-infinity whilst the former turns at a maximum value. Thus the optimum voltages for the two conditions show an inverse relationship.
- (2) At load angles near those for which the rotor voltage is infinite (unity stator power factor condition only) a region of instability exists (see figure 3.8) which does not exist in the unity rotor power factor condition (figure 3.9).

- (3) The variation of torque with load angle is unaffected by the machine speed under the unity rotor power factor condition but shows a phase shift with speed under the unity stator power factor condition (this is easily explained by examining the variation of load angle with speed, for constant Δwt).
- (4) Maximum available torques in the two conditions are very different. Under the unity rotor power factor condition maximum torques are of the order of twice rated value, independent of speed, at load angles which are 90° apart. Under unity stator power factor conditions, however, maximum torques can rise from twice the rated value at the fundamental speed to seven or eight times the rated value near the stability boundary. These torques are set at load angles which are set 170° to 180° apart and which lie within the unstable band mentioned in (2) above.
- (5) Under the unity stator power factor condition the variation of load angle with torque, at speeds remote from the fundamental, is very small and the machine is very rigid (or stiff).

In summarising these differences, it can be said that the criterion where the stator power factor is unity has several advantages over that where rotor power factor is unity. These are mainly in terms of the magnitude of the machine torque. They do not include any variation in the stable speed range, between the two conditions, as there is none. The most important

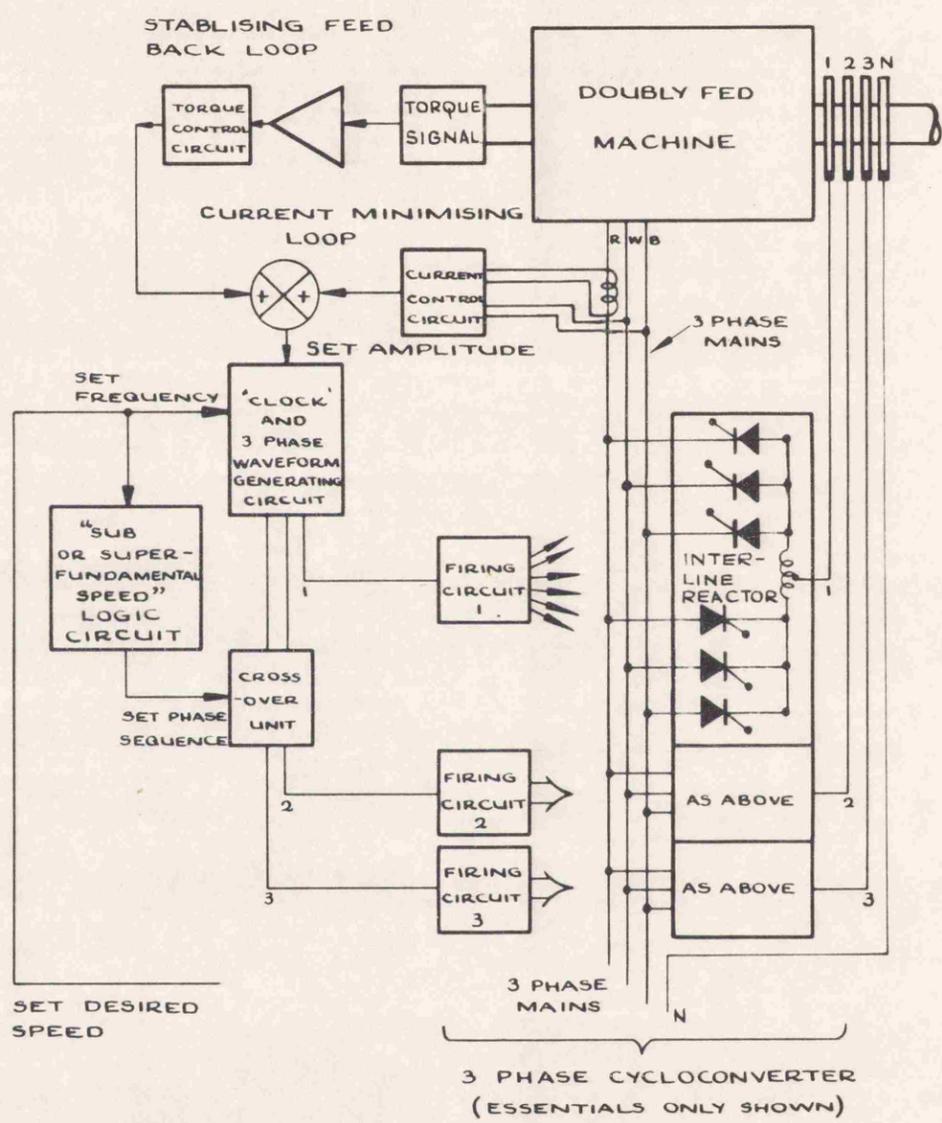


FIG. 5.1. SCHEMATIC DIAGRAM OF FUTURE PRACTICAL VARIABLE SPEED DRIVE INVOLVING A DOUBLY FED MACHINE AND A CYCLOCONVERTER

parameter when deciding which power factor should be made to be unity is of a practical nature, i.e. which is easier and what requirements are imposed by the rotor supply frequency convertor. The possibility of the rotor voltage passing through zero (see (1) above), however, is not good.

