Title: Pollution Abatement as a Source of Stabilisation and Long-Run Growth

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Proposed Running Title: Pollution Abatement as a Source of Growth

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Abstract

In a two-period overlapping generations model with production, we consider the damaging impact of environmental degradation on health and, consequently, life expectancy. Despite the presence of social constant returns with respect to capital, which would otherwise generate unbounded growth, when pollution is left unabated, the economy cannot achieve such a path. Instead, it converges either to a stationary level of capital per worker or to a cycle in which capital per worker oscillates permanently. The government's involvement in environmental preservation proves crucial for both short-term dynamics and long-term prospects of the economy. Particularly, an active policy of pollution abatement emerges as an important engine of long-run economic growth. Furthermore, by eliminating the occurrence of limit cycles, pollution abatement is also a powerful source of stabilisation.

JEL classification: O44; Q56

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1 Introduction

In recent years, environmental issues have gained prominence in both academic and political discussions. At the same time, they have received considerable media attention. Problems such as the emission of greenhouse gases, the depletion of natural resources, and the presence of hazardous chemicals have become major issues of concern. This is of course not surprising, given their significant direct and indirect repercussions on our health and therefore our overall quality of life.

Naturally, economic growth has been an indispensable aspect of all the discussion in relation to environmental problems. Given that pollution is a by-product of human activities such as production and consumption, can economies sustain ever increasing levels of GDP per capita without reaching a tipping point that makes long-run growth environmentally unsustainable? From a different point of view, the preceding discussion has additional implications for the interactions between health indicators and economic growth/development. For example, what are the long-term economic prospects (in terms of capital accumulation) in an economy where environmental degradation has a negative effect on health and life expectancy? Can environmental policy alter these prospects, despite the fact that growth is still detrimental to environmental quality?

To analyse these issues, we build a two-period overlapping generations model in which pollution affects a person's prospects of survival to the next period[1] and labour productivity is enhanced by an aggregate learning-by-doing externality. Despite the fact that this type of externality is the source of social constant returns with respect to capital and could potentially allow for an equilibrium path with a positive growth rate in the long-run, when pollution is left unabated in our model, the economy cannot achieve such a path. Instead, it will either converge to a stationary level of capital per worker (possibly a poverty trap) or to a stable cycle in which capital per worker oscillates permanently. Nevertheless, when resources are devoted towards pollution abatement, the equilibrium outcomes change drastically. Specifically, by influencing longevity and saving behaviour, public policy (in the form of pollution abatement) can put the economy on a sustainable growth path. Economic growth is environmentally sustainable because a sufficient level of environmental quality is maintained. These outcomes occur even when economic growth has a net damaging effect on environmental quality – with or without environmental policy – and even though the quality of the environment is essential for supporting longevity and, therefore, saving and capital accumulation. Furthermore, by eliminating long-run cycles, environmental policy smoothens income, thereby becoming a source of stabilisation, albeit one whose scope and implications are quite different from the more conventional countercyclical policies designed against short-term fluctuations.

Our model shares similarities with other studies that have combined elements of environmental quality and health in dynamic economies. Gutiérrez (2008) examines how income taxation can restore dynamic efficiency in an economy where pollution affects the health profile of agents belonging to the old generation. In a similar model, in which agents save more in response to pollution-induced health risk, Wang *et al.* (2015) demonstrate that an economy may experience high capital accumulation and high pollution levels. They also show how the social optimum can be replicated by a combination of emission taxes on producers, public health spending and an appropriately designed private health insurance scheme. Balestra and Dottori (2012) examine the politico-economic implications of population aging in a set-up where the relative preference weight for environmental care versus direct health expenditures differs between the young and the old generations. Using a model that accounts for two conflicting health externalities from pollution, Jouvet *et al.* (2010) show that taxes on both income and private health expenditures are necessary to decentralise the social optimum. Mariani *et al.* (2010) show that when environmental quality leads to higher life expectancy, then the economy may admit multiple, path-dependent equilibria, whereas Varvarigos (2014) shows that such multiplicity also requires the combination of emission taxes with the endogenous choice of environmental technology by producers. Finally, in Pautrel (2008) the natural environment has an amenity value in addition to its beneficial effect on longevity. He shows that greener preferences can support a higher optimal growth rate despite the fact that they are associated with an optimal allocation of time that does not favour human capital accumulation.[2]

The aforementioned papers account for the two-way causal effects between economic activity and the environment. Our paper differs from these studies in two aspects. First, it gives emphasis to the endogenous cycles that arise with respect to output and environmental quality. Hence, it can account for the cyclical interactions between life expectancy and capital accumulation.[3] Second, the presence of endogenous long-run cycles, allows us to explore any scope for pollution abatement as a source not only of growth but also of stabilization.

Naturally, our paper is also related to models of capital accumulation and environmental quality that have identified the possibility of endogenous fluctuations, such as Zhang (1999), Ono (2003) and Seegmuller and Verchère (2004). In terms of set-up, the difference of our model in relation to these ones is threefold. Firstly, we consider the health effects of deteriorating environmental conditions. Secondly, their mechanisms of endogenous cycles differ from ours.[4] In our model, cycles may emerge because unbounded environmental degradation, and its impact on longevity, introduces non-monotonicity in the dynamics of capital accumulation. Thirdly, we analyze both process-integrated abatement technology, which reduces the economy's emission intensity (i.e., the emission-to-output ratio), and end-of-pipe abatement technology, which reduces the amount of already formed pollutants.

The link between policies of pollution abatement and economic growth is an important element of our results and implications. This link is also analyzed in Bovenberg and Smulders (1995). Particularly, the authors develop a two-sector representative-agent model with pollution-augmenting technical change and derive technical conditions under which sustainable growth is both feasible and optimal. They then explore optimal environmental policies. Closer to our setting is the analysis of Smulders and Gradus (1996). They use a one sector growth model in which pollution reduction has a direct benefit to the economy's productivity. They find that pollution abatement allows the economy to sustain growth in the long-run. Nevertheless, this is possible only when appropriate parameter restrictions allow abatement to grow at a faster rate compared to pollution. In our model, this type of environmental policy allows the economy to sustain long-run growth despite the fact that the environment is essential for survival and output growth may have a monotonically negative effect on environmental quality, irrespective of whether pollution is abated or not.[5]

The rest of the paper is organised as follows. In Section 2 we set up the economic model. In Section 3, we analyse the different equilibrium outcomes of the model, according to whether process-integrated pollution abatement is active or not. In Section 4, we extend our framework to introduce the case where environmental quality is a stock variable. In Section 5, we consider an alternative specification concerning the effect of pollution on health and life expectancy and analyze the effects of end-of-pipe abatement technology. Finally, in Section 6 we summarise our results and draw conclusions.

2 The Economic Framework

We construct an overlapping generations economy in which time, $t = 0, 1, 2, \ldots$, is measured in discrete intervals that represent periods. The economy is populated by an infinite sequence of agents who face a potential lifetime of two periods. In particular, an agent will live with certainty during the period following her birth, i.e., her youth, but she may or may not survive to her old age. We assume that, before her survival prospect is realised, each agent gives birth to an offspring. Thus, the prospect of untimely death does not have any repercussions on the population mass of newly-born agents, whose size we normalise to one.

During youth, each agent is endowed with one unit of labour. She supplies her labour to firms inelastically and receives the competitive wage, w_t. Even if she survives to maturity, nature does not bestow to her the ability to work when old. For this reason, and in order to satisfy her future consumption needs, she deposits an amount s_t, when young, to a financial intermediary that promises to repay it next period, augmented by the gross interest rate r_{t+1}.

As mentioned earlier, survival to maturity is not certain. Particularly, we assume that a young person will survive to maturity with probability $\beta_t \in [0,1)$, whereas with probability $1-\beta_t$ she dies prematurely. Furthermore, we assume that life expectancy is endogenous in the sense that the agent's survival prospect depends on her health characteristics (or health status), denoted as h_t, according to[6]

$$\beta_t = \mathrm{B}(h_t), \tag{1}$$

where, following Chakraborty (2004), we assume that $\mathrm{B}'(h_t) > 0$, $\mathrm{B}''(h_t) < 0$, $\mathrm{B}(0) = 0$, $\mathrm{B}(\infty) = \lambda$, $\lambda \in (0,1)$, $\mathrm{B}'(0) = \psi$, $\psi \in (0,1)$, and $\mathrm{B}'(\infty) = 0$.

We delve further into the determinants of life expectancy by assuming that an agent's health status depends positively on the extent to which the government supports the provision of health services g_t (e.g., public hospitals; the presence of a national health system; preventive measures; the design and implementation of health and safety rules, etc.) and on the quality of the natural environment e_t (e.g., the cleanliness of air, soil and water;

the relative abundance of natural resources such as forestry and other forms of plantation, etc.). Formally, these ideas are captured by[7]

$$h_t = g_t^{\varphi} e_t^{\chi}, \quad \varphi, \chi > 0. \tag{2}$$

The assumption that the arguments affecting health status are introduced through a Cobb-Douglas specification is not new (for example, see van Zon and Muysken 2001). Obviously, one main reason is the tractability associated with its form. However, this tractability does not come at the cost of intuitive reasoning; after all, the assumed form implies that the positive health impact of public spending is more pronounced in conducive natural environments.[8]

Agents maximize their *ex ante* (i.e., expected) lifetime utility function

$$V^t = \ln c_t + \beta_t \ln d_{t+1}, \tag{3}$$

where c_t and d_{t+1} denote the levels of consumption during youth and old age, respectively.

There is a single, perishable commodity. It is produced by perfectly competitive firms, who combine physical capital, K_t (which they rent from financial intermediaries at a price of R_t per unit), and labour, L_t, so as to produce output Y_t. The production function is

$$Y_t = \mathrm{A} K_t^{\gamma} (L_t \overline{K}_t)^{1-\gamma}, \ 0 < \gamma < 1, \tag{4}$$

where $\mathrm{A} > 0$ and $\overline{K}_t$, which denotes the economy's average amount of capital, captures an economy-wide, learning-by-doing externality, as in Romer (1986).

One unfortunate by-product from firms' activities is pollution. We assume that one unit of output generates $\widetilde{p}_t > 0$ units of pollutant emissions; therefore, total pollution is

$$P_t = \widetilde{p}_t Y_t. \tag{5}$$

The amount of pollution can be mitigated by government-funded activities that are designed and implemented so as to reduce the extent of environmental damage. For the purposes of our analysis, we shall refer to them as pollution abatement activities. Following the literature, we distinguish between two types of abatement technologies. The first type is *process-integrated* technology, which alters the production process in a more environmental-friendly way and hence reduces the emission-to-output ratio (or the emission intensity), $\widetilde{p}_t$. In other words, this type of abatement technology reduces the amount of contaminants, per unit of output, **before** they form. It can be represented by $\widetilde{p}_t = \widetilde{p}(a_t)$, where a_t denotes abatement activities that are of the process-integrated type and $\widetilde{p}' < 0$. An example of the functional form $\widetilde{p}$ that we use below is

$$\widetilde{p}_t = \frac{p}{1+a_t}, \; p > 0. \tag{6}$$[9]

For further details regarding this type of abatement technology see Requate (2005), Clemens and Pittel (2011), Endres and Friehe (2013), to name but a few.

The second type of abatement activities, which we analyze in Section 5, uses *end-of-pipe* technology. This technology does not modify the production process and hence does not change the emission-to-output ratio, $\widetilde{p}_t$. Instead, it is used **after** the production process to remove already formed contaminants. End-of-pipe abatement technology is analyzed in, among others, Diamantoudi and Sartzetakis (2006), Economides and Philippopoulos (2008) and Marianni *et al.* (2010).

Restricting the analysis for now only to process-integrated abatement, we write

$$P_t = \frac{pY_t}{1+a_t}. \tag{7}$$

Moreover, the quality of the natural environment, $e_t \geq 0$, depends on the extent of environmental degradation. We capture this idea through

$$e_t = \begin{cases} E - P_t & \text{if } P_t < E \\ 0 & \text{otherwise} \end{cases} , \tag{8}$$

where $E > 0$ allows us to consider an environmental tipping point. This is a situation that occurs when the environmental impact of human activities exceeds a critical threshold, after which the conditions for a meaningful existence of the Earth's species (including humans) are severely undermined. In our model, this occurs whenever $P_t \geq E$. The importance of such tipping points for various environmental indicators is now well established among scientists (see, for example, Barnosky *et al.* 2012) and it has been used in existing analyses examining the interactions between economic growth and the environment (e.g., Acemoglu *et al.* 2012).

