Gamma-ray bursts, supernova kicks, and gravitational radiation

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ABSTRACT

We suggest that the collapsing core of a massive rotating star may fragment to produce two or more compact objects. Their coalescence under gravitational radiation gives the resulting black hole or neutron star a significant kick velocity, which may explain those observed in pulsars. A gamma–ray burst can result only when this kick is small. Thus only a small fraction of core–collapse supernovae produce gamma–ray bursts. The burst may be delayed significantly (hours – days) after the supernova, as suggested by recent observations. If this picture is correct, core–collapse supernovae should be significant sources of gravitational radiation with a chirp signal similar to a coalescing binary.

Subject headings: accretion, accretion discs — binaries: close stars: evolution — stars: neutron — gamma rays: bursts — supernovae — gravitational radiation.

1. Introduction

It is now widely believed that long ($\gtrsim 5$ s) gamma–ray bursts are produced by a class of supernovae, known as collapsars or hypernovae (Woosley 1993; MacFadyen & Woosley, 1999; Paczyński, 1998). The collapse of the core of a massive star is assumed to lead to the formation of a black hole, the remaining core material having enough angular momentum to form a massive accreting torus around it. The gravitational energy of the torus is radiated as neutrinos or converted to a beamed outflow by MHD processes. An evacuated channel forms along the rotation axis of the core, allowing the expulsion of matter with high Lorentz factors.

Direct evidence for the association of SNe and GRBs comes from the detection of bumps in the afterglow of several gamma–ray bursts (e.g. Price et al. 2002 and references therein) and the recent detection of SN ejecta in the X–ray afterglow of GRB 011211 by Reeves et al. (2002). However the hypernova class is extremely small: even allowing for the probable beaming of gamma–ray bursts (Frail et al. 2001), the fraction of HNe among SNe cannot be greater than about 10^{-3}. Evidently the production of a gamma–ray burst by a supernova is a very rare event. What causes this rarity is unclear.

The X–ray observation of a SN–GRB association by Reeves et al. (2002) throws up a further puzzle. Light travel arguments give a size $10^{14} - 10^{15}$ cm for the reprocessing region

producing the X–ray spectrum, depending on the beaming. This is much larger than the radius of the progenitor star, and must be associated with the supernova outflow. Indeed the measured blueshift of the spectrum with respect to the known GRB redshift implies an outflow velocity $\sim 0.1c$. But these two measurements together require that the GRB occurred between 10 hr and 4 d after the supernova. This is clearly incompatible with the simplest version of how a hypernova proceeds.

In this paper we offer a solution to both problems. We reconsider the collapse of a rotating core and suggest by analogy with simulations of star formation that this may produce two or more compact objects. The subsequent coalescence of these objects can power a gamma–ray burst, accounting for the SN–GRB delay. The merger itself will generally give the black hole resulting from the collapse a significant velocity ('kick'). This may be the explanation for the kicks observed in pulsars (Arzoumanian, Chernoff & Cordes, 2002). Following the suggestion of MacFadyen and Woosley (1999) that GRB production will be adversely affected by such kicks, we show that only a small fraction of core–collapse supernovae will produce gamma–ray bursts. These are likely to be a subset of those producing a massive black hole ($\gtrsim 12$ M$_\odot$).

