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Three Essays on Natural Resources, Climate Change and Economic Growth

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Abstract

This thesis contains three main chapters that explore various issues related to natural resources, climate change and long-term economic growth. In Chapter 2, we examine the conditions under which endogenous long-term economic growth will emerge in an overlapping-generation model with productive non-renewable resources. We formally prove that the elasticity of substitution between labour and natural resource input plays a crucial role in generating endogenous long-term economic growth. In Chapter 3, we examine how the substitutability between renewable and non-renewable resources will affect long-term economic growth. To achieve this, we develop two discrete-time endogenous growth models with both renewable and non-renewable natural resources. These two types of resources enter into the production function through a constant-elasticity-of-substitution (CES) aggregator. The ease or difficulty in substituting between these two inputs is then captured by a single parameter, namely the elasticity of substitution. We then analyse how changes in this elasticity will affect long-term economic growth. Finally, in Chapter 4, we develop a multi-sector neoclassical growth model to analyse the effects of climate change on economic growth and the allocation of productive inputs and outputs across sectors. A novel feature of this model is that it takes into account the differential impact of global warming on agricultural and non-agricultural productivity growth. In particular, the effect of climate change on long-run growth is well-characterised. However, the impacts on long-run allocation of labour across sectors as well as value-added shares are ambiguous due to ambiguity effect of the climate impact on capital return.

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Declaration

Chapter 2 is joint work with Richard Suen.

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Chapter 1

Introduction

This thesis consists of three self-contained essays on issues related to natural resource utilisation, climate change and economic growth. We focus on three research questions. The first question concerns the conditions under which endogenous long-term economic growth can emerge in neoclassical growth models with productive non-renewable resources. The second question is about how the substitutability between renewable and non-renewable resources will affect long-term economic growth. The last question is about how global warming will affect economic growth and sectoral allocation of productive inputs. These questions will be addressed in Chapter 2-4, respectively. Chapter 5 provides a brief summary of the entire thesis.

In Chapter 2, we present an overlapping-generation model in which non-renewable resources are used as input of production. The main objective of this chapter is to investigate the conditions under which endogenous long-term economic growth will emerge. As is well-known in the macroeconomics literature, endogenous growth models predict that long-run economic growth is endogenously determined by the primitives (or deep parameters) of the model economy. This type of model is in stark contrast to the standard neoclassical model which predicts that long-run economic growth is solely determined by exogenous technological improvements (Solow, 1956). Endogenous growth models have proved to be useful in furthering our understanding of the factors and government policies that would affect the long-run growth performance of an economy.

The study in Chapter 2 is motivated by the results in Agnani et al. (2005). These authors extend a standard overlapping-generation model to take into account the necessity of the flow of non-renewable resources in producing final goods. Assuming a Cobb-Douglas production function in three inputs (physical capital, labour and natural resources), they show that a unique balanced growth path exists in this economy under certain conditions, and the long-term growth rate is endogenously determined. This raises a natural follow-up question of whether the same result can be obtained under a more general specification of production function. The main objective of Chapter 2 is to address this question. To this end, we replace the Cobb-Douglas production function in Agnani et al. (2005) with a more general specification. Our main findings can be summarised as follows: If the elasticity of substitution between labour and natural resource is unitary, then long-term growth rate is endogenously determined as in Agnani et al. (2005). However, if this elasticity is not equal to one (or bounded away from one), then long-term economic growth is solely determined by an exogenous technological factor as in the standard neoclassical growth model. These results are useful because a unitary elasticity of substitution (i.e., the Cobb-Douglas specification) is frequently rejected by empirical studies [see Henningsen et al. (2018) and the references therein].

Chapter 3 considers how the long-run growth of output per capita will be affected by the elasticity of substitution between renewable and non-renewable resources. It is now widely accepted that renewable resource utilisation is important in reducing our dependency on fossil energy. However, empirical evidence suggests that the process of switching from non-renewable to renewable energy is no easy feat (Papageorgiou et al., 2017). Also, the ability to switch is likely to differ across countries (Malikov et al., 2018). If this difference reflects the heterogeneous supply-side structures across countries, how could it affect the rate of long-run economic growth?

To address this question in Chapter 3, we extend two endogenous growth models à la Romer (1986) and Barro (1990) by introducing renewable and non-renewable resources as inputs of production. These two types of resources are combined in the production function through a constant-elasticity-of-substitution (CES) aggregator. After developing the models, we characterise the long-run growth rate of output per capita and examine how changes in the elasticity of substitution between the two natural inputs will affect the macroeconomy. To ensure a consistent comparison between different CES functions, we adopt the CES normalisation procedure as proposed by Klump and de La Grandville (2000). Under this normalisation, we find that changing the elasticity of substitution will have no effect on the long-term growth rate in the Romer-style model, but it can affect growth in the Barro-style model. The growth effect arises because changing the elasticity of substitution between the two natural resource inputs will affect the allocation of labour across sectors. We also provide the conditions under which increasing the substitutability between these two inputs will promote growth.

Chapter 4 examines the macroeconomic implications of climate change for poor countries. By poor countries, we refer to those countries that rely heavily on agricultural output and have insignificant contribution to global pollution and hence climate change. We choose to focus on low-income countries because a number of reports have revealed an unfair burden of climate change between advanced economies and under-developed economies (Althor et al., 2016; IMF, 2017, among others). For example, IMF (2017) reports how low-income countries will bear the economic brunt of climate change; despite contributing very little to global greenhouse gas (GHG) emissions. On a deeper level, the empirical investigation by Burke et al. (2015) suggests that climate change hits the poor countries more heavily due to their reliance on agricultural activities and their geographical location. In the same vein, the estimates from econometric models according to Dell et al. (2012), and Letta and Tol (2019) show that the adverse consequences of higher temperatures will hit poorer countries the hardest while the impacts on the rich are negligible and inconclusive.

To investigate the long term impacts of climate change, we develop a multi-sector neoclassical growth model along the line of Kongsamut et al. (2001), Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008). Based on the empirical evidence in Burke et al. (2015), we postulate that a rising global temperature will negatively affect productivity growth in both agricultural and non-agricultural production, but the effect on agricultural production is more damaging. We then investigate the effects of global warming on economic growth and sectoral allocation. The comparison of two steady states reveals that climate change always hurt the long-run growth rate of the poor. However, the structural change effects in terms of employment share and value-added share are ambiguous.

Finally, Chapter 5 concludes and summarises the results in this thesis.

Chapter 2

Endogenous Growth Implication in Standard Neoclassical Growth Models with Productive Non-Renewable Resources

2.1 Introduction

The interdependence between long-run economic growth and non-renewable resource utilisation has been extensively studied since the 1970s, a time when natural resource scarcity was viewed as a potent threat to economic growth. Since then, most of the theoretical studies have focused on the conditions under which the depletion of non-renewable resources would not limit long-run economic growth.[1] Most of these studies have developed an endogenous growth model that features a Cobb-Douglas production technology. However, recent empirical studies [such as Kemfert (1998), Kemfert and Welsch (2000), van der Werf (2008) and Henningsen et al. (2019)] have provided evidence showing that the elasticity of substitution between different productive inputs are not equal to one. This raises the question of whether the results of previous studies will continue to hold under a general form of production function. The main objective of this paper is to examine the conditions under which endogenous long-term economic growth will emerge in an overlapping-generation model with productive non-renewable resources.

The general framework of non-renewable resources based economy is that the flow of the natural resources will be combined with capital and labour to produce composite goods

[1] The analysis is conducted in both overlapping generations and infinitely-lived representative agent models. See Dasgupta and Heal (1974), Solow (1974), Stiglitz (1974a), Stiglitz (1974b), Hartwick (1978), Barbier (1999), Grimaud and Rouge (2003), Agnani et al. (2005), Groth and Schou (2007), Valente (2011) , Benchekroun and Withagen (2011), Antony and Klarl (2019) among many others.

that can be either consumed or invested to accumulate capital and then potentially generate perpetual growth. Existing studies show that even without exogenous technological progress, perpetual growth in per-capita output is possible under certain conditions on consumers' preferences and production technology. For example, Solow (1974) extends an infinitely-lived agent model of neoclassical growth to show that non-declining in per capita consumption is possible if and only if the share of capital in production exceeds that of the non-renewable resources in constant returns to scale Cobb-Douglas production function. Hartwick (1978) extended this and showed how increasing returns to scale in Cobb-Douglas production economy can 'override' scarcity. Another example is Agnani et al. (2005) who analyse the problem in an overlapping generations (OLG) model. They show that discount factor must be sufficiently high to avoid global contraction, which refers to the situation under which per capita consumption is declining overtime. The general mechanism of these models is that the long-run depletion rate is constant and endogenously determined by model parameters and this endogenous property is transferred to determining the long-run economic growth. This property is common among existing studies and is valid regardless of whether the fundamental structure of the model is an exogenous growth model or an endogenous growth model.[2]

In this paper, we show that the elasticity of substitution between labour and natural inputs play a crucial role in generating endogenous long-run economic growth. To achieve this, we replace the Cobb-Douglas production function in Agnani et al. (2005) with a two-level nested constant elasticity of substitution (CES) production function. The rest of the model economy is the same as in Agnani et al. (2005). More specifically, we consider a two-level CES function where the inner function combines effective natural input (natural resource flow times exogenous resource-augmenting technology) with effective labour input (labour force times exogenous labour-augmenting technology) and the outer function combines physical capital with the composite input generated from the inner CES. We then show that if the elasticity of substitution of the inner CES (i.e., between labour and natural resource) is one, then the endogenous growth solution in Agnani et al. (2005) is preserved. This result holds even if the outer CES is not Cobb-Douglas. But on the other hand, if the elasticity of substitution between labour and natural resource is not equal to one, then the long-run economic growth is solely determined by the exogenous labour-augmenting technological factor. In this case, the model will feature exogenous economic growth.

An intuitive explanation of this finding is as follows. When the combination of the effective flow of non-renewable input and the effective labour is under Cobb-Douglas, the flow of non-renewable input can be combined with the combination of the labour and the

[2]By the fundamental structure of the model, we refer to that without natural input essentiality the model is fundamentally exogenous or endogenous. See Stiglitz (1974b), Stiglitz (1974a), Benchekroun and Withagen (2011), among others, for exogenous growth setting. For endogenous growth setting, see Barbier (1999), Grimaud and Rouge (2003) and Groth and Schou (2007), for example.

natural resource augmenting technologies as another layer of technologies. This property turns the production technology out to be an equivalent technology that combines capital and labour with a modified labour augmenting technology. The modified labour augmenting technology contains natural resource decay rate as a component of the growth factor. Since the depletion rate is determined by various factors within and beyond the parameters containing in the inner CES, the endogenous growth feature emerges. In contrast, if Cobb-Douglas assumption is ruled out, the effective flow of non-renewable input and the effective unit of labour cannot be combined but has to grow at a common growth rate which is exogenously determined by the growth rate of population and the labour augmenting technology. This turns out that the long-run depletion rate will be determined by the parameters in the inner CES. More importantly, the long-run output per capita must grow by the rate of growth of the labour-augmenting technology along the long-run growth path, and then, the long-run economic growth is exogenously given.

As a sensitivity analysis, we also consider two other possible ways of nesting the two-level CES. Assuming that one and only one between the inner and the outer function is Cobb-Douglas while the other is assumed to be a CES that the elasticity of substitution differs from unity, our sensitivity examination reveals that if a long-run growth path exists, it is an exogenous growth. This results arise because the pure combination of labour and natural input in Cobb-Douglas fashion is impossible.

The analysis we introduce here can create some connections to Uzawa's Steady State Theorem (Uzawa, 1961). The Uzawa Theorem states that, within a neoclassical growth model of two inputs including capital and labour, the existence of the long-run growth equilibrium requires that the production technology must be either Cobb-Douglas or labour-augmenting. The main similarity between the two studies is that our and his studies try to find a condition under which a long-run stationary growth path exists. However, these two studies have two main differences. First, our study focuses on the existence of endogenous growth engine under exogenous growth setting; whereas, Uzawa highlights the condition under which a long-run stationary growth path exists no matter the engine of growth is exogenous or endogenous . Second, not only does our analysis concern capital and labour but also the role of productive natural resources.

The rest of this chapter is organized as follows: In section 2.2, we describe the behaviour of economic agents, and define the competitive equilibrium. We illustrate the conditions under which the long-run competitive equilibrium exists and unique in subsection 2.3.1 and compare our results with related literature. In subsection 2.3.2, we extend the analysis by considering alternative production specifications. Some discussion is contained in subsection 2.3.3. Finally, section 2.4 summarises the study.

2.2 The Model

The basic assumptions of this framework are virtually identical to those of the Agnani et al. (2005) model (AGI, herein after) except that the functional form of the production function is generalised. The notations used here are identical to those given in AGI unless otherwise stated, so we can compare our results with theirs.[3]

Consider a perfectly competitive economy where economic activity is demonstrated over infinite discrete time periods, indexed by $t \in \{0, 1, 2, ...\}$. On the demand-side, we consider a two-period OLG model, in which young and old agents coexist in each period. On the supply-side, we considers a neoclassical growth model with two commodities, a composite good and non-renewable natural resources (e.g., fossil fuels). The composite good is produced with three factors of production, namely labour, physical capital and a flow of non-renewable resources. This good can be either consumed in the same period, or accumulated as capital for the next period. The flow of non-renewable resources is extracted from the the entire stock of natural resources held by the old agents. The extraction cost is nil in this model. All prices are expressed in units of the composite goods.

2.2.1 Consumers

In each time period t, a generation which consists of N_t consumers is born. The population size of generation t is $N_t = (1+n)^t$, where $n > 0$ is the rate of population growth. Consumers are identical within as well as across generations.

There is one representative consumer in each generation t who derives her lifetime utility from consumption when young $c_{1,t}$ and consumption when old $c_{2,t+1}$. Her lifetime utility function is a log-linear function:

$$U(c_{1,t}, c_{2,t+1}) \equiv \text{ln}c_{1,t} + \frac{1}{1+\theta}\text{ln}c_{2,t+1} \tag{2.1}$$

where $\theta > 0$ is the rate of time preference. The representative agent maximises the above intertemporal utility function subject to her budget constraints faced all periods. In the first period of life, the young receives a real wage income w_t from supplying a unit of labour supply inelastically to the composite good firms. A part of the real wage is then spent on current consumption $c_{1,t}$. The rest is devoted to buy assets in order to transfer the income to the retirement period. There are two assets available including saving related to capital accumulation s_t and the purchase of m_t ownership rights of non-renewable resource stock.

[3]Fundamental contributions on the subject has been illustrated in both continuous-time infinite horizon formulation and discrete-time overlapping generations setting [such as Stiglitz (1974b), Stiglitz (1974a), Groth and Schou (2002), Agnani et al. (2005) and Benchekroun and Withagen (2011)]. We choose to address the issue via a discrete time OLG model with productive non-renewable resources according to Agnani et al. (2005). This extension allows us to compare our results to those of the existing literature. As we will show later, our main results can apply to both formulations.

This forms the first period budget constraint:

$$c_{1,t} + s_t + p_t m_t = w_t. \tag{2.2}$$

where p_t is the price of the resource stock.

In the second period, the representative agent is old. She retires from labour market and consumes $c_{2,t+1}$ out of her entire income from sale of all assets. This means that the second period budget constraint is

$$c_{2,t+1} = (1 + r_{t+1})s_t + p_{t+1}m_t \tag{2.3}$$

where r_{t+1} is the real interest rate.

Taking prices $\{w_t, r_{t+1}, p_t, p_{t+1}\}$ as given, the problem of the representative agent born in generation t is to choose a consumption profile $\{c_{1,t}, c_{2,t+1}\}$ and an investment portfolio $\{s_t, m_t\}$ so as to maximise her lifetime utility (2.1), subject to the budget constraints (2.2) and (2.3) and the non-negativity constraints $c_{1,t}, c_{2,t+1}, s_t, m_t \geq 0$. The first order conditions of this problem imply the following conditions:

$$\frac{(1+\theta)c_{2,t+1}}{c_{1,t}} = 1 + r_{t+1}, \tag{2.4}$$

$$\frac{p_{t+1}}{p_t} = 1 + r_{t+1}. \tag{2.5}$$

Equation (2.4) is the well-known Euler condition which describes the inter-temporal optimal consumption choice of the individual between the current and the future period. This condition indicates that the individual will choose her consumption path so that the marginal rate of substitution between current and future consumption is equal to the relative price of current consumption in terms of future consumption, which is the real interest rate. Equation (2.5) is the Hotelling condition[4] which contains an important intuition about the equilibrium allocation of wealth between natural and physical capitals. Since capital investment yields the rate of interest r_{t+1} while holding a unit of non-renewable resources yields capital gain $\frac{p_{t+1}}{p_t}$, equation (2.5) establishes an equilibrium decision rule in the sense that the young will allocate her assets holding so that she will be indifferent at the margin between (extracting and) selling and holding natural resources. Strictly speaking, at the equilibrium point, the resource price rises at the rate of interest. Using (2.2)-(2.5), we can derive the optimal level of consumption,

$$c_{1,t} = \left(\frac{1+\theta}{2+\theta}\right)w_t \quad \text{and} \quad c_{2,t+1} = \left(\frac{1+r_{t+1}}{2+\theta}\right)w_t, \tag{2.6}$$

[4]This condition is widely discussed in the literature concerning intertemporal allocation of natural resources. See, for instance, Stiglitz (1974a, p.124), among others.

and the optimal level of investment in physical capital,

$$s_t = \frac{w_t}{2+\theta} - p_t m_t. \tag{2.7}$$

2.2.2 Composite Good Production

The composite good sector comprises a large number of identical competitive firms. In each period t, each firm hires labour (N_t), rents physical capital (K_t) and acquires extracts of non-renewable resource (X_t) from the competitive factor markets, and produces output (Y_t) according to the following production function:

$$Y_t = F\Big(K_t, G(Q_t X_t, A_t N_t)\Big) \tag{2.8}$$

The terms Q_t and A_t are the indexes of a resource-augmenting technological factor and a labour-augmenting technological factor, respectively. Both factors are assumed to grow at some constant exogenous rate, denoted by $q > 0$ and $a > 0$, such that $Q_t = (1+q)^t$ and $A_t = (1+a)^t$, for all $t \geq 0$.

