UNIVERSITY OF
LEICESTER

Modelling the stock market using a multi-scale approach

Thesis submitted for the degree of Master of Philosophy at the University of Leicester

By

Fan Li

School of Management

University of Leicester

March 2017

Abstract

Mathematical modelling is one of the fundamental elements in the modern financial industry, playing vital roles in terms of decision making, risk management, financial innovation, and government regulation. The financial market has attracted extensive research interest from academics due to its strategic importance to the global economy. With the increased understanding of the market, the financial industry has developed towards a direction of a structured and refined system, thanks to a large number of financial instruments carefully engineered to meet the demand from the investors. On the other hand, fundamental research such as risk modelling remains challenging. The advancement of financial models does not prevent the market from financial crisis or mitigate the consequence of crash. Bearing in mind that modelling is an abstract of reality, the current research takes a step back to examine one of the corner stones of financial modelling: the efficient market hypothesis, the Gaussian statistics and the Brownian motion, as well as the process of data analyses for the modelling inputs. Using the context of the stock market, a systematic approach is adopted based on a variety of data from different stock markets during different periods. Some interesting statistical findings are presented in a quantitative manner, providing both the confirmation of the non-Gaussian statistics and empirical understandings to the market movements. Meanwhile, two different research methodologies are adopted to model the empirical findings: 1) macroscopic and phenomenological modelling based on analysing statistical data, and 2) microscopic and mechanism-based modelling based on understanding the behaviours of the market players. By taking advantage of both modelling methodologies, a multi-scale modelling approach is proposed in the current research. A step by step method is used to pin down the essential mechanisms that lead to the market inefficiency and non-Gaussian statistics. It is shown that the proposed approach requires a small number of input parameters by maximising the information obtained from market performance data and market microstructure. It is also shown that the multi-scale modelling approach, facilitated by a systematic empirical study, will greatly enhance both our understandings on the micro-foundations of the stock market and the applicability of the classical models widely used by the modern financial industry.

Key words: financial modelling, stock market, risk modelling; risk management; multi-scale modelling; agent based method.

Acknowledgements

List of Figures

List of Tables

Table 2.1 Comparing risk measures using different VaR limits, between empirical distributions, normal distribution using statistical volatility and normal distribution using modelled volatility.

Table 2.2 The slopes of the curves in Figure 2.10 (the value for Gaussian process equals -0.5; the value found by Mantegna and Stanley (1995) is equal to -0.712 ± 0.025)

Table 3.1 Values for normalising the empirical distributions and inputs of the analytical distributions

Table 4.1 Values of volatilities that are used in Figures 4.2-4.5.

Contents

Chapter 1 Introduction – the modern financial market

1.1 The evolution of the financial market

1.1.1 Overview

Over the past 50 years, the financial market has grown considerably in terms of size, geographical regions and transaction volume, as well as its significance to the global economics as a whole. For example, the Standard and Poor's 500 Stock Index (S&P 500) has grown in size 50 times over between 1970 and 2010, and the daily transaction volume has increased over 80 times (normalised by inflation, source: S&P and Capital IQ, 2012). Similar figures can be found in other major stock indices, such as FTSE 100 and NASDAQ. In terms of functionalities, too, the financial markets have moved a long way, and include trading (e.g., stocks and bonds, commodities and foreign exchanges), lending (e.g., money market), risk management (insurance, derivatives, swaps and future market). From the macro-economical perspective, the financial market affords huge benefit to economics, such as saving mobilisation, national wealth growth, entrepreneurship and industrial development. Many countries have nowadays adopted the strategic view that the financial market is the "barometer of the economy".

At the same time, those who participate in the financial market demand access to a wider range of investment opportunities, and flexibility of trading between different markets. Such demands have been met by development of financial services and financial products. Apart from the traditional ones - such as mortgages, loans, pensions, etc. - in recent years, the global financial market has experienced a huge boom in the financial service industry. Some examples of the financial products include money market funds, NOW accounts, ATMs, index mutual funds, ETFs, tax-exempt funds, emerging-market funds, target-date funds, floating rate funds, volatility derivatives, inflation protection securities, equity REITs, asset-back securities, Roth IRAs, zero-coupon bonds, financial and commodity futures and options, and many more (Malkiel, 2011).

1.1.2 The role of financial research

It was not until the 1950s that the study of financial market became a defined research field, separate from the study of economics (Fox, 2011). The hallmark of the financial research is the extensive utilisation of mathematical modelling techniques. The financial market has since drawn much attention of academics from both natural and social sciences. The ability to predict numerically the return on investments makes modelling based decision making a popular practice in the financial sector, particularly thanks to technological development, including the electronic trading system and mass-storage devices. Each financial contract, which records the prices traders are willing to buy or sell, is available on a real-time basis to the rest of the market, and this enormous dataset has triggered a huge amount of study including modelling and analytics. In the past 50 years, the financial sector has witnessed numbers of attempts to understand and predict the market: from the earlier chartists and technicians who dedicated their efforts to discover statistical trends, to the fundamentalist who believes that price is always rationalised by the intrinsic value; from the purists' theory that the "bubble can't be detected until it bursts", to the behaviour finance which concludes that human's investment activities can be extreme and irrational; from the random walkers who believe that no speculators can beat the market consistently, to the modern portfolio theory focusing on risk measurements and risk management.

There are a number of reasons as to why financial research has been so popular among academics, who are deemed not to be the social group chasing profit and speculating for short term gains. Theoretically, a financial market is both well-defined and with a large number of uncertainties - the research of which also implies huge socio-economic impact. Using stock markets as an example: they constitute a relatively self-contained system that can be described using mathematical models, but they do have enough unknowns to drive sufficient research interest. Comparing with items traded in other markets, such as consumables, those stocks can be relatively easily described mathematically using clear attributes such as prices and volumes of shares. Those attributes enable scientific calculations as well as further mathematical deduction, leading to models that require moderate levels of assumption. A perfect example is the Black-Scholes option pricing model (see e.g., Franke et al., 2008). On the other hand, the dataset that describes every detail of the market makes it attractive to statistical analysts. Those datasets are normally sufficient and complete, therefore with minimum bias. In the modern financial market, statistical studies are extensively used in risk analysis and modelling (e.g., portfolio selection and credit rating).

2

1.1.3 Some research results and applications

One of the foundations of much financial research is the rational market theory (see e.g., Muth, 1961), with the best known element being the efficient market hypothesis. Formalised in the 1960s at the University of Chicago, and with reference to the US stock market, the hypothesis states that the financial price always justifies itself and is as unpredictable as a completely random process. As a scientific constructivist approach, the efficient market hypothesis may be a "simplification and abstract of reality"; however, the idea of an efficient market is so powerful that it forms the basis of the modern portfolio theory which has revolutionised the mutual fund service, the risk-adjusted performance measure that shaped the money management business, the rise of financial derivatives, as well as the structured finance and even the free market ideology (Fox, 2011). Particularly in the 1970s and 1980s, the thinking rooted from efficient market hypothesis became increasingly influential. It was regarded as an advanced practice by which governments could promote competition while removing regulation boundaries. An example is the Depository Institutions Deregulation and Monetary Control Act in 1980 (see e.g., Millon-Cornett and Tehranian, 1989) in the US, which provided a basis of removing limitations on the interest rates of retail banking. It is generally believed that competition in the market is far more efficient than government intervention. The theoretical description of the financial market and deregulation, together, promote innovative financial products (such as securitisation and default swaps, which will be discussed in a later context) which have added huge value to the financial sector and the global economy.

The dark side of the moon is the financial crisis. Modern financial crises are generally of large scale and heavy impact, and are both unpredictable and periodical. The first ten years of the 21st century are often referred to as the "lost decade" for the stock market, due to a number of major financial crises. The financial market made its appearance in the spotlight too frequently for the wrong reasons. A recent example is the mortgage crisis in the years 2007-2008. The US housing market's bubble burst; this caused severe disruption to the global economy and long term recession. There are many published research papers and books discussing the crisis (e.g., Buiter 2007; Lowenstein 2010). The root causes are largely to do with the financial sector and can be generally regarded as: the 1) the ability and freedom of shifting risk (e.g., shadow banking system); 2) the loosened of leading standard (e.g. the

subprime market); 3) the imperfection of quantitative financial instruments, such as the credit rating system; 4) the lack of regulatory experience and political reasons (e.g., it was argued that promoting house ownership would strengthen democracy); and 5) human factors (e.g., wrong incentives, culture and fraud).

1.1.4 Future research trends

These financial crises have led to more de-constructivism and critical views on the modern financial market, questioning fundamentals of the structured financial system. It is argued that more than 100 years' trading experience can't be captured by mathematical modelling alone, and that the market is a "devilish thing", and that "much is lost" by using a simplified theory such as the efficient market hypothesis. Any mathematical model has its domain of applicability; those scholars who came up with the hypothesis did not "mean for it to be taken as a literal description of reality" (Fox, 2011). It is, however, extremely difficult, if not impossible, for anyone to control how and under what context those models are applied, especially in an environment of market competition. For example, in the mortgage crisis of 2007-8, it may be perfectly justifiable using mathematical models to issue mortgage loans to higher-risk applicant groups, as long as the risks are compensated by charging a higher interest rate. However, when the risks materialised, the domino effect soon proved too powerful to have been captured by any known risk model, or to be controlled by any known risk management framework.

Human behaviours have also been extensively studied in the context of the financial market. This has posed some fundamental challenges to the efficient market hypothesis: 1) individual investors may be less than rational, but those behaviours are random, therefore cancel each other out in the sense of the whole market; 2) even where irrational behaviours are not random - i.e., have some trends - the efficient market will ensure those mispricings are corrected by professional and smart investors, i.e. through arbitrage opportunities. However, those behaviourists (who study behaviour finance) have found that many financial market investors are far from fully rational (Sewell, 2007), and that the market as a whole can be far from inefficient. This is because: 1) trades of investors can be highly irrational and correlated; and 2) there are substantial barriers to efficient arbitrage in the modern financial market. Behaviour finance takes this a step further to quantify those irrational behaviours; this will also be discussed further in a later part of the thesis.

1.1.5 The structure of this thesis

The thesis is structured in the following way. The rest of this chapter will be discussing the main elements in the financial industry and the financial crisis, with the emphasis on financial modelling principles and techniques. In Chapter 2, the system of the stock market is described, together with its mathematical description, empirical characteristics, as well as some models that are widely adopted in the industry. In Chapter 3 some recent advancements in the field of mathematical modelling are introduced. Such models are then compared with those standard models in the context of the stock market, and also compared with the empirical evidence. Chapter 4 will discuss microscopic modelling, focusing on the investors' behaviours and the impact on the market as a whole. A numerical model is built, incorporating a number of microscopic mechanism and evaluate their effect on the financial market. The conclusions of the thesis can be found in Chapter 5, together with future work and insights for financial engineering, regulation, and risk management.

1.2 Financial risks and a structured financial market

1.2.1 The evolution of the concept of risk

There has been a revolution in the understanding and perception of risk in the past two decades. When economic growth is driven by the financial sector, the concept of risk is exposed to more market participants and has received much attention. Financial products, such as insurance and derivatives, are utilised to slice large risks into affordable chunks, and to spread them to those who are willing to take risks (D'Arcy and Brogan, 2001). It becomes a common belief that "risks are no longer the dark side of profits, they are also market opportunities" (Beck, 1992). Furthermore, the terminology of risk has also moved much beyond financial risk. Twenty years ago, the job of a risk manager of a company was typically a low-level position in charge of insurance purchasing (Nocco and Stulz, 2006). Nowadays, risk is on the table in almost every area of the economy – including operational risk, reputational risk, strategic risk, environmental risk, and many more. "Risk management of everything" (Power, 2004) has become a new field of management research and shapes modern organisational design. It is generally agreed that "risk cannot be reduced to zero", and that "there is a cost to reducing risk" (HM Treasury, 2006). In contrast, "taking risk strategically" is a positive concept which looks at risks not only for the potential loss, but also focusing on the potential benefit.

1.2.2 Risk and performance

Risk itself is a most slippery and elusive concept (Malkiel, 2011). The American Heritage Dictionary defines risk as "the possibility of suffering harm or loss". Due to the fluctuation of the financial market, financial risks are often illustrated using the long term mean and standard deviations of the price movements. A financial security whose future return is more likely to deviate from the expected value is said to have a higher risk. This definition enables the quantification of financial risks: the future returns are generally uncertain and can be described using probabilistic models. The well-defined statistical terms of mean and standard deviation can be used to measure the risk levels, and can be achieved by performing statistical studies on the historical price return data. Figure 1.1 shows one of the studies on financial risks from Gibson (2013), using data from 1926 to 2009 and covering different areas in the financial market including stocks, bonds and treasury bills. It is assumed (and also supported by a large number of empirical studies) that, for a portfolio containing a large number of securities, the long term returns follow a symmetric distribution. That is, the annual gain and loss shown in Figure 1.1 are symmetric and the magnitudes are measured by the standard deviation. If the investors can survive some short term (annual) loss in the stock market, they will be more likely to see long term reward in the stock market as a reward of bearing the risk than buying US treasury bills, which is essentially risk-free. This leads to one of the most celebrated risk-related propositions in the financial market: on average, investors receive higher returns by bearing a greater level of risk.

1.2.3 The quantification of risks

The mathematical description closely related to mean and standard deviation is the random walk theory. It assumes that the financial price return is unpredictable; the near future price of a financial security has no dependence on the historical price, and has the same chance of going up as going down. The random walk models the financial price return using the Brownian motion, which can be described by the normal distribution in conjunction with a Gaussian diffusion process. The random walk theory is intrinsically linked with the efficient market hypothesis and has been tested by many empirical studies (e.g., Fama, 1959). It also has a mathematically closed form with calculus for analytical deduction. One of the features of the model is the explicit measure of financial risk - generally regarded as an intrinsic property for a single financial security or the whole financial market. The random walk

model provides a mathematical instrument to the financial market with an important risk measure. It enables a number of widely-used analytical and modelling studies in the financial industry. One of the most influential applications is the modern portfolio theory (MPT) (Markowitz, 1952).
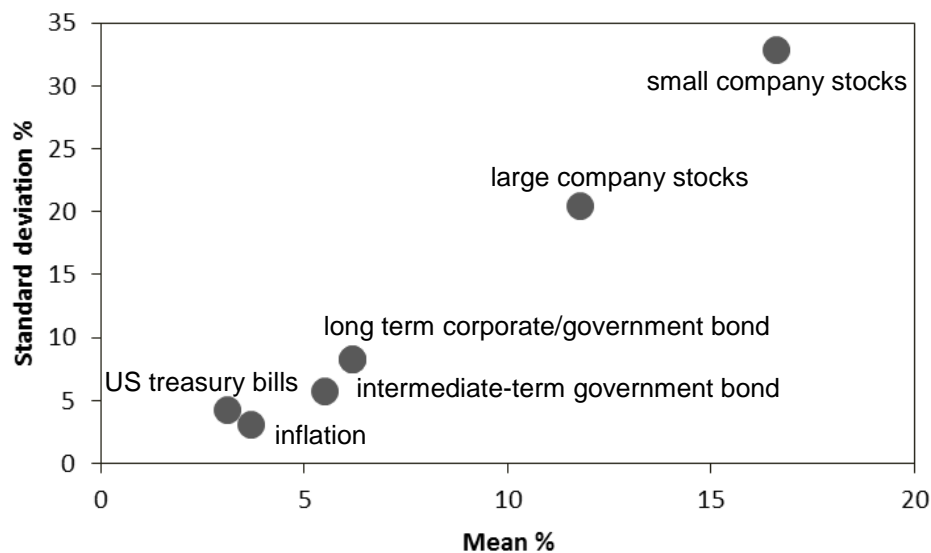
Figure 1.1 Summary of statistical indicators of annual returns from 1926 to 2009 (source: Gibson (2013)

1.2.4 The modern portfolio theory

The modern portfolio theory (MPT) accepts the fact that the financial return between different securities can be (either positively or negatively) correlated. Such correlation can be used to mitigate financial risk by carefully selecting a number of financial securities to form a portfolio. The risk reduction can be calculated with known correlation of the price returns. At first glance, the existence of correlation in the market is somehow incompatible with the efficient market hypothesis: the future price is unpredictable, but the correlation between the prices is not. The rationales are: 1) it is well known that different industries and sectors can be both positively and negatively correlated, due to those intrinsic characteristics of the industries. A famous example is that gold is normally considered as a safe haven of the equity market during financial crises (see e.g., Chiang *et al.* 2013); 2) for a large selection of diversified portfolios, empirical evidence suggests that there are always correlations to be identified at some stage during a sustained period (e.g., Stivers and Sun, 2002); and 3) even if

there is zero correlation between the financial securities, some risk mitigation is still possible (Malkiel, 2011). It is widely accepted that using the modern portfolio theory is one of the most efficient ways of risk mitigation: at any time in the future, loss from one security may be compensated by gain from another security. If the portfolio is well selected, e.g., from a large variety of sectors and global financial markets, the total risk can be reduced by over 60 percent (Malkiel, 2011).

1.2.5 The capital asset pricing theory

The risk mitigation from the MPT approach and risk bearing (closely linked with rate of return, as illustrated in Figure 1.1) can be further articulated in conjunction with the Capital Asset Pricing Model (CAPM) (Sharpe, 1964). The CAPM posits that the financial risk consists of some systematic risk such as interest rate, economic recession, etc.; that they can't be mitigated by diversification of investment; and that only bearing the systematic risk will lead to a higher rate of returns. The concept of CAPM is intuitive: a) not all risks can be mitigated by the approach introduced by MPT - there are risks intrinsically linked with the global economy and the characteristics of the underlying security or portfolio; b) if bearing risks that can be mitigated using portfolio generates a higher rate of return, investors will work against the portfolio selection and the investment opportunity will disappear. Furthermore, the CAPM quantifies such systematic risk using the "beta measure" (see e.g., Mullins, 1982), which reflects the "sensitivity" between the systematic risk of individual security or the portfolio and the risk of the whole market. The beta measure provides a single index on the risk with theoretically sound justifications, and is widely adopted in the financial market. Using the beta measure, investors are able to determine whether or not to accept the extra risk to pursue the higher future market return, i.e., the risk premium of the equity market. For example, mutual fund managers who believe they have predictability of the market will hold high beta securities when they think the market is rising, while switching to low beta securities when they think the market is falling.

From a quantitative prospective, the beta measure also provides the input of the asset pricing model, which assumes that the loss of marginal utility from purchasing assets at current price is compensated by the discounted future payoff for the investor (Luenberger, 1997). The discount factor is the most important input for the pricing model, and is associated with future uncertainties. The CPAM believes that the future discount factor, as a utility function for

investors of the financial security, as the whole, can be represented by a single index beta. This is a strong statement. The fact that financial institutions and academics are still seeking to understand more about risks, and that people are not rushing to take loans to buy stocks suggest that beta is still not a universally valid risk metric. For example, Fama and French (2004) studied share price and returns for traded stocks between 1963 and 1990, and found that beta measures over a long period have little/no correlation with returns. There are also similar findings from studies on mutual funds. (e.g., Malkiel, 1995). This does not necessarily suggest the CAPM is wrong, but it does highlight the difficulty of risk measures. Particularly: a) the systematic risk for a security, a portfolio - and even a market - is difficult to measure, and can be unstable over time; b) systematic risks from the macroeconomics perspective, such as national income, interest rate and inflation, are difficult to predict; and c) some secondary factors, such as size and momentum, cannot be captured by the single-factor beta measure. However, it is generally accepted that bearing higher risk leads to higher return, in a long term investment strategy. The financial market is still on its journey to find a precise and quantitative risk measure. At the same time, the measure of risks and returns, the pricing theory and the portfolio theory have provided a basis for the modern structured financial market, which will be discussed and detailed in a later context.

1.2.6 The risk management system

Another important element in the modern financial industry is the risk management system. One of the most widely adopted risk management frameworks globally is the enterprise risk management (ERM, see e.g., COSO, 2004) theory. It was initially designed to understand and mitigate risks in a portfolio sense (normally at the corporate level), and to address risks that might not be captured by the quantitative models (recalling that any model is only an "abstractive of reality"), but was soon used in a much wider context. On the one hand, the refined credit rating system leads to a dramatic change of a company's balance sheet (Becker and Milbourn, 2010). With the fast growth of economics, raising capital becomes crucial for the corporations (Vitek and Roger, 2012). As investors are free to diversify their portfolios, management boards have to prove to stakeholders that the business is worth investing in – higher value with less risk. On the other hand, investors are obsessive about chasing risk-related profit, such as returns and dividends. It becomes an obligation that the management board keep the dividend promise each year (Baker, 2009), evoking some intriguing headlines such as "managers becoming the agent of stakeholders" (Hill and Jones, 1992). Thanks to the

boom of information broadcasts and mass media, investors are able to access much more information than only financial reports (Demirgüç-Kunt et al., 2006). They would also like to see effective management and mitigation of a company's risk, which may not be reflected in the balance sheet until several years later. A company adopting a contemporary and systematic business model would normally send a strong and positive signal to investors.

The Value at Risk (VaR) is another popular risk management framework adopted by a large number financial institutions. Instead of focusing on the volatility of the underlying securities, VaR monitors the percentile value (normally 1% or 5%, known as the VaR limit) of the portfolio loss for the probability distribution (see e.g., Philippe, 2006). The total gain and loss over the portfolio is monitored on a daily basis, providing a reasonable measure of the risk profile of the financial institution. Inside the VaR limit, the risks can be traded-off between different investment vehicles and strategies. Taking risks in this regime is treated as normal operational practice, and is considered to generate risk-related profit. Outside the VaR limit, however, the probabilistic measure of gain and loss is no longer meaningful, because much less data is available in the regime. The risk trade-off is less effective and those risks must be analysed on a case-by-case basis, using information beyond the gain and loss data. This regime is also normally associated with large amounts of uncertainty. Instead of imposing uncertainties on top of uncertainties (i.e., the probabilistic measure of gain and loss), techniques such as stress tests are often used, and risk mitigation strategies are then triggered to increase the resilience of the investment institution (e.g., increasing capital reserves). VaR provides a simple and structured way of managing risks and is widely used by both financial institutions and regulators. It assumes that the historical gain and loss data offer a reasonable representation of the risk profile of an investment portfolio. According to the nature of application, data during financially turbulent periods may be excluded from the model. The VaR method also implicitly assumes that the price movement of the securities in the portfolio represents the risk of the security issuers. This is ultimately related back to the efficient market hypothesis: any under- or over-pricing will be immediately corrected by the market.

1.2.7 Summary of the section
The modern financial market has developed significantly in the past decades, moving towards a structured and risk-centred direction. Thanks to the advancement of information technology

and involvement of academics, the quantification of performance and risks in various aspects makes it possible for financial investors to chase risk-related profit and at the same time survive under the risk-related damage. The modern portfolio theory promotes the practices of risk trade-off and the development of new financial products, especially financial derivatives. Those financial products enhance the ability to manipulate risks, such as through the practice of "slides and spread". The structured finance is heavily reliant on modelling and data. At the same time, making the financial performance and financial risk explicit and transparent has significantly increased acceptance of the financial industry, by both public and the government. This has further expedited the growth of the financial industry. Nowadays, the global economy is closely linked with the financial market. Creation or revision of financial policy has a huge impact on almost every sector of the national and global economy. It is, however, important to understand that those underlying models that enable everyday operations in the structured financial market are based on simplified assumptions, such as efficient market hypothesis. Any model is only an abstract of reality and has its own domain of applicability. This is easily overlooked by the financial industry, which is renowned for the risk-hungry and profit-chasing culture. For example, it is generally believed that a lot of models in the structured financial market break down during financial crises, a topic to be discussed in the next section.

1.3 Financial crises

1.3.1 The overview

In the year 2007, the JPMorganChase (JPMorganChase&Co, 2007) included a statement in the company's annual report that financial crises had been occurring [roughly] every five to seven years. Some examples include: the severe recession of 1982 (Moy, 1985), the 1987 stock market crush (also known as "Black Monday", see, Carlson, 2007 for details), the S&L and commercial real estate bust of 1990-91 (Geltner, 2013), the LTCM collapse (1998) (Edwards, 1999), the internet bubble burst 2000-2001 (Ofek and Richardson, 2003), and the latest mortgage crisis of 2007-2008 (Lowenstein, 2010). Each time, a financial crisis has seen massive boom of the market and has ended with burst bubbles, followed by severe disruption in real economic activities. Financial crises have been examined extensively by both academics and the financial sector itself, from technical, political, legal, psychological and even philosophical aspects. There are some commonalities observed, however a financial

crisis always originates from a collection of complicated unexpected reasons: "the perfect storm". For example, the well-known Y2K bug did not fully materialise and only had modest impact compared with those financial crises mentioned above (Malkiel, 2011). To predict and prevent a future financial crisis still remains the holy grail of most financial researchers. The current thesis is trying to look at the financial market from a multi-scaled viewpoint. It is therefore worth examining, using known theoretical frameworks, how the investors make their decisions and how they have behaved differently in financial turbulence, leading to those market-wide ruptures.

1.3.2 Intrinsic values and "castle-in-the-air"

There are essentially three types of investors in the financial market: speculators and arbitrageurs who seek information about the market movement in temporal and spatial domains, and hedgers who create portfolios to offset market risk (Voit, 2000). The performance of a financial investor is ultimately dependent on his/her ability to predict the future: to buy when the price is low and sell when the price is high. Accordingly to the pricing theory introduced in Section 1.2, the correct price of a financial security equals the summation of the stream of discounted future returns, i.e., the intrinsic value (Donaldson and Preston, 1995). Investors who follow this investment strategy will buy when they believe the actual price is lower than the intrinsic value and sell when they believe the actual price is higher. As a cumulative effect, the market will always price a financial security correctly, which justifies the efficient market hypothesis. The twist comes from the difficulties of calculating the intrinsic value. There are generally two reasons: a) it is difficult to forecast the extend and duration of the future growth - the styles and fashions of the pricing analyst play a critical role in the process of pricing securities (Jegadeesh et al., 2004); and b) the future discount rate is difficult to estimate and is different for different investors (i.e. different utilisation). The interest rate related discount only contributes to ~10% of the future discount (Cochrane, 2009) rate, while each investor has a different perception of risk, and such perception will change according to both the macro-economic factors (such as GNP and inflation) and market-specific factors (such as historical price-return performance). As a result, financial securities are not always priced correctly by the market. Bubbles can form when the link between the intrinsic value and the price is lost. Such bubbles may be explained by the "castle-in-the-air" theory.

The "castle-in-the-air" theory argues that analysing the intrinsic value has limited use in terms of financial investment. Instead, the psychic values, i.e., how other investors believe the value is and how they will behave in the future, matters most in a successful investment strategy. Keynes (1936) states that most people are "largely concerned not with making superior long term forecast of the probability yield of an investment", but "with foreseeing changes in the conventional basis of valuation a short time ahead of the general public". The psychological principle that "the crowd are willing to pay more" may dominate in financial trading, instead of finding the intrinsic value of the financial securities. "It is perfectly alright to pay three times what something is worth, as long as later on you can find someone innocent to pay five times". Eventually, the "castle in the air" is built by the investors and "skyrocketing markets that depend on purely psychic support have invariably succumbed to the financial law of gravitation" (Malkiel, 2011). In the modern era, the principles related to the "castle-in-the-air" theory has been further developed and established the research field of behaviour finance, which has won the Nobel Prize in Economics in 2002 (Kahneman and Tversky, 1979).

1.3.3 The internet bubble 2000-2001

It may be instructive to examine the internet bubble in the early 2000s using both the intrinsic value theory and the "castle-in-the-air" theory. Being the greatest technological advancement in the 20th century, the internet has fundamentally changed people's lifestyles and has provided huge business opportunities - initially for the IT industry and later all industry sectors. Companies involved in early internet development experienced a period of fast growth which was reflected in their share price. NASDAQ, the index that represents technology and new economy companies, more than tripled the total returns from 1998 to early 2000 (Brunnermeier and Nagel, 2003). From the intrinsic value point of view, the price rise in the "dot-com" stocks reflects expectations from investors on the future growth of this sector, therefore the price-earning (P-E) multiples are higher than traditional sectors. The capital raised from the financial market is further reinvested into the industry, such as in building new IT infrastructure and R&D activities around new technologies, which are expected to promote further growth. The financial market plays an important role in this "steady-growth" phase. Such a positive feedback loop is captured by the analysts of the financial market and taken into account in their analyses of intrinsic values. This positive feedback loop is soon overwhelmed by another positive feedback loop in the financial

market: when early investors on the "dot-com" stocks are seen and praised, having made huge fortunes overnight, others will start to follow the "herd", hoping the price will rise even further. Those group buying activities will push the price higher and drive more people to rush to the stock market. The intrinsic value no longer matters and a "castle-in-the-air" is eventually built. The market enters the bubble phase.

.

The financial market's positive feedback loop is a typical feature of speculation activities (Szado, 2011), and can be observed in a number of historical financial crises, from the early-years tulip mania, to the Tokyo real-estate bubble, from the speculation using new concepts of technologies such as blue chips and bio-technologies, to the speculation using new financial instruments such as the LTCM collapse and the latest mortgage crisis. The question is: why are the lessons learned so soon forgotten? During the internet bubble in the early 2000s, when the price of the stocks could no longer be justified by conventional intrinsic analyses, in the climate of free market and de-regulation, financial analysts (for example, from the highly technical Wall Street), invented a new value metric to justify those high P-E values and claim that the price was appropriate. Such "technical justification" rockets the share price by forecasting unsustainable future growth rates. Those analysts become "public cheerleaders" by "steering the investment" and "promising the uncertain future" (Malkiel, 2011). When speculation activities dominate the market, a large proportion of the general public becomes active in the market, hungry for more investment opportunities. Such demand is so high and there is research showing that companies' stock performances increase on average 125% by simply changing the trade names (Cooper *et al*., 2001). Meanwhile, analysts producing negative or sceptical forecasts are unfavoured by the investors, the public, the media, the financial institutions and the company in question. No analyst is incentivised to do so when huge personal fortune can be made by simply "finding reasons to bullish the market" (Mehmood and Hanif, 2014). Another feature during the internet bubble is the high frequency of trading. Research has shown that the average holding period of a stock changes from months during the steady phase, to days and hours during the bubble phase (Sornette and Becke, 2011). Such a short trading window is nowhere near the investment cycle on constructing infrastructure or implementing R&D outcomes, making the intrinsic value analyses even less meaningful. The market eventually realises that the expected growth is no longer sustainable and the bubble pops. Investors who have purchased high multiples of stocks can't find another buyer without dropping the price. The market moves to a decline phase that shortly leads to the catastrophic crash which is not foreseen by most analysts. For

example, Goldman Sachs argued in mid-2000 at the start of the decline, that the signals from the market were primarily due to "investor sentiment" and not a "long term risk" (Malkiel, 2011). In fact, panic spreads even faster than good news in the financial market. The positive feedback loop becomes a fictitious loop. Hundreds of "dot-com" companies went bankrupt in a few months and the predicted "soft landing" never happened. The NASDAQ index fell below what it was before 1998 and it is estimated that \$5.7 trillion of market value evaporated (Hellwig, 2008).

1.3.4 Efficient market hypothesis during financial crisis

It is generally recognised that the efficient market hypothesis is less valid during turbulent periods, such as financial crises (e.g., Cooper 2010; Lim *et al.,* 2008). At a certain point in time, it is unclear whether the price is too high or too low due to uncertainties around the intrinsic value calculation. Assessing retrospectively, however, before the burst of the internet bubble, the financial market does not adjust the over-valued prices fast enough - although it later corrected itself in a fairly dramatic fashion. Statistical analysis shows high intensity of volatilities and autocorrelations, as well as inter-correlations between share prices of different companies. It will be shown later in the thesis that this empirical evidence violates the mathematical description behind the efficient market hypothesis. On the other hand, as the market driver is switched from the intrinsic values to the "castle-in-the-air", the behaviours of investors switch from "fundamentalist" to "speculators", who can be irrationally optimistic or irrationally "herd-following" (see Bowe and Domuta, 2004). Such behaviours violate the assumptions of rational market theory, that the majority of market players are rational and "make decisions to maximise their own utility" (see e.g., Rubinstein, 2001). Another interesting application of the efficient market hypothesis during the boom of the internet stocks is around the IPO (initial public offering). It is used to justify venture capitalists' issuing a large number of "dot-com" IPOs, to satisfy the demand from the public for more investment vehicles in this sector. Instead of judging which company had better growth potential hence intrinsic value, those venture capital firms hoped that the market would be in the best position to do so. It is proved that when "castles-in-the-air" dominate, the intrinsic values matter very little. Despite accumulating huge personal fortunes, most of those companies no longer existed after the crisis in 2000 and 2001 (Scherbina, 2013).

It might worth expanding the above discussion regarding the "validity of the efficient market hypothesis". Recalling that the market is said to be "efficient" with respect to an information set if the price "fully reflects" that information set (Fama, 1970). When examining the market at a whole (macroscopic view point), it might be difficult (especially in a quantitative way) to identify whether the market is currently efficient, or whether it is dominated by the intrinsic value theory or the "castle-in-the-air" (recalling that an asset's intrinsic value can be difficult to obtain). This is often referred to as the "joint hypothesis" in the literature (see e.g., Lo and MacKinlay, 1999; Sewell, 2012). For example, in the asset pricing models, the risk discount factor may vary significantly. The collapse of the market during the financial crisis can be attributed either due to the re-adjustment of the risk factor or due to the market in-efficiency. According to the joint hypothesis, using the market-wise data alone is impossible to identify whether the market is inefficient or the risk perception from the investors has changed. Such "joint hypothesis" supports the argument that "bubbles cannot be detected before burst" and it is also extremely difficult to justify any intervention plans is "more efficient than the market itself". From the microscopic point of view, however, there are literatures suggesting that the market efficiency can be observed from "hallmarks" at the investor's level (market microstructure). For example, the market heterogeneity, including informational asymmetries, differing reaction speeds to information innovations, and etc., are regarded as essential for the market inefficiency in the foreign exchange market (Sager and Taylor, 2006). Another example is that when the herding of the investors as a result of the intent by market participants to imitate the actions of other's, the market may be inefficient as a whole (Bikhchandani and Sharma, 2001). It will be demonstrated in a later context of the thesis that those microscopic mechanisms at the investor level can be examined using microscopic modelling techniques (e.g. agent based modelling).

1.3.5 Financial engineering and investment banking

As discussed in Section 1.2, the modern financial industry operates in a structured manner based on defined risk and performance metrics. For example, the credit rating is a system used to determine credit worthiness and inform creditors and investors in the capital market. Under a structured finance model, more and more quantitative methods are adopted in determining risk, and it is possible to turn the relative risk measure (e.g., ranking) into absolute risk measure (e.g., probability of default, see The Basel Committee on Banking Supervision, 2014 for more details). It enables financial institutions to assign a monetary

value to a risk grade by calculating the probability of default, and the cost, if default happens. Using the modern portfolio theory, such risks can be managed in a portfolio sense by working out exactly how much is the premium for the risk of default, i.e., trading off between risk and cost. Another type of risk is uncertainty. The pricing theory of derivatives calculates the monetary values associated with future uncertainties using volatility as an input (e.g., Bouchaud and Potters, 2003). Financial derivative products are issued against "underlyings" (financial securities such as stocks and currencies in the context of derivatives) and the issuer takes the risk of the price fluctuations by charging the uncertainty-related risk premium. What is more, those financial derivatives are traded in a similar way to a standard financial security. The risks of the fluctuation of "underlyings" are then effectively transferred to the investors who purchase those derivative products. The ability to quantify, mitigate and transfer risks using mathematical modelling techniques has laid the foundation of modern financial engineering and investment banking. Different from traditional banks, investment banks specialise in producing structured and packaged financial products using quantified risk metrics. Those financial products are designed to either facilitate capital-raising, or meet demand of investors such as by: accessing large varieties of standard financial products, risk diversification, insuring against or betting on market movements, etc. In the 21st century, the investment banking industry plays an important role in understanding the market, providing quantitative information, managing risk, and financial innovation activities (Reinholdson and Henrik, 2012).

1.3.6 Mortgage crisis 2007-2008

The mortgage industry is well established in servicing home buyers with long term finance. Traditionally, the mortgage endorsers (e.g., banks) bear the risks of mortgage default. The mortgage applicants are required to go through the underwriting processes to get the credit risk assessed for their creditworthiness. If a default happens, the original underwriting documents will be investigated. The process may be reviewed to refine the analysis of risks and capital reserves may be affected. This situation changed in the early 2000s. Thanks to the deregulation in the 1980s and further in 1990s, there is much more freedom to take, trade or transfer financial risks (Hou and Skeie, 2014). For the investment banks, the mortgage market is seen as a perfect place to apply financial engineering practices. Those loans are backed up by real estate and the value, which is governed very much by the law of demand and supply. It has predictability; it is a steadier, more generic and larger portfolio than loans to firms

which are individual to each business. After a mortgage loan is issued, it will be sold immediately to an investment bank, where a group of mortgages are packaged with an overall credit risk and mortgage-backed bonds issued in the security market. Such a "securitisation process" is a new invention of Wall Street, and achieved great success in the early 2000s. The future growth of bonds relies on interest repayments on the mortgage; thus, the risk of mortgage defaults is transferred to the bond purchasers who are deemed as "willing to take the risk" (Feldstein M., 1999). To maximise the utilisation of the mortgages, a group of mortgages are carefully "engineered" and multiple bonds can be issues against a single "mortgage pool". A waterfall of credit ratings (often referred to "waterfalls") is assigned to those bonds to attract different types of investors. Those lower-rated bonds can be further bundled together and re-securitised to issue another product called collateralised debt obligations (CDO). By bundling again, it is regarded that the individual risk of default is further reduced (at least theoretically). Another financial instrument that is "engineered" to utilise mortgage-backed bonds is the credit default swap (CDS). It is issued by financial insurers who calibrate the probability of default and work out the risk premiums of bonds. It is believed that, by using the portfolio theory, the insurers charge appropriate premiums to smaller investors who require certain level of risks assurance. Like CDOs, CDS can also be traded in the security market. To make the situation more complicated, a type of synthetic CDO can be further issued against CDS which generates future cash flow through risk premium payments. During the year 2007, the US market value of CDOs reached 641 billion USD (Cordell et al., 2012) while CDS was over 5 trillion USD (Siciliano et al., 2012), as much as ten time more than the value of the underlying bonds (Malkiel, 2012).

According to those established mathematical models and accounting rules, banks and investment banks essentially take very small risks, after they are transferred to other investors or mitigated by purchasing other financial products. In fact, most investors in the financial market regard themselves as being in a safe place for similar reasons. The whole market is heated up and starts to see more speculation. At the same time, the investors demand new investment opportunities and the number of mortgages soon becomes a constraint. The underwriting process becomes redundant in the equation: if those risks can be transferred immediately, it is irrelevant how much risk is involved in the loan, especially when the underwriting process potentially stops mortgages from being issued and reduces the sale to investment banks. High risk mortgage borrowers (i.e., subprimes), previously unable to access the credit, are likely to be successful in the new banking system. For North America,

as a percentage of annual volume [of the mortgage market], subprimes [by 2006] topped 25 percent, while they were still below 10 percent in 2003, and "hardly anything in the '90s" (Furlong et al., 2007). Many investment banks have acquired their own mortgage endorsers and form a "pipeline of the securitisation process". The loosened mortgage lending standards have triggered the growth of the real estate market. To compete for market share, some mortgage endorsers have created "innovative mortgage products" (such as the famous adjust rate mortgage (ARM)) to attract more mortgage borrowers. The housing price has risen far ahead of the household income curve (Lowenstein, 2010). At the same time, the US government policy is toward promoting home ownership, which is believed to have the effect of "strengthening democracy" (Lowenstein, 2010). A large number of speculators came into the housing market, and even many ordinary homeowners were able to buy multiple properties in the hope that their value would shoot up in the future. In the year 2005, the US housing market saw the largest bubbles in history (Shiller, 2005). The "castle-in-the-air" was built and many mortgage borrowers held a loan contract with a value much higher than that of their home.

The subprime market by definition lacks a long term plan and is far from unstainable. When the interest rate increased, people found themselves facing monthly repayments higher than their total income and were forced to default. This had a domino effect on the security market – more and more mortgage-backed securities were defaulted, and the CDOs and CDS had very little use in terms of shielding risk (as they were designed to do). The decline was devastating – when the bubble popped, many homeowners had to abandon their houses. House prices fell by two-thirds in some areas of US. Big players in the market suffered from even bigger hit. Those investment banks found themselves holding too many "toxic assets" that couldn't be shifted out. Some of them went bankrupt, and some had to be rescued by government, evoking extensive social, economic, legal and moral debates about "too big to fail" (e.g., Gelinas, 2009; Department of the Treasury, 2010). Compared to the internet bubble, the mortgage crisis had a much wider and longer-term impact on the global economy. Family utilities were heavily damaged. Consumers who used to finance their consumptions became more cautious. Credit markets were shut to small- and medium-sized businesses. The public became less confident about future growth and more reluctant to consume or invest. The government was also short of cash flow and had to rationalise funding in a number of sectors. The global economy experienced the biggest recession since the Great Depression of the 1930s.

1.3.7 Lessons learned from the mortgage crisis 2007-2008

There are many literature documents and studies of the mortgage crisis in 2007-2008. We are not going to dive into details of events or analyse every aspect of the root cause. Instead, it is worth examining in a systematic way the role played by financial institutions (e.g. financial engineers and investment banks), to gain some fundamental understanding of financial modelling. Firstly, any model is only an abstract of reality and only works within the domain that it is designed for. However, once the models are developed and handed over to the financial engineers, it is extremely difficult to control where they are applied to. Efficient market hypothesis breaks down when speculation level and herd effect are high. However, it is difficult to detect bubbles and the level of bubbles, as well as how this will affect those modelling outputs; it is certainly not down to those operations-focused financial engineers to do so. For example, the models used to value the price of derivatives heavily rely on the historical distribution of the price movement. When the volatility of the market is high, the model starts to break down due to the higher probability towards the tails of the distribution (i.e., fat tails) (Merton, 1975). The credit default swaps (CDS) based on calculations of option pricing found themselves lacking capital cushions when the market crashed (Mollenkamp et al., 2008). What is more, during market turbulence, different modelling methods which used to agree with each other during calm periods will produce significantly different outputs (Danielsson et al., 2014), with no obvious way to identify which method is the best.

Secondly, models used in the graded rating system require some deterministic measurement of risks (Thomson Reuters, 2013). The rated grade is a quantitative abstract of risks which normally contains both quantitative and qualitative information: in the case of the mortgage crisis, the risk profile of each mortgage holder or a group of mortgage holders. The intention of using the graded system is to inform better decision-making without diving into too much underwriting details. When the rating is used literally, to calculate the probability of default and then issue financial products, it is proved to be an ill-equipped and over-simplified risk indicator (Stein, 2006). Take credit ratings, for example: when the methodology adopted by the credit rating agents becomes transparent to the financial engineers who package mortgages, they are able calibrate the profile of the mortgage group to put as much subprime mortgage into the "mortgage pool" as possible, as long as the desired credit rating is achieved (Lowenstein, 2010). The credit ratings have therefore become even more flawed due to such

a "conflict of interests" between investors and financial engineers. On the other hand, risks will change over time and according to market movements. During the mortgage crisis, it is difficult to identify whether those risk changes are due to the loose borrowing standards or the behavioural changes of the homeowners. As speculators are more likely to abandon their house when the price falls, the categories originally rated as "safe" may have a much higher probability of default when the market consists of a larger number of speculators (Ferreira and Gyourko, 2015). Another factor influencing the risk rating is data and statistics. The securitisation of groups of mortgages is a relatively new practice, and the credit rating system has only been applied to this process since the 2000s. Statistically, to calculate the probability of default for a group requires autocorrelation analysis, which needs enough default data (i.e., statistical significance). When those data are lacking, a popular approach adopted by those trusted rating agents is to use "expert judgement" (Lowenstein, 2010), making those credit rating scores even less objective.