2. Core collapse and fragmentation

It is well known that dynamical collapse of a self–gravitating gas cloud increases the importance of rotation. The ratio of kinetic to gravitational binding energy grows as $\sim 1/r$, where r is the lengthscale of the collapsing object. Many authors (see Bonnell & Pringle, 1995 and references therein) have suggested that this probably leads to fragmentation, seen for example in the collapse of molecular clouds to form pre–main–sequence stars. Fragmentation requires that the collapsing core becomes bar–unstable, and that any bar lives a few dynamical times. In core collapse to nuclear densities the second requirement is very likely to be met (Bonnell & Pringle, 1995) while the first depends on the equation of state and the initial conditions. Thus determining the precise conditions under which fragmentation occurs requires large–scale numerical simulations, which are under way. For the remainder of this paper we consider the case where two compact objects form with masses and radii M_1, M_2 and R_1, R_2, with $M_1 > M_2$. The minimum mass of these objects is set by the requirement that a nucleon fluid exists in their cores. In Fig 1 we plot R_2 as a function of M_2 for the equation of state (EOS) of Shen et al. (1998a,b). From Figure 1 we see that we require a mass $\gtrsim 0.2$ M$_\odot$ – at lower masses nuclei form even in the center of the stars and the object has a much larger radius. A similar low–mass limit is obtained for other equations of state.
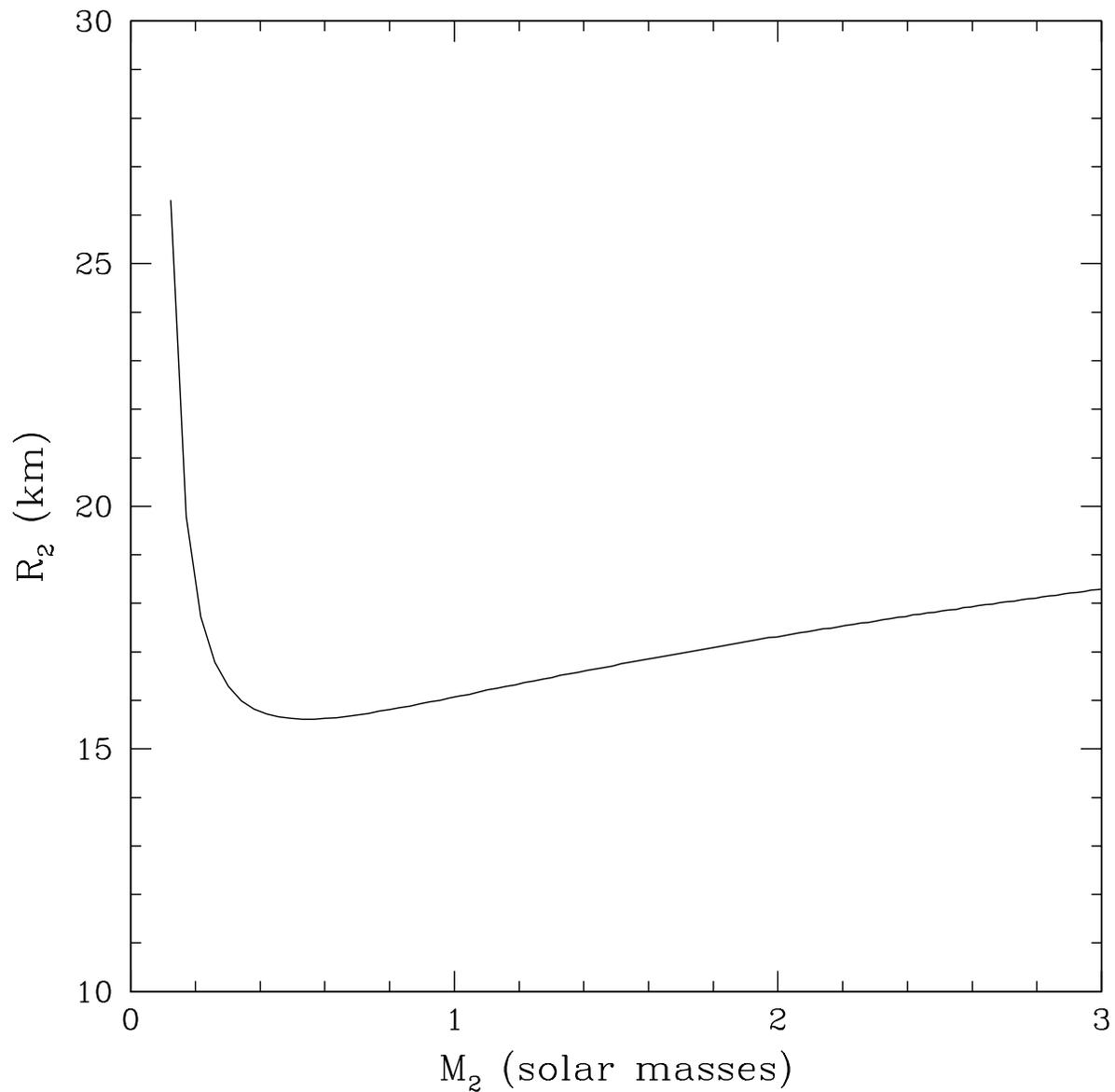
Fig. 1.— Mass–radius relation for cold neutron stars in beta-equilibrium using the EOS of Shen et al. (1998). The end point on the low mass side is reached once nuclei start to form in the center of the star.