The production function (2.8) is a version of two-stage, three-factor production technology. The first stage is given by a function $G(\cdot)$ that combines effective natural input ($Q_t X_t$) and effective labour ($A_t N_t$) to form a composite input, denoted by $Z_t = G(Q_t X_t, A_t N_t)$. The inner function is then nested into the outer function $F(\cdot)$ that combines the composite input (Z_t) with physical capital. In the terminology of Leontief (1947a, p.363), Leontief (1947b, p.343) and Blackorby and Russell (1976, p.286), the duplet $\{Q_t X_t, A_t N_t\}$ is said to be functionally separable from K_t. Allowing this kind of functional separability will prove to be useful in our analysis without changing the main implications of AGI solution. We will discuss about this later. Note that the two-stage nesting structure presented in Eq. (2.8) is only one possible structure. In general, the production function can be nested in three ways. For the purpose of illustration we focus in this subsection on this structure, the other two nesting structures will be considered in subsection 2.3.2.

The properties of $F(\cdot)$ and $G(\cdot)$ are summarised in Assumption 2.1 and Assumption 2.2. Recall that an input is said to be essential for production if no output can be produced without some positive amount of this input (Dasgupta and Heal, 1974; Solow, 1974). Throughout this paper, we will use $F_i(\cdot)$ to denote the partial derivative of $F(\cdot)$ with respect to the ith argument, and $F_{ij}(\cdot)$ to denote the partial derivative of $F_i(\cdot)$ with respect to the jth argument, $i, j \in \{1, 2\}$. The partial derivatives of $G(\cdot)$ are similarly represented.

Assumption 2.1. *Both $F : \mathbb{R}_+^2 \to \mathbb{R}_+$ and $G : \mathbb{R}_+^2 \to \mathbb{R}_+$ are twice continuously differentiable, strictly increasing, strictly concave and exhibits constant returns to scale (CRTS) in its arguments.*

Assumption 2.2. *Each input $I \in \{K, X, N\}$ is either essential for production or its* *marginal product is unbounded when I is arbitrarily close to zero.*

Assumption 2.1 is a list of regularity conditions, all of which are commonly used in the economic growth literature. These conditions imply that the composite function in (2.8) is twice continuously differentiable, strictly increasing, strictly concave and exhibits CRTS in all three inputs. In neoclassical growth models (without natural resources), it is also common to impose two other assumptions on the production function: First, both physical capital and labour are essential for production. Second, the marginal product of these inputs are unbounded as their quantity drop towards zero. These assumptions, however, can be quite restrictive. For instance, among those production functions with constant elasticity of substitution (CES), only Cobb-Douglas production functions (with unitary elasticity of substitution) satisfy both of these assumptions.[5] Our Assumption 2.2 gets around this issue by requiring only one of these conditions to hold. This suffices to ensure that in any equilibrium all three inputs will be used in all time periods.[6]

In Appendix A.1, we show that Assumption 2.2 is satisfied by various forms of nested CES production functions, including those that are frequently used in empirical studies. [7] For instance, the production function in (2.8) is a nested CES production function if $F(\cdot)$ and $G(\cdot)$ are given by

$$F(K_t, Z_t) = [\alpha K_t^{\eta} + (1-\alpha) Z_t^{\eta}]^{\frac{1}{\eta}}, \quad \text{with} \quad \alpha \in (0,1) \quad \text{and} \quad \eta < 1,$$

$$G(Q_t X_t, A_t N_t) = [\varphi (Q_t X_t)^{\psi} + (1-\varphi)(A_t N_t)^{\psi}]^{\frac{1}{\psi}}, \quad \text{with} \quad \varphi \in (0,1) \quad \text{and} \quad \psi < 1.$$

This specification encompasses the production function in AGI as a special case.[8] Specifically, these authors assume that both $F(\cdot)$ and $G(\cdot)$ take the Cobb-Douglas form, i.e., $\eta = \psi = 0$:

$$F(K_t, Z_t) = K_t^{\alpha} Z_t^{1-\alpha}, \quad \text{with} \quad \alpha \in (0,1), \tag{2.9}$$

[5] The same point has also been made by Dasgupta and Heal (1974, p.14) and Solow (1974, p.34) in natural resource economics. These studies consider production functions that use physical capital and natural resources as inputs.

[6] The argument goes like this: It seems natural and reasonable to focus on equilibrium with strictly positive output in every period. If an input is deemed essential for production, then a strictly positive amount must be used in every period in this kind of equilibrium. On the other hand, since both factor markets and goods markets are competitive, the price of any input must be equated to its marginal product in equilibrium. If the marginal product of an input is unbounded at or around zero, then the marginal benefit of increasing this input from zero to $\epsilon > 0$, where ϵ is infinitesimal, will be infinitely large but the marginal cost will be finite. Hence, it is never optimal to use a zero amount of this input.

[7] Several existing studies have estimated the input elasticities using all possible kinds of the two-level nested CES. It seems that "double Cobb-Douglas" specification is statistically rejected. Examples of these empirical studies include Kemfert (1998), Kemfert and Welsch (2000), van der Werf (2008) and Henningsen et al. (2019). For instance, Henningsen et al. (2019) use German data for the years 1991-2014 to estimate the CES production function with capital, labour, and energy as inputs for the nesting CES structure (2.8); K(XN). They found perfect substitution for the inner CES and found gross substitution at the rate of 1.42 for the outer CES. For estimations of the other specifications; X(KN) and N(XK), see their Table 4.

[8] See equation (5) in Agnani et al. (2005, p.391).

$$G(Q_tX_t, A_tN_t) = (Q_tX_t)^{\frac{v}{1-\alpha}}(A_tN_t)^{\frac{\beta}{1-\alpha}}, \quad \text{with} \quad \beta, v \in (0,1), \tag{2.10}$$

and $\beta + v = 1 - \alpha$. Under this *double Cobb-Douglas* specification, the two technological factors A_t and Q_t are observationally equivalent to a single Hicks neutral technological factor (total factor productivity), $B_t \equiv Q_t^v A_t^\beta$. Hence, the separate effects of A_t and Q_t are not considered in AGI.

Because all the composite good firms have the same CRTS production function, an aggregate production function immediately appears and turns out to be the same as the identical CRTS production functions of the individual firms. We are able to focus on just a single representative firm whose production represents the aggregate production in this sector. Let R_t be the rental price of physical capital and $\delta \in (0,1)$ be the depreciation rate. The representative firm hires inputs to maximise profit:

$$\max_{\{K_t, X_t, N_t\}} \left\{ F\Big(K_t, G(Q_tX_t, A_tN_t)\Big) - R_tK_t - p_tX_t - w_tN_t \right\}$$

and the first-order conditions imply that each factor is paid according to its marginal product:

$$R_t = r_t + \delta = F_1\Big(K_t, G(Q_tX_t, A_tN_t)\Big), \tag{2.11}$$

$$p_t = Q_tF_2\Big(K_t, G(Q_tX_t, A_tN_t)\Big)G_1(Q_tX_t, A_tN_t), \tag{2.12}$$

$$w_t = A_tF_2\Big(K_t, G(Q_tX_t, A_tN_t)\Big)G_2(Q_tX_t, A_tN_t). \tag{2.13}$$

2.2.3 Natural Resources

At time $t = 0$, the economy is endowed with a (finite) stock of non-renewable resources, $M_0 > 0$.[9] As in Agnani et al. (2005), we assume *a grandfathering process*: at the beginning of period t, the entire stock of natural resources M_t is held by the old agents. Then, the old agents sell the entire stock of natural resources at the unit price p_t to support their consumptions. Part of M_t is extracted costlessly as natural input in composite good production, X_t, while the remaining stock is sold to the young agents to constitute resource assets, M_{t+1}.[10] Define the extraction rate at time t as $\tau_t \equiv \frac{X_t}{M_t} \in (0,1)$. The stock of natural

[9] At time 0, the initial stock of physical capital and non-renewable resources are owned by a group of "initial old" consumers. The decision problem of these comsumers is trivial and does not play any role in the analysis of balanced growth equilibrium.

[10] This notation is slightly different from the one in AGI. Specifically, these authors define M_t as the stock remaining at the end of time t (after extraction). This difference is immaterial since we both focus on balanced growth path along which M_t depletes at a constant rate.

resources then evolves over time according to

$$M_{t+1} = M_t - X_t = (1 - \tau_t)M_t. \tag{2.14}$$

2.2.4 Competitive Equilibrium

The competitive equilibrium can be defined as follows.

Definition 2.1. *Given the initial conditions: $K_0 > 0$ and $M_0 > 0$, a competitive equilibrium of this economy includes sequences of allocation $\{c_{1,t}, c_{2,t+1}, s_t, m_t\}_{t=0}^{\infty}$, aggregate inputs $\{K_t, N_t, X_t\}_{t=0}^{\infty}$, natural resources $\{M_t\}_{t=0}^{\infty}$ and prices $\{w_t, R_t, p_t, r_{t+1}\}_{t=0}^{\infty}$ such that,*

(i) Given prices, $\{c_{1,t}, c_{2,t+1}, s_t, m_t\}$ solves the consumer's problem in each period $t \geq 0$.

(ii) Given prices, $\{K_t, N_t, X_t\}$ solves the firm's problem in each period $t \geq 0$.

(iii) The stock of natural resources evolves according to (2.14).

(iv) All market clear in every period, i.e., $K_{t+1} = N_t s_t$ and $M_{t+1} = N_t m_t$ for all $t \geq 0$.

2.3 Balanced Growth Path Competitive Equilibria

2.3.1 Theoretical Results

As in AGI, we focus on a balanced growth path (BGP) competitive equilibrium which is defined as follows.

Definition 2.2. *A BGP competitive equilibrium is a competitive equilibrium that satisfy three additional conditions:*

(v) Output per worker (Y_t/N_t) grows at a constant rate $\gamma^* - 1$, for some $\gamma^* > 0$.

(vi) The physical capital-output ratio is constant over time, i.e., $K_t = \kappa^* Y_t$, for some $\kappa^* > 0$.

(vii) The rate of return from physical capital is constant over time, i.e., $r_t = r^*$ for some $r^* > 0$.

(viii) The extraction rate of non-renewable resources is positive and constant over time, i.e., $\tau_t = \tau^*$, for some $\tau^* \in (0, 1)$.

Strictly speaking, a BGP competitive equilibrium is a competitive equilibrium along which output per worker grows steadily, capital to output ratio and the rate of return on capital are constant and the extraction rate is time-invariant.

Combined with Assumption 2.1, Assumption 2.2 and the BGP restrictions (v)-(vii), a competitive equilibrium (if exists) constitutes the long-run trajectories of all quantities and prices. Conditions (v) and (vi) are very common restrictions in the literature of long-run economic growth: these conditions imply that the long-run behavior of the model is consistent with the long-term economic growth observations made by Kaldor (1961) and many subsequent studies.[11] Taken together, these two conditions imply that Y_t and K_t must be growing steadily at the common growth rate $\gamma^*(1+n)$ in any BGP competitive equilibrium, i.e.,

$$\frac{K_{t+1}}{K_t} = \frac{Y_{t+1}}{Y_t} = \gamma^*(1+n)$$

Condition (viii) is a common assumption in economic growth models with natural resources.[12] An immediate implication of this is that X_t and M_t must be decreasing at the same rate $(1-\tau^*)$ in any BGP competitive equilibrium, i.e.,

$$\frac{X_{t+1}}{X_t} = \frac{M_{t+1}}{M_t} = 1-\tau^*.$$

Before proceeding any further, it will prove useful to review some fundamental results in AGI in order to create comparison between our results and theirs. In AGI study, they propose the Cobb-Douglas production function:

$$Y_t = B_t K_t^{\alpha} N_t^{\beta} X_t^{v}, \quad \alpha, \beta, \gamma > 0 \quad \alpha+\beta+\gamma = 1$$

where B_t refers to total factor productivity (TFP) and is assumed to grow at an exogenous constant rate b. Based on this specification, a BGP competitive equilibrium exists if there exists a stationary growth rate $\gamma^* > 0$ and a stationary extraction rate $\tau^* \in (0,1)$ satisfying

$$\frac{\gamma^*(1+n)}{1-\tau^*} = \frac{\alpha(1+n)(2+\theta)\gamma^*}{\beta - (2+\theta)v\left(\frac{1-\tau^*}{\tau^*}\right)} + 1 - \delta, \tag{2.15}$$

$$\gamma^* = (1+b)^{\frac{1}{1-\alpha}} \left(\frac{1-\tau^*}{1+n}\right)^{\frac{v}{1-\alpha}}. \tag{2.16}$$

As illustrated in their Lemma 1 and Proposition 1, a BGP competitive equilibrium exists and unique.[13] In addition, the stationary extraction rate is strictly greater than $\frac{(2+\theta)v}{\beta+(2+\theta)v} \equiv \overline{\tau}_{AGI}$ and less than one, i.e., $\tau^* \in (\overline{\tau}_{AGI}, 1)$. Once (τ^*, γ^*) is obtained, we can get the steady state real interest rate r^* and the stationary capital to output ratio κ^*:

$$1+r^* = \frac{\gamma^*(1+n)}{1-\tau^*} \quad \text{and} \quad \kappa^* = \frac{\alpha}{r^*+\delta}. \tag{2.17}$$

In the sequel, we will refer to this as the AGI solution.

A crucial implication of the AGI solution is that long-run economic growth engine is endogenous. The long-run growth rate of output per worker is endogenously determined

[11] According to Kaldor's facts, in an economy, (i) the growth rate of output, (ii) the capital-labor ratio, (iii) the capital-income ratio, (iv) capital and labor shares of income and (v) the rate of return on capital are constant (in an approximate sense) over a long period of time. For subsequent studies, see Jones and Romer (2010), for example.

[12] See, Stiglitz (1974a), Barbier (1999), Grimaud and Rouge (2003), Groth and Schou (2007), among many others.

[13] See Agnani et al. (2005, p.394-395).

by a number of factors; including the TFP growth rate (b), population growth rate (n), depreciation rate (δ), the share of factor incomes in total output (α, β, v), and the consumers' rate of time preference (θ).

For clarity, consider two alternative economies. For the AGI economy, if we decompose B_t according to $B_t \equiv Q_t^v A_t^\beta$ and define $\hat{k}_t \equiv \frac{K_t}{A_t N_t}$ as physical capital per effective unit of labour, and $\hat{x}_t \equiv \frac{Q_t X_t}{A_t N_t}$ as effective unit of resource input. Then, the AGI solution implies that

$$\frac{\hat{k}_{t+1}}{\hat{k}_t} = \Big(\frac{\hat{x}_{t+1}}{\hat{x}_t}\Big)^{\frac{v}{1-\alpha}} = \Big[\frac{(1+q)(1-\tau^*)}{(1+a)(1+n)}\Big]^{\frac{v}{1-\alpha}}, \tag{2.18}$$

where $\frac{A_{t+1}}{A_t} \equiv 1+a$ and $\frac{Q_{t+1}}{Q_t} \equiv 1+q$. Depending on (2.15) - (2.16), $\hat{k}_t$ and $\hat{x}_t$ can be monotonically increasing, monotonically increasing or constant over time along the unique BGP competitive equilibrium. Since the output per worker is:

$$\frac{Y_t}{N_t} = A_t \hat{k}_t^\alpha \hat{x}_t^v,$$

the growth rate of output per worker is not necessarily equal to that of the factor A_t. In particular, if the solution of (2.18) satisfies $(1+q)(1-\tau^*) > (1+a)(1+n)$, then the long-term growth rate under the AGI solution is strictly greater than $1+a$. Alternatively, consider an economy without productive natural input, i.e., an economy with $v = 0$ in AGI production. In this economy, non-renewable resource is not essential in the production process and $B_t \equiv A_t^{1-\alpha}$.[14] In any BGP competitive equilibrium, the constancy of r_t immediately implies a constant $\hat{k}_t$. As a result, capital per worker and output per worker must be growing at the rate $\frac{A_{t+1}}{A_t} \equiv 1+a$, so that $\gamma^* = 1+a$. Obviously, this is in stark contrast to the results from AGI. Nevertheless, this result is nothing new. When natural input necessity is muted, the AGI model turns out to be a standard OLG model with exogenous technological progress. As is widely known, the neoclassical growth model predicts that, in the long run, output per capita will grow at the exogenously determined rate of technological progress.[15]

An interesting question is whether the endogenous growth feature is still preserved under more general production function specifications. To answer the question, we introduce a more generalised functional form within the class of two-level CES production functions into the general form (2.8). We, then, show our main findings in Theorem 2.1 and Theorem 2.2.

To begin with, let us briefly review the CES functional form. Define $g(\hat{x}) \equiv G(\hat{x}, 1)$, for all $\hat{x} \geq 0$. Under Assumption 2.1, $g(\cdot)$ is twice continuously differentiable, and satisfies

[14]It follows immediately that $\tau_t = \tau^* = 0$ for all t.

[15]This result holds in both overlapping generations models and models with infinitely lived consumers.

$g'(\cdot) > 0$ and $g''(\cdot) < 0$. Due to the linear homogeneity of $G(\cdot)$, we can write

$$G(QX, AN) = AN \cdot g(\hat{x}).$$

As shown in Arrow et al. (1961), Palivos and Karagiannis (2010) and Moysan and Senouci (2016) , the elasticity of substitution of $G(\cdot)$ can be expressed as[16],

$$\sigma_G(\hat{x}) = -\frac{g'(\hat{x})}{\hat{x}g(\hat{x})} \cdot \frac{g(\hat{x}) - \hat{x}g'(\hat{x})}{g''(\hat{x})} > 0, \quad \text{for all} \quad \hat{x} > 0. \tag{2.19}$$

If $\sigma_G(\cdot)$ is a constant function, say $\sigma_G(\hat{x}) = \frac{1}{1-\psi}, \psi < 1$, for all $\hat{x} > 0$, the $G(\cdot)$ takes the CES form, i.e.,

$$G(QX, AN) = \Big[\varphi(QX)^{\psi} + (1-\varphi)(AN)^{\psi}\Big]^{\frac{1}{\psi}}, \quad \text{with} \quad \varphi \in (0,1) \quad \text{and} \quad \psi < 1. \tag{2.20}$$

If, in addition, $\psi = 0$ so that $\sigma_G(\cdot)$ is identical to unity, then $G(\cdot)$ has the Cobb-Douglas form. We can illustrate the CES form of $F(\cdot)$ using the same argument.