As it is evident in (8), we abstract from the dynamics of environmental quality by assuming that e_t is a flow and not a stock variable. This choice has been dictated by the need for analytical tractability. As it will become clear later, even in its current form the model is very complicated and any analytical solutions that allow the reader to understand the intuition and the mechanisms involved are made possible only when environmental quality is a flow variable. Nevertheless, in order to show that our results survive under more general settings, in Section 4 we solve the model for the case where environmental quality is a stock that evolves according to $e_t = \eta e_{t-1} + (1-\eta)E - P_t$ $(0 < \eta < 1)$, a specification that follows the seminal work by John and Pecchenino (1994). As we shall see, our main results and their implications survive even under this more general setting. [10]

We complete our description of the economy's structure with a discussion on the process under which the government finances its activities. We utilise the widely used assumption that the government imposes a flat tax rate $\tau \in (0,1)$ on firms' revenue. Assuming that the government abides by a balanced budget rule in each period, our previous assumptions imply that $g_t + a_t = \tau Y_t$. If we denote the fixed fraction of revenue devoted towards abatement by $\upsilon \in [0,1)$, it is straightforward to establish that

$$g_t = (1-\upsilon)\tau Y_t, \tag{9}$$

and

$$a_t = \upsilon \tau Y_t, \tag{10}$$

give the levels of public health spending and abatement activities in relation to the economy's total output.

2.1 Temporary Equilibrium

We begin our analysis with a description of the economy's temporary equilibrium.

Definition 1. *The temporary equilibrium of the economy is a set of quantities* $\{c_t, d_t, d_{t+1}, s_t, L_t, Y_t, \beta_t, h_t, e_t, P_t, a_t, g_t, K_t, K_{t+1}\}$ *and prices* $\{w_t, \mathrm{R}_t, \mathrm{R}_{t+1}, r_{t+1}\}$ *such that:*

(i) *Given w_t, r_{t+1} and β_t, the quantities c_t, d_{t+1} and s_t solve the optimisation problem of an agent born at time t;*

(ii) *Given w_t and R_t, all firms choose quantities for L_t and K_t in order to maximise profits;*

(iii) *All markets clear.*

(iv) *The government budget is balanced.*

The objective of a young agent is to choose her intertemporal consumption profile so as to maximise V^t subject to $c_t = w_t - s_t$ and $d_{t+1} = r_{t+1}s_t$. Alternatively, given (3), the problem can be modified to $\max_{0 \le s_t \le 1}\{\ln(w_t - s_t) + \beta_t \ln(r_{t+1}s_t)\}$. The solution to this problem is

$$s_t = \frac{\beta_t}{1+\beta_t} w_t. \tag{11}$$

Naturally, the prospect of premature death modifies an agent's saving behaviour. An increase in longevity raises the marginal utility of an agent's consumption when old; to restore the equilibrium, the marginal utility derived from her first period consumption must increase as well. She can achieve this by choosing to save more and consume less while she is young.

Profit maximisation implies

$$w_t = (1-\tau)(1-\gamma)\mathrm{A}K_t^{\gamma}L_t^{-\gamma}\overline{K}_t^{1-\gamma}, \tag{12}$$

and

$$\mathrm{R}_t = (1-\tau)\gamma\mathrm{A}K_t^{\gamma-1}L_t^{1-\gamma}\overline{K}_t^{1-\gamma}. \tag{13}$$

In equilibrium, $L_t = 1$ and $k_t = K_t = \overline{K}_t$, where $k_t = K_t / L_t$ is capital per worker. Consequently, we can write (12) and (13) as

$$w_t = (1-\tau)(1-\gamma)\mathrm{A}k_t, \tag{14}$$

and

$$\mathrm{R}_t = (1-\tau)\gamma\mathrm{A} \equiv \hat{\mathrm{R}}. \tag{15}$$

There are two conditions that describe the financial market equilibrium. We assume that perfectly competitive financial intermediaries undertake the task of channelling capital from depositors to firms. Specifically, they transform saving deposits into capital by accessing a technology that transforms time-t output into time-$t+1$ capital on a one-to-one basis. They, subsequently, supply this capital to firms that manufacture the economy's single commodity. Hence, $K_{t+1} = L_t s_t$ or, in intensive form,

$$k_{t+1} = s_t. \tag{16}$$

To resolve the issue of saving under an uncertain lifetime, we assume, following Chakraborty (2004), that financial intermediaries represent mutual funds that offer contingent annuities. Specifically, when accepting deposits, intermediaries promise to offer retirement income (in our case, $r_{t+1}s_t$) provided that the depositor survives to old age.

Otherwise, the income of those who die is shared equally among surviving members of the mutual fund. Considering this assumption, and the fact that financial intermediaries operate under perfect competition, we have

$$\beta_t r_{t+1} = \mathrm{R}_{t+1} = \hat{\mathrm{R}} , \tag{17}$$

which translates into the equilibrium condition requiring costs to be equal to revenue.

Next, we substitute $L_t = 1$ in (4) to obtain output per worker $y_t = Y_t / L_t$:

$$y_t = \mathrm{A}k_t . \tag{18}$$

If we combine the expression in (18) together with (1), (2), (5), (6), (7), (8), (9) and (10), and substitute together with (11) and (14) in equation (16), we can eventually derive

$$k_{t+1} = (1-\tau)(1-\gamma)\mathrm{A}\frac{\mathrm{B}\left([(1-\upsilon)\tau\mathrm{A}k_t]^{\varphi}\left(E-\dfrac{p\mathrm{A}k_t}{1+\upsilon\tau\mathrm{A}k_t}\right)^{\chi}\right)}{1+\mathrm{B}\left([(1-\upsilon)\tau\mathrm{A}k_t]^{\varphi}\left(E-\dfrac{p\mathrm{A}k_t}{1+\upsilon\tau\mathrm{A}k_t}\right)^{\chi}\right)}k_t = z(k_t) , \tag{19}$$

which is a first-order difference equation in capital per worker. The analysis of this equation will facilitate us in understanding the dynamics and the long-run equilibrium of the economy. This is the issue to which we now turn our attention. Before we proceed, we should note that all the proofs are presented in the Appendix.

3 Dynamic Equilibrium

The economy's dynamic equilibrium is formally described through

Definition 2. *Given* $k_0 \geq 0$ *, the dynamic equilibrium is a sequence of temporary equilibria that satisfy* $k_{t+1} = z(k_t)$ *for every* t *.*

We can facilitate our subsequent analysis by defining a new variable, θ_{t+1}, which denotes the growth rate of physical capital per worker, k. That is,

$$\theta_{t+1} = \frac{k_{t+1}}{k_t} - 1. \tag{20}$$

Furthermore, our subsequent results will be further clarified with the use of

Definition 3. *Consider* $k_0 \geq 0$. *An equilibrium orbit* $\{k_t\}$ *is a 'no growth' equilibrium if there exists* $M > 0$ *such that* $k_t < M \quad \forall t$. *If* $\lim_{t \to \infty} k_t = \hat{k}$ *then we call* $\hat{k}$ *a 'no growth' steady-state equilibrium. If, in addition,* $\hat{k} = 0$ *then the equilibrium is a 'poverty trap'. If there does not exist such an* M, *then the equilibrium orbit is called a 'long-run growth' equilibrium and satisfies* $\lim_{t \to \infty} \frac{k_{t+1}}{k_t} = \lim_{t \to \infty}(1 + \theta_{t+1}) > 1$.

Our purpose is to examine two scenarios that differ with respect to the government's provision of pollution abatement services. As we shall see, the public sector's stance on environmental protection has significant repercussions for both the economy's dynamics and its long-term prospects. Furthermore, the subsequent analysis will be utilising

Assumption 1. $(1-\tau)(1-\gamma)\mathrm{A}\frac{\mathrm{B}(\Omega)}{1+\mathrm{B}(\Omega)} > 1$, *where* $\Omega = \left(\frac{\varphi\tau}{p}\right)^{\varphi} \chi^{\chi} \left(\frac{E}{\varphi+\chi}\right)^{\varphi+\chi}$.

Assumption 1 is essential for the existence of a meaningful long-run equilibrium and is very common in overlapping generations economies. As we shall see later, the slope of the phase line at the origin is below unity – an outcome that raises the possibility that the only steady-state equilibrium entails the corner solution of a zero capital stock. Assumption 1 eliminates this possibility and ensures the existence of an interior equilibrium (see Figure 1) by requiring that the structural parameters conducive to the economy's capital formation (such as the productivity parameter A; the parameters of the health function $\mathrm{B}(\cdot)$; or the environmental parameter E) be sufficiently high to guarantee a positive rate of capital accumulation for at least some range on the capital stock's domain. In economic terms, Assumption 1 requires that, over some range of the capital stock, the saving of the young suffice to buy the existing capital stock.

3.1 Dynamic Equilibrium without Abatement

We begin our analysis with the case where the fraction of revenue devoted to abatement $\upsilon = 0$, i.e., the government does not actively engage in policies of environmental preservation. Given (19), we have

$$z(k_t) = (1-\tau)(1-\gamma)\mathrm{A}\frac{\mathrm{B}\left((\tau \mathrm{A} k_t)^{\varphi}(E - p\mathrm{A}k_t)^{\chi}\right)}{1+\mathrm{B}\left((\tau \mathrm{A} k_t)^{\varphi}(E - p\mathrm{A}k_t)^{\chi}\right)}k_t. \tag{21}$$

First, we are interested in obtaining the model's steady-state equilibria. These are fixed points of the map $z(\cdot)$, i.e., values $\hat{k}$ of capital per worker that satisfy $\hat{k} = z\left(\hat{k}\right)$. A formal analysis of (21) allows us to derive

Lemma 1. *There exist three steady-state equilibria* $\hat{k}'$, $\hat{k}''$ *and* $\hat{k}'''$, *where* $\hat{k}' = 0$ *and* $\hat{k}''' > \hat{k}'' > 0$. *The steady state* $\hat{k}'$ *is locally asymptotically stable,* $\hat{k}''$ *is unstable and* $\hat{k}'''$ *may be either locally asymptotically stable or unstable.*

The result from Lemma 1 facilitates us in tracing the economy's dynamic behaviour and transitional dynamics. We can formally present these ideas in the form of

Proposition 1. *Consider* $k_0 > 0$. *Then:*

(i) *If* $k_0 < \hat{k}''$, *the economy will converge to the poverty trap* $\hat{k}' = 0$;

(ii) *If* $k_0 > \hat{k}''$, *the economy will converge to a 'no growth' equilibrium. Particularly, if* $\hat{k}'''$ *is locally asymptotically stable, then it will also be the stationary equilibrium for the stock of capital per worker – otherwise, the economy will asymptotically converge to an equilibrium where capital per worker displays permanent cycles around* $\hat{k}'''$.

The different possible scenarios are depicted in Figures 1-3. In all three cases, we see that the point $\hat{k}''$ acts as a natural threshold, which allows history (approximated by the initial capital stock) to determine the long-term prospects of the economy. The model's ability to generate multiple steady-state equilibria rests on the beneficial effect of publicly provided health services on saving behaviour – an effect that lies on the idea that health services promote longevity. Specifically, for some levels of k_t, capital accumulation and saving complement each other. Thus, for relatively low levels of initial capital endowment, saving is not sufficient enough to guarantee a positive rate of capital accumulation: capital per worker declines constantly until it rests on an equilibrium which is, essentially, a poverty trap. If, however, the initial endowment is sufficient enough, the economy can escape the poverty trap because saving allows growth at positive (albeit declining) rates during the early stages of transition.

INSERT FIGURE 1 ABOUT HERE

So far, the results and their intuition are similar to those discussed in Chakraborty (2004). Nevertheless, our model is able to generate richer implications for the dynamics of an economy whose history allows it to move to the right side of the natural threshold $\hat{k}''$. The reason for such implications is economic activity's contribution to environmental degradation and the corresponding repercussions for health status and longevity. Particularly, for sufficiently high values of k_t the negative effect of pollution on life expectancy and saving dominates the positive effect of publicly provided goods and services on health. Hence, the dynamics of capital accumulation are non-monotonic and $\hat{k}'''$ may actually lie on the downward sloping part of $z(k_t)$ (see Figures 2 and 3). Furthermore, as Figure 3 illustrates, when the slope of the graph at the steady state $\hat{k}'''$ is steep enough, the economy may converge to an equilibrium in which capital per worker oscillates permanently around $\hat{k}'''$, i.e., an equilibrium with a permanent, endogenously determined cycle. In terms of intuition, a relatively high level of capital per worker implies relatively high pollution. The health status is affected negatively and, consequently, saving is reduced. Capital accumulation is mitigated, but this also implies that the extent of environmental degradation is mitigated as well. Next period's health status improves and so is saving, which promotes capital accumulation. This sequence of events may ultimately become self-repeating, thus generating equilibrium with persistent cycles.[11]

INSERT FIGURE 2 ABOUT HERE

INSERT FIGURE 3 ABOUT HERE

Concerning the dynamic behaviour of environmental quality, it should be obvious that this will be dictated by the dynamics of the capital stock. More specifically, if the economy converges to a poverty trap, then environmental quality approaches its maximum level E given that economic activity is the ultimate cause of environmental deterioration;

nevertheless, the severe limitation of resources towards public health means that agents cannot benefit from the improved environmental conditions and, hence, they live essentially for one period. If, on the other hand, the capital stock converges to a stationary or a periodic equilibrium, then so does environmental quality (the dynamics of environmental quality are analysed formally in the Appendix; see Section A5).