3.   Mergers and gamma–ray bursts

To have the potential of powering a gamma–ray burst, the merger of the two orbiting lumps must produce a central object surrounded by a torus. This will happen if at least one of the lumps is a neutron star rather than a black hole, and mass transfer eventually becomes dynamically unstable. For a corotating object less massive than the accretor and filling its Roche lobe, of radius (Paczyński 1971)

$$R_{\rm L} = 0.462\Big(\frac{M_2}{M_1 + M_2}\Big)^{1/3} a \tag{1}$$

(where a is the separation) this requires that $R_{\rm L}$ moves inwards with respect to its radius R_2. The standard result (e.g. van Teeseling & King, 1998) is

$$\frac{\dot{R}_{\rm L}}{R_{\rm L}} - \frac{\dot{R}_2}{R_2} = -\frac{2\dot{M}_2}{M_2}\Big(\frac{5}{6} + \frac{\zeta}{2} - \frac{M_2}{M_1}\Big) + 2\frac{\dot{J}}{J}, \tag{2}$$

where the mass–radius relation is taken as $R_2 \propto M_2^\zeta$, and $\dot{J}$ includes all forms of orbital angular momentum loss. This expression shows that dynamical instability must occur if $\zeta < -5/3$, since $\dot{M}_2, \dot{J} < 0$. The mass–radius relation Fig. 1 now shows that the instability is inevitable for any lobe–filling object, since it will occur at the latest once its mass is reduced to $M_2 \simeq 0.2{\rm M}_\odot$ and the lump begins to expand rapidly on mass loss. The tidal lobe of a non–corotating object is similar in size, so again instability will occur for $M_2 \simeq 0.2{\rm M}_\odot$. Instability may well happen before this point for other reasons: for example, the orbit may be so close that the accreting matter cannot form a disc around the accretor, adding a dynamical–timescale term to $\dot{J}$. For sufficiently stiff equations of state Newtonian tidal effects can also lead to an instability on a dynamical time scale (Lai, Rasio and Shapiro 1993).

The only way that dynamical instability can be avoided is if (a) both lumps are already black holes, or (b) the accretor is a black hole, and the accreting object spirals within its horizon before filling its Roche (or more generally tidal) lobe. This occurs if $R_2 < R_{\rm L}$ for $a = \eta GM_1/c^2$. With $R_2 = 10^6 R_6$ cm, we find the condition

$$\frac{M_1^3 M_2}{M_1 + M_2} > \Big(\frac{7.2R_6}{\eta}\Big)^3. \tag{3}$$

Only the most massive black holes can swallow neutron stars whole. This is true for a wide range of neutron–star radii, including $R_6 = 1$. Most mergers result in dynamical instability of the neutron lump. Thus fragmentation and subsequent coalescence release enough energy to power a gamma–ray burst.

4. Mergers and kicks

Simulations of unequal–mass neutron star mergers show that the mass loss from the system is asymmetric. The escaping material originates from the lower–mass star and is ejected on a timescale shorter than the orbital period. This provides a thrust to the merged object, which is found to have a velocity $V_{\text{kick}} \sim 800$ km s^{-1} for the case $M_1 = 0.8$ M$_\odot$ and $M_2 = 0.7$ M$_\odot$ (Rosswog & Davies, in prep; but see also Rosswog et al, 2000).