In what follows, we will state two main theorems of our study. In Theorem 2.1, we will show that if $\sigma_G(\cdot)$ is equal to one, then the long-term growth factor γ^* and the extraction rate τ^* can be determined similarly as in the AGI solution. In this case, endogenous growth feature is preserved while the Cobb-Douglas specification of $F(\cdot)$ is sufficient but not necessary. This finding provides a partial generalisation of AGI result. However, Theorem 2.2 show that if $\sigma_G(\cdot)$ differs from one, then any balanced growth equilibrium (if exists) must satisfy $\gamma^* = (1+a)$ and $(1+q)(1-\tau^*) = (1+a)(1+n)$. This means that γ^* and τ^* are determined exogenously and, thus, the AGI solution is no longer valid.

Theorem 2.1. *Suppose that Assumptions 2.1 and 2.2 are satisfied and $G(\cdot)$ takes the following Cobb-Douglas form:*

$$G(Q_tX_t, A_tN_t) = (Q_tX_t)^{1-\phi}(A_tN_t)^{\phi}, \quad \textit{with} \quad \phi \in (0,1). \tag{2.21}$$

Then any BGP competitive equilibrium (if exists) must satisfy

$$\gamma^* = (1+a)^{\phi}\Big[\frac{(1+q)(1-\tau^*)}{1+n}\Big]^{1-\phi}, \tag{2.22}$$

$$F_1(1, \chi^*) = r^* + \delta, \tag{2.23}$$

$$(1+r^*)(1-\tau^*) = \gamma^*(1+n), \tag{2.24}$$

[16] As explained in (Arrow et al., 1961, p. 228–229) this expression is derived under two assumptions: (i) both the factor markets and goods markets are competitive and (ii) $G(\cdot)$ exhibits CRTS. Both assumptions are satisfied in our model.

$$\gamma^*(1+n) = \chi^* F_2(1,\chi^*)\Big[\frac{\phi}{2+\theta} - \Big(\frac{1-\tau^*}{\tau^*}\Big)(1-\phi)\Big], \tag{2.25}$$

where $\chi_t \equiv \frac{\hat{x}_t^{1-\phi}}{\hat{k}_t}$. *In addition, if a BGP competitive equilibrium exists, then*

$$\tau \in (\overline{\tau}, 1)$$

where $\overline{\tau} \equiv \frac{(2+\theta)(1-\phi)}{\phi+(2+\theta)(1-\phi)} \in (0,1)$.

Theorem 2.1 imposes four necessary conditions that must be jointly fulfilled in any BGP competitive equilibrium. In effect, the theorem provides a system of non-linear equations that can be used to characterise the steady state values of four key variables including the growth factor of per-worker output (γ^*), the extraction rate of natural resources (τ^*), the net rate of return on physical capital (r^*) and the transformed variable χ_t (χ^*). If a BGP competitive equilibrium exists, the trajectories of all other variables in that equilibrium can be derived using these four values. Similar to the AGI solution, the existence of a BGP competitive equilibrium implies that the steady state extraction rate τ^* must be strictly greater than a certain threshold $\overline{\tau}$.

If $F(\cdot)$ takes the CD form as in AGI, then

$$\chi^* = \Big(\frac{r^*+\delta}{\alpha}\Big)^{\frac{1}{1-\alpha}} \quad \text{and} \quad \chi^* F_2(1,\chi^*) = \frac{1-\alpha}{\alpha}(r^*+\delta).$$

Upon substitution these into the system (2.22) - (2.25) and setting $\phi = \frac{\beta}{1-\alpha}$ and $1-\phi = \frac{v}{1-\alpha}$, we can obtain

$$\frac{\alpha(2+\theta)(1+n)\gamma^*}{\beta-(2+\theta)v\Big(\frac{1-\tau^*}{\tau^*}\Big)} = r^*+\delta = \frac{\gamma^*(1+n)}{1-\tau^*} - (1-\delta),$$

which is the same equation that appears in AGI's Lemma 1 part (i). Their Proposition 1 then establishes the existence and uniqueness of a BGP competitive equilibrium.

We now show that similar results exist when $F(\cdot)$ is a CES function with elasticity of substitution greater than one.

Proposition 2.1. *Suppose* $F(\cdot)$ *is given by*

$$F(K_t, Z_t) = \Big[\alpha K_t^{\eta} + (1-\alpha) Z_t^{\eta}\Big]^{\frac{1}{\eta}}, \tag{2.26}$$

with $\alpha \in (0,1)$ *and elasticity of substitution* $\sigma_F \equiv (1-\eta)^{-1} \geq 1$. *Suppose* $G(\cdot)$ *takes the Cobb-Douglas form as in (2.21). The the economy has at least one BGP competitive* *equilibrium. If, in addition,*

$$\left[(1+a)^{\phi}(1+q)^{1-\phi}\left(\frac{1+n}{1-\overline{\tau}}\right)^{\phi}-(1-\delta)\right]^{\eta}>\alpha(1-\eta)^{1-\eta} \tag{2.27}$$

where $\overline{\tau}$ is defined as stated in Theorem 2.1, then a unique BGP competitive equilibrium exists.

Under certain situations, the inequality (2.26) is always satisfied. For example, when $\eta = 0$ the condition (2.26) is immediately implied. In this case our and AGI economies turned out to be identical. This proposition subsumes the AGI solution as a special case. Thus, assuming double Cobb-Douglas is sufficient but not necessary when concerning endogenous growth feature induced by an introduction of productive non-renewable resources. Another example is when capital fully depreciates at the end of each period; $\delta = 1$, which is a common assumption in OLG models.

However, it seems to be more difficult to verify whether a BGP competitive equilibrium exists and unique when $\sigma_F < 1$; or equivalently $\eta < 0$. In this situation, slight changes in $\sigma_F < 1$ can often lead to drastic changes in the results. We will demonstrate this via the following numerical example. To do so, we combine (2.21)-(2.26) to form a single non-linear equation of τ. A BGP competitive equilibrium exists if there is $\tau^* \in (\overline{\tau}, 1)$ satisfying:

$$\underbrace{\frac{(2+\theta)(1+a)^{\phi}(1+q)^{1-\phi}(1-\tau)^{1-\phi}(1+n)^{\phi}}{\phi-\left(\frac{1-\tau}{\tau}\right)(2+\theta)(1-\phi)}}_{\equiv\Lambda(\tau)}=\underbrace{\left[\left(\frac{r(\tau)+\delta}{\alpha}\right)^{\frac{\eta}{1-\eta}}-\alpha\right]\frac{r(\tau)+\delta}{\alpha}}_{\equiv\Gamma(\tau)} \tag{2.28}$$

where $r(\tau) \equiv (1+a)^{\phi}(1+q)^{1-\phi}(1-\tau)^{-\phi}-1$. Next, we assign value to the model parameters. Suppose that one period lasts for 25 years. We set $\theta = 1.775$ to ensure the annual subjective discount factor of 0.96. As the average annual employment growth rate of U.S. employment during 1953 and 2008 is 1.6 %, we choose $n = (1 + 0.016)^{25} - 1 = 0.4857$. The average annual growth rate of TFP is taken to be 1.05%. The estimate is based on Feng and Serletis (2008) who estimate productivity measurement in United States manufacturing based on annual data over the period from 1953 to 2001. The implied value of $(1+a)^{\phi}(1+q)^{1-\phi}-1$ is $(1+0.0105)^{25}-1 = 0.2984$. Also, we set $\delta = 1, \phi = 0.38$ and $\alpha = 0.24$. Finally, we assign $\sigma_F = 0.62$ and 0.65. These values are chosen according to empirical studies concerning estimations of CES production functions with capital, labour and energy. Some of the estimations are summarised in Henningsen et al. (2019, Table 4).[17] Once all the parameter are assigned their value, we can verify numerically whether the economy possesses its BGP or not.

[17] In Henningsen et al. (2019, Table 4), the elasticity of substitution between the inputs of $F(\cdot)$ is denoted by $\sigma_{(LE)K}$. Also, in the existing empirical studies, it is conventional to use commercial energy consumption as a proxy for natural resource input. See section 3.3 for a brief discussion on the empirical literature.

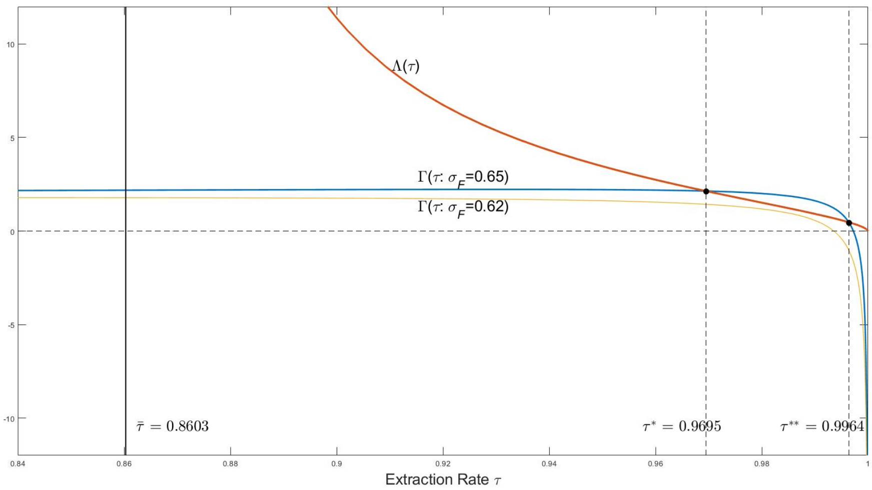

Figure 2.1: *Existence of BGP competitive equilibria when $\sigma_F < 1$. Parameter values are:* $\theta = 1.775, n = 0.4857, (1+a)^{\phi}(1+q)^{1-\phi} - 1 = 0.2984, \delta = 1, \phi = 0.38, \alpha = 0.24, \sigma_F = 0.62, 0.65$.

The numerical example is displayed in Figure 2.1. In the figure, we plots $\Lambda(\tau)$ and $\Gamma(\tau)$ under two different values of σ_F, namely 0.62 and 0.65. As can be seen in the diagram, (2.28) has no solution when $\sigma_F = 0.62$ (or equivalently, $\eta =$ -0.6129). This means that there is no BGP competitive equilibrium in this situation. In contrast, when $\sigma = 0.62$ (or equivalently, $\eta =$ -0.5385), (2.28) has two solutions including $\tau^* = 0.9695$ and $\tau^{**} = 0.9964$. In this situation, two BGP competitive equilibria exist.

Before moving forward, the first finding is reminded here. When considering a two-level CES production technology where the inner function combines effective natural input and effective labour while the outer one combines capital with corresponding input derived from the inner function, we have shown that restricting the elasticity of substitution of the inner production to unitary plays a crucial role in preserving the AGI solution of neoclassical growth model with productive non-renewable resource. This result is valid no matter what is the assigned value of the elasticity of substitution of the outer production. An interesting question is what will happen when we deviate the elasticity of substitution of the inner production from one. This is the next task we are going to investigate.

We now turn to the case when $\sigma_G(\hat{x}) \neq 1$ for all $\hat{x} > 0$. The following theorem provide the main finding in this case.

Theorem 2.2. *Suppose that Assumption 2.1 and 2.2 are satisfied. Suppose that the elasticity of substitution of $G(\cdot)$ is never equal to one. Then, any balanced growth path (if exists) must satisfy*

$$\gamma^* = (1+a), \quad r^* = q, \quad \textit{and} \quad 1 - \tau^* = \frac{(1+a)(1+n)}{1+q} \tag{2.29}$$

In addition, such an equilibrium will have $\hat{k}_t = \hat{k}^*$ and $\hat{x}_t = \hat{x}^*$ for all t, where $\hat{k}^*$ and $\hat{x}^*$ are jointly determined by

$$F_1\Big(\hat{k}^*, G(\hat{x}^*, 1)\Big) = q + \delta, \tag{2.30}$$

$$(1+a)(1+n)\hat{k}^* = F_2\Big(\hat{k}^*, G(\hat{x}^*, 1)\Big)\Big[\frac{G_2(\hat{x}^*, 1)}{2+\theta} - \Big(\frac{1-\tau^*}{\tau^*}\Big)\hat{x}^* G_1(\hat{x}^*, 1)\Big]. \tag{2.31}$$

An explanation of Theorem 2.2 is as follows. If the elasticity of substitution of $G(\cdot)$ is not equal to one; i.e. if $G(\cdot)$ is not a Cobb-Douglas function, then either there is no BGP competitive equilibrium or any BGP competitive equilibria will always have a common growth rate in per-capita variables that is solely determined by the exogenous growth factor A_t. Thus, endogenous growth feature as in AGI does not hold. In other words, there is no room for the AGI solution. The theorem also clarifies two major differences between the two exogenous growth factors A_t and Q_t. Firstly, the growth rate of per worker variables are entirely determined by the growth rate of A_t, while the real interest rate is entirely determined by the growth rate of Q_t. Secondly, all else unchanged, a higher growth rate in A_t will suppress the extraction rate τ^* whereas a higher growth rate in Q_t will promote it. Since τ^* must be confined between zero and one, it is necessary to impose the restriction $1+q > (1+a)(1+n)$. Note that the solution in Theorem 2.2 is still valid even if there is no population growth (i.e., $n=0$) and no labour-augmenting technological improvement (i.e., $a=0$). But the growth rate of resource-augmenting technological factor Q_t must remain strictly positive. This shows that a sufficiently high growth rate of this factor is most crucial for this type of solution.

We use specific forms in here to prove existence and uniqueness. To do so, let both $F(\cdot)$ and $G(\cdot)$ take the CES forms (2.26) and (2.20), respectively. But here we do not impose any restriction on $\sigma_F = (1-\eta)^{-1}$. Define the notation Θ according to

$$\Theta \equiv \frac{q+\delta}{\alpha(2+\theta)}\left[\Big(\frac{q+\delta}{\alpha}\Big)^{\frac{\eta}{1-\eta}} - \alpha\right].$$

Then, a characterization of the existence and uniqueness of a BGP competitive equilibrium is summarized in the following proposition.

Proposition 2.2. *Suppose that $F(\cdot)$ takes the CES form in (2.26), with $\alpha \in (0,1)$ and $\eta < 1$, and $G(\cdot)$ takes the CES form in (2.20). Suppose further that min$\{\Theta, 1+q\} >$ $(1+a)(1+n)$. Then, there exists a unique BGP competitive equilibrium that satisfies (2.29)-(2.31).*

It is worth noting that the above result covers the special case in which $F(\cdot)$ and $G(\cdot)$ have the same constant elasticity of substitution, i.e., $\psi = \eta$. In this case, the overall production becomes

$$Y_t = \Big[\alpha K_t^\eta + (1-\alpha)\varphi(Q_tX_t)^\eta + (1-\alpha)(1-\varphi)(A_tN_t)^\eta\Big]^{\frac{1}{\eta}}$$

which is a familiar a Dixit-Stiglitz aggregator.

2.3.2 Alternative Specifications of Production Function

In this subsection we consider two alternative ways of combining the three inputs in the composite good production. These alternative specifications are:

$$Y_t = F\Big(A_tN_t, G(K_t, Q_tX_t)\Big), \tag{2.32}$$

$$Y_t = F\Big(Q_tX_t, G(K_t, A_tN_t)\Big). \tag{2.33}$$

As can be seen, we still use $G(\cdot)$ to represent the "inner" aggregate function and $F(\cdot)$ to represent the "outer" aggregate function in (32) and (33). This primitive is nothing but used in order for consistency preservation across all three specifications. If both $G(\cdot)$ and $F(\cdot)$ take the Cobb-Douglas form, then the production functions (2.8), (2.32) and (2.33) coincide with AGI's production function. The long-run endogenous growth feature arises in these specifications. However, an interesting question is whether the long-run endogenous growth feature is still preserved if either $G(\cdot)$ or $F(\cdot)$ but not both is allowed to take the Cobb-Douglas form.

In order to investigate this, we consider four different parametric production functions based on (2.32) and (2.33). In the first two specifications, the inner aggregator function is Cobb-Douglass but the outer one is given by a CES function, so that

$$Y_t = \left\{\varphi(A_tN_t)^\psi + (1-\varphi)\Big[K_t^\alpha(Q_tX_t)^{1-\alpha}\Big]^\psi\right\}^{\frac{1}{\psi}}, \tag{2.34}$$

$$Y_t = \left\{\varphi(Q_tX_t)^\psi + (1-\varphi)\Big[K_t^\alpha(A_tN_t)^{1-\alpha}\Big]^\psi\right\}^{\frac{1}{\psi}}, \tag{2.35}$$

with $\alpha \in (0,1)$, $\varphi \in (0,1)$ and $\psi < 1$. For the other two specifications, the inner aggregator function is a CES function but the outer one is given by a Cobb-Douglas function:

$$Y_t = \Big[\varphi K_t^\psi + (1-\varphi)(Q_tX_t)^\psi\Big]^{\frac{1-\beta}{\psi}}(A_tN_t)^\beta, \tag{2.36}$$

$$Y_t = (Q_tX_t)^v\Big[\varphi K_t^\psi + (1-\varphi)(A_tN_t)^\psi\Big]^{\frac{1-v}{\psi}}, \tag{2.37}$$

with $\beta \in (0,1)$, $v \in (0,1)$, $\varphi \in (0,1)$ and $\psi < 1$.[18] Even though the functional forms (2.34) - (2.37) may look very different in appearance, they all share the same implications for the long-run growth engine. In particular, the existence of any BGP competitive equilibrium would imply that $\gamma^* = 1 + a$, $r^* = q$ and $(1 - \tau^*) = (1 + a)(1 + n)/(1 + q)$. It follows that the two transformed variables $\hat{k}_t$ and $\hat{x}_t$ are time-invariant in this type of equilibrium. Strictly speaking, the long-run endogenous growth feature does not hold in these cases. These results are formally stated in the following theorem.