These results, as well as the intuition behind them, merit some discussion in relation to their empirical relevance. As we can see, the equilibrium behaviour of all variables, including environmental quality and life expectancy/mortality, can be cyclical under some circumstances. With respect to the former, there is evidence to show that indicators of environmental quality display such cyclical movements (e.g., Mayer, 1999). With respect to the latter, the analyses of Chay and Greenstone (2003) and Rolden *et al.* (2014) make an explicit connection between pollution and procyclical mortality rates. In any case, the subsequent section of our analysis will show that in the presence of environmental policy, the economy's dynamics and the repercussions for life expectancy become drastically different. Thus, an additional implication will be the identification of the possible importance of environmental policy in preserving the positive, *on average*, link between longevity and per capita GDP that we observe in cross-section data.

3.2 Dynamic Equilibrium with Active Pollution Abatement

Next, we analyze the scenario where $0 < \upsilon < 1$, i.e., the government pursues a policy of environmental preservation. Therefore, the dynamics of capital accumulation are represented by the difference equation we originally obtained in (19). Once more, we begin our analysis with the derivation of the model's steady-state equilibrium.

Lemma 2. *Suppose that* $\tau > p/\upsilon E$ *holds. Then, there exist two steady-state equilibria* $\hat{k}_1$ *and* $\hat{k}_2$*, such that* $\hat{k}_2 > \hat{k}_1 = 0$*. The steady state* $\hat{k}_1$ *is locally asymptotically stable, while the steady state* $\hat{k}_2$ *is unstable.*

The condition $\tau > p/\upsilon E$ imposes a lower bound on the share of government expenditure that is allocated to abatement. Equivalently, it imposes an upper bound on the maximum emission rate *p*, so that, even at very high levels of output, the effect of degradation due to pollution does not exceed the natural capacity of the environment (i.e., *E*). If this condition does not hold, the dynamic equilibrium of the economy resembles the one derived for $\upsilon = 0$. Using Lemma 2, we can identify the economy's dynamic behaviour. We do this through

Proposition 2. *Consider* $k_0 > 0$*. Then:*

(i) *If* $k_0 < \hat{k}_2$*, the economy will converge to the poverty trap* $\hat{k}_1 = 0$;

(ii) *If* $k_0 > \hat{k}_2$*, then the economy will converge to a 'long-run growth' equilibrium in which both capital and output per worker grow at the rate* $\hat{\theta} = (1-\tau)(1-\gamma)\mathrm{A}\dfrac{\lambda}{1+\lambda} - 1 > 0$.[12]

The dynamics of the economy with pollution abatement are illustrated in Figure 4. The steady state $\hat{k}_2$ emerges as an endogenous threshold that determines long-term prospects according to the initial capital stock. Once more, an economy which is initially endowed with resources below this threshold will degenerate towards the poverty trap, where capital and output are very low – so low, in fact, that the reduced pollution cannot be translated into improvements in the health characteristics of the population. Naturally, the intuition behind this result is identical to the one provided in the case without pollution abatement.

Of particular interest is the situation that occurs when the initial capital stock is above $\hat{k}_2$. Contrary to the $\upsilon = 0$ case, in which capital converges to an equilibrium without growth (i.e., either a positive level or a limit cycle), in this case the economy is able to sustain a positive growth rate in the long-run. The reason is that abatement limits the extent to which economic activity causes environmental damage. Thus, it protects the population's health against the damage from environmental degradation; therefore, agents' saving behaviour is not impeded as the economy grows. Combined with the effect of the learning-by-doing externality in production, a policy of environmental preservation allows the social marginal return of capital to be high enough so as to guarantee a positive rate of capital accumulation that, eventually, allows the economy to achieve balanced growth as an equilibrium outcome. Moreover, as the economy grows without bound, environmental quality approaches from above the constant level $E-(p/\upsilon\tau)$; for this to be positive it must be the case that $\tau > p/\upsilon E$, which we assumed in Lemma 2.[14]

INSERT FIGURE 4 ABOUT HERE

3.3 Some Important Implications

In the preceding sections, we have examined the transitional dynamics and the long-term equilibrium of an economy under two opposite scenarios concerning the government's engagement in policies that are designed to mitigate pollution and promote environmental quality. Apart from the common theme of multiple steady states and the existence of poverty traps, the predictions from the two scenarios concerning the long-term prospects of economies that escape such poverty traps are strikingly different. The purpose of this section is to compare and contrast these predictions in order to derive important implications that arise as a result of the government's stance on activities of pollution abatement.

We begin with the implications concerning economic growth. Equation (4) implies that the labour's contribution to aggregate production is augmented by a productivity variable which is driven by the presence of an economy-wide, learning-by-doing externality as in Romer (1986). It is well known that, in standard dynamic general equilibrium models with production, such externalities allow the emergence of an equilibrium with ongoing output growth. In our framework, however, we have established that the learning-by-doing mechanism is not by itself sufficient to guarantee growth in the long-run. Indeed, such an equilibrium exists only when the government commits sufficient resources towards activities that abate pollution. Therefore, one significant implication from our analysis is the following:

Corollary 1. *For an economy that avoids the poverty trap, pollution abatement is a complementary engine of long-run economic growth.*

This idea comes in stark contrast to previously held views concerning the macroeconomic repercussions of pollution. In her influential paper, Stokey (1998) argued that the prospects of long-run growth may be hampered as a result of the society's need to implement policies that support the quality of the natural environment – policies that are costly and, therefore, reduce the marginal product of capital to the extent that capital accumulation cannot be permanently sustained. By taking account of the well-documented effects of environmental quality to the overall health characteristics of the population, and their consequence for saving behaviour, our model has reached a different conclusion: policies that preserve some degree of environmental quality are, actually, essential for the existence of an equilibrium with ongoing output growth. Furthermore, notice that for environmental policy to achieve this outcome, we do not require an equilibrium in which pollution declines constantly over time (see, for example, Smulders and Gradus 1996). In fact, pollution abatement supports long-run growth even though it is only capable of reducing the rate of environmental degradation, rather than eliminating it altogether.

Another important implication of our analysis is related to the existence of limit cycles. As we have seen, when pollution abatement is absent, it is possible for capital per worker to oscillate permanently around its positive steady state. Of course, such persistent fluctuations are different in nature from cycles whose impulse sources may be exogenous demand and/or supply disturbances. In our model, both the impulse source and the propagation mechanism of cycles rest on the presence of non-monotonicity in the dynamics of capital accumulation. Naturally, policies that could eradicate such fluctuations are policies that would address the source of non-monotonicities rather than counter-cyclical rules designed to mitigate temporary fluctuations around a given trend. With this in mind, a straightforward comparison between our two different scenarios allows us to infer:

Corollary 2. *For an economy that avoids the poverty trap, pollution abatement is a source of stabilisation, in the sense that it eliminates the possibility of permanent cycles.*

Given that environmental policy has an indirect positive effect on health and, consequently, life expectancy, our model derives implications that differ from those of Bhattacharya and Qiao (2007). In their model, the positive complementarities between private and public health spending imply that there is a trade-off between saving and private health expenditure. This trade-off generates non-monotonic capital dynamics, hence rendering health-enhancing public policy a source of endogenous fluctuations. In our model, a policy that facilitates health improvements (albeit indirectly through pollution abatement) actually eliminates such fluctuations.

At this point it is important to discuss a conceptual issue that is related to Corollaries 1 and 2. Given the combined modelling of life expectancy, environmental quality and pollution abatement, one may wonder why is the environmental policy of abatement, and its link to pollution, critical for the results. After all, pollution abatement affects saving behaviour and capital accumulation in the same manner as the direct expenditure on health (g_t): both of them improve health and life expectancy. The reason why environmental policy is critical, and its effects quite different from the direct effect of public health spending, is the following. In the absence of pollution abatement, an increase in the capital stock, production, and thus income has two conflicting effects. On the one hand, it raises tax revenue and hence spending on health services, and, on the other, it raises pollution and worsens environment degradation. The first effect improves agents' health status and hence increases their saving rate, while the second deteriorates their health status and decreases the saving rate. It is exactly this interplay between environmental degradation and health services that environmental policy breaks. It mitigates the source of non-monotonicities, i.e., the rate of environmental degradation that leads to endogenous cycles and does not allow the economy to grow in the long-run. This becomes obvious from the fact that when $a_t = 0$, the no-growth equilibrium (and, possibly, cycles) emerge even in the presence of g_t, i.e., the part of public spending directly devoted to health.

Another way to see the point made in the previous paragraph is to consider a case where the link between a_t and environmental quality is removed. For example, consider the case where the health function is given by $h_t = g_t^{\varphi} e_t^{\chi} a_t^{\zeta}$, $\zeta > 0$, $P_t = pY_t$ and hence $e_t = E - pY_t$.

Life expectancy then equals $1+B(h_t)=1+B\left((1-\upsilon)^{\varphi}\upsilon^{\zeta}(\tau \mathrm{A})^{\varphi+\zeta}k_t^{\varphi+\zeta}(E-p\mathrm{A}k_t)^{\chi}\right)$. Substituting this in the transition equation, one can easily establish that even when $\upsilon>0$, the model would behave qualitatively identically to the scenario we presented in Section 3.1 (summarized in Proposition 1). Nevertheless, this would be a scenario in which there is conceptually nothing to specify a_t as pollution abatement. Instead, a_t behaves identically to g_t, as just another item of spending that improves health directly – in fact, one can consider $g_t^{\varphi}a_t^{\zeta}$ as a composite term manifesting the direct effect of public spending on health. On the contrary, although the effect of a_t is ultimately a benefit in terms of health and life expectancy, the transmission of this effect in our original formulation entails a direct reduction of emissions per unit of output. It is for this reason that a_t can be conceptually associated with pollution abatement in the first place, while our set-up and implications differ from a typical $\mathrm{A}K$ growth model with public health expenditure.

4 Environmental Quality as a Stock Variable

Next, to test the robustness of our results, we consider environmental quality as a stock, instead of a flow, variable. We should note that using environmental quality as the variable of concern, and assuming that its dynamic behaviour is affected by the flow of emissions, is not an alien assumption. On the contrary, a similar approach has been used by seminal analyses within the context of either OLG economies (e.g., John and Pecchenino 1994; Mariani *et al.* 2010) or representative-agent ones (e.g., Bovenberg and Smulders 1995; Acemoglu *et al.* 2012). To that end, we replace equation (8) with the following equation:

$$e_t=\begin{cases}(1-\eta)E+\eta e_{t-1}-P_t & \text{if } P_t<(1-\eta)E+\eta e_{t-1}\\ 0 & \text{otherwise}\end{cases}, \tag{22}$$

where P_t is given in equation (7) (an alternative specification, mentioned in footnote 10, is presented in the working paper version). Equation (22) follows John and Pechennino (1994).[14] Accordingly, environmental quality is now a convex combination of the maximum long-run level of environmental quality E and the environmental quality of last period. Notice that this formulation can also encompass the case analysed in Sections 2 and 3; specifically, if $\eta = 0$, then equation (22) is reduced to (8).