Experimental tests agreed with theoretical predictions showing a doubling of the stable speed range when the stator supply was operating at unity power factor. Other interesting results were a high efficiency (which fell off slightly at speeds more remote from the fundamental) and a high degree of stability. To maintain the rotor voltage at this optimum value, a simple servo system was produced. This worked well. With this servo system the machine could be run up as an induction motor and then synchronised by merely applying the rotor voltage.

Thus the doubly fed machine can be used as a variable speed drive, over a restricted speed range (around the fundamental synchronous speed), with good utilization and fair efficiencies. The (stable) speed range, the efficiency and the utilization are increased by proportioning the rotor and stator voltages so that one of the supplies is operating at unity power factor. This requires the rotor, or the stator voltage to be adjusted for every change in speed and load but this is not a difficult operation to automate.

5.2 Future work

It is expected that the doubly fed machine will be used, with a cycloconvertor, in a variable speed drive system similar to that shown in figure 5.1. The analysis of the doubly fed machine has reached a sufficient point for this to be done

with safety provided the output of the cycloconverter can be assumed to be sinusoidal. Various rotor supplies containing a high percentage of 50 Hz ripple have been applied to the doubly fed machine with some success but if the cycloconverter output is far from sinusoidal it may be necessary to analyse the machine for nonsinusoidal supplies. Stabilization of the machine, where necessary, would be by feedback of either torque (as shown here) or acceleration (in a manner similar to that of Albertson²). This feedback would affect the voltage, and not the frequency, of the rotor supply to maintain the true speed-independent-of-torque characteristic of the system. The cycloconverter is ideal for use in a feedback system as it requires little power to control its output. It also has the advantage of being able to handle varying amounts of reactive power and thus it does not apply the restraint on the system which a d.c. - to - a.c. inverter would.

The design of machines for use in the doubly fed mode would be greatly helped by the development of a simple criterion relating the stable speed range of the machine (under conditions of unity power factor of one supply) to the machine parameters by extending the theory in a manner similar to that which Prescott and Raju¹ applied to the double speed machine.

It is envisaged that doubly fed machines may be used in applications where several machines are required to remain in step with each other (such as a steel mill or a printing press). To do this would require a study of the multi-machine system to investigate the possibility of disturbances being transmitted from one machine to another and the possibility of large currents circulating between machines on differing loads. It is most

likely that, in this application, the machines would have a common stator supply (which may or may not be of a variable frequency) and separate rotor supplies which would just be used for the trimming of machine speeds and angles. If this were the case the multi-machine problems mentioned above would not arise and the machine would only be used over a 'restricted speed range'.

APPENDIX 1

Derivation of Machine Torque Expression from Rotor emf's
and Currents

Equation 3.20 restated in rotor terms is:

$$2T = \underline{\text{Re}} \left(\frac{e_{R1}^* i_{R1}}{(\omega_R - \Delta\omega)} + \frac{e_{R1}^* i_{R2}}{(\omega_R - \Delta\omega)} \right) + \underline{\text{Re}} \left(\frac{e_{RS2}^* i_{R1}}{\omega_R} + \frac{e_{RS2}^* i_{R2}}{\omega_R} \right)$$

A1.1

$$= 2T_A + 2T_B + 2T_C + 2T_D \text{ say.}$$

Equation A1.1 will be evaluated in four parts as follows:

The first component

$$2T_A = \underline{\text{Re}} \left(\frac{e_{R1}^* i_{R1}}{\omega_R - \Delta\omega} \right) \quad \text{from equn. A1.1}$$

or, substituting for e_{R1}^*

$$2T_A = \underline{\text{Re}} \left(-jM e^{j\omega t} i_{S1}^* i_{R1} \right)$$

and, substituting for i_{S1}^* and i_{R1} (from equn.'s 3.8 and 3.7)

$$2T_A = \underline{\text{Re}} \left(\frac{-M^2 (\omega_R - \Delta\omega) \{R_R - j(\omega_R - \Delta\omega)L_R\} v_S^* v_S}{A^2 + B^2} \right)$$

now $v_S^* v_S = V_S^2$

$$\therefore 2T_A = \underline{\text{Re}} \left(\frac{-M^2 (\omega_R - \Delta\omega) \{R_R - j(\omega_R - \Delta\omega)L_R\} V_S^2}{A^2 + B^2} \right)$$

$$= \frac{-M^2 R_R V_S^2 (\omega_R - \Delta\omega)}{A^2 + B^2} = -2T_1$$

A1.2

The second component

$$2T_B = \underline{\text{Re}} \left(\frac{e_{R1}^* i_{R2}}{(w_R - \Delta w)} \right) \quad \text{from equn. A1.1}$$

$$= \underline{\text{Re}} \left(-jMe^{j\omega t} i_{S1}^* i_{R2} \right)$$

or, substituting for i_{S1}^* and i_{R2} (from equns. 3.8 and 3.14)

$$2T_B = \underline{\text{Re}} \left[\frac{-jM \{R_S + j(w_S + \Delta w)L_S\} \{R_R - j(w_R - \Delta w)L_R\} e^{j\omega t} v_R v_S^*}{(C + jD)(A - jB)} \right]$$

which, by comparison with equn. 3.27, shows that

$$T_B = -T_3 \quad \text{A1.3}$$

The third component

$$2T_C = \underline{\text{Re}} \left(\frac{e_{RS2}^* i_{R1}}{w_R} \right) \quad \text{from equn. A1.1}$$

or, substituting for e_{RS2}^* ,

$$2T_C = \underline{\text{Re}} \left(-jMe^{j\omega t} i_{S2}^* i_{R1} \right)$$

and substituting for i_{S2}^* and i_{R1} (from equations 3.13 and 3.7)

$$2T_C = \underline{\text{Re}} \left(\frac{-jM^3 (w_S + \Delta w)(w_R - \Delta w) e^{-j\omega t} v_S v_R^*}{(C - jD)(A + jB)} \right)$$

