Risk portofolio management under Zipf analysis based strategies

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Summary. A so called Zipf analysis portofolio management technique is introduced in order to comprehend the risk and returns. Two portofoios are built each from a well known financial index. The portofolio management is based on two approaches: one called the "equally weighted portofolio", the other the "confidence parametrized portofolio". A discussion of the (yearly) expected return, variance, Sharpe ratio and β follows. Optimization levels of high returns or low risks are found.

1 Introduction

Risk must be expected for any reasonable investment. A portofolio should be constructed such as to minimize the investment risk in presence of somewhat unknown fluctuation distributions of the various asset prices [1,2] in view of obtaining the highest possible returns. The risk considered hereby is measured through the variances of returns, i.e. the β. Our previous approaches were based on the "time dependent" Hurst exponent [3]. In contrast, the Zipf method which we previously developed as an investment strategy (on usual financial indices) [4,5] can be adapted to portofolio management. This is shown here through portofolios based on the $DJIA30$ and the $NASDAQ100$. Two strategies are examined through different weights to the shares in the portofolio at buying or selling time. This is shown to have some interesting features. A key parameter is the *coefficient of confidence.* Yearly expected levels of returns are discussed through the Sharpe ratio and the risk through the β.

2 Data

Recall that a time series signal can be interpreted as a series of words of m letters made of characters taken from an alphabet having k letters. Here below $k = 2$: u and d, while the words have a systematic (constant) size ranging between 1 and 10 letters.

Prior to some strategy definition and implementation, let us introduce a few notations. Let the probability of finding a word of size m ending with a u in the i (asset) series be given by $P_{m,i}(u) \equiv P_i([c_{t-m+2}, c_{t-m+1}, ..., c_{t+1}, c_t; u])$ and correspondingly by $P_{m,i}(d)$ when a d is the last letter of a word of size m. The character c_t is that seen at the end of day t.

In the following, we have downloaded the daily closing price data available from the web: (i) for the $DJIA30$, 3909 data points for the 30 available shares, i.e. for about 16 years[1]; (ii) for the $NASDAQ100$, 3599 data points[2] for the 39 shares which have been maintained in the index, i.e. for about 14.5 years. The first 2500 days are taken as the preliminary *historical* data necessary for calculating/setting the above probabilities at time $t = 0$. From these we have invented a strategy for the following 1408 and 1098 possible investment days, respectively, i.e. for ca. the latest 6 and 4.5 years respectively. The relevant probabilities are recalculated at the end of each day in order to implement a *buy* or *sell* action on the following day. The daily strategy consists in buying a share in any index if $P_{m,i}(u) \geq P_{m,i}(d)$, and in selling it if $P_{m,i}(u) \leq P_{m,i}(d)$.

However the weight of a given stock in the portofolio of n assets can be different according to the preferred strategy. In the equally weighted portofolio (EWP), each stock i has the same weight, i.e. we give $w_{i \in B} = 2/n_u$ and $w_{i \in S} = -1/n_d$, where n_u (n_d) is the number of shares in B (S) respectively such that $\Sigma[w_{i \in B} + w_{i \in S}] = 1$, with $n_u + n_d = n$ of course. This portofolio management strategy is called $ZEWP$.

In the other strategy, called $ZCPP$, for the confidence parametrized portofolio (CPP), the weight of a share depends on a confidence parameter $K_{m,i} \equiv P_{m,i}(u)$ - $P_{m,i}(d)$. The shares i to be bought on a day belong to the set B when $K_{m,i} > 0$, and those to be sold belong to the set S when $K_{m,i} < 0$. The respective weights are then taken to be $w_B = \frac{2K_{m,i \in B}}{\Sigma K_{m,i \in B}}$, and $w_S = \frac{-K_{m,i \in S}}{\Sigma K_{m,i \in S}}$.

3 Results

The yearly return, variance, Sharpe ratio, and β are given in Table 1 and Table 2 for the so called $DJIA30$ and so called $NASDAQ39$ shares respectively as a function of the word length m. The last line gives the corresponding results for the $DJIA30$ and the $NASDAQ100$ respectively. We have calculated the average (over 5 or 4 years for the $DJIA30$ and $NASDAQ39$ respectively) yearly returns, i.e. $E(r_P)$ for the portofolio P. The yearly variances σ_P result from the 5 or 4 years root mean square deviations from the mean. The Sharpe ratio SR is given by $SR = E(r_P)$ / σ_P and is considered to measure the portofolio performance. The β is given by $cov(r_P, r_M)/\sigma_M^2$ where the P covariance

[1]From Jan. 01, 1989 till Oct. 04, 2004

[2]From June 27, 1990 till Oct. 04, 2004

$cov(r_P, r_M)$ is measured with respect to the relevant financial index, so called market (M), return. Of course, σ_M^2 measures the relevant *market* variance. The β is considered to measure the portofolio risk. For lack of space the data in the tables are not graphically displayed.

It is remarkable that the $E(r_P)$ is rather low for the $ZEWP$, and so is the σ_P, but the $E(r_P)$ can be very large, but so is the σ_P in the $ZCPP$ case for both portofolios based on the $DJIA30$. The same observation can be made for the $NASDAQ39$. In the former case, the highest $E(r_P)$ is larger than 100% (on average) and occurs for m =4, but it is the highest for m=3 in the latter case. Yet the risk is large in such cases. The dependences of the Sharpe ratio and β are not smooth functions of m, even indicating some systematic dip near $m = 6$, in 3 cases; a peak occurs otherwise.