Using $m_{t+1} = e_t$ and following the same steps as before, we can reduce the model to the following planar system of difference equations:

$$k_{t+1} = (1-\tau)(1-\gamma)\mathrm{A}\frac{\mathrm{B}\left([(1-\upsilon)\tau\mathrm{A}k_t]^{\varphi}\left[(1-\eta)E + \eta m_t - \frac{p\mathrm{A}k_t}{1+\upsilon\tau\mathrm{A}k_t}\right]^{\chi}\right)}{1+\mathrm{B}\left([(1-\upsilon)\tau\mathrm{A}k_t]^{\varphi}\left[(1-\eta)E + \eta m_t - \frac{p\mathrm{A}k_t}{1+\upsilon\tau\mathrm{A}k_t}\right]^{\chi}\right)}k_t, \tag{23}$$

and

$$m_{t+1} = (1-\eta)E + \eta m_t - \frac{p\mathrm{A}k_t}{1+\upsilon\tau\mathrm{A}k_t}. \tag{24}$$

Unlike the one-dimensional system analyzed in Section 3, the two-dimensional system of difference equations in this case is quite complex. To simplify the analysis, we adopt the functional form $B(h_t) = \lambda h_t/(1+h_t)$ $(0 < \lambda < 1)$ that was suggested by Chakraborty (2004). This is a functional form that satisfies all the properties listed after equation (1). Moreover, we let $\varphi = \chi = 1,$ so that equation (23) becomes

$$k_{t+1} = \frac{(1-\tau)(1-\gamma)\lambda(1-\upsilon)\tau\mathrm{A}^2k_t^2\left[(1-\eta)E + \eta m_t - \frac{p\mathrm{A}k_t}{1+\upsilon\tau\mathrm{A}k_t}\right]}{1+(1+\lambda)(1-\upsilon)\tau\mathrm{A}k_t\left[(1-\eta)E + \eta m_t - \frac{p\mathrm{A}k_t}{1+\upsilon\tau\mathrm{A}k_t}\right]}. \tag{25}$$

Combining (24) and (25), the steady-state loci $k_{t+1}=k_t=k$ and $m_{t+1}=m_t=m$ are given by

$$m=\frac{1}{\eta}\left\{\frac{1}{\left[(1-\tau)(1-\gamma)\mathrm{A}\lambda-(1+\lambda)\right]\mathrm{A}\tau(1-\upsilon)k}-\left((1-\eta)E-\frac{p\mathrm{A}k}{1+\upsilon\tau\mathrm{A}k}\right)\right\}, \tag{26}$$

and
$$m=E-\frac{1}{1-\eta}\frac{p\mathrm{A}k}{1+\upsilon\tau\mathrm{A}k}. \tag{27}$$

Proposition 3. i) *If $\upsilon=0$ then for sufficiently high value of p there is no positive steady state, while for sufficiently low value of p there are two positive steady states ii) If $\upsilon>p/(\tau(1-\eta)E)>0$, then there is only one non-degenerate positive steady state.*

Proposition 3 is illustrated in Figures 5 and 6. If there is no abatement and the maximum emission rate p is sufficiently high, then there is no positive steady state (the two loci $\Delta m_t=0$ and $\Delta k_t=0$ in Figure 5 do not intersect). In fact, as a straightforward analysis of the phase diagram shows, the economy converges towards the steady state $(k,m)=(0,E)$. This case then represents that of a poverty trap (recall that a similar result was obtained in Section 3). If, on the other hand, p is sufficiently small, then there will be two steady states. For example, if we use the following values: $\tau=0.4, \gamma=0.4, p=0.3, \mathrm{A}=25, E=10,$ and $\eta=0.8,$ there are two steady-state equilibria. If we specify $\lambda=0.4$, then the two equilibria are $(k_1,m_1)=(0.005,9.827)$ and $(k_2,m_2)=(0.262,0.173)$. Moreover, when $\lambda=0.4$, at both equilibria the eigenvalues are real; one of them has modulus greater and the other less one. Thus, both equilibria are saddle-path stable. Nevertheless, the stability properties of the equilibria change with λ. In fact, as shown in the Appendix (see Section A8), an attracting two-period cycle may emerge. For example if, again, $\lambda=0.4$, then the two-period cycle includes the points: (0.202, 0.075) and (0.315, 0.547). Hence, given that $m_{t+1}=e_t$, the economy converges to an equilibrium where it oscillates between $(k,e)=(0.202,0.547)$ and $(k,e)=(0.315,0.075)$. Just as in Section 3, where e was a flow variable, low (high) capital stock implies low (high) pollution and high (low) environmental quality.

INSERT FIGURE 5 ABOUT HERE

INSERT FIGURE 6 ABOUT HERE

Consider, next, the case where there is pollution abatement, that is, $\upsilon > 0$. If, as in Lemma 2, the percentage of tax revenue that the government allocates to abatement (υ) is high enough ($\upsilon > p/(\tau(1-\eta)E)$), then there will be a unique steady-state equilibrium (see Figure 6). For the values specified above, $\upsilon = 0.5$, and $\lambda = 0.4$, the unique steady-state equilibrium is $(k_1, m_1) = (0.009, 9.663)$ and it is saddle-path stable. In this case, no cycle emerges. An economy whose initial capital stock is greater than k_1, and not on the saddle path, will be able to grow unboundedly.

We conclude that extending the analysis to consider the stock of the natural environment (i.e., a form of natural capital) has not affected the qualitative implications of our original set-up. This outcome is not surprising. In both scenarios regarding the set-up for e_t (flow and stock), the model's mechanisms, as well as the manner through which pollution abatement impinges on them, are the same. Without pollution abatement, the impact of capital accumulation on total emissions has a strong negative effect on the evolution of e_t (i.e., the dynamics of environmental quality). Given the two-way causal nature behind the joint dynamics of k_t and e_t, the deteriorating environmental conditions will impinge on life expectancy and saving, inhibiting the rate of capital accumulation and the flow of emissions, thus improving the evolution of environmental quality and perpetuating the cyclical dynamics. Pollution abatement can mitigate the rate of environmental deterioration, hence eliminating the magnitude of the forces that inhibit the prospects of long-run growth and lead to cycles. The lagged effect associated with treating e_t as a stock variable does not change the model's mechanisms and intuition when it comes to the issue of pure economic dynamics – the issue which is the main concern of our framework.

5 Alternative Specifications

As further tests of robustness, in this section we examine two alternative specifications.[15] First, we replace process-integrated technology with end-of-pipe technology. For this, we set $a_t = 0$ in equations (6) and (7), so that $P_t = pY_t$. Recall that the end-of-pipe abatement technology does not modify the production process and hence does not change the emission-to-output ratio. Instead, it can clean *already* formed contaminants. Hence, we have to distinguish now between pollution and environmental degradation (net pollution). The latter is denoted by D_t and it is given by

$$D_t = P_t - f(P_t, a_t) = D(P_t, a_t), \tag{28}$$

where $D_P > 0$ and $D_a < 0$, so that environmental degradation increases with the flow of pollution (P) and decreases with abatement activities (a). To be able to derive analytical solutions, we assume the functional form $f(P_t, a_t) = P_t^{-\sigma} a_t^{\varepsilon}$, where $\sigma \geq 0$ and $\varepsilon > 0$, so that (28) is written as

$$D_t = P_t - P_t^{-\sigma} a_t^{\varepsilon}. \tag{29}$$

Substituting (10) and (18) in (29), we see that

$$D_t = p\mathrm{A}k_t - p^{-\sigma}(\upsilon\tau)^{\varepsilon}\mathrm{A}^{\varepsilon-\sigma}k_t^{\varepsilon-\sigma}. \tag{30}$$

According to Grossman and Krueger (1995), the Environmental Kuznets Curve (EKC) arises as a result of countries applying "*more stringent environmental standards and stricter enforcement of their environmental laws*" (p. 372). From (30), one can establish that there is an inverse U-shape relation between D_t and k_t as long as $\varepsilon - \sigma > 1$, which we henceforth assume. Given the presence of the EKC, we impose a non-negativity constraint on D_t, i.e., $D_t = 0 \ \ \forall k_t \geq \varsigma$, where ς is the solution of the equation $D(k) = 0$, that is $\varsigma \equiv [\,p\mathrm{A}/\delta]^{1/(\varepsilon-\sigma-1)}$ where $\delta \equiv p^{-\sigma}\mathrm{A}^{\varepsilon-\sigma}(\upsilon\tau)^{\varepsilon}$.

The second alternative specification is in regard to environmental quality and the health status. To avoid the criticism that in reality environmental quality is a stock and not a flow variable, we let the health status and life expectancy be functions of environmental degradation. Thus, we replace (2) with

$$h_t = g_t^{\varphi} D_t^{-\chi}, \quad \varphi, \chi > 0 . \tag{31}$$

Similarly to the analysis in Sections 2 and 3, a worsening of the natural environment has negative repercussions for the population's health status and life expectancy.

To simplify the technical aspects of the analysis, we shall employ again the functional form $B(h_t) = \lambda h_t / (1 + h_t)$. Substituting (1), (9), (11), (30) and (31) in (16), we can express the process of capital accumulation according to

$$k_{t+1} = z(k_t) = (1-\tau)(1-\gamma)\mathrm{A}\frac{\lambda[(1-\upsilon)\tau\mathrm{A}k_t]^{\varphi}\left[\,p\mathrm{A}k_t - p^{-\sigma}(\upsilon\tau)^{\varepsilon}\mathrm{A}^{\varepsilon-\sigma}k_t^{\varepsilon-\sigma}\right]^{-\chi}}{1+(1+\lambda)[(1-\upsilon)\tau\mathrm{A}k_t]^{\varphi}\left[\,p\mathrm{A}k_t - p^{-\sigma}(\upsilon\tau)^{\varepsilon}\mathrm{A}^{\varepsilon-\sigma}k_t^{\varepsilon-\sigma}\right]^{-\chi}}k_t \, , \tag{32a}$$

an expression that can be used to examine the implications for economic dynamics under different scenarios concerning the government's stance towards environmental abatement.

We begin the analysis with the case where $\upsilon = 0$, i.e., there is absence of pollution abatement. In this case, (32a) can be rewritten as

$$k_{t+1} = z(k_t) = (1-\tau)(1-\gamma)\mathrm{A}\frac{\lambda\tau^{\varphi}\mathrm{A}^{\varphi-\chi}p^{-\chi}k_t^{\varphi-\chi}}{1+(1+\lambda)\tau^{\varphi}\mathrm{A}^{\varphi-\chi}p^{-\chi}k_t^{\varphi-\chi}}k_t \, . \tag{33}$$

Proposition 4. *i) If $\varphi > \chi$ then there exists only one positive steady-state equilibrium, which is unstable. ii) If $\chi > \varphi$, then there exists again only one positive steady-state equilibrium, which now may be either* *stable or unstable. If the steady state is stable, then the economy converges either monotonically or with damped oscillations to the unique positive state. On the other hand, if the steady state is unstable, then the economy converges to a limit cycle.*

INSERT FIGURE 7 ABOUT HERE

When $\varphi > \chi$, there is a unique positive steady state, which acts as a threshold. Below it the economy will fall into the poverty trap and above it will have a positive growth rate in the long-run. These outcomes are not surprising, given that when $\varphi > \chi$ (i.e., when the effect of pollution on life expectancy is not strong enough) the model becomes qualitatively identical to that of Chakraborty (2004), amended with a learning-by-doing externality. On the other hand, if $\chi > \varphi$ then saving is not sufficient enough to guarantee a positive growth rate of economic growth. Depending on the parameter values, the economy will either converge to a positive *level* of capital or will oscillate permanently between two different levels of capital. Figure 7 illustrates these results. If the steady state lies on the upward-sloping part of $z(k_t)$, then the economy converges monotonically to a positive level of capital stock (such as $\hat{k}'$ in Figure 7). If, on the other hand, it lies on the downward sloping part, then, depending on the magnitude of the slope of $z(k_t)$ at the steady state, the economy either converges to a positive level of capital with damped oscillations or it oscillates permanently around an unstable steady state. Obviously, these results and the intuition are similar to those in Section 3.

We examine next the effect of abatement in the scenarios for which, without abatement, the economy cannot sustain an equilibrium with long-run growth and it may be subjected to permanent cycles, i.e., the cases where the negative effect of pollution is sufficiently strong ($\chi > \varphi$). The transition equation $k_{t+1} = z(k_t)$ is given by (32a) for values of $k_t < \varsigma$ and by

$$k_{t+1} = z(k_t) = (1-\tau)(1-\gamma)\mathrm{A}\frac{\lambda}{(1+\lambda)}k_t\,, \tag{32b}$$

for values of $k_t \geq \varsigma$.[16] Importantly, the map $z(\cdot)$ is continuous. The following proposition then is similar to Proposition 2 in Section 3.

Proposition 5. *If* $\varphi < \chi$ and k_0 *is sufficiently high, then the economy will converge to a 'long-run growth' equilibrium in which both capital and output per worker grow at the rate* $\hat{\theta} = (1-\tau)(1-\gamma)\mathrm{A}\dfrac{\lambda}{1+\lambda} - 1 > 0$.

Hence, once again pollution abatement acts as a source of long-run growth and (possibly) stabilisation. Even under circumstances where, in the absence of abatement, the economy cannot sustain a positive growth rate in the long-run and can possibly converge to a periodic equilibrium, once pollution abatement becomes active the possibility of limit cycles disappears as capital per worker and output grow over time.