The real part of a complex number and its conjugate are the same. Hence, taking the conjugate:

$$2T_C = \underline{\text{Re}} \left[\frac{jM^3 (w_S + \Delta w)(w_R - \Delta w) e^{j\omega t} v_S^* v_R}{(C + jD)(A - jB)} \right]$$

which by comparison with equn. 3.24, shows that

$$T_C = -T_2 \quad \text{A1.4}$$

The fourth component

$$2T_D = \underline{\text{Re}} \left(\frac{e_{RS2}^* i_{R2}}{\omega_R} \right)$$

$$= \underline{\text{Re}} (-jM e^{j\omega t} i_{S2}^* i_{R1})$$

or, substituting for i_{S2}^* and i_{R2} (from equations 3.13 and 3.14)

$$2T_D = \underline{\text{Re}} \left(\frac{M^2 (\omega_S + \Delta\omega) \{R_S + j(\omega_S + \Delta\omega)L_S\} v_R^* v_R}{c^2 + D^2} \right)$$

and, as $v_R^* v_R = V_R^2$

$$2T_D = \underline{\text{Re}} \left(\frac{M^2 (\omega_S + \Delta\omega) \{R_S + j(\omega_S + \Delta\omega)L_S\} V_R^2}{c^2 + D^2} \right)$$

$$= \frac{M^2 V_R^2 R_S (\omega_S + \Delta\omega)}{c^2 + D^2} = -2T_4 \quad \text{A1.5}$$

The total torque

Eqns. A1.2 to A1.5 show that:

$$T_A = -T_1$$

$$T_B = -T_3$$

$$T_C = -T_2$$

$$T_D = -T_4$$

Thus (as the total torque in stator terms is $T_1 + T_2 + T_3 + T_4$ and the total torque in rotor terms is $T_A + T_B + T_C + T_D$) it can be concluded that the airgap torque, as evaluated in rotor terms, is equal and opposite to the airgap torque, as evaluated in stator terms. This agrees with Newtons law of action and reaction and demonstrates the truth of the equations relating currents and voltages, and the basic torque equation - as suggested in section 3.2.

APPENDIX 2

Computer Program for Machine Simulation

```

DOUBLYFEDMACHINETORQUES;
"BEGIN"
"INTEGER"Q,K,N,FS,FR,F1,F2,F3;
"REAL"VS,RR,RS,LR,LS,M,WS,WR,W1,W2,W3,DEL1,DEL2,DEL3,DELT,U;
"READ"VS,RR,RS,LR,LS,M,FS,F1,F2,F3,DEL1,DEL2,DEL3,Q,N,U;
WS:=6.28318*FS;
W1:=6.28318*F1;
W2:=6.28318*F2;
W3:=6.28318*F3;
"PRINT"VS,RR,RS,LR,LS,M,FS,F1,F2,F3,DEL1,DEL2,DEL3,Q,N,U,WS;
"FOR"WR:=W1"STEP"W2"UNTIL"W3"DO" "BEGIN"
"REAL""ARRAY"A,B,C,D,DW,T[1:2];
"PRINT"WR;
"FOR"K:=1"STEP"1"UNTIL"2"DO""BEGIN"
DW[K]:=0.001*(1-K);
A[K]:=RS*RR+WS*(WR-DW[K])*(M*M-LS*LR);
B[K]:=RS*(WR-DW[K])*LR+RR*WS*LS;
C[K]:=RS*RR+(WS+DW[K])*WR*(M*M-LS*LR);
D[K]:=RS*WR*LR+RR*(WS+DW[K])*LS;
"END";
"FOR"DELT:=DEL1"STEP"DEL2"UNTIL"DEL3"DO""BEGIN"
"REAL"VR,CO,SI;
CO:=COS(DELT);
SI:=SIN(DELT);
"IF"N=1 "THEN"
    VR:=VS*(B[1]*RR-A[1]*LR*WR)/(M*WS*(A[1]*CO-B[1]*SI))
"ELSE""IF"N=2"THEN"
    VR:=VS*M*WR*(A[1]*CO+B[1]*SI)/(B[1]*RS-A[1]*LS*WS)
"ELSE"VR:=U*(WR+6.0);
"FOR"K:=1"STEP"1"UNTIL"2"DO""BEGIN"
T[K]:=(M*M*RR*VS*VS*(WR-DW[K]))/(A[K]*A[K]+B[K]*B[K])
-(M*M*RS*VR*VR*(WS+DW[K]))/(C[K]*C[K]+D[K]*D[K])
+M*VS*VR*((WS+DW[K])*(WR-DW[K])*(M*M-LS*LR)-RS*RR)*
((A[K]*D[K]-B[K]*C[K])*CO+(A[K]*C[K]+B[K]*D[K])*SI)
+(RR*(WS+DW[K])*LS-RS*(WR-DW[K])*LR)*
((A[K]*C[K]+B[K]*D[K])*CO-(A[K]*D[K]-B[K]*C[K])*SI)
/((A[K]*A[K]+B[K]*B[K])*(C[K]*C[K]+D[K]*D[K]));
"END";
"IF"Q=1"THEN""PRINT" VR,SAMELINE,T[1],T[2],(T[2]-T[1]) "ELSE"
"IF"(T[2]-T[1])<0"THEN""PRINT"SAMELINE,'U'
"ELSE""PRINT"SAMELINE,'S' ;

"END";
"END";
"END";