In general we can assume that the ejected material, M_{lost} is ejected at a speed $\propto V_{2,\text{orb}}$ (where $V_{2,\text{orb}}$ is the orbital velocity of the primary, M_2) when the donor finally gets shredded. Combining expressions for $V_{2,\text{orb}}$ and the orbital separation a (assuming the donor fills its Roche lobe), and applying conservation of momentum, we obtain the following expression for the kick given to the merged object:

$$
\begin{aligned}
V_{\text{kick}} \quad \propto \quad & \left[\frac{GM_1^2}{(M_1 + M_2)R_2} \left(\frac{M_2}{M_1 + M_2} \right)^{1/3} \right]^{1/2} \\
& \times \frac{M_{\text{lost}}}{M_1 + M_2 - M_{\text{lost}}}
\end{aligned}
\tag{4}
$$

For systems where M_2 has been reduced to the mininum mass of ~ 0.2 M$_\odot$, with $M_1 \gg M_2$, we see that $V_{\text{kick}} \propto M^{-2/3}$ where $M = M_1 + M_2$. We use the result of Rosswog & Davies (as stated above), for $M = 1.5$ M$_\odot$, to write

$$
V_{\text{kick}} = 800 \left(\frac{1.5 M_\odot}{M} \right)^{2/3} \text{ km s}^{-1}
\tag{5}
$$

The kick may be slightly lower if the final shredding occurs before M_2 reaches 0.2 M$_\odot$. It should also be noted that the speed of the compact object at infinity will be reduced as it is decelerated by the gravitational force of the ejected material. Likely values of the speed of the merged object at infinity lie in the range $100 - 300$ km s^{-1}, although the merged object and the ejecta may be bound in some cases.

A remarkably similar kick occurs if both merging objects are black holes, because of the effect of gravitational radiation reaction on the final plunge orbit (Bekenstein, 1973; Fitchett, 1983; Fitchett & Detweiler, 1984). For rapidly spinning holes, as are likely in core collapse, the kick velocity may approach 1500 km s^{-1} (Fitchett, 1983). The basic reason for the similarity is that in both cases the recoil velocity is of order the primary's centre–of–mass–velocity immediately before the plunge phase.

5. Supernova kicks and GRBs

The torus surrounding the compact object releases its energy into the region along its rotation axis. As pointed out by MacFadyen & Woosley (1999), the production of a GRB will be inhibited if the volume into which the $\nu - \bar{\nu}$ or MHD energy is deposited is increased significantly by the motion of the central object and torus with respect to the surrounding gas. This motion may be the recoil described in the previous section, or it may be a kick derived from some other physical mechanism (see e.g. Lai 2001).

A potential GRB will be extinguished if $V_{\text{kick}} \gtrsim d/\tau_{\text{er}}$, where d is the lengthscale for energy deposition into a potential fireball, and τ_{er} is timescale for the energy to be released from the torus which is set by the viscous timescale of the torus. Assuming $\tau_{\text{er}} \sim 1$ s and taking $d = 15(M_1/M_\odot)$ km (see figures in Fishbone & Moncrief, 1976), a GRB will fail if:

$$V_{\text{kick}} \geq 15\left(\frac{M_1}{M_\odot}\right) \text{ km s}^{-1} \qquad (6)$$

The expression above is plotted in Fig 2 along with the likely kick received by the central object and torus (as given in equation 5; note here that for the interesting range of values of M, $M_2 \ll M_1$, hence $M_1 \simeq M$). This figure suggests that the recoil velocity is likely to extinguish any potential GRB when the total mass $M \lesssim 12$ $M_\odot$. In other words GRBs will be extinguished when V_{kick} exceeds some particular value, ~ 200 km s^{-1}. The exact limiting mass for a gamma–ray burst is uncertain, but the important point here is that gamma–ray bursts will only occur above some limiting mass.