Theorem 2.3. *Suppose that the production function takes one of the forms in (2.34)-(2.37). Then any balanced growth equilibrium (if exists) must satisfy $\gamma^* = 1 + a$, $r^* = q$, and*

$$(1 - \tau^*) = \frac{(1 + a)(1 + n)}{1 + q}.$$

2.3.3 Discussions

Our analysis in sub-sections 2.3.1 and 2.3.2 reveal that the AGI solution will be valid in a more generalised OLG model with productive non-renewable resources only if the elasticity of substitution between labour and natural resource is constant and equal to one. If we write the function (21) as

$$G(Q_t X_t, A_t N_t) = \left[A_t (Q_t X_t)^{\frac{1-\phi}{\phi}} N_t\right]^{\phi}$$

then the variable $\tilde{X}_t \equiv A_t (Q_t X_t)^{\frac{1-\phi}{\phi}}$ can be viewed as a labour augmenting technological factor. This means that the AGI solution is preserved in our more generalised model only when the effective unit of natural input is labour augmenting in the final good production. In particular, the composite good production function must take any form that is consistent with

$$Y_t = F\Big(K_t, (\tilde{X}_t N_t)^{\phi}\Big).$$

Our finding seems to be consistent with Uzawa's Steady State Growth Theorem (Uzawa, 1961) which states that if a neoclassical growth model (and its variants) exhibits BGP, then technical change must be labour-augmenting, at least along the BGP.

Nevertheless, there are at least two key differences between our study and the Uzawa Growth Theorem. Firstly, the Uzawa Growth Theorem and its variants are typically derived from a CRTS production function with only two inputs, namely capital and labour (Uzawa, 1961; Schlicht, 2006; Jones and Scrimgeour, 2008; Grossman et al., 2017; Irmen, 2018, among others). It is not immediately clear how the Uzawa Growth Theorem can be applied when a CRTS production has more than two inputs, such as the one studied here.

[18] The parameters β and v have the same economic meaning as in AGI. Specifically, they represent the share of total output distributed as labour income and spent on natural resource input, respectively.

Secondly, and more importantly, in our analysis we show that under certain conditions the standard neoclassical growth with productive non-renewable resources can exhibit endogenous growth feature. This is in contrast with the Uzawa Growth Theorem and the related studies as these studies aim at showing conditions under which a BGP competitive equilibrium exists, regardless of whether the engine of growth is exogenous or endogenous.

An interesting question is whether the elasticity of substitution between labour and natural input is unity. There are a number of empirical studies providing estimated values of the elasticities of substitution between capital, labour and energy of nested CES production functions.[19] Regarding the elasticity of substitution for the nest between labour and energy, these studies usually report a less-than-unity elasticity of substitution [see, for example, Kemfert (1998), Kemfert and Welsch (2000) and van der Werf (2008)].

2.4 Conclusion

In this study we investigate under which conditions the long-run engine of growth in neoclassical growth model with productive non-renewable resources will be endogenous. While many studies suggest that deep parameters in both preference and production sides could affect the long-run behaviour, we have shown that this endogenous growth feature is implied only if production function is formed in certain ways. In particular, the combination of effective labour and effective flow of non-renewable resources in the basis of our Cobb-Douglas is necessary. Since the interpretation strongly depends on model specifications, policy implications should come with reasonable choice of functional forms that claims to supporting empirical evidence.

[19] See van der Werf (2008) and Henningsen et al. (2019) for literature review and discussions on different estimation strategies.

Chapter 3

Natural Resource Substitution and Long-Run Economic Growth: the Role of Labour Allocation Effect

3.1 Introduction

There is no doubt that natural resources are essential in the production process for any economy. To produce a unit of output, it is unavoidable to exploit an amount of natural resources as fuel or intermediate inputs. The overwhelming majority of natural resources consumption are non-renewable such as oil, coal, natural gas, etc (BP, 2019). To illustrate this, the data provided by Eurostat (Eurostat, 2019) and U.S. Energy Information Administration (EIA, 2019) show that more than 85 percent of final energy consumption within 28 countries in the EU and the US between 1990 and 2016 had been provided by non-renewable resources. Since the entire stock of non-renewable resource is finite, a serious concern for our society is how to overcome this constraint to maintain a sustainable growth in per-capita output. One possibility is to rely more heavily on renewable resources such as solar and wind. There are signs that this shift is already underway. For example, the EU has planned that by 2030, 27.0 percent of gross final energy consumption will be generated by renewable resource (EU, 2018). The figure in 2016 was 14.2 percent. As evident from Figure 3.1, a rising importance in renewable resources can also be seen in the U.S. data.

Even though the importance of renewable energy is widely acknowledged, changes are likely to be slow in some countries. For instance, BP (2019) projects that the EU will be the world leading renewable resource user in coming decades. In particular, it is estimated that the share of renewable resource in primary energy consumption will be 29 percent in this region in the year 2040. However, the share for other major economies are projected

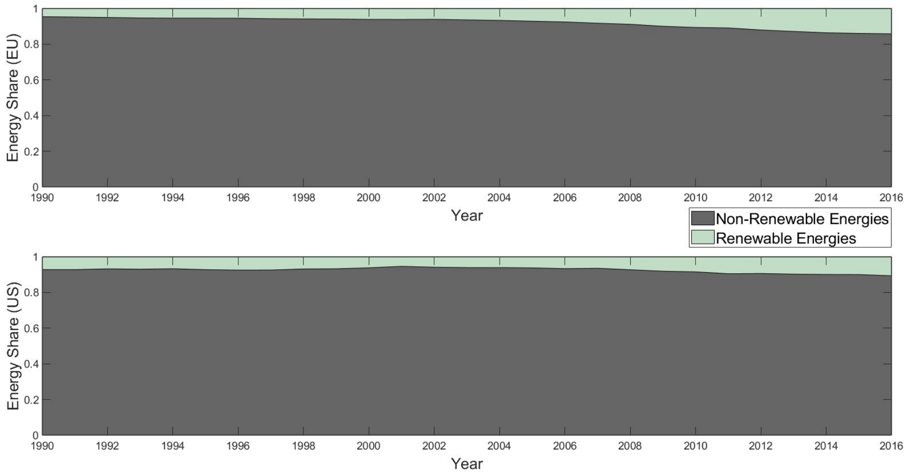

Figure 3.1: *Natural resource shares in final energy consumption in the EU and the US between 1990 and 2016.*

Source: *Eurostat and EIA.*

to be much lower (18 percent for the U.S. and China, and 16 percent for India). At least part of these difference is likely due to technical difficulties in substituting between non-renewable and renewable resources. This raises the question of how the substitutability between these two resources will affect long-run economic growth. The main objective of this chapter is to analyse this question theoretically using a dynamic general equilibrium model.

At the heart of our analysis is a constant-elasticity-of-substitution (CES) aggregator that combines renewable and non-renewable resources in the production function. The ease or difficulty in substituting between these two resources is thus captured by a single parameter, namely the elasticity of substitution. There are now a number of growth-theoretic studies that adopt a similar CES function to combine the flows of renewable and non-renewable resources. These studies mainly focus on three aspects of the theoretical model. The first aspect is the role of the elasticity of substitution on the potential trajectories of the two natural inputs (Andre and Cerda, 2005). The second aspect, analysed by Silva et al. (2013), Golosov et al. (2014) and Engström and Gars (2016), concerns the dynamic general equilibrium impact of pollution externalities generated by non-renewable resource extraction. These works allow for various possibilities of natural resource substitution in the CES aggregator and show numerically how the degree of natural resource substitution will affect macroeconomic variables along the transitional path. Finally, Growiec and Schumacher (2008) have analysed how an exogenous technical change in the elasticity of substitution between the two natural resources will affect the short and long-term trajectories of macroeconomic variables. This study, however, assumes that these two types of natural inputs are perfect substitute in the long run. Thus, there is no room for comparing two economies that differ in the elasticity of substitution. In sum, the growth effect of natural resource substitution remains largely unexplored in the existing literature. Our study attempt to fill in this research gap.

For this purpose, we develop two discrete-time endogenous growth models with natural resource substitution to examine how changes in the degree of natural resource substitutability affects long-run economic growth. These two models share the following common features. In both economies, there are three production sectors, namely the final-good sector, the non-renewable resource extraction sector, and the renewable resource extraction sector. Producers in final-good sector rent capital, hire labour, and buy energy from perfectly competitive input markets to produce final output. Energy input is produced by the two natural inputs via a CES aggregator. The natural resources are extracted by private firms using labour as input. The two models differ only in the source of long-run economic growth. In the first setup, perpetual growth in per-capita variables is made possible by the positive externalities created by learning-by-doing as in Romer (1986). In the second model, perpetual growth is made possible by the introduction of productive government spending as in Barro (1990). One novelty of our analysis is that the allocation of labour across three sectors is explicitly considered. This specification allows the models to be more realistic as we can capture the evidence of labour allocation among final-good sector and the two natural resource extraction sectors.

The present study is close in spirit to Golosov et al. (2014) but differs from it in two important ways. Firstly, economic growth in their model is driven by exogenous improvements in total factor productivity, whereas in our models economic growth is endogenously determined. This allows us to examine how changes in the degree of natural resource substitutability would affect the long-term growth via adjusting the rate of return on capital. Secondly, we do not consider any negative externalities (pollution) created by non-renewable resources. In this context, the long-run growth effect (if exists) resulting from changing in the elasticity of substitution will be driven by scarcity problem not pollution externalities.

The present study is also related to the work by Grandville (1989) and Klump and de La Grandville (2000). These authors argue that a normalisation procedure is necessary when analysing the potential influences of the elasticity of substitution on long-run economic growth. This normalisation procedure has been frequently adopted in macroeconomic research to address a wide range of issues related to capital-labour substitution. Nevertheless, to the best of our knowledge, this method has not yet been applied in natural resource substitution analysis.

Our main findings can be summarised as follows. When economic growth is driven by externalities and learning-by-doing, we show that changing the degree of natural resource substitutability will not affect the long-run allocation of labour across sectors. However, this does not mean that changing the elasticity of substitution will not affect economic growth. In fact, we show numerically that when we do not normalise the CES function, an increase in the degree of natural resource substitutability will promote economic growth when the share of labour employed in renewable resource extraction is sufficiently high. But once we adopt the normalisation procedure, the long-run growth rate is independent of the degree of natural resource substitutability.

In the model with productive government spending, changing in the elasticity of substitution creates a labour allocation effect, i.e. equilibrium labour allocation is changed in response to such parameter changes. This effect continues to hold when normalisation is imposed. As a result, long-run growth effect remains. Under plausible parameter values, we can numerically show that an increase in the elasticity of substitution can exert positive influences on economic growth if the labour share devoted to renewable resource extraction sector is sufficiently high, the two natural inputs are gross substitute and the growth rate of non-renewable resource augmenting technology is strictly positive but not too large.

This chapter is structured as follows. The second section presents a discrete-time version of learning-by-doing endogenous growth model with natural resource substitution. In the third section, we examine the growth effect via the normalisation procedure. Then, the fourth section introduces a discrete-time version of productive government spending endogenous growth model and then examine the growth effect via the normalisation procedure. Some discussions will be addressed in the fifth section. The conclusion reassesses the main findings of the chapter.

3.2 Benchmark Model: A Romer (1986) Model with Natural Resource Substitution

In this section, we develop a discrete-time version of endogenous growth model à la Romer (1986) where natural resource input is essential in the final-good production process. After setting up the model, we characterise conditions that are necessary for existence of a balanced growth path. Also, we provide sufficient conditions for the existence of a unique balanced growth path. Lastly, we analyse analytically the impact of changes in the degree of natural resource substitution on the long-run economic growth.

3.2.1 The Model Setting

Time is discrete with an infinite horizon. We assume that there are two types of agents: households and firms. The households are identical and infinitely-lived, and they decide how much to consume in each period in order to maximise their lifetime utilities. Firms are operating in three sectors including final-good sector, renewable resource extraction sector and non-renewable resource extraction sector. The firms in the final-good sector produce a homogeneous product that can be either consumed or invested. The firms in the renewable resource extraction sector produce renewable energy used as an input in the final-good sector. The firms in non-renewable resource extraction sector produce non-renewable energy used as an input in the final-good sector. All markets are perfectly competitive. The model nests the Romer (1986) framework with strong spillovers in the final-good production at the aggregate level.

Households

The size of the representative household is constant over time and normalised to one. In each period, the household supplies one unit of labour endowment to labour market inelastically. In the initial period, $t = 0$, it has capital endowment $K_0 > 0$. In addition, we assume that the household owns all firms in the economy and these firms return profits to the household in every period.

Denote by c_t the individual's consumption at time t. Individual utility is defined over sequences of consumption $\{c_t\}_{t=0}^{\infty}$. Assuming instantaneous log-utility with constant discount factor $\beta \in (0, 1)$, the preference of the representative household is represented by the utility function

$$U_0 = \sum_{t=0}^{\infty} \beta^t lnc_t. \tag{3.1}$$

Denote by π_t the aggregate profit at time t, w_t the real wage at time t, $\delta \in [0, 1]$ the depreciation rate of capital and R_t the rental price of capital at time t. The household's sequential budget constraint is

$$c_t + K_{t+1} - (1 - \delta)K_t = R_t K_t + w_t + \pi_t. \tag{3.2}$$

Equation (3.2) states that in each period a typical household will allocate income to either consumption, c_t, or saving in terms of gross investment, $K_{t+1} - (1 - \delta)K_t$.

Given the sequence of prices $\{w_t, R_t\}_{t=0}^{\infty}$ and initial asset K_0, the household solves

$$\max_{\{c_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t lnc_t$$

subject to Equation (3.2) and non-negativity constraints; $c_t, K_{t+1} \geq 0$. This problem leads to the following first order condition:

$$\frac{1}{c_t} = \beta\Big(1 + R_{t+1} - \delta\Big)\frac{1}{c_{t+1}} \tag{3.3}$$

and the transversality condition (TVC):

$$\lim_{T\to\infty}\left[\prod_{t=1}^{T}(1+R_t-\delta)\right]^{-1}(K_{T+1})=0. \tag{3.4}$$

Equation (3.3) is the well-known Euler condition which states that an optimal (intertemporal) consumption plan $\{c_t\}_{t=0}^{\infty}$ must equalise the present-value marginal utility from consumption across periods. Combined with the sequential budget constraint (3.2), Equation (3.3) implies that any admissible sequence of consumption and next period's capital stock $\{c_t, K_{t+1}\}_{t=0}^{\infty}$ that maximises the lifetime utility must satisfy these conditions. Equation (3.4) states that the present discounted value of wealth (physical capital) at infinity must be zero as there is no reason to leave any wealth behind after $T=\infty$ is reached. In sum, we say that, given initial wealth K_0, if there exists a sequence $\{c_t, K_{t+1}\}_{t=0}^{\infty}$ that satisfies the sequential budget constraint (3.2), the Euler equation (3.3) and the TVC (3.4), then such sequence is the solution of this utility maximisation problem.[1]

Final-Good Sector

In the final-good sector, there is a continuum of identical firms, each indexed by $i\in[0,1]$. In any period t, firm i accesses the following production technology

$$Y_{i,t}=k_{i,t}^{\alpha_1}L_{i,Y,t}^{\alpha_2}\Omega_{i,t}^{1-\alpha_1-\alpha_2}H_t^{1-\alpha_1};\alpha_j\in(0,1),0<\sum_j\alpha_j<1,j=1,2 \tag{3.5}$$

where $Y_{i,t}$ is the amount of final goods produced by the firm i, $k_{i,t}, L_{Y,i,t}, \Omega_{i,t}$ are the amounts of three private inputs including physical capital, labour input, and energy input hired by the firm i, respectively, and finally H_t is the aggregate knowledge of the economy. Note that α_1, α_2 and thus $1-\alpha_1-\alpha_2$ are the elasticities of output with respect to capital, labour working in the final-good sector, and energy, respectively. The production technology is strictly concave and homogeneous of degree one in $k_{i,t}, L_{i,Y,t}$ and $\Omega_{i,t}$.

Two other assumptions regarding the above production function require some explanation. First, as in Romer (1986), Le Van et al. (2002), and Acemoglu (2009), among others, we consider the knowledge stock H_t as a by-product of a learning-by-doing process. In particular, we assume

$$H_t=K_t=\int_0^1 k_{i,t}di \tag{3.6}$$

Intuitively, following Romer (1986) suggestion, capital input used in each firm is associated with a certain fraction of the state of the knowledge of the economy. Transforming a unit of consumption good to a unit of capital input requires some knowledge in order to achieve

[1]The derivation of this optimization problem is illustrated in Appendix B.1

the task. Even if the capital input itself is purely private, this knowledge cannot be kept secret; an individual's knowledge will be embodied into the capital and by learning and spillover process this knowledge will be finally available to other firms. Accordingly, the stock of aggregate knowledge can be measured by the aggregate stock of capital. The second assumption is relating to the structure of the energy input. In this context, we assume that the energy input of a particular firm i , $\Omega_{i,t}$, is a CES aggregate of the flow of renewable resource used in the firm, denoted by $Z_{i,t}$, and the flow of non-renewable resource used in the firm, denoted by $X_{i,t}$, as follows:

$$\Omega_{i,t} = \Omega[A_t X_{i,t}, Z_{i,t}] = D\Big[\varpi(A_t X_{i,t})^{\rho} + (1-\varpi)(Z_{i,t})^{\rho}\Big]^{\frac{1}{\rho}}; \rho \leq 1, D > 0 \tag{3.7}$$

where $D > 0$ is a productivity parameter, $\varpi \in (0, 1)$ is a distribution parameter which determines the relative importance between the two natural factors, $\rho \in (-\infty, 1]$ is the substitution parameter which is related to the elasticity of substitution between the two natural factors $\epsilon \equiv \frac{1}{1-\rho} \in [0, \infty)$, and $A_t \equiv (1+g_A)^t$, where $g_A > 0$, is a technological factor that augments the use of non-renewable resource. Other things being equal, an increase in A_t lowers the amount of non-renewable resource needed to generate the same amount of energy input. The variable A_t can be interpreted as a fossil energy saving technology.[2]

All prices are expressed in units of final goods. Let $p_{x,t}$ and $p_{z,t}$ be the unit prices of non-renewable and renewable resources, respectively. Given the sequence of prices $\{R_t, w_t, p_{x,t}, p_{z,t}\}_{t=0}^{\infty}$ and the state of aggregate knowledge $\{H_t\}_{t=0}^{\infty}$, the firm i solves

$$\max_{\{k_{i,t}, L_{i,Y,t}, X_{i,t}, Z_{i,t}\}} \{\pi_{i,Y,t} \equiv Y_{i,t} - R_t k_{i,t} - w_t L_{i,Y,t} - p_{x,t} X_{i,t} - p_{z,t} Z_{i,t}\},$$

subject to Equations (3.5) and (3.7). Under the assumption that input markets are perfectly competitive, the first order conditions for the maximum entail

$$\alpha_1 \frac{Y_{i,t}}{k_{i,t}} = R_t, \tag{3.8}$$

$$\alpha_2 \frac{Y_{i,t}}{L_{i,Y,t}} = w_t, \tag{3.9}$$

$$\frac{(1-\alpha_1-\alpha_2)Y_{i,t}}{\left(1 + \left(\frac{1-\varpi}{\varpi}\right)\left(\frac{Z_{i,t}}{A_t X_{i,t}}\right)^{\rho}\right)} \cdot \frac{1}{X_{i,t}} = p_{x,t}, \tag{3.10}$$

[2]See Smulders et al. (2014) and Casey (2017) for more detail.

$$\frac{(1-\alpha_1-\alpha_2)Y_{i,t}}{\left(\left(\frac{\varpi}{1-\varpi}\right)\left(\frac{A_t X_{i,t}}{Z_{i,t}}\right)^{\rho}+1\right)} \cdot \frac{1}{Z_{i,t}} = p_{z,t}. \tag{3.11}$$

The above conditions state that the final good firms will hire each private input up to the point at which its marginal cost equals its marginal product in order to maximise its profit. Since the final goods production function exhibits constant returns to scale and all markets are perfectly competitive, profit is zero; i.e. $\pi_{i,Y,t} = 0$.