```

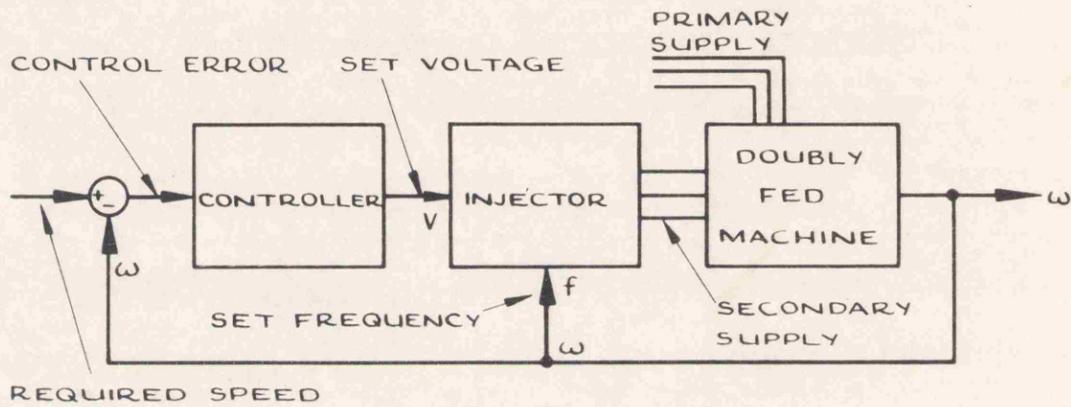


FIG.1. USUAL CONTROL SYSTEM ASSOCIATED WITH DOUBLY FED MACHINES

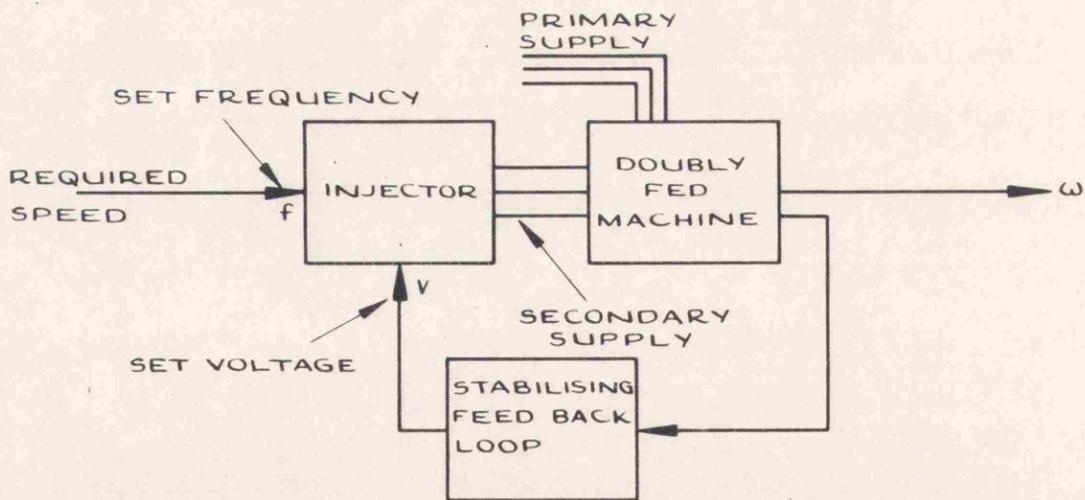


FIG.2. A PRECISE CONTROL SYSTEM WITHOUT CONTROL ERROR

APPENDIX 3Special cases of the doubly fed machine

All, non salient, electrical machines are, in one form or another, special cases of the doubly fed machine. These fall into two classes: speed variable machines and fixed speed machines.

Speed Variable Machines

All speed variable machines and machine systems contain two basic components: the doubly fed machine and the injector or variable frequency supply. They use a closed loop control on the secondary power-frequency-injection device to maintain a secondary frequency compatible with the speed (see fig.A.1). In simple systems (universal commutator motors and schrage motors) this is achieved by making the injector in the form of a commutator and mounting it on the same shaft as the rotor. Other systems (such as the Scherbuis system and all previous systems using semiconductors) use an injection device which is external to the machine but which is designed so that it "generates" power at a frequency dictated by the machine. Speed control in all cases is achieved by regulating the amount of energy being fed in or drawn out of the secondary windings (this usually is achieved by voltage control). Thus these machines all have the doubly fed machine as a component, this component generally being just the windings and magnetic circuit of the machine. The doubly fed machine dictates its secondary frequency to its injector (upon which it also depends for control) and so depends on its injector for stability. Machines in which a commutator is used operate at a fixed load angle (as the commutator is mechanically connected to the windings). Consequently ' $\Delta\omega$ ' is always zero and the machine cannot become unstable. Load, and speed, variations

are made by way of the voltage-torque (or voltage-speed) relationships of Chapter 3. Where commutators are not used the injection system must be designed in such a way that the load angle remains constant. Thus overall stability is maintained by the injection system. Torque and speed variations are achieved by the same means as in commutator machines.