6. The SN–GRB delay and gravitational wave emission

In the picture presented here, core collapse only produces a gamma–ray burst in a few cases. Even in these cases, the burst may not follow the collapse immediately, but may be delayed while gravitational radiation brings the orbiting fragments into contact. For an initial circular orbit of separation a_0, it is easy to show that this requires a time

$$\tau_{\text{gr}} = 0.18 \times \frac{(a_0/1000 \text{ km})^4}{m_1 m_2 (m_1 + m_2)} \text{ hours.} \qquad (7)$$

where $m_1 = M_1/$ $M_\odot$ etc. We see that a delay of hours is quite possible. Since $\tau_{\text{gr}} \propto a_0^4$, only a small increase in the value of a_0 (say to 3000 km), will produce a large (factor ~ 100) increase in the delay between formation of the two lumps and their coming into contact. Thus SN–GRB delays of the order inferred by Reeves et al in GRB 011211 are quite reasonable in this picture.

An obvious corollary of this is that our picture predicts that core–collapse supernovae
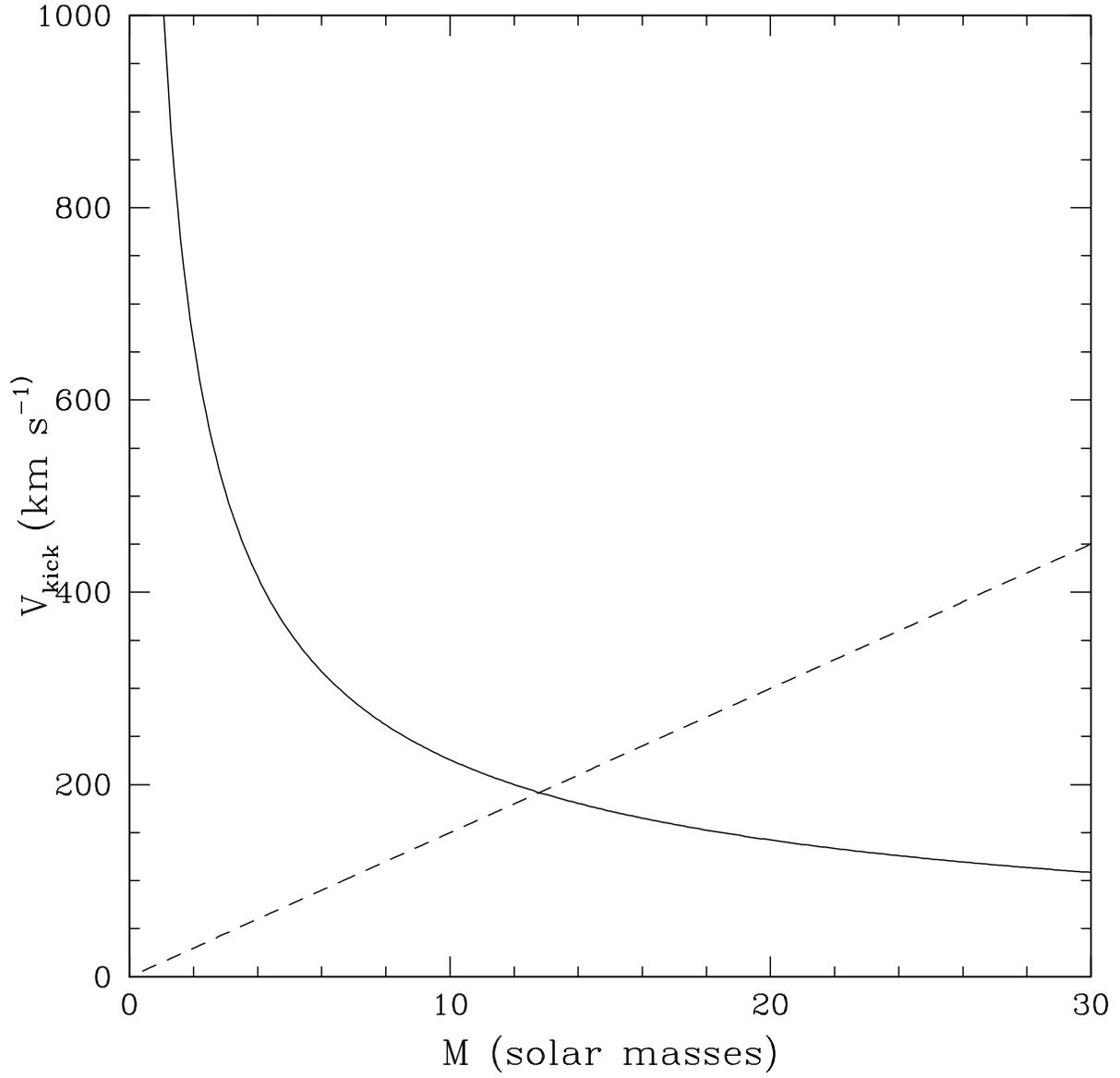
Fig. 2.— The kick received (in km s^{-1}) by the central object and torus (solid line) and the kick required to extinguish a GRB (dashed line). Both are plotted as a function of total mass, M (in solar masses).