Finally, the modern portfolio theory is the theoretical foundation behind grouping mortgages and bonds to mitigate risks. This has implicitly created a much higher inter-dependence between credits and debts. When one part of the market collapses, it is the domino effect that takes down the whole market. Under such a system, sufficient capital (in case risks materialise) are no longer required, but are assured by transferring those risks to third parties using financial products, or by purchasing additional financial products issued by other parties. Each time a structured financial product is "engineered" by grouping a number of securities, for example CDOs, part of the information contained in the original individual security is lost, either due to the averaging process or the financial product being too remote from the underlying asset. The individual mortgages may be risky, but the investors are given a false sense of safety using the credit ratings derived from the whole portfolio. Overall, during the mortgage crisis 2007-2008, the housing bubble may have been no different to those historical speculation bubbles. Those risks (e.g., subprime mortgages and junk bonds) found their way into the financial market, turning them into imperfect and over-simplistic mathematical measures. The structured finance and modern portfolio methods cannot mitigate all risks which are "abstracted" by the financial models. The later crash demonstrates the existence of the bubble, which is amplified significantly, not only through the positive feedback loops but also through the false perception of risk.

1.3.8 The risk management system during financial crisis

Some of the literatures attribute the mortgage crisis 2007-2008 as "the failure of risk management systems" (e.g., Ashby, 2010). Risk management models that passed the back test or stress test are found to be insufficient in the new round of crisis. On the one hand, the risk management systems were believed to be so robust and sophisticated, and some were designed to pick up those risks that couldn't be captured by mathematical modelling. However, talking about risks, in corporate terms, requires a huge culture challenge referred to as "intellectual confrontation" (Kaplan et al., 2012). Identifying risks is counter-intuitive: it is not only a rule-based or a compliance issue (such as legislation risks). When chasing risks is incentivised by financial returns, it becomes a strategic issue that requires informed forward thinking and group discussion. In the financial sector where the bonus culture prevails, when the market sees a high rate of returns, it is difficult to steer the corporate strategy to stay in a risk-adverse position by sacrificing both personal income and corporate market share, especially when the board members take comfort from the mathematical models showing that the risks are effectively mitigated. The corporate risk officers often found themselves sitting in an awkward position against the board, and many of them became marginalised (Lowenstein, 2010).

On the other hand, the market-wise systematic risk is not effectively managed by the regulators. This type of risk is not easily managed for each institution, and relies heavily on the regulators to oversee and control in a pro-active manner. However, during the mortgage crisis, those risks inherited from structured financial products such as CDO and CDS were not sufficiently regulated (Crotty, 2009). Recalling that the efficient market hypothesis suggested those financial products will be valued correctly by the market, it is ethically correct to have a market without intervention/regulation. In fact most of the financial products are heavily reliant on financial modelling which has got deep roots of the random walk, i.e. Gaussian framework, which has also an ethical justification since the statistical normality can justify that markets are fair (Jovanovic, 2001). It has been argued that the Gaussian framework leads to a symmetrical probabilistic distribution for which negative changes occur with the same probability than positive ones. In other words, all investors face the same trading conditions and any intervention/regulation will have high chance of destroying this symmetry therefore regarded as inefficient (Schinckus, 2015). It will be however shown from later chapters of the thesis that there are other symmetrical distributions

that can also serve the purpose of the "ethical fairness", which does not exclusively require a solution using the Gaussian framework.

Those de-regulation (or not-regulation-at-all) approaches that originated from the efficient market hypothesis lose their designed functionality during high market turbulence. Also, the de-regulation does not mean that financial regulators should not monitor the risk and promote sustainability. In fact, not only did the regulators fail to identify early signs and warnings (Minsky and Kaufman, 2008); they also assured investors that that "the high yields and low-risk spreads of the period were permanent, and thus the risk of crisis was minimal" (Crotty, 2009). In the modern financial industry with high degrees of complexity, the regulators are heavily reliant on the financial institutions themselves to report the risk profile, using innovative risk management models such as value at risk (VaR) (Soros, 2008). As introduced in the above section, the VaR model uses the percentile loss of the portfolio as a risk indicator, and is ultimately related back to the efficient market hypothesis. It has been argued that: 1) such a measure will "understate potential risk" and lead to a "false sense of complacency" (Partnoy, 2003); 2) the results from the VaR framework are sensitive to input parameters, especially in the extreme confidence intervals; and 3) it is difficult to estimate the interactions between future risk exposures in stressed conditions, due to the uncertainty of investors and overall economy (Geithner 2006). The widely adopted VaR model effectively allows the banks to self-regulate, and eventually leads to a stage without sufficient capital reserve when the risks materialised (Crotty, 2009).

1.3.9 A summary of the section

As a summary, the financial crises are always associated with some form of speculation. Bubbles form when the link between supply and demand is lost. The market is not always rational and does not always price assets correctly, especially when experiencing high turbulence. The efficient market hypothesis has however been taken for granted and made into almost every building block of the financial market. The ability to monetise, measure and mitigate risks enables the structured financial system to use advanced mathematical modelling approaches. However, risk is still an untamed monster to human beings and hits us when we are most unprepared. Humans' understanding of risk is far from sufficient, and risk itself cannot [ever] be measured as a single "definitive interpretation" (Stirling, 2010). Bearing this in mind, while our knowledge about the financial market and financial risks has

improved significantly over the past decades, many people ask the question why advancement of understanding the market becomes the hot air that keeps the bubble floating and gives false sense of security. During the internet bubble, the high P-E risk was justified by "innovative methodology of forecasting future growth". During the mortgage crisis, the risks originating from subprime market were believed to be "sliced and transferred to those who are willing to take risks" (Greenspan 1999).

Understanding human behaviour is crucial to explain these phenomena, not only in terms of building the "castle-in-the-air", but also in a wider sense of responding to risks and unknowns. Firstly, the perception of risks has dramatically changed. In the modern financial market, the concept of risks can be largely summarised as follows: 1) bearing risk comes with reward; 2) risks can be mitigated using portfolios; and 3) risks can be traded with costs. Those ideas are so popular among investors and implicitly promoted the growth of the credit market. However, when investors become concerned about a cessation of credit, they are more liable to panic, causing failure in a catastrophic way (Gorton, 2008). Secondly, when risks are made explicit, the competitive nature of the market will push the market players to chase market gaps, i.e., in high-risk areas. This might be safe if those risk models are perfect (and, in fact, believing in they are is regarded as a quality of leadership (Lowenstein, 2010)). When those models get more complicated, the risks originating from both the market and the imperfection of the models accumulated and eventually manifested. Finally, risk management is counter-intuitive and is not appropriately incentivised, for example, through the "bonus system". It is much easier to talk about numbers and compliance than by diving into complicated discussions about future hazards. Human are naturally optimistic before risks materialise. An effective risk management system requires both culture change and the reorganisation of corporate structures.

1.4 The purpose of this thesis

The thesis will not solve the problem of financial crisis and it is not going to touch on economic fundamentals, such as supply and demand or bubble formation. It is also not aiming to develop a completely new mathematical model that leads to a more sophisticated financial product. Instead, this thesis will focus on evaluating those fundamental assumptions of quantitative financial models, and provide some insights using a multi-scale approach.

Choosing the stock market as a representative of the financial market, the thesis attempts to critically examine the popular efficient market hypothesis, while at the same time providing some possible alternatives. The quantitative links between human behaviours and the financial market response are explored. Specifically, the macroscopic measures of the market movement, e.g., price returns, are analysed using both empirical methods and some recent advancement of mathematical modelling. The microscopic studies focus on investors' decision making process using agent based simulation methods. The results from both scales are compared and discussed, in the context of both calm and turbulent market periods. Some interesting phenomena about the financial market are also modelled and validated. It is demonstrated that, by using the combined approach from both the macroscopic and microscopic aspects, the enhanced findings can be greatly beneficial while applying the modelling techniques to the modern financial modelling practices

Chapter 2 The stock market: models and empirical evidence

2.1 Introduction

2.1.1 Overview of the stock market

The stock market is one of the largest components of the financial sector. It is a well regulated marketplace equipped with secure and efficient transaction facilities and with relatively low transaction costs. The stock market has the advantages of liquidity and accessibility. Compared to other investment vehicles such as corporate bonds, equity returns are also less affected by inflation (e.g., Bodie, 1976). On the one hand, selling shares of the ownership in the stock market is one of the most important ways for companies to raise capital. By transferring available funds from investors to the equity issuers, stock markets play a crucial role in businesses development, and ultimately affect the real economy (Naik and Padhi, 2012). On the other hand, the high percentage of participants in the stock market (in terms of the general public) means that the share prices' rise and fall is closely related to the wealth of family utility and the ability of consumption. As discussed in Chapter 1, the stock market helps to create a positive feedback loop that leads to healthy and sustained economic growth. Increased share price is a sign of investors' confidence in future economic growth. In reality, the performance of major stock markets is often considered the primary indicator of a country's economic strength and prospects (Mahipal, 2011).

Stock shares are traded in stock exchanges, between market players including individual investors and institutional investors such as mutual funds, banks, and hedge funds. Similar to all marketplaces, the share price is agreed between a buyer and a seller, and facilitated by the price-matching service provided by the stock exchange. The share prices listed by the stock exchange can be therefore understood as a cumulative effect contributed from all market players (Hamilton, 1922). In the context of efficient market hypothesis, it assumes that the investors are rational enough to exploit the arbitrage opportunities, and at the same time correct the mispriced stocks. The future price movements are therefore as unpredictable as

external information shocks (some examples of those information are outlook for margins, profits or dividends). Recalling that the "intrinsic value theory" is not always honoured by the market, a "weak" form of the efficient market hypothesis is proposed such that, after paying transaction cost, the speculators can't benefit from market inefficiencies (e.g., arbitrage, information inefficiency, etc.) (Malkiel and Fama, 1970). The present chapter starts with introducing a mathematical description of the stock market, followed by theoretical frameworks, and then the modelling approaches based on efficient market hypothesis. This chapter will also examine some classical models together with empirical evidence.

2.1.2 The pricing theory of stocks

Recalling that the asset price is determined by the discounted future stream of cash flow, a simple model (see e.g., Fuller and Hsia, 1984) of the stock price value P can be calculated using earnings per share E, growth rate G (based on earnings per share), and dividend payment D, and then discounted by the risk-adjusted discount rate $\bar{k}$:

$$P = \frac{GE}{\bar{k}^2} + \frac{D}{\bar{k}}, \tag{2.1}$$

in which $GE/\bar{k}$ represents the future value that the current earning will generate, and $\bar{k}$ is closely linked to risk premium of the individual company (as an illustration, the ratio between the company risk to the market risk can be referred to as the beta measure in the capital asset pricing model introduced in Chapter 1). The two governing terms in Eqn. (2.1) is the future growth rate G and the risk-adjusted discount rate $\bar{k}$, which can be captured by an important indicator of the stock market: P-E ratio. Specifically, by separating $\bar{k}$ into $\bar{k}_{current}$ and $\bar{k}_{future}$, the P-E ratio equals to:

$$P/E = \frac{G}{\bar{k}^2_{future}} + \frac{D/E}{\bar{k}_{current}}. \tag{2.2}$$

As shown in Eqn. (2.2), the P-E ratio includes two terms. A fixed term can be calculated from a company's balance sheet, and a predictive term including the future growth rate and risk adjusted discount rate for which forecast techniques are required. A relatively high P-E ratio indicates either a lower risk perception or a higher expected growth rate. A share price with high P-E ratio will be regarded as overpriced if there is no obvious reason to justify so.

Consistent with the statement that bearing higher risk will lead to higher returns, research evidence shows that companies with low P-E ratios and smaller-sized companies have a tendency to outperform the market (Fama and French, 1992). A follow-up statement is that the P-E ratio should be relatively stable over a longer period due to the company's and industry's "intrinsic value", therefore the long term stock price return is determined only by the growth rate and the dividend pay-out (Malkiel, 2011).

The validity of using dividend D in the valuation model Eqn. (2.1) is challenged due to a number of developments of corporate finance and new pay-out policies. On the one hand, dividend pay-out is a decision made to signal to the market future growth expectations. The executives of the company would do everything before "slaying the sacred cow by cutting dividends". The dividend payment eventually becomes a "high-cost way of signalling", and no longer conveys as much information as when it is invented, especially for the companies with a long history of dividend pay-out (Brav et al., 2005). On the other hand, the repurchase, defined as the company buying shares from its shareholders with a price higher than the market price, is offered as an alternative way of paying-out (see e.g., Jagannathant et al., 2000). The repurchase has the advantage of flexibility and tax reduction, and is preferred by a number of companies and investors. Depending on the assumptions made and empirical data examined, there are mixed research results of the impact of repurchases on stock valuation models; either "it is still valid using traditional dividend discount models" (Stowe, et al., 2009; Pástor and Veronesi, 2003), or "the stock price rise much higher in terms of repurchase than the dividend pay-out" (Ofer and Thakor, 1987).

It is, however, not the aim of the current thesis to go through the details and give an appraisal of those stock price valuation models. The complications and uncertainties involved in those models are used to illustrate that quantitative methods are far from perfect, and that the investors' decisions could be affected by the perception of the "correct price".

2.1.3 The pricing theories of derivatives

Another important aspect of the stock market is the derivatives issued against the equities. Strictly speaking, the stock index can be regarded as a stock derivative that index funds can be issued. Major stock exchanges publish stock indices using a selection of representative stocks traded in the exchange (price weighted by market capitalisation). Two other important

financial derivatives in the stock market are the stock futures contract and the stock option. These financial derivatives can be either traded in the stock market or through specialised markets (e.g., over the counter). Compared with standard equities (EBA, 2012), many financial derivatives are also associated with financial innovation initiatives and are relatively less regulated,. The futures contract allows traders to purchase or sell stocks at a fixed price, i.e., the value of the contract, at a future point in time - expiry time. It can be used as a way of mitigating against future price fluctuations. The value P for the futures contract at time t is simply the expected price discounted by the risk free rate r over the period until the expiry time $\overline{T}$ (see e.g., Bingham and Kiesel, 2013, for details):

$$P = E\{Y(t)\}e^{r(\overline{T}-t)},\qquad(2.3)$$

in which $Y(t)$ is the stock price at time t, $E\{\}$ represents the expectation (of a data set or a probability distribution). Based on the understanding that stock prices are unpredictable, i.e., have the same chance of going up and of going down, the value of the futures contract becomes:

$$P = Y(t)e^{r(t-\overline{T})}.\qquad(2.4)$$

Valuation of the stock option is more complicated. An option holder has the freedom to either exercise the option at a future point in time to buy/sell stocks at a prescribed price K (delivery price) on gains, or not exercise it on losses. Such freedom comes with the premium cost to purchase the option that can also be traded in the market. Taking the European call option as an example, the probability of the option holder exercising the option depends on the probability of the price of the stock at a future point of time $\overline{T}$ (mature time) is higher or lower than the delivery price K. The option price C can be modelled as a function of the stock price, the delivery price, and the maturity time that:

$$C = Y\overline{N}(d_1) - Ke^{r(t-\overline{T})}\overline{N}(d_2),\qquad(2.5)$$

in which $\overline{N}$ is the stand normal distribution in the cumulative form. Looking at Eqn. (2.5), if the option holder will 100% exercise the option, the probabilistic terms $\overline{N}$ will disappear and the option price becomes the difference between (the discounted) future and the current price - i.e., equivalent to the futures contract (Eqn. (2.4)). At the same time, the option price must reflect the risk premium associated with the future uncertainties, i.e., the volatility of the

underlying stocks σ. This is included in the parameters d_1 and d_2 (see e.g., Higham, 2004 for details):

$$d_1 = \frac{\ln(Y/K) + (r + \sigma^2/2)(\overline{T} - t)}{\sigma\sqrt{T - t}} \text{ and } d_2 = \frac{\ln(Y/K) + (r - \sigma^2/2)(\overline{T} - t)}{\sigma\sqrt{T - t}}. \qquad (2.6)$$

Eqns. (2.5) and (2.6) are the solution of the celebrated Black-Scholes stochastic differential equations (Black and Scholes, 1973). Qualitatively speaking, Eqn. (2.6) indicates a higher option price C for higher $\overline{T} - t$ (longer time to mature), for higher σ (more volatile), and for higher $\ln(Y/K)$ (bigger difference between current price and delivery price). Those option pricing equations are, however, strongly dependent on the assumptions made under the efficient market hypothesis, when constructing an arbitrage-free portfolio and when using the "implied volatility" (Beckers, 1981). Instead of performing statistical analysis on historical data, the implied volatility assumes the options are priced correctly by the market (during trading) and back-calculates the volatility using option pricing models such as Eqns. (2.5) and (2.6). The implied volatility, however, implicitly assumes that the option pricing models sufficiently capture the decision-making process of the option traders. Those models are generally derived from using both stochastic calculus and an analytical description of the statistical properties of the stock price. The remaining part of the chapter will take a step back and will examine some fundamental assumptions used to model the price of stocks and derivatives, including mathematical descriptions, calculus and theorems, as well as some statistical analyses.

2.2 Modelling statistical properties of the stock market

2.2.1 The mathematical description of the stock returns
The stock market exhibits typical properties of a complex system. It consists of a large number of investors interacting in a non-linear way, with the existence of feedback loops. It is also an open system, subject to external shocks such as information and the macro-economic climate. The governing rules for the market are relatively stable and the information is well recorded, detailing each transaction in the stock exchanges (Mantegna and Stanley, 2000). Figure 2.1 shows the time series of the S&P 500 index for 2015. It can be seen from the figure that the time series is a stochastic but non-stationary one. The time series

data is non-differentiable everywhere, with no recognisable pattern. It is difficult to plot trends using statistical methods such as auto regression or moving averages (or more advance techniques such as machine learning) to accurately predict the time series. This can be also demonstrated using the algorithm complexity theory. A complexity of the algorithm can be understood as a measure how predictable the data series is. A predictable time series data can be represented by an algorithm the size of which is considerably shorter than the size of the data, while the algorithm that "predicts" the financial time series is roughly, the same size of the data itself (see e.g., Bürgisser et al., 2013, for details). These observations are generally consistent with the efficient market hypothesis that price movement is unpredictable. Using mathematical terms, the share price $Y(t)$ is a function of trading time t. The price change $Z(t)$ and logarithm price change $S(t)$ can be written as:
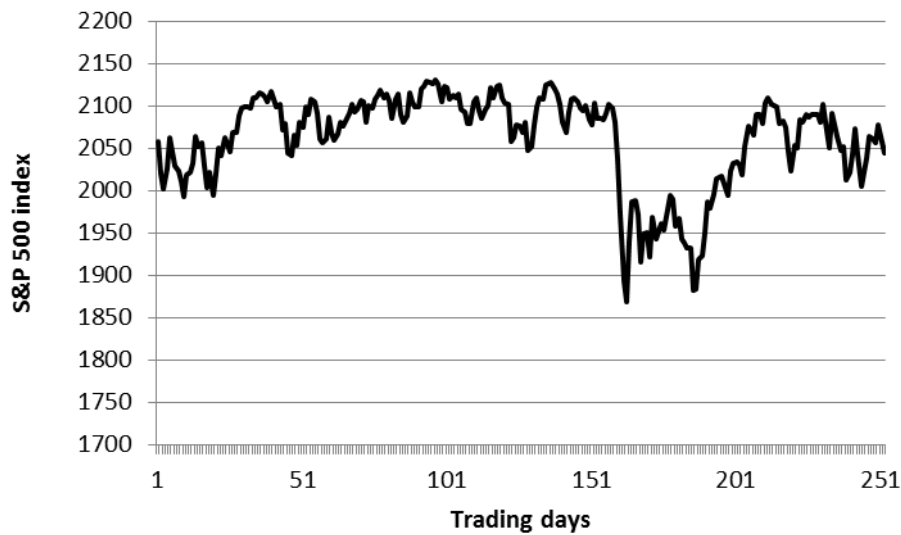
Figure 2.1 The S&P 500 index plotted in time series for 2015

$$Z(t) = Y(t + \Delta t) - Y(t), \tag{2.7}$$

and

$$S(t) = \ln Y(t + \Delta t) - \ln Y(t) = \ln \frac{Y(t + \Delta t)}{Y(t)}. \tag{2.8}$$

Compared to the definitions used in natural science, such as time and velocity, the price recorded in the stock market is affected by factors such as inflation and currency exchange rate, and the time recorded is affected by the open and close time of the stock exchanges - bearing in mind, for example, weekends (Fama, 1965). This may cause some bias when performing statistical analysis (French, 1980) as well as mathematical models, making it

difficult to compare like-for-like between different studies from different researchers (Mantegna and Stanley, 2000). For example, Eqns. (2.7) and (2.8) can be rewritten as:

$$S(t) = \ln \frac{Y(t + \Delta t)}{Y(t)} = \ln(1 + \frac{Y(t + \Delta t) - Y(t)}{Y(t)}) = \ln(1 + \frac{Z(t)}{Y(t)}) \approx \frac{Z(t)}{Y(t)} . \qquad (2.9)$$

When $t \rightarrow 0$, and define price return as $R(t) = Z(t)/Y(t)$, it can be concluded from Eqn. (2.9) that $R(t) = Z(t)/Y(t)$ is valid for an infinitesimal time horizon t (therefore, the difference of logarithm price is often termed as logarithm returns). In the real stock market, however, different trading time horizon t may cause differences between the different definitions of returns, therefore creating a modelling bias. Empirical studies using S&P 500 index and analysing such biases are shown in Figure 2.2, in which the absolute difference:

$$\wp_1(\Delta t) = \frac{\Delta t \sum_i |R(t_i) - S(t_i)|}{\sum_i \Delta t} , \qquad (2.10)$$

and the relative difference:

$$\wp_2(\Delta t) = \frac{\Delta t \sum_i |R(t_i) - S(t_i)| / |Z(t_i)|}{\sum_i \Delta t} , \qquad (2.11)$$

are calculated using different trading frequencies.

As shown in Figure 2.2, these differences are generally acceptable (<1% of absolute return). However, depending on the applications of the modelling, the bias may accumulate, especially for lower-frequency trading. The current thesis assumes the difference is sufficiently small unless addressed explicitly. Any bias caused will also be discussed where appropriate.
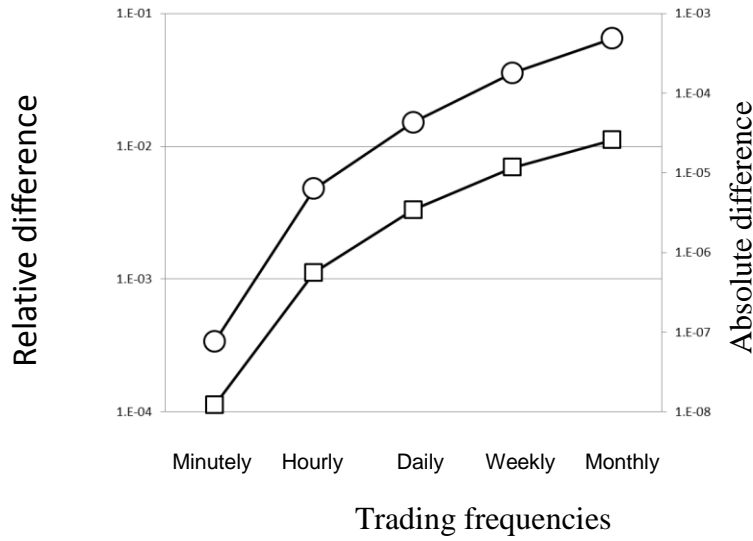
Figure 2.2 Differences between price return and logarithm price change in terms of different time horizons. The results are calculated from S&P 500 index for the year of 2015 using trading times and assuming 6.5 hours a day, five days a week, and four weeks a month. The circular symbols are the mean value of absolute differences (Eqn. 2.10), and the square symbols are the mean percentage of absolute returns (Eqn. 2.11).

2.2.2 The formulation of the efficient market hypothesis and the random walk theory

Recalling that the theoretical foundation of the modern stock market is the efficient market hypothesis, the random walk theory, as a special case, is worth examining the mathematical descriptions and assumptions around it. Samuelson (1965) is among the pioneers of mathematic formulation of the efficient market hypothesis. Given the value of the stock price Y_t at time t, the future price is stochastic and independent of the observed price values Y_0, Y_1, …,Y_t that:

$$E\{Y_{t+1}|Y_0, Y_1 ..., Y_t\} = Y_t . \tag{2.12}$$

The stochastic process obeying the conditional probability described in Eqn. (2.12) is called martingales (the formal definition can be found in Doob, 1953), which models the price movement in the next time step as unpredictable and governed by a probabilistic distribution function. A widely used martingale is the Wiener process (see Merton and Samuelson, 1999 for details). In the context of the stock market, it can be formulated as below. At a certain time t_0, assuming a number of future time steps for which the stock price Y fluctuates

following the definition of the martingales, i.e., independent of the previous prices of the stock:

$$Y(t_0 + \Delta t) = Y(t_0) + x_i,$$ (2.13)

in which x_i is an independent identically distributed random variable. By further assuming $t = n\Delta t$ $(\Delta t \rightarrow 0)$, the price change during time horizon t will be:

$$Z(t) = Y(t_0 + t) - Y(t_0) = Z(n\Delta t) = x_1 + x_2 + ... + x_n \quad .$$ (2.14)

Here we use the special case random walk as an illustration, whilst the more generic cases will be discussed later. At each time step, the price change will be the same distance $x_i = \pm s$, in which s is the distance of the random walk. The expected price change is therefore:

$$E\{Z(t)\} = 0,$$ (2.15)

and the variance (noted as Var{ }) is determined by:

$$Var\{Z(t)\} = ns^2 = \frac{t}{\Delta t}s^2.$$ (2.16)

Eqn. (2.16) represents a diffusion process that the variance is a function of time t. There is a higher probability in finding the stock price away from the expected position (Eqn. (2.15)) for a longer time period of diffusion. The relevance of Eqn. (2.16) to the physical diffusion process will be apparent in a later context. The evolvement of the probability function is governed by the evolvement of the standard deviation with a constant time characteristic $\sqrt{t}$, and with a constant coefficient $\sigma = s^2/\Delta t$, which is also termed as the volatility in a defined time horizon of price returns (see e.g., Bartram et al., 2012, and also Eqn. (2.6)). The probability distribution function satisfying this diffusion process is found in the form of:

$$P(Z(t) = z) = \frac{1}{\sigma\sqrt{2\pi t}}\exp(-\frac{z^2}{2\sigma^2 t}),$$ (2.17)

which can also be written as $N(0, \sigma)$, representing the normal distribution with mean zero and standard deviation σ. One way of deriving Eqn. (2.17) is to use the central limit theorem (CLT), which guarantees that the normal distribution is not only restricted to the random walk case, but is also valid for any random variables that are independent, identically distributed and with finite variance. The efficient market hypothesis assumes the price is

always corrected by the market (semi-strong form) or can't be predicted for speculation (weak form). The stock price change is therefore due to external random information shock that can be modelled by Eqn. (2.14). What is more, any information will be absorbed by the market immediately (i.e., no memory), therefore those random shocks are independent. The price change during the time horizon t follows a normal distribution Eqn. (2.17) through the CLT. Using mathematical representation, the price of the stock can be modelled by a stochastic Wiener process $\{W_t\}$ (Etheridge, 2002):

$$Y(t_0 + t) = Y(t_0) + Z(t) = Y(t_0) + \mu t + \sigma W_t, \tag{2.18}$$

in which μ is the drift constant and W_t is a random variable following a standard normal distribution with mean zero and variance t. As Eqn. (2.18) will not prevent the stock price from falling below zero, a popular treatment is to model the price dynamics using a geometric normal distribution:

$$Y(t_0 + t) = Y(t_0)S(t) = Y(t_0)(\mu t + \sigma W_t), \tag{2.19}$$

in which the two constants μ and σ may be of different scales between Eqns. (2.18) and (2.19). Recalling the relationship $S(t) = Z(t)/Y(t)$, where Y is a slowly changing variable compared to Z and S, the above two equations can recover from each other. Eqn. (2.19) is also referred to as a geometric Gaussian process or geometric Brownian motion, the meaning of which will become apparent in section 2.2.5.

2.2.3 Volatility and the measure of uncertainties

The geometric Gaussian process Eqn. (2.19) is the foundation of the option pricing model introduced in section 2.1. However, finding the value of volatility as the input parameter is not a trivial task. Statistically, the volatility is defined using the price return period between time t and maturity time $\overline{T}$, by calculating the variance of the statistical distribution from the historical stock price returns. Such statistical volatility assumes that the historical volatility value is a fair representation of future volatility. This assumption is, however, not fully supported by empirical evidence. As shown in Figure 2.3, the monthly volatility of the S&P 500 index shows a large degree of fluctuation during the year. Similar to the time series of the stock index itself, the time series of statistical volatility is also non-differentiable and non-stationary. What's more, Figure 2.3 also shows the feature of "volatility clustering", which

indicates that large stock price movements are likely to be followed by large price movements. Such "volatility clustering" is found to exist widely in most stock markets (Ding, et al., 1983; Breidt et al., 1998; Cont, 2001); this will be discussed in detail in section 2.3. There are other approaches forecasting the volatility time series using stochastic methods, such as auto regressions (e.g., GARCH model, Bollerslev, 1992). These methods take a phenomenological approach, assuming volatility is an intrinsic property of a stock or a stock market. The validity of this type model (which is difficult to benchmark using back-tests) is heavily reliant on whether the model captures the market's underlying principles as well as the calibration of input parameters through historical data. Despite the sophistication and success in some areas, critics claim that the overall predictive power of those models is similar to the simple statistical volatility (Cumby et al., 1993; Jorion, 1995), especially for the situation of "out-of-sample, where different data are used to estimate the models and to test them" (Andersen and Bollerslev, 1998). As an important measure of market uncertainties, the research around volatility is still among the most challenging tasks in the field of financial market research.

An alternative method of calculating the volatility is to determine the implied volatility from the prices of options traded in the market. As discussed in section 2.1, the approach of using implied volatility assumes that the option pricing model and the stochastic representation of stock price return are relatively "perfect" and capture accurately both the market's statistical behaviours and the (heterogeneous) decision making processes of the option traders. This is a strong assumption that must be tested extensively against real market data. For example, volatility is a measure of stock price movement and should not be affected by the option strike price. It is, however, shown that the implied volatility is lower when the strike price is closer to the stock price. Such empirical evidence is often referred as "volatility smile", and is observed universally in the stock and option markets (Hull, 2012). Similar to simple statistical volatility, implied volatility also uses historical values for future predictions, and assumes that volatility is constant over the period of consideration. Adib (2009) performed a study comparing the statistical volatility from the S&P 500 and implied volatility issued by VIX (CBOE Volatility Index); striking similarities are observed.

Figure 2.3 Statistical volatility of monthly logarithm return, S&P 500, 2015.

Another interesting area is the time dependence of the volatility values. The random variables of the Wiener process follow a normal distribution that diffuses with time. As the time horizon t increases, the variance of the normal distribution becomes larger and the shape of the distribution becomes flatter. As shown in Eqn. (2.16), the standard deviation (volatility) is governed by a constant time characteristic $t^{1/2}$, which is the key to deriving stochastic differential equations. Figure 2.4 shows the change in volatility with different time horizons, each of which is plotted with eleven different data points corresponding to 0-100% percentiles, showing the range of the volatility values. It can be observed that this range increases with increased length of return period, representing an increased uncertainty (of volatility) over time. This indicates that not only the uncertainty of stock price increases over time (measured by volatility), but also the uncertainties of the volatility itself. The theoretical curve $\sigma \sim t^{1/2}$ is also superimposed as the broken line on the figure. Comparing the empirical analysis with the theoretical model in Figure 2.4, the medians of the volatilities for different time horizons generally follow this theoretical curve. However, it is less the case for higher and lower percentiles. Specifically, some volatility percentiles show a diffusion process with no obvious characteristic time scales. The statistical analysis around volatilities highlights that the simplified assumptions of the Gaussian models are not always supported by empirical evidence. The risk around future uncertainties can be either over- or under-estimated, depending heavily on the choice of model and the calibration of input parameters.

Figure 2.4 Statistical volatility, with ranges of percentiles calculated for different time horizons of return. Data used: S&P 500, 2015. The broken line represents the steady state diffusion process $\sigma \sim t^{1/2}$.

2.2.4 The probabilistic distribution and the measure of risks

Apart from the risk around uncertainties, another risk measure is the loss of the portfolio that can be taken as a percentile limit of the probabilistic distribution, for example, those adopted in VaR (value at risk) practices (Jorion, 2007). Figure 2.5 shows the daily logarithm returns of the S&P 500 index, with empirical data plotted using discrete symbols. A transformation of the dataset is performed by subtracting the mean value from each data point. It can be observed from Figure 2.5 that the distribution is roughly symmetrical, with highest probability for small returns and small probability for larger returns. Two normal distributions with zero mean value are also plotted in the figure using continuous lines. The broken line uses the statistical volatility, calculated from the standard deviation of the whole dataset, while the solid line uses modelled volatility by calculating the maximum likelihood between the empirical data and a normal distribution. It can be observed that the tails of the distribution are longer and fatter for the empirical distribution than for the normal distribution. Table 2.1 shows the loss side of the tails for different VaR limits. It can be found immediately in the table that the magnitude of loss (as a risk measure) is underestimated using a normal distribution, especially when modelled volatility is used. By varying the VaR limit from 99% to 97.5% (inspired by the fact that the Basel II (2014) standard moved the VaR limit from 99% to 97.5% in 2011), the underestimation is reduced, but is indeed still substantial. It is demonstrated in Figure 2.5 and Table 2.1 that the risk measure adopting the

normal distribution can deviate significantly from empirical evidence. A risk management framework completely relying on those quantitative models may lead to underestimated risk levels and a "false sense of risk perception".

It is worth noting that the case study based on the stock price return and VaR methodology is only for illustration purposes. Some VaR models (or risk management models) adopted in the financial industry are less dependent on the assumed shape of the distribution (Rockafellar and Uryasev, 2000). Unlike the option pricing model derived from a closed form of mathematical deduction, risk management models normally take a constructive approach focusing on data collection and risk reporting (Jorion, 1996). As no financial institution has its portfolio completely relying on the stock market, the mechanisms of financial returns can be considerably different from what is shown in Figure 2.5. The current thesis looks at the stock price movements and does not go into details of risk management frameworks such as VaR. The phenomenon of fat-tail distribution is, however, observed in stock markets across the world and is considered by academics as a major disadvantage of the Gaussian-type models. The fat-tail distribution is a signature of both non-ideal market and inter-dependence of investor behaviours. It will be discussed extensively in the remaining parts of the thesis.
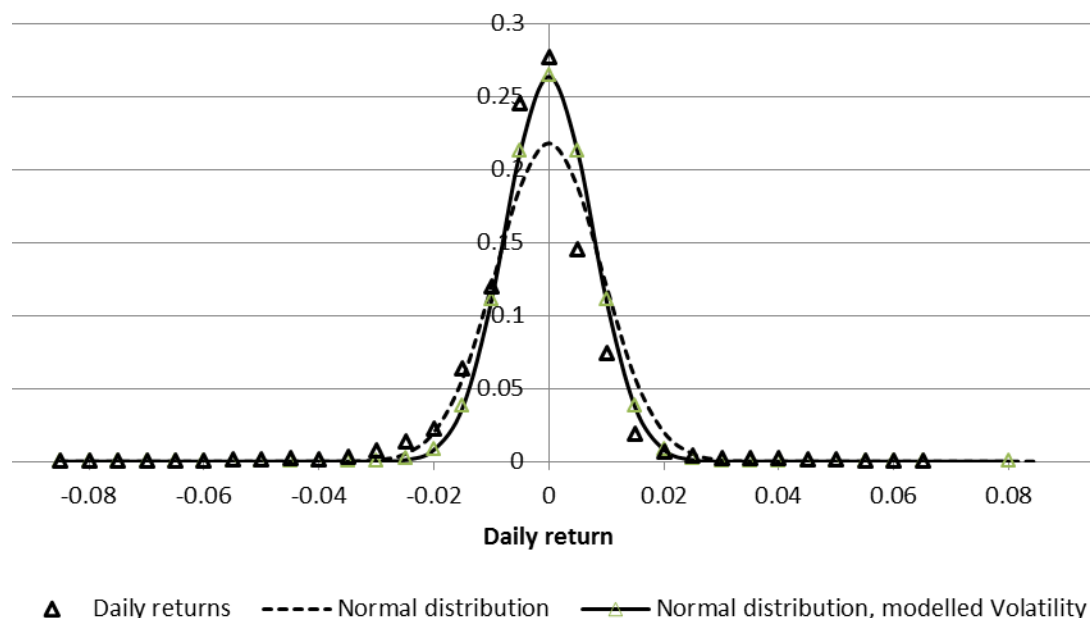
Figure 2.5 Empirical probabilistic distribution (discrete symbols) of daily logarithm returns, S&P 500, 2015, compared to normal distribution with different values of volatility; broken line: statistical volatility; solid line: implied volatility.

Table 2.1 Comparing risk measures using different VaR limits, between empirical distributions, normal distribution using statistical volatility and normal distribution using modelled volatility.

	Empirical distribution	Normal distribution - statistical volatility	Normal distribution - modelled volatility
Volatility used	n/a	0.009152	0.008565
99% VaR limit	-0.0325	-0.0239	-0.019
97.5% VaR limit	-0.02537	-0.021	-0.0175

2.2.5 The Brownian motion and statistical mechanics

The Gaussian process introduced above is based on a stochastic approach, originating from the central limit theorem. Another way of constructing such Gaussian-type models is to use theories developed to model chaos systems (Alligood et al., 1997). Thanks to advancements in physics research, especially statistical mechanics, it has been recognised that unpredictable time series in the macroscopic scale can be generated from deterministic nonlinear systems in the microscopic scale. The most famous example is to use the Brownian motion to describe the price movements of stocks. The Brownian motion is referred to as the phenomenon whereby fine particles in a system (such as pollen powders suspended in liquid) undergo random movements due to thermal motions of molecules or atoms. Such a physical phenomenon is captured using the standard diffusion equation (Voit, 2003). For a one-dimension case, assuming $\rho(x,t)$ the probability of a particle in position x at time t:

$$\frac{\partial \rho}{\partial t} = \tilde{D} \frac{\partial^2 \rho}{\partial x^2}, \qquad (2.20)$$

in which $\tilde{D}$ is the diffusion coefficient. It is easy to verify that the solution satisfying Eqn. (2.20) is the normal probabilistic density function:

$$\rho(x,t) = \frac{1}{\sqrt{4\pi\tilde{D}t}} \exp(-\frac{x^2}{4\tilde{D}t}), \qquad (2.21)$$

which is the same format of Eqn. (2.17) with $\tilde{D} = \sigma^2 / 2$. Thermodynamics and statistical mechanics assign a physical meaning to the diffusion coefficient $\sigma^2 = k_B T / 2$, in which k_B is the Boltzmann constant and T is the temperature in Kelvin. To understand this relationship, we start from the second law of thermodynamics. During heat transfer, the change of heat Q is related to the change of system entropy $\bar{S}$ and the absolute temperature T that:

$$\Delta \bar{S} = \frac{\Delta Q}{T} .$$
(2.22)

In the above equation, entropy is the measure of how chaotically the microsystems are arranged. Using ideal gas as an example, a larger volume of gas will have a larger number of microstates, as each particle in the gas will have more possible positions at a certain point in time. We can understand the entropy using the macroscopic gas compression equation:

$$\bar{P}\bar{V} = \bar{n}\bar{R}T = Nk_B T ,$$
(2.23)

in which $\bar{P}$ is the pressure and $\bar{V}$ the gas volume, $\bar{n}$ the molar volume of the gas and $\bar{R}$ the gas constant, N is the number of molecules in the gas. Under a gas compression process, assuming all pressure increase will convert to heat when the volume changes from $\bar{V}_2$ to $\bar{V}_1$, i.e.,

$$\Delta Q = \int_{\bar{V}_1}^{\bar{V}_2} \bar{P} d\bar{V} .$$
(2.24)

And at the same time, using Eqns. (2.22) and (2.23), this leads to:

$$\Delta \bar{S} = \frac{\Delta Q}{T} = Nk \int_{\bar{V}_1}^{\bar{V}_2} \frac{1}{V} d\bar{V} = Nk_B \ln(\bar{V}_2 / \bar{V}_1) .$$
(2.25)

Following the argument that the number of microstates is associated with the volume of the gas, for a single particle, it is easy to understand that this relationship is linear if the particle is unconstrained (Zurek, 1989). Letting w represent the number of microstates for one particle, we have:

$$\bar{V}_2 / \bar{V}_1 = w_2 / w_1 .$$
(2.26)

Using W the number of microstate for N particles, it is can be seen that the total possible ways of arranging those particles will be $W = w^N$ (Braunsterin, 1969), and Eqn. (2.26) becomes:

$$\overline{V}_2 / \overline{V}_1 = w_2 / w_1 = (W_1 / W_2)^N. \tag{2.27}$$

The expression of entropy linked with the microstate can be expressed from Eqn. (2.25) that:

$$\Delta \overline{S} = k \ln(W_2 / W_1). \tag{2.28}$$

Further noticing that the entropy will be zero when there is a single state of the particles (W=1, at absolute zero temperature), the absolute value of entropy can be expressed as:

$$\overline{S} = k_B \ln W, \tag{2.29}$$

which is the famous Boltzmann entropy. Gibbs (1878) further extended the entropy function to include the different probability p_i of each possible microstates that:

$$\overline{S} = k \sum_{i=0}^{W} \left[p_i \ln p_i \right]. \tag{2.30}$$

In order to understand Eqn. (2.30), we consider the following two cases: assuming two possible destinations of a particle - x_1 and x_2 - for the first case, there is only one possibility $p_1(x_1) = 1$ and $p_2(x_2) = 0$ which leads to $\overline{S}(p) = 0$; for the second case there are two possibilities with $p_1(x_1) = 1/2$ and $p_2(x_2) = 1/2$ which lead to $\overline{S}(p) = k \ln 2$. It is easy to prove that case 1 indicates the minimum entropy because it is certain where the particle is, while case 2 is the maximum entropy because there is nothing known beforehand of where the particle is likely to be (i.e., minimum prior information). For a more general case, i.e., a certain number of possible destinations for a large number of particles, it is straightforward to validate that the normal distribution maximises the entropy of Eqn. (2.30) that:

$$p(x) = e^{-\beta x^2} / Z, \tag{2.31}$$

in which $Z = \int_{-\infty}^{+\infty} e^{-\beta x^2} dx = (\pi / \beta)^{1/2}$ is a scaling factor. β is the linkage between energy and entropy (see Nelson, 1967 for details) with $\beta = 1/k_B T$ (thermo-dynamic expression) or $\beta = 1/2\sigma^2$ (statistical expression). Similar to the law of gravity in Newtonian mechanics,

nature always has the tendency to maximise the entropy level of a system (Truesdell and Muncaster, 1980). The normal distribution is therefore the solution to an ideal system expressed in Eqn. (2.30). It is, however, the above derivation which assumes the particles are independent and unconstrained, with no inter-particle friction and no memory (i.e., each time step is independent, sometimes referred to as infinitesimal relax time (Van Mieghem, 2009)). In the past decades, the development of computational power has enabled microscopic simulation using particles governed by physical laws to represent a macroscopic chaos system. Monte Carlo methods such as molecular dynamics are used to simulate the diffusion process, as well as Brownian motion, for which some statistical mechanics parameters (such as kinetic energy, temperature, entropy) can be calculated explicitly using local information such as velocity vector and mass of the particles. It is generally concluded from those studies that the normal distribution still holds for a chaotic systems with local or weak inter-dependence (Umarov, et al., 2008.). Those relaxed conditions are still not always applicable in the financial market. The feedback loops and the herd effect remain difficult to explain using statistical mechanics alone. Before attempting to address the impact of those factors, the macroscopic representatives of the financial market are discussed in the next section.