Figure A.2 shows a block diagram for the doubly-fed machine system as discussed in this work. This contrasts with figure A.1 in that the machine can no longer control the injector frequency, which results in varying load angles. The machine, rather than the injector, is now relied upon for overall stability of the system.

Fixed Speed Machines

This category includes all machines which roughly have a fixed speed for a fixed primary frequency and machines which rely on (apparent) changes in pole numbers to vary their speeds. Three examples of fixed speed machines will be discussed. These are: the transformer, the induction motor and the 'synchronous' motor.

(1) The transformer can be considered as a doubly-fed machine which is rotating at zero speed. The 'rotor' and 'stator' frequencies are the same and power may be transmitted from the 'stator' to the 'rotor' as the windings of these are each 'moving' relative to the magnetic field (as is always the case in doubly-fed machines). It is this 'transformer effect' which causes the necessity to study the ratio of rotor to stator voltage in the doubly-fed machine to prevent large circulating currents.

(2) The induction motor, whilst not being a true fixed speed machine, is most easily studied in this section. It is a special case of the doubly-fed machine for the condition of one of the supply voltages being reduced to zero. The machine therefore operates under conditions of zero V_R (say), ~~and constant ϕ_R~~ . Load (and speed) variations are achieved by the voltage-torque relationship of Chapter 3. This relationship can be reduced to the classical one for an induction motor as shown below.

Of the four torque components only one remains, this being the one which does not involve V_R in the numerator:

$$\text{Thus } T = \frac{V_S^2 R_R M^2 (w_R - \Delta w)}{(A^2 + B^2)} \quad \text{Nw.m/phase}$$

(V_S is the R.M.S. voltage here)

$$\therefore T = \frac{V_S^2 R_R M^2 (w_S - w)}{(A^2 + B^2)}$$

$$= \frac{V_S^2 R_R M^2 (w_S - w)}{[R_S R_R - L_S L_R (w_S - w) + M^2 (w_S - w) w_S]^2 + [R_R L_S w_S + R_S L_R (w_S - w)]^2}$$

$$\text{putting slip } = s = \frac{w_S - w}{w_S} \quad \text{or } (w_S - w) = s w_S$$

$$T = \frac{V_S^2 R_R M^2 s w_S}{[R_S R_R - L_S L_R s w_S^2 + M^2 s w_S^2]^2 + [R_R L_S w_S + R_S L_R s w_S]^2}$$

$$= \frac{V_S^2 R_R}{s w_S} \frac{M^2}{\left(\frac{R_R}{s L_S} + R_S L_R \right)^2 + \left(\frac{R_S R_R}{s w_S} - (L_S L_R - M^2) w_S \right)^2}$$

putting $L_S = M + L_1$ $L_R = M + L_2$ (see fig.3.4 for definition
of L_1 and L_2)

and then assuming $L_S \approx M$

$$L_R \approx M$$

$$\frac{R_S R_R}{S w_S} \ll (L_S L_R - M^2) w_S^2$$

$$\begin{aligned} \text{Then } T &\approx \frac{V_S^2 R_R}{S w_S} \frac{M^2}{(R_{R/S} + R_S)^2 M^2 + (L_S L_R - M^2)^2 w_S^2} \\ &\approx \frac{V_S^2 R_R}{S w_S} \frac{M^2}{(R_{R/S} + R_S)^2 M^2 + (L_1 L_2 + L_1 M + L_2 M)^2 w_S^2} \end{aligned}$$

using $(L_1 + L_2) w_S = X_e$

$$\therefore [L_1 L_2 + (L_1 + L_2) M]^2 w_S^2 = [L_1 L_2 w_S + X_e M]^2 \approx X_e^2 M^2$$

$$\text{Then } T \approx \frac{V_S^2 R_R}{S w_S} \frac{1}{(R_{R/S} + R_S)^2 + X_e^2} \quad Nw \text{ m/phase-pole pair}$$

which is the classical induction motor equation for torque.

(3) The 'synchronous' machine is a special case of the doubly fed machine for the condition of zero rotor frequency. Torque variations result from variation in load angle or voltage and the stator power factor can be adjusted by the stator to rotor voltage ratio. The classical expression for torque in a synchronous machine (along with a term for damping due to the rotor field) can be derived from the equations for torque in the doubly fed machine as follows. The torque equation (for $w_R=0$ and $\Delta w=0$) reduces

to two components:

$$T = \frac{-V_R^2 R_S M^2 w_S}{C^2 + D^2} - \frac{M V_S V_R}{(A^2 + B^2)(C^2 + D^2)} \left\{ \begin{aligned} &[-R_R L_S w_S (AC + BD) \cos \delta] \\ &+ [R_S R_R (AC + BD) \sin \delta] \end{aligned} \right\}$$

$$= T_i + T_S$$

(V_S and V_R are R.M.S. voltage here)