should be strong sources of gravitational radiation (cf Bonnell & Pringle, 1995; van Putten, 2001; Fryer, Holz & Hughes, 2002). A neutron star merger should be detectable by LIGO out to 20 Mpc, and by LIGO II out to 300 Mpc. The gravitational wave signal strength for two point masses in circular orbits with a separation a is given by $h \propto \Omega^2 \mu a^2$ where $\Omega^2 = G(M_1 + M_2)/a^3$ and $\mu = M_1 M_2/(M_1 + M_2)$. Hence the detectability of a merger of two compact objects is a sensitive function of their masses. As an example, we consider here the merger of two half neutron stars (ie $M_1 = M_2 = 0.7$ $M_\odot$.) will be detectable to a distance of 6 Mpc with LIGO and 100 Mpc with LIGO II. We can derive a predicted event rate from an assumed event rate per galaxy (see Phinney 1991). Assuming a formation rate of 10^{-2} yr^{-1} per galaxy, LIGO II should see ~ 400 mergers per year. This is much larger than the number of neutron–star merger events per year LIGO II should detect (~ 10).

7. Conclusions

We have suggested by analogy with large–scale simulations of star formation (see e.g. Bate, Bonnell & Bromm, 2002, in prep, and http://www.ukaff.ac.uk/starcluster/) that core collapse of a massive rotating star may lead to fragmentation of nuclear–density lumps. The subsequent coalescence of these lumps under gravitational radiation gives the resulting black hole or neutron star a significant kick velocity, compatible with those observed in pulsars (see for example Arzoumanian, Chernoff, & Cordes, 2002). A gamma–ray burst can result only when this kick is small. Thus only a small fraction of core–collapse supernovae produce gamma–ray bursts. The most likely candidates are those containing massive black holes ($M_1 \gtrsim 12$ $M_\odot$) which have not formed via the merger of two lower–mass black holes. The burst may be delayed significantly (hours – days) after the supernova, as suggested by recent observations.

The complexity seen in star formation studies suggests that a large variety of behaviours is likely in core collapse. A gamma–ray burst appears to require a rather high degree of symmetry and alignment, and is therefore a rather unusual outcome. We note that in the case of a kick driven by mass expulsion in a double neutron–star merger, the expelled gas may have up to 10 times the energy of the kinetic energy of the merger product. Given likely initial neutron star kick velocities of ~ 1000 km s^{-1}, these energies may approach 10^{50} erg, and thus have noticeable effects on the early development of the supernova outburst.

A clear test of our picture will be given by gravitational wave experiments. An observed chirp signal where the *total* mass $\simeq 1.4$ $M_\odot$ would be easily explained in our model but practically impossible to explain via standard neutron–star mergers (as predicted from the observed binary pulars [Phinney, 1991]).

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