Finally, due to linear homogeneity of final goods production function, the corresponding function capturing the marginal product of each private input (the LHS of Equations (3.8) - (3.11)) will be homogeneous of degree zero. Using this fact, the aggregate demand functions for capital, labour, renewable resource, and non-renewable resource are, respectively,

$$\alpha_1 \frac{Y_t}{K_t} = R_t \tag{3.12}$$

$$\alpha_2 \frac{Y_t}{L_{Y,t}} = w_t \tag{3.13}$$

$$\frac{(1-\alpha_1-\alpha_2)Y_t}{\left(1+\left(\frac{1-\varpi}{\varpi}\right)\left(\frac{Z_t}{A_t X_t}\right)^{\rho}\right)} \cdot \frac{1}{X_t} = p_{x,t} \tag{3.14}$$

$$\frac{(1-\alpha_1-\alpha_2)Y_t}{\left(\left(\frac{\varpi}{1-\varpi}\right)\left(\frac{A_t X_t}{Z_t}\right)^{\rho}+1\right)} \cdot \frac{1}{Z_t} = p_{z,t} \tag{3.15}$$

where $Y_t \equiv \int_{i=1}^{1} Y_{i,t} di$, $L_{Y,t} \equiv \int_{i=1}^{1} L_{i,Y,t} di$, $X_t \equiv \int_{i=1}^{1} X_{i,t} di$, and $Z_t \equiv \int_{i=1}^{1} Z_{i,t} di$ are the aggregate output, the aggregate labour input employed in final-good sector, the aggregate flow of non-renewable resource, and the aggregate flow of renewable resource, respectively.

Renewable Resource Extraction Sector

The stock of renewable resource is available in infinite supply, but it is costly to extract.[3] These resources are extracted by private extraction firms. After having extracted, they sell the resources to the firms in the final-goods sector. Similar to Golosov et al. (2014), the renewable resource can be extracted according to the following technology:

$$Z_t = L_{z,t}, \tag{3.16}$$

[3]Examples of this type of renewable resources include solar energy and wind.

where $L_{z,t}$ is the labour input devoted to renewable resource extraction sector. Regarding the choices of renewable resource extraction technologies, it is worth mentioning that the usage of non-renewable resources might be needed for renewable resource provisions (Acemoglu et al., 2012). For example, we might need the non-renewable resources in terms of batteries to store the energy from renewable. We abstract from this issue and apply the same specification of renewable resource extraction technology as in Golosov et al. (2014) in order to focus on the role of labour allocations among different sectors.

Given the prices $\{p_{z,t}, w_t\}$, for any period t, the representative firm solves:

$$\max_{L_{z,t}} \{\pi_{z,t} \equiv p_{z,t} L_{z,t} - w_t L_{z,t}\}.$$

This leads to the following first order condition:

$$p_{z,t} = w_t, \tag{3.17}$$

which states that the renewable resource price equals to the marginal cost of extraction because of the perfectly competitive nature of this sector. Since the renewable resource market is perfectly competitive and the stock of this kind of resource is infinite, then all firms in this sector make zero profit in equilibrium, i.e. $\pi_{z,t} = 0$ for all t.

Non-Renewable Resource Extraction Sector

Consider the representative firm in the non-renewable resource extraction sector. Let M_t be the (finite) stock of non-renewable resources available at time t. We assume that the stock is owned by the extraction firms. Given the initial stock of non-renewable resources $M_0 > 0$, in each period the representative firm hires labour, $L_{x,t}$, to extract a flow of non-renewable resource, X_t, according to

$$X_t = L_{x,t} M_t. \tag{3.18}$$

The same extraction technology is also used in Engström and Gars (2016) and Le and Van (2016).[4] Equation (3.18) implies that it is more costly to extract a unit of resource when the stock size M_t becomes smaller. The stock of non-renewable resources available in the next period, M_{t+1}, is the remainder after extraction, i.e.,

$$M_{t+1} = M_t - X_t. \tag{3.19}$$

We now turn to the firm's problem. After extracting the resources, the firm sells its output to the firms in the final-goods sector at a competitive price $p_{x,t}$. Also, since labour

[4] In Golosov et al. (2014), non-renewable resource extraction is assumed to be costless.

is the only private input used in the production process, the total cost of extraction is then given by $w_t \frac{X_t}{M_t}$. Hence, its profit at time t is $\pi_{x,t} \equiv p_{x,t} X_t - w_t \frac{X_t}{M_t}$. These profits are discounted at the market interest rate, i.e., $q_t = \prod_{j=1}^{t}(1 + R_j - \delta)^{-1}$. The representative firm wishes to identify the sequence $\{X_t, M_{t+1}\}_{t=0}^{\infty}$ that maximises the total present value of its profits, i.e.,

$$\max_{\{X_t, M_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} q_t \left[p_{x,t} X_t - w_t \frac{X_t}{M_t} \right],$$

subject to the evolution of the stock of non-renewable resources; Equation (3.19), and non-negativity constraints; $X_t, M_{t+1} \geq 0$, where the initial stock $M_0 > 0$ is given.[5]

Let λ_t be the Lagrange multiplier with respect to Equation (3.19). The first order conditions with respect to X_t and M_{t+1} are, respectively, given by

$$q_t \left[p_{x,t} - w_t \frac{1}{M_t} - \lambda_t \right] = 0 \tag{3.20}$$

and

$$\lambda_t = \frac{q_{t+1}}{q_t} \left[\frac{w_{t+1}}{M_{t+1}} \frac{X_{t+1}}{M_{t+1}} + \lambda_{t+1} \right]. \tag{3.21}$$

Since $q_t > 0$ for all t, Equation (3.20) states that the Lagrange multiplier must be equal to the difference between price and unit cost, i.e. $\lambda_t = p_{x,t} - w_t \frac{1}{M_t}$. Intuitively, λ_t can be interpreted as the current value scarcity rent of (a unit of) the remaining stock of the non-renewable resource. This value is strictly positive as current use of the non-renewable resources diminishes future opportunities. In each period the unit price will be set higher than its extraction cost reflecting the fact that the resource is limited in all future time periods. Next, Equation (3.21) states that the current value of a unit of resources left unused must equal to the discounted value of future value of a unit of resources left in the ground plus an additional extraction cost resulting from declining in future stock; note that $\frac{q_{t+1}}{q_t} = \frac{1}{1+R_{t+1}-\delta}$. By combining Equations (3.20) and (3.21), we will get the so called Hotelling condition

$$p_{x,t} - \frac{w_t}{M_t} = \frac{1}{1 + R_{t+1} - \delta} \left[p_{x,t+1} - \frac{w_{t+1}}{M_{t+1}} + \frac{X_{t+1} w_{t+1}}{(M_{t+1})^2} \right]. \tag{3.22}$$

Intuitively, this condition states that if an extraction profile $\{X_t\}_{t=0}^{\infty}$ maximises the firm's objective function, in any period t the current value scarcity rent - measured by $p_{x,t} - \frac{w_t}{M_t}$ - must equal to the discounted value of the future scarcity rent sacrificed - measured by $\frac{1}{1+R_{t+1}-\delta} \left[p_{x,t+1} - \frac{w_{t+1}}{M_{t+1}} \right]$ - plus the discounted value of the marginal increase in extraction cost resulting from a smaller size of resource stock - measured by $\frac{1}{1+R_{t+1}-\delta} \left[\frac{X_{t+1} w_{t+1}}{(M_{t+1})^2} \right]$.

[5] The derivation of this optimization problem is illustrated in Appendix B.

Finally, the transversality condition is given by

$$\lim_{T\to\infty} q_T \lambda_T M_{T+1} = 0. \tag{3.23}$$

Intertemporal Equilibrium

Since the size of population is normalised to unity, the labour market clearing condition requires that

$$L_{x,t} + L_{Y,t} + L_{z,t} = 1. \tag{3.24}$$

This states that the sum of the labour demands in the three sectors must be equal to the exogenously given endowment constraint, provided by the representative household. Next, by the linear homogeneity of final goods production and perfect competition, it is straightforward to show that

$$Y_t = w_t + R_t K_t + \pi_t \tag{3.25}$$

where $\pi_t = \pi_{x,t} + \pi_{Y,t} + \pi_{z,t}$. As a result, the economy-wide resource constraint dictates that, at each date $t \geq 0$, total output is the sum of private consumption c_t and gross investment $K_{t+1} - (1-\delta)K_t$:

$$c_t + K_{t+1} - (1-\delta)K_t = Y_t. \tag{3.26}$$

Lastly, by using Equations (3.6), (3.16), (3.18), with linear homogeneity property of the final-good production function, we obtain the aggregate (final-good) production function which is

$$Y_t = L_{Y,t}^{\alpha_2} D^{(1-\alpha_1-\alpha_2)} \left[\varpi (B_t L_{x,t})^{\rho} + (1-\varpi)(L_{z,t})^{\rho} \right]^{\frac{1}{\rho}(1-\alpha_1-\alpha_2)} K_t \tag{3.27}$$

where $B_t \equiv A_t M_t$ can be seen as the effective stock of non-renewable resources.

Using Equations (3.1) - (3.27) and all assumptions imposed so far, an intertemporal equilibrium can be defined as follows:

Definition 3.1. *Given initial conditions* $(K_0, M_0) \in \mathbb{R}^2_{++}$ *and the exogenous process* $\{A_t\}_{t=0}^{\infty}$*, an intertemporal equilibrium of the economy is a sequence of prices, profit, and allocations*

$$\{w_t, R_t, p_{x,t}, p_{z,t}, \pi_t, L_{x,t}, L_{Y,t}, L_{z,t}, X_t, Z_t, Y_t, c_t, K_{t+1}, H_{t+1}, B_{t+1}, M_{t+1}\}_{t=0}^{\infty}$$

so that

(i) the allocation of consumption and capital $\{c_t, K_{t+1}\}_{t=0}^{\infty}$ *chosen by consumer satisfies Equations (3.2)-(3.4), taking the sequence* $\{w_t, R_t, \pi_t\}_{t=0}^{\infty}$ *as given;*

(ii) the aggregate inputs and final goods choices $\{K_t, L_{Y,t}, X_t, Z_t, Y_t\}_{t=0}^{\infty}$ *chosen by a continuum of identical firms in final-good sector satisfies Equations (3.12)-(3.15), taking the sequence* $\{w_t, R_t, p_{x,t}, p_{z,t}\}_{t=0}^{\infty}$ *as given;*

(iii) the input and output choices $\{L_{z,t}, Z_t\}_{t=0}^{\infty}$ *made by the representative firm in renewable resource extraction sector satisfies Equation (3.17), taking the sequence* $\{w_t, p_{z,t}\}_{t=0}^{\infty}$ *as given;*

(iv) the input and output choices $\{L_{x,t}, X_t\}_{t=0}^{\infty}$ *made by the representative firm in non-renewable resource extraction sector satisfies Equations (3.19), (3.22)-(3.23), taking the sequence* $\{w_t, p_{x,t}\}_{t=0}^{\infty}$ *as given;*

(v) all markets clear.

In subsequent subsections, we will focus on a special type of the intertemporal equilibrium which is known as balanced growth path.

3.2.2 Balanced Growth Path

Since the aim of this study is to examine the long-run economic growth effect of different degrees of natural resource substitutability, it suffices to focus on a balanced growth path (BGP hereafter). Regarding this matter, we provide a formal definition of a BGP and sufficient conditions ensuring the existence and uniqueness of such a long-run growth path.

A formal definition of a BGP is as follows.

Definition 3.2. *A BGP is an intertemporal equilibrium such that*

(i) final goods Y_t*, capital stock* K_t*, consumption* c_t*, and aggregate knowledge stock* H_t *all grow at the same constant rate, say* g^**, i.e.*

$$\frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}}{K_t} = \frac{c_{t+1}}{c_t} = \frac{H_{t+1}}{H_t} = 1 + g^*,$$

(ii) the rate of returns on capital is constant, i.e. $R_{t+1} = R_t = R^*$*,*

(iii) labour allocation is stationary, i.e. $L_{i,t+1} = L_{i,t} = L_i^*$ *where* $i \in \{x, Y, z\}$*.*

In other words, a BGP is an intertemporal equilibrium such that Y_t, K_t, c_t and H_t grow at a common (constant) growth rate while the rate of return on capital and labour allocation are stationary.

In order to characterise a BGP associated with the economy, we focus on eight variables:

$$L_{x,t}, L_{Y,t}, L_{z,t}, B_t, R_t, \Theta_t, g_t \text{ and } \Omega_t$$

where $\Theta_t \equiv \frac{p_{x,t}X_t}{w_t L_{x,t}}$. These variables will be stationary along a BGP and jointly determined by a set of eight non-linear equations. These equations will be illustrated in Lemma 3.1.

Lemma 3.1. *If a BGP exists, then there is a stationary point*

$$(L_x^*, L_Y^*, L_z^*, B^*, R^*, \Theta^*, \Omega^*, g^*) \in (0,1)^3 \times \mathbb{R}_{++}^4 \times \mathbb{R}$$

such that all quantities are jointly determined by

$$L_x^* = \frac{g_A}{1+g_A}, \tag{3.28}$$

$$L_Y^* = 1 - L_x^* - L_z^*, \tag{3.29}$$

$$g^* = \beta\Big(1 + R^* - \delta\Big) - 1, \tag{3.30}$$

$$\Omega^* = D\Big(\varpi(B^* L_x^*)^\rho + (1-\varpi)(L_z^*)^\rho\Big)^{\frac{1}{\rho}}, \tag{3.31}$$

$$R^* = \alpha_1 (L_Y^*)^{\alpha_2} (\Omega^*)^{1-\alpha_1-\alpha_2}, \tag{3.32}$$

$$\Theta^* - 1 = \frac{1}{1+R^*-\delta} \cdot \frac{1+g^*}{1-L_x} \cdot (\Theta^* - 1 + L_x^*), \tag{3.33}$$

$$\frac{1-\alpha_1-\alpha_2}{\alpha_2} \cdot \frac{L_Y^*}{L_z^*} = 1 + \frac{\varpi}{1-\varpi}\Big(\frac{B^* L_x^*}{L_z^*}\Big)^\rho, \tag{3.34}$$

$$\Theta^* = \frac{1-\alpha_1-\alpha_2}{\alpha_2} \cdot \frac{L_Y^*}{L_x^*} \cdot \left(1 + \frac{1-\varpi}{\varpi}\Big(\frac{L_z^*}{B^* L_x^*}\Big)^\rho\right)^{-1}. \tag{3.35}$$

The proof of this lemma and other theoretical results in this study are relegated to appendices.

Again, suppose that a BGP exists, Lemma 3.1 states that the long-run behaviour of the economy can be characterised by using the set of equations (3.28)-(3.35). By equation (3.28), labour employed in non-renewable resource extraction sector is exogenously determined by the growth rate of energy efficiency. Equation (3.29) guarantees that labour market clears along the BGP. Next, Equation (3.30) is Euler condition that holds along the BGP. This condition determines the long-run growth rate of output per head. Equation (3.31) quantifies the stationary value of the aggregate energy input used in the final-good sector. Equation (3.32) determines the long-run equilibrium rate of return on physical capital. Equation (3.33) is another expression of the Hotelling condition. This condition also expresses the no-arbitrage condition according to the equalised labour costs of the final-good and non-renewable resource extraction sectors. Equation (3.34) is another no-arbitrage condition ensuring that labours woking in final-good and renewable resource extraction sectors will be paid equally. Lastly, Equation (3.35) determines the revenue-to-cost ratio of the non-renewable resource extraction firm. This ratio shows the level of revenue generated by every dollar of labour cost.

Our next proposition provides a *sufficient* condition that establishes the existence and uniqueness of a BGP.

Proposition 3.1. *Suppose the following condition is satisfied:*

$$g_A < \left(\frac{1}{\beta} - 1\right) \frac{1}{1 + \left(\frac{\alpha_2}{1-\alpha_1-\alpha_2}\right)\left(\frac{1}{\beta} - 1\right)}. \tag{3.36}$$

Then there exists a unique balanced-growth equilibrium and the allocation of labour is given by (3.28), $L_Y^* = 1 - l_z^* - L_x^*$*, and*

$$L_z^* = \frac{1}{1+g_A}\left\{1 - \frac{\alpha_2}{1-\alpha_1}\left[1 + \frac{g_A(1-\beta)}{1-\beta(1+g_A)}\right]\right\} < \frac{1}{1+g_A}. \tag{3.37}$$

In other words, Proposition 3.1 provides parameter restriction (3.36) that guarantees the existence and uniqueness of the labour market equilibrium allocation. Intuitively, the inequality states that the growth rate of non-renewable resource saving technology cannot be too large in order to ensure the existence of a unique BGP. Once the labour allocation $\{L_x^*, L_Y^*, L_z^*\}$ is uniquely determined, the other stationary values $\{g^*, \Omega^*, R^*, B^*, \Theta^*\}$ can be uniquely determined by (3.30)-(3.32) and (3.34)-(3.35). This proves that a unique balanced growth equilibrium exists. Based on the long-run equilibrium allocation, it is remarkable that the parameter ρ disappears from (3.37). Combined with the fact that L_x^* is solely determined by g_A, we can see that a change in ρ will never alter labour allocation among the three sectors.