2.3 Empirical studies of the stock market

2.3.1 Data set and analytics methodology

In order to understand the market behaviour in a systematic manner, the current chapter will examine three stock indices from three major stock markets, over a reasonably long period of time. The time series under consideration include: S&P 500 from New York Stock Exchange from January 2001 to December 2015; FTSE 100 from London Stock Exchange from January 2003 to December 2015 and Shanghai Composite from Shanghai Stock Exchange from January 2008 to December 2015. Compared to the study of individual share prices, the advantage of using these stock indices is to capture the overall movements of the world's stock markets. As shown in Figure 2.6, the time series studied contain data for calm and turbulent periods of each market, both of which will be discussed in this section. The turbulent market period is during 2008 and 2009, and the calm market period is 2013 to 2014. This is to ensure the same periods for all three markets, and to enable a like-for-like comparison. The frequency of the time series is up to 1 minute and can be transformed to

analyse lower frequency data. Some models discussed in the previous sections are applied in this section, where appropriate.
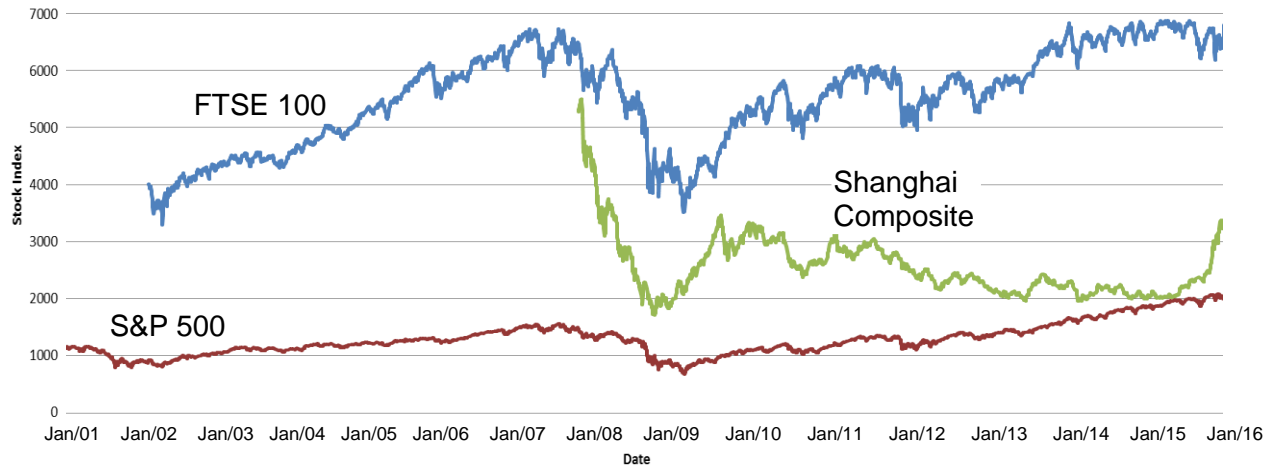
Figure 2.6 Daily stock indices from S&P 500, FTSE 100, and Shanghai Composite, plotted for different periods of time.

Looking at the stock indices plotted in Figure 2.6, the S&P 500 and FTSE 100 indices are correlated for almost the whole period. Both indices show long term growth with periodical sharp declines. Specifically, the market crash in 2000-01 after the internet bubble burst, and that after the 2007-08 mortgage crisis can be clearly observed from the figure. The Shanghai Composite index, on the other hand, shows an overall decline from 2008 to 2015, with the market spiking up at the start and end of the period under study. Between those spikes, the market also experienced long term deterioration, which is negatively correlated with the S&P 500 and FTSE 100. The double-figure growth rate of the Chinese economy in the 21th century is not clearly reflected in the stock market. Qualitatively speaking, the sharp rise and fall represents a market dominated by speculation activities, which are more liable to panic. The lack of long term growth in the Shanghai Composite index is also not supported by the theory of risk premium illustrated in Figure 1.1 (Malkiel, 2011). The rest of this chapter will analyse these data quantitatively, and such empirical study will focus on statistical properties of the stock markets, especially those that are not sufficiently captured by the efficient market hypothesis. Section 2.3.2 will examine the probability distributions for the three stock markets, for both calm and turbulent periods, and will analyse the impact of different time horizons of return. The diffusion behaviour of the empirical data will be discussed in Section

2.3.3, while section 2.3.4 will discuss the higher order factors an correlations which are also important in terms of statistical modelling and multi-scale modelling, such as volatility time series, memory effects (auto-correlation of stock return) and volatility clustering (auto-correlation of volatility time series).

2.3.2 Probability distributions

Figure 2.7 shows the probability distributions for different stock markets with different time horizons of return. Both calm and turbulent periods are plotted in the figure. The statistical treatment leading to the figures are detailed as follows. Firstly, the stock index values against each point of time corresponding with each time horizon of return are obtained from the raw data; then, the logarithmic returns $S(t)$ are calculated using Eqn. (2.8). The mean value of the returns is removed from the dataset following the convention of analysing the probability distribution (e.g. Lux, 1998). A series of intervals of returns covering all possible returns values are constructed as: $\{I_1, I_2, \ldots, I_{i, \ldots}, I_{n-1}, I_n, \} = \{S(t)_{min}, S(t)_{min} + \Delta S, \ldots, S(t)_{max} - \Delta S, S(t)_{max}\}$, which is then determined using the maximum and minimum returns, as well as the number of data points in the time series. Methodologies of determining the number of intervals are considered (Freedman and Diaconis, 1981) while all the plots in Figure 2.7 are obtained using a fixed size of intervals $\Delta S = 0.001$, in order to enable a like-for-like comparison. Finally, the number of values falling into each interval are calculated and normalised by the total number of data points. The probability associated with each interval can be understood as the product between the probability density function and the size of the interval: $\tilde{P}(I_i) = \Delta S \times P(S=S_i)$, in which $\tilde{P}$ the probability and P the probability density (note that P is used in a different context to represent price, however this will not cause confusion here). An analytical solution of normal distribution is also plotted, using the best fit of all data points for illustration purposes. Looking across the figures, the probability distributions are highly self-similar in the high probability areas, while they diverge in the low probability areas. Figures 2.7(a1) and (a2) show the one-minute return for the market's calm (2013-14) and turbulent periods (2008-09), respectively.

Figure 2.7 Probability distribution of logarithm returns for three different markets and different time horizon of returns, plotted for two different time windows reflecting a calm (column on the left-hand side) and a turbulent market (column on the right-hand side). An analytical normal distribution is also plotted for illustration purposes. Specifically: (a1): one-minute, market calm; (a2): one-minute, market turbulent; (b1): one-hour, market calm; (b2): one-hour, market turbulent; (c1): one-day, market calm; (c2): one-day, market turbulent. The probabilities in the figures are plotted using log plot. All the statistical analysis is plotted using the same interval of return $\Delta S = 0.001$.

Noting that the probabilities are plotted onto a logarithmic scale, it can be observed from the figures that the high-frequency data plot is symmetric and leptokurtic. Most of the returns are of small values; however, the rare events are much larger than those modelled using the normal distribution. As shown by Figure 2.7(a1), the FTSE 100 and S&P 500 indices follow almost the same distribution during the calm market period, whilst the Shanghai Composite index deviates further from the normal distribution with fatter tails. In Figure 2.7(a2), on the other hand, the market turbulent period suggests a much higher volatility, with empirically about 2.5 times the standard deviation to that of the calm market period. The probability distributions for the three stock indices overlap with each other and are almost indistinguishable. The empirical distributions using high-frequency data are also studied by Mantegna and Stanley (1995), and similar findings are published. The current thesis performs a systematic approach, covering both calm and turbulent periods as well as different stock markets. If the normal distribution can be regarded as the ideal market, how much the empirical distribution deviates from the normal distribution might be regarded as the measure of non-ideal market. From Figures 2.7(a) and 2.7(b), we can draw the conclusions that the market is less ideal in turbulent periods than in calm periods. What is more, the market is less ideal in the Shanghai stock exchange than the New York and London markets, during the same period. This may be due to different regulatory market rules (for example, the daily gain and loss in the Shanghai Stock Exchange is capped at a 10% limit (Mostowfi, 2015)). During the market turbulent period, on the other hand, no significant difference can be observed for stock markets with a similar level of deviations.

Figures 2.7 (b1, b2, c1 and c2) can be also examined using similar logic. It can be observed that the market is more "Gaussian-like" with the increase of time horizons of returns, for the calm market period as opposed to the turbulent market period. Figure 2.7(c1) is the closest to the normal distribution of all the figures plotted. The Shanghai Stock Exchange is the most "non-ideal" of the three stock markets studied. However, with the increase of the return horizon or in the case of market turbulence, it become closer to the other two stock markets. Recalling that the central limit theorem suggested that a large number of independent random variables will converge to a normal distribution, a larger time horizon of returns includes more random variables of this type, and is therefore closer to the convergence. Inter-dependences between time series and investors will lead to a non-Gaussian distribution. A market turbulent period is considered to have more such inter-dependencies and therefore deviates more from the normal distribution. The empirical evidence presented in Figure 2.7 is

qualitatively consistent with the above understandings, which can be tested against longer time horizons of returns, as plotted in Figure 2.8 and Figure 2.9 for weekly and monthly values respectively.
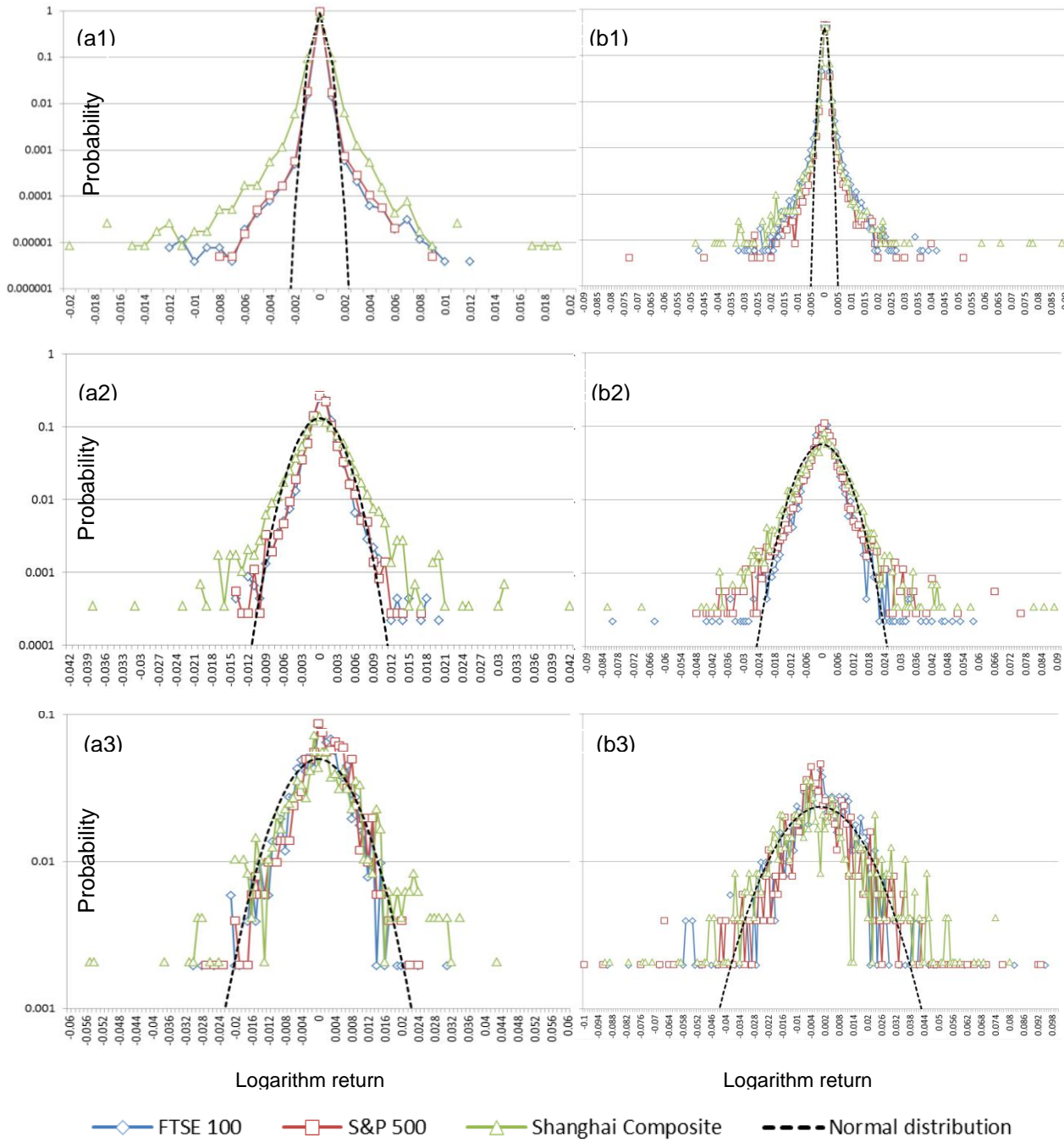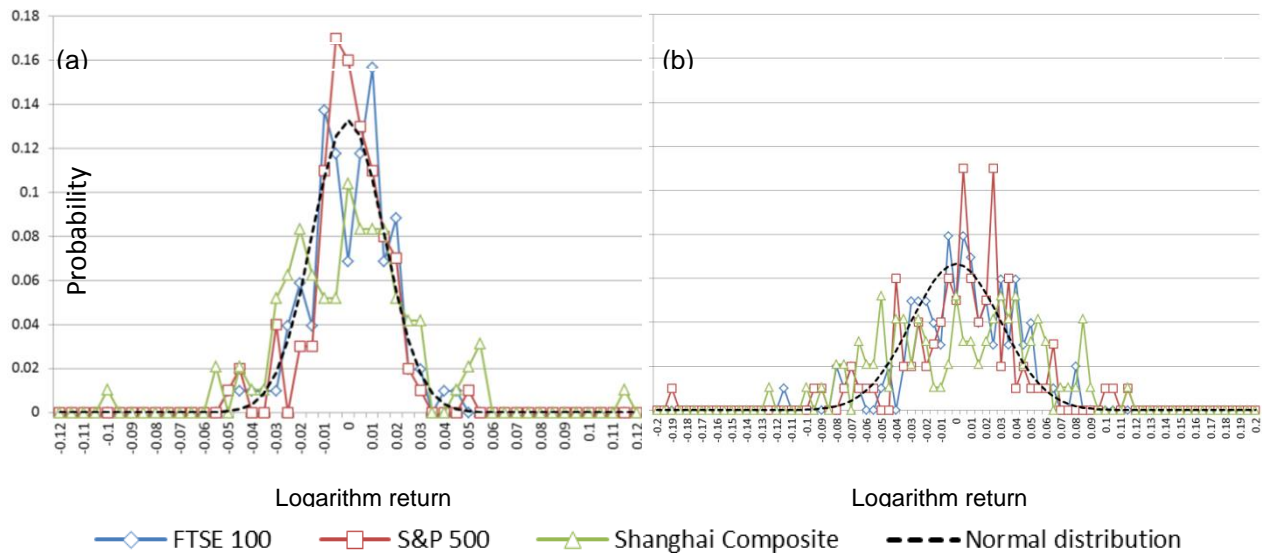

Figure 2.8 Probability distribution of logarithmic returns for three different markets and weekly return plotted for two different time windows reflecting market calm (a) and market turbulence (b). An analytical normal distribution is also plotted for illustration purpose. All the statistical analysis is plotted using the same interval of return $\Delta S = 0.005$.
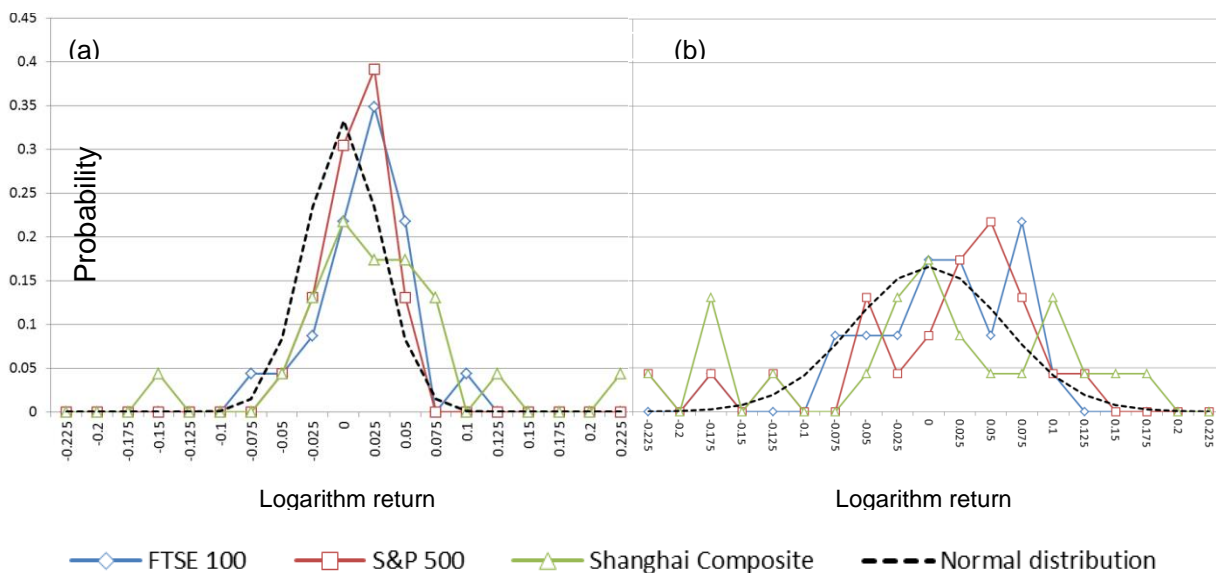
Figure 2.9 Probability distribution of logarithmic returns for three different markets and monthly returns plotted for two different time windows reflecting a calm market (a) and a turbulent market (b). An analytical normal distribution is also plotted for illustrative purposes. All the statistical analysis is plotted using the same interval of return $\Delta S = 0.025$.

As can be observed from Figure 2.8, the probability distributions of weekly returns for the three stock indices overlap with each other and the feature of fat-tail starts to vanish. The distribution for the market turbulent period (Figure 2.8(b)) is considerably flatter than that of the market calm period (Figure 2.8(a)), representing much larger volatility values. Due to a different interval of return ΔS being used, the probabilities are not directly comparable with those in Figure 2.7. The figures also show a higher noise level, resulting from the smaller sample size. Such noise level is most prominent in Figure 2.9 which plots the probability distribution of monthly returns. The distributions in the figure are highly "Gaussian-like", showing that the market is relatively efficient for a longer duration, despite high inefficiency for a short time horizon.

2.3.3 The diffusion process and its scaling

Another way of understanding the probability distribution function is through the diffusion process regarding different time horizons of returns. Mantegna and Stanley (1995) suggested that such a diffusion process can be identified by the governing time scale of the different probabilities of zero returns. According to the Gaussian process in Eqn. (2.17), the normal distribution leads to the probability distribution function $P^{\tilde{}}(S=0) = P(S=0) \times \Delta S \sim t^{-1/2}$. The empirical values of $P^{\tilde{}}(S=0)$ are plotted in Figure 2.10. As shown in Figure 2.10, the probability of zero return follows the power law with respect to different time scales, shown as straight lines in the log-log plot of the figure. The slopes of the best fit lines are detailed in Table 2.2, representing the time scale of the diffusion process. For the market calm period in Figure 2.10(a), the slope is close to -0.5, which is very similar to the Gaussian process. According to Mantegna and Stanley (2000), such a value indicates a long term weak correlation of stock price returns. Similar values are found in previous studies for the New York Composite (0.52), DAX index (Frankfurt, 0.53), and MIB index (Milan, 0.57) (Dacorogna et al., 1993; Mantegna and Stanley, 1996). This finding is also in line with the diffusion process shown in Figure 2.4, despite the relatively high noise level in the volatility analysis. For the market turbulent period, the slopes are slightly steeper (increases in terms of the absolute values), while still between -0.5 and -0.6. It is somehow different from the value reported in a previous study using the S&P 500 high frequency (the value -0.712±0.025 is found by Mantegna and Stanley, 1995). This is an interesting finding. It will be shown in the next chapter that the "supper diffusion" (i.e. the value of the slope is considerable higher than

0.5) can be due to a large number of statistical reasons such as heterogeneous time scales by mixing the calm and turbulent periods (which are governed by different time scales, as demonstrated by Figure 2.7). On the one hand, the volatilities are of different magnitudes for the calm and turbulent market periods (as shown in Figure 2.7); on the other hand, the calm market period and the turbulent period are identified from Figure 2.6 without a quantitative measure. It seems that the "super-diffusion" claimed by previous literature can be caused by an artefact mixing calm and turbulent periods with significantly different magnitudes of volatilities (e.g., Mantegna and Stanley, 1995). Such "super-diffusion processes" are observed in Micelles molecules (using fluorescent tracer, see e.g. Ott et al.,1990 for details), however it has later been proven that those molecules are still following Brownian motion. It is the combination of different Brownian motions governed by heterogeneous time scales that conveys an apparent super-diffusive character (Voit, 2005).

2.3.4 Higher order statistical factors and correlations

The empirical evidence discussed above focuses on the probability distribution and its diffusion process. This section will discuss other factors that may cause inconsistencies between empirical data and "Gaussian-type" models. These factors are identified using correlation and covariance analysis, and are referred to as higher order and correlations factors in this thesis. An important higher order factor is the volatility (and its clustering). Some important correlations include autocorrelation of time series of stock returns (memory effect) and autocorrelation of volatility (volatility clustering). The autocorrelation of time series of stock returns are widely discussed in the literature (Guillaume et al., 1997; Cont, 2001), using empirical data from both individual share prices and stock indices. It is generally concluded that the autocorrelations of price returns are often "insignificant", except for very small return periods (Cont, 2005). The time period of decay for the autocorrelation function is generally less than 20 minutes (Plerou et al., 1999). Based on such findings, the assumption that the stock returns is a Martingale process (Eqn. (2.12)) is justified for modelling stock price returns. Additionally, Plerou et al., (1999) studied high-frequency data for the S&P 500 stock index and found that autocorrelation is decaying at an exponential rate that:
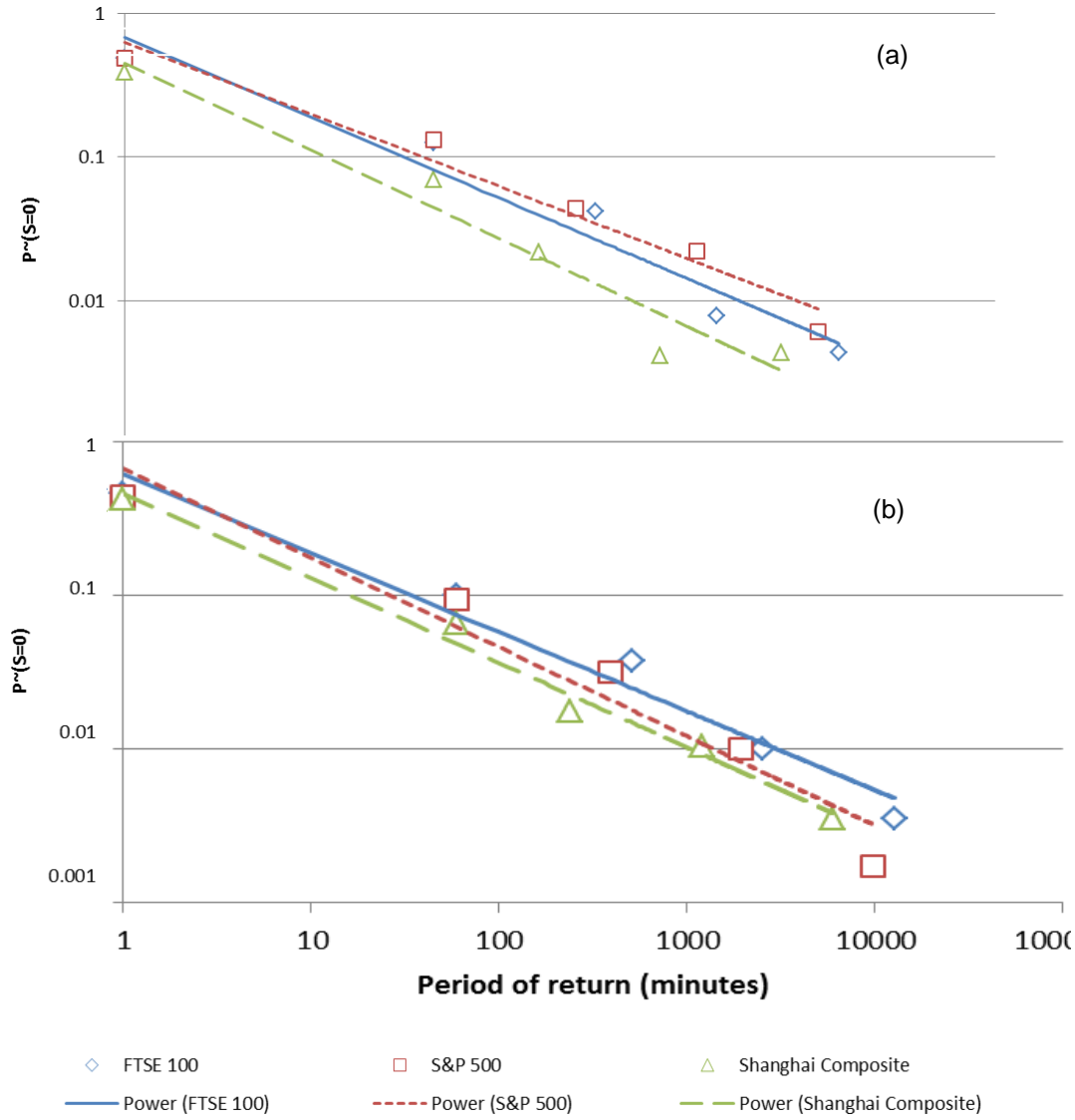
Figure 2.10 Probability of zero return as a function of period of return, plotted using three stock markets and two time windows of (a) calm market, and (b) turbulent market. The figure uses log-log plot and the best fit of power law curves are also plotted.

$$\mathrm{ACF}(\tau) \sim e^{-\tau/\tau_0}, \tag{2.32}$$

in which the ACF denotes the autocorrelation function and τ is the time lag of the autocorrelation with τ_0 the characteristic time governing the rate of decay. The exponential decay of the autocorrelation function is a signature of "short (temporal) term dependence" (Cont, 2005), that may be also used to measure the market efficiency.

Table 2.2 The slopes of the curves in Figure 2.10 (the value for Gaussian process equals -0.5; the value found by Mantegna and Stanley (1995) is equal to -0.712±0.025)

	Calm	Std. error (Calm)	Turbulent	Std. error (Turbulent)
FTSE 100	-0.536	±0.084	-0.513	±0.062
S&P 500	-0.494	±0.075	-0.58	±0.133
Shanghai	-0.562	±0.126	-0.551	±0.078

The current thesis studied the autocorrelations using high frequency returns from different stock markets, during both calm and turbulent market periods. As shown in Figure 2.11, those in the left-hand column use a log-log plot to highlight the absence of a long term correlation. This is generally in agreement with previous findings. However, the time scale of the decay varies significantly from market to market, while it is not strongly affected by whether the market is experiencing turbulence or not. The long term autocorrelation for the turbulent period is marginally higher for all markets, but without statistical significance. The column on the right-hand side is the zoom-in for the figures on the left to highlight the short term autocorrelation, with only the y-axis log plotted. The exponential decay rate of Eqn. (2.32) is generally supported by all the data presented in Figure 2.11. It can be identified that the S&P 500 has the shortest memory - around five minutes for both calm and turbulent periods. The FTSE 100 index has a memory of around five minutes for calm periods and around ten minutes in turbulent periods. For the Shanghai Composite, the decay is much longer: 15 minutes for calm market periods, and 20 minutes for turbulent periods. This is interesting: as in the statistical analysis in Figure 2.7, the Shanghai stock market stood out from London and New York, which is consistent with the autocorrelation analysis.

The statistical behaviour of volatility, on the other hand, is more complicated. It is one of the most striking pieces of empirical evidence, and it has intrigued a number of academics (Mandelbrot, 1963, LeBaron, 2001a, Kirman and Teyssiere, 2002). An example is the volatility clustering that can be observed qualitatively in Figure 2.3. Such inter-dependence between volatility time series suggests that the linear autocorrelation analysis of stock returns is not sufficient to prove that there is lack of long term correlation in the stock market. In fact, many empirical studies have found long term autocorrelation of the absolute (unsigned)
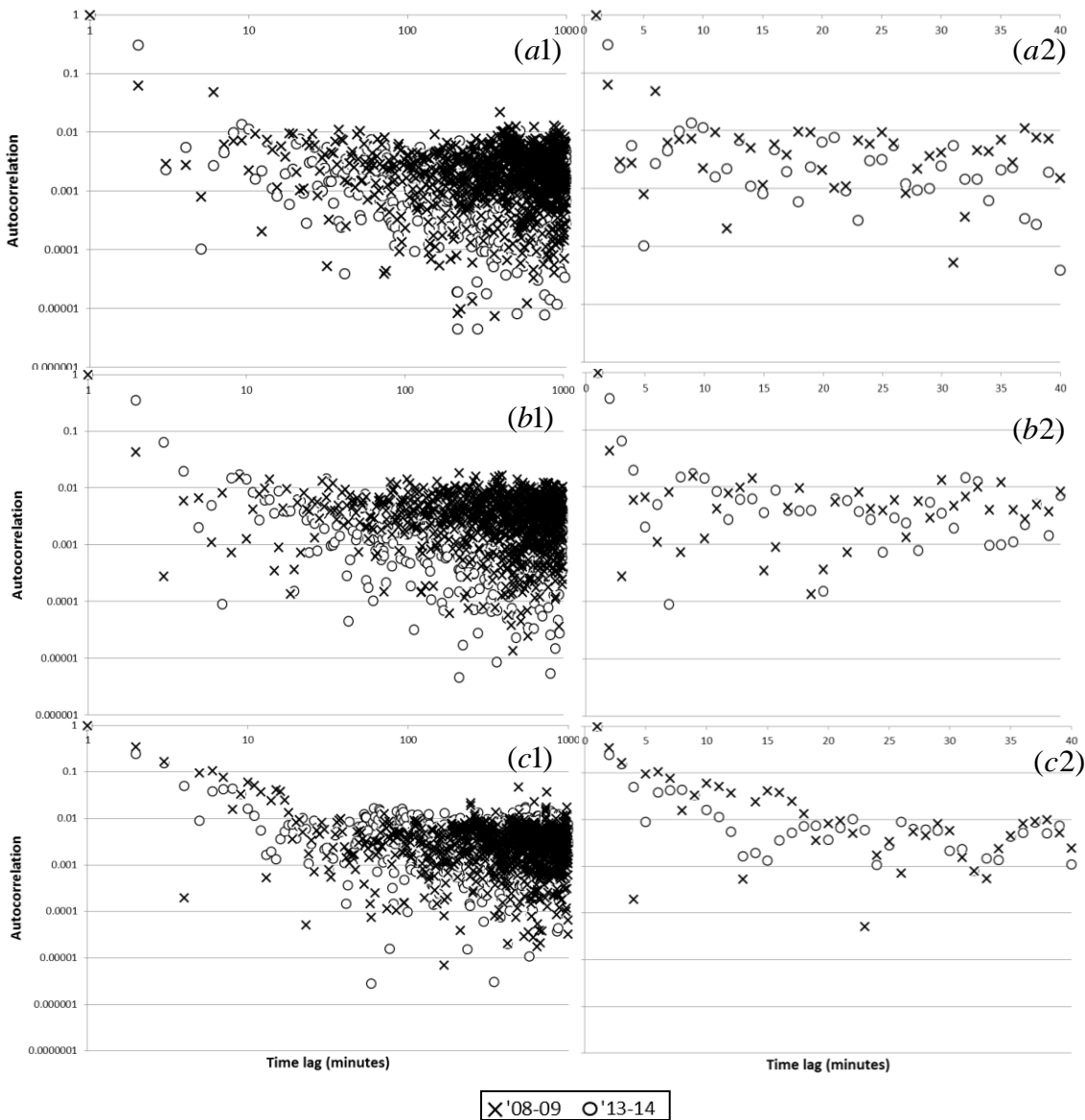
Figure 2.11 Autocorrelation of logarithmic (one-minute) return for different stock markets. The left-hand column uses log-log plot showing long term correlation while the right-hand column uses log plot on the y-axis only showing the decaying time in terms of memory effect. Rows (a), (b) and (c) are corresponding to different stock indices: FTSE 100, S&P 500, and Shanghai Composite, respectively. The different symbols represent different market periods: i.e., calm and turbulent.

values of stock price returns with significant slower decay rate compared with the (signed) values of price returns. The long term correlation has a signature of a power law decay rate (Cizeau et al., 1997) and the time lag of such a correlation could be in the order of sample size (LeBaron, 2001a). It is worth noting that a market with a long term correlation does not

mean it is not arbitrage-free. Cheridito (2004) gives examples of stochastic processes with long term dependence which are also semi-Martingales. For example, a stochastic process can be constructed as a function of the fractional Brownian Motion using the stochastic calculus of variations. The compatibility between arbitrage free and long term dependence is qualitatively explained by Mantegna and Stanley (2000): the (long term) autocorrelation of volatility is affected by higher order probability conditional densities, while the statistical distribution of return (which is linked with arbitrage-free) is by the first order factors. There are a number of attempts to model the volatility cluster, long term autocorrelation of volatilities or autocorrelation of absolute returns, such as fractional Brownian motion (Mandelbrot and Van Ness, 1968) and FIGARCH model (Arthur et al., 1997). However it is still unclear how and why such long term correlation prevails in the empirical studies (Mikosch and Starica, 2003; Resnick, 1998). The volatility clustering will be further discussed in the following chapters, and our empirical studies are firstly presented here, using the same approach to examine different markets and different periods.

Figure 2.12 shows the autocorrelation functions for the volatility time series, calculated from the stock exchange indices of FTSE 100, S&P 500, and Shanghai Composite. Both periodical and moving average calculations are tested to calculate the volatility over a defined time horizon, and no significant difference is found. The left-hand column shows the market calm period while the right shows the market turbulent period. The initial power law decay against the time lag can be identified in all figures. The rate of the initial decay is much slower than that for the price return, which can last several days and weeks. This is consistent with some other empirical studies (Cont, 2005). For the autocorrelation of the volatility, the decay rate is much faster during the calm period than the turbulent period. After the initial decay, the autocorrelation functions experience a complex shape with rising values of either positive or negative correlations, persistent for the whole period of analysis. Using the FTSE 100 index (Figures 2.12(a1) and (a2)) as an example, the negative autocorrelation follows the initial positive one and then varies as a cyclic function. The decay rate in each cycle is also much faster for the market calm period than for the turbulent period. This observation is generally valid for the other two markets, too. Interestingly, unlike the autocorrelation of returns, the Shanghai market shows the smallest differences between the two periods. The long term correlation for the turbulent period is also much smaller than the other two markets.

Figure 2.12 The autocorrelation functions of volatility of return for the data used in Figure 2.11, plotted against the time lag (number of trading days). The left-hand column represents the market calm period and the right-hand column represents the market turbulent period. Rows (a), (b) and (c) correspond to different stock indices: FTSE 100, S&P 500, and Shanghai Composite, respectively.

Figure 2.13 shows the autocorrelation function for the absolute values of price returns, which is considered as another method of examining the long term correlation. Similarities can be observed for those plotted in Figure 2.12 and Figure 2.13. The decay of the correlation is of similar rate for both figures. For the FTSE 100 and S&P 500, the autocorrelation of both volatility and absolute returns are similar for the market turbulent period. For the calm market period, however, the negative correlation can happen much earlier for the absolute return than for the volatility. Specifically, the cycle (of switching between positive and negative correlation) in the left-hand column (for market calm period) is of a much higher frequency in Figure 2.12, but the envelope of those cycles is similar to those in Figure 2.13. Again, the Shanghai market shows a distinct feature in Figure 2.13 that a very small autocorrelation is observed after the initial decay period. Following the argument of efficient market hypothesis, if we consider the memory effect of the stock return as a measure of how efficient the market will be at absorbing information, it seems like the market is extremely inefficient in terms of absorbing the long term correlation of volatilities. The phenomenon that positive correlations are followed by negative correlations can be better understood by thinking those correlations are linked with volatility clustering. The fact that large volatilities are followed by large volatilities - and smaller volatilities followed by small volatilities - can be a signature of a feedback loop in the market. As discussed by Mantegna and Stanley (2000), volatility can be directly related to the amount of information arriving in the market at a given time. Such information will be absorbed in the stock price returns efficiently; however, in terms of volatility, it has the effect of a feedback loop: a high volume of trading will trigger another, higher volume of trading. It is a self-catalyst effect and therefore it is understandable that the autocorrelation is much higher in terms of both volatility and the absolute return for the market turbulent period. As discussed by Cont (2005), the long term correlation of volatility and absolute returns can also be due to the combination of a number of factors, including non-stationary time series (Mikosch and Starica, 2003), heterogeneous information shock (Ding et al., 1993; LeBaron, 2001b), the switching of investors' behaviours (Lux and Marchesi, 2000), and/or the inertia of investors (Liu, 2000). Those factors are extremely difficult to model through a stochastic process but can be effectively studied through a multi-scale approach.
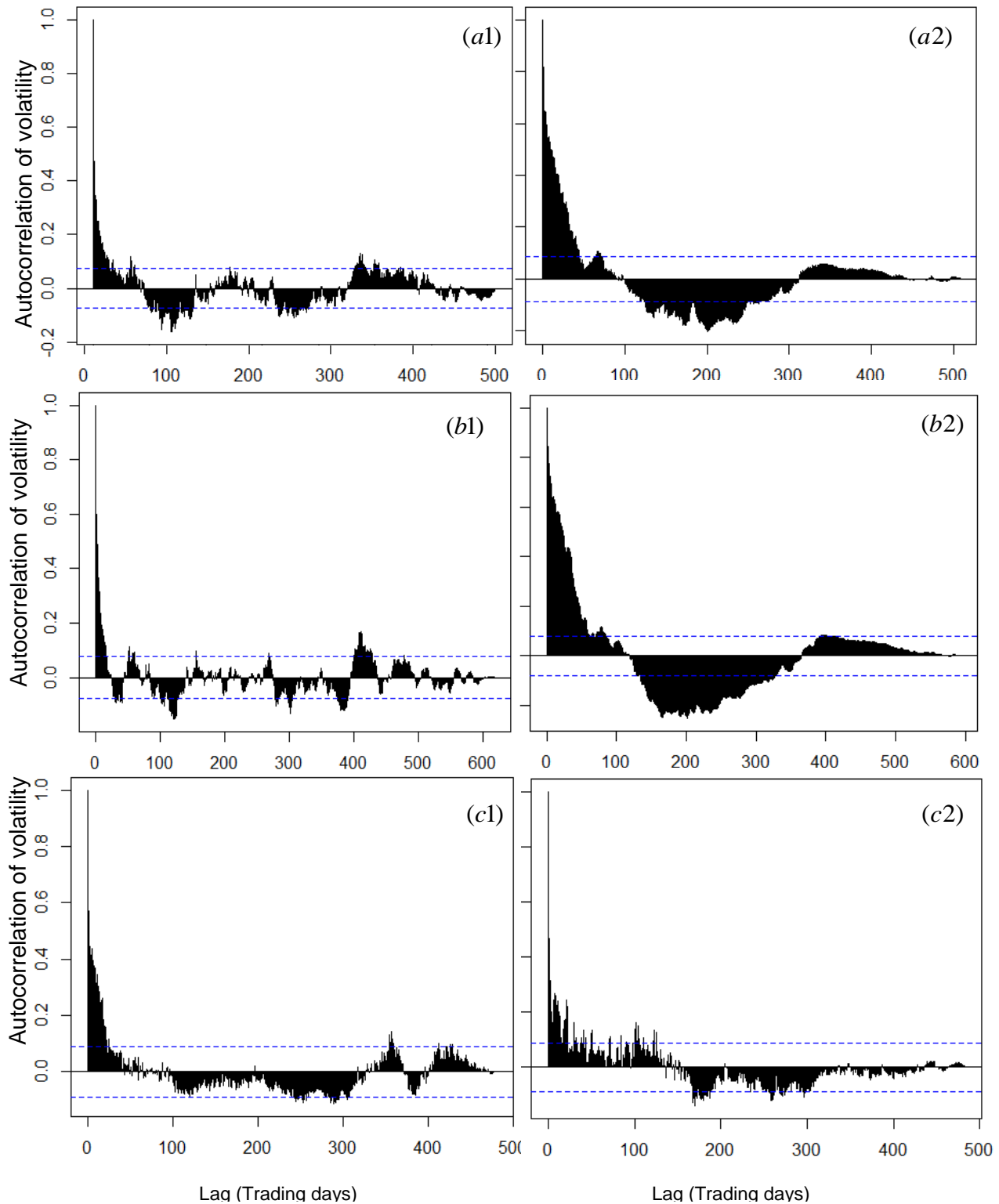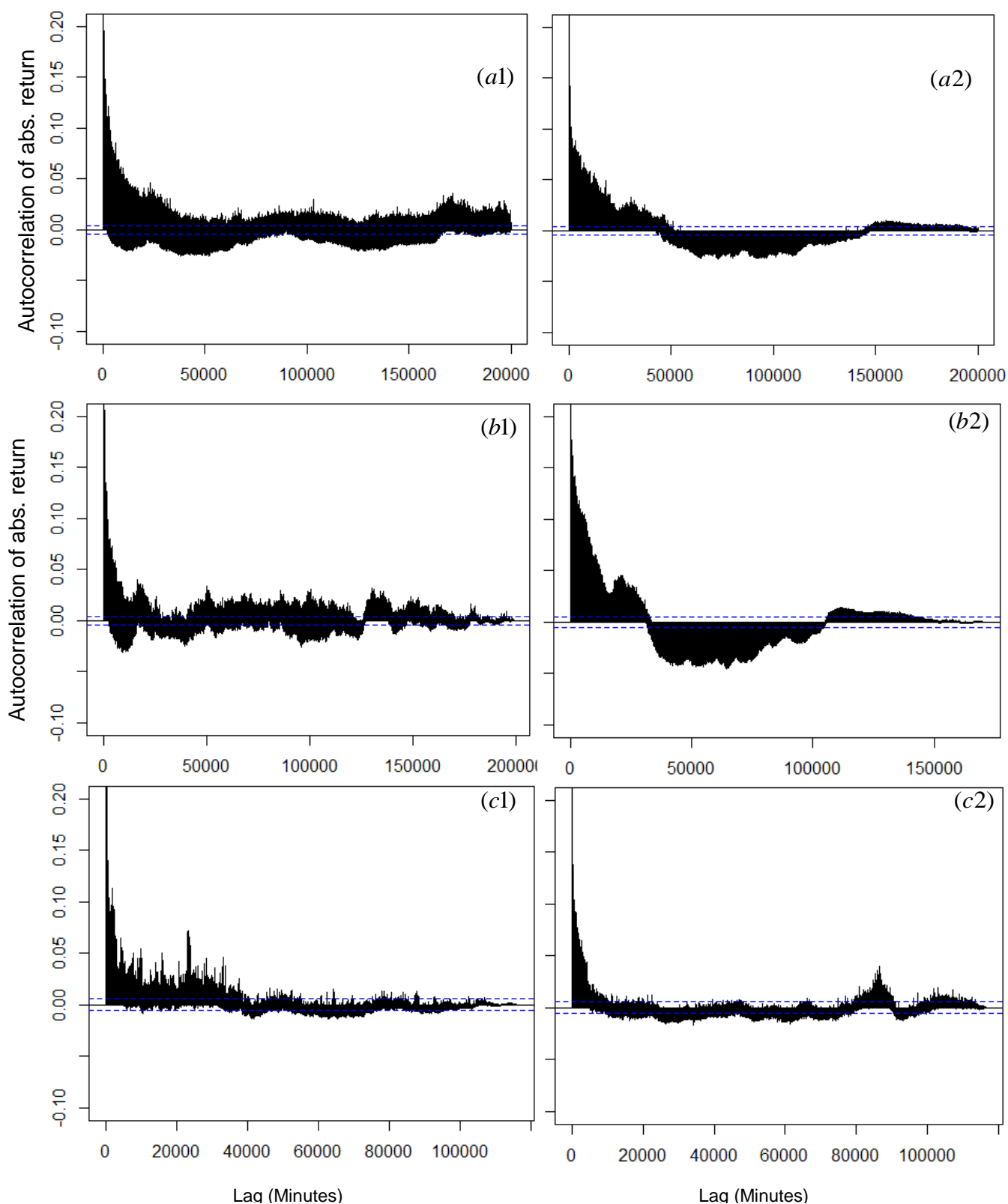
Figure 2.13 The autocorrelation functions of absolute return for the data used in Figure 2.11, plotted against time lag in minutes. The left-hand column represents the calm market period and the right-hand column represents the turbulent market period. Rows (a), (b) and (c) correspond to different stock indices: FTSE 100, S&P 500, and Shanghai Composite, respectively.