6 Summary and Conclusions

We have constructed a two-period overlapping generations model where life expectancy is positively affected by the provision of public health services and by the quality of the natural environment. We showed that, despite the presence of an aggregate learning-by-doing externality, the economy cannot sustain a positive growth rate in the long-run if resources are not devoted towards environmental preservation. As the environment deteriorates without bound, the negative impact on life expectancy causes a reduction in saving and, therefore, the rate of capital formation: the economy's capital stock either converges to a stationary level or oscillates permanently. An equilibrium with on-going output growth is possible only if the government commits a sufficient amount of resources towards pollution abatement. Given that the possibility of cycles disappears in the latter scenario, we concluded that an active policy of environmental preservation is not only an important complementary engine of long-run growth, but a powerful tool of stabilisation as well. These results pinpoint the importance of environmentally-oriented policies as a means of supporting not only the environment, but also the economy's prospects for sustained economic growth.

Obviously, our framework can be enriched with respect to several aspects that could broaden its scope and implications. For example, an obvious direction is to consider private resources in support of abatement activities, in addition to the public ones. Despite the fact that such an extension generates free-riding issues and requires a crucial assumption regarding the degree at which individuals internalise the effect of their own activities on an aggregate outcome such as environmental quality, it would allow us to examine the trade-off between saving and environmental spending. This trade-off would most probably allow an additional channel through which environmental factors impinge on saving and capital accumulation. Moreover, a similar trade-off exists between saving and individual health spending. Finally, when environmental quality is treated as a stock variable, then the externalities associated with environmental outcomes can have significant intergenerational effects, i.e., the extent to which the current generation pollutes may have long lasting implications for the welfare of future generations.[17] As long as agents do not possess strongly altruistic characteristics, such a scenario could lie at the core of arguments suggesting that countries do not invest sufficient resources towards environmental improvements. Naturally, there are significant implications for optimal policy under such a setting.[18] We view these extensions as important topics for future research.

Appendix

A1 Proof of Lemma 1

Using equation (21), we define the function

$$J(k_t) = \frac{z(k_t)}{k_t} = (1-\tau)(1-\gamma)\mathrm{A}\frac{\mathrm{B}\left((\tau \mathrm{A} k_t)^{\varphi}(E - p\mathrm{A}k_t)^{\chi}\right)}{1+\mathrm{B}\left((\tau \mathrm{A} k_t)^{\varphi}(E - p\mathrm{A}k_t)^{\chi}\right)}. \tag{A1.1}$$

Clearly, any interior steady state must satisfy $J(\hat{k}) = 1 \Leftrightarrow \hat{k} = z(\hat{k})$. From (A1.1), we have $J(0)$ =0 and, by virtue of (8), $J(k_t) = 0 \ \forall k_t \geq E/p\mathrm{A}$. Thus, for an interior steady state to exist, there must be at least one $\tilde{k}$ such that $J(\tilde{k}) \geq 1$. When this condition holds with strict inequality, there will be at least two interior steady states; otherwise, there will not be any interior equilibrium at all (see Figure A1).

Combining (A1.1) with (1), (2), (7), (8) and (9) allows us to derive

$$J'(k_t) = (1-\tau)(1-\gamma)\mathrm{A}\frac{\mathrm{B}'(h_t)}{[1+\mathrm{B}(h_t)]^2}\frac{dh_t}{dk_t}, \tag{A1.2}$$

where
$$\frac{dh_t}{dk_t} = \varphi\tau\mathrm{A}(\tau\mathrm{A}k_t)^{\varphi-1}(E - p\mathrm{A}k_t)^{\chi} - p\mathrm{A}\chi(\tau\mathrm{A}k_t)^{\varphi}(E - p\mathrm{A}k_t)^{\chi-1}. \tag{A1.3}$$

INSERT FIGURE A1 ABOUT HERE

For $0 \leq k_t \leq E/p\mathrm{A}$, the sign of (A1.3) determines the sign of $J'(k_t)$. Straightforward factorisation allows us to write (A1.3) as

$$\frac{\partial h_t}{\partial k_t} = (\tau\mathrm{A}k_t)^{\varphi}(E - p\mathrm{A}k_t)^{\chi}\left(\frac{\varphi}{k_t} - \frac{\chi p\mathrm{A}}{E - p\mathrm{A}k_t}\right),$$

which means that $\frac{\partial h_t}{\partial k_t} \geq 0$ iff

$$\frac{\varphi}{k_t} \geq \frac{\chi p\mathrm{A}}{E - p\mathrm{A}k_t} \Rightarrow \qquad k_t \leq \frac{\varphi}{\varphi+\chi}\frac{E}{p\mathrm{A}} \equiv \tilde{k}.$$

The preceding analysis implies that there exists a unique $\tilde{k} \in (0, E/p\mathrm{A})$ such that

$$J'(k_t)\begin{cases} >0 & \text{for} \quad k_t < \tilde{k} \\ =0 & \text{for} \quad k_t = \tilde{k}, \\ <0 & \text{for} \quad k_t > \tilde{k} \end{cases}$$

i.e., $J(\tilde{k})$ is a global maximum. We can use this result to identify the parameter combination that allows the existence of interior equilibria. Particularly, we can solve $(\tau\mathrm{A}\tilde{k})^{\varphi}(E - p\mathrm{A}\tilde{k})^{\chi}$ using $\tilde{k} = \varphi E/(\varphi+\chi)p\mathrm{A}$. Doing so, we derive $(\varphi\tau/p)^{\varphi}\chi^{\chi}[E/(\varphi+\chi)]^{\varphi+\chi} \equiv \Omega$. Hence, by the Intermediate Value Theorem, Assumption 1 is a sufficient condition for the existence of interior equilibria. Moreover, if this condition holds, then there exist two interior steady-state equilibria $\hat{k}'''$ and $\hat{k}''$ satisfying $\hat{k}''' > \tilde{k} > \hat{k}'' > 0$; thus, $J'(\hat{k}'') > 0$ and $J'(\hat{k}''') < 0$.

Using (A1.1) we can derive

$$J'(k_t) = \frac{z'(k_t)k_t - z(k_t)}{\left(k_t\right)^2}. \tag{A1.4}$$

Given (A1.4), $J'(\hat{k}'') > 0$ implies $z'(\hat{k}'') > \dfrac{z(\hat{k}'')}{\hat{k}''} \Rightarrow \quad z'(\hat{k}'') > J(\hat{k}'') \Rightarrow \quad z'(\hat{k}'') > 1$,

because $J(\hat{k}'') = 1$. Thus, $\hat{k}''$ is an unstable equilibrium. Similarly, (A1.4) implies that $J'(\hat{k}''') < 0$ is equivalent to $z'(\hat{k}''') < 1$. In this case, however, we cannot reach any definite conclusions concerning the stability of this equilibrium as we do not yet know whether the dynamics generated by equation (21) are monotonic. For this reason, let us return to the transition equation $k_{t+1} = z(k_t)$. Given (21), we can see that $z(0) = 0$, $z(k_t) = 0 \ \forall k_t \geq E/p\mathrm{A}$ and $z(k_t) > 0$ for $k_t \in (0, E/p\mathrm{A})$. Thus, the dynamics of capital accumulation may not be non-monotonic which means that, indeed, the stability properties of $\hat{k}'''$ cannot be determined with certainty. Particularly, $\hat{k}'''$ is a stable long-run equilibrium if $z'(\hat{k}''') > -1$; otherwise, i.e., if $z'(\hat{k}''') < -1$, the equilibrium $\hat{k}'''$ is unstable.

In our preceding analysis, we have established that $z(0) = 0$. Of course, this result indicates that $\hat{k}' = 0$ is a steady state. Moreover, $z'(k_t) = J'(k_t)k_t + J(k_t)$, and since, from equations (A1.2) and (A1.3), $\lim_{k \to 0}\left(\frac{dh_t}{dk_t}k_t\right) = 0$ and $J'(k_t)k_t = 0$, it follows that $z'(\hat{k}') = z'(0) = 0$, i.e., $\hat{k}' = 0$ is a super-stable equilibrium. ■

A2 Proof of Proposition 1

Part (i) follows from Lemma 1 in which we have shown that $\hat{k}' = 0$ is an asymptotically stable equilibrium, while $\hat{k}'' > 0$ is an unstable one. Hence, given $\hat{k}'' > \hat{k}'$, we can safely conclude that, for any $k_0 < \hat{k}''$, it is $k_{t+1} = z(k_t) < k_t$, i.e., the economy's capital per worker will constantly decline until it converges to the poverty trap $\hat{k}' = 0$.

To prove part (ii), we can once more utilise Lemma 1. In particular, let us consider the case where $\hat{k}'''$ is an asymptotically stable equilibrium, i.e., the case for which $\left|z'(\hat{k}''')\right| < 1$. Given $\hat{k}''' > \hat{k}''$, we may conclude that for $k_0 > \hat{k}''$ the transitional dynamics imply that $\lim_{t \to \infty} k_t = \hat{k}'''$. Also, using (20), we have $\theta_{t+1} = (k_{t+1}/k_t) - 1$ and, thus,

$$\lim_{t \to \infty}\theta_{t+1} = \lim_{t \to \infty}\left(\frac{k_{t+1}}{k_t}\right) - 1 = \lim_{t \to \infty}\left(\frac{z(k_t)}{k_t}\right) - 1 = \lim_{t \to \infty} J(k_t) - 1 = J(\hat{k}''') - 1 = 0. \qquad \text{(A2.1)}$$

Therefore, the economy will converge (either monotonically or through damped oscillations) to a long-run equilibrium with a positive stock for capital per worker, but zero growth.

Now, let us consider the possibility that $z'(\hat{k}''') \le -1$. Although $\hat{k}'''$ is an unstable steady-state equilibrium, it is well known that when the transition equation is non-monotonic and its slope at the steady state is negative and sufficiently steep (that is, below -1), then the dynamical system may exhibit periodic equilibria. In terms of our model, consider a sequence of n discrete points along the 45^0 line, denoted $\breve{k}_\eta$ for $\eta = \{1, 2, ..., i-1, i, i+1, ...n\}$, such that $\breve{k}_1 < ... < \breve{k}_{i-1} < \breve{k}_i < \hat{k}''' < \breve{k}_{i+1} < ... < \breve{k}_n$ and

$$z(k_t)\begin{cases} > k_t & \text{for} \quad \eta \in [1, i] \\ \\ < k_t & \text{for} \quad \eta(i, n] \end{cases}.$$

If, for $k_0 > \hat{k}''$, the capital stock passes repeatedly through the points $\breve{k}_\eta$ during its transition, then the economy converges to a period-n cycle, where the sequence $\breve{k}_\eta$ represents periodic (rather than stationary) equilibria. Indeed, as long as $z'(\hat{k}''') < -1$, the function $z(k_t)$ satisfies the following

Theorem (Azariadis, 1993, pp. 86-88). *Suppose* 0 *and* $\hat{k} > 0$ *are fixed points of the scalar system* $k_{t+1} = z(k_t)$ *in which* $z : \mathrm{R}_+ \supseteq X \to \mathrm{R}_+$ *and* $z \in C^1$. *Suppose also that there exists a* $b > \hat{k}$ *such that* $b > z(b)$ *and* $b > z^2(b)$, *where* z^2 *is the second iterate of* z. *Then* $z'(\hat{k}) < -1$ *is a sufficient condition for the existence of a period-2 cycle* $\{\breve{k}_1, \breve{k}_2\}$ *that satisfies* $\breve{k}_1 < \hat{k} < \breve{k}_2 < b$.