3.2.3 Natural Resource Substitution and Long-Run Economic Growth

In this subsection, we will examine the role of substitution parameter ρ on the long-run economic growth. Throughout this part we assume that the condition (3.36) is met so that a unique BGP always exists. The analysis will proceed as follows.

From Euler condition (3.30), after differentiating both sides with respect to ρ, we have

$$\frac{dg^*}{d\rho} = \beta \frac{dR^*}{d\rho}. \tag{3.38}$$

This means that an increase in the substitution parameter ρ can affect the long-run economic growth if and only if it causes a change in the long-run rate of return on capital. Moreover, because β is positive, the impacts on both R^* and g^* work in the same direction as well. Accordingly, it suffices to examine the influence of ρ on R^*.

The following lemma provides an alternative expression for R^*. Especially, this expression depends only on the model parameters and L_z^*.

Lemma 3.2. *In the long-run, the rate of return on capital can be evaluated by*

$$R^* = O[L_z] \times \left((1-\varpi)\Big(\frac{1-\alpha_1-\alpha_2}{\alpha_2} \frac{1}{1+g_A} \frac{1}{L_z^*} - \frac{1-\alpha_1-\alpha_2}{\alpha_2} \Big) \right)^{\frac{1-\alpha_1-\alpha_2}{\rho}} \tag{3.39}$$

where $O[L_z] \equiv \alpha_1 (DL_z^*)^{1-\alpha_1-\alpha_2} \left(\frac{1}{1+g_A} - L_z^* \right)^{\alpha_2} > 0.$

As mentioned before, a crucial implication of Proposition 3.1 is that the equilibrium allocation in labour market is essentially unchanged while the substitution parameter changes arbitrarily. This implies that any change in ρ will never cause a change in L_z^*. In other words, natural resource substitution has no effect on labour reallocation among the three production sectors. Accordingly, Lemma 3.2 tells us that $O[L_z]$ remains unchanged as ρ changes and, thus, the impact of ρ on R^* can be explained by the second term on the RHS of Equation (3.39) only.

From Equation (3.39) and the fact that $L_z^* \in \left(0, \frac{1}{1+g_A}\right)$, we can show that

$$\frac{dR^*}{d\rho} \begin{cases} > 0 & \text{iff } L_z^* \in \left(\tilde{L}_z, \frac{1}{1+g_A}\right) \\ = 0 & \text{iff } L_z^* = \tilde{L}_z \\ < 0 & \text{iff } L_z^* \in \left(0, \tilde{L}_z\right). \end{cases} \tag{3.40}$$

where $\tilde{L}_z = \frac{(1-\varpi)(1-\alpha_1-\alpha_2)}{\alpha_2+(1-\varpi)(1-\alpha_1-\alpha_2)} \cdot \frac{1}{1+g_A} \in (0,1)$ can be seen as a cut-off share of labour employed in renewable resource sector beyond which the substitution parameter is growth-enhancing.[6] This means that an increase in the degree of the natural resource substitution matters for enhancing the rate of return on capital if and only if the share of labour employed in the renewable resource extraction sector L_z^* is sufficiently high. Combined with Equation (3.38) the parameter ρ is growth enhancing if and only if the share of labour employed in the renewable resource extraction sector L_z^* is sufficiently high.

Measuring whether and how the elasticity of substitution between the two natural inputs could create the long-run growth effect require some careful treatments. The sensitivity analysis that we have done before is based on the primitive that any change in the substitution parameter ρ would cause a change in the long-run economic growth through changing in the rate of return on capital. The conclusion drawn on this primitive is valid as long as a change in the rate of return on capital is driven solely by the change in the state of the elasticity of substitution. However; as mentioned in Klump and de La Grandville (2000), Klump and Saam (2008) and Alvarez-Cuadrado et al. (2018), among many others,

[6] See Appendix B.6.

without CES normalisation the results created by changes in the substitution parameter ρ does not derive purely from the different states of the substitutability among inputs. In the next section, we therefore work with a normalised version of the CES aggregated energy input to re-examine if results in this section will remain valid and to draw a general conclusion based on this model.

3.3 Reassessment of the Growth Effect via CES Normalisation

3.3.1 Theoretical Analysis

As suggested by Klump and de La Grandville (2000), a "valid" comparison between two CES functions with different elasticity of substitution can be conducted only if two conditions are satisfied: (i) the marginal rate of technical substitution (MRTS) between the two inputs must be kept constant, and (ii) the output of the CES function must also be kept constant. To explain this more formally, we start with a CES aggregator between renewable and non-renewable resources with elasticity of substitution $\epsilon_1 \equiv 1/(1-\rho_1)$, distribution parameter ϖ_1 and scaling parameter D_1. Using these parameter values and the procedure described in the previous sections, we can derive a unique balanced growth equilibrium. To highlight the dependence on these parameter values, we will denote the stationary values as $g^*(\rho_1, \varpi_1, D_1)$, $R^*(\rho_1, \varpi_1, D_1)$, $B^*(\rho_1, \varpi_1, D_1)$, etc. It is important to mention that the allocation of labour $\{L_x^*, L_Y^*, L_z^*\}$ is not only independent of ρ_1, but also ϖ_1 and D_1. Thus, this allocation is unaffected by the normalisation procedure. The corresponding marginal rate of technical substitution between renewable and non-renewable resources can now be expressed as

$$m^*(\rho_1, \varpi_1, D_1) \equiv \frac{1-\varpi_1}{\varpi_1}\left[\frac{B^*(\rho_1, \varpi_1, D_1)\, L_x^*}{L_z^*}\right]^{1-\rho_1}, \tag{3.41}$$

and the output of the CES aggregator is

$$\Omega^*(\rho_1, \varpi_1, D_1) \equiv D_1\left\{\varpi_1\left[B^*(\rho_1, \varpi_1, D_1)\, L_x^*\right]^{\rho_1} + (1-\varpi_1)\left(L_z^*\right)^{\rho_1}\right\}^{\frac{1}{\rho_1}}. \tag{3.42}$$

Suppose now the elasticity of substitution is changed to $\epsilon_2 \equiv 1/(1-\rho_2)$. Then under the normalisation procedure, the scaling parameter and the distribution parameter need to be adjusted in order to maintain the same MRTS and the same level of output at the same level of inputs, i.e., $B^*(\rho_1, \varpi_1, D_1)\, L_x^*$ and L_z^*. Formally, the new parameters ϖ_2 and D_2 must satisfy

$$\frac{1-\varpi_2}{\varpi_2}\left[\frac{B^*(\rho_1, \varpi_1, D_1)\, L_x^*}{L_z^*}\right]^{1-\rho_2} = m^*(\rho_1, \varpi_1, D_1), \tag{3.43}$$

$$D_2 \left\{ \varpi_2 \left[B^* \left(\rho_1, \varpi_1, D_1 \right) L_x^* \right]^{\rho_2} + \left(1 - \varpi_2 \right) \left(L_z^* \right)^{\rho_2} \right\}^{\frac{1}{\rho_2}} = \Omega^* \left(\rho_1, \varpi_1, D_1 \right). \tag{3.44}$$

Once ϖ_2 and D_2 are determined, we can then solve for the corresponding balanced growth equilibrium. We will denote the variables in this equilibrium using $g^* \left(\rho_2, \varpi_2, D_2 \right)$, $R^* \left(\rho_2, \varpi_2, D_2 \right)$, $B^* \left(\rho_2, \varpi_2, D_2 \right)$, etc.

Note that the normalisation procedure does not necessarily imply that $\Omega^* \left(\rho_1, \varpi_1, D_1 \right) = \Omega^* \left(\rho_2, \varpi_2, D_2 \right)$. This is because they are evaluated under different input values [specifically, $B^* \left(\rho_1, \varpi_1, D_1 \right)$ may be different from $B^* \left(\rho_2, \varpi_2, D_2 \right)$]. But, in the current framework, we can show that $B^* \left(\rho_1, \varpi_1, D_1 \right) = B^* \left(\rho_2, \varpi_2, D_2 \right)$ and $\Omega^* \left(\rho_1, \varpi_1, D_1 \right) = \Omega^* \left(\rho_2, \varpi_2, D_2 \right)$ are both true. First, consider equation (3.34). Since the left-hand side is independent of the parameters of the CES aggregator, we can get

$$\frac{\varpi_1}{1 - \varpi_1} \left[\frac{B^* \left(\rho_1, \varpi_1, D_1 \right) L_x^*}{L_z^*} \right]^{\rho_1} = \frac{\varpi_2}{1 - \varpi_2} \left[\frac{B^* \left(\rho_2, \varpi_2, D_2 \right) L_x^*}{L_z^*} \right]^{\rho_2}.$$

On the other hand, (3.41) and (3.43) imply

$$\frac{\varpi_2}{1 - \varpi_2} \left[\frac{B^* \left(\rho_1, \varpi_1, D_1 \right) L_x^*}{L_z^*} \right]^{\rho_2} = \frac{\varpi_1}{1 - \varpi_1} \left[\frac{B^* \left(\rho_1, \varpi_1, D_1 \right) L_x^*}{L_z^*} \right]^{\rho_1}.$$

These two conditions immmediately imply $B^* \left(\rho_1, \varpi_1, D_1 \right) = B^* \left(\rho_2, \varpi_2, D_2 \right)$. Substituting this into (3.44) gives $\Omega^* \left(\rho_1, \varpi_1, D_1 \right) = \Omega^* \left(\rho_2, \varpi_2, D_2 \right)$. Furthermore, substituting this into (3.32) gives $R^* \left(\rho_1, \varpi_1, D_1 \right) = R^* \left(\rho_2, \varpi_2, D_2 \right)$. Since there is no change in the rate of return, it follows from (3.30) that $g^* \left(\rho_1, \varpi_1, D_1 \right) = g^* \left(\rho_2, \varpi_2, D_2 \right)$. This leads to the following proposition.

Proposition 3.2. *Under the normalisation procedure, any change in ρ will neither increase nor decrease the long-run economic growth. Then ρ is a growth-neutral parameter in Romer (1986) model with natural resource substitution.*

Clearly, the rate of return on capital remains unchanged - and thus the long-run growth rate - due to the lack of labour resource reallocation induced by changes in the state of natural resource substitutability. Labour reallocation plays a crucial role in explaining the long-run growth effect when the normalisation procedure is introduced. In the next section, we introduce an alternative model in order to show that labour-allocation effect appears under the normalisation procedure and, thus, the change in the degree of natural resource substitution can cause changes in the long-run growth rate. Before moving on to the next section, we give a numerical example aimed at delivering important messages initiated by the model.

3.3.2 Numerical Example

To gain more insight into the story of the long-run economic growth and natural resource substitutability that emanate from the model, we resort to numerical analysis. In particular, our numerical illustrations are drawn on the US data and some relevant literature.

Parameterisation

We calibrate a benchmark economy using the U.S. data and relevant literature. The parameters to be calibrated consists of the income share of physical capital (α_1), the income share of labour employed in final-good sector (α_2), the growth rate of exhaustible input augmenting technology (g_A), the discount factor (β), the depreciation rate (δ), the distribution parameter (ϖ) and the substitution parameter (ρ). For the scale parameter (D), we normalise to unity.

Our calibration procedure is to choose the bundle of parameters $(\alpha_1, \alpha_2, g_A, \beta, \delta, \varpi, \rho)$ so that the benchmark economy can match some targeted real-world statistics. The targets include GDP per capita growth, capital to output ratio, real interest rate (rate of return on capital minus depreciation rate), the share of renewable resource sector employment, the share of non-renewable resource sector employment and labour income relative to GDP. These values are provided in Table 3.1.

Target	Value	Source
Labour Share	0.6211	Giandrea and Sprague (2017)
GDP per capita growth	0.0200	BEA (2019)
Capital-Output ratio	3.2255	BEA (2019)
Real interest rate	0.0385	WDI (2019)
Renewable resource sector employment (%)	0.0045	BLS (2019) and DOE (2017)
Non-renewable resource sector employment (%)	0.0084	BLS (2019) and DOE (2017)

Table 3.1: Selected the US economy Indicators

The source of the data and a description of the targets are as follows. Firstly, we use the U.S. labour share of income on average between 1961 and 2016 as a target. We adopt Giandrea and Sprague (2017)'s estimation which reveals that the U.S. labour share is 0.6211 on average. Secondly, using the data provided by the U.S. Bureau of Economic Analysis (BEA, 2019), real gross domestic product per capita of the U.S. increased at an annual rate of 2 percent between 1961 and 2016 while the average ratio aggregate fixed assets plus consumer durables to gross domestic product is 3.2255. Thirdly, from World Development Indicator (WDI, 2019), the value for real interest rate in the United States was on average 3.85 percent during 1961-2016. Finally, in Quarter 1 2016, the employed labour force of the United States numbered 150.959 million (BLS, 2019). Of these, the data provided by U.S. Department of Energy (DOE, 2017) reveal that the renewable energy sector and non-renewable energy sector accounted for 677.544 and 1,266.074 thousand jobs in such period, respectively.[7] This implies that the share of renewable resource sector employment and non-renewable resource sector employment are 0.45 and 0.84 percent, respectively.[8]

Table 3.2 summarises the parameterisation for the calibration targets. Firstly, we calibrate g_A by using Equation (3.28):

$$L_x = \frac{g_A}{1+g_A} \Rightarrow g_A = \frac{L_x}{1-L_x}.$$

When L_x is set to satisfy the fact that the share of labour working in non-renewable resource sector is 0.0084, we obtain $g_A = 0.0085$. Secondly, we calibrate β by using the condition

$$\beta = \frac{1 + \text{GDP per capita growth}}{1 + \text{real interest rate}}$$

where this condition can be derived by Euler condition. Using 0.0200 for GDP per capita growth and 0.0385 for real interest rate, we get 0.9822 for β. Thirdly, we calibrate α_2 by arguing that if total income is distributed to all labour force by 62.11 percent, then it will be for labour work for non-energy extraction activities by

$$\alpha_2 = \text{Labour Share} \times (1 - 0.0045 - 0.0084)$$

which implies that $\alpha_2 = 0.6131$. Fourthly, the value of α_1 is chosen as follows. We know from Proposition 3.1 that L_z^* is determined by (3.37), provided that (3.36) holds. If we fix the parameters $g_A = 0.0085, \beta = 0.9822, \alpha_2 = 0.6131$ and the worker share $L_z = 0.0045$, then Equation (3.37) can be seen as an equation with one unknown, i.e. α_1. A single root of the equation is 0.3743. Accordingly, we set $\alpha_1 = 0.3743$. Up until now, we have $g_A = 0.0085$, $\beta = 0.9822$, $\alpha_1 = 0.3743$ and $\alpha_2 = 0.6131$. Then we calibrate δ by using another variation of Euler condition:

$$\delta = \alpha_1 \Big(\text{Capital to Output ratio}\Big)^{-1} + 1 - \Big(\frac{1 + \text{GDP per capita growth}}{\beta}\Big)$$

to obtain $\delta = 0.0776$. Finally, we calibrate ρ and ϖ simultaneously to ensure that the steady state match the US moments. To do so, we follow recent empirical studies which suggest that the the two types of natural inputs are gross substitute. Evidently, Papageorgiou et al. (2017) estimates that the substitution parameter between clean (renewable) and dirty (fossil) inputs in energy-generating sector to be around 0.46; based on the data

[7] The proxy of renewable resource extraction employment are the employment for electric power generation and fuel extraction activities relying on solar, wind, geothermal, bioenergy/CHP, low impact hydro, and traditional hydro. Also, the proxy of non-renewable resource extraction employment are the employment for electric power generation and fuel extraction activities based on nuclear, coal, natural gas, oil/petroleum, advanced gas, and others.

[8] The data for labour allocation among energy sectors and non-energy sector is very limited. In particular, DOE started to collect the data in 2015.

of 26 countries (including the U.S.) for the year 1995 to 2009. Moreover, the estimated cross-price elasticities of the U.S. industrial sector demand for fuels by Suh (2016) reveals the substitutable relationships between coal and biomass and between natural gas and biomass. Thus, it seems that assuming the two types of natural inputs are gross substitute is sensible. We choose $\rho = 0.01$ as a benchmark. Combined with the choice of $\varpi = 0.9070$, we obtain all parameters required to illustrate our numerical example. Table 3.2 provides the set of parameter values selected here.

Parameter	Value	Source
ϖ	0.9070	Calibration
ρ	0.0100	Calibration
g_A	0.0085	Calibration
β	0.9822	Calibration
α_2	0.6131	Calibration
δ	0.0776	Calibration
α_1	0.3743	Calibration
D	1.0000	Normalisation

Table 3.2: Benchmark Parameter Values

Numerical Illustration of the baseline BGCE

After obtaining the benchmark economy, we can demonstrate the balanced growth equilibrium corresponding to the benchmark economy.

Variable	Steady State Result	U.S. Data
Non-renewable resource sector employment (L_x)	0.0084	0.0084
Renewable resource sector employment (L_z)	0.0044	0.0045
Capital-Output Ratio (K/Y)	3.1492	3.2255
Real Interest Rate (r)	0.0413	0.0385
Long-Run Output per capita growth (g)	0.0227	0.0200

Table 3.3: Benchmark Result

The stationary values of the real interest rate, the long-run growth rate, capital-output ratio as well as the corresponding shares of employment in renewable resource and non-renewable resource sectors are listed in Table 3.3. It can be observed that these values match the targets very well.