2.4 Summary of this chapter

This chapter introduces some useful concepts that are essential in understanding the stock markets and performing mathematical modelling. It starts with the introduction of the market and popular models that are used on a daily basis in making investment decisions. We have then taken a step back to examine the fundamental assumptions used to construct those models, focusing on the exploration of the efficient market hypothesis. It is also interesting to see that, based on parallel assumptions, the stochastic modelling approach and the chaos system modelling approach derive similar models of the Gaussian process and Brownian motion. It is also shown in this chapter, using those models, how popular risk measures and risk management practices widely adopted in the financial industry are still far from ideal. The current chapter uses examples of the option pricing model and the value at risk model to demonstrate the success and pitfalls of those classical models, and to highlight the importance of understanding the modelling limitations. At the same time, the inputs used in those models are also discussed.

The second part of the chapter examines empirical data in a systematic manner, with a particular focus on the shape of the distribution, the memory effect of the price returns and the long term correlation of the market. A comparative study is performed, detailing different markets for calm and turbulent periods. Some interesting findings are presented. Particularly, the combined feature of both fat-tail distribution and long term correlation of volatility is still among the most active topics in financial research. The current research gives particular attention to the high frequency data due to its ability to reflect the microscopic mechanisms of the market. Such analysis provides insight to the investor's behaviours, and hence some inputs and directions for the following multi-scale modelling approach. The empirical study performed in the current chapter is generally in agreement with previous studies. However, some differences are found, particularly for the Shanghai stock market, which deviates from the other markets and from previous understandings. This could be due to a number of reasons associated with market microstructure. As introduced by some previous studies (e.g. Zhang, 2012), the Shanghai market is seen as an "emerging market" with heavier regulations. Particularly, it does not allow short selling and off-market trading, with less structured investors mainly from domestic capital source (in comparison to international capital source in S&P and FTSE). Qualitatively speaking, the short selling reduces deviations of transaction prices from the market value by accelerating the process of incorporating public information

into prices (Boemer and Wu, 2009). What is more, a daily price threshold of 10% is also enforced in the Shanghai market. According to Ryoo and Graham, (2002), the price threshold will limit the market's ability to reflect on the newly released information and will have negative impact on the market efficiency. On the other hand, Lagorade-Segot (2009) suggested that microstructures of emerging markets (Shanghai market is regarded as an emerging market) such as informational efficiency, volatility, and liquidity levels are more affected by economic and political changes which are also more apparent in China for the past 10 years than in UK or US.

Chapter 3 Macroscopic modelling using non-Gaussian approaches

3.1 Overview

It is demonstrated in the preceding chapters that the stock market has the following properties: 1) not always as efficient as suggested by the intrinsic value theories or efficient market hypothesis; 2) non-Gaussian (fat-tail) distribution of price return especially for high-frequency data; 3) Gaussian-type diffusion process for non-Gaussian distributions; 4) time-dependant volatility and its clustering; 5) short term and long term correlations. Empirical evidence from different stock exchanges and different periods suggests that the stock market is far from ideal, as assumed by the geometric Brownian motion. Depending on the context of applications, the financial models and financial products (e.g., those introduced in chapter 1) derived based on the efficient market hypothesis may stop functioning (Voit, 2003), especially during financial crisis. On the other hand, the results from statistical analyses are strongly dependent on the dataset examined and how it is manipulated. Empirical models and input parameters devised from statistical analyses of historical information might not be universally valid. What is more, the modern financial industry heavily relies on complicated modelling techniques for everyday decision making. Apart from understanding the assumptions and limitations of the existing models, a more accurate quantitative understanding of the fundamental properties of the stock market is relevant to both financial modelling and risk management frameworks.

One of the reasons for the popularity of the Gaussian-type approach is the mathematical closed formulation of the normal distribution. It is a one-parameter model (volatility) with a relatively simple analytical expression. It is a stable distribution with the feature of shape-invariance that is guaranteed through the central limit theorem. It is also governed by a diffusion process, the time scale of which can be clearly identified, leading to analytical calculus that is essential in deriving stochastic differential equations. What is more, it has the theoretical foundation from statistical mechanics with defined physical representatives, such as the Gibbs-Boltzmann entropy. Any non-Gaussian models must have similar properties in

order to be applicable in the industry. Bearing these in mind, the current chapter will attempt to step away from the efficient market hypothesis and the geometric Brownian motion. Facilitated by the empirical methodology established in Chapter 2, some non-Gaussian approaches using stochastic process and statistical mechanics are examined, with the focus on their applicability to the stock market. This type of approach is generally derived from analysing properties of the macroscopic factors of the market such as price returns and volatility. Specifically, the Lévy statistics (Mandelbrot, 1963) derived from a generalised central limit theorem will be discussed in the context of stochastic processes; the Tsallis statistics (Tsallis, 1988) derived from non-extensive entropy will be discussed in the context of statistical mechanics. Phenomenological models such as ARCH and GARCH processes will also be discussed in the context of the higher order factors (Eagle, 1982; Bollerslev, 1986)[1]. Those non-Gaussian approaches studied in the current chapter are also compared with each other, together with the empirical data examined in Chapter 2. The objectives of the chapter are to provide better insight into the empirical discoveries of the stock market and to function as a benchmarking exercise to those non-Gaussian approaches. Based on the findings from the current chapter, some simple but powerful models are also proposed.

3.2 The Lévy statistics

3.2.1 Introduction to non-Gaussian distributions

An active research stream of the non-Gaussian approaches is to find a probability distribution that captures the fat-tail behaviour observed in the empirical data. Figure 3.1 plots the probability distribution tail for the one minute price returns as used in Figure 2.7. Note that the price return in Figure 3.1 is normalised by the volatility of the time series. It can be observed from the log-log plot that the distribution tail is governed by a power law distribution instead of the exponential law suggested by the normal distribution. As a general agreement in this research field, a non-Gaussian distribution adopted to model the stock market must be able to capture such power law distribution tails. Furthermore, despite differences in the value of volatility for the different datasets in the figure, those distribution tails plotted in Figure 3.1 all decay at a similar rate, which seems to be insensitive to market-

---

[1] The definition of non-Gaussian can be regarded as loose because the ARCH and GARCH process may be constructed based on a standard Gaussian process. In the current thesis, the non-Gaussian is defined as the unconditional probability distribution that is used to model the empirical data from the financial market. As a result, a process or model that has got a root of Gaussian distribution can be non-Gaussian in the sense of unconditional probability distribution.

by-market or period-by-period differences. The power law distribution for the tails, however, seems to be sensitive to different time horizons of price returns. As discussed in Chapter 2, normal distribution becomes valid for weekly or monthly returns (Figures 2.8 and 2.9), which indicates that the power law distribution must break down when the time horizon of return increases. Gopikrishnan et al. (1999) suggested that price returns of the stock market follow a power law distribution when the time horizon of returns is smaller than four days, while following a normal distribution when the time horizon of returns is larger than four days. As shown in Figure 3.1, the slope of the straight line in the log-log plot is -3.5. Following the convention we use $-\bar{\mu}$ to represent the slope for the cumulative distribution while $-\bar{\mu}-1$ is the slope for the tail of distribution density function. For the one-minute price return shown in Figure 3.1, the power law distribution is governed by $\bar{\mu}=2.5$. The cases for larger time horizons of return are plotted in Figure 3.2, in which the power law distribution can also be observed; it is, however, not as statistically significant as in Figure 3.1.

Figure 3.1 The tail behaviours of the probability distribution that are plotted in Figure 2.7, for the one-minute return, which is normalised by the standard deviation of each data series. The normal distribution and a power law function are superimposed, showing the tail behaviour is following a power law distribution. The standard error (around the best fit power law distribution) is ±0.31.

The findings in Figures 3.1 and 3.2 may suggest that the shape of the distribution tail moves from a power law to normal distribution in a continuous manner, instead of at a point of singularity (four days as suggested by Gopikrishnan et al., 1999). What is more (and by accepting the validity of the power law distribution), the slope of the lines in Figure 3.2 are less steep than in Figure 3.1, with $\overline{\mu} = 2$ for Figure 3.2(a) (one-hour return) and $\overline{\mu} = 1.5$, for Figure 3.2(b) (one-day return). The slope of the power law distribution, i.e. the $\overline{\mu}$ values are also studied extensively in previous literature: $\overline{\mu} = 1.66$, for 15-minute returns (Bouchaud and Potters, 1997); $\overline{\mu} = 2.33$ for 15-second returns (Voit, 2003); $1.7 \leq \overline{\mu} \leq 3$ for daily returns (Gopikrishnan et al., 1999), $1.42 \leq \overline{\mu} \leq 1.75$ for daily returns (Lux, 1996). The common (and qualitative) element of those studies is that the distribution tail is moving from a power law distribution to a normal distribution with the increase of the time horizon of return. This is also consistent with some empirical discoveries that the tail is "fatter than the normal distribution but thinner than the power law distribution" (Lux, 1996). It is however extremely difficult to quantitatively compare those previous studies based on different markets and time periods. An analytical framework is required to facilitate the understandings on those empirical findings.
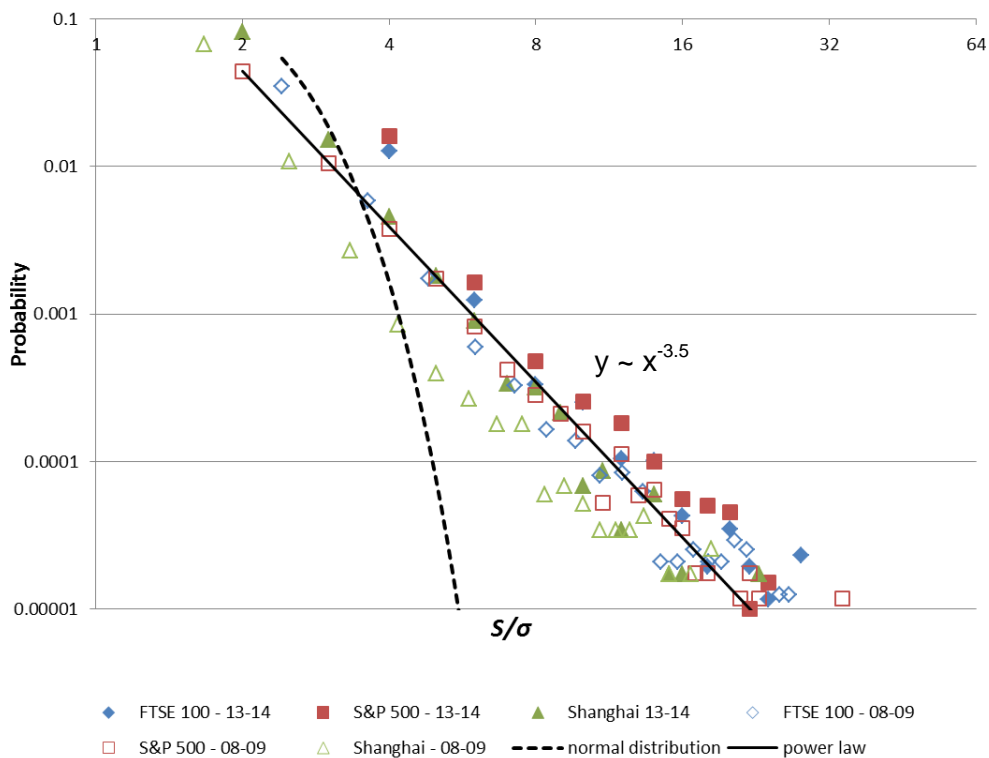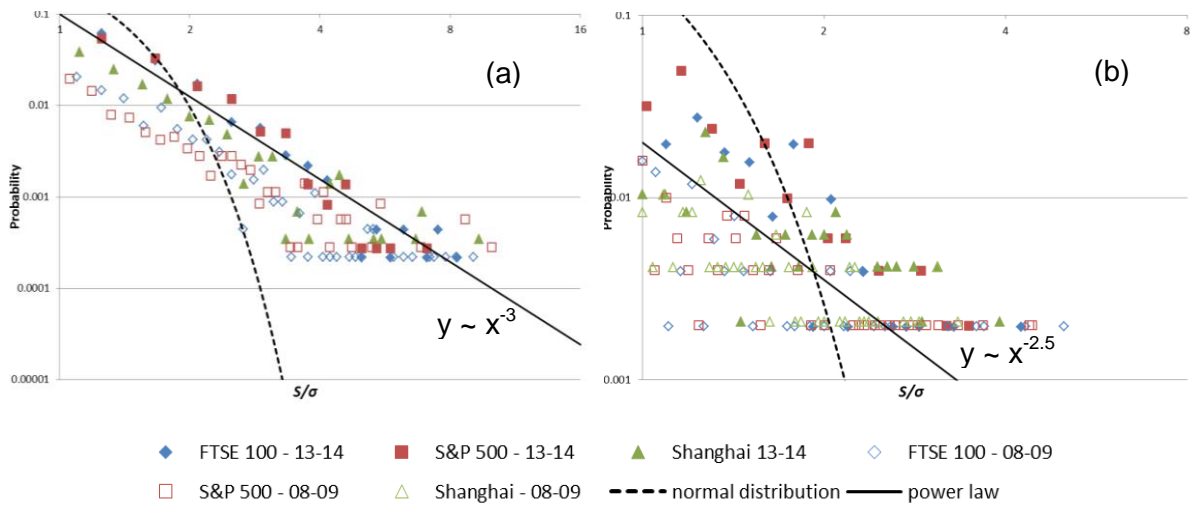

Figure 3.2 The tail behaviours of the probability distribution that are plotted in Figure 2.7, for the one-hour return (a) and one-day return (b). The normal distribution and a power law function are superimposed, showing the tail behaviour is following a power law distribution. The standard errors (around the best fit power law distributions) are: (a) ±0.62, and (b) ±1.25.

3.2.2 The theoretical framework

The power law distribution is discovered in a large number of social and scientific phenomena such as the Pareto law (Pareto, 1964), the turbulence of fluids (Solomon et al., 1993, 1994), and the dynamics of human heartbeat (Peng et al., 1995). If the normal distribution describes an ideal situation (e.g., efficient market, ideal gas, friction free diffusion process, etc.), the power law distribution seems to be more prevalent in nature where non-ideal situations are encountered. Important contributions are from Lévy (1925) and Khintchine (1936) who solve a group of probability distributions with power law tails. Recalling that the normal distribution is stable with the shape of the distribution function remains invariant with diffusion process, the work of Lévy leads to a generalised central limit theorem for a group of stable distributions. For a symmetric and zero mean probabilistic distribution, all those stable distributions will have the format of the Lévy distribution:

$$P_L(z) = \int_0^{+\infty} e^{-\gamma|\xi|^{\mu_L}} \cos(\xi z) d\xi , \qquad\qquad (3.1)$$

in which P_L denotes the Lévy probability distribution function (in analogy with the Gaussian case Eqn. (2.17)), ξ is the variable in the frequency space as used in the Fourier transformation (see Gnedenko and Kolmogorov, 1954 for details), μ_L is the decay rate associated with slope of the tail in a log-log plot, and γ the scaling factor associated with the width of the distribution. It is easy to validate that the Lévy distribution Eqn. (3.1) will reduce to Eqn. (2.17), i.e. normal distribution, when $\mu_L = 2$ and $\gamma = \sigma^2/2$. It can be also proven that Eqn. (3.1) will converge asymptotically to a power law distribution when the value of z is large (i.e., at distribution tail):

$$P_L(z) \sim \frac{\gamma \mu_L}{|z|^{1+\mu_L}} , \qquad \text{for } |z| \gg 0 . \qquad\qquad (3.2)$$

Eqn. (3.1) requires that $1 \le \mu_L \le 2$, in order to be suitable for a probability distribution function (Bouchaud and Potters, 1997). This means the power law distribution of the tail (i.e., the slope in the log-log plot) will be subject to the constraint $-3 \le -(1+\mu_L) \le -2$. It is clearly not the case in Figures 3.1 and 3.2, as well as for previous empirical studies (see section 3.2.1 for details). At the same time, there are contradictory research findings on the behaviour of

$\overline{\mu}$ values with different time horizons t of returns. Gopikrishnan et al. (1999) suggested (through empirical studies) that $\overline{\mu}$ does not depend on the time horizon of return t, so long as t is smaller than four days. Dresel (2001) suggested that $\overline{\mu}$ increases dramatically with the increase of t, from a value of 2 up to 14. The current study, on the other hand, shows a slightly decreased value of $\overline{\mu}$ when t increases.

To better understand the relationship between the Lévy distribution and the empirical evidence, we plot in Figure 3.3 the distribution tail of function Eqn. (3.1), together with the asymptotical power law that is described by Eqn. (3.2). As shown in the figure, μ_L is the parameter governing the shape the distribution tail, which changes dramatically from a power law when $\mu_L = 1$ to the normal distribution when $\mu_L = 2$. Curves of Eqn. (3.1) with different values of μ_L intersect at the same value when $z \approx 4\sigma$. It is shown in the figure that the asymptotic power law distribution is only valid for very large values of z/σ: $z/\sigma \geq 4$, for $\mu_L = 1$, $z/\sigma \geq 8$, for $\mu_L = 1.5$, and $z/\sigma \geq 16$ for $\mu_L = 1.9$, indicating that the speed of convergence reduces when the values of μ_L increases. In particular, for $\mu_L = 2$ there is no convergence (the Gaussian case). Before the convergence is achieved, the rate of decay can be either slower or faster than the power law described by Eqn. (3.2).

3.2.3 The success and limitation of applying Lévy distribution
Comparing Figure 3.3 with the empirical studies in Figures 3.1 and 3.2, it can be immediately discovered that the slope of the empirical power law $\overline{\mu}$ decreases from $\overline{\mu} = 2.5$ to $\overline{\mu} = 1.5$ with the increased time horizon of returns. However, Figure 3.3 suggests that μ_L should increase with the increase of time horizon of returns, and the long term return will converge to normal distribution, when μ_L reaches its theoretical maximum value $\mu_L = 2$. One reason for such a discrepancy can be attributed to the definition of the "distribution tail", which is not clearly articulated and may be different from one study to another. As shown in Figure 3.3, the "tail" may be defined as $z/\sigma \geq 4$ for $\mu_L = 1$, $z/\sigma \geq 8$ for $\mu_L = 1.5$, and $z/\sigma \geq 16$ for $\mu_L = 1.9$. Following this definition, the daily return distribution in Figure 3.2(b) has no "distribution tail", therefore can't be described by a Lévy distribution. For minutely and hourly returns, whether the "distribution tail" exists is strongly dependent on the value of μ_L

which is also dependent on the value of γ in Eqn. (3.1). Such a finding challenges the two-step approach adopted by previous literature: firstly, the value of μ_L is determined by finding the power law factor $\overline{\mu}$ (the slope of the log-log return); then, by equation $\mu_L = \overline{\mu}$, the scaling parameter γ in Eqn. (3.1) can be determined by equation $P_L(0)$ with the empirical data (Mantegna and Stanley, 1995). Such a two-step approach will become invalid if $\mu_L \neq \overline{\mu}$, as demonstrated by the current research. Since the empirical findings of $\overline{\mu}$ values often fall outside the Lévy regime (i.e., between the values 1 and 2), the inconsistencies between different research findings in the literature can be therefore understood.
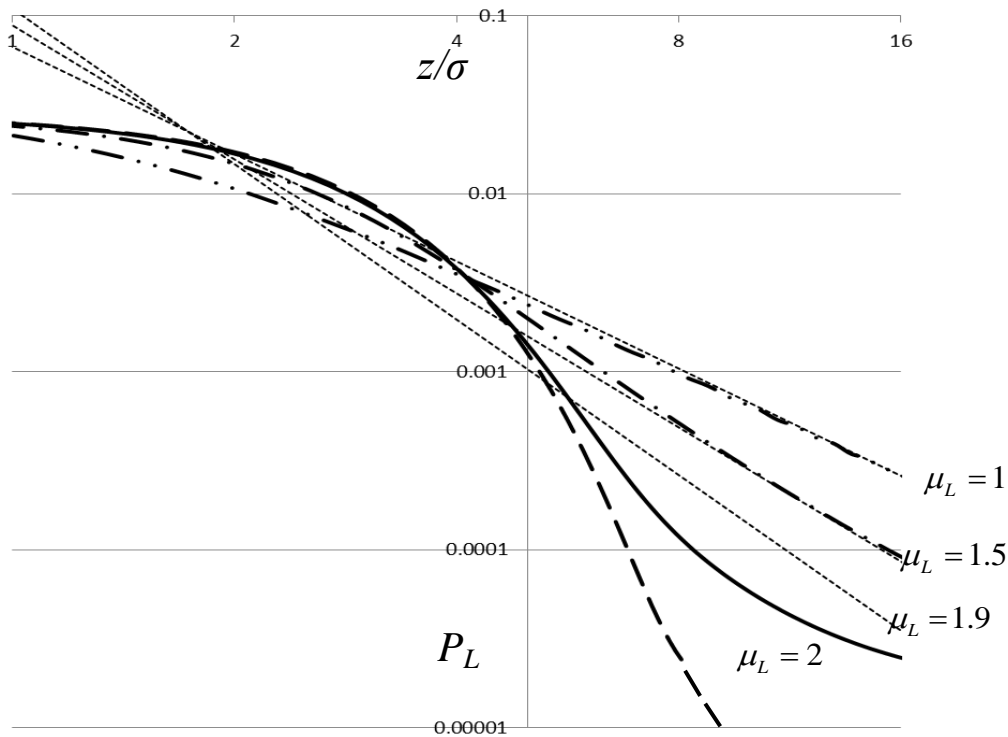
Figure 3.3 The Lévy distribution plotted against different values of μ_L; the dotted line represents the asymptotical power law that is described by Eqn. (3.2). The log-log plot is used and the x-axis is normalised using the standard deviation.

To make the situation even more complicated, there seems to be a limit beyond which the distribution tail can no longer be described by the Lévy distribution. Cont et al., (1998) found that "the probability of extreme returns is not a pure power law but rather contains multiplicative corrections varying more slowly than the power law". Gopikrishnan et al., (1999) also found that that "the end of the distribution tail is decaying faster than the Lévy

regime". For those extreme return values towards the end of the distribution tails, it is difficult to identify whether those values are genuinely suggesting a non-Lévy regime or whether they are simply too few statistical samples. Another major disadvantage of the Lévy distribution comes from the theoretical framework that the Lévy distribution has an infinite variance, which is difficult to justify using financial data (Matacz, 2000). For the group of probability distributions that are governed by the generalised (Lévy) central limit theorem, the normal distribution is the only stable distribution with a finite variance. Some modifications, such as the truncated Lévy distribution (Mantegna and Stanley, 1994), are proposed to mitigate such limitations. Those models, however, introduced additional ad hoc parameters that may not be valid universally for different datasets. As a general conclusion, there are many uncertainties and complications which severely limit the applicability of the Lévy distribution on financial modelling. For those high frequency price returns where the feature of power law tail is statistically significant, a three-step approach is proposed here, using both the Lévy distribution and the Gaussian distribution, to estimate the range of the probability for rare events. As shown in Figure 3.4, the one-minute returns of FTSE 100 stock index during 2013-14 are plotted using a log plot on the y-axis, while the x-axis is normalised by the volatility of the dataset. In the three-step approach, the Lévy distribution can be obtained without assuming whether the convergence to the power law is achieved (i.e., without knowing the value of μ_L). It, however, assumes that the empirical Lévy regime is when the return S is in the region of $\sigma \leq S \leq 8\sigma$ (a similar regime is used by Mantegna and Stanley, 1995). Firstly, the value of μ_L is obtained not by calculating $\overline{\mu}$ but by finding the shape of the Lévy distribution closest to the empirical distribution inside the Lévy regime. Secondly, the value of γ in Eqn. (3.1) can be obtained by comparing $P_L(0)$ with the peak of the empirical distribution. Such a Lévy distribution is considered as an upper limit of the empirical distribution. Finally the normal distribution can be obtained by using $\gamma = \sigma^2/2$ as a lower limit of the empirical distribution. The results are plotted in Figure 3.3 with $\mu_L = 1.8$ and $\gamma = 0.4$. The figure is very similar to that is found in previous studies using the S&P 500 index and the Xerox stock price returns (Mantegna and Stanley, 1995, 2000)
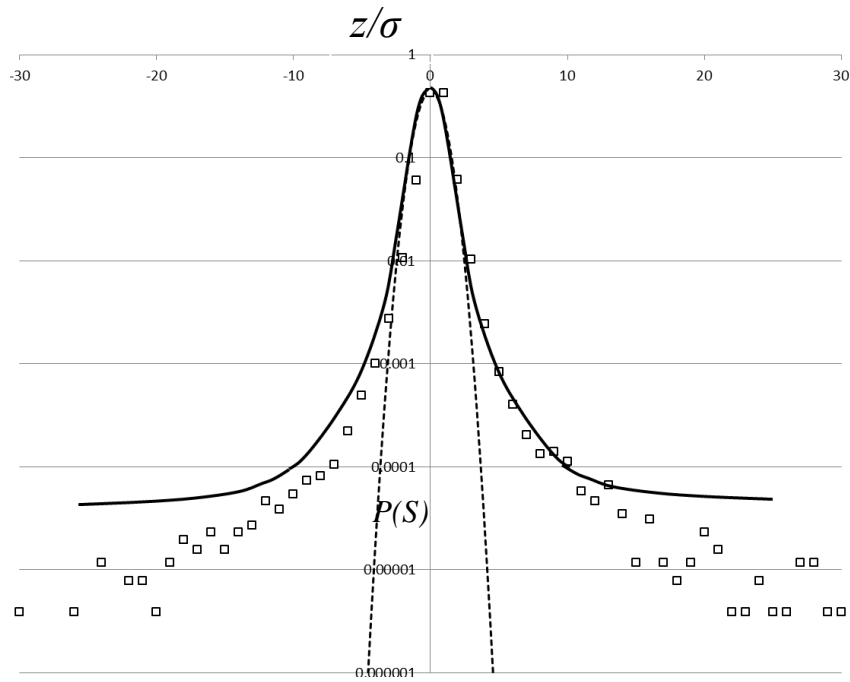
Figure 3.4 The upper and lower limit of the distribution for high-frequency data from the FTSE stock index. The dataset of choice is one-minute returns for the period 2013-14. The three-step approach introduced in the current section is used to determine the Lévy distribution (solid line) and the normal distribution (broken line). The empirical data are plotted using discrete symbols.

3.3 The Tsallis statistics

3.3.1 The theoretical framework

The motivation for applying the Lévy distribution on financial modelling can be attributed to the following reasons: 1) the self-similarity (stable distribution) that is governed by the generalised central limit theorem; and 2) the power law distributed tails that are found in a large number of social and natural phenomena. However the applicability of the Lévy distribution on financial modelling is restricted by some complications discussed in the previous section - recalling that the generalised central limit theorem states that all independently distributed random variables will converge to the Lévy distribution, with the normal distribution a special case for finite variance. Comparing this to our understandings of the stock market, the assumptions behind the Lévy distribution do not capture the underlying principles leading to the empirical features. Particularly: 1) the infinite variables are difficult to be detected in the stock market; and 2) those random variables behind the stochastic process are not always independent (see e.g., Eqn. (2.14)). Tsallis et al. (1988) have proposed a group of distributions that describe the behaviour of random variables with a certain level

of inter-dependence and with both finite and infinite variances. They have the theoretical foundation of statistical mechanics with defined expression of entropy (referred to as the Tsallis entropy). In comparison to the classical statistical mechanics, the Tsallis statistics are sometimes referred to as non-extensive statistical mechanics in the literature (e.g., Tsallis and Bukman, 1996). This has been observed in a number of physical and numerical studies (Moyano et al., 2006; Kaniadakis et al., 2002) and is also adapted for financial modelling (e.g., Borland, 2002a; Iliopoulos et al., 2015). This section will study the Tsallis statistics from the angles of both stochastic process and statistical mechanics. Following a similar research method to that used in section 2.3.2 and section 3.3.2, the applicability of the Tsallis distribution on the empirical data is also examined.

Similar to the Lévy statistics, the Tsallis statistics also introduce an additional parameter (namely, the parameter q) to the classical Gaussian model. It also has a closed mathematical form and defined calculus (Umarov et al., 2008). The value of q is a measure of the correlation of certain type and controls the shape of the probability distribution function and the diffusion time scale. Assuming x_i are inter-dependent identically distributed variables with zero mean, and the degree of inter-dependence can be described using a single parameter q, the distribution of the sum:

$$Z_q(t) = Y_q(t_0 + t) - Y_q(t_0) = Z_q(n\Delta t) = x_1 + x_2 + ... + x_n \qquad (3.3)$$

will converge to a Tsallis distribution:

$$P_q(Z_q = z) = \left[1 - \beta(1-q)z^2\right]^{1/(1-q)} / C_q , \qquad (3.4)$$

in which the scaling factor $C_q = \int_{-\infty}^{+\infty}[1 - \beta(1-q)z^2]^{1/(1-q)} dz$, β is of the same thermodynamic meaning as in Eqn. (2.31) that $\beta = 1/k_B T$. The Tsallis distribution and Tsallis statistics are therefore referred to as q-distribution and q-statistics in some literature (e.g. Picoli et al., 2009). The expression using the statistical terms for β under q-statistics becomes $\beta = \Delta_q / \sigma^2$, in which σ is the standard deviation/volatility, and Δ_q has the form that (Tsallis, et al., 1995):

$$\Delta_q = \begin{cases} \dfrac{1}{2}\left[\left(\dfrac{1-q}{2\pi}\right)^{1/2} \dfrac{\Gamma\big((5-3q)/2(1-q)\big)}{\Gamma\big((2-q)/(1-q)\big)}\right]^{2(q-1)/(3-q)} & , \ \ if \ \ q<1, \\[4mm] \dfrac{1}{2}, & if \ \ q=1, \\[4mm] \dfrac{1}{2}\left[\left(\dfrac{q-1}{2\pi}\right)^{1/2} \dfrac{\Gamma\big(1/(q-1)\big)}{\Gamma\big((3-q)/2(q-1)\big)}\right]^{2(q-1)/(3-q)} & , \ \ if \ \ 1<q\leq 3. \end{cases} \qquad (3.5)$$

In Eqn. (3.5) Γ is the gamma function that $\Gamma(s) = \int_0^\infty x^{s-1}e^{-x}dx$. It can be seen from Eqns. (3.4) and (3.5) that the q-distribution will reduce to the normal distribution Eqn. (2.17) when $q = 1$. Using numerical techniques, it is shown that Eqn. (3.4) will have a finite variance when $1 \leq q \leq 5/3$ (Tsallis, et al., 1995) - this is the range of q values that will be focused on in the current research. The level and type of inter-dependence of the random variables x_i are extensively studied by Umarov et al., (2008) with detailed derivation of a q-central limit theorem, governing the group of distributions for correlated random variables. For independent random variables following the q-distribution, the classical central limit theorem applies and the final distribution converges to a Gaussian or Lévy distribution, depending on whether the variance is finite or not.

3.3.2 The non-extensive statistical mechanics

From a statistical mechanics point of view, the Tsallis statistics can be also derived using a chaos system with a number of particles that are inter-dependent. The Boltzmann-Gibbs entropy (Eqn. (2.30)) can be generalised as (Tsallis, 1988):

$$\overline{S}_q = k_B \frac{1-\sum_i P_i^q}{q-1} \ . \qquad (3.6)$$

It can be verified that the probability distribution function Eqn. (3.4) maximises the entropy Eqn. (3.6), with $q = 1$ being the special case Boltzmann-Gibbs entropy. The level of inter-dependence between the particles can be illustrated as follows: by sub-dividing the system into two correlated subsets A and B, and the Tsallis entropy can be re-written as (Umarov et al., 2008):

$$\overline{S}_q(A+B) = \overline{S}_q(A) + \overline{S}_q(B) + (1-q)\overline{S}_q(A)\overline{S}_q(B), \qquad (3.7)$$

in which q is a measure of inter-dependence between the two subsets and will be reduced to 1 for the independent case. Further details characterising such inter-dependence can be found in Umarov et al. (2008). It has been shown that a single parameter q serves the purpose of quantitative measure for the inter-dependence, the fat-tail, and the time scale of the diffusion process. In the context of financial economics, the concept of entropy is also explored by a number of researchers (see e.g. Gulko,1999; Darbellay & Wuertz,2000). A general statement is that the concept of entropy it related to the market efficiency. The investor, as an information receiver and process, can also use entropy as a measurement of uncertainty about the future information received (Brissaud, 2005). It is shown in the current thesis that information represented in the price dynamics does not fully following the Brownian Motion but with some correlation and feedback mechanisms, with is measured using a single parameter q.

The q-distribution can be also obtained from the dynamic equation in a similar manner, using Eqn. (2.20) (Borland, 1998; Tsallis and Lenzi, 2002). Another way of re-constructing Eqn. (2.20) is through the non-linear Fokker-Planck equation (Tsallis and Lenzi, 2002) that:

$$\frac{\partial \rho(x,t)}{\partial t} = \frac{\partial \rho(x,t)}{\partial x} + \tilde{D}\frac{\partial^2 \rho^\nu(x,t)}{\partial x^2} , \tag{3.8}$$

in which ν is a measure of non-linearity and $\tilde{D}$ is the (constant) diffusion coefficient. It can be proven that at the microscopic level the evolution of the particle position x follows the Langevin equation (Borland, 1998):

$$\frac{dx}{dt} = \sqrt{2\tilde{D}}\rho(x,t)^{\nu/2}W_t \tag{3.9}$$

in which $\{W_t\}$ is the Werner process introduced in Eqn. (2.18). It can be observed from Eqn. (3.9) that the diffusion process is no longer governed by a constant diffusion coefficient $\tilde{D}$ alone, but is also dependent on the macroscopic density distribution $\rho(x,t)$. The chaos system is therefore governed by a feedback loop described by Eqns. (3.8) and (3.9), leading to the so called non-extensive statistical mechanics. From the Brownian motion point of view, it can be understood that the amplitude of the movement of a Brownian particle is dependent on the macroscopic environmental variables, not only through temperature, but

also through its past visits to the specific regions of space (Voit, 2003). It can be further demonstrated that $v = 2 - q$ and the diffusion process is governed by a time scale: the variance $\{x^2\}$ is proportional to a function of the time $t^{2/(3-q)}$.

After examining the Gaussian statistics and the Lévy statistics, the Tsallis statistics seem a good candidate to replace the Gaussian statistics in modelling the price return distribution of the stock market. It has a relatively simple analytical solution with defined mathematical formulae that can be used to derive stochastic differential equations. It also has a solid statistical mechanics foundation, consistent with the classical Boltzmann-Gibbs framework. What is more, the Tsallis statistics take into consideration correlation and feedback loops, a major feature of the stock market, while the classical models fail to recognise this. It is therefore worth examining how much and to what extent the analytical framework of the Tsallis statistics captures the complexity of the distribution tails generated from complicated interactions in the stock market. Following the methodology used in section 2.3, the suitability of using the q-distribution to describe the empirical distribution is studied in the next section.

3.3.3 Empirical study using price return data from the stock market

Figure 3.5 shows the plot using the data discussed in Figure 2.7(a) with high-frequency price return, further normalised by the standard deviation of each individual dataset. The values of the normalisation scheme are detailed in Table 3.1. It can be immediately observed from the figures that empirical data from the 'calm' and 'turbulent' market periods collate for each market, forming a similar empirical distribution. It is also noted that, during the market turbulent period, there are more rare events recorded, represented by fatter and longer tails on either side of the distribution. When comparing across different markets, it can be seen that Figures 3.5(a) and 3.5(b) generally agree with each other, while the Shanghai Composite index in Figure 3.5(c) shows a more leptokurtic shape after the normalisation, with a higher value of $\tilde{P}(S=0)$. Such a difference is also reflected by the values used for normalisation detailed in Table 3.1. This is consistent with the analyses in section 2.3.3, that the Shanghai Composite index in Figure 3.5(c) stands out from the other two markets. Two analytical functions using the normal distribution (Eqn. (2.17)) and the q-distribution (Eqn. (3.4)) are also plotted in Figure 3.5. The approach used to determine the input of the equations is described as follows: firstly, the modelled volatility is used in Figure 3.4 instead of using the

volatilities of the dataset, using the maximum likelihood technique to fit the empirical data. It can be seen from Table 3.1 that the (normalised) modelled volatility is close to 1, which is consistent with that shown in Table 2.1. After the volatility value is determined, the value of q is found by assessing the tail behaviour of the empirical distribution. It can be verified numerically that the impact of σ and q on the q-distribution is relatively independent, with σ governing the behaviour of the distribution centre and q the distribution tails. Finally, those values detailed in Table 3.1 can be verified using the normal distribution as a reference function. According to Eqn. (3.5), the modelled volatility used for the normal distribution (σ_n) and the q-distribution (σ_q) has the relationship $\sigma_n{}^2 / 2 = \beta = \Delta_q / \sigma_q{}^2$. As can be found in Table 3.1, such a relationship holds for all values of input parameters: the value of σ_n and σ_q are the same for Figures 3.5(a) and 3.5(b) but smaller for Figure 3.5(c), which is required to capture the higher value of (normalised) probability distribution $\tilde{P}(S=0)$. Comparing to the Lévy distribution shown in Figure 3.4, the q-distribution shown in Figure 3.5 captures much better the tail behaviour using a unified value of q for all markets and all periods. However, for small values of price returns close to the distribution centre, the normal distribution describes the distribution better. This may be due to the fact a single and unified volatility value is used, while in the real market, the (absolute value of) price return and volatility are positively correlated (Glosten et al., 1993). The behaviour of volatility falls into the area of higher order financial factors - discussed in section 2.3.4 - while using a single value of volatility for the global probability distribution is still a popular approach in finance. (Moreover, the fact that the empirical distributions from market calm and turbulent periods collate supports the validity of using a single volatility value.) The current section will employ a single volatility and focus on the behaviour of q, while the time series of the volatility will be discussed later.
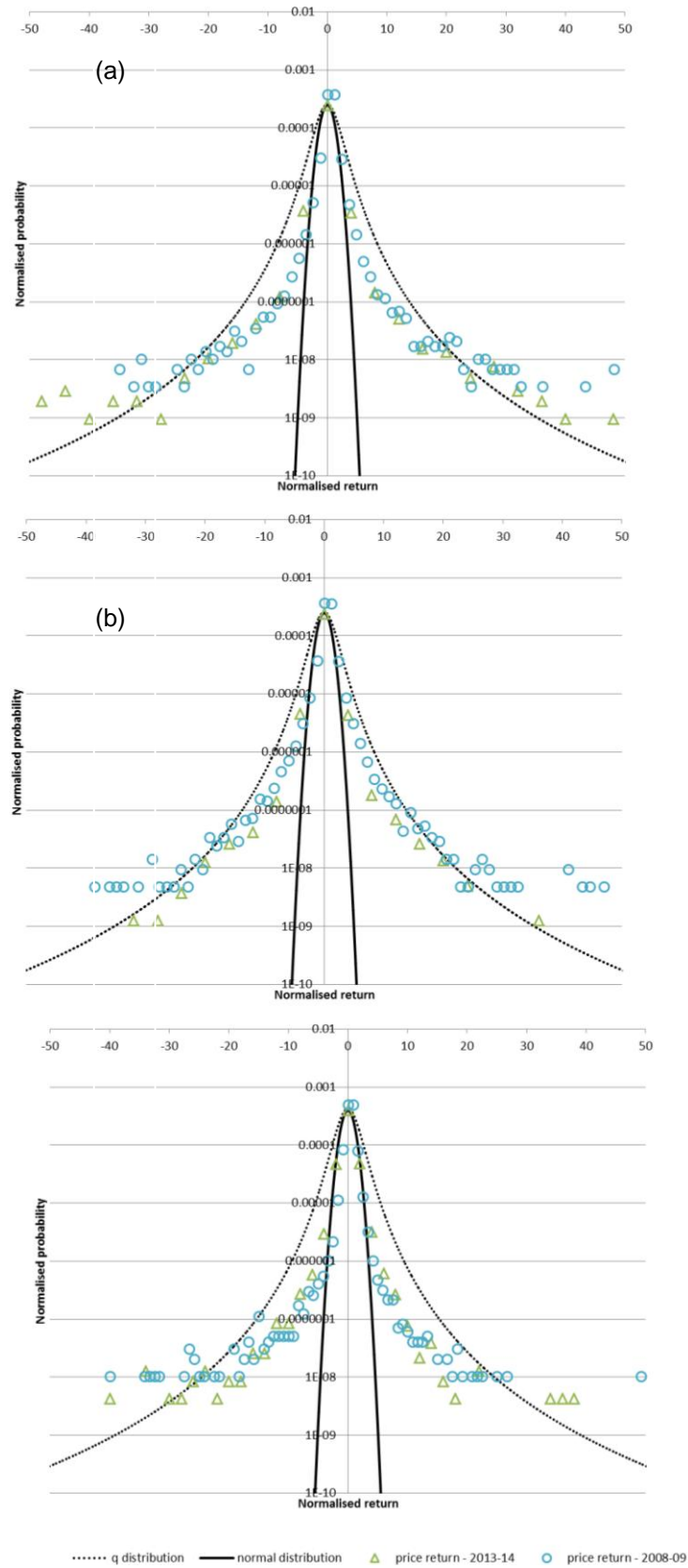
Figure 3.5 Comparing *q*-distribution with the empirical distributions of one-minute stock return, for market turbulent and market calm periods, and for different markets: (a) FTSE 100; (b) S&P 500, and (c) Shanghai Composite. Both normal distribution and *q*-distribution ($q = 1.4$) are plotted in the charts.

The data plotted in Figures 2.7(b) and 2.7(c) are reproduced in Figures 3.6 and 3.7, respectively, using the normalisation scheme detailed in Table 3.1. Specifically, Figure 3.6 shows the hourly data for all three markets and the two (calm and turbulent) periods under consideration. It can be observed that the normalised empirical distributions collate with each other, with the only exception that market turbulent periods experience higher probabilities of rare events. Similar findings can be also observed for the normalised daily returns plotted in Figure 3.7. Despite the different number of trading hours for different markets, the empirical values of daily returns still collate into a single distribution. The advantage of using such a normalised plot is to remove the impact of the volatility differences between markets and (calm and turbulent) periods, therefore highlighting the distribution shape and the effect of different values of q. It can be noticed from Figures 3.5 to 3.7, that the value of q is not sensitive to different markets and periods, but rather is strongly related to the time horizon of the return. The value of q decreases with the increased time horizon, from a value close to the theoretical upper limit $q = 1.67$ to the normal distribution $q = 1$. This is consistent with previous findings in Figures 2.8 and 2.9 that the normal distribution is a good representation for weekly and monthly price returns. This is however slightly inconsistent with some pioneering research findings. Osorio et al. (2004) studied the daily return of the S&P 500 Index and minute returns of Nasdaq 10 high-volume stocks. It was shown that, for both cases, the q value equals 1.43. It was later claimed by Borland (2002a) that $q \approx 1.5$ provides a good fit for realistic financial data. The present research has posed some question marks around those empirical findings: the study of Osorio et al. (2004) considers different financial data groups (S&P 500 and Nasdaq) with different time frequencies (daily and minutely). It is uncertain whether returns for a same dataset of different time frequencies will lead to a constant value of q as, in the later publication of Borland herself (2002b), it is commented that as the time intervals get larger, the whole distribution will converge to Gaussian, i.e., $q \rightarrow 1$, which somewhat conflicts with her own $q \approx 1.5$ statement. As demonstrated by Eqn. (3.3), a constant value of q requires a persistent degree of inter-dependence between the random variables. However, such correlation is unlikely to be time-invariant, as demonstrated in the autocorrelation study from section 2.3.4. Eqn. (3.9) also shows that the diffusion coefficient is a function of the macroscopic distribution density, which also evolves with time. The current study shows that $q \rightarrow 1$ when $t \rightarrow \infty$, which is consistent with the previous studies from the author (Li, 2013).