Thus, the system $k_{t+1} = z(k_t)$ exhibits (at least) a period-2 cycle. To apply this Theorem to our case, let $\hat{k} = \hat{k}'''$ and $b = E / p\mathrm{A}$. Naturally, the growth rate θ_{t+1} will be positive during phases of the transition for which $\eta \in [1, i]$ but negative during phases of the transition for which $\eta \in (i, n]$. Hence, a long-run equilibrium with a constantly positive growth rate does not exist. ■

A3 An Example of an Economy with Cycles

We illustrate the results in Proposition 1 with a simple numerical example. Suppose that

$$\mathrm{B}(h_t) = \frac{\lambda h_t}{1+h_t}, \ 0 < \lambda < 1.$$

This functional form satisfies the properties of $\mathrm{B}(\cdot)$. Let also $\tau = 0.2$, $\gamma = 0.3$, $p = 0.3$, $\mathrm{A} = 10$, $E = 1$, $\varphi = 0.7$, $\chi = 0.2$. Then at $\lambda = 0.682$ a saddle-node bifurcation occurs; that is, the number of fixed points (steady states), except from the origin, is none for $\lambda < 0.682$, one for $\lambda = 0.682$ and two for values of $\lambda > 0.682$. In particular, if $\lambda < 0.682$ the origin is the only steady-state equilibrium (Assumption 1 is not satisfied). At $\lambda = 0.682$ the function $z(k_t)$ is tangent to the 45^0 degree line and hence there is only one interior steady state. If $\lambda > 0.682$ there are two interior steady-state equilibria, say $\hat{k}''$ and $\hat{k}'''$. The lower equilibrium, $\hat{k}''$, is repelling, whereas the stability of the higher equilibrium, $\hat{k}'''$, depends on the value of λ. For example, if $\lambda = 0.7$ then any orbit that starts in the neighbourhood of $\hat{k}'''$ converges to it monotonically, since $0 < z'(\hat{k}''') < 1$. On the other hand, if we let $\lambda = 0.75$, then the convergence to $\hat{k}'''$ occurs through damped oscillations since $0 > z'(\hat{k}''') > -1$. Next, suppose that we let $\lambda = 0.78$. Simple calculations show that the stability of the equilibrium $\hat{k}'''$ changes since $z'(\hat{k}''') < -1$; i.e., $\hat{k}'''$ becomes a repelling equilibrium. At the same time there is a period-2 cycle $\{0.306, 0.326\}$, which is stable since its multiplier is $z^{2\prime}(0.306) = z^{2\prime}(0.326) = z'(0.306)z'(0.326) = -0.452 > -1$ (z^2 denotes the second iterate of z, i.e., $z^2(k_t) = z(z(k_t))$). Next, suppose that we raise λ to 0.8. Then again simple calculations reveal that, while $\hat{k}'''$ remains a repelling equilibrium, the period-2 cycle has become an unstable one (the value of its multiplier is lower than -1). Instead, there is a period-4 cycle now, which is stable. This process continues as λ increases. In other words, the system undergoes a sequence of period-doubling bifurcations; that is, there is an increasing sequence of bifurcation points, such that for values of λ between any two consecutive members of the sequence λ_n and λ_{n+1} the prime 2^n-period solution is stable, while the periodic solutions of all other periods $2,4,\ldots,2^{n-1}$ become unstable. ■

A4 Proof of Lemma 2

Consider again the function

$$J(k_t)=\frac{z(k_t)}{k_t}=(1-\tau)(1-\gamma)\mathrm{A}\frac{\mathrm{B}\left([(1-\upsilon)\tau\mathrm{A}k_t]^{\varphi}\left(E-\frac{p\mathrm{A}k_t}{1+\upsilon\tau\mathrm{A}k_t}\right)^{\chi}\right)}{1+\mathrm{B}\left([(1-\upsilon)\tau\mathrm{A}k_t]^{\varphi}\left(E-\frac{p\mathrm{A}k_t}{1+\upsilon\tau\mathrm{A}k_t}\right)^{\chi}\right)}. \tag{A4.1}$$

Given the properties of $\mathrm{B}(h_t)$ and the restriction $\tau > p/\upsilon E$, it can be easily established that $J(0)=0$ and $J(\infty)=(1-\tau)(1-\gamma)\mathrm{A}\lambda/(1+\lambda)$. An interior steady state must satisfy $J(\hat{k})=1\Rightarrow \hat{k}=z(\hat{k})$. Therefore, Assumption 1 represents a sufficient condition for the existence of an interior equilibrium. This is because $\mathrm{B}(\infty)=\lambda$ and $\mathrm{B}(h_t)/[1+\mathrm{B}(h_t)]$ is increasing in h_t; therefore $\lambda > \mathrm{B}(\Omega)$. Differentiating (A4.1) yields

$$J'(k_t)=(1-\tau)(1-\gamma)\mathrm{A}\frac{\mathrm{B}'(h_t)}{[1+\mathrm{B}(h_t)]^2}\frac{dh_t}{dk_t},$$

where

$$\begin{aligned}\frac{dh_t}{dk_t}&=\varphi(1-\upsilon)\tau\mathrm{A}[(1-\upsilon)\tau\mathrm{A}k_t]^{\varphi-1}\left(E-\frac{p\mathrm{A}k_t}{1+\upsilon\tau\mathrm{A}k_t}\right)^{\chi}\\&\quad-\chi[(1-\upsilon)\tau\mathrm{A}k_t]^{\varphi}\left(E-\frac{p\mathrm{A}k_t}{1+\upsilon\tau\mathrm{A}k_t}\right)^{\chi-1}\frac{p\mathrm{A}}{(1+\upsilon\tau\mathrm{A}k_t)^2}.\end{aligned} \tag{A4.2}$$

Substituting (A4.2) in the expression for $J'(k_t)$ gives us

$$J'(k_t) = \frac{(1-\tau)(1-\gamma)\mathrm{A}\mathrm{B}'(h_t)}{[1+\mathrm{B}(h_t)]^2}[(1-\upsilon)\tau\mathrm{A}k_t]^{\varphi}\left(E - \frac{p\mathrm{A}k_t}{1+\upsilon\tau\mathrm{A}k_t}\right)^{\chi}\Xi(k_t)\,. \tag{A4.3}$$

where
$$\Xi(k_t) = \frac{\varphi}{k_t} - \frac{\chi p\mathrm{A}}{(1+\upsilon\tau\mathrm{A}k_t)^2}\frac{1}{E - \dfrac{p\mathrm{A}k_t}{1+\upsilon\tau\mathrm{A}k_t}}\,. \tag{A4.4}$$

Obviously, the sign of $J'(k_t)$ depends on the sign of $\Xi(k_t)$ in (A4.4). Particularly, for this to be non-negative, it must be $\Xi(k_t) \geq 0$. After some algebraic manipulation, the inequality $\Xi(k_t) \geq 0$ is reduced to a quadratic expression

$$(k_t)^2 + \frac{(\upsilon\tau E - p) + \left(\upsilon\tau E - \dfrac{p\chi}{\varphi}\right)}{(\upsilon\tau E - p)\upsilon\tau\mathrm{A}}k_t + \frac{E}{(\upsilon\tau E - p)\upsilon\tau\mathrm{A}^2} \geq 0\,. \tag{A4.5}$$

As long as $2\upsilon\tau E > p(\varphi+\chi)/\varphi$, which is true if $\tau > p/\upsilon E$ and $\chi \leq \varphi$, the above expression holds with strict inequality and, by virtue of (A4.3) and (A4.4), $J'(k_t) > 0\ \forall k_t$. Hence, there is only one interior steady state $\hat{k}_2$ with $J'(\hat{k}_2) > 0$. Moreover, it can be easily checked that $J'(\hat{k}_2) > 0 \Rightarrow z'(\hat{k}_2) > 1$, i.e., the interior steady state is unstable.[19]

Next, notice from equation (19) that $z(0) = 0$; hence, $\hat{k}_1 = 0$ is a steady state. Moreover,

$$z'(k_t) = J'(k_t)k_t + J(k_t)\,,$$

and, since from equations (A4.3) and (A4.4)

$$\lim_{k\to 0}\Xi(k_t)k_t = 0 \quad \text{and} \quad J'(k_t)k_t = 0,$$

it follows that $z'(\hat{k}_1) = z'(0) = 0$, i.e., $\hat{k}_1 = 0$ is a super-stable equilibrium. ■

A5 The Dynamics of Environmental Quality

The dynamics in k dictate the dynamics in e. More specifically, the dynamic behaviour of environmental quality e_t is described by the contemporaneous equation $e_t = E - \frac{pAk_t}{1+v\tau Ak_t}$.

Note that e_t is a continuous function of k_t and $de_t/dk_t < 0$. Thus, it follows, from the equation directly above, that if $k_t \to k^*$ then $e_t \to E - \frac{pAk^*}{1+v\tau Ak^*}$. In particular, if $k^* \to 0$ then $e^* \to E$, while if $k^* \to \infty$ then $e^* \to E - \frac{p}{v\tau}$. On the other hand, if k oscillates between two values, say $\hat{k}_1$ and $\hat{k}_2$, where $\hat{k}_2 > \hat{k}_1$, then e oscillates between $\hat{e}_1$ and $\hat{e}_2$, where $\hat{e}_1 = E - \frac{pA\hat{k}_1}{1+v\tau A\hat{k}_1} > \hat{e}_2 = E - \frac{pA\hat{k}_2}{1+v\tau A\hat{k}_2}$. ■

A6 Proof of Proposition 2

Part (i) follows from Lemma 2. Specifically, given that $\hat{k}_1 = 0$ is an asymptotically stable equilibrium and $\hat{k}_2 > 0$ is an unstable one, for any $k_0 < \hat{k}_2$, we have $k_{t+1} < k_t$ for all subsequent steps of the transition. Hence, the economy's stock of capital per worker will constantly decline until it reaches the poverty trap $\hat{k}_1 = 0$.

To prove part (ii), we can use (19) and (20) to write the gross growth rate as

$$\frac{k_{t+1}}{k_t} = 1+\theta_{t+1} = (1-\tau)(1-\gamma)A\frac{B\left([(1-\upsilon)\tau Ak_t]^{\varphi}\left(E-\frac{pAk_t}{1+\upsilon\tau Ak_t}\right)^{\chi}\right)}{1+B\left([(1-\upsilon)\tau Ak_t]^{\varphi}\left(E-\frac{pAk_t}{1+\upsilon\tau Ak_t}\right)^{\chi}\right)}, \qquad \text{(A6.1)}$$

for which Appendix A4 establishes that $k_{t+1} > k_t \Rightarrow 1+\theta_{t+1} > 1$ (as long as $k_0 > \hat{k}_2$), because the dynamics of capital accumulation are monotonic. Therefore, (A6.1) can be written as

$$k_t = \prod_{\varepsilon=0}^{t}(1+\theta_\varepsilon)k_0 . \tag{A6.2}$$

From equation (A6.2) we can verify that $\lim_{t\to\infty} k_t = k_\infty \to \infty$. Therefore, we can use equation (A6.1) to establish that

$$\lim_{t\to\infty}\theta_{t+1} = \theta_\infty =$$

$$\lim_{t\to\infty}\left[(1-\tau)(1-\gamma)\mathrm{A}\frac{\mathrm{B}\left([(1-\upsilon)\tau\mathrm{A}k_t]^{\varphi}\left(E-\frac{p\mathrm{A}k_t}{1+\upsilon\tau\mathrm{A}k_t}\right)^{\chi}\right)}{1+\mathrm{B}\left([(1-\upsilon)\tau\mathrm{A}k_t]^{\varphi}\left(E-\frac{p\mathrm{A}k_t}{1+\upsilon\tau\mathrm{A}k_t}\right)^{\chi}\right)}-1\right]=$$

$$(1-\tau)(1-\gamma)\mathrm{A}\frac{\lambda}{1+\lambda}-1=\hat{\theta} .$$

Since $(1-\tau)(1-\gamma)\mathrm{A}\lambda/(1+\lambda)>1$ (from Assumption 1), $\hat{\theta}>0$: asymptotically, the economy will converge to a balanced growth path where capital per worker grows at a rate $\hat{\theta}$. ■

A7 Proof of Proposition 3

If there is no pollution abatement, i.e., $\upsilon = 0$, the planar system of equations (24) and (25) simplifies to

$$k_{t+1} = \frac{(1-\tau)(1-\gamma)\lambda\tau\mathrm{A}^2 k_t^2\left[(1-\eta)E+\eta m_t - p\mathrm{A}k_t\right]}{1+(1+\lambda)\tau\mathrm{A}k_t\left[(1-\eta)E+\eta m_t - p\mathrm{A}k_t\right]},$$

and
$$m_{t+1} = (1-\eta)E+\eta m_t - p\mathrm{A}k_t .$$

As for the steady-state loci, we have

$$m = \frac{1}{\eta}\left\{\frac{1}{\left[(1-\tau)(1-\gamma)\mathrm{A}\lambda-(1+\lambda)\right]\mathrm{A}\tau k}-\left((1-\eta)E-p\mathrm{A}k\right)\right\}, \tag{A7.1}$$

and $$m = E - \frac{1}{1-\eta} p\mathrm{A}k. \tag{A7.2}$$

Combining equations (A7.1) and (A7.2) we find that the steady-state values of k are the solutions of the quadratic equation

$$\left[(1-\tau)(1-\gamma)\mathrm{A}\lambda-(1+\lambda)\right]p\mathrm{A}^2\tau k^2-\left[(1-\tau)(1-\gamma)\mathrm{A}\lambda-(1+\lambda)\right](1-\eta)E\mathrm{A}\tau k+1-\eta=0.$$

This equation does not have a real root if $4p>\left[(1-\tau)(1-\gamma)\mathrm{A}\lambda-(1+\lambda)\right](1-\eta)E^2\tau$. On the other hand, if p is sufficiently low, it has two positive roots. ii) This follows right away from the limiting properties of equations (26) and (27) (see also Figure 6). ■

A8 Existence of a Limit Cycle when e is a Stock Variable

If we use the following values: $\tau = 0.4, \gamma = 0.4, p = 0.3, \mathrm{A} = 25, E = 10, v = 0.5,$ and $\eta = 0.8$, there are two steady-state equilibria. For example, if $\lambda = 0.4$, then the two equilibria are $(k_1, m_1) = (0.005, 9.827)$ and $(k_2, m_2) = (0.262, 0.173)$. Moreover, when $\lambda = 0.4$, at both equilibria the eigenvalues are real; one of them has modulus greater and the other less one. Thus, both equilibria are saddle-path stable.