Numerical Illustrations of Growth-Resource Substitutability Interaction

We have shown analytically in subsection 3.2 that if normalisation is taken into account, the variation of the natural resource substitutability will never affect the long-run economic growth in this model. Nevertheless, illustrating the numerical analysis here will

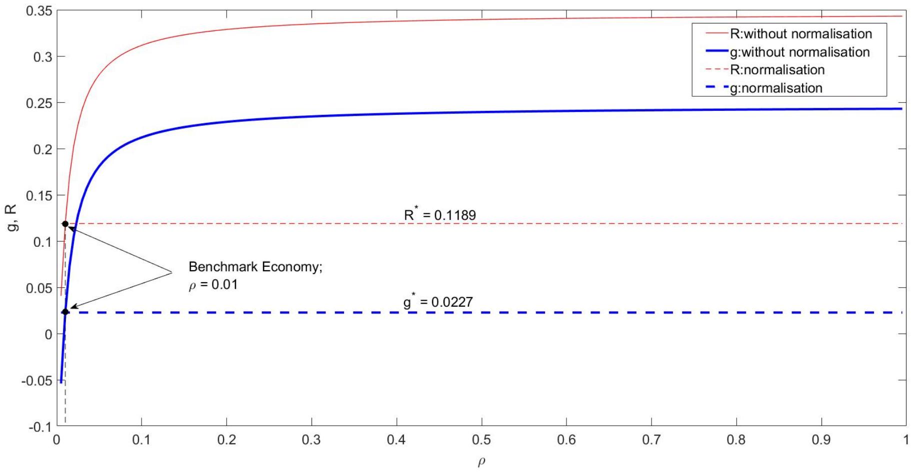

Figure 3.2: Growth Sensitivity induced by Natural Resource Substitutability

be beneficial since it can emphasise the importance of normalisation as it can isolate the effect of different degrees of natural resource substitution of others.

Figure 3.2 shows the sensitivity to natural resource substitution variability within a certain range. To illustrate this, we vary the state of the substitution parameter within the range $(0, 1)$ while maintaining all the other parameters constant, i.e. we do not re-calibrate the remaining parameters but keep all as in Table 3.2. In this figure, the thin solid line is the rate of return on capital and the bold solid line is the per capita output growth when normalisation is ignored. Clearly, as the substitution parameter ρ increases from the benchmark value, the long-run growth rate increases. This result arises because the equilibrium employment in renewable resource extraction sector L_z^* is greater than the cut-off level $\tilde{L}_z$ so that the rate of return on capital increases when the state of the substitution parameter changes.[9] However, if normalisation is taken into account growth rate will neither increase nor decrease as the substitution parameter changes. The bold dashed line passes through the vertical axis at 0.0227 which is equal to the initial growth rate. This conclusion is drawn on the fact that under normalisation the rate of returns on capital will never change as it remains constant at the rate 0.1189 despite the variation of the states of natural resource substitutability (See the thin dashed line).

3.4 Alternative Model: A Barro (1990) Model with Natural Resource Substitution

In this section, an alternative endogenous growth model is introduced in order to examine the effects of an increase in the degree of natural resource substitution on long-run economic

[9]In the benchmark $\tilde{L}_z \cong 0.0019$ while $L_z^* = 0.0044$.

growth. In particular, long-run economic growth in this model is endogenously determined and is generated by productive government spending as in Barro (1990).

3.4.1 The Model Setting

Consider an economy formulated in discrete time ($t = 0, 1, ..., +\infty$). We assume that there are three types of agents: households, firms and a government. The households are infinitely-lived individuals who decide how much to consume in each period in order to maximise their lifetime utilities. Firms are operating in three sectors including final-good sector, renewable resource extraction sector and non-renewable resource extraction sector. The firms in the final-good sector produce a homogeneous product that can be either consumed or invested. The firms in the renewable resource extraction sector produce renewable energy used as an input in the final-good sector. The firms in the non-renewable resource extraction sector produce non-renewable energy used as an input in the final-good sector. The government imposes income tax and spends all the tax revenues in productive ways. All markets are perfectly competitive. Finally, we assume that all agents are identical so that the economy can be characterised by means of a representative agent model.

In order to present the model environment and the optimising behaviours, we begin by formulating the economic structures of the representative household, the representative firm in final-good sector and the government. We omit the presentations of the two natural resource extraction sectors here because the precise formulations of these two sectors are in fact identical to their counterparts in the previous model. After getting the general structure, we define an intertemporal equilibrium of this economy.

Households

The representative household in this model is defined almost identical to that in Section 3.2. The only difference is that in this model the representative household have to pay income taxes to the government by a constant rate $\tau \in (0, 1)$.

As a result, the household's optimising behaviour will be governed by the household's sequential budget constraint:

$$c_t + K_{t+1} - (1-\delta)K_t = (1-\tau)\Big(R_t K_t + w_t + \pi_t\Big), \tag{3.45}$$

Euler condition:

$$\frac{c_{t+1}}{c_t} = \beta\Big(1 + (1-\tau)R_{t+1} - \delta\Big), \tag{3.46}$$

and the transversality condition (TVC):

$$\lim_{T\to\infty}\left[\prod_{t=1}^{T}(1+(1-\tau)R_t - \delta)\right]^{-1}(K_{T+1}) = 0. \tag{3.47}$$

Final-Good Sector

In any period t, the representative firm in the final-good sector maximises profits subject to a technology that produces a single commodity, Y_t. The stock of physical capital, K_t, together with productive services provided by the government, G_t, are used together with the labour force devoted to the final-good sector, $L_{Y,t}$, and the energy input, Ω_t, as production inputs in a technology:

$$Y_t = K_t^{\alpha_1} L_{Y,t}^{\alpha_2} \Omega_t^{1-\alpha_1-\alpha_2} G_t^{1-\alpha_1}; \alpha_j \in (0,1), 0 < \sum_j \alpha_j < 1, j = 1,2 \tag{3.48}$$

where α_1, α_2 and thus $1 - \alpha_1 - \alpha_2$ are the elasticities of output with respect to capital, labour working in the final-good sector, and energy, respectively. Note that the production technology exhibits constant returns to scale with respect to the private inputs, namely physical capital, labour and energy.

As in Barro (1990) and Cazzavillan (1996), we assume that the public service G_t is a public good, i.e., it is nonrival and nonexcludable. Hence, public services provided to private producers by the government is not only available without financial cost but the use by a firm will never diminish the quantity available to others.[10]

All prices are expressed in units of final goods. Let $p_{x,t}$ and $p_{z,t}$ be the unit prices of non-renewable and renewable resources, respectively. Given the sequence of prices $\{R_t, w_t, p_{x,t}, p_{z,t}\}_{t=0}^{\infty}$ and the sequence of public service $\{G_t\}_{t=0}^{\infty}$, the representative firm solves

$$\max_{\{K_t, L_{Y,t}, X_t, Z_t\}} \{\pi_{Y,t} \equiv Y_t - R_t K_t - w_t L_{Y,t} - p_{x,t} X_t - p_{z,t} Z_t\},$$

subject to Equations (3.7) and (3.48). Under the assumption that input markets are perfectly competitive, the firm's optimality conditions are the same as in Equations (3.12)-(3.15).

Government

For any period t, the government provides productive services, G_t, to final good firms. We denote by $\tau \in (0,1)$ the proportion of the proceeds from income taxes that are used each period to finance public production services, G_t. The government is assumed to run a balanced budget, thus the government budget constraint is given by:

$$G_t = \tau \Big[w_t + R_t K_t + \pi_t \Big]. \tag{3.49}$$

[10] In this study, we assume that there is no congestion even if it might be possible in the real world. For more details about productive spending with congestion, see, for example, Glomm and Ravikumar (1994).

Intertemporal equilibrium

Both Equations (3.24) and (3.25) are still satisfied in the present model. In addition, the government balanced budget condition

$$G_t = \tau Y_t \tag{3.50}$$

holds in equilibrium. As a result, the economy-wide resource constraint dictates that, at each date $t \geq 0$, total output is the sum of private consumption c_t, gross investment $K_{t+1} - (1-\delta)K_t$ and the productive government spending G_t:

$$c_t + K_{t+1} - (1-\delta)K_t + G_t = Y_t \tag{3.51}$$

Lastly, we use Equations (3.7), (3.16), (3.18) and (3.48) to obtain another expression of final-good production which is

$$Y_t = K_t^{\alpha_1} G_t^{1-\alpha_1} L_Y^{\alpha_2} D^{1-\alpha_1-\alpha_2} \left(\varpi (B_t L_{x,t})^\rho + (1-\varpi)(L_{z,t})^\rho \right)^{\frac{1}{\rho}(1-\alpha_1-\alpha_2)}. \tag{3.52}$$

An intertemporal equilibrium of this economy can be formulated almost exactly the same way as in the Romer (1986) model with natural resource substitution but the source of growth is different in the two economies. In the next subsection we will consider behaviours of the intertemporal equilibrium of this economy along a balanced growth path.

3.4.2 Balanced Growth Paths

In this subsection, a BGP in Barro (1990) model with natural resource substitution will be defined precisely. Not only that, necessary and sufficient conditions for the existence and uniqueness of the BGP will be formulated.

Definition 3.3. *A BGP is an intertemporal equilibrium such that*

(i) final goods Y_t, capital stock K_t, consumption c_t, and productive government spending G_t all grow at the same constant rate, say g^*, i.e.,

$$\frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}}{K_t} = \frac{c_{t+1}}{c_t} = \frac{G_{t+1}}{G_t} = 1 + g^*,$$

(ii) the rate of return on capital is constant, i.e., $R_{t+1} = R_t = R^*$,

(iii) labour allocation in each sector is constant, i.e., $L_{i,t+1} = L_{i,t} = L_i^*$ where $i \in \{x, Y, z\}$.

In other words, a BGP is an intertemporal equilibrium such that Y_t, K_t, G_t and c_t grow at a constant rate while the rate of return on capital and labour allocation across sectors are time-invariant.

Similar to the previous model, we focus on eight variables including

$$L_{x,t}, L_{Y,t}, L_{z,t}, B_t, R_t, \Theta_t, g_t \text{ and } \Omega_t$$

where these variables will be stationary along a BGP and jointly determined by a set of eight non-linear equations. The system of equations will be stated in the following lemma.

Lemma 3.3. *If a BGP exists, then there is a stationary point*

$$(L_x^*, L_Y^*, L_z^*, B^*, R^*, \Theta^*, \Omega^*, g^*) \in (0,1)^3 \times \mathbb{R}_{++}^4 \times \mathbb{R}$$

such that all quantities are jointly determined by

$$L_x^* = \frac{g_A}{1+g_A} \in (0,1), \tag{3.53}$$

$$L_Y^* = 1 - L_x^* - L_z^*, \tag{3.54}$$

$$\beta\Big(1 + (1-\tau)R^* - \delta\Big) = 1 + g^*, \tag{3.55}$$

$$\Omega^* = D\Big(\varpi(B^* L_x^*)^\rho + (1-\varpi)(L_z^*)^\rho\Big)^{\frac{1}{\rho}}, \tag{3.56}$$

$$R^* = \alpha_1 \tau^{\frac{1-\alpha_1}{\alpha_1}} (L_Y^*)^{\frac{\alpha_2}{\alpha_1}} (\Omega^*)^{\frac{1-\alpha_1-\alpha_2}{\alpha_1}}, \tag{3.57}$$

$$\Theta^* - 1 = \frac{1}{1+R^*-\delta} \cdot \frac{1+g^*}{1-L_x^*}(\Theta^* - 1 + L_x^*), \tag{3.58}$$

$$\frac{1-\alpha_1-\alpha_2}{\alpha_2} \cdot \frac{L_Y^*}{L_z^*} = 1 + \frac{\varpi}{1-\varpi}\Big(\frac{B^* L_x^*}{L_z^*}\Big)^\rho, \tag{3.59}$$

$$\Theta^* = \frac{1-\alpha_1-\alpha_2}{\alpha_2} \cdot \frac{L_Y^*}{L_x^*} \cdot \left(1 + \frac{1-\varpi}{\varpi}\Big(\frac{L_z^*}{B^* L_x^*}\Big)^\rho\right)^{-1}. \tag{3.60}$$

By comparing this lemma with Lemma 3.1, these two non-linear systems are almost identical. It is clear that the government tax and productive expenditure will play a role here in the growth process so that the Euler condition (3.55) and the rate of return on capital condition (3.57) differ from Equations (3.30) and (3.32), respectively. Similar to the previous model, we can apply Lemma 3.3 to obtain an equation determining the long-run employment share in renewable resource extraction sector and this equation is stated in the following lemma.

Lemma 3.4. *Let $S \equiv (0, s_M)$ such that $s_M \equiv \frac{1}{1+g_A} \cdot \frac{1-\alpha_1-\alpha_2}{1-\alpha_1}$. If a BGP exists, then there exists $L_z^* \in S$ satisfying*

$$\frac{(1-\alpha_1-\alpha_2)-(1-\alpha_1)(1+g_A)L_z^*}{\alpha_2 g_A} = 1 + \frac{g_A}{\frac{1+\Phi[L_z^*]-\delta}{\beta(1+(1-\tau)\Phi[L_z^*]-\delta)} - (1+g_A)} \tag{3.61}$$

where $\Phi[L_z^*] \equiv \alpha_1 \tau^{\frac{1-\alpha_1}{\alpha_1}} D^{\frac{1-\alpha_1-\alpha_2}{\alpha_1}} \left(\frac{(1-\varpi)(1-\alpha_1-\alpha_2)}{\alpha_2}\right)^{\frac{1-\alpha_1-\alpha_2}{\rho\alpha_1}} \left(\frac{1}{1+g_A} - L_z^*\right)^{\frac{1-\alpha_1-\alpha_2+\rho\alpha_2}{\rho\alpha_1}} (L_z^*)^{\frac{1-\alpha_1-\alpha_2}{\alpha_1}(1-\frac{1}{\rho})}$.

By comparing Lemma 3.2 and Lemma 3.4, the substitution parameter ρ appears in Equation (3.61) whereas it does not have in Equation (3.37). This implies that the states of such parameter matter in determining L_z^* in this model. Thus, labour allocation effect appear and growth effect could exist under normalisation.

Under a certain set of parameters, if we are able to find a root of non-linear equation (3.61), then a BGP exists. The following lemma shows sufficient conditions ensuring the existence and uniqueness of a BGP.

Lemma 3.5. *Suppose that $\beta < \beta(1+g_A) < 1$ is satisfied. Then the following results hold:*

(i) Suppose that $\rho \in (0,1)$. Then a unique balanced-growth equilibrium exists if and only if

$$g_A < \frac{\left(\frac{1-\alpha_1-\alpha_2}{\alpha_2}\right)(\beta^{-1}(1-\tau)^{-1}-1)}{(\beta^{-1}(1-\tau)^{-1}-1)+\left(\frac{1-\alpha_1-\alpha_2}{\alpha_2}\right)}. \tag{3.62}$$

(ii) Suppose that $\rho < 0$. Then a balanced-growth equilibrium exists if

$$g_A < \frac{\left(\frac{1-\alpha_1-\alpha_2}{\alpha_2}\right)(\beta^{-1}-1)}{(\beta^{-1}-1)+\left(\frac{1-\alpha_1-\alpha_2}{\alpha_2}\right)}. \tag{3.63}$$

Put differently, a sufficient condition for the existence of a BGP is that the growth rate g_A is strictly positive but cannot be too large. The set of sufficient conditions to ensure the uniqueness is more restrictive as it requires that the growth rate g_A cannot be too large and the two types of natural resources are gross substitute.

3.4.3 Natural Resource Substitution and Long-Run Economic Growth

This subsection is devoted to examine the influence of natural resource substitution on the long-run growth rate under the normalisation.

Normalisation

As in the previous model, we apply the normalization technique to capture the effect of the elasticity of substitution on the long-run economic growth. To do so, we assume that

Lemma 3.5 holds so that there exists a BGP for any ρ. Then, a BGP associated with a particular ρ will be chosen as a reference point.

Before analysing the long-run growth effect, we define some useful notations and normalise fundamental structure of the model. To begin with, we define $e_t \equiv \frac{B_t L_{x,t}}{L_{z,t}}$ and by linear homogeneity of $\Omega[\cdot,\cdot]$ we have $\Omega[e_t, 1] = \frac{\Omega[B_t L_{x,t}, L_{z,t}]}{L_{z,t}}$. As in subsection 3.1, given the parameter values ρ_1, ϖ_1, D_1 and the rest, we can have a BGP associated with

$$e_1^* \equiv e^*(\rho_1, \varpi_1, D_1) > 0, \Omega[e_1^*, 1] > 0, m_1^* \equiv m^*(\rho_1, \varpi_1, D_1) > 0.$$

Suppose that ρ changes arbitrarily from ρ_1 to ρ_2. The normalisation procedure requires that both D and ϖ have to adjust such that

$$D[\rho] = \Omega[e_1^*, 1]\left(\frac{(e_1^*)^{1-\rho} + m_1^*}{e_1^* + m_1^*}\right)^{\frac{1}{\rho}}$$

and

$$\varpi[\rho] = \frac{(e_1^*)^{1-\rho}}{(e_1^*)^{1-\rho} + m_1^*}.$$

For more details of such derivations, see Klump and de La Grandville (2000). Next, we normalise our economy by replacing D and ϖ with the right-hand side of above two conditions, respectively, in all conditions stated in Lemma 3.3 and Lemma 3.4.

Based on the normalised economy, we can analyse the problem using two key conditions. The first equation is the normalised version of $\Phi[\cdot]$:

$$\Phi[L_z, \rho] = \tilde{O}[L_z]\Big(\frac{m_1^*}{m_1^* + e_1^*}\Big)^{\frac{1-\alpha_1-\alpha_2}{\rho\alpha_1}}\Big(\frac{1-\alpha_1-\alpha_2}{\alpha_2}\frac{1}{1+g_A}\frac{1}{L_z} - \frac{1-\alpha_1-\alpha_2}{\alpha_2}\Big)^{\frac{1-\alpha_1-\alpha_2}{\rho\alpha_1}} \tag{3.64}$$

where $\tilde{O}[L_z] \equiv \alpha_1 \tau^{\frac{1-\alpha_1}{\alpha_1}} (\Omega[e_1^*, 1] L_z)^{\frac{1-\alpha_1-\alpha_2}{\alpha_1}} \left(\frac{1}{1+g_A} - L_z\right)^{\frac{\alpha_2}{\alpha_1}}$. The second equation is the normalised version of (3.61):

$$\underbrace{\frac{(1-\alpha_1-\alpha_2) - (1-\alpha_1)(1+g_A)L_z}{\alpha_2 g_A}}_{\equiv LHS[L_z]} = 1 + \underbrace{\frac{g_A}{\frac{1+\Phi[L_z,\rho]-\delta}{\beta(1+(1-\tau)\Phi[L_z,\rho]-\delta)} - (1+g_A)}}_{\equiv RHS[L_z,\rho]}. \tag{3.65}$$

Obviously, the left-hand side of the above expression is independent of the substitution parameter ρ whereas the right-hand side depends on such parameter. This observation tells us that a change in the state of natural resource substitution could create labour allocation effect if it induces a change in the value of the $RHS[L_z, \rho]$.