The Tsallis statistics have been adopted to develop the financial models, for example, the option pricing formula introduced in section 2.1.3. Instead of assuming the price return follows the geometric Brownian motion and normal distribution, the q-distribution is used as the governing distribution for the stochastic process (Borland, 2002a). In order to derive the stochastic differential equations required for the option pricing model, the diffusion properties of the q-distribution have to be understood. It has been shown that the diffusion process of the q-distribution is different from that of the Brownian motion, in that $\{x^2\} \sim t^{2/(3-q)}$ (Umarov, 2008). The option pricing model based on Tsallis statistics therefore has two parameters that must be determined: the volatility σ and the measure of correlation q. It is essential to understand the behaviour of q before evaluating the models that are derived based on the Tsallis statistics. For example, Boland (2002a) uses a fixed value of $q = 1.5$ to derive the option pricing model for European call options. It is, however, shown in the paper that the artefact of "volatility smile" is stronger after using the Tsallis statistics. Following the hypothesis that the effect of "volatility smile" is a measure of how far away the model is from the realistic situation, the Tsallis statistics should better capture the statistical properties of the market and would have led to a weaker "smile". Apart from the philosophical argument that the "model is only a metaphor of reality", one of the reasons may be that q should be a variable that reduces with the time horizon of returns, instead of being a constant. Recalling that q is a measure of underlying correlation, it is difficult to justify, at least in the stock market, that such correlation will stay constant with increasing time horizon of returns. As demonstrated in section 2.3.4, the short and long term autocorrelation decays with increased time lag. It has also been demonstrated that the value of q is not sensitive to different markets and periods; in the current research we assume the value of q is a only function of the time horizon of returns t, and a random selected market and period is sufficient to study the function $q(t)$. The non-constant q value has not been studied explicitly in the literature. It will be demonstrated in the this section, from theoretical analysis, data analysis, as well as numerical modelling that he value of q is a function of the time horizon of returns, with limiting values from both ends. Besides the option pricing models, the Tsallis statistics is also extensively used in other areas in financial modelling, e.g. capital asset pricing model (Gulko, 1997), portfolio theory (Gupta and Campanha, 2002), as well as risk management (VaR, Mattedi, et al., 2008). The fundamental study performed in the current section is a crucial input factor in terms of better implementing those models.
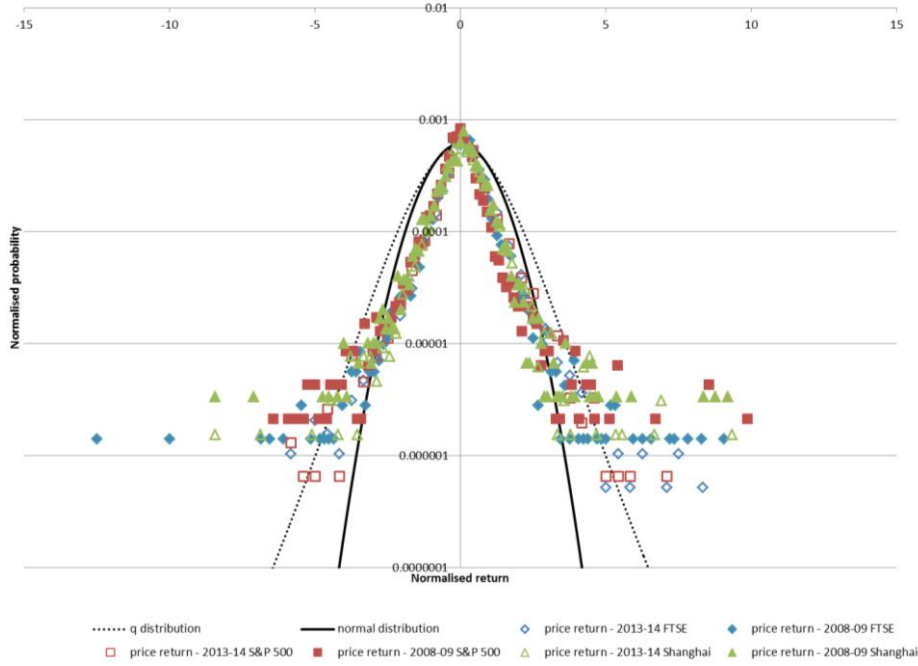
Figure 3.6 Comparing *q*-distribution with the empirical distributions of one-hour stock return, for market turbulent and market calm periods, and for different markets. Both normal distribution and *q*-distribution (*q* = 1.15) are plotted in the charts.

The value of *q* is obtained using the price return from the FTSE 100 index for the 2013-14, by using the same maximum likelihood techniques as adopted in Figure 3.5. The results are plotted in Figure 3.8, in which the *x*-axis is the log-plot. It can be seen from the figure that the function *q*(*t*) follows a power law function, which increases and decreases asymptotically on both directions when $t \to 0$ and $t \to \infty$, for $q \to 1.66$ and $q \to 1$, respectively. Using these boundary conditions together with the *q* values plotted in Figure 3.8, an empirical model for finding the *q* values when performing the Tsallis statistics is proposed:

$$q = 0.66\exp[-(t/\tau_0)^\alpha]+1, \tag{3.10}$$
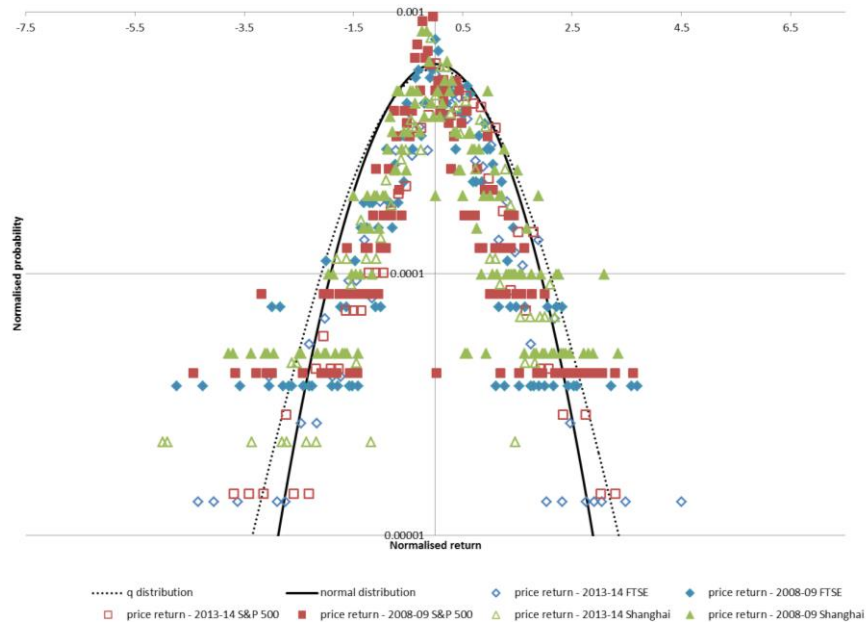
Figure 3.7 Comparing *q*-distribution with the empirical distributions of one-day stock return, for market turbulent and market calm periods, and for different markets. Both normal distribution and *q*-distribution (*q* = 1.1) are plotted in the charts.

Table 3.1 Values for normalising the empirical distributions and inputs of the analytical distributions that are used in Figures 3.5 to 3.7.

Values of σ_0 that is used for normalisation $\sigma^* = \sigma / \sigma_0$ and $\tilde{P}^* = \tilde{P} \cdot \sigma_0$ (as per Figures 2.7, 3.5-3.7)				Values for inputs of Eqns. (2.17) and (3.4)		
One-minute price returns	2013-14	FTSE 100	0.00025	$q=1.4$	$\sigma_q = 1.5$	$\sigma_n = 1$
		S&P 500	0.00025		$\sigma_q = 1.5$	$\sigma_n = 1$
		Shanghai	0.0005		$\sigma_q = 1$	$\sigma_n = 0.6$
	2008-09	FTSE 100	0.001		$\sigma_q = 1.5$	$\sigma_n = 1$
		S&P 500	0.001		$\sigma_q = 1.5$	$\sigma_n = 1$
		Shanghai	0.0012		$\sigma_q = 1$	$\sigma_n = 0.6$
One-hour price returns	2013-14	FTSE 100	0.0024	$q=1.15$	$\sigma_q = 0.6$	$\sigma_n = 0.65$
		S&P 500	0.0024			
		Shanghai	0.0045			
	2008-09	FTSE 100	0.0064			
		S&P 500	0.0076			
		Shanghai	0.0097			
One-day price returns	2013-14	FTSE 100	0.0069	$q=1.1$	$\sigma_q = 0.6$	$\sigma_n = 0.63$
		S&P 500	0.0073			
		Shanghai	0.011			
	2008-09	FTSE 100	0.019			
		S&P 500	0.021			
		Shanghai	0.024			

in which τ_0 is a characteristic time scale governing the asymptotic behaviour of power law and α is the factor governing the slope of the power law. Eqn. (3.10) is plotted in Figure 3.8 using $\tau_0 = 1000$ seconds and $\alpha = 0.3$. It can be observed from the figure that the proposed model accurately captures the behaviour of the q value as a function of time t. As demonstrated earlier, although the empirical model is derived from the FTSE 100 index, it can be justified that such a q value is not a market-specific parameter and is valid for all markets and periods (calm and turbulent).
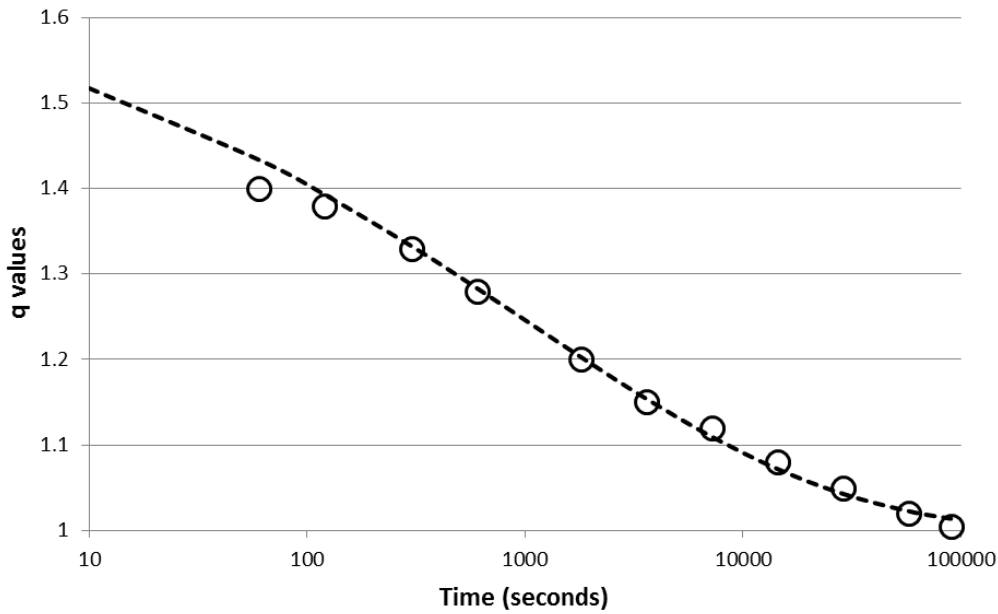
Figure 3.8 The empirical values of q obtained using the price return from the FTSE 100 index for years 2013 and 2014, as a function of different time horizons of returns. The dashed line superimposed is the proposed model of $q(t)$ detailed in Eqn. (3.10).

An alternative way of validating the proposed empirical model is to compare the price return distributions between different time horizons. As shown in Figures 3.5 to 3.7, the empirical distributions collate after being normalised by the volatility/standard deviation σ. It is also shown that the variance is $\sigma^2 \sim t^{2/(3-q)}$ from the literature (Umarov, 2008). By using Eqn. (3.10), the scaling property between the standard deviation and time horizon of returns can be made explicit. Figure 3.9 plots the price return for the FTSE 100 index for different values of time horizon of returns t. Instead of being normalised by a single value of standard deviation, the price return is normalised by:

$$S^* = \frac{S}{t^{1/(3-q)}}, \quad and \quad P^*(S) = P^*(S)t^{1/(3-q)}. \tag{3.11}$$
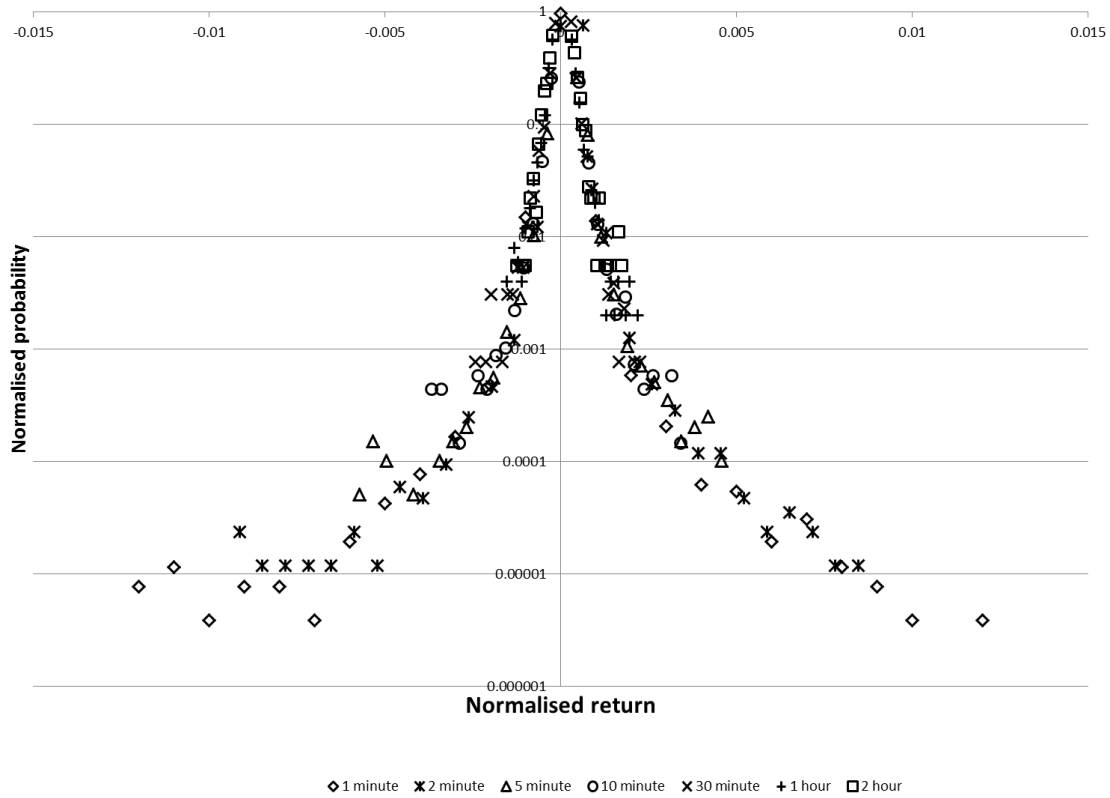
Figure 3.9 Normalised price return distributions for different time horizons of returns. All results are normalised by Eqns. (3.10) and (3.11). The different time horizons of returns are shown by different discrete symbols.

It can be clearly observed from Figure 3.9 that the distributions for different time horizons of the price return collapse into a single distribution. This proves that there is a continuous varying time scale governing the diffusion process, however still maintaining the shape of the distribution invariant (after normalisation). It is an important finding that when the Tsallis statistics are originally derived from statistical mechanics of a chaos system, it considers an equilibrium system where the level of correlation of the system generally maintains itself. As a measure of correlation, a constant value of q is generally assumed. However, for the financial market, it is difficult to determine whether the system reaches equilibrium - especially in terms of the level of interaction and the feedback loop. The studies of the

correlation suggest a decaying correlation with time, and the q-distribution can be used to describe an "anomalous diffusion process" (e.g., Zanette and Alemany, 1995; Bologna et al., 2000; Chen, 2006). A third way of validating the proposed Eqn. (3.10) is to use the $\tilde{P}(0)$ analysis presented in Figure 2.10. It can be also found from Figure 3.8 that the q value reduces quickly at smaller time horizons - $q \leq 1.15$ after 1 hour - leading to a diffusion scaling that $\sigma \sim t^{1/(3-q)} \leq t^{0.54}$. This is consistent with the empirical findings detailed in Table 2.2. The "super diffusion" when $\sigma \sim t^{0.7}$, as found by Mantegna and Stanley (2000), will correspond to the regime of $q \approx 1.6$ and may only be observed for extremely high-frequency returns (i.e., when the time horizon is much less than one minute).

3.4 The time series of volatility and correlations

3.4.1 The ARCH processes

The models discussed so far in this chapter assume a constant value of volatility. It is demonstrated that this assumption is reasonable in terms of capturing the probability distribution functions. It is, however, shown in section 2.2.3 that volatility itself is a time-dependent stochastic variable, with properties of volatility clustering and long term correlations following power law decay. The ARCH process (auto-regressive conditional heteroskedasticitity, Eagle, 1982) is derived to capture those features of the volatility time series. It is a simple process that describes a stochastic process with local conditional probability density function controlled by a fluctuating local volatility. The mathematical expression of the volatility can be written as:

$$\sigma_t^2 = \alpha_0 + \alpha_1 x_{t-1}^2 + ... + \alpha_{\bar{p}} x_{t-\bar{p}}^2 , \qquad (3.12)$$

in which σ_t is the local volatility fluctuates with time t, α_0, α_1 …, $\alpha_{\bar{p}}$ are positive constants, and x_t is a random variable following the distribution function governed by the local volatility σ_t. In the context of the stock market, Eqn. (3.12) suggests that the local volatility is dependent on previous returns. The memory effect of the returns can be controlled by varying the value of $\bar{p}$ in an ARCH ($\bar{p}$) process. The most commonly used probability density function for x_t is the Gaussian distribution, and in this section, we also

use this choice to examine how the Gaussian local distribution with varying control parameter σ_t will lead to a non-Gaussian global distribution, and at the same time will enable a like-for-like comparison to previous research results (e.g., Bollerslev et al., 1992; Drost and Nijman, 1993).

The simplest ARCH process is when the volatility depends only on the return from the last time step, namely ARCH (1), which is defined by:

$$\sigma_t^2 = \alpha_0 + \alpha_1 x_{t-1}^2.$$
(3.13)

It can be found that the global variance of the process is given by (Eagle, 1982):

$$\sigma^2 = \frac{\alpha_0}{1-\alpha_1},$$
(3.14)

with $0 \le \alpha_1 < 1/\sqrt{3}$. The two cases $\alpha_1 = 0$ and $\alpha_1 = 0.575$ (a value that is chosen to be close to $1/\sqrt{3}$) are shown in Figure 3.10, with the global volatility equal to 1. The q-distributions with best fit are also plotted in the figure. It can be observed that the probability distribution from the ARCH (1) process agrees well with the q-distribution, especially in the tail regions. When $\alpha_1 = 0$ the ARCH (1) process is reduced to the Gaussian process, while when the value of α_1 is close to the theoretical upper bound of $1/\sqrt{3}$, the value $q = 1.33$ can be found. Applying the results obtained from Figure 3.8, it can be concluded that the ARCH (1) process with $\alpha_1 = 0.575$ represents the stock price return period of $t = 5$ minutes that corresponds to $q = 1.33$. Facilitated by the q-distribution and the diffusion model proposed in section 3.3, a connection between the time steps of the ARCH process and the physical time in the stock market is established. Such connection can be then used to plot the time series of the volatility and compare with empirical data from the real market. The time series of simulated "monthly volatility" is plotted in Figure 3.11. The empirical feature of volatility clustering is clearly illustrated in the figure, and is qualitatively consistent with the time series plotted in Figure 2.3. As discussed earlier, the ARCH (1) process models the feedback loop - the key ingredient leading to a fat-tail distribution. Compared to the real market, the variance of the volatility for a monthly return in the ARCH (1) process is considerably

smaller, indicating that there are some higher order conditional probabilities in the stock market which are not captured by the ARCH (1) process (Mantegna and Stanley, 2000).
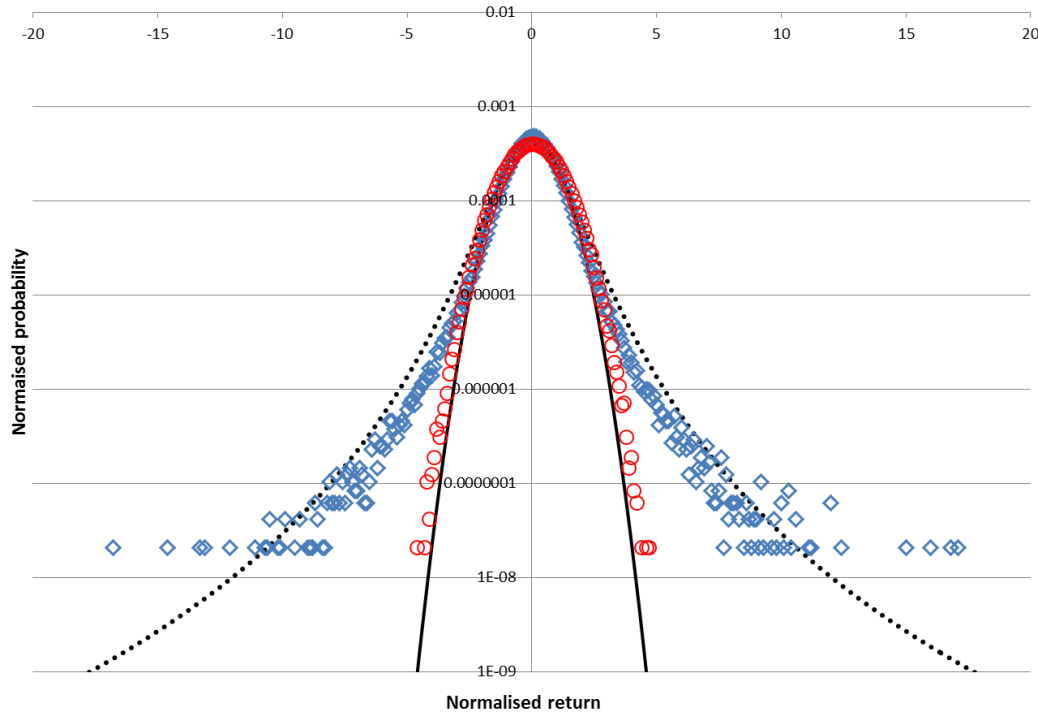
Figure 3.10 The distribution of ARCH (1) process with the global volatility $\sigma = 1$, for different values of α_1 in Eqn. (3.13). The circles show $\alpha_1 = 0$ and the diamonds show $\alpha_1 = 0.575$. The q-distributions discussed in section 3.3 are also plotted in the figure, with the solid line $q = 1$ and dotted line $q = 1.33$.

Those higher order conditional probabilities can be examined using the similar techniques introduced in section 2.3.4, i.e., the autocorrelation study of the volatility and absolute returns. Those autocorrelations for the time series in Figure 3.11 are plotted in Figure 3.12, which shows that the autocorrelation functions decays with exponential rate and becomes insignificant very quickly (five time steps, corresponding to 25 minutes in the current case). The behaviour of the ARCH (1) process is inconsistent with the empirical findings in Figures 2.11 and 2.12, in which the long term correlation with power law decay is demonstrated. This is, however, in line with previous theoretical work which suggests an exponential decay for the ARCH (1) process with the characteristic time scale of several time steps (Andersen et al., 2003). It is therefore proven that the phenomenological observation of volatility clustering may be generated without long term autocorrelations. Increasing the value of $\bar{p}$ for an

ARCH ($\bar{p}$) process will increase the long term autocorrelation, but the short memory for the price returns limits the max value of $\bar{p}$. At the same time, optimising the $\bar{p}+1$ constants of α_0, α_1 ..., $\alpha_{\bar{p}}$ creates further complications in terms of implementing the ARCH model. To overcome those difficulties, Bollerslev (1986) introduced the generalised ARCH process, namely the GARCH model.
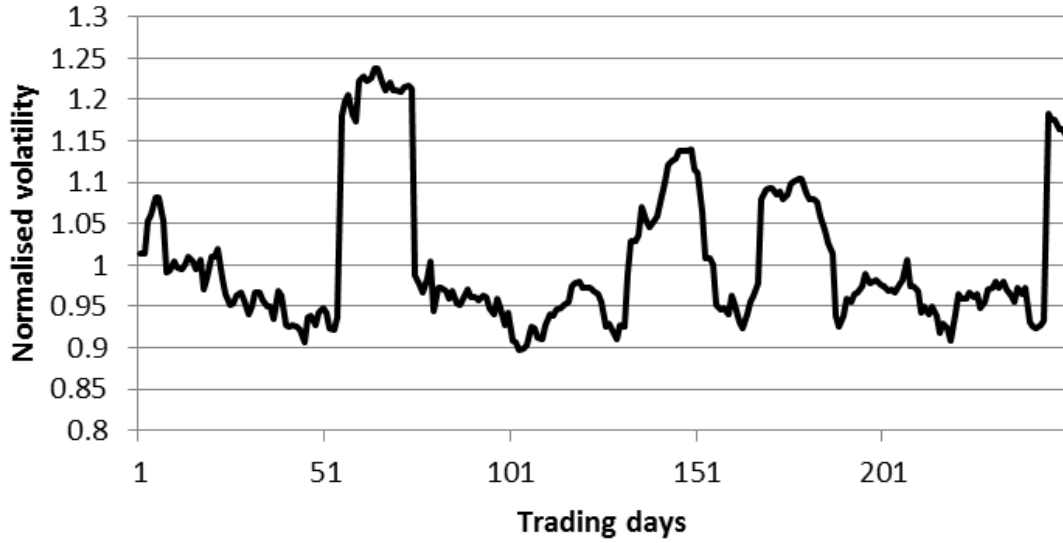
Figure 3.11 Time series of monthly (normalised) volatility simulated using the ARCH (1) process using the parameters that are same with those in Figure 3.10.

3.4.2 The GARCH process

The GARCH process models the volatility as a function of historical time steps that:

$$\sigma_t^2 = \alpha_0 + \alpha_1 x_{t-1}^2 + ... + \alpha_{\bar{p}} x_{t-\bar{p}}^2 + \beta_1 \sigma_{t-1}^2 + ... + \beta_{\bar{q}} \sigma_{t-\bar{q}}^2 \quad , \tag{3.15}$$

where α_0, α_1 ..., $\alpha_{\bar{p}}$, and β_1 ..., $\beta_{\bar{q}}$ are positive constants and Eqn. (3.15) is referred to as a GARCH ($\bar{p}, \bar{q}$) process. x_t is a random variable following the distribution function (which can be arbitrary, but a default choice is Gaussian) and can be understood as price returns in the context of the stock market. Comparing to the ARCH process, the local volatility σ_t for the GARCH process is not only dependent on historical volatilities, but also on the historical returns that are governed by historical volatilities. This additional dependence enables a long

term autocorrelation of the volatility, while it still maintains the feature of short memory for the price returns. We consider the simplest GARCH (1, 1) process that:
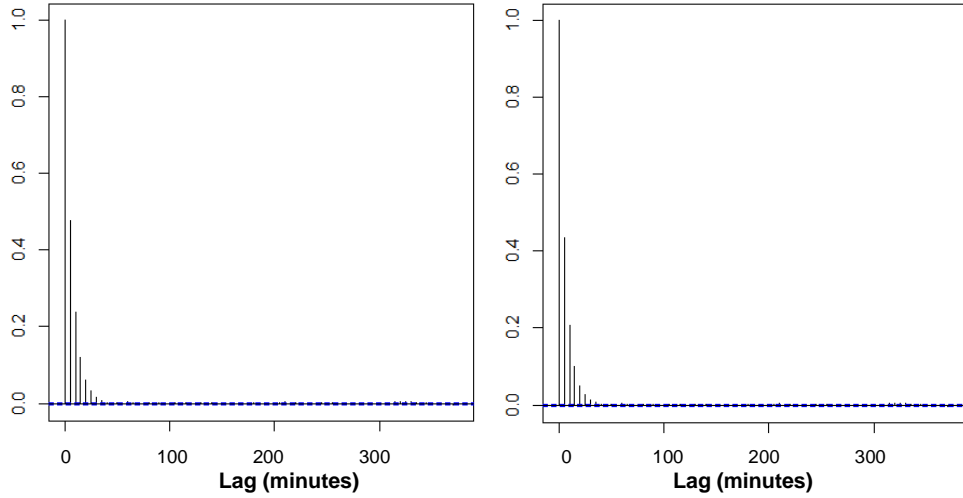
Figure 3.12 The autocorrelation functions for the ARCH (1) process using the parameters that are same with those in Figure 3.10; left-hand side: autocorrelation of the volatility; right-hand side: autocorrelation of the absolute return.

$$\sigma_t^2 = \alpha_0 + \alpha_1 x_{t-1}^2 + \beta_1 \sigma_{t-1}^2. \tag{3.16}$$

To model the stock price returns, the constants in the equation are chosen as $\alpha_0 = 0.01$, $\alpha_1 = 0.08$, and $\beta_1 = 0.91$. These values are obtained by optimising the likelihood to the q-distribution with $q = 1.4$, i.e., corresponding to the one-minute return for the stock market. Very similar values are chosen in other literature to model the stock returns (Poon and Granger, 2003). A global volatility value $\sigma = 1$ also satisfies that (Baillie and Bollerslev, 1992):

$$\sigma^2 = \frac{\alpha_0}{1 - \alpha_1 - \beta_1}. \tag{3.17}$$

The probability distribution function of the above GARCH (1, 1) process is plotted in Figure 3.13, in which the q-distribution ($q = 1.4$) is also superimposed. A good agreement can be observed between the two models, indicating that the GARCH (1, 1) process is sufficient in terms of modelling the probability distribution of high-frequency returns. Unlike the ARCH (1) process (which is unable to model returns with less than five minutes), the numerical test

shows that the GARCH (1, 1) process is able to capture probability distribution functions with q up to 1.6 (corresponding to tick-by-tick returns, according to Eqn. (3.10)).
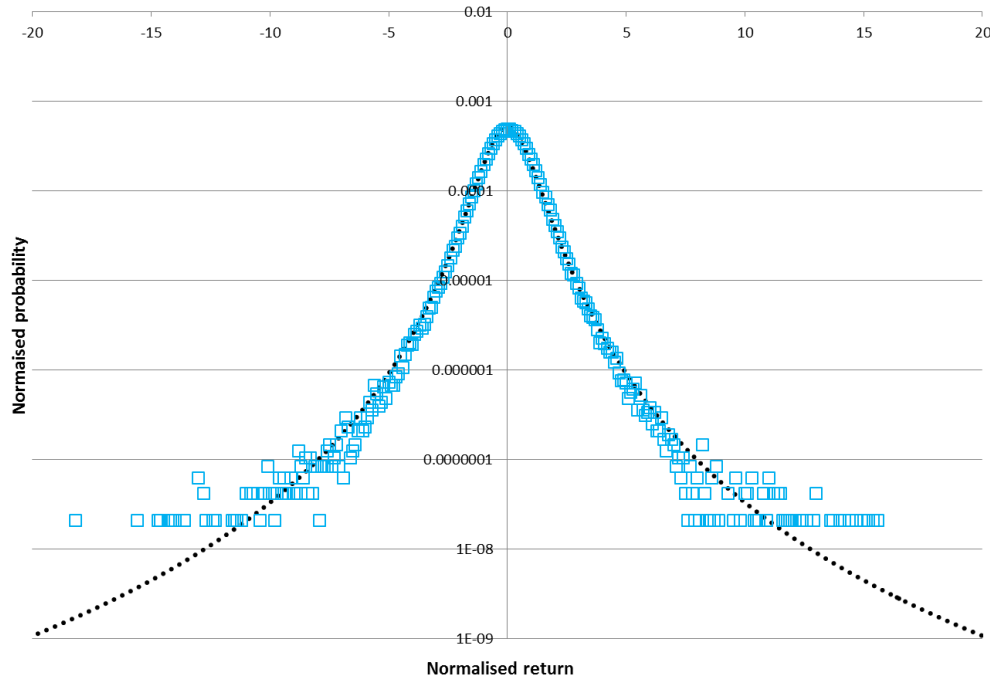
Figure 3.13 The distribution of GARCH (1, 1) process with the global volatility $\sigma = 1$, using values $\alpha_0 = 0.01$, $\alpha_1 = 0.08$, and $\beta_1 = 0.91$. The q-distributions discussed in section 3.3 are also plotted in the figure, with the solid line $q = 1.4$.

The time series of the volatility modelled by the GARCH (1, 1) process is shown in Figure 3.14. The qualitative feature of volatility clustering is captured by the model and is very similar to that of the ARCH (1) model shown in Figure 3.11. The variance of the volatility in Figure 3.14 is slightly higher than in Figure 3.11, but still much smaller than the real market. Bollerslev (1986) pointed out that the autocorrelation of the volatility for the GARCH (1, 1) process follows an exponential decay rate with the characteristic decay rate proportional to $\left| \ln(\alpha_1 + \beta_1) \right|^{-1}$. The autocorrelation functions for the time series generated from the GARCH (1, 1) process in Figure 3.14 are plotted in Figure 3.15, both for volatility and absolute returns. It can be observed in the figure that such autocorrelation functions decay much more slowly than those by the ARCH (1) process in Figure 3.12 - but still much faster than the real market shown in Figure 2.13. The GARCH process is a powerful model with simple non-linear numerical representations, and it captures some features of the secondary parameters of

the real financial market. In this thesis, we are not going to discuss the GARCH ($\bar{p}, \bar{q}$) process with $\bar{p}$, $\bar{q}$ values larger than 1. It has already been demonstrated in the literature that it is possible to use the GARCH model to mimic the long term autocorrelation with characteristic time scale up to 62.3 days (Akgiray, 1989).
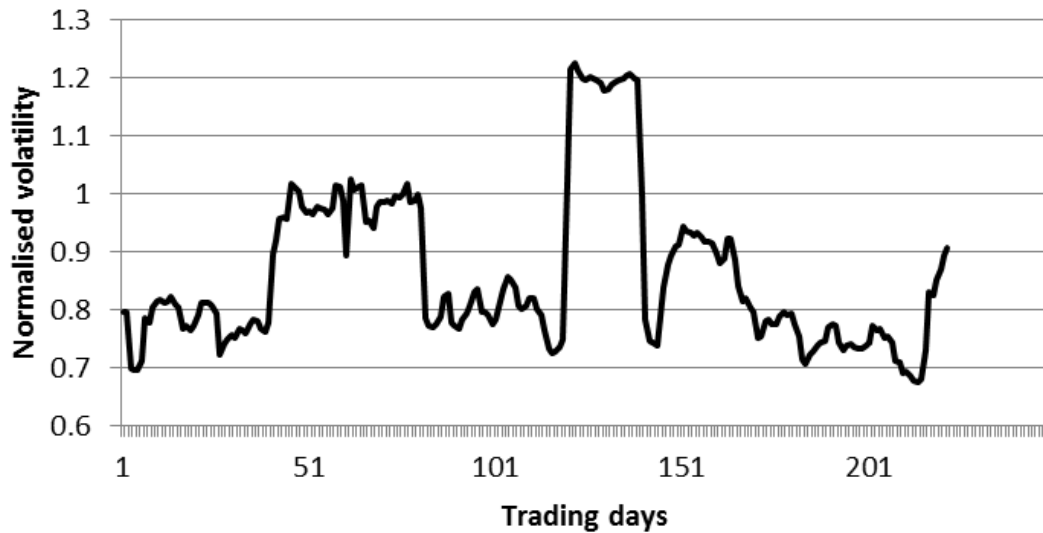
Figure 3.14 Time series of monthly (normalised) volatility simulated using the GARCH (1, 1) process using the parameters that are the same as those in Figure 3.13.
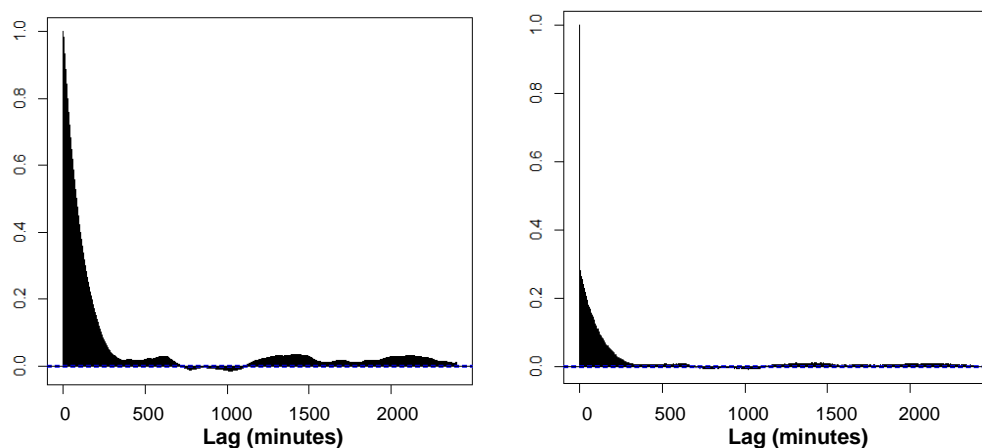
Figure 3.15 The autocorrelation functions for the GARCH (1, 1) process using the parameters that are the same as those in Figure 3.13; left-hand side: autocorrelation of the volatility; right-hand side: autocorrelation of the absolute return.

It is instructive to study the diffusion behaviour of the GARCH process. It has been shown in the current thesis that the Gaussian statistics follow a constant diffusion rate $\sim t^{-0.5}$ and the q-distribution follows a non-constant diffusion rate from $\sim t^{-0.7}$ to $\sim t^{-0.5}$, while the empirical observation is close to $\sim t^{-0.5}$. The (anomalous) diffusion behaviour of the GARCH (1, 1) process can be examined by aggregating the GARCH (1, 1) process that:

$$S_t^n = \sum_{i=0}^{n-1} x_{t-i} \tag{3.18}$$

Frost and Nijman (1993) have demonstrated that the process obtained through Eqn. (3.18) is also a GARCH (1, 1) process but with different values of α_0, α_1, and β_1. Moreover, it is shown that the GARCH (1, 1) process will converge to the process of $\alpha_1 = \beta_1 = 0$, i.e., a Gaussian process with the increased value of n in Eqn. (3.18). Figure 3.16 plots the probability of return to mean value for the probability distribution function that is generated for the GARCH (1, 1) process, for different time horizon of returns. It can be found that such probability reduces with an increase in time horizon of returns generated from the GARCH (1, 1) process. A remarkable similarity can be found between the simulated data in Figure 3.16 and the real data in Figure 2.10. The slope of the log-log plot is -0.53 in Figure 3.16, which also agrees with previous findings. It is however interesting to plot Eqn. (3.10), which suggested a non-constant diffusion rate. As shown in the figure, for the commonly used time horizons of returns, i.e., $t = 1$ minute to 1000 minutes, the assumption of a constant diffusion rate is generally valid, while the findings in section 3.3 for the q-distribution give a more precise solution of the diffusion process for a much wider range of time horizons of returns. The current research demonstrates that the q-distribution describes the ARCH/GARCH process rather well in terms of both the probability distribution and the diffusion process. The understanding of the q-distribution - especially its diffusion properties - provides a method to translate the time steps in the simulation to the physical trading time of the stock market. There may be some intrinsic relationship between the ARCH/GARCH process and the q-distribution through some measure of inter-dependence, such as that used in the q-central limit theorem (Umarov et al., 2008). It is, however, beyond the scope of the current thesis to establish the theoretical limit theorem; this will be left for future research.
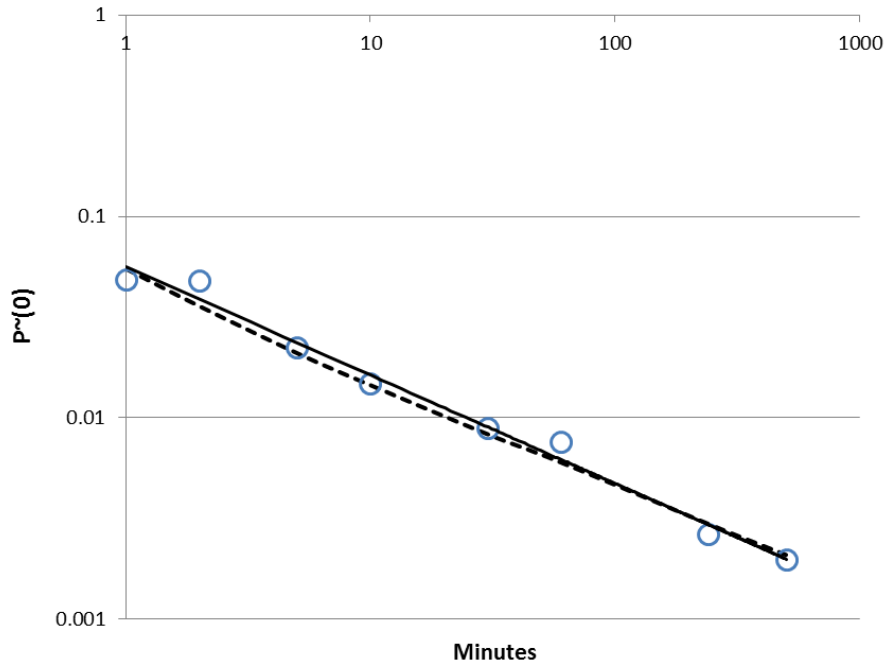
Figure 3.16 The diffusion process studied using the probability of zero returns simulated from a GARCH (1, 1) process with the same input parameters to those in Figure 3.13. The solid line is the power law function with constant diffusion rate of -0.53 and the broken line shows the varying diffusion rate following Eqn. (3.10).

3.5 Summary

This chapter studies a number of macroscopic approaches that have previously been used to model the stock market. The empirical evidence studied in Chapter 2 is revisited using the methodology of normalised parameters. Specifically, the probability distribution function, the diffusion behaviour, and the higher order factor of volatility and its autocorrelations are examined in the context of the Lévy statistics, Tsallis statistics, and the ARCH/GARCH process. At first, the distribution tails are examined in the context of a Lévy distribution. The universal existence of the power law distribution might originate from the generalised central limit theorem, but might also originate from different statistical techniques and different empirical data that are selected in different markets with different time horizons of returns. The different (and controversial) power law decay rates found in the literature are explained in terms of statistical bias and the convergent behaviour of the model. Thanks to the framework established in Chapter 2, those tail behaviours are studied in detail following a systematic method. This chapter then focuses on the Tsallis statistics, characterised by the q-distribution. The importance of the Tsallis statistics is its statistical mechanics foundation that

captures a group of inter-dependent random variables. The agreement between the q-distribution and the empirical data indicates that the fat-tails in the stock return distribution are related to some form of inter-dependence and feedback loop, which is concluded as the main ingredient leading to the fat-tail distributions. The importance of the Tsallis statistics in the financial modelling is its ability capturing the stylised statistical facts of financial data, at the same time its root in the statistical mechanics which lead to a full suite of mathematical calculus. It is also compatible with most existing financial models that is built on the Gaussian statistics, therefore minimises the implementation effort. Overall, the Tsallis statistics is a promising area with both theoretical merit and practical potential hence attracted extensive attentions from both the financial industry and academics. However, as a relatively new area of statistical modelling, especially in terms of the application on the financial market, the Tsallis statistics requires extensive research in terms of understanding the empirical relationship between different input parameters, as well as their microscopic implications, for example, the relationship between the value of q and the volatility. Using the framework of a systematic empirical study, it is concluded that the q-inter-dependence gradually diminishes with the increase of the time horizon of returns, resulting in time-dependent q values associated with an anomalous diffusion process. An empirical model to describe the diffusion process is proposed and validated. The model can be also used for further modelling practices such as stochastic differential equations, based on non-Gaussian statistics.

The remaining part of the chapter introduces a numerical macroscopic approach, namely ARCH and GARCH processes, which are extensively used in financial modelling. Instead of repeating previous research findings or performing heavy parameter optimisation exercises, the fundamental properties of this pedigree of models are illustrated in this chapter, facilitated by the q-statistics. The ARCH and GARCH processes incorporate some correlations in the secondary factors (i.e., the volatilities) and it can be demonstrated that fat-tail behaviours and the first order variable (i.e., the returns) can be generated from non-fat-tail Gaussian distributions. Remarkable agreements between the q-distribution and the ARCH/GARCH processes are found, suggesting some intrinsic linkage between the two types of models through the measurement of inter-dependence. The behaviour of higher-order statistical factors is, however, less promising. This is expected, though, as those processes are only designed to capture the first order factors.

As a summary, the empirical findings of probability distribution functions can be accurately modelled by some macroscopic approaches. Those models are generally non-Gaussian and show promise and capability to capture the high impact low probability events. The key factor to the success of those models is to take into consideration temporal and spatial correlation. However, it still remains unclear how those correlations should be calculated explicitly from the empirical data, and thus how they can be connected with the decision making processes of the investors in the financial market. What is more, it is relatively difficult for those models to capture the higher order properties of the market, which is related to more detailed microstructure of the market. On the one hand, the macroscopic models are the most widely used models in the financial industry, due to their simplicity and applicability to the established structured finance. On the other hand, it requires a microscopic model to provide insight regarding investment activities and how they contribute to the macroscopic representations. This task will be carried out in the next chapter in order to provide further understanding of the models introduced in this chapter and, at the same time, to establish a multi-scale modelling approach to enable a better system of financial modelling and risk management practices.