Thus:

$$T_i = \frac{-V_R^2 R_S M^2 w_S}{C^2 + D^2}$$

$$= \frac{-V_R^2 R_S M^2 w_S}{R_S^2 R_R^2 + R_R^2 L_S^2 w_S^2}$$

$$= \frac{-I_R^2 R_S M^2 w_S}{R_S^2 + L_S^2 w_S^2} \quad (\text{as } I_R = V_R / R_R \text{ for D.C.})$$

$$= \frac{-I_R^2 R_S M^2 w_S}{R_S^2 + X_S^2} \quad (\text{as } L_S w_S = X_S)$$

$$= \frac{-E_S^2 R_S}{w_S (R_S^2 + X_S^2)} \quad \text{Nw.m./phase-pole pair} \quad \text{A3.1}$$

$$\text{also } T_S = \frac{-M V_S V_R}{(A^2 + B^2)(C^2 + D^2)} \left\{ \begin{aligned} &[-R_R L_S (w_S) (AC + BD) \cos \delta] + [R_S R_R (AC + BD) \sin \delta] \end{aligned} \right\}$$

$$= \frac{-M V_S V_R}{R_S^2 R_R^2 + R_R^2 L_S^2 w_S^2} \left\{ \begin{aligned} &[-R_R L_S w_S \cos \delta] + [R_S R_R \sin \delta] \end{aligned} \right\}$$

$$= \frac{-M V_S V_R}{R_R^2 (R_S^2 + X_S^2)} \left\{ \begin{aligned} &-R_R X_S \cos \delta + R_S R_R \sin \delta \end{aligned} \right\} \quad (\text{as } L_S w_S = X_S)$$

$$= \frac{-M V_S V_R R_R}{R_R^2 (R_S^2 + X_S^2)} (R_S - jX_S) \sin\delta \quad (\text{as } \cos\delta = j\sin\delta \text{ by convention})$$

$$= \frac{-M V_S V_R}{R_R (R_S + jX_S)} \sin\delta$$

$$= \frac{-M V_S V_R}{R_R Z_S} \sin\delta \quad (\text{as } Z_S = R_S + jX_S)$$

$$= \frac{-M I_{S1} V_R}{R_R} \sin\delta = -M I_{S1} I_R \sin\delta$$

and using $E_S = j\omega_S M I_R$

and $I_{S1} \approx V_S / jX_S$

$$\text{Then } T_S = \frac{-V_S E_S \sin\delta}{j^2 X_S \omega_S} = \frac{V_S E_S \sin\delta}{X_S \omega_S} \quad \text{Nw.m./ phase-pole pair} \quad \text{A3.2}$$

combining A3.1 and A3.2 we get

$$T = T_S + T_i = \frac{V_S E_S \sin\delta}{X_S \omega_S} - \frac{E_S^2 R_S}{\omega_S (R_S^2 + X_S^2)}$$

The first term here is the classical equation for synchronous machine torque whilst the second is the classical equation for induction motor torque with zero supply frequency (rotor supplied) and explains the instable tendencies in synchronous machines (it is torque due to stator conductors cutting the rotor field). The expression for $\phi_S = 0$ (min stator current) reduces to the expected expression for the case of the synchronous machine:

For the case of $w_R = 0$

$$V_S B R_R = V_R M w_S (A \cos \delta - B \sin \delta) \text{ for minimum } I_S$$

When substituting for 'A' and 'B' this expression becomes

$$V_S R_R^2 w_S L_S = V_R M w_S (R_S R_R \cos \delta - R_R w_S L_S \sin \delta)$$

$$\text{or } V_S = \frac{V_R M w_S}{R_R} \left(\frac{R_S}{w_S L_S} \cos \delta - \sin \delta \right)$$

$$\text{or } V_S \approx \frac{-V_R M w_S}{R_R} \sin \delta \quad (\text{assuming } R_S \ll w_S L_S)$$

$$\text{But } E_S = j M w_S I_R = j M w_S V_R / R_R$$

$$\therefore -j E_S = \frac{M w_S V_R}{R_R}$$

$$\text{Which gives } V_S = j E_S \sin \delta$$

$$\text{or } V_S = E_S \cos \delta \quad \text{A3.3}$$

for minimum stator current (or $\phi_S = 0$)

It is easy to show, by sketching a right angled triangle to illustrate equation A3.3, that this is, indeed, the case for stator current in phase with stator voltage (or minimum stator current - as the third side of the triangle is $I_S X_S$)

Thus it has been shown that many electrical machines are just special cases of the doubly-fed-machine. Other examples could be used (such as the double-speed machine) but those which have been used demonstrate quite clearly that the doubly-fed machine is merely a generalisation of all non-salient pole machines.

APPENDIX 4Practical difficulties and precautions

In any series of experiments there arise several practical problems which must be overcome to achieve a successful outcome to the experiments. The tests described in chapter four were no exception and below are listed some of the more significant problems (or apparent problems) and how they were overcome.

Metering

There were two problems here:

(1) Measurements of r.m.s. values of currents and voltages and measurements of power had to be made over a wide range of currents and, for the rotor supply, voltages and frequencies. To measure r.m.s. voltages and currents, moving iron and dynamometer type meters were used throughout. The stator frequency remained constant at 50Hz so transformers could be used in conjunction with the stator ammeter and voltmeter permitting wide ranges of currents and voltages to be measured. The rotor meters, however, could not employ transformers and other arrangements had to be made. The rotor ammeter had a 1 amp movement, and to permit readings of higher currents to be taken, a 5 amp and a 10 amp shunt were made (and calibrated at 50Hz against the stator ammeter). The rotor voltmeter possessed two ranges but proved inaccurate at low voltages (and frequencies). So rotor voltage measurements were made using an unearthened cathode ray oscilloscope. Power was measured in each of the stator and rotor circuits by the "two wattmeter" method. A shunt was attached to each wattmeter

enabling it to take 5 amp and various power ranges were achieved by use of the voltage range switch on the wattmeter.