The expected yearly returns $E(r_P)$ *vs.* σ are shown for both portofolios and for both strategies in Figs.1-2, together with the equilibrium line, given by $E(r_M)(\sigma/\sigma_M)$, where it is understood that σ is the appropriate value for the investigated case. Except for rare isolated points below the equilibrium line, data points fall above it. They are even very much above in the $ZCPP$'s. cases.

	ZEWP				ZCPP			
m	$E(r_P)$	σ_P	SR	β	$E(r_P)$	σ_P	SR	β
1	20.00	16.98	1.18	0.97	20.16	17.95	1.12	1.02
2	18.10	16.21	1.12	0.92	20.36	17.66	1.15	1.00
3	22.00	14.05	1.57	0.79	65.24	39.52	1.65	0.08
4	24.93	11.90	2.09	0.57	104.85	47.02	2.23	-1.11
5	22.60	9.16	2.47	0.38	95.96	56.54	1.70	-1.58
6	18.37	11.68	1.57	0.47	67.97	40.55	1.68	0.09
7	17.33	8.93	1.94	-0.06	65.27	30.18	2.16	-0.50
8	9.84	7.73	1.27	0.11	53.83	37.52	1.43	0.32
9	11.23	4.91	2.29	-0.01	44.23	38.12	1.16	0.58
10	6.46	7.11	0.91	0.15	37.40	61.05	0.61	1.92
	$E(r_M)$	σ_M	SR	β				
DJIA30	17.09	17.47	0.98	1				

Table 1. Statistical results for a portofolio based on the 30 shares in the $DJIA30$ index for two strategies, i.e. $ZEWP$ and $ZCPP$ based on different word sizes m for the time interval mentioned in the text. The last line gives the corresponding results for the $DJIA30$. All quantities are given in %

4 Conclusion

We have translated the time series of the closing price of stocks from two financial indices into letters taken from a two character alphabet, searched for

	ZEWP				ZCPP			
m	$E(r_P)$	σ_P	SR	β	$E(r_P)$	σ_P	SR	β
1	12.68	22.01	0.58	0.89	5.41	26.30	0.21	0.55
2	11.43	19.99	0.57	0.81	2.25	28.12	0.08	0.63
3	20.25	16.92	1.20	0.24	149.27	192.91	0.77	-1.87
4	27.08	15.74	1.72	-0.04	131.69	149.75	0.88	-1.70
5	27.84	11.49	2.42	-0.18	106.63	103.30	1.03	-1.08
6	24.89	8.77	2.84	-0.05	90.11	68.89	1.31	-0.26
7	15.99	9.19	1.74	-0.10	67.28	32.58	2.07	0.37
8	13.93	12.39	1.13	-0.25	68.34	44.33	1.54	0.06
9	17.52	11.13	1.57	-0.32	99.20	38.84	2.55	0.21
10	14.77	10.81	1.37	-0.32	71.42	32.09	2.23	0.30
	$E(r_M)$	σ_M	SR	β				
NASDAQ100	7.36	24.11	0.31	1				

Table 2. Statistical results for a portofolio based on 39 shares from the $NASDAQ100$ index for two strategies, i.e. $ZEWP$ and $ZCPP$ based on different word sizes m for the time interval mentioned in the text. The last line gives the corresponding results for the $NASDAQ100$. All quantities are given in %

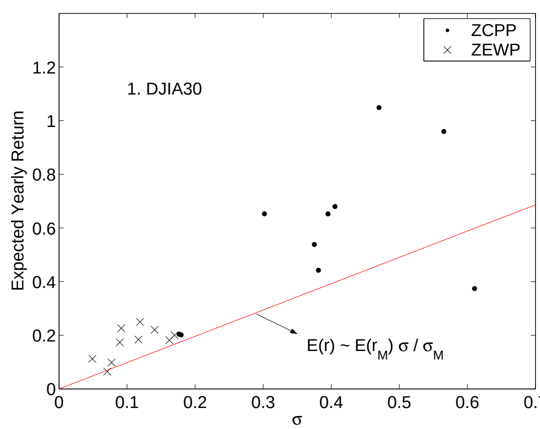

Fig. 1. Expected yearly return as a function of the corresponding variance for two investment strategies involving the shares in the $DJIA30$. The time of investigations concerns the latest 5 yrs

words of m letters, and investigated the occurrence of such words. We have invented two portofolios and maintained them for a few years, buying or selling shares according to different strategies. We have calculated the corresponding yearly expected return, variance, Sharpe ratio and β. The best returns and weakest risks have been determined depending on the word length. Even though some risks can be large, returns are sometimes very high.

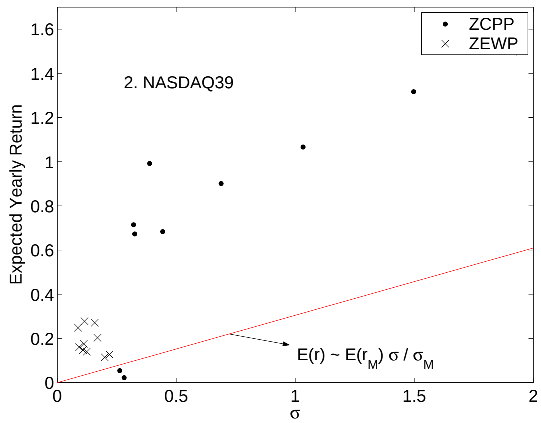

Fig. 2. Expected yearly return as a function of the corresponding variance for two investment strategies involving 39 shares taken from the $NASDAQ100$. The time of investigations concerns the latest 4 yrs

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