Chapter 4 microscopic modelling using agent based methods

4.1 Introduction

The preceding chapter focused on the models able to capture the non-Gaussian statistics of the time series from the stock market. The macroscopic data, such as price returns and volatilities, are the main information source, based on which macroscopic phenomenological models are derived. It has been also demonstrated on various occasions that the micro-foundation - e.g., those from statistical mechanics - can be beneficial in understanding the applicability of a model. As discussed in Chapter 1, this is an important issue in the context of the financial crisis. For example, when a macroscopic model is applied to the stock market, only the macro-variables such as price returns and volatilities are studied explicitly. The microscopic insights of the financial market are either unclear, or cannot be directly related to those from the statistical mechanics, leading to insufficient understanding about the financial mechanisms behind the market movements. As the microscopic mechanisms are often leading indicators, and are followed by dramatic changes in the whole market (Christianson, 2002), it is therefore crucial to study the micro-foundation of the market in order to make better investment decisions and to manage financial risk. Some specific microscopic mechanisms will be examined in the current chapter: 1) the origin of the non-Gaussian statistics; 2) the impact of the external information and the trading activity itself; 3) the relationship between market behaviour and the decision-making behaviours of investors; 4) mechanisms causing volatility clustering and the long term correlation of volatility and absolute returns; and 5) mechanisms differentiating between the calm market period and turbulent market period. We will attempt to answer some of the above questions at the end of this chapter.

The macroscopic modelling studied in the previous chapter made assumptions on microscopic mechanisms, which would ideally be tested through scientific experiments. It is, however, extremely difficult to perform a large-scale experiment for the stock market under

realistic conditions (Kamenica, 2012). Alternatively, computer simulations inspired by the Monte Carlo methods are adopted to perform bottom-up studies. With the advancement of computational power in the past two decades, complicated interactions between a large number of investors can be simulated for a reasonably large timescale. A popular method is to model each investor or investment institution as an "agent", which makes investment decisions, and at the same time interacts with each other and reacts to external information. There are numerous clear advantages of applying such computational agent based methods on the stock market. Firstly, issues of information (e.g., share price movements; public announcements; government reports) tend to be universally available to all investors with a similar format. All the agents have clear objectives; therefore, decisions based on those pieces of information are made in a relatively quantitative (and "sharp") way (LeBaron, 2000). Secondly, the heterogeneity of investors can be explicitly modelled by agents with different sizes and trading strategies. The effect of heterogeneity can be studied in conjunction with the macroscopic time series, without too many assumptions about the microscopic market constitution. Finally, there are increasing developments in the area of experimental financial markets (Smith et al., 1988), in which carefully controlled environments can be compared with agent based simulations. The fruitful area of behaviour finance can be explicitly incorporated into the decision-making modelling to assess the impact on macroscopic properties.

This chapter is organised as follows: in section 4.2, the previous literature regarding the agent based method is reviewed, with particular emphasis on the assumptions made in terms of modelling microscopic mechanisms (such as investors' decision-making strategies and interactions between investors). In section 4.3 a baseline model is proposed and a numerical scheme used in order to simulate the stylised (non-Gaussian) price dynamics. Section 4.4 aims to understand the microscopic mechanisms in the baseline model by performing a detailed parametric study. Section 4.5 proposes a multi-scale approach that can be used to quantitatively and qualitatively model the empirical findings from the stock market, including in both calm and turbulent periods. The whole chapter will be summarised in section 4.6.

4.2 Modelling microscopic market mechanisms – the agent based method

4.2.1 Adaptive agents

The early agent based method models the investors as agents who update their decision-making strategies periodically, according to their performance (Grimm et al., 2005). A pioneering piece of research carried out by Lettau (1997) developed a genetic algorithm to model how mutual fund agents adjust their portfolios based on historical profit. At each stage in time, an agent's task is to determine how many stocks it holds based on its decision-making rule. A binary string is assigned to each agent to represent their individual decision boundaries. In order to evaluate the fitness of the decision rules, the agent will observe for a period of time (i.e., a number of time steps) and calculate the cumulative profit before determining whether to update the decision rules. Those rules that fit for the market will be inherited; the rules that make losses will be abandoned. At the same time, the model also introduces new investment strategies by either combining or flipping existing decision-making rules to mimic the genetic mutation. After a period of time, the decision-making rules will converge to some optimised pedigree following the principles of "evolution theory" and "survival of the fittest".

The major contribution from the work of Lettau (1997) is that agent based modelling explains some of the observations in the mutual fund market which deviate from the rational market hypothesis. Such irrational phenomena are often captured in the area of behavioural finance, but agent based modelling also provides some explanation by using the genetic algorithm instead of psychological factors. Agent based modelling also shows that the agent will take more risks than determined by a rational model. This is because the optimised portfolio in reality requires the agent to observe a large number of events before adjusting the trading strategy; those strategies that involve risk and considerable luck are more likely to be inherited due to the fitness rules than those optimised from rational models. Due to such bias, it is more likely that the agents adjust the portfolio under loss than under profit (Dowd, 2000). Those observations are in agreement with the research results from the behaviour finance, to which investors are often considered as far from rational, e.g., taking excessive risk and exhibiting asymmetric loss aversion behaviours (Kahneman and Tversky, 1979).

Following the early research using the adaptive agents approach, a comprehensive model that simulates the artificial stock market has been developed; it is known as the Santa Fe Stock Market (Arthur et al., 1997; LeBaron et al., 1999). Compared to the simple agent based model by Lettau (1997), the agents in the Santa Fe Stock Market are able to forecast the future value of the equity as well as the price dynamics. Such a forecast is based on information obtained by individual agents, including price history, trading volume, economic state, etc. A number of decision-making rules determining an agent's short- and long term strategies can be also modelled. Each agent will make investment decisions - buying, selling, or holding shares - at each time step. The clear price of the share can be calculated as an aggregate effect from all agents through an auction procedure. At the same time, the agents' decision-making rules are evaluated based on how accurate the forecast is; thus, the agents' methodologies of forecast will be updated through a genetic algorithm similar to those adopted by Lettau (1997). The Santa Fe Stock Market is one of the most complicated agent based models and has all the ingredients commonly used in this type of modelling. The price return time series generated by the artificial stock market is shown to follow non-Gaussian statistics, and to have long term autocorrelation of volatilities (LeBaron, 2000). Some studies have later argued that the artificial market allows agents with a wide range of forecasting techniques to often converge to one type of trading strategy (Raberto et al., 2001), or even that the model allows the introduction of new trading techniques through the genetic algorithm. On the other hand, the inputs into the artificial financial market - i.e., the microscopic data at the agent level - are not as well recorded as the macroscopic data. It therefore still remains unclear how valid the observations and forecasts using the agent based simulations are, when informing the real market (LeBaron, 2000)

4.2.2 The agents' decision-making process

The current research focus is on the statistical properties of the stock market, namely, the statistical distributions, the diffusion process, and the short- and long term correlations of the price returns and volatilities. Instead of trying to simulate the market using a group of complicated agents, we are more interested in those micro-foundations behind the persistent and universal statistical properties discussed in the previous chapters. The task is therefore to examine those mechanisms that can't be explicitly captured by the macroscopic models. The first mechanism is the agents' decision-making rules, related to the interactions between agents and the feedback loop between agents and the market. One of the early works of

research was to understand the impact of speculation activities on the market; Gode and Sunder (1993) performed a study employing both human and computational agents to test the difference between random and guided trading strategies. A single asset is traded in a double auction system where the selling and bidding prices are matched to create the asset price. The market efficiency can be calculated by modelling the intrinsic value of the asset as a piece of public information. It is interesting to find that the market is much more efficient when the traders act according to the asset values, instead of using random trading strategies. Speculative activities (modelled as random trading) drive the asset prices away from the intrinsic value, and at the same time create excessive volatility. It is worth noticing that, in the model proposed by Gode and Sunder (1993), both trading strategies follow a simple decision-making rule, i.e., they are zero-intelligence agents. The authors therefore argue that it is difficult to distinguish whether the price dynamics of an asset are due to carefully constructed rules or due to the agents' learning and adaptation in their decision-making process (LeBaron, 2000).

The above study leads to an active research field to identify whether a financial market is experiencing a calm or a turbulent period. It has been shown by a number of researchers (Kirman, 1993; Lux and Marchesi, 2000; Kirman and Teyssiere, 2002) that speculative agents will create market turbulence, when compared with agents who trade according to the asset's intrinsic values. Lux and Marchesi (2000) studied those different trading behaviours by allowing agents to switch between a "fundamentalist" and a "chartist". It has been shown that, when the market is driven by the intrinsic value of an asset, the price movement is determined by supply and demand imbalances and distributes according to Gaussian random walk. An "outbreak of volatility" occurs when a fraction of agents start to move away from the intrinsic value and use the market trends to speculate for profit. Such a behaviour switch gives rise to excessive volatility and also causes non-Gaussian statistics, volatility clustering as well as long term memory of the volatility (Ghoulmie et al., 2004; Cont, 2005). It is also argued by Liu (2000) that investor inertia (which is defined as the time spent by each agents before making new investment decisions) is an important factor. Giardina and Bouchaud (2003) pointed out that the switching of behaviours is triggered by the performance of an agent. Therefore, the investor inertia and the behaviour switch may be interpreted as a unified mechanism. Cont (2005) interpreted the investor inertia as the non-trading periods of agents, and suggested that it will decrease during the turbulent market period. This is in agreement with the general understanding that market turbulence is related to high trading activities

(Iori, 2002). As a difficult-to-measure microscopic mechanism, the quantitative effect of investor inertia is still unclear.

4.2.3 The role of market microstructure

The second mechanism is the agents' interactions with each other. The agents collectively form a heterogeneous system with communication channels and feedback loops. The feedback loop is generally incorporated in the agents' decision-making process, as discussed above. The explicit interactions between agents can be also modelled using the microscopic interactions between agents, such as communication and information exchanges, which are strongly dependent on the market microstructure. One of the many methods to model the agents' interactions is to use topological methods, such as percolation theory (Stauffer and Aharony, 1994; Stauffer and Sornette, 1999). Cont and Bouchaud (2000) have successfully applied the topological model on the agent based method. As shown in Figure 4.1, a lattice system with nodes and boundaries is defined in a 2D space (theoretically extendable to 3D). A node in the lattice system may be randomly occupied (dark circle) or left empty (hollow circle). Neighbouring nodes will form clusters with different sizes, representing agents with a heterogeneous arrangement. Each cluster may grow or diminish, and clusters will further separate or merge to form smaller or bigger agents, mimicking the life cycle of the agent or the communication channel. An agent is the unit to make investment decisions, according to their own decision-making strategy. The microstructure of the market and the effect of market heterogeneity can, as a whole, be evaluated by allowing the topology of the market to develop on a Monte Carlo basis. It has been demonstrated that the heterogeneous microstructure of the market is capable of creating non-Gaussian statistics (Yang et al., 2005), however the quantified relationship between market microstructure and the stylised price dynamics from the stock market is still a puzzle yet to be solved.

Another source of market heterogeneity is the fact that the agents have different access to information, different investment preferences, expectations, and forecast techniques, etc. Harrison and Kreps (1978) performed an analytical framework following the argument that such heterogeneity will add some "chartist" features to the "fundamentalists". This effectively drives the market away from the efficient market hypothesis and leads to non-Gaussian price dynamics. Levy et al. (1995) performed a comparison study between homogenous and heterogeneous investors, and concluded that the use of a "representative

investor" will lead to unrealistic market phenomena; hence, this type of heterogeneity is essential in terms of simulating a realistic financial market. Lebaron (2001) demonstrated, using agent based modelling, that the heterogeneity of time horizons in which agents make their investment decisions will increase the volatility of the time series. Granger (1980) suggested that the long term correlation of the volatility can be attributed to the aggregate effect from "different time series with different persistence levels". Cont (2005) concluded that the time horizon heterogeneity and the feedback loop are both essential ingredients in order to simulate the non-Gaussian statistics and the long term autocorrelations of the volatility.
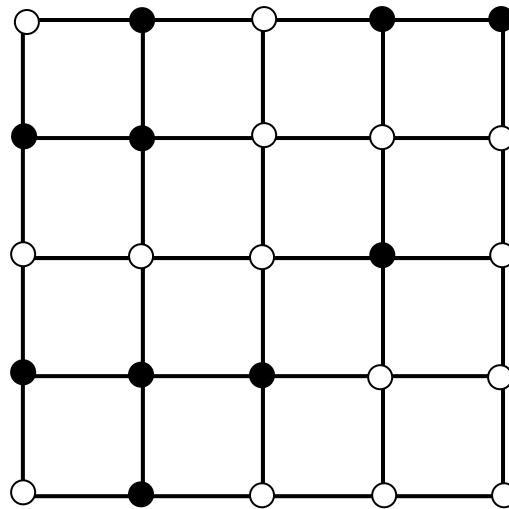
Figure 4.1 Illustration of the topological configuration of the model from Cont and Bouchaud (2000). Dark circles represent investors while interconnected investors form an institutional investment agent. Reproduced from Yang et al. (2005).

4.2.4 The herd effect

One of the most discussed mechanisms when seeking the market micro-foundations is the herd effect of investors (e.g., Scharfstein 1990; Lux, 1995; Chang et al., 2000). In the adaptive agents' approach, most investors seek to imitate the successful strategies adopted by many others, leading to the herd behaviour (Barberis and Thaler, 2003). In the feedback loop model, the herd effect is implicitly captured by modelling a higher impact of the feedback, due to market-wide information flows that are broadcast to all agents (Ghoulmie et al., 2004; Cont, 2005). In a market microstructures model, meanwhile, the herd behaviour is amplified through either the growth of the cluster or reprinting the same investment strategy to an increased number of agents (Yang et al., 2005). There is empirical evidence to suggest that,

during turbulent market periods, the herd effect is stronger and leads to higher market volatilities (Chiang and Zheng, 2010). Raberto et al. (2001) modelled the herd effect as a link between market volatility and agent uncertainty, to represent the fact that an agent is less constrained to the intrinsic value during a turbulent market period. The general conclusion from the literature is that the herd effect is a key ingredient in the non-Gaussian statistics (Cont and Bouchaud, 2000). For many agent based models, the herd effect is not always modelled explicitly; it is a factor that represents the combined effect from the agents' decision-making process, the market heterogeneity, and the feedback loop to the information flow.

4.2.5 The current research methodology

One of the major challenges of the agent based method is that most of the research outcomes still remain qualitative. The agent based method often adopts a numerical scheme includes a large number of input factors. Unlike the microscopic simulations used in physics, where the interactions between particles can be derived either from the first principles, or from empirical approaches, the input parameters adopted in the agent based modelling are often *ad hoc* to suit the numerical needs. It is extremely difficult to compare the outcomes of one piece of research with those of another, making it difficult to demonstrate the model's applicability in a wider context. Moreover, those input factors are also often inter-dependent, with little data support from the microscopic level. At the same time, the interplay between those factors and complicated numerical schemes makes it often impossible to identify the key underlying microscopic mechanisms. For the agent based modelling approach, the challenge is not only to qualitatively simulate the non-Gaussian statistics, but also to pinpoint to what extent each mechanism contributes to those stylised facts. For example, in the adaptive agent approach, the genetic algorithm makes it difficult to trace the individual agents as a carefully controlled numerical experiment.

The current research will take a step back to a baseline model which only includes the essential mechanisms with simple mathematical representations. We adopt the approach of the Genoa artificial market (Marchesi et al., 2003) in which the asset price is determined by the supply-demand relationship, and in which each agent makes buy or sell decisions in a probabilistic manner. The agents' trading strategies are relatively defined, while how one chooses between the rules is randomly determined by individual agents. Compared to other

approaches such as the self-adaptive agents, the Genoa market uses relatively simple decision-making rules, making it easy to trace the impact of the mechanisms. Other microscopic mechanisms can be also introduced to the baseline model in order to understand their impact on the macroscopic behaviours of the market. A multi-scale approach is proposed, based on the systematic understandings of both the agent based and q-distribution models introduced in Chapter 3. The data from the real stock market presented in Chapter 2 are also used to facilitate the study in terms of making systematic and quantitative evaluations. As an attempt to contribute to the micro-foundations of the financial market, the multi-scale approach is used to study the empirical findings from both calm and turbulent market periods.

4.3 The baseline model and the numerical scheme

4.3.1 The baseline model with simple mechanisms

We start with a simple modelling approach that follows the same route as Cont (2005). In the agent based model, a single asset is traded between N agents at time steps $t = 0, 1, 2, \ldots N_T$. The asset price is denoted by $Y(t)$ at each time step. The relationship between the numerical time step and physical time will be apparent in a later context. At each time step t, each agent i submits a buy or sell order to trade a single unit of the asset. The order is represented by a step function $\phi_i(t)$ that $\phi_i(t) = 1$ denotes the buy order and $\phi_i(t) = -1$ denotes the sell order. The function $\phi_i(t)$ is also allowed to be equal to zero, in order to represent no trading for agent i at time step t. The price fluctuation due to the supply and demand relationship is modelled using the excess demand function (Mantel, 1974) that:

$$d(t) = \sum_{i=1}^{N} \phi_i(t) , \tag{4.1}$$

and the logarithm return function $S(t)$ (see Eqn. (2.8)) is calculated through:

$$S(t) = g(\frac{d(t)}{N}) = \frac{d(t)}{N\lambda} , \tag{4.2}$$

in which λ $(\lambda > 1)$ is the market depth. Eqn. (4.2) takes in to consideration the relationship between excess demand, the market's ability of meeting that demand, and the price fluctuation. In the current research, the market depth is modelled as a linear function, while

other choices are possible (Goldstein and Kavajecz, 2000; Engle and Lange, 2001). Instead of using an auction system to match each buy and sell order, Eqn. (4.2) is an empirical approach that simplifies the relationship between the demand-supply and the price fluctuation, and is widely used in a number of agent based studies (Yang et al., 2005; Ghoulmie et al., 2004; Cont, 2005).

The decision-making process of the agents is modelled as an agent's response to the external information flow, denoted as $\varepsilon_i(t)$. It can be also understood as the agents' forecast about the movement of the intrinsic values of the asset. In the baseline model, the external information flow $\varepsilon_i(t)$ is assumed to be a random variable following the normal distribution $\varepsilon_i(t) \sim N(0, \sigma')$, in which σ' is the volatility of the information flow. The heterogeneity of the agents' decision-making process is modelled by assigning a different threshold $\theta_i(t)$ for each agent. The thresholds can be understood as each agent's view on future price movements. The agent's decision-making process is mathematically modelled as rule-based, so that:

$$\phi_i(t) = \begin{cases} 1, & \text{if } \varepsilon_i(t) > \theta_i(t), \\ -1, & \text{if } \varepsilon_i(t) < -\theta_i(t), \\ 0, & \text{if } |\varepsilon_i(t)| \leq \theta_i(t). \end{cases} \tag{4.3}$$

In the baseline model - even in reality - each agent has different expectations as to the future value of the assets, the step function of Eqn. (4.3) effectively makes it equivalent to using the same value of $\varepsilon_i(t)$, as long as the agents have different forecasts of the future price $\theta_i(t)$. It is later demonstrated that the simplified approach reduces the number of input parameters and makes it easier to identify the underlying mechanisms. At each time step, an agent is comparing the value change of the asset with the decision threshold - the agent's expectation on future price movements. Such an expectation may be updated periodically (but not at every time step), according to the agent's latest learning about the asset price movements. Using the mathematical description, the threshold of each agent's decision-making process $\theta_i(t)$ is updated as:

$$\theta_i(t+1) = |S(t)|, \text{ if } s > \mu_i(t), \text{ and } \theta_i(t+1) = \theta_i(t), \text{ if } s \leq \mu_i(t), \tag{4.4}$$

in which μ is a uniformly distributed random variable. At each time step, an agent i has got a certain probability s to update its threshold $\theta_i(t)$. Eqn. (4.4) provides a simple modelling approach to the agents' forecasting techniques. When an agent updates its future expectations, it will equal the latest price movement. This is supported by some empirical evidence (Zovko and Farmer, 2002), where a common future expectation of the investors is the most recent price movement. Judging from Eqn. (4.4), the average time it takes for an agent to update the decision-making threshold is equal to 1/s, which means s is a small number representing how quickly the agent learns from the market. The decision-making rule, Eqn. (4.3), and the updating scheme, Eqn. (4.4), are based on a small number of input parameters which capture both the threshold nature of the agent's behaviours and the random nature of updating the decision-making rules.

4.3.2 The numerical algorithm and implementation

The algorithm of the baseline model is summarised as below:

1) A number of N agents are configured to trade on a single asset.

2) An initial decision-making rule is assigned to each agent according to a normal distribution: $\theta_i(0) \sim N(0, \sigma'')$.

3) At each time step, each agent receives the same external market signal as the forecast asset value fluctuation: $\varepsilon_i(t) \sim N(0, \sigma')$.

4) Each agent makes buy and sell decision by comparing their thresholds with the external market signal using Eqn. (4.3).

5) After submitting the buy or sell order, each agent will update their decision-making rules, with a constant probability s, using Eqn. (4.4).

6) The asset price is updated using the excess demand model Eqns. (4.1) and (4.2), by collecting all the orders from each agent, with a constant market depth λ.

The algorithm is translated into an Excel VBA script which iterates for a large number of time steps to generate the time series of price return. Other macroscopic market indices (e.g., market activity level and trading volume) can be simulated at the same time. The evolution of the agents' decision-making rules can be also traced through the time steps. Before performing any valid numerical experiments, it is important to make sure that the algorithm converges to a consistent result by choosing appropriate numerical parameters - namely, the

number of agents N and the number of time steps N_T. By examining Eqns. (4.1) and (4.2), it can be found that the underlying distribution of $\phi_i(t)$ follows an identical distribution; the two are dependent on each other. Such dependency is similar to the interactions in non-extensive statistical mechanics, and will remain constant due to the persistent feedback loop. According to the q-central limit theorem (section 3.3.1), $S(t)$ must converge to a q-distribution which is independent of the number of agents N. For a real financial market, it is believed that there are enough market participants and it is assumed to generate a converged empirical distribution.

The effect of number of agents N on the statistical distribution of the price return is shown in Figure 4.2, with the other input parameters detailed in the figure caption. Using the same techniques of analysing the empirical data (see section 3.3.2 for details), the statistical distributions of the simulated time series are normalised by the volatility of each time series (which are detailed in Table 4.1). It can be observed from the table that the increasing of the number of agents leads to a converged overall volatility of the price return. After being normalised by those volatility values, Figure 4.2 shows a fatter distribution tail with an increased number of agents. For a small number of agents ($N = 500$), the price return distributes following a tri-modal distribution, with three peak values around -1, 1 and 0. Such a tri-modal distribution represents the agents' decisions of buy, sell, or hold (Eqn. 4.3). This is not observed in any real market; however, Cont (2005) reports using a similar type of agent based modelling technique. The tail behaviour is very close to the normal distribution, which is super-imposed in the figure. When the number of agents increases, the feature of tri-modal distribution starts to disappear while the feature of fat-tails starts to appear. The distributions converge to a unified q-distribution for the cases $N \geq 8000$, which validates the numerical code and informs the appropriate range of the numerical parameter N. It can be further found that the converged distribution is related, where the q value equals 1.4. As demonstrated by the empirical analysis of the real data (Figure 3.5), the value $q = 1.4$ corresponds to the one-minute return from the stock market. It is therefore possible to link the time steps from the simulation with physical trading time from the stock market.
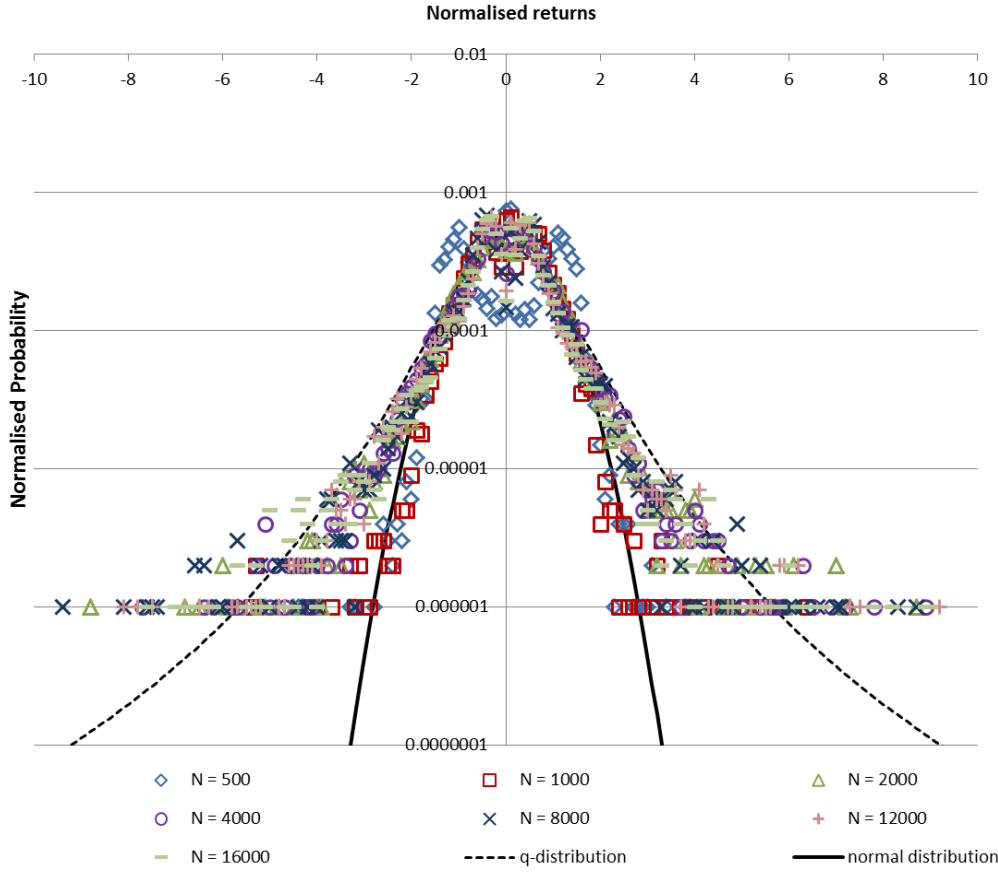
Figure 4.2 The statistical distribution that is obtained from the agent based model described in section 4.3.1. The distribution is normalised by the standard deviation of the whole time series for each case (detailed in Table 4.1). Different symbols represent different numbers of agents used in the simulation. Other parameters are: the probability of updating decisions rules for each agent: $s = 0.01$; the volatility of the external information $\sigma' = 0.001$; the market depth $\lambda = 10$. The q-distribution with $q=1.4$ and the normal distribution are also plotted.

The total number of time steps N_T does not impact on the convergence of the price return statistics but the resolution of the distribution, i.e., the ability of generating rare events. The smallest probability from the simulated distribution will be $1/N_T$. As shown in Figure 4.2, for which $N_T = 10000$ time steps are simulated, this is sufficient to pick up most rare events in the distribution tail outside the Gaussian regime. This choice of the value of N_T is also made based on computational cost: a simulation for 10000 time steps with 8000 agents requires four computational hours on a mini-super computer. Before the simulation starts, there is also a pre-processing procedure to set up the initial configuration of the agents' initial decision-making threshold. This is in order to minimise the impact from the initial distribution of the agents' decision-making threshold $\theta_i(0) \sim N(0, \sigma'')$ on the simulation results. After a number of time steps, agents adjust their decision-making threshold according to the price fluctuation

following $\theta_i(t+1) = |S(t)|$. Recalling that the average number of time steps that an agent waits before updating the threshold is $1/s$, the numerical scheme in the current research allows the agents to overwrite their initial randomly-chosen thresholds $\theta_i(0)$ for a time period of $10/s$ before starting the numerical experiments. Therefore, in total, $10/s + 10000$ time steps are required in all the numerical experiments performed.

4.4 The parametric study on microscopic mechanisms

The numerical scheme introduced above includes a small number of microscopic parameters, namely the probability of agent's update threshold s, the standard deviation of the external information flow σ', and the market depth λ. The baseline model only includes some basic mechanisms represented by simple parameters. It allows the non-Gaussian statistics from the stock market to be reproduced using a simulation method. Using a smaller group of parameters also enables us to pin down the essential ingredients leading to both the non-Gaussian statistics and the long term correlations. We perform a parametric study around those inputs to establish a quantitative understanding of the microscopic mechanisms. Due to its linkage with the macroscopic model of q-distribution, the statistical distribution for the time series of the price return is the main simulation output examined in this section. The secondary factors, such as short- and long term correlations and the diffusion properties, will be studied in a later section.

4.4.1 The agents' decision-making thresholds

In the baseline model, the decision-making process of each agent is a simple rule-based process. At each time step, a threshold value is used by individual agents to make buy, sell, or non-trade decisions. Those trading activities from all agents create the cumulative effect of the price fluctuation; the price movement will then feed back to the agents, who update their decision-making threshold, which further affects the agents' trading and non-trading decisions. This is the feedback loop from the market to each agent, and its intensity is measured by the value of s. Instead of directly modelling the interface between agents, the baseline model implicitly creates the interactions between agents through the feedback loop. Another dimension of s is the heterogeneity of the agents' decision-making rules, as discussed in section 4.2. According to Eqn. (4.4), each agent will randomly process the price

return information. Such randomness will cause heterogeneity in the decision threshold used by the agents to make investment decisions. For a very large value of s, each agent will update the threshold simultaneously to the latest share price, leading to homogenous threshold values. For a small value of s, the threshold will not be updated; hence, there is little feedback loop. As demonstrated in Chapter 3, the shape of the probability distribution can be characterised by the q-distribution, with the value of q measuring the level of interactions between agents. On the other hand, Cont (2005) pointed out that both feedback loop and heterogeneity are essential in producing the non-Gaussian statistics. A correlation between the value of s and q demonstrates that a combination of heterogeneity and the feedback loop creates persistent interactions between the agents.

The impact of such a combined effect can be tested by varying the value of s in the baseline model. The simulation results are then compared with the theoretical q-distribution, which quantifies the level of interaction between the agents. As shown in Figure 4.3, the distribution tail is fatter for smaller values of s, which is negatively correlated with the value of q in the q-distribution. Recalling that the average time step for an agent to update the threshold will be $1/s$, the interactions between agents are therefore weaker when the agents update their decision-making rules more frequently (larger values of s) and homogeneously. In a turbulent market, agents are more likely to change their decision-making rules. Such behaviour can be linked to the herd effect of investors, because those decision-making rules are more homogeneous in a turbulent market than in a calm market. As detailed in Table 4.1, the simulated volatilities of the price returns (which are used to normalise the distribution in Figure 4.3) increase asymptotically with the frequency of updating the decision-making rules. This validates the hypothesis between the herd effect and the turbulent market. It is also shown by the numerical experiments that when the value of s is very close to 1 or 0, the statistical distribution fails to converge to a realistic shape (therefore is not plotted in the figure). As an illustration, for the case $s = 1$, all agents will change their decision rules simultaneously, and according to the external information flow. The market acts as one big agent trading around the asset. For a small number of agents, Figure 4.2 has already shown that the distribution is unrealistic and behaves as a tri-modal distribution with thin tails. It is worth mentioning that, while the distribution shape can be used to connect the simulation time step with physical time, the microscopic parameter s is not suitable to be interpreted as physical frequency. This is because the baseline model adopts simplified factors to highlight the effect of mechanisms instead of trying to capture the complicated market constitutions.

Figure 4.3 The statistical distribution that is obtained from the agent based model described in section 4.3.1. The distribution is normalised by the standard deviation of the whole time series for each case (detailed in Table 4.1). Different symbols represent different values of s used in the simulation. Other parameters are: the number of agents: $N = 8000$; the volatility of the external information $\sigma' = 0.001$; the market depth $\lambda = 10$. The q-distribution with $q=1.4$ and the normal distribution are also plotted.

4.4.2 The external information flow

The impact of the external information flow $\varepsilon_i(t)$ is plotted in Figure 4.4. The normalisation parameters can be found in Table 4.1. As shown in section 4.3.1, the external information is modelled as a random variable that follows a normal distribution $N(0, \sigma')$. Each agent will receive the same information at each time step, and will then process such information using their own decision-making rules and perform buy / sell / hold trading activities. If agents do not process the information (i.e., they simply buy or sell according to external information), the price return will follow the same distribution as the external information. It can be seen from Figure 4.4 that the trading activities transform both the amplitude of the volatility and the shape of the distribution. This means that the trading activities of the investors create both excessive volatility and the fat-tail distribution observed in the stock market. What is more,

107

the impact of trading activity is much larger when the external information is relatively uniform, i.e., there is a small value of σ'. For the case $\sigma'=0.005$, the volatility of the price return has been magnified by more than 10 times (detailed in Table 4.1) than that of the external information flow. The statistical distribution is clearly non-Gaussian with fat tails characterised by $q >1.4$. When the external information starts to intensify, i.e., the value of σ' increases, the market seems to be less affected by the trading activities, while the price return distribution moves towards a Gaussian distribution. For the volatility value, Table 4.1 shows a modest magnification effect with increased intensity of the external information. If we understood the external information as the fluctuation of the intrinsic value, it can be revealed from the agent based simulations how such intrinsic value is manifested into the excessive volatility of the stock market. If we regard the deviation from the asset intrinsic value as bubbles, the baseline model provides invaluable insight into such market efficiencies.

The interplay between the trading activities and the external information flow can be understood using the theory of investors' inertia. Due to the threshold nature of the agents' decision-making process (modelled in Eqn. (4.3)), the investors will make decisions not to trade in a number of time steps. With the increase of the intensity of external information, the investor inertia starts to vanish due to strong external influences suggesting to trade. According to Bayraktar et al. (2006), the fat-tail distribution found in the stock market is closely correlated with the investor inertia, while diminishing such an effect will lead to a Gaussian distribution shown in Figure 4.4. This is in agreement with the current findings that a strong external information flow will reduce the investor inertia, and the market then moves more according to the external information, i.e. towards the Gaussian distribution. It is also understood that investors in a turbulent market are subjecting to smaller inertia, which is confirmed by the values of volatilities shown in Table 4.1.
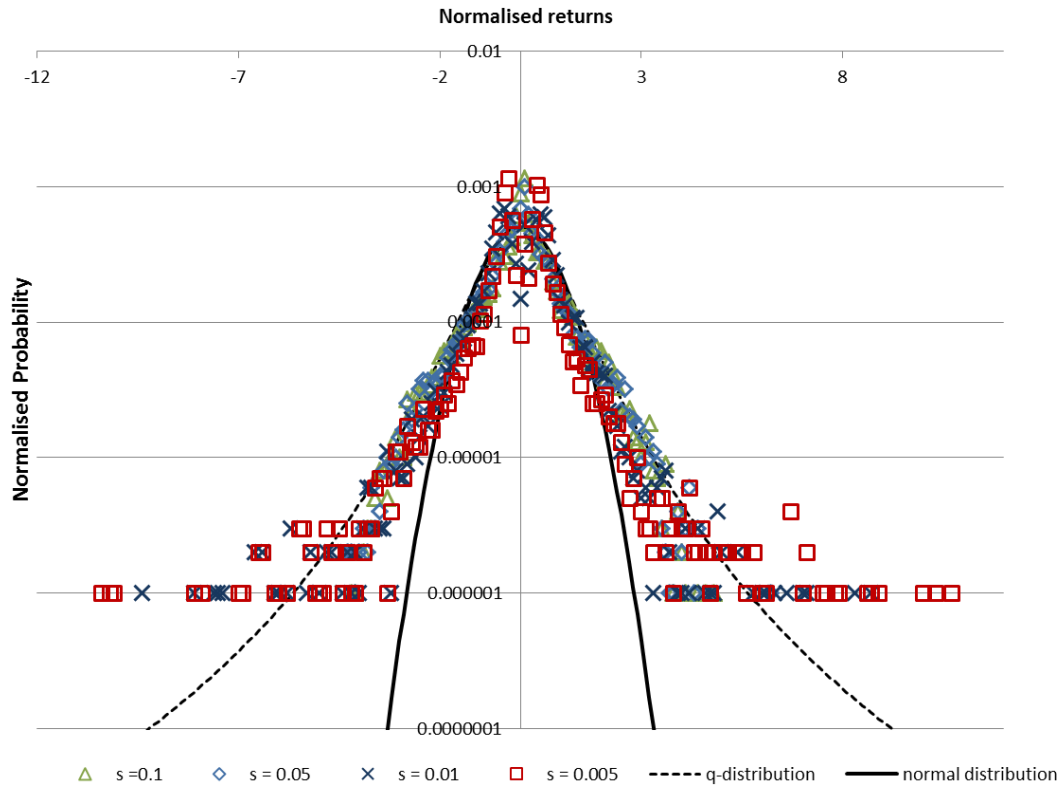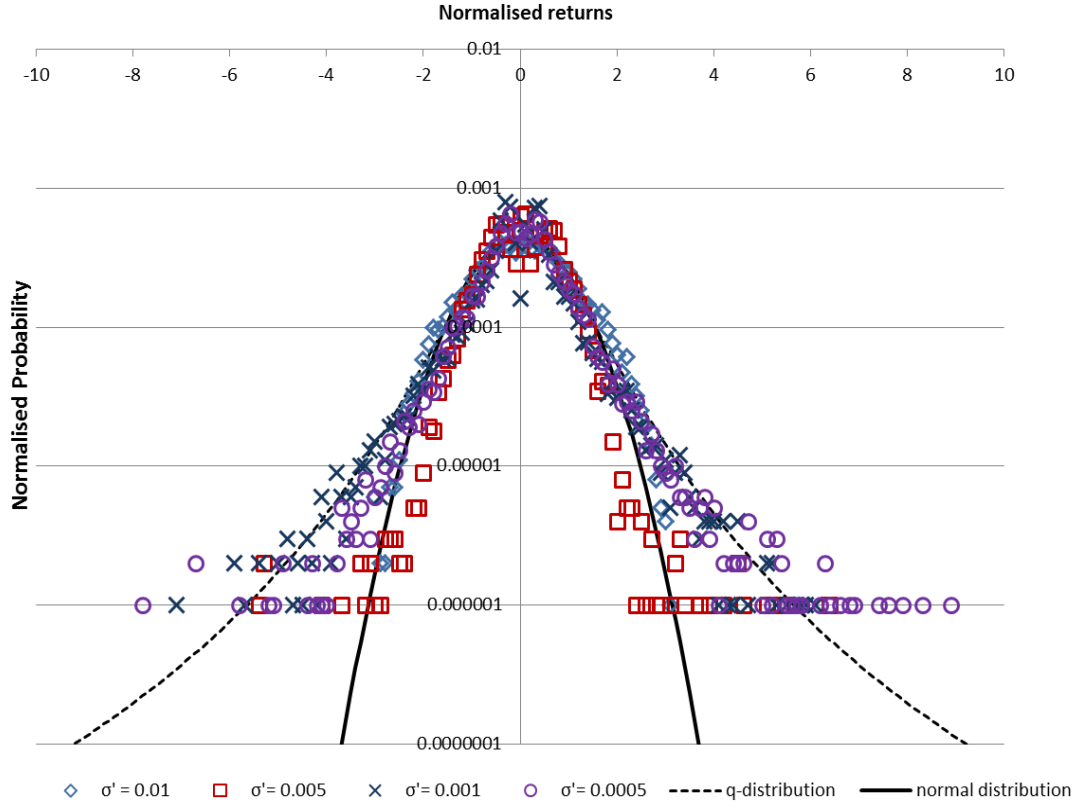
Figure 4.4 The statistical distribution that is obtained from the agent based model described in section 4.3.1. The distribution is normalised by the standard deviation of the whole time series for each case (detailed in Table 4.1). Different symbols represent different values of σ' used in the simulation. Other parameters are: the probability of updating decisions rules for each agent: $s = 0.01$; the number of agents: $N = 8000$; the market depth $\lambda = 10$. The q-distribution with $q=1.4$ and the normal distribution are also plotted.

4.4.3 The excess demand and the market depth

The third input parameter of the baseline model is the market depth λ, which is an important microscopic factor that draws substantial research interest (e.g., Kyle, 1885; Engle and Lange, 2001; Hautsch, 2003). It is a parameter measuring the market liquidity - the ability that an excess demand can be met by the market at a certain price. If the demand can't be met, the buyers or sellers might want to raise or drop the price, which is believed to cause price fluctuations. Empirically, the market depth can be calculated statistically using the transaction-by-transaction data and is shown to be negatively correlated with market volatility (Engle and Lange, 2001). Using the mechanism-based interpretation, a turbulent market with higher market volatility will have lower market depth. This is because the buyers and sellers in a turbulent market are less patient and more willing to pay extra to secure the deal (Hautsch, 2003). A simple market depth model (Eqn. (4.2)) is adopted in the current

research, whereby the market responses linearly with excess demand. Some of the excess demand can be met by the market, while the remainder will cause price fluctuation. The proportion that can't be met by the market is $1/\lambda$, with λ being the market depth. In the baseline model, the (negative) correlation between the market depth and market volatility is not explicitly modelled. Instead, the understanding from the parametric study is used to determine an appropriate market depth. From the modelling point of view, it is also important to understand the impact of the market depth to evaluate the simulation results. Table 4.1 shows how the market volatility decays with the increase of market depth. Figure 4.5, meanwhile, shows that a reduced market depth will lead to a fatter distribution tail after normalisation. It is also interesting to look at the extreme values of the price returns plotted in the figure. Those extreme values are related to the cases that all agents submit a buy or sell order at the same time step. According to Eqn. (4.2), the extreme values should be linearly correlated to the market depth λ, and this can be observed in both Figure 4.5 and Table 4.1. It is however inappropriate to directly estimate the appropriate value range of market depth from the parametric study, due to its correlation with the other two input parameters through the market volatility.

Table 4.1(a) Values of volatilities that are used in Figures 4.2.

Number of agents	$N = 500$	$N = 1K$	$N = 2K$	$N = 4K$	$N = 8K$	$N = 12K$	$N = 16K$
	0.0127	0.0107	0.0109	0.0099	0.0094	0.01	0.0098

Table 4.1(b) Values of volatilities that are used in Figures 4.3-4.5.

| Update threshold	$s = 0.005$	$s = 0.01$	$s = 0.05$	$s = 0.1$
	0.0088	0.0094	0.0147	0.0166
External information	$\sigma'=0.0005$	$\sigma'=0.001$	$\sigma'=0.005$	$\sigma'=0.01$
	0.0072	0.0094	0.0203	0.0275
Market depth	$\lambda = 5$	$\lambda = 10$	$\lambda = 15$	$\lambda = 25$
	0.0154	0.0094	0.0081	0.0068
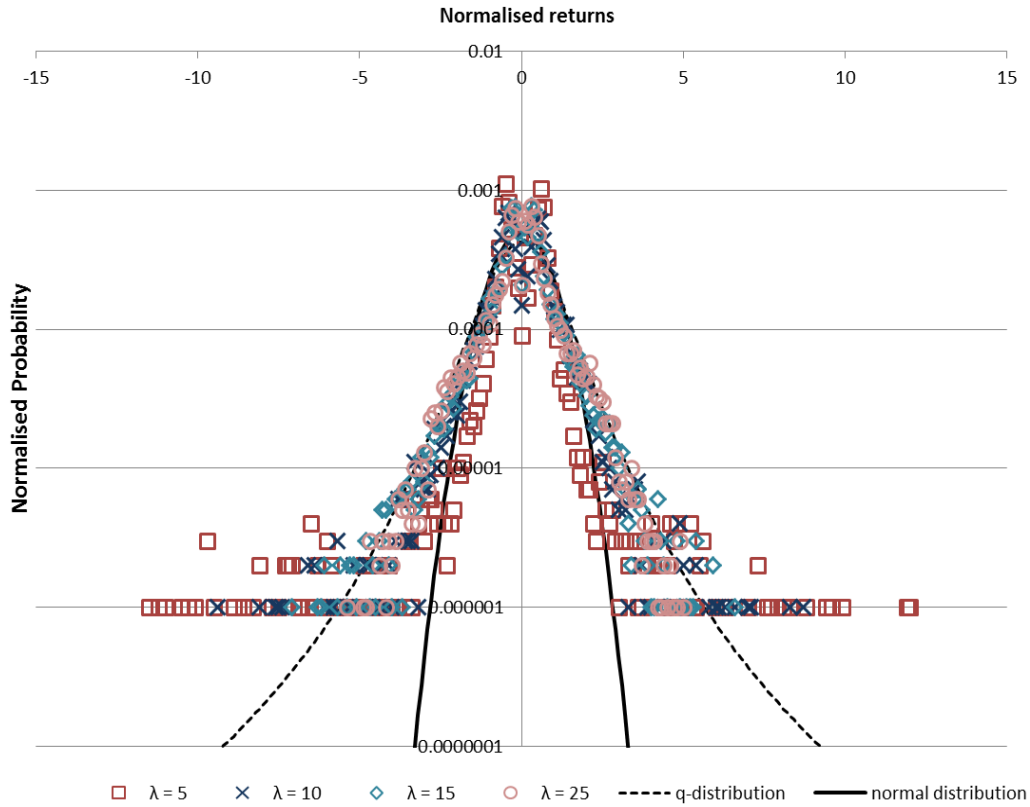
Figure 4.5 The statistical distribution that is obtained from the agent based model described in section 4.3.1. The distribution is normalised by the standard deviation of the whole time series for each case (detailed in Table 4.1). Different symbols represent different values market depth λ used in the simulation. Other parameters are: the probability of updating decisions rules for each agent: $s = 0.01$; the volatility of the external information $\sigma' = 0.001$; the number of agents: $N = 8000$. The q-distribution with $q=1.4$ and the normal distribution are also plotted.