At low rotor frequencies the rotor ammeter and wattmeter needles followed the rotor supply waveform in a semi sinusoidal manner. This prevented the taking of readings with a rotor frequency below 1Hz.

(2) One set of meters was provided for each of the rotor and stator supply circuits and three phase measurements were made by use of a plugboard arrangement. The impedance of the ammeters and the wattmeter current coils would cause an impedance unbalance in each of the two circuits (as they would only be inserted into one of the three lines at any one time). This unbalance would move with the meters and cause erroneous readings to be taken. To overcome this the impedance of the current carrying meter coils had to be reduced as much as possible. Meters with a current loading of less than 1 volt amp at f.s.d. were used throughout. All meters except the stator ammeter were used with current shunts - which were retained even when reading low currents to reduce the loading effect further. The stator ammeter had a loading of .5 volt amp at f.s.d. for all ranges (the current transformer being part of the meter). For ranges of 10 amp f.s.d. this does not represent a significant voltage drop but for ranges of .5 amp it is significant. This ammeter was therefore used on as high a range setting as was sensible and (to reduce the stator meter loading further) a shorting changeover switch was fitted to the ammeter and the wattmeter current coils. The rotor meters presented a negligible unbalance to the circuit but, at low frequencies - where the rotor voltage was low

compared to the current, the cables connecting them to the plug-board introduced a significantly high impedance. These cables were, therefore, made short and heavy.

With these precautions, readings could be taken with an accuracy within that required by the experiment.

Protection

Protection against short circuit currents was already provided by the various fuses and magnetic trips in the supplies. Short circuit currents would be very large and would require an immediate brake in the circuits to prevent damage. There was a second class of fault, however, which caused excessive currents to flow but these currents were not as great as short circuit currents would be and would only cause damage (due to overheating) after a time. This fault will be referred to as 'induction-motoring'. Induction-motoring occurred whenever the machine was desynchronised or in the event of failure of one of the supplies. At certain speeds an induction motor draws many times the current of a properly synchronised doubly fed machine and it was these currents that the machine was to be protected against. This protection could not be provided by merely down-rating the supply fuses as similar currents flowed, during oscillations or when the rotor voltage was far from optimum, and time had to be allowed for these currents to be reduced by other means before disconnecting the supplies. Thus a thermal trip, set to brake circuit only if the machine was in danger of overheating, would be suitable. Both supply circuits had to be tripped simultaneously as the tripping of one circuit only would cause further induction-motoring. Thus the protection

system was comprised of:

- (1) a thermal overload trip connected in supply 'a'
- (2) a contactor on supply 'a' which drew its holding current from supply 'a' via the overload trip,
- and (3) a contactor on supply 'b' which drew its holding current from the same point as (2) above.

Thus, in the event of persistent overcurrents in supply 'a', or in the event of failure of supply 'a', both the doubly fed machine supplies would be disconnected. In practice 'supply a' above was the stator supply and 'supply b' was the rotor supply. Only one such protection system was necessary to provide full protection as can be seen by examining the mechanisms of the causes of 'induction-motoring', which were:

- (1) Rotor supply failure - here the rotor circuit required no further protection and the stator circuit was protected by the overload trip.

- (2) Desynchronisation, persistent oscillations or prolonged rotor voltage discrepancies - here high currents would flow in both circuits and would be sensed by an overload trip in either circuit.

- (3) Stator supply failure - here both contactors would open due to lack of holding voltage.

Trip buttons were provided in case it was thought that oscillations of the machine might be mechanically detrimental to the machine (or in case of any other failure) and the rotor contactor was designed to short circuit the rotor windings when open to permit the machine to be used as an induction motor.

Starting the machine

It is commonly known that the double speed doubly fed machine is not self starting and has to be run-up on the load machine and synchronised in the manner described in section 4.2. This is not the case with the variable speed doubly fed machine as it can be run up to a speed just below fundamental as an induction motor (all connections having been made but the rotor voltage being zero) and synchronised at (or about) this speed by merely raising the rotor voltage. When automatic rotor voltage optimisation was used this synchronising was merely a matter of energising the rotor voltage control system and utilising the property of the system to raise the rotor voltage for the duration of an inductive power factor in the stator circuit. After synchronising the machine speed may be set to the desired value.

Generation of the rotor frequency

The method of producing the rotor supply has already been described (chapter 4) but the reasons for the choice of this method have not been given.

It was stipulated at the outset of the project that the doubly fed machine should not be able to affect the frequency of either of its supplies (either by feedback or by loading the supply plant). This stipulation was made so that the problem of stability of the doubly fed machine itself could be studied.

The rotor supply could not be produced by a static convertor for practical reasons so some form of rotating generator or convertor was required. The ideal source would

be such that currents in the windings of this source could not produce any torque which would tend to alter its speed. Thus the doubly fed machine rotor power would have to be taken from the 50Hz mains and the frequency of the voltage waveform would have to be changed by a rotating convertor. The ideal convertor consisted of a rotor with a set of slip rings, a commutator and some inductive winding connecting the two. This is what was used with the 50Hz supply connected to the sliprings (to prevent there being a possibility of d.d. in the convertor windings) and a voltage regulator on the 50Hz side of the convertor. The speed of rotation of the convertor would be related to the output frequency and, with judicious connection of the supplies, this speed could be made to correspond to the synchronous speed of the doubly fed machine*. This arrangement was used, with the convertor driven by a variable speed universal commutator motor, in the experimental work on the doubly fed machine.