Nevertheless, the stability properties of the equilibria change with λ. In particular, at $\lambda = 0.174$ there are also two equilibria: (0.029, 9.829) and (0.238, 1.071). Evaluated at the second equilibrium, the system has two complex eigenvalues, $0.817 \pm 0.576i$, which have modulus 1, i.e., the equilibrium is non-hyperbolic. Neither of these eigenvalues is a second, third, nor fourth root of unity.[20] Moreover, the derivative of the modulus of the eigenvalues, evaluated at $\lambda = 0.174$, is equal to 1.468. It follows then from the Hopf Bifurcation Theorem (see Azariadis 1993, Theorem 8.5, p. 100) that there exists a limit cycle in the neighbourhood of the equilibrium for either $\lambda > 0.174$ or $\lambda < 0.174$. To find out whether the cycle emerges for values of λ higher or lower than 0.174 we need to apply some additional rather technical tests (see Devaney 2003, Theorem 8.8, p. 249). Nevertheless, using numerical methods, we can find that the cycle emerges for values of λ > 0.174. For example if $\lambda = 0.4$, then there is a two-period cycle: (0.202, 0.075) and (0.315, 0.547). Moreover, since the steady-state equilibrium $(k_2, m_2) = (0.262, 0.173)$ surrounded by the cycle is unstable in the saddle-path sense, it follows that the cycle is attracting. Hence, given that $m_{t+1} = e_t$, the economy converges to an equilibrium where it oscillates between $(k, e) = (0.202, 0.547)$ and $(k, e) = (0.315, 0.075)$. Just as in Section 3, where e was a flow variable, low (high) capital stock implies low (high) pollution and high (low) environmental quality. Finally, numerical investigations for a wide range of parameter values can show that cycles of higher periodicity may also emerge. ■

A9 Proof of Proposition 4

i) If $\varphi > \chi$ then it follows from (33) that $z(0) = 0$, $z(\infty) = \infty$, and $z'(k_t) > 0$. In addition the gross growth rate $J(k_t) = z(k_t)/k_t$ exhibits the following properties: $J(0) = 0$, $J(\infty) > 1$, by virtue of Assumption 1, and $J'(k_t) > 0$. Using then arguments similar to those in the proof of Lemma 1, the result follows. ii) In the case where $\chi > \varphi$ we have to distinguish between two cases. Case 1: $1 + \varphi > \chi > \varphi$. Straightforward calculations show that $z(0) = 0$, $z(\infty) = \infty$, and $z'(k_t) > 0$. Moreover, $J(0) > 1$, by virtue of Assumption 1, $J(\infty) = 0$ and $J'(k_t) < 0$. Hence, there exists a unique positive steady-state equilibrium, which is stable. Case 2: $\chi > 1 + \varphi$. In this case, $z(0) = 0$, $z(\infty) = 0$, while initially $z'(k_t) > 0$ and then $z'(k_t) < 0$. Also, $J(0) > 1$, $J(\infty) = 0$ and $J'(k_t) < 0$. If the 45 degree line intersects the phase line $k_{t+1} = z(k_t)$ on the upward sloping part, then the equilibrium is stable and the economy approaches monotonically. If the equilibrium lies on the downward sloping part and

$z'(k) > -1$ then the economy approaches it with damped oscillations. Finally, if the equilibrium lies on the downward sloping part of the phase line and $z'(k) < -1$ then the equilibrium is unstable. In this case the economy will approach a stable limit cycle.[21] ■

A10 Proof of Proposition 5

From (32a) and (32b), the gross growth rate is given by

$$J(k_t) = \frac{k_{t+!}}{k_t} = \frac{z(k_t)}{k_t} = \frac{(1-\tau)(1-\gamma)\mathrm{A}\lambda[(1-\upsilon)\tau\mathrm{A}]^{\varphi}}{\pi(k_t)+(1+\lambda)[(1-\upsilon)\tau\mathrm{A}]^{\varphi}},$$

if $k_t < \varsigma$, where $\pi(k_t) = (p\mathrm{A} - \delta k_t^{\varepsilon-\sigma-1})^{\chi} k_t^{\chi-\varphi}$, and $\delta \equiv p^{-\sigma}(\upsilon\tau)^{\varepsilon}\mathrm{A}^{\varepsilon-\sigma}$, and by

$$J(k_t) = (1-\tau)(1-\gamma)\mathrm{A}\frac{\lambda}{(1+\lambda)} > 1,$$

if $k_t \geq \varsigma$ (recall that $\varsigma \equiv [p\mathrm{A}/\delta]^{1/(\varepsilon-\sigma-1)}$, $\delta \equiv p^{-\sigma}\mathrm{A}^{\varepsilon-\sigma}(\upsilon\tau)^{\varepsilon}$, is the solution to the equation $D_t = 0$). It follows that $J(0) = \frac{(1-\tau)(1-\gamma)\mathrm{A}\lambda}{1+\lambda} > 1$, $J(k_t) = \frac{(1-\tau)(1-\gamma)\mathrm{A}\lambda}{1+\lambda} > 1$ $\forall k_t \geq \varsigma$.

Moreover, $J'(k_t) = -\frac{(1-\tau)(1-\gamma)\mathrm{A}\lambda[(1-\upsilon)\tau\mathrm{A}]^{\varphi}}{\{\pi(k_t)+(1+\lambda)[(1-\upsilon)\tau\mathrm{A}]^{\varphi}\}^2}\pi'(k_t)$, $\forall k_t < \varsigma$ and $J'(k_t) = 0$ $\forall k_t \geq \varsigma$, where $\pi'(k_t) = (p\mathrm{A} - \delta k_t^{\varepsilon-\sigma-1})^{\chi-1} k_t^{\chi-\varphi-1}\{(\chi-\varphi)p\mathrm{A} - [\chi(\varepsilon-\sigma)-\varphi]\delta k_t^{\varepsilon-\sigma-1}\}$. Let $\omega \equiv \{(\chi-\varphi)p\mathrm{A}/[(\chi(\varepsilon-\sigma)-\varphi)\delta]\}^{1/(\varepsilon-\sigma-1)}$. Note that $\omega < \varsigma$. Hence, $\pi'(k_t) > 0$ and $J'(k_t) < 0$ $\forall k_t \in (0,\omega)$, $\pi'(k_t) < 0$ and $J'(k_t) > 0$ $\forall k_t \in (\omega,\varsigma)$ and $\pi'(k_t) = J'(k_t) = 0$ $\forall k_t \geq \varsigma$.

Given the previous analysis, we can distinguish between two possible scenarios regarding the economy's dynamics and long-run equilibrium. First, consider the case where $\frac{(1-\tau)(1-\gamma)\mathrm{A}\lambda[(1-\upsilon)\tau\mathrm{A}]^{\varphi}}{\overline{\pi}+(1+\lambda)[(1-\upsilon)\tau\mathrm{A}]^{\varphi}} > 1$, $\overline{\pi} \equiv \pi(\omega)$. If this condition holds, then $J(k_t) = \frac{k_{t+1}}{k_t} > 1$ $\forall k_t$, i.e., the economy will grow at positive rates (see Figure A2). The second possibility regarding the economy's dynamics is when $\frac{(1-\tau)(1-\gamma)\mathrm{A}\lambda[(1-\upsilon)\tau\mathrm{A}]^{\varphi}}{\bar{\pi}+(1+\lambda)[(1-\upsilon)\tau\mathrm{A}]^{\varphi}}<1$. Taking account of previous results, we have two steady-state solutions, say k^{LOW} and k^{HIGH} ($k^{LOW}<k^{HIGH}$), such that $J(k^{LOW})=J(k^{HIGH})=1$, $J'(k^{LOW})<0$ and $J'(k^{HIGH})>0$. In this case, k^{HIGH} emerges as a threshold above which the economy will grow at positive rates in the long-run: once more, pollution abatement acts as a source of long-run growth and stabilisation in economies for which $k_0>k^{HIGH}$. ■

INSERT FIGURE A2 ABOUT HERE

Endnotes

1. There is a large number of existing theoretical analyses that incorporate endogenous longevity in dynamic general equilibrium models. See, among others, Chakraborty (2004); Bhattacharya and Qiao (2007); and Strulik and Weisdorf (2014).
2. Other analyses on the environment-growth nexus include Prieur *et al.* (2013), Prieur and Brechét (2013) and Vella *et al.* (2014).
3. Although life expectancy is higher and mortality rates are lower in more developed countries, this negative relation between income and mortality does not appear to hold for a given country and within shorter intervals. In fact, empirical evidence suggests that mortality rates are procyclical. Such evidence is presented by Ruhm (2000); Chay and Greenstone (2003); and Rolden *et al.* (2014), among others. What is of particular interest to our analysis is that some of these investigations argue that pollution can be an important explanatory factor for the procyclicality of mortality rates – both for the cases of infant (Chay and Greenstone 2003) and adult mortality (Rolden *et al.* 2014). In our framework, in the presence of limit cycles, mortality rates co-move with output.
4. Ono (2003) introduces environmental quality in a model of cycles and growth. The implication for environmental policy in his framework is different from ours in that he obtains a critical level of tax above which higher growth and improved environmental quality may actually require a less stringent environmental policy, that is, a reduction in abatement efforts.
5. In Smulders and Gradus (1996), pollution declines constantly along the balanced growth path because abatement is sufficiently strong. In one version of our model, abatement can only reduce the rate of environmental degradation. As a result, pollution increases even in the presence of abatement efforts.

6. An agent's expected lifetime at birth is equal to $\beta_t 2 + 1 - \beta_t = 1 + \beta_t$ periods (recall that she lives 2 periods with probability β and 1 with probability $1-\beta$). For this reason, we shall be using such terms as 'life expectancy', 'longevity' and 'survival probability' interchangeably. In fact, an alternative interpretation is that in principle all agents survive to the second period, but are alive only a fraction $\beta = \mathrm{B}(h_t) \in [0,1)$ of the period as, for example, in Bhattacharya and Qiao (2007).
7. Note that the health status is a flow and not a stock variable. Although a health stock would be more appropriate in an environment where agents live for three or more periods, our current assumption seems more suitable in a setting where an agent's potential lifetime is divided in two broad periods. Of course, even under a two period setting, one can argue that a health stock may make sense once we consider the intergenerational transmission of genetic attributes. Such issues, however, go way beyond the scope of our paper; that is why we have decided to abstract from them.
8. Further support for this idea is provided by Balestra and Dottori (2012), who argue that "it seems reasonable to assume that health expenditure and environmental quality are only imperfect substitutes and exhibit some complementarity to effectively improve health status" (p. 1068).
9. A similar assumption is used in the paper by Clemens and Pittel (2011), namely, $\tilde{p}_t = p / a_t$. Our functional form has the advantage that it eliminates the possibility of infinite pollution when $a_t = 0$.
10. In the working paper version of the manuscript we also analyze the case where $e_t = e_{t-1}^{\eta}(E - P_t)^{1-\eta}$. Our results remain qualitatively the same.
11. In Appendix A3, we present an example of an economy with cycles, which illustrates further the results of Proposition 1.

12. By virtue of Assumption 1, $\hat{\theta} > 0$.

13. Note that $\lim_{k_t \to \infty} e_t = \lim_{k_t \to \infty}\left(E - \frac{P_t}{1+a_t}\right) = \lim_{k_t \to \infty}\left(E - \frac{p\mathrm{A}k_t}{1+\upsilon\tau\mathrm{A}k_t}\right) = E - \frac{p}{\upsilon\tau} > 0$, if $\tau > p/vE$.

14. We note that E in John and Pecchenino (1994) is equal to zero. This is so because, in their model, the index of environmental quality is allowed to take negative value. Given the role of environmental quality (*e*) in our model, we impose that it takes non-negative values. Moreover, one can easily show that if $E = 0$, then Assumption 1 and the assumptions embedded in Lemma 2 and Proposition 3 below are all violated.