4.5 A Multi-scale approach to model the stock market

4.5.1 The calm market and the turbulent market

The above sections provide a level of quantitative understanding to the agent based models as well as the microscopic mechanisms in the stock market. It is therefore possible to provide a controlled and quantified model to explain the phenomenological observations of the calm and turbulent periods of the markets. As introduced in section 3.3.3 (and Figure 3.5), if normalised by the volatility of the time series, the statistical distributions of the price return from the calm and turbulent periods of the market will collapse to the same shape. At the same time, the volatility of the turbulent period is much higher than that of the calm period. In terms of microscopic mechanisms, the understandings from the parametric study and the

previous research activities suggest a turbulent market is subject to a) a larger value of s, which indicates an agent is more likely to update their decision-making rules with less heterogeneity (herd effect), and leads to a higher volatility (Table 4.1) and thinner tails (Figure 4.3); b) a larger value of σ', which indicates more volatile external information and agents are more likely to perform a buy or sell activity, i.e., smaller investor inertia, and which leads to a higher volatility (Table 4.1) and thinner tails (Figure 4.4); and c) a smaller value of λ, indicating the market has less depth because the agents are less patient, and this leads to a higher volatility (Table 4.1) and fatter tails (Figure 4.5). It is therefore possible to manipulate the three input parameters to achieve the macroscopic phenomena and the magnitude of volatility while maintaining the tail shape. It is also possible to simulate the physical timescale and the range of returns and volatilities using a multi-scale approach, i.e., by comparing the microscopic simulation results and the macroscopic modelling. From section 3.3.3, the tail behaviour is characterised by the q-distribution with $q = 1.4$, for the time horizon of return $t = 1$ minute (Figure 3.5). The volatility of the time series is in the range of ~0.0025-0.005 for the calm period and in the range of ~0.01 for the turbulent period, for the time horizon of return $t = 1$ minute (Table 3.1). The above qualitative and quantitative observations are simulated using the agent based model and the results are shown in Figure 4.6. The specific input parameters are detailed in the figure caption. It can be observed in the figure that the calm and turbulent market periods are reproduced by the agent based model, using the quantified understandings about the microscopic mechanisms of the stock market.

The approach used to produce Figure 4.6 is an important one. Firstly, it provides quantitative connections between the macroscopic phenomenological models and the microscopic mechanism-based models. Although those mechanisms are modelled in a simplified manner, it proves that we have captured those essential ingredients leading to the stylised facts from the stock market. Secondly, unlike much previous agent based research, the current approach starts with a systematic understanding from the empirical data. It proves that the macroscopic data and modelling contain rich information, which is essential in terms of pinning down the effect of the individual mechanisms, as well as in fine-tuning the input parameters. Finally, the multi-scale modelling approach used here provides a way of forecasting the future evolution of the stock market. For example, the trading activities of the market may evolve dramatically after a financial crisis. Decisions from both the trading institutions and the regulators may impact the mechanisms instantly, but may only manifest themselves market-wide after a long period of time. It is difficult to use phenomenological modelling alone to

predict the effect of those changes. By introducing a mechanism-based approach, those decisions can be evaluated in a more controlled and justifiable manner. It might be worth noticing that the current agent based modelling is a simplified one; it is also possible that the sets of input parameters used to produce the market calm and turbulent periods are not unique. Extensive empirical research on markets' microscopic parameters is required to understand more about these factors.
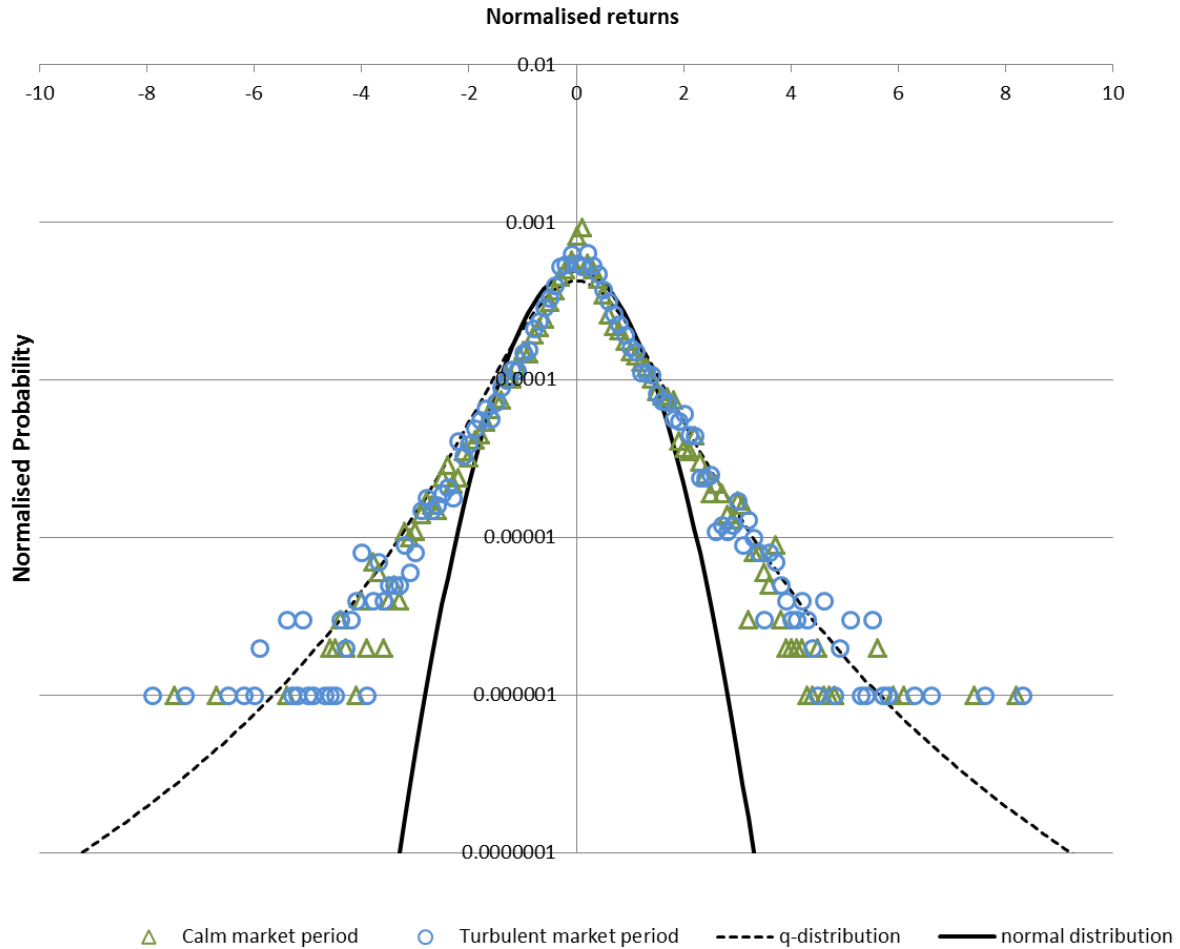
Figure 4.6 The simulation results for a calm and a turbulent market period, using the agent based model described in section 4.3.1. The distribution is normalised by the standard deviation of the whole time series for each case, the value for the calm period is 0.0046, the value for the turbulent period is 0.0094. Other parameters are: market calm period: the probability of updating decisions rules for each agent: $s = 0.005$; the volatility of the external information $\sigma' = 0.0005$, market depth $\lambda = 20$; the number of agents: $N = 8000$. Market calm period: the probability of updating decisions rules for each agent: $s = 0.01$; the volatility of the external information $\sigma' = 0.001$, market depth $\lambda = 10$; the number of agents: $N = 8000$. The q-distribution with $q=1.4$ and the normal distribution are also plotted.

4.5.2 The diffusion process

The diffusion process of the price return distribution simulated from the agent based method is shown in Figure 4.7. Due to the scalable behaviour of the statistical distribution shown in Figure 4.6, only the turbulent market is used to study the diffusion process here. The different time horizons of returns are generated by summing across different time horizons using Eqn. (3.18), and are then normalised by the volatility of each time series. It can be seen from Figure 4.7 that the agent based model shows statistical distributions following an anomalous diffusion process, where the distribution shape changes with time. As discussed in section 3.3, such an anomalous diffusion process can be characterised by the q-distribution, with the q values following Eqn. (3.10). Specifically, the q values vary from $q = 1.4$ for the one-minute return, to almost the normal distribution with $q = 1$ for the weekly return. This is consistent with the understanding from empirical data detailed in section 3.3, and is also in quantitative agreement with Eqn. (3.10) and Figure 3.8. Such a diffusion process can also be represented by the evolution of the volatility. As proposed in section 3.3, the anomalous diffusion process in the stock market is characterised by the relationship between the volatility and the time horizons of return t:

$$\sigma^2 \sim t^{2/(3-q)}.$$ (4.5)

The simulated volatility values are plotted in Figure 4.8, for different time horizons of returns. It can be observed from the figure that the slope of the log-log plot is very close to 0.5, for both calm and turbulent market periods. According to Figure 3.16, the power law plot with the constant factor 0.5 is a good approximation of Eqn. (4.5), for the region 1 minute $\leq t$ ≤ 1 week. Consistent to studies performed in this thesis so far, no such phenomenon of "super-diffusion", i.e., with a power law factor in the region of ~0.7, is observed in the agent based simulation results.
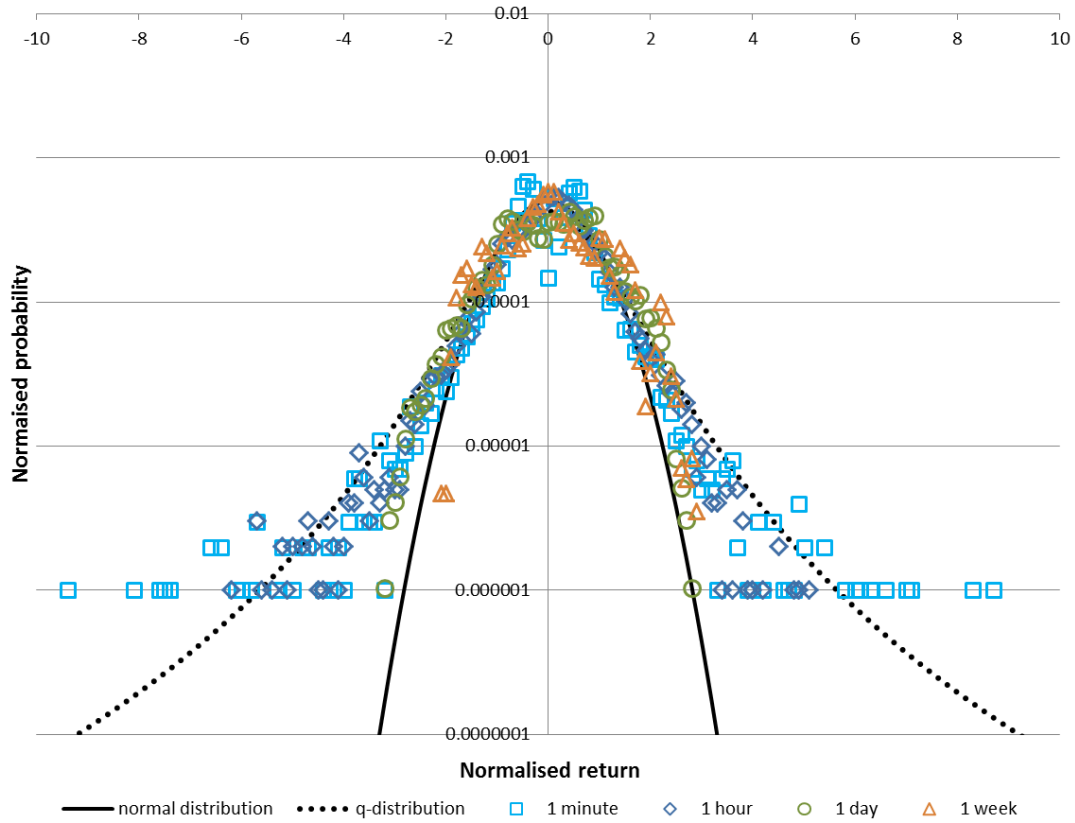
Figure 4.7 The statistical distributions for different horizons of return periods normalised by the volatility of each time series. The price return time series are simulated using the agent based model, with the parameters used in producing Figure 4.6 (turbulent period). The q-distribution with $q=1.4$ and the normal distribution are also plotted.
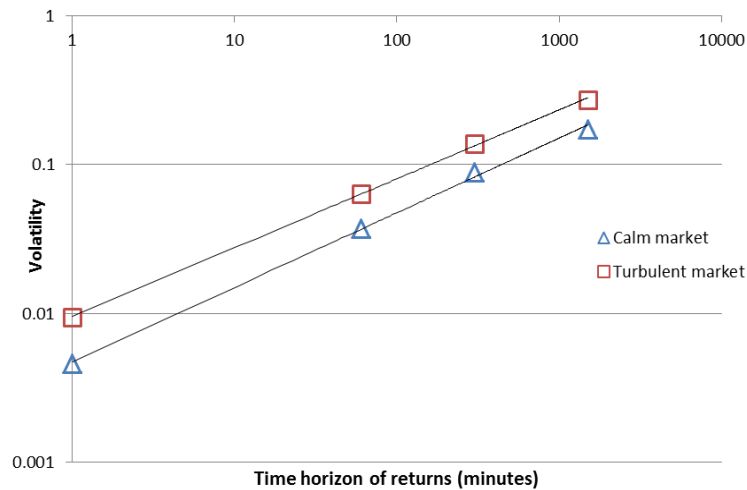
Figure 4.8 The values of the volatilities plotted against the different time horizons of returns. The price return time series are simulated using the agent based model, with the parameters used in producing Figure 4.6. The best fit lines represent the power law diffusion process with factor 0.5 ± 0.05 for the calm market while 0.46 ± 0.05 for the turbulent market.

4.5.3 The higher order factors and the correlations

As demonstrated in the above sections, agent based modelling is an excellent approach in terms of capturing the statistical behaviours of the price returns. At the same time, it provides some quantitative agreement with the existing macroscopic models and enables a multi-scale approach. This section will study the correlations and the higher-order factors that are sensitive to more detailed market microscopic mechanisms. It can be seen from the study performed in this section that the agent based model is still able to capture the stylised facts introduced in section 2.3: the short term correlation of price return; the volatility clustering; and the long term autocorrelation of volatility and absolute return. It can, however, be seen that the relationship between the physical time and the simulated time step determined from the multi-scale approach fails to agree with existing empirical findings, suggesting that more research effort is required to understand the more detailed market mechanisms. For this reason, the discussions in this section will remain qualitative; no parametric study will be performed. The market turbulent case studied in section 4.5.1 is used in this section as a baseline scenario. The memory effect of the market is shown in Figure 4.9, where the autocorrelation of price returns is plotted against different time lags. It can be observed that no memory effect exists for the simulated results. This can be understood from Eqn. (4.3), in that the agents' decisions on buy or sell depend on either the external information flow, which is uncorrelated, or the updated decision-making threshold, which is only related to the previous time step (latest price return). The lack of autocorrelation of price return is qualitatively in agreement with the empirical findings regarding short memory of the stock market. However, section 2.3 shows that there is a weak autocorrelation of the time series that decays within several minutes. By introducing a short memory to Eqn. (4.3), such a memory effect can be controlled in the model, in a similar way of controlling the memory effect in the ARCH/GARCH model (section 3.4). This will, however, require additional parameters in the model, which in turn demands further microscopic market research.

Figure 4.10 shows the time series of the simulated volatility, which is calculated as a moving average for every 300 time steps. The volatility clustering can be observed in the figure which is qualitatively in agreement with the empirical findings. A period of high price fluctuation will cause the agents to update their decision thresholds to a more divergent range which will lead to increased heterogeneity of the decision-making rules. According to understanding from the parametric study, a higher heterogeneity will gradually transform the market to a lower volatility period. The rate of such transformation is characterised by the

frequency with which the agents update their decision-making rules, i.e., $1/s$. By contrast, a period of low volatility will gradually transform to a period with high volatility, due to a similar effect of reduced heterogeneity. Such a mechanism guarantees that the persistent price fluctuation and the clustering of volatility, which is in agreement with the empirical findings. As a measure of the volatility clustering, the autocorrelations of the volatility and the absolute price return are plotted in Figure 4.11. It can be seen that the agent based modelling is capable of producing a long term autocorrelation for both the volatility and the absolute return. However, both plots shown in Figure 4.11 follow an exponential decay with the increase of time lag. This is inconsistent with the empirical findings from the stock market. We will demonstrate in a later context that the simplified model used in the current research requires refinement in capturing some other mechanisms which may also impact upon the higher order factors and correlations.
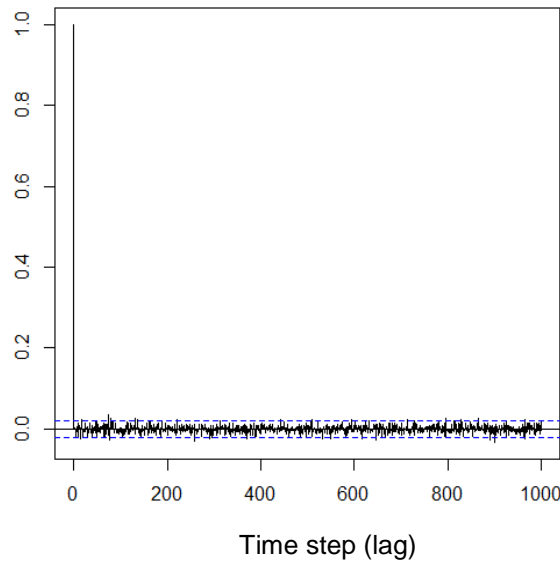
Figure 4.9 The autocorrelation function of price return, for the case of turbulent market in Figure 4.6.

4.5.4 Other microscopic mechanisms

One mechanism that has not been considered yet is the market microstructure discussed in section 4.2.3. In the baseline model, all agents are of the same size and the same buying power. The size effect of agents can be introduced into the baseline model by assigning a random size to each agent, which determines their buying/selling power. As a simplified treatment, if an agent submits a buy-or-sell order, the volume of the order will be

proportional to its size. We model the initial size of the agent ω_i as a random variable following the exponential distribution:
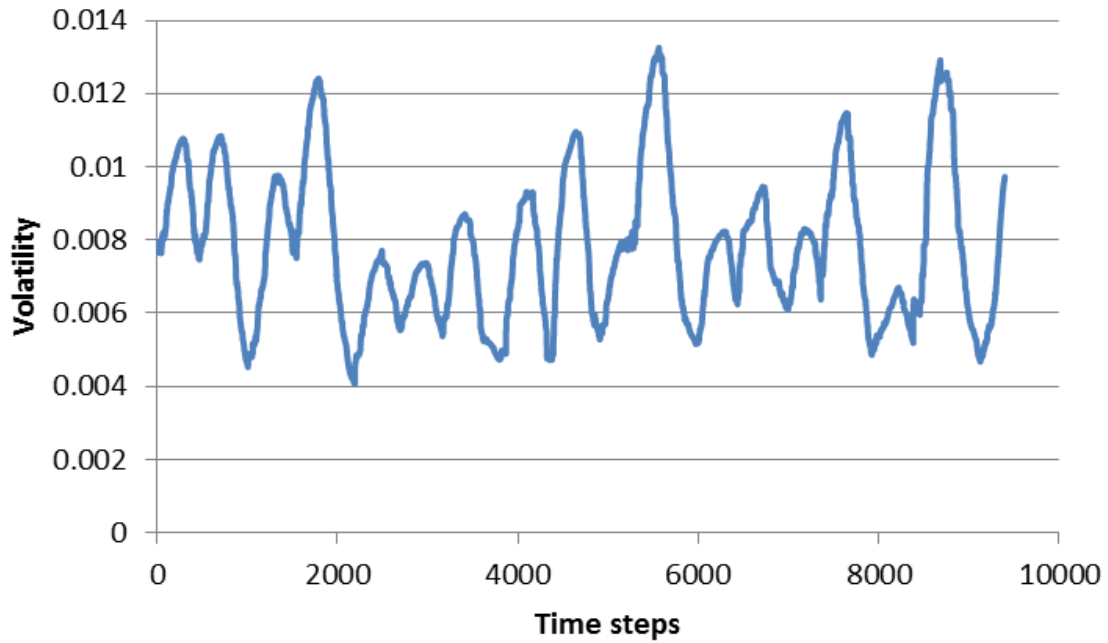
Figure 4.10 The time series of volatility calculated per 300 time steps, and plotted against the simulated time steps, for the case of turbulent market in Figure 4.6.
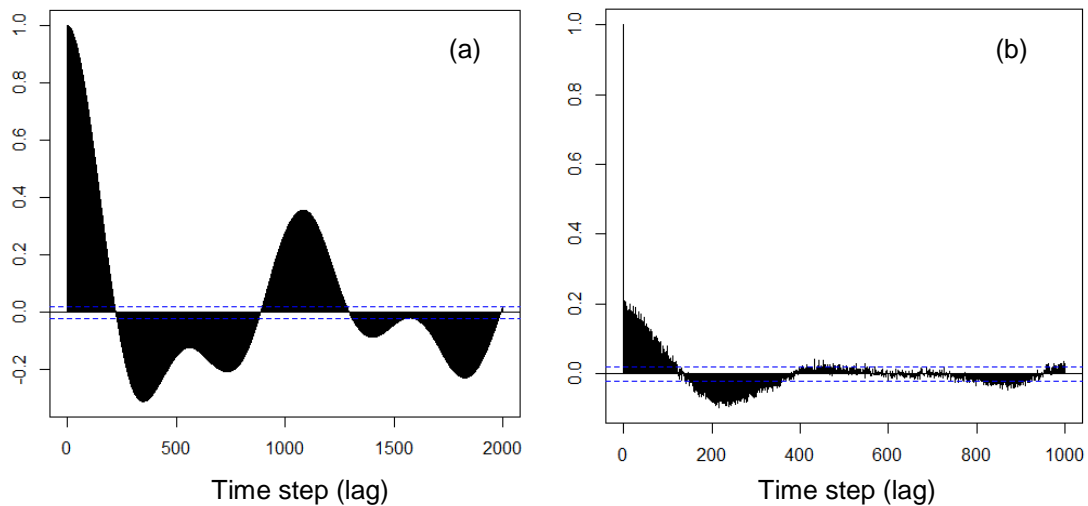
Figure 4.11 The autocorrelation function of (a) volatility, and (b) the absolute value of price return, for the case of turbulent market in Figure 4.6.

$$P(\omega_i) = \exp(-\omega_i / \varpi)/\varpi, \tag{4.6}$$

in which ϖ is a scalar representing the homogeneity of the agent size distribution, with all agents the same size when $\varpi \to 0$. After including the size effect, Eqn. (4.1) becomes:

$$d(t) = \sum_{i=1}^{N} \omega_i \phi_i(t). \tag{4.7}$$

The numerical experiments incorporating Eqn. (4.7) in the baseline model are performed and the results plotted in Figure 4.12. We choose the scalar $\varpi = 10$, indicating that the median size of the agents is 0.7; meanwhile, the mean size is 1, to enable comparison with the baseline model in which all the agents will have the same size of 1. The simulated market consists of a large number of small-size agents and a small number of large-size agents. As shown in Figure 4.12, the simulated market including the agents' size effect acts similarly with the market with agents of the same size, in terms of both the distribution shape and the volatility of the price return. It is, however, noted that the size effect of agents will have an impact on the market liquidity, hence the market depth (Biais et al., 2005). The market will react with high volatility whenever those large agents decide to buy or sell. According to section 4.4.3, this will lead to a fatter distribution tail and smaller volatilities. An interesting finding of the size effect is shown in Figure 4.13, in which the decay rate of the long term autocorrelation of the price return is much slower after the size effect is introduced into the model. As discussed earlier, the long term autocorrelation is linked with volatility clustering, which is more sensitive to detailed market microstructure. Instead of directly impacting on the first-order factors such as price returns and the diffusion, the size of agents seems to have an impact on higher-order factors, such as the long term correlations. It might be worth noting that the modest impact from the agents' size heterogeneity could be due to the fact that the interactions between agents are not modelled explicitly, but rather through the feedback loop. More microscopic empirical research is required to understand the size effect of the investors.

Another mechanism that is worth mentioning here is the psychological bias when agents make their investment decisions. As discussed by a number of previous studies (e.g., Kahneman and Tversky, 1979; Daniel et al., 1998), investors often make asymmetrical decisions while responding to market movements. They are more likely to change their

decision-making rules after negative returns than positive returns. We model such biases by making the threshold value s of updating the decision-making rules after negative returns twice the value of that after positive returns (similar values are used by Yang et al., 2005):

$$s = \bar{s}, \text{ if } S(t) >= 0 \text{ and } s = 2\bar{s}, \text{ if } S(t) < 0, \tag{4.8}$$
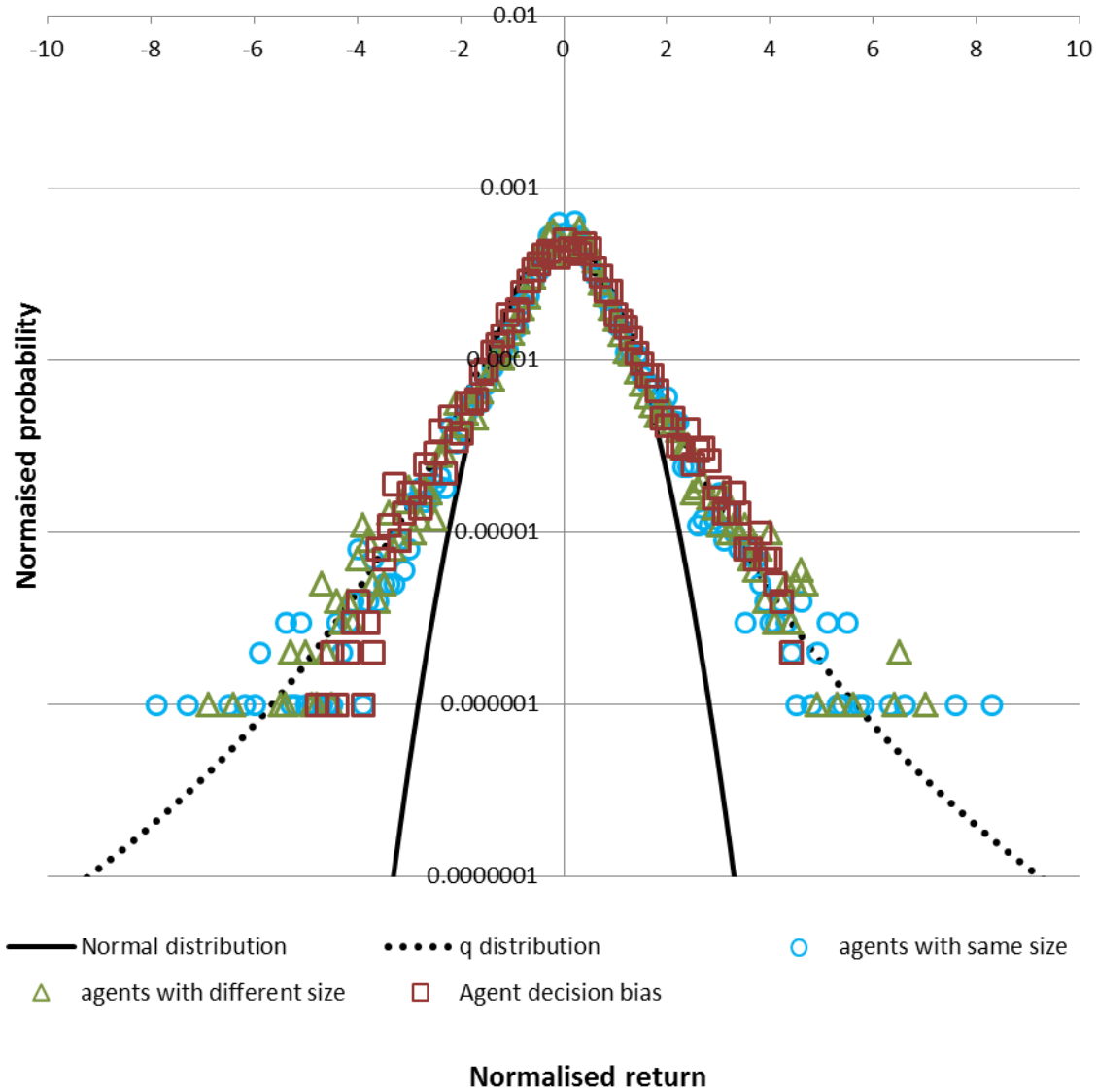
Figure 4.12 The impact of size effect and asymmetric decision bias using the agent based model. The input parameters are the same with the market calm period case in Figure 4.6. The volatility values used for normalisation are: 0.0046 for the case agents with same size but no size effect or decision bias; 0.0043 for the case agents with different size but no decision bias, and 0.0044 for the case agents with decision bias but no size effect.

in which $\bar{s}$ is a positive number governing the step function Eqn. (4.8). The simulation results of incorporating Eqn. (4.8) in the baseline model are plotted in Figure 4.12, with the value $\bar{s}$ = 0.0033. The choice of the parameter is to make the average probability that agents update their decision-making rules s = 0.005, which is the same as the input value to simulate the calm market period. As shown in Figure 4.12, the introduction of the agents' decision bias will change the shape of the statistical distribution, but not the overall volatility of the market. However, such bias is not an essential ingredient to generate the fat-tail distribution or the long term correlation. It is also noted in Figure 4.13 that by introducing the decision bias of the agents, the decay rate of the autocorrelation function becomes slower when compared with the baseline model. This is similar to the size effect of the agents: the higher-order statistical factors are more sensitive to detailed microscopic market mechanisms.
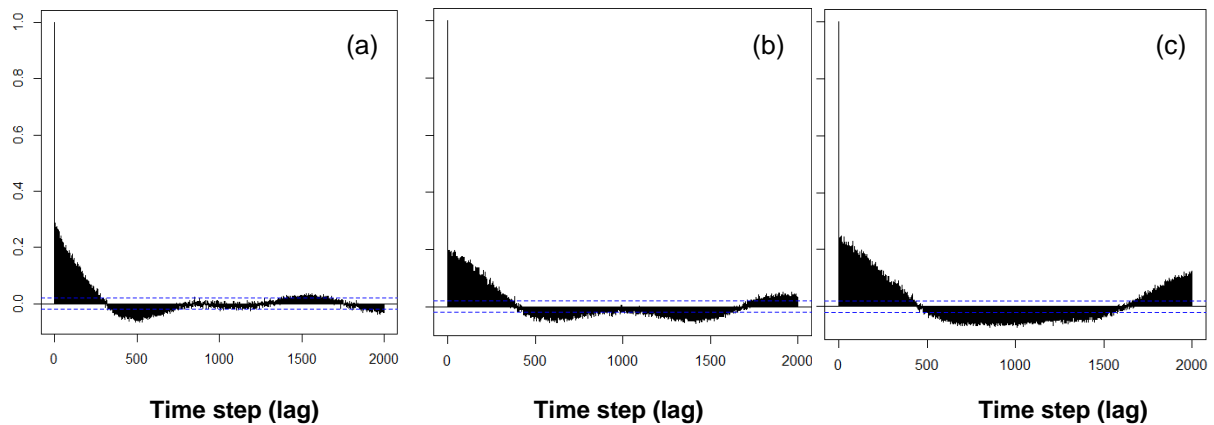
Figure 4.13 The autocorrelation function of the absolute value of price return, for the case of (a) same agent size, (b) different agent size, and (c) agents' decision update bias, which are plotted in Figure 4.12.

4.6 Summary of this chapter

The understanding of the micro-foundations of the financial market is a popular area that has drawn extensive research efforts in recent years. The agent based method adopts market-specific mechanisms, and it is demonstrated that it can produce stylised statistics, as discussed in the previous chapters. A number of agent based models are reviewed in the current chapter together with the understanding of the microscopic models. We start with a simple agent based model with only the essential ingredients required to produce the fat-tail

distribution and the long term correlations. A baseline numerical scheme is established and validated against empirical data that are discussed in Chapter 2. The results of the baseline model are also shown to be consistent with the macroscopic understandings from Chapter 3. With the help of the agent based model, we hereby attempt to answer the questions asked in the beginning of this chapter: 1) the non-Gaussian statistics are demonstrated to originate from the combined effect of the feedback loop, the investor inertia, and the heterogeneous decision-making process of the agents; 2) the trading activities of investors generates excess price fluctuations that can't be explained by external information alone; 3) the change in decision-making behaviours may lead to changes in microscopic factors, which may be difficult to measure by traditional modelling techniques, but can be studied using the agent based method. The higher-order factors of the financial market (e.g., volatility and its correlation) are more sensitive to changes in agents' behaviours than the first-order factors (e.g., the statistical distributions of the price return and its diffusion process); 4) the volatility clustering and the long term correlation of volatility and absolute returns are connected with the essential ingredients that are required to generate the fat-tail distribution, as well as other mechanisms such as agent size and the decision bias; 5) the microscopic factors will impact on the market volatility while the shape of the (normalised) statistical distribution and the diffusion process are maintained, both of which are deemed as key characteristics of the calm market period and turbulent market period.

In this chapter, we have also proposed a multi-scale modelling approach that can be used to quantitatively evaluate the impact of microscopic mechanisms. Due to complications of the real market, the mechanisms considered in an agent based model will always include some form of abstract. The realistic microscopic data obtained from the empirical market study may not be suitable to be directly fed into the agent based model. Instead, the large quantity of macroscopic data can be used to back-calculate and validate the input parameters. For example, the physical timescale and the numerical time step can be made relevant using the q-distribution introduced in Chapter 3. Such a multi-scale approach has demonstrated its capacity by successfully replicating the empirical findings and can be used to quantitatively study the first-order factors. However, it is also shown that the higher-order factors can be only modelled in a qualitative manner. This suggests that other microscopic mechanisms may be needed to model those factors, illustrated by introducing the size effect and the decision bias effect.

The multi-scale financial modelling framework proposed in this chapter has got a number of advantages. Firstly, while the market efficiency may be difficult to identify from the market (macroscopic) point of view (Lo and MacKinlay, 1999; Sewell, 2012), those "hallmarks" of market efficiency can be identified by observing the microscopic perspectives especially investors behaviours (Bikhchandani and Sharma, 2001; Sager and Taylor, 2006). Secondly, while the microscopic modelling is a strong tool to understand the market mechanisms, the validity of the inter-correlated microscopic models and associated factors is difficult to justify. The market-wise (macroscopic) data is often required to validate and calibrate those models, which might be different from market to market, and period to period. Thirdly, the multi-scale approach strikes a balance between the complication of modelling human behaviours and the simplicity required in terms of model implementation. A test can be carried out to understand whether some microscopic complications have got material impact for different financial applications. Finally, the flexibility of the microscopic simulation means it is able to take advantages of the latest modelling advancement of market microstructure and individual dataset. Such bottom-up approach ''starts from the investors, and then tries to understand how the market (macroscopic) properties emerge from the interactions" (Grimm 1999). Consequently, this multi-scale approach can efficiently deal with many research needs that traditional approaches may find difficult or impossible to deal with, and may 'provide more accurate predictions' (Li et al., 2005).

From the management point of view, the risk management practices will greatly benefit by the extra dimension provided by the multi-scale approach. There is a gap between the traditional risk management using a bottom-up approach to escalate risk from the operational level (and often based on local knowledge and industrial experiences), and the modern risk management using a top-down approach to understand the corporate risk using data-driven methods (e.g. VaR). The microscopic modelling provides a way to bridge this gap by taking into consideration microscopic risk indicator before the risk manifested itself and materialised at the macroscopic domain. This will effectively avoid the "false sense of security" that is discussed in Chapter 1. On the other hand, the microscopic modelling provides an insight of "forward looking" to the "unpredictable market", by understanding the mechanisms of the inter-connected microstructures. It is important when making managerial decisions such as resource, technology, and policy. For example, when decisions are made to modify a certain component of the market structure, e.g. changing tick size or introducing off-market trading, the impact on the whole market might not be intuitive due to the

complicated interactions between different components. It would be too late that those market-wise indicators can be observed In contrast, the agent based modelling provides a way of "experimenting" different scenarios regarding different managerial decisions hence much stronger "justification" to the stakeholders and investors. An example is given in the next chapter specifying a case study in terms of the impact of price cap on a stock market.

Chapter 5 Conclusions and Future work

5.1 Conclusions of the current research

Modelling the financial market may be one of the most challenging tasks in the financial sector and academic research. On the one hand, the financial market is a complicated chaos system which is not only governed by a large number of non-linear interactions and feedback loops, but also an open system subjecting to evolvement of economical climates, sector specific information, as well as the financial market players. On the other hand, the application oriented financial engineering require some simple, quantified and non-sticky measures of the market, which are also deemed by those stakeholders with non-mathematical background as credible and transparent. Mathematical modelling, while is heavily relied-on by the modern financial industry, has also been assigned much social-political implications such as liability, leadership, regulation and trust. The ideology of the risk based decision making are so popular that it moves beyond the boundary of the financial industry to other sectors (utility, see Pollard et al., 2004; health sector, see Montgomery et al., 2000). From the fundamental research point of view, the efficient market hypothesis has been carrying too much duty than it is originally designed to. Even the risk management frameworks, which are created to capture the information missed out by the simplified risk-based decision tools, are also increasingly influenced by the efficient market hypothesis (e.g. Value at Risk). Meanwhile, there are voices from the academics, especially after the financial crisis 2007-2008, that "plural and conditional expert advice" is not only beneficial but also essential during the decision making processes (Stirling, 2010). Understanding the microscopic mechanisms is in a similar thinking stream of the above arguments: instead of only based on statistical data and back-tests, we dive into some detailed subject matter information using domain specific languages. The multi-scale modelling approach proposed by the current thesis is therefore particularly important: a) the agent based method provides a platform to incorporate expert opinion in the world dominated by modelling based decision making; b) the non-Gaussian models provide an interface between the microscopic understandings and the industrial practices; c) a combined approach provides a framework of validating and

quantification while maximising the information from both macroscopic and microscopic perspectives; and d) the systematic empirical data analyses provide a framework obtaining the input parameters of the models at the same time understands the context of modelling applicability.

What is difficult to convey is the importance of performing the above studies from the first principles using a systematic approach. The statistical analyses in Chapter 2 follow a standard method. It is however demonstrated that the results of the statistical analysis can be heavily reliant on the details of the dataset and the analytical fashion. The current research compares different stock markets, different time horizons of return, and different periods of the market (by separating the calm and turbulent periods). It is discovered that, not only the fat-tail distribution prevails universally; the shape is also dependent on the time horizons of returns but not specific market information or whether the market is experiencing turbulence. What is more, the "super-diffusion" (time scale governed by $\sim t^{-0.7}$) that is claimed by a number of previous literatures might be due to mixing data of different periods therefore a statistical artefact. Another possibility of super-diffusion is from statistical analyses using extremely small time horizons of returns (such as tick-by-tick data). This is a consistent finding throughout the current research using a number of different modelling techniques (ARCH/GARCH, agent based simulation, etc.). The empirical analyses inspire us to use a data transformation technique of normalisation, which have greatly reduced the statistical effort required for mathematical modelling. More importantly, while the agent based modelling is still a less-well-established area, the proposed multi-scale modelling approach requires minimum input parameters by using the understanding of empirical data and its parametric transformations. In Chapter 3 we have examined the classical non-Gaussian statistics such as Lévy distribution and Tsallis distribution. Our main contributions include: 1) the definition of the Lévy distribution tail and the speed of convergence; 2) the explanation to some controversial empirical findings in the literature while quantifying the power law distribution; 3) relating the distribution shape with the q value in the Tsallis distribution; and 4) the quantification of the anomalous diffusion and an empirical model with validations. This chapter also provides some quantitative linkage between the macroscopic phenomena and microscopic insight, which highlights the importance of understanding the micro-foundations of the stock market. Chapter 4 adopts the agent based method for a microscopic simulation approach. It is demonstrated that the non-Gaussian statistics is a result of combined effects of feedback loop, decision heterogeneity, and the market depth. A

considerable amount of effort in Chapter 4 is dedicated to the model validation and parametric studies, at the same time utilising as much as possible the understandings from the empirical analyses and macroscopic modelling. This is naturally a multi-scale approach, which is demonstrated to be powerful to establish a quantitative multi-scale model explaining the differences between the calm periods and the turbulent periods of the market.

Implementing such a multi-scale approach in the financial industry is however far from straightforward. The agent based modelling requires relatively complicated numerical schemes. Despite in theory as many microscopic mechanisms can be incorporated in the numerical algorithm, it is difficult to obtain input parameters directly from the market or by performing empirical analyses alone. A number of input parameters require back-calculation from the simulation results due to either data unavailability or simplistic assumptions made to establish microscopic models. The aim of the current research is not to re-create another Santa Fe (artificial) Stock Market, but to use the agent based method to conduct fundamental research towards the statistical properties of the stock market, at the same time to instruct the development of the macroscopic models. To achieve this objective, the financial industry needs to recognise the skill sets and operational processes that are required by performing studies similar to those presented in the current thesis, i.e. both data science and modelling expertise, together with subject matter knowledge. When decisions are made, it is important to take into consideration not only the modelling results and modelling assumptions, but also opinions from someone who understands the statistical details and modelling details, in order to provide assurance that the mathematical techniques are applied within their designed limitations.

5.2 Future research work

Following the discussion in section 4.5.1, one of the immediate applications of the current multi-scale approach is to test whether imposing a macroscopic and market-wise trading rules, such as a regulatory constraint, will be effective and produce desired outcome. This type of exercise generally does not demand accurate modelling outputs (ballpark figures are normally acceptable), while does not require difficult-to-obtain modelling inputs such as detailed market microstructure or the investor's irrational behaviours. A case study using the current research framework is proposed here, to evaluate the effect of introducing the daily

cap of return in the equity market. As discussed in section 2.3, a 10 percent cap (for maximum daily return, on a stock by stock basis) is imposed in the Shanghai Stock Exchange to stabilise the volatility. By limiting the size of the rare events, the risk of (daily) maximum loss can be managed in a controlled manner. It is however a controversial policy that due to the efficient market hypothesis, human interventions are always less efficient than the market itself. Based on established understandings on the microscopic mechanisms, such daily cap will stop the daily price from excessive fluctuation. However, this will lead to smaller investor inertia, due to a smaller threshold value to buy and sell. Plus, the investors are more likely to trade in the next trading day if their order cannot fulfilled due to the cap, leading to more demand for market liquidity hence impact to the market depth. According to section 4.4, both the smaller investor inertia and a smaller market depth will cause higher volatilities, which is obviously not the intention of introducing the price cap. It is however extremely difficult to use empirical analysis on the market data to evaluate the impact of the complicated interactions of a number of mechanisms, while it is a clear advantage of using the agent based simulation. For the baseline model used to produce the turbulent market and calm market cases in Figure 4.6, the daily return can be calculated for the asset price. When an accumulative price return reaches ±10% within a single day, the market will be closed until the next trading day. The results can be compared with the case study without the market cap, for both calm and turbulent periods of the market. The impact on the market liquidity from the cap can be also evaluated by varying the market depth value in the agent based model, at the same time compare with the empirical data obtained from the Shanghai Stock Exchange.