It has been said that a static frequency convertor was not used in the rotor supply for practical reasons (none were available). It is realised, however, that a static convertor would be the ideal rotor supply as such a convertor offers a higher degree of controllability (as regards voltage optimisation) and a shorter time constant than an electro-mechanical system. The doubly fed machine is also the ideal load for the static convertor (providing stability is assured and some form of current limit is fitted) as the power handled by the converting plant is only a fraction of the total power output of the machine.

*here 'synchronous speed' means the speed of synchronous operation of the doubly fed machine and is not necessarily $\frac{3000}{n}$ rev/m (which is its fundamental speed).

The torsional vibration transducer

The fitting of a torsional vibration transducer resulted from an attempt to stabilize the machine by the method of Albertson². A series of drag-cup accelerometers (as described by Albertson) were tried and, though they produced an output proportional to acceleration at low speeds, they became subject to axial and transpositional vibrations (as well as torsional ones) when attached to the machine. The output signal also became swamped by noise which could not be filtered out with the variable speed machine.

A precision d.c. tachogenerator was then fitted and its output differentiated. This produced a similar low signal to noise ratio.

Finally a transducer employing a seismic mass was used and proved successful. This is described below. The transducer was a "torsional vibration transducer (type G318-A)" made by Southern Instruments Limited for use with their "frequency modulation" system. A permanent magnet formed the seismic mass, which was mounted on bearings (axial with the machine shaft). This magnet was enclosed by a copper cylinder surrounded by a steel casing, the magnetic forces between this combination and the magnet itself provided the elastic coupling and the damping (about 60% critical) in the transducer. The position of the magnet (or seismic mass) relative to the casing was measured by the changing inductance of a coil which resulted when a probe (attached to the seismic mass) was moved in or out of it. The entire transducer was attached to the end of the machine shaft so that movements between the casing of the transducer and the

seismic mass would provide a measure of the acceleration or the variations in load angle of the machine. This transducer was used in conjunction with an oscillator and a detector-amplifier unit (as prescribed by the manufacturers) to produce a d.c. voltage level proportional to the relative position of the seismic mass to the transducer casing (or the machine shaft). Other parameters of this transducer system were:

Natural frequency	5Hz
Frequency range	Flat <u>+10%</u> from 5Hz upwards
Amplitude range	Flat 4° peak to peak (end stops set somewhat further apart)

The oscillations of the test machine were of a frequency considerably less than 5 Hz. But this appeared to present no difficulties. The signal to noise ratio of the transducer output was much lower than that produced by previous transducers and the device could be used, in conjunction with a d.c. meter, or an ultra-violet recorder, to measure oscillations of the machine shaft. Traces from this transducer are shown in all the dynamic recordings discussed in sections 4.3 and 4.4 where they are labelled (to some extent incorrectly) 'load angle' or 'acceleration'.

One disappointing result shown by the production of these torsional vibration traces was that, in the event of instability, the machine did not oscillate sufficiently before desynchronising for Albertson's method² of stabilization to be used.

The remaining practical problems were of a trivial nature and are not considered to be of interest to the reader.

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References

1. Prescott, J.C. and Raju, B.P.: "The Inherent Instability of Induction Motors under Conditions of Double Supply" Proc. I.E.E. Vol. 105C, Aug. 1958, pp.319-29.
2. Albertson, V.D.: "Analysis and Stabilization of the Doubly-Fed Wound Rotor Polyphase Machine", Ph.D. Thesis, University of Wisconsin 1962.
3. Schmitz, N.L. and Albertson, V.D.: "Analysis and Dynamic Behaviour of the Doubly-Fed Synchronous-Induction Machine", I.E.E.E. Trans. on Power Apparatus and Systems Vol. 82, 1964, (Special supplement), pp.155-68.
4. Schmitz, N.L. and Albertson, V.D.: "The Stabilized Doubly-Fed Synchronous-Induction Machine", *ibid.*, Vol. 83, 1964, pp.858-64
5. Bird, B.H. and Burbidge, R.F.: "Analysis of Doubly-Fed Slip-Ring Machines", Proc. I.E.E., Vol. 113, June 1966, pp.1016-20.
6. Bird, B.H., McCloy, K. and Chalmers, B.J.: "New Methods of Stabilizing Doubly-Fed Slip-Ring Machines", *ibid.* Vol. 114, June 1967, pp.791-96.
7. Chalmers, B.J.: "A Stabilized Doubly-Fed Synchronous Machine", Ph.D. Thesis, University of Bristol 1960.
8. Concordia, C., Gary, S.B. and Kron, G.: "The Doubly-Fed Machine", Trans. A.I.E.E., Vol. 61, May 1942, p.286.
9. Alger, P.L.: "The Nature of Induction Machines", Gordon and Breach 1965, pp.393-425.

10. Lyon, W.V. and Edgerton, H.E.: "Transient Torque-Angle Characteristics of Synchronous Machines", Trans. A.I.E.E. Vol. 49, April 1930, p.686.
11. MacFarlane, A.G.J.: "Engineering Systems Analysis", Harrap 1964, pp.207-9.