15. We thank a referee for suggesting these alternative specifications to us.

16. Upon reaching $k = \varsigma$ (where $D_t = 0$) the agents' health status achieves its maximum possible level. From there on, as long as the government spends enough on pollution abatement to maintain $D_t = 0$, there is no need to spend resources on health services (more formally, it could just spend an infinitesimal amount). Nevertheless, we assume that the government continues to tax private incomes at the rate τ and spends the resources over what is needed to maintain $D_t = 0$ on health services or perhaps on other non-productive activities. Needless to say, our argument, that pollution abatement is a source of long-run growth, holds **all the more** in the case where the government reduces the tax rate so that it raises enough resources every period just to maintain $D_t = 0$.

17. We thank an anonymous referee for suggesting this extension.

18. In the working paper version, we consider the optimal allocation of public spending between direct health expenditures and pollution abatement. Particularly, we examine the choice of υ that maximises the lifetime utility of a representative generation for the

model presented in Section 2. Gutiérrez (2008) provides a formal treatment and analysis of the intergenerational issues.

19. The restriction $\chi \leq \varphi$ is sufficient but not necessary for the results of Lemma 2 and Proposition 2 below. Effectively, it ensures that only one endogenous threshold separates the two opposite convergence scenarios. In the working paper version, we show that when this assumption is relaxed, it is possible that more equilibria emerge between the poverty trap and the long-run growth equilibrium. Nevertheless, the implication regarding the economy's ability to sustain a positive growth rate in the long-run remains intact.
20. An n-th root of unity, $n = 1, 2, \ldots$, is a complex number z satisfying the equation $z^n = 1$.
21. For example, if $p = 0.25, \mathrm{A} = 5, \gamma = 0.2, \tau = 0.1, \chi = 5, \varphi = 0.3, \lambda = 0.95,$ then there exists a 2-period cycle $(k_1, k_2) = (0.772, 0.862)$.

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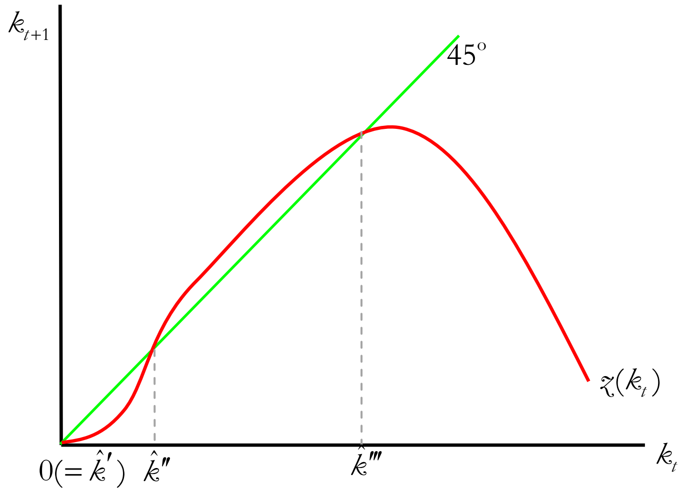

Figure 1. $\upsilon = 0$ and $0 < z'(\hat{k}''') < 1$

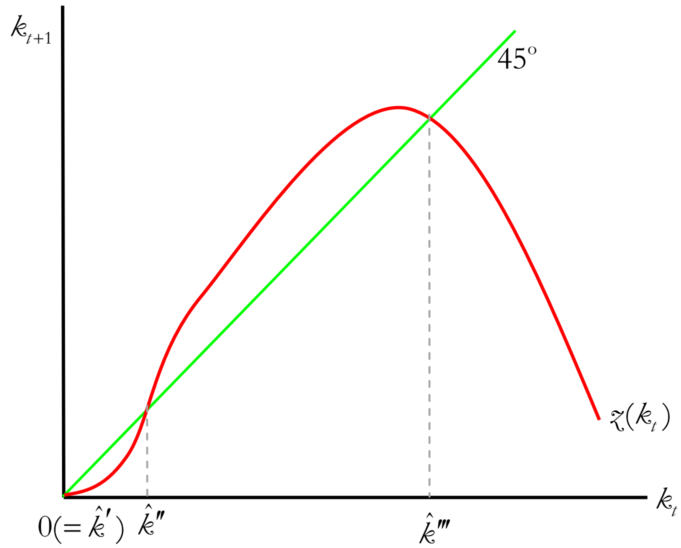

Figure 2. $\upsilon = 0$ and $-1 < z'(\hat{k}''') < 0$

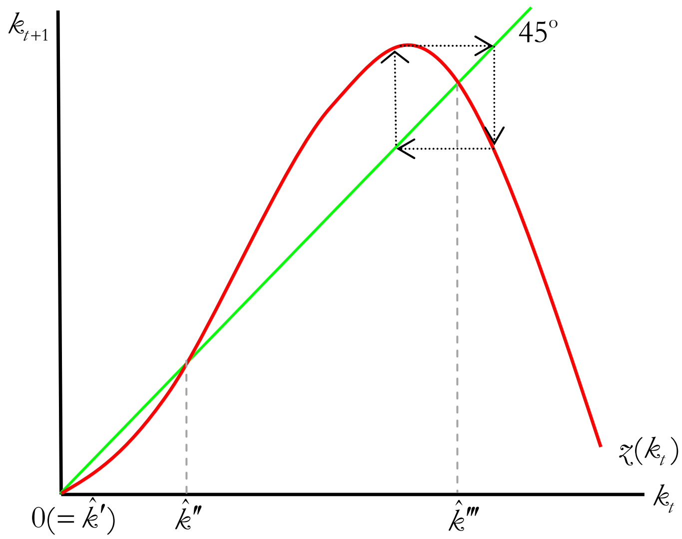

Figure 3. $\upsilon = 0$ and $z'(\hat{k}''') < -1$: an example with a period-2 cycle

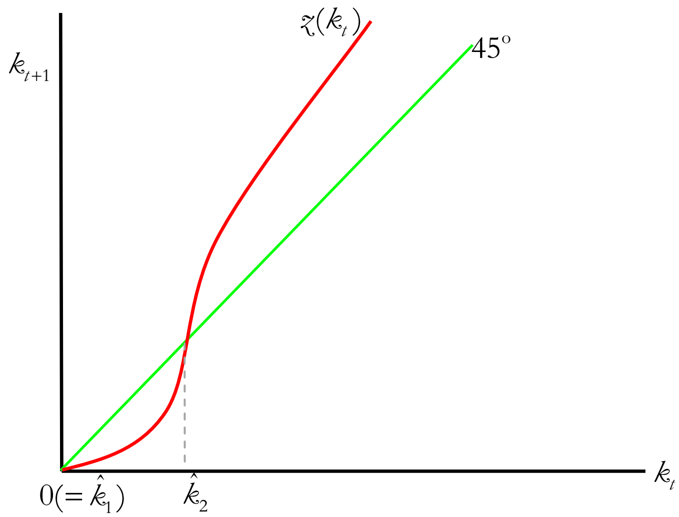

Figure 4. $0 < \upsilon < 1$

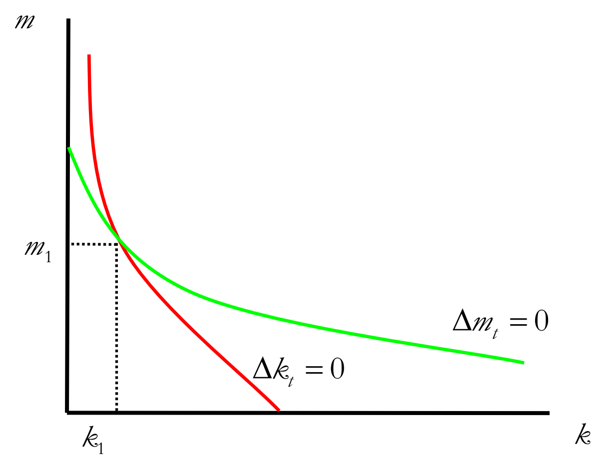

Figure 6. Pollution abatement $(\upsilon > 0)$ with environmental quality as in eq. (22)

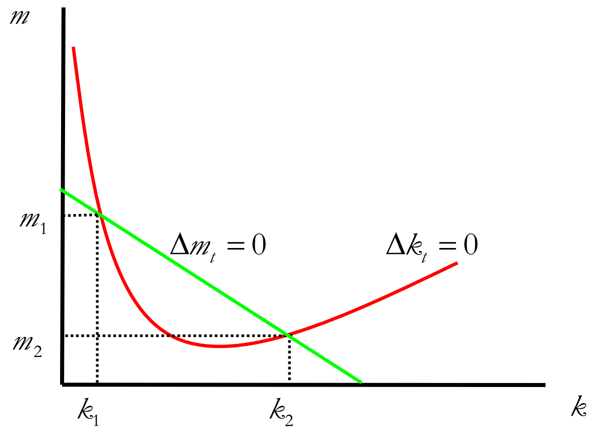

Figure 5. Multiple equilibria with environmental quality as in eq. (22) and $\upsilon = 0$

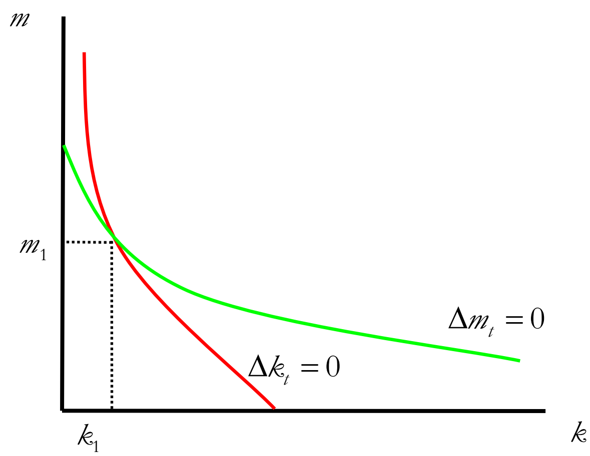

Figure 6. Pollution abatement $(\upsilon > 0)$ with environmental quality as in eq. (22)

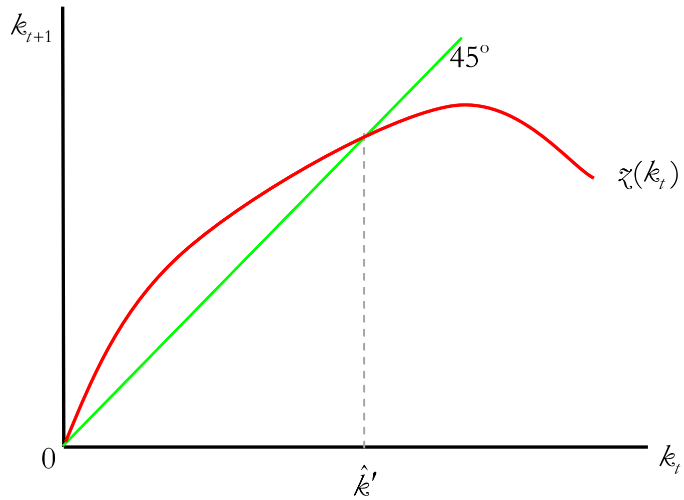

Figure 7. $\upsilon = 0$, $\chi > \varphi$ and $0 < z'(\hat{k}') < 1$

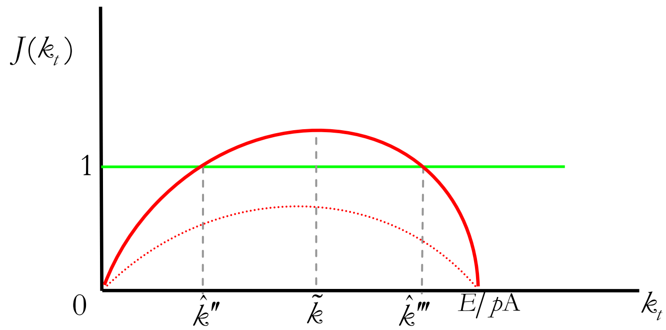

Figure A1. The existence of interior steady states require $J(\tilde{k}) > 1$

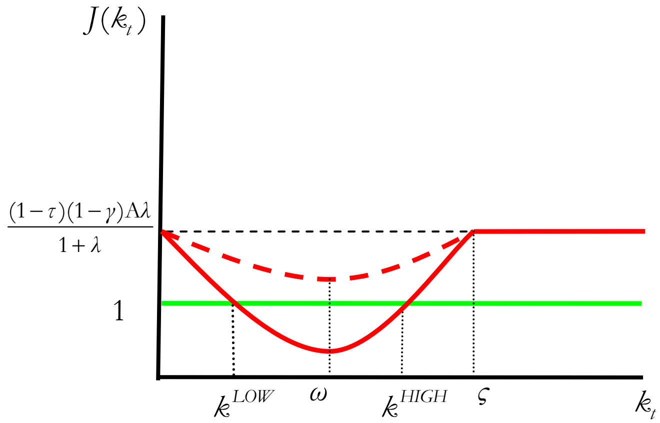

Figure A2. The existence of interior steady states requires $J(\omega) < 1$