Another possible direction of future work is to carefully introduce other microscopic mechanisms to the agent based model. The goal of including such mechanisms is to evaluate the impact of those mechanisms instead of forecast the financial returns. For example, it is easy to introduce agent specific information such as cash constraints or performance measures. As discussed in section 4.5.3, those mechanisms will be more likely to affect those more sensitive higher order factors. The trading volume, which is another macroscopic datasets in the stock market, is not directly studied in the current research. It contains similar message with volatility on level of information arrived to the market, while also reflects some microscopic message such as market liquidity and market depth. The trading volume data is expected to provide an additional information source to facilitate back calculation and validation, at the same time pose challenges to parametric study and quantitative agreements.

This type of work however will not be possible without optimised numerical code and enhanced computational power (such as cluster computational methods using, e.g. MPI (message passing interface) library. The third example of the future work is refining the financial instruments by using the q-distribution together with the aid from mechanism based modelling. The q-distribution only introduces a single additional parameter to the classical Gaussian model therefore with promising applicability. One significant piece of work is to quantitatively connect the value of q with explicit financial microscopic mechanisms, together with standard statistical treatment of macroscopic or microscopic data. On the one hand, the q-distribution is capable of capturing the rare events in the market hence a better representative of financial risks. On the other hand, a better informed value of q will enable the models better capture the market specific information such as turbulence. It is expected that the value of q can be obtained by the similar way to the volatility value σ, through either statistical analysis or through implied methods using the trading price of the financial derivatives.

References

Adib A., (2009). Performance of the VIX index as a volatility predictor. Online Source. https://arturscorner.wordpress.com/2009/09/12/vix-silliness/

Akgiray, V., (1989). 'Conditional heteroscedasticity in time series of stock returns: Evidence and forecasts.' *Journal of business*, 62(1): pp.55-80.

Alligood, K.T., Sauer, T.D., Yorke, J.A. and Crawford, J.D., (1997). Chaos: an introduction to dynamical systems. *Physics Today*, *50*, p.67.

Andersen, T.G. and Bollerslev, T., (1998). 'Answering the skeptics: Yes, standard volatility models do provide accurate forecasts.' *International economic review*, 39(4): pp.885-905.

Andersen, T.G., Bollerslev, T., Diebold, F.X. and Labys, P., (2003). 'Modeling and forecasting realized volatility.' *Econometrica*, 71(2):pp.579-625.

Arthur W., Holland J. H. , LeBaron B., Palmer R.G. and Tayler, P. (1997). 'Asset pricing under heterogeneous expectations in an artificial stock market.' In Arthur, W.B., Durlauf, S.N. and Lane, D.A. (eds.) *The Economy as an Evolving Complex System*, Reading, MA: Addison-Wesley, pp15-44.

Ashby A., (2010). 'The 2007-09 Financial Crisis Learning the Risk Management Lessons.' *Financial Services Research Forum*, The open University, BB841 Managing financial risk, January 2010.

Baillie, R.T. and Bollerslev, T., (1992). 'Prediction in dynamic models with time-dependent conditional variances.' *Journal of Econometrics*, 52(1): pp.91-113.

Baker, H. K., (2009). *Dividends and dividend policy*. New Jersey: John Wiley & Sons.

Barberis, N. and Thaler, R., (2003). A survey of behavioral finance. In Bruce L. Gardner Gordon C. Rausser *(eds.) Handbook of the Economics of Finance*, *1*, pp.1053-1128.

Bartram, S.M., Brown, G. and Stulz, R.M., (2012). 'Why are US stocks more volatile?' *The Journal of Finance*, *67*(4): pp.1329-1370.

Basel Committee on Banking Supervision, (2006). Basel II: International Convergence of Capital Measurement and Capital Standards A Revised Framework Comprehensive Version. Available at http://www.fsb.org/2006/06/cos_060601/, accessed on the 1st April, 2013.

Bayraktar, E., Horst, U. and Sircar, R., (2006). 'A limit theorem for financial markets with inert investors.' *Mathematics of Operations Research*, 31(4): pp.789-810.

Beck, U., (1992). *Risk society: Towards a new modernity (Vol. 17)*. London: Sage.

Becker, B. and Milbourn, T. (2011). 'How did increased competition affect credit ratings?' *Journal of Financial Economics*, *101*(3): pp.493-514.

Beckers, S., (1981). 'Standard deviations implied in option prices as predictors of future stock price variability.' *Journal of Banking & Finance*, 5(3):pp.363-381.

Biais, B., Glosten, L. and Spatt, C., (2005). 'Market microstructure: A survey of microfoundations, empirical results, and policy implications.' *Journal of Financial Markets*, 8(2):pp.217-264.

Bikhchandani, S. and Sharma, S. (2001), "Herd Behavior in Financial Markets" *IMF Staff Papers* 47(3): pp279-310.

Bingham, N.H. and Kiesel, R., (2013). *Risk-neutral valuation: Pricing and hedging of financial derivatives*. London: Springer Science & Business Media.

Black, F. and Scholes, M., (1973). 'The pricing of options and corporate liabilities.' *The journal of political economy*, 81(3): pp.637-654.

Bodie, Z., (1976). 'Common stocks as a hedge against inflation.' *The Journal of Finance*, *31*(2): pp.459-470.

Bollerslev, T., (1986). 'Generalized Autoregressive Conditional Hetero-skedasticity.' *Journal of Econometrics*. 31(3): pp.307–327.

Bollerslev, T., Chou, R.Y. and Kroner, K.F., (1992). 'ARCH modeling in finance: A review of the theory and empirical evidence.' *Journal of econometrics*, *52*(1-2): pp.5-59.

Bologna, M., Tsallis, C., Grigolini, P., (2000). 'Anomalous diffusion associated with nonlinear fractional derivative Fokker-Planck-like equation: Exact time-dependent solutions.' *Physical Review E*. 62(2): pp. 2213.

Boehmer, E. and Wu, J., 2009. Short selling and the informational efficiency of prices. Unpublished manuscript. https://www.researchgate.net/profile/Ekkehart_Boehmer/publication/228971875_Short_Selling_and_the_Informational_Efficiency_of_Prices/links/54d551bd0cf24647580765b5.pdf

Borland, L., (1998). 'Microscopic dynamics of the nonlinear Fokker-Planck equation: A phenomenological model.' *Physical Review E*, *57*(6), p.6634.

Borland, L., (2002a) 'Option Pricing Formulas Based on a Non-Gaussian Stock Price Model.' *Physical Review Letters*, 89(9): 098701 1-4.

Borland, L., (2002b). 'A theory of non-Gaussian option pricing.' *Quantitative Finance*. 2(6):pp415-431.

Bouchaud, J.P. and Potters, M., (1997). *Théorie des risques financiers*. Commissariat à l'' énergie atomique.

Bouchaud, J.P. and Potters, M., (2003). *Theory of financial risk and derivative pricing: from statistical physics to risk management*. Cambridge: Cambridge university press.

Bowe, M. and Domuta, D., (2004). 'Investor herding during financial crisis: A clinical study of the Jakarta Stock Exchange.' *Pacific-Basin Finance Journal*, *12*(4): pp.387-418.

Braunsterin J., (1969). 'States, Indistinguishability, and the Formula $S = k \ln W$ in thermodynamics.' *Journal of Chemical Education*, 46(11): 719-721.

Brav, A., Graham, J.R., Harvey, C.R. and Michaely, R., (2005). 'Payout policy in the 21st century.' *Journal of financial economics*, *77*(3): pp.483-527.

Breidt, F.J., Crato, N. and De Lima, P., (1998). 'The detection and estimation of long memory in stochastic volatility.' *Journal of econometrics*, *83*(1): pp.325-348.

Brissaud,J. (2005).The meanings of entropy. *Entropy*, 7(1), pp.68-96.

Brunnermeier, M.K. and Nagel, S.,(2004). 'Hedge funds and the technology bubble.' *The Journal of Finance*, *59*(5): pp.2013-2040.

Buiter W.H. (2007). *'Lessons from the 2007 Financial Crisis.'* CEPR Discussion Paper No. DP6596. Available at SSRN: http://ssrn.com/abstract=1140525.

Bürgisser, P., Clausen, M. and Shokrollahi, A., (2013). *Algebraic complexity theory*. London: Springer Science & Business Media.

Carlson, Mark A. (2007). *'A Brief History of the 1987 Stock Market Crash with a Discussion of the Federal Reserve Response'*. FEDS Working Paper No. 2007-13. Available at SSRN: http://ssrn.com/abstract=982615or http://dx.doi.org/10.2139/ssrn.982615

Chang, E.C., Cheng, J.W. and Khorana, A., (2000). An examination of herd behavior in equity markets: An international perspective. *Journal of Banking & Finance*, *24*(10):pp.1651-1679.

Chen, W., (2006). 'Time–space fabric underlying anomalous diffusion.' *Chaos, Solitons & Fractals*, 28(4): pp.923-929.

Chiang, S.M., Lin, C.T. and Huang, C.M., (2013). *'The Relationships Among Stocks, Bonds and Gold: Safe Haven, Hedge or Neither?'* In Diversity, Technology, and Innovation for Operational Competitiveness: Proceedings of the 2013 International Conference on Technology Innovation and Industrial Management (pp. S2_164-180). ToKnowPress.

Chiang, T.C. and Zheng, D., (2010). 'An empirical analysis of herd behavior in global stock markets.' *Journal of Banking & Finance*, *34*(8): pp.1911-1921.

Cheridito P., (2004). 'Gaussian moving averages, semimartingales and option pricing.' Stochastic Processes and their Applications. 109(1): pp47-68.

Christianson K., (2002). *Sustainability through the market: A brief. Government of Norway as an input into the Norwegian "Global Markets & Governance" project*. Published by: Regjeringen Bondevik II.

Cochrane, J.H., (2009). *Asset Pricing:(Revised Edition)*. Woodstock, Oxfordshire: Princeton university press.

COSO (Committee of Sponsoring Organizations of the Treadway Commission), (2004). *Enterprise risk management – Integrated Framework*. Committee of Sponsoring Organizations of the Treadway Commission.

Cont, R., (1999). 'Modeling economic randomness: statistical mechanics of market phenomena.' In M. Batchelor and LT Wille (eds.) *Statistical Physics on the eve of the 21st century,* Singapore: World Scienti.

Cont, R., (2001). 'Empirical properties of asset returns: Stylized facts and statistical issues.' *Quantitative Finance*. 1(2): pp1-14.

Cont, R., (2002). 'Empirical properties of asset returns: Stylized facts and statistical Issues.' *Quantitative Finance*. 1(2): 223-236.

Cont, R., (2005). 'Volatility Clustering in Financial Markets: Empirical Facts and Agent–Based Models.' In Kirman A. and Teyssiere G. (eds.): *Long memory in economics*: pp. 289-309. Berlin: Springer.

Cont, R. and Bouchaud, J.P., (2000). 'Herd behavior and aggregate fluctuations in financial markets.' *Macroeconomic dynamics*, *4*(02): pp.170-196.

Cooper, G., (2010). *The origin of financial crises: central banks, credit bubbles and the efficient market fallacy*. Peterfield: Harriman House Limited.

Cooper, M.J., Dimitrov, O. and Rau, P.R., (2001). 'A rose. com by any other name.' *The journal of Finance*, *56*(6): pp.2371-2388.

Cordell L., Yilin Huang Y. and Williams M., (2012). 'Collateral Damage: Sizing and Assessing the Subprime CDO Crisis.' Federal Reserve Bank of Philadelphia, Working Paper No.11-30/R.

Crotty, J., (2009). 'Structural causes of the global financial crisis: a critical assessment of the 'new financial architecture'. *Cambridge Journal of Economics*, *33*(4): pp.563-580.

Cumby, R., Figlewski, S. and Hasbrouck, J., (1993). 'Forecasting volatilities and correlations with EGARCH models.' *The Journal of Derivatives*, *1*(2): pp.51-63.

Dacorogna, M.M., Müller, U.A., Nagler, R.J., Olsen, R.B. and Pictet, O.V., (1993). 'A geographical model for the daily and weekly seasonal volatility in the foreign exchange market.' *Journal of International Money and Finance*, 12(4): pp.413-438.

Daniel, K., Hirshleifer, D. and Subrahmanyam, A., (1998). 'Investor psychology and security market under-and overreactions.' *the Journal of Finance*, 53(6): pp.1839-1885.

Danielsson J., James K., Valenzuela M. Zer I., (2014). 'Model Risk of Risk Models.' SRC Discussion Paper No 11.

Darbellay,G.A. and Wuertz, D. (2000). The entropy as a tool for analysing statistical dependences in financial time series. *Physica A*, 287, pp. 429-439.

D'Arcy, S.P. and Brogan, J.C., (2001). 'Enterprise risk management.' *Journal of Risk Management of Korea*, *12*(1): pp.207-228.

Demirgüç-Kunt, A., Detragiache, E. and Gupta, P., (2006). 'Inside the crisis: An empirical analysis of banking systems in distress.' *Journal of International Money and Finance*, 25(5): pp.702-718.

Department of the Treasury, (2010). *Financial regulatory reform: Rebuilding financial supervision and regulation.* Washington, DC: Department of the Treasury.

Ding, Z., Granger, C.W. and Engle, R.F., (1993). 'A long memory property of stock market returns and a new model.' *Journal of empirical finance*, *1*(1): pp.83-106.

Donaldson, T. and Preston, L.E., (1995). 'The stakeholder theory of the corporation: Concepts, evidence, and implications'. *Academy of management Review*, *20*(1): pp.65-91.

Doob JL., (1953). *Stochastic Processes (Revised Edition)*. New York: J. Wiley & Sons.

Dowd, K., (2000). 'Adjusting for risk: An improved Sharpe ratio'. *International review of economics & finance*, *9*(3): pp.209-222.

Dresel, S., (2001). *Die Modellierung von Aktienmärkten durch stochastische Prozesse* (Doctoral dissertation, Diplomarbeit, Universität Bayreuth,(unpublished).

Drost, F. C., Nijman, T. E., (1993). 'Temporal aggregation of GARCH processes.' *Econometrica: Journal of the Econometric Society*, 61(4): pp.909-927.

Edwards, F.R., (1999). 'Hedge funds and the collapse of long term capital management.' *Journal of Economic Perspectives*, *13*: pp.189-210.

Engle, R. F., (1982). 'Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation.' *Econometrica: Journal of the Econometric Society*, 50(4): pp.987-1007.

Engle, R.F. and Lange, J., (20010.' Predicting VNET: A model of the dynamics of market depth.' *Journal of financial markets*, 4(2): pp.113-142.

Etheridge A., (2002). *A Course in Financial Calculus*. Cambridge: Cambridge University Press.

European Banking Authorty (EBA), (2012). *Financial Innovation and Consumer Protection: An overview of the objectives and work of the EBA's Standing Committee on Financial Innovation (SCFI).* European Banking Authorty.

Fama, E.F., (1965). 'The behavior of stock-market prices.' *The journal of Business*, *38*(1): pp.34-105.

Fama, E.F., (1995). 'Random walks in stock market prices'. *Financial analysts journal*, *51*(1): pp.75-80.

Fama, E. and French, K., (1992). 'The cross-section of expected stock returns.' *Journal of Finance*, 47(2): 427-465.

Fama, E.F. and French, K.R., (2004). 'The capital asset pricing model: Theory and evidence.'*Journal of Economic Perspectives*, *18*: pp.25-46.

Feldstein M., (1999). *International Capital Flows*. London: University of Chicago Press.

Ferreira, F. and Gyourko, J., (2015). *A new look at the US foreclosure crisis: Panel data evidence of prime and subprime borrowers from 1997 to 2012* (No. w21261). National Bureau of Economic Research.

Fox, J., (2011). *The myth of the rational market: a history of risk, reward, and delusion on Wall Street*. Petersfield: Harriman House Limited.

Franke, J., Härdle, W.K. and Hafner, C.M., (2004). *Statistics of financial markets* (Vol. 2). Heidelberg, Germany: Springer.

Freedman, D. and Diaconis, P., (1981). 'On the histogram as a density estimator: L 2 theory.' *Probability theory and related fields*, *57*(4): pp.453-476.

French, K.R., (1980). 'Stock returns and the weekend effect.' *Journal of financial economics*, *8*(1): pp.55-69.

Fuller, R.J. and Hsia, C.C., (1984). 'A simplified common stock valuation model.' *Financial Analysts Journal*, *40*(5): pp.49-56.

Furlong, F.T. and Krainer, J., (2007). 'The subprime mortgage market: national and Twelfth District developments.' *Annual Report*, Federal Reserve bank of San Francisco, pp.6-17. Available at http://www.frbsf.org/files/2007annualreport.pdf, accessed on the 6th July, 2013.

Geithner T. (2008). 'Comments by New York Fed President Timothy Geithner,.' *Wall Street Journal*, 30 May.

Gelinas, N., (2009). 'Too big to die must fail.' *City Journal* , Vol. 19.

Geltner, D. (2013). '*Commercial Real Estate and the 1990-91 Recession in the United States*' Working paper for the Korea Development Institute, Available at https://mitcre.mit.edu/wp-content/uploads/2013/10/Commercial_Real_Estate_and_the_1990-91_Recession_in_the_US.pdf, accessed March 17, 2014.

Ghoulmie, F., Cont, R. and Nadal, J. P., (2005). 'Heterogeneity and feedback in an agent based market model.' *Journal of Physics: condensed matter*, *17*(14), p.S1259.

Giardina, I. and Bouchaud, J.P., (2003). 'Bubbles, crashes and intermittency in agent based market models. *The European Physical Journal B-Condensed Matter and Complex Systems*, *31*(3): pp.421-437.

Gibbs J.W., (1878). 'On the Equilibrium of Heterogeneous Substances.' *Transactions of the Connecticut Academy of Arts and Sciences.* Vol. 3:108-248.

Gibson R. C. (2013). *Asset Allocation: Balancing Financial Risk*, (Fifth Edition). The U.S.A.: McGraw-Hill Professional.

Grimm, V. 1999. 'Ten years of individual-based modeling in ecology: What have we learned and what could we learn in the future?' *Ecological Modeling*. 115: pp129–48.

Glosten, L.R., Jagannathan, R. and Runkle, D.E., (1993). 'On the relation between the expected value and the volatility of the nominal excess return on stocks.' *The journal of finance*. 48(5): pp1779-1801.

Gnedenko, B.V. and Kolmogorov, A. N., (1954). *Limit Distributions for Sums of Independent Random Variables*. Cambridge MA: Addison-Wseley.

Gode, D.K. and Sunder, S., (1993). 'Allocative efficiency of markets with zero-intelligence traders: Market as a partial substitute for individual rationality.' Journal of political economy, 101(1) :pp.119-137.

Goldstein, M.A. and Kavajecz, K.A., (2000). 'Eighths, sixteenths, and market depth: changes in tick size and liquidity provision on the NYSE'. Journal of Financial Economics, 56(1), pp.125-149.

Gopikrishnan, P., Plerou, V., Amaral, L.A.N., Meyer, M. and Stanley, H.E., (1999). 'Scaling of the distribution of fluctuations of financial market indices.' Physical Review E, 60(5), p.5305.

Gorton, G.B., (2008). The panic of 2007 (No. w14358). National Bureau of Economic Research.

Granger, C.W., (1980). 'Long memory relationships and the aggregation of dynamic models.' Journal of econometrics, 14(2): pp.227-238.

Greenspan, A. (1999). Financial Derivatives. Speech to the futures industry Association, Boca ,Raton, Florida, March, 1999, Available at http://www.federalreserve.gov/boarddocs/speeches/1999/19990319.htm, accessed 5th November, 2012.

Grimm, V., Revilla, E., Berger, U., Jeltsch, F., Mooij, W.M., Railsback, S.F., Thulke, H.H., Weiner, J., Wiegand, T. and DeAngelis, D.L., (2005). 'Pattern-oriented modeling of agent based complex systems: lessons from ecology.' *Science*, *310*(5750): pp.987-991.

Guillaume, D.M., Dacorogna, M.M., Davé, R.R., Müller, U.A., Olsen, R.B. and Pictet, O.V., (1997). 'From the bird's eye to the microscope: A survey of new stylized facts of the intra-daily foreign exchange markets.' *Finance and stochastics*, 1(2): pp.95-129.

Gulko, L. (1997). Dart boards and asset prices introducing the entropy pricing theory. *Adv. Econom. 12*, pp.237–276.

Gulko, L., (1999). The entropic market hypothesis. *International Journal of Theoretical and Applied Finance*, 2, pp.293-329.

Gupta, H. M., Campanha, J. R. (2002). Tsallis statistics and gradually truncated Lévy flight-distribution of an economical index *Physica A: Statistical Mechanics and its Applications*, 309(3), pp. 381-387.

Harrison, J.M. and Kreps, D.M., (1978). 'Speculative investor behavior in a stock market with heterogeneous expectations.' *The Quarterly Journal of Economics*, 92(2): pp.323-336.

Hamilton, W.P., (1998). *The stock market barometer*. New York: John Wiley & Sons.

Hautsch, N., (2003). 'Assessing the risk of liquidity suppliers on the basis of excess demand intensities.' *Journal of Financial Econometrics*, 1(2): pp.189-215.

Hellwig, M., (2008), *The causes of the financial crisis*. In *CESifo Forum*, Ifo Institute for Economic Research at the University of Munich, 9 (4): pp. 12-21. Available at http://www.coll.mpg.de/sites/www/files/text/ifo_forum.pdf, accessed on the 12th, July, 2014.

Higham, D., (2004). *An introduction to financial option valuation: mathematics, stochastics and computation* (Vol. 13). Cambridge: Cambridge University Press.

Hill, C.W. and Jones, T.M. (1992),'Stakeholder-agency theory.' *Journal of management studies*, *29*(2), pp.131-154.

HM Treasury, (2006), *Thinking about your risk, setting and communicating your risk appetite,* Available at https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/191519/Setting_and_communicating_your_risk_appetite.pdf, accessed on the 8th October, 2013.

Hou D. and Skeie D., (2014). LIBOR: Origins, Economics, Crisis, Scandal, and Reform. Federal Reserve Bank of New York. Staff Report No. 667, Federal Reserve Bank of New York.

Hull J.C., (2012). *Options, Futures, and Other Derivatives*. Harlow,England: Pearson.

Iliopoulos, A.C., Pavlos, G.P., Magafas, L., Karakatsanis, L., Xenakis, M. and Pavlos, E., (2015). 'Tsallis q-triplet and Stock Market Indices: The cases of S & P 500 and TVIX.' *Journal of Engineering Science and Technology Review*, 8(1): pp.34-40.

Iori, G., (2002). 'A microsimulation of traders activity in the stock market: the role of heterogeneity, agents' interactions and trade frictions.' *Journal of Economic Behavior & Organization*, *49*(2): pp.269-285.

Jagannathan, M., Stephens, C.P. and Weisbach, M.S., (2000). 'Financial flexibility and the choice between dividends and stock repurchases.' *Journal of financial Economics*, *57*(3): pp.355-384.

Jegadeesh, N., Kim, J., Krische, S.D. and Lee, C., (2004). 'Analyzing the analysts: When do recommendations add value?' *The journal of finance*, *59*(3): pp.1083-1124.

Jorion, P., (1995). 'Predicting volatility in the foreign exchange market.' *The Journal of Finance*, *50*(2): pp.507-528.

Jorion, P., (1996). Risk2: Measuring the risk in value at risk. *Financial Analysts Journal*, *52*(6): pp.47-56.

Jorion, P., (2007). *Value at risk: the new benchmark for managing financial risk* (Vol. 3). New York: McGraw-Hill.

Jovanovic, F. (2001). 'Pourquoi l'hypothèse de marche aléatoire en théorie financière? Les raisons historiques d'un choix éthique'. *Revue d'Economie Financière*, 61, pp.203-211.

JP Morgan Chase & Co, (2007), *Annual Report 2007: Building a Strong Foundation*, Available at http://files.shareholder.com/downloads/ONE/0x0x184756/31e544ec-a273-4228-8c2a-8e46127783f8/2007ARComplete.pdf, accessed on the 8th, July, 2012.

Kahneman, D. and Tversky, A., (1979). 'Prospect theory: An analysis of decision under risk.' *Econometrica: Journal of the Econometric Society,* 47 (2): pp.263-291.

Kamenica, E., (2012). 'Behavioral economics and psychology of incentives.' *The Annual Review of Economics*, *4*(1): pp.427-452.

Kaniadakis, G., Lissia, M. and Rapisarda, A., (2002). 'Non extensive thermodynamics and physical applications.' *PHYSICA. A*, 305: pp.XV-XVII.

Kaplan, R.S. and Mikes, A., (2012). 'Managing risks: a new framework.' *Harvard Business Review* 90(6):pp48-60.

Keynes J.M., (1936). The *general theory of employment, interest and money*. New York: Palgrave Macmillan.

Khintchines, A. Y. and Levy, P., 1936, Sur les loi stables. C.R Acad. Sci. Paris. 202(1936): 374-376.

Kirman, A., (1993). 'Ants, rationality, and recruitment.' *Quarterly Journal of Economics.* 108(1): pp.137–156.

Kirman, A. and Teyssiere, G., (2002). 'Microeconomic models for long memory in the volatility of financial time series.' *Studies in Nonlinear Dynamics & Econometrics*, *5*(4):281-302.

Kyle, A.S., (1985). 'Continuous auctions and insider trading'. *Econometrica: Journal of the Econometric Society,* pp.1315-1335.

Lagoarde-Segot,T. (2009). Financial reforms and time-varying microstructure in emerging equity markets.*Journal of Banking & Finance*, 33,pp.1755-1769.

LeBaron, B., Arthur, W.B. and Palmer, R., (1999). 'Time series properties of an artificial stock market.' *Journal of Economic Dynamics and control*, *23*(9): pp.1487-1516.

LeBaron, B., (2000). 'Agent based computational finance: Suggested readings and early research.' *Journal of Economic Dynamics and Control*. *24*(5): pp.679-702.

LeBaron B., (2001a). 'Stochastic volatility as a simple generator of apparent financial power laws and long memory.' *Quantitative Finance*. 1(6): 621-631.

LeBaron B., (2001b). 'Evolution and time horizons in an agent based stock market.' *Macroeconomic Dynamics*. 5(02): pp. 225–254.

Lettau, M., (1997). 'Explaining the facts with adaptive agents: The case of mutual fund flows.' *Journal of Economic Dynamics and Control*, *21*(7): pp.1117-1147.

Levy, M., Levy, H. and Solomon, S., (1995). 'Microscopic simulation of the stock market: the effect of microscopic diversity.' *Journal de Physique I*, 5(8): pp.1087-1107.

Levy P., (1925). *Calcul des probabilities*, Paris :Gauthier-Vallars.

Li A., Marc L., Qi J., Ashton S., Liu J., (2005). 'Exploring Complexity in a Human–Environment System: An Agent-Based Spatial Model for Multidisciplinary and Multiscale Integration'. *Annals of the Association of American Geographers*, 95(1): pp. 54-79.

Li, F., (2013). *Modelling the statistical behaviour of stock market using a multi-scale approach.* PhD coferences, University of Leicester, April. 2013.

Lim, K.P., Brooks, R.D. and Kim, J.H., (2008). 'Financial crisis and stock market efficiency: Empirical evidence from Asian countries.' *International Review of Financial Analysis*, *17*(3): pp.571-591.

Liu, M., (2000). 'Modeling long memory in stock market volatility.' *Journal Econometrics*, 99(1): 139-171.

Lo. A.W, and Mackinlay A.C., (2002). 'Non-Random Walk Down Wall Street. Princeton University Press.

Lowenstein R. (2010). The End of Wall Street (First Edition) New York: Penguin.

Luenberger, D.G. (1997). Investment Science. Oxford: Oxford University Press.

Lux, T., (1995). 'Herd behaviour, bubbles and crashe's. *The economic journal*, 105(431): pp.881-896.

Lux, T., (1996). 'The stable Paretian hypothesis and the frequency of large returns: an examination of major German stocks.' *Applied financial economics*, 6(6):pp 463-475.

Lux, T., (1998). 'The socio-economic dynamics of speculative markets: interacting agents, chaos, and the fat tails of return distributions.' *Journal of Economic Behavior & Organization*, 33(2): pp.143-165.

Lux, T., and Marchesi, M., (2000). 'Volatility clustering in financial markets : a micro simulation of interacting agents.' *International Journal of Theoretical and Applied Finance*. 3(04): 675-702.

Mahipal S., (2011). *Security Analysis With Investment And Portfolio Management*. New Delhi : Isha Books.

Malkiel, B.G. and Fama, E.F., (1970). 'Efficient capital markets: A review of theory and empirical work.' *The journal of Finance*, 25(2): pp.383-417.

Malkiel, B.G. (1995). 'Returns from investing in equity mutual funds 1971 to 1991.' *The Journal of finance*, *50*(2): pp.549-572.

Malkiel, B.G. (2011). *A Random Walk Down Wall Street: The Time-Tested Strategy for Successful Investing (Tenth Edition)*, London: W. W. Norton & Company.

Mandelbrot B. B., (1963). 'The variation of certain speculative prices.' *Journal of Business*. 36(4): pp394–419.

Mandelbrot, B.B. and Van Ness J., (1968). 'Fractional Brownian motion, fractional noises and applications.' *SIAM Review*. 10(4): pp422–437.

Mantegna, R.N. and Stanley, H.E., (1994). 'Stochastic process with ultraslow convergence to a Gaussian: the truncated Lévy flight'. *Physical Review Letters*, *73*(22):pp.2946.

Mantegna, R.N. and Stanley, H.E., (1995). 'Scaling behaviour in the dynamics of an economic index.' *Nature*, 376(6535): pp.46-49.
Mantegna, R.N. and Stanley, H.E., (1996). 'Turbulence and financial markets.' *Nature*, 383(6601): pp.587-588.

Mantegna, R.N. and Stanley, H.E., (2000). *An introduction to econophysics: correlation and complexity in finance*. Cambridge, UK: Cambridge University.

Mantel, R.R., (1974). 'On the characterization of aggregate excess demand.' *Journal of Economic Theory*, *7*(3): pp.348-353.

Marchesi, M., Cincotti, S., Focardi, S.M. and Raberto, M., (2003). 'The genoa artificial stock market: Microstructure and simulations.' In *Heterogenous Agents, Interactions and Economic Performance* .Berlin Heidelberg : Springer. pp. 277-289.

Markowitz H. (1952). 'Portfolio Selection.' *The journal of finance*, 7(1): pp77–91.

Matacz, A., (2000). 'Financial Modelling and Option Theory with the Truncated Levy Process.' *International Journal of Theoretical Applied Finance*. 03(01): pp143-160.

Mattedi A.P., Ramos F.M., Rosa R.R, Mantegna R.N., (2008) Value-at-Risk and Tsallis statistics: risk analysis of the aerospace sector. *Unpublished*.
https://arxiv.org/abs/cond-mat/0402654v3

Mehmood, Y. and Hanif, W.,(2014). 'Impact of Bullish and Bearish Market on Investor Sentiment.' *International Journal of Innovation and Applied Studies*, *9*(1): pp142-151.

Merton, R. C., (1976). 'Option pricing when underlying stock returns are discontinuous.' *Journal of financial economics*, *3*(1-2): pp.125-144.

Merton, R.C. and Samuelson, P.A., (1992). *Continuous-time finance.* Oxford: Blackwell Publishers Inc.

Mikosch, T., and Starica C., (2003). Long-range dependence effects and ARCH modeling, in Doukhan, P., Oppenheim, G., and Taqqu, M. (eds). *Theory and applications of long-range dependence*. Boston, MA: Birkh¨auser : pp439–459.

Millon-Cornett, M.H. and Tehranian, H. (1989). 'Stock market reactions to the depository institutions deregulation and monetary control act of 1980.' *Journal of Banking & Finance*, *13*(1): pp.81-100.

Minsky, H.P. and Kaufman, H., (2008). *Stabilizing an unstable economy* (Vol. 1). New York: McGraw-Hill.

Mollenkamp, C., Ng, S., Pleven, L. and Smith, R., (2008). 'Behind AIG's Fall, Risk Models Failed to Pass Real-World Test.' *Wall Street Journal*. . October 21, 2008, Available at http://www.wsj.com/articles/SB122538449722784635, accessed on the 8th June 2014.

Montgomery, A.A., Fahey, T., Peters, T.J., MacIntosh, C. and Sharp, D.J., (2000). Evaluation of computer based clinical decision support system and risk chart for management of hypertension in primary care: randomised controlled trial. *Bmj*, *320*(7236), pp.686-690.

Mostowfi S., (2015). *Managing Market Structure Risk: Flash Back.* U.S.A.: The TABB Group, LLC.

Moy, J.T. (1985). 'Recent trends in unemployment and the labor force, 10 countries.' *Monthly Labor Review*, *108*(8): pp.9-22.

Moyano, L.G., Tsallis C., and Gell-Mann, M., (2006). 'Numerical indications of a q-generalised central limit theorem.' *Europhyics. Letters*, 73 (6): pp813-819.

Mullins, D.W. (1982). 'Does the capital asset pricing model work.' *Harvard Business Review*, *60*(1): pp.105-114.

Muth J.F. (1961). 'Rational Expectations and the Theory of Price Movements.' *Econometrica: Journal of the Econometric Society,* 29 (3): pp 315-335.

Naik, P.K. and Padhi, P., (2012). 'The impact of macroeconomic fundamentals on stock prices revisited: evidence from Indian data.' *Eurasian Journal of Business and Economics*, *5*(10): pp.25-44.

Nelson, E., (1967). *Dynamical theories of Brownian motion*. Princeton: Princeton university press.

Nocco, B.W. and Stulz, R.M. (2006). 'Enterprise risk management: theory and practice.' *Journal of applied corporate finance*, *18*(4): pp.8-20.

Ofek, E. and Richardson, M. (2003). 'Dotcom mania: The rise and fall of internet stock prices.' *The Journal of Finance*, *58*(3): pp.1113-1137.

Ofer, A.R. and Thakor, A.V., (1987). 'A theory of stock price responses to alternative corporate cash disbursement methods: Stock repurchases and dividends.' *The Journal of Finance*, *42*(2): pp.365-394.

Osorio, R., Borland, L. and Tsallis, C., (2004) 'Distributions of high-frequency stock-market observables' In Tsallis, C. and Gell-Mann, M. (eds.) *None-extensive Entropy: Interdisciplinary Applications* Oxford: Oxford University Press: pp321-334.

Ott, A., Bouchaud, J.P., Langevin, D. and Urbach, W., (1990). 'Anomalous diffusion in ''living polymers'': A genuine Levy flight?'. *Physical review letters*, *65*(17), p.2201.

Pareto V. (1964). *Cours d'econemie plolitique.* Librairie Droz.

Partnoy F., (2003). *Infectious Greed: How Deceit and Risk Corrupted the Financial Market*s. New York:Times Books.

Pástor, Ľ. and Veronesi, P., (2003). 'Stock valuation and learning about profitability.' *The Journal of Finance*, *58*(5): pp.1749-1790.

Peng, C.K., Havlin, S., Hausdorff, J.M., Mietus, J.E., Stanley, H.E. and Goldberger, A.L., (1995). 'Fractal mechanisms and heart rate dynamics: long-range correlations and their breakdown with disease.' *Journal of electrocardiology*, 28: pp.59-65.

Picoli Jr, S., Mendes, R.S., Malacarne, L.C. and Santos, R.P.B., (2009). 'q-distributions in complex systems: a brief review.' *Brazilian Journal of Physics*,39(2A): pp.468-474.

Plerou, V., Gopikrishnan, P., Rosenow, B., Amaral, L.A.N. and Stanley, H.E., (1999). 'Universal and nonuniversal properties of cross correlations in financial time series.' *Physical Review Letters*, 83(7): pp.1471-1474.

Pollard, S.J., Strutt, J.E., MacGillivray, B.H., Hamilton, P.D. and Hrudey, S.E., (2004). Risk analysis and management in the water utility sector: a review of drivers, tools and techniques. *Process Safety and Environmental Protection*, *82*(6), pp.453-462.

Poon,S.H. and Granger, C.W., (2003). 'Forecasting volatility in financial markets: A review.' *Journal of economic literature*, 41(2): pp.478-539.

Power, M. (2004). *The risk management of everything: Rethinking the politics of uncertainty*. London: Demos.

Raberto, M., Cincotti, S., Focardi, S.M. and Marchesi, M., (2001). 'Agent based simulation of a financial market.' *Physica A: Statistical Mechanics and its Applications*. 299(1): pp.319-327.

Reinholdson, F. and Olsson, H. ., (2012). 'The Separation of Commercial and Investment Banking: A Literature Review'. Master Thesis. Industrial and Financial Management. University of Gothenburge, School of Business, Economics, and Law.

Resnick, S.I., (1998). 'Why non-linearities can ruin the heavy-tailed modeler's day.' In Adler, R. J., Feldman, R. E., and Taqqu, M. S.(eds.) *A Practical Guide to Heavy Tails: Statistical Techniques and Applications.* Boston: Birkhäuser, pp.219-239.

Rockafellar, R.T. and Uryasev, S., (2000). 'Optimization of conditional value-at-risk.' *Journal of risk*, *2*: pp.21-42.

Ryoo,H.,and Graham,S., (2002). Korean stock prices under price limits: Variance ratio tests of random walks. *Applied Financial Economics*, 12, pp.545-553.

Rubinstein, M., (2001). 'Rational markets: yes or no? The affirmative case.' *Financial Analysts Journal*, *57*(3): pp.15-29.

S&P and Capital IQ, (2012). *History of the financial market. Financial communications*.

Sager, M.J. and Taylor, M.P., (2006). 'Under the microscope: the structure of the foreign exchange market'. *International Journal of Finance & Economics*, 11(1): pp.81-95.

Samuelson, P.A., (1965). Proof that properly anticipated prices fluctuate randomly. *IMR; Industrial Management Review (pre-1986)*, *6*(2): pp.41-45.

Scharfstein, D.S. and Stein, J.C., (1990). 'Herd behavior and investment.' *The American Economic Review*, 80(3): pp.465-479.

Scherbina A., (2013). 'Asset Price Bubbles: A Selective Survey.' *IMF Working Paper WP/13/45*. Institute for Capacity Development, International Monetary Fund.

Sewell, M., (2007),'Behavioural finance.' *University of Cambridge* (revised April 2010). From http://www.behaviouralfinance.net/behavioural-finance.pdf.

Sewell M., (2012). 'The Efficient Market Hypothesis: Empirical Evidence'. *International Journal of Statistics and Probability*; 1 (2): pp.164-178.

Sharpe, W.F., (1964), 'Capital asset prices: A theory of market equilibrium under conditions of risk.' *The journal of finance*, *19*(3): pp.425-442.

Shiller, R.J., (2015). *Irrational exuberance*. Oxford: Princeton university press.

Siciliano G., Andrews P., Arce A., Demartini A., Goebel S., Lewis C. and Nakagawa S., (2012). *The Credit Default Swap Market. The Board of the International Organisation of Securities Commissions* FR05/12.

Smith, V.L., Suchanek, G.L. and Williams, A.W., (1988). 'Bubbles, crashes, and endogenous expectations in experimental spot asset markets.' *Econometrica: Journal of the Econometric Society*: pp.1119-1151.

Solomon, T. H., Weeks, E.R., and Swinney, H. L., (1993). 'Observation of anomalous diffusion and Lévy flights in a two-dimensional rotating flow.' *Physical Review Letters*. 71(24): pp3975.

Solomon, T. H., Weeks, E. R., Swinney, H. L., (1994). 'Chaotic advection in a two-dimensional flow: Levy flights and anomalous diffusion.' *Physica D: Nonlinear Phenomena*, 76(1): pp70-84.

Sornette, D. and Von der Becke, S., (2011), 'Crashes and High Frequency Trading: An evaluation of risks posed by high speed algorithmic trading.' *Foresight, the future of computerised trading in financial markets, Driver Review*, 7. Available at https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/289016/11-1226-dr7-crashes-and-high-frequency-trading.pdf, accessed 8th, December, 2012.

Stauffer, D. and Aharony, A., (1994). *Introduction to Percolation Theory*, London: Taylor & Francis.

Stauffer, D. and Sornette, D., (1999). 'Self-organized percolation model for stock market fluctuations.' *Physica A: Statistical Mechanics and its Applications*, 271(3): pp.496-506.

Stein, R.M., (2006). 'Are the probabilities right? Dependent defaults and the number of observations required to test for default rate accuracy.' *Journal of Investment Management*, 4(2): p.61-71.

Stirling, A., (2010). 'Keep it complex.' *Nature*, 468(7327), pp.1029-1031.

Stivers, C.T. and Sun, L., (2002), *Stock market uncertainty and the relation between stock and bond returns* (No. 2002-3). Federal Reserve Bank of Atlanta, Available at https://www.frbatlanta.org/-/media/Documents/filelegacydocs/wp0203.pdf, accessed on 12th, June, 2013.

Stowe, J. D., McLeavey, D. W. and Pinto, J. E., (2009). 'Share repurchases and stock valuation models.' *Journal of Portfolio Management*, 35(4): pp170-179.
Szado, E., (2011), 'Defining speculation: The first step toward a rational dialogue.' *The Journal of Alternative Investments*, 14(1):p.75-82.

Schinckus C. (2015). 'Positivism in finance and its implication for the diversification finance research. Diversifying finance research: From financialization to sustainability'. *International Review of Financial Analysis*, 40. Pp. 103-106.

The Basel Committee on Banking Supervision (2014) *Basel II, Fundamental Review of the Trading Book: A Revised Market Risk Framework.* Consultative Document. The Basel Committee on Banking Supervision.

Thomson Reuters, (2013). *Thomson Reuters Corporate Responsibility Ratings (TRCRR) Rating and Ranking Rules and Methodologies.* Available at: http://thomsonreuters.com/content/dam/openweb/documents/pdf/tr-com-financial/methodology/corporate-responsibility-ratings.pdf, accessed on the 11th April, 2014.

Truesdell, C. and Muncaster, R.G., (1980). Fundamentals of Maxwell's kinetic theory of a simple monatomic gas: treated as a branch of rational mechanics. New York: Academic Press, Inc.(Pure and Applied Mathematics, No. 83).

Tsallis, C., (1988). 'Possible Generalization of Boltzmann-Gibbs Statistics.' *Journal of Statistical Physics*. 52(1-2): pp 479-487.

Tsallis, C. and Bukman, D.J., (1996). 'Anomalous diffusion in the presence of external forces: Exact time-dependent solutions and their thermostatistical basis.' *Physical Review E*, *54*(3), p.R2197-2200.

Tsallis, C., Levy, S.V., Souza, A.M. and Maynard, R., (1995). 'Statistical-mechanical foundation of the ubiquity of Lévy distributions in nature'. *Physical Review Letters*, *75*(20), p.3589.

Tsallis, C. and Lenzi, E.K., (2002). 'Anomalous diffusion: nonlinear fractional Fokker–Planck equation.' *Chemical Physics*, 284(1): pp.341-347.

Umarov, S., Tsallis, C. and Steinberg, S., (2008). 'On a q-central limit theorem consistent with nonextensive statistical mechanics.' *Milan journal of mathematics*, 76(1): pp.307-328.

Van Mieghem, P., (2009). *Performance analysis of communications networks and systems*. Cambridge: Cambridge University Press.

Vitek, Francis and Roger, Scott (2012). The Global Macroeconomic Costs of Raising Bank Capital Adequacy Requirements. IMF Working Paper No. NO.12/44. Available at SSRN:http://ssrn.com/abstract=2012229, accessed on 4th, July, 2014.

Voit, J. (2003). 'From Brownian motion to operational risk: Statistical physics and financial markets.' *Physica A: Statistical Mechanics and its Applications*. 321(1): 286-299.

Voit, J., (2005). *The statistical mechanics of financial markets*. Berlin: Springer Science & Business Media.

Yang, C., Wang, J., Zhou, T., Liu, J., Xu, M., Zhou, P. and Wang, B., (2005). 'Financial market model based on self-organized percolation.' *Chinese Science Bulletin*, *50*(19): pp.2140-2144.

Zanette, D.H. and Alemany, P.A., (1995). 'Thermodynamics of anomalous diffusion.' *Physical review letters*, *75*(3): pp366.

Zhang, X., 2012. Role of Market Microstruture in the Relationship between Market Co-movement and Market Efficiency. Masters Thesis, Wuhan University, School of Graduate Studies of the University of Lethbridge

Zovko, I. and Farmer, J.D., (2002). 'The power of patience: a behavioural regularity in limit-order placement.' *Quantitative finance*, *2*(5), pp.387-392.

Zurek W.H., (1989). 'Algorithmic randomness and physical entropy.' *Physical Review A*. 40(8): 4731- 4751.