In the next part, we will investigate the growth effect of the substitution parameter in Barro (1990) model with natural resource substitution under the normalisation. As mentioned before, our analysis will rely on Equations (3.64) and (3.65).

Growth effect under normalisation

From now, we will provide a heuristic discussion on how changing the elasticity of substitution between renewable and non-renewable resources would affect the rate of return of physical capital in a balanced growth equilibrium, and hence long-term economic growth. To facilitate a precise analysis, we focus our attention on the case that the two natural inputs are gross substitutes. Throughout the analysis we assume that $\beta < (1+g_A)\beta < 1$ and the statement (i) in Lemma 3.5 holds. As mentioned before that the gross-substitutability has received some empirical supports, we argue that our theoretical analysis would present a theoretical contribution within real-world consistent environment.[11]

Changing in the degree of natural resource substitutability creates both direct and indirect effects on the rate of return on capital. On the one hand, define

$$\overline{L}_z \equiv \frac{(1-\alpha_1-\alpha_2)}{\left(\frac{m_1^*+e_1^*}{m_1^*}\right)\alpha_2 + (1-\alpha_1-\alpha_2)} \frac{1}{1+g_A} \in S.$$

Then, using Equation (3.64),

$$\frac{\partial \Phi[L_z,\rho]}{\partial \rho} \begin{cases} > 0 & \text{iff } L_z^* \in \left(\overline{L}_z, s_M\right) \\ = 0 & \text{iff } L_z^* = \overline{L}_z \\ < 0 & \text{iff } L_z^* \in \left(0, \overline{L}_z\right) \end{cases} . \tag{3.66}$$

While keeping L_z unchanged, condition (3.66) states that raising the parameter ρ increases (decreases) the term $\Phi[L_z,\rho]$ if $L_z > (<)\overline{L}_z$. In addition, $\Phi[L_z,\rho]$ remains constant as ρ changes if $L_z = \overline{L}_z$. Intuitively, an increase in the degree of natural input substitution can stimulate the rate of return on capital only if the share of renewable energy employment is sufficiently high, say greater than $\overline{L}_z$. We refer to this effect as *a direct effect* (or general equilibrium effect) of natural resource substitution on R^*. On the other hand, variations in natural resource substitution could also create *an indirect effect* on R^* via labour reallocation between the final good and renewable extraction sectors. In particular, a shifting in $\Phi[L_z,\rho]$ could also affect the equilibrium value of L_z, which will create an indirect effect on R^*. These two effects, however, often lead to opposite changes in R^*, rendering the overall effect ambiguous. In what follows, we will explain these two effects and discuss the conditions under which changing ρ is growth-enhancing, growth-neutral or growth-reducing by using a graphical analysis.

In Figure 3.3 we depict the effect of a permanent increase in the state of substitution parameter from ρ_1 to ρ_2. The upper panel illustrates the impact on the rate of return on capital for any level of renewable resource extraction employment. As the rate of

[11] As mentioned before, Papageorgiou et al. (2017) estimates that the degree of natural inputs substitutability is larger than one, using data from developed countries.

natural resource substitution increases, the marginal product of capital curve $\Phi[L_z, \rho]$ rotates around the point $\overline{L}_z$ in the counter-clockwise direction. The properties of $\Phi[L_z, \rho]$ reflect the features of $RHS[L_z, \rho]$. The lower panel of the figure illustrates the impact on the value of the right-hand side of Equation (3.65) for any level of renewable resource extraction employment. Such increase in the rate of natural resource substitution cause a clockwise rotation in the $RHS[L_z, \rho]$ curve around the cut-off $\overline{L}_z$. This property arises because $RHS[L_z, \rho]$ is decreasing in $\Phi[L_z, \rho]$; see Equation (3.65). Capturing this variation is crucial as it can tell us about the indirect effect (the labour allocation effect) due to changes in the substitution parameter. Adding $LHS[L_z, \rho]$ to the framework, we can specify the long-run equilibrium and thus the growth effect of the substitution parameter.

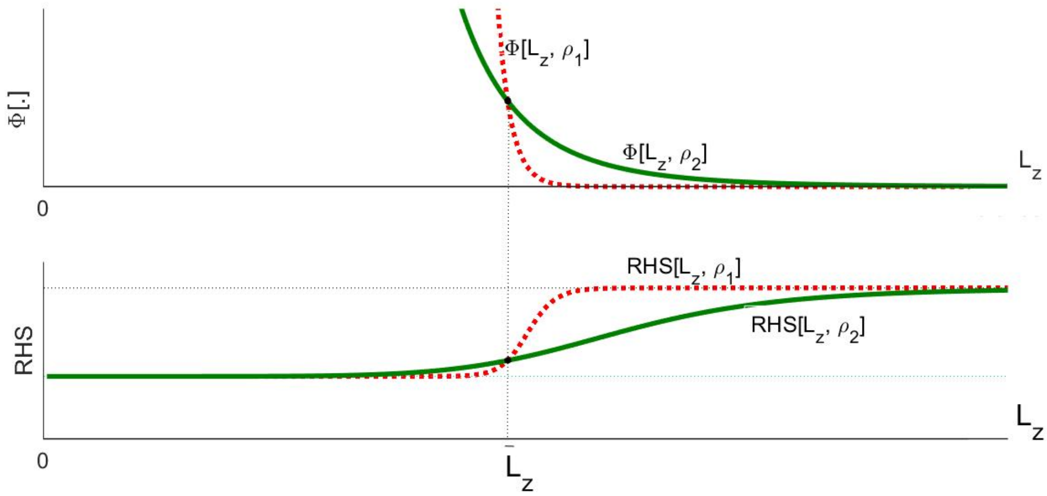

Figure 3.3: Graphical Illustrations of $\Phi[L_z, \rho]$ and $RHS[L_z, \rho]$.

Three possible long-term implications including growth-detrimental, growth-neutral and growth-enhancing scenarios can be classified regarding the location of the initial share of renewable extraction employment relative to the cut-off level. Firstly, if the initial employment share is lower than the cut-off one, growth-detrimental scenario is revealed. As illustrated in Figure 3.4 we can see that without the indirect effect, i.e. keeping the labour allocation unchanged, an increase in ρ from ρ_1 to ρ_2 causes a decrease in R from R_1^* to a rate lower than R_2^*. This capital return change captures *the direct effect* of an improvement in the degree of natural resource substitutability. However, such improvement will alter the equilibrium allocation in labour market in this case. As can be seen from the lower panel of Figure 3.4, the equilibrium L_z^* will decrease to a point which is strictly greater than $\underline{\gamma}$,i.e. $L_2^* \in (\underline{\gamma}, L_1^*)$. This labour market reallocation leads to an increase in the rate of return on capital which is *the indirect effect* in our context. Summing up these two isolated effects yields the net effect of the change in the degree of natural resource substitutability. Since the two effects act in opposite directions, the sign of the net effect will depend on the relative strength of these two forces. It is noteworthy that if $L^*_{z,2}$ were less than or equal to $\underline{\gamma}$, then R^*_2 would be greater than or equal to R^*_1 and the total effect would be positive. However, this situation would happen only if the slope of $LHS[L_z, \rho]$ were positive or zero. Thus, *the direct effect always dominates the indirect effect* and an improvement in the degree of natural resource substitutability will always reduce the long-run economic growth when the initial share of renewable extraction employment share is too low. Graphically, the economy moves from point P_1 to point P_2, where $LHS[L_z]$ and $RHS[L_z, \rho]$ curves intersect.

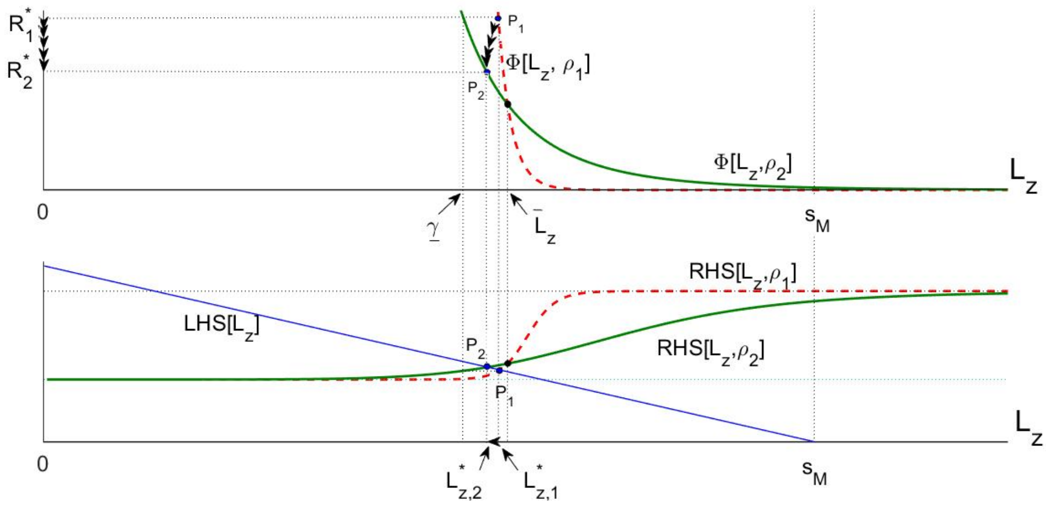

Figure 3.4: Growth-detrimental scenario

Secondly, if the initial employment share is equal to the cut-off one, growth-neutral scenario is revealed. As illustrated in Figure 3.5 an increase in ρ from ρ_1 to ρ_2 will neither cause a change in the return on capital and the labour market allocation. In other words, both direct and indirect effects are nil in this case. Hence, an improvement in the degree of natural resource substitutability will exhibit no long-run economic growth effect when the initial share of renewable extraction employment share is exactly the same as the cut-off allocation.

Finally, if the initial employment share is greater than the cut-off one, growth-enhancing scenario is revealed. As illustrated in Figure 3.6 we can see that without the indirect effect, i.e. keeping the labour allocation unchanged, an increase in ρ from ρ_1 to ρ_2 causes an increase in R from R^*_1 to a rate higher than R^*_2. This capital return change captures *the direct effect* of an improvement in the degree of natural resource substitutability. However, such improvement will alter the equilibrium allocation in labour market as well. As shown in the lower panel of Figure 3.6, the equilibrium L^*_z will increase to a point which is strictly lower than $\overline{\gamma}$,i.e. $L^*_2 \in (L^*_1, \overline{\gamma})$. This labour market reallocation leads to a decrease in the rate of return on capital which is *the indirect effect* in our context. As in previous

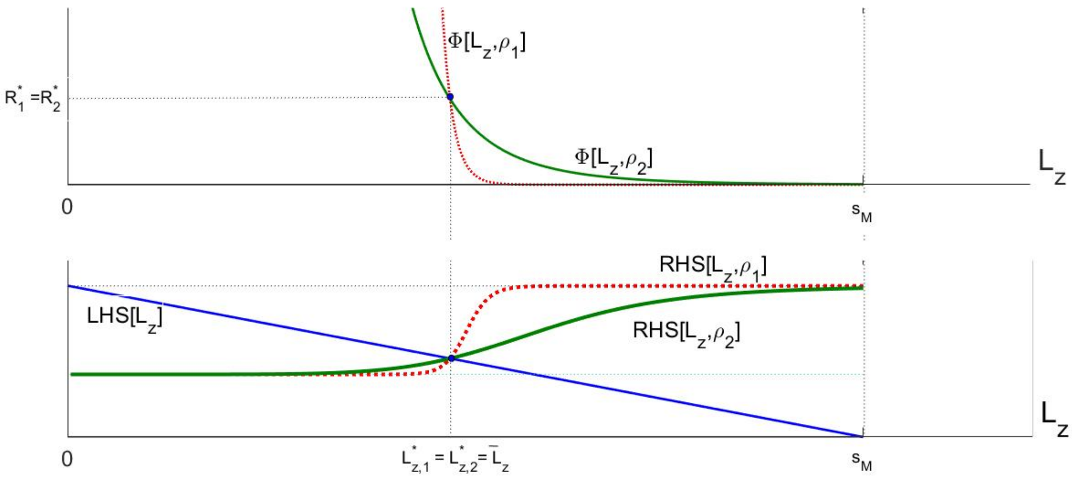

Figure 3.5: Growth-neutral scenario

cases, summing up these two isolated effects yields the net effect. It is noteworthy that if $L^*_{z,2}$ were greater than or equal to $\overline{\gamma}$, then R^*_2 would be lower than or equal to R^*_1 and the total effect would be negative. However, this situation would happen only if the slope of $LHS[L_z, \rho]$ were positive or zero. Thus, *the direct effect always dominates the indirect effect* and an improvement in the degree of natural resource substitutability will always stimulate the long-run economic growth when the initial share of renewable extraction employment share is sufficiently high. Graphically, the economy moves from point P_1 to point P_2, where $LHS[L_z]$ and $RHS[L_z, \rho]$ curves intersect.

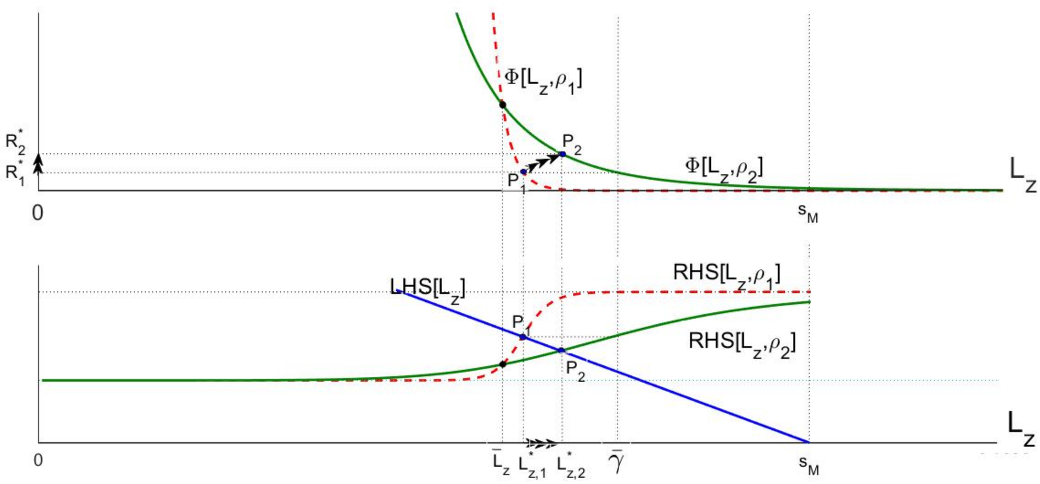

Figure 3.6: Growth-enhancing scenario

In sum, what we have learned from this experiment is as follows: An increase in the ability to substitute renewable resource for non-renewable resource will stimulate the long-run economic growth if the two types of natural inputs are gross substitute, the growth rate of non-renewable resources augmenting technology is positive but not too large and the initial stationary fraction of worker employed in renewable resource sector is sufficiently high.

3.4.4 Numerical Example

To get a better understanding of the role of labour allocation effect in determining long-term economic growth as the degree of natural resource substitution changes we resort to numerical illustration. To do this, we will use the U.S. data and relevant literature to construct a benchmark economy. Nevertheless, the economy presented here do not aim at approximating the U.S. economy, it is intended for illustrative propose only.

Benchmark Case

Table 3.4 summarises the benchmark parameters upon which our economy relies on. Firstly, we normalise D whereas we calibrate α_2, β and g_A on the same data as in the previous model.[12] Applying the strategies used in the previous model[13] to get $D = 1, \alpha_2 = 0.6131, \beta = 0.9822$ and $g_A = 0.0085$. These parameter values are identical to those of the previous model.

Secondly, the choice of parameter ϖ is drawn from the existing literature whereas the specific value of τ is assigned using the U.S data. The distribution parameter ϖ is set at 0.5008, following Golosov et al. (2014). In addition, the choice of benchmark value for τ is assigned the number 0.2. According to BEA (2019), the productive government spending to GDP ratio is around 0.14 - 0.22 between 1961 and 2016.[14] In our numerical illustration, we use 0.20 which is consistent with literature (e.g., Escobar-Posada and Monteiro (2015)).

Finally, three parameters including ρ, α_1 and δ are calibrated simultaneously to make the model predictions as close as possible to the selected real-world statistics in Table 3.1. This implies a value 0.0005, 0.3740, 0.0550 for ρ, α_1 and δ, respectively.

Table 3.5 summarises the model predictions using the benchmark parameters. The model predictions in the baseline case seem to be consistent with the U.S. economy in many aspects. Especially, the predictions of the percentage of labour input devoted to non-renewable resource extraction activity, capital to output ratio and long-run growth rate. The benchmark predictions of the percentage of labour input devoted to renewable

[12] See subsection 3.3.2 for clarifications of the sources and the periods of the data used in our calibration.

[13] Under this framework, the real interest rate depends on the gap between *after-tax* marginal productivity of capital and depreciation rate so that we pick a set of parameters so that $(1-\tau)R-\delta$ sufficiently close to 0.0385.

[14] Irmen and Kuehnel (2009) suggests that the productive spending includes the government current expenditures on public order and safety, economic affairs, health, education and government investment in fixed assets. Under this definition, the productive government spending will take place around 0.17 - 0.22 during 1961 - 2016.

Parameter	Value	Source
α_2	0.6131	Calibration, as in the previous model
β	0.9822	Calibration, as in the previous model
g_A	0.0085	Calibration, as in the previous model
D	1.0000	Normalisation
ϖ	0.5008	Golosov et al. (2014)
τ	0.2000	BEA (2019)
α_1	0.3740	Calibration
ρ	0.0005	Calibration
δ	0.0550	Calibration

Table 3.4: Benchmark Parameter Values

resource extraction activity seems to be a bit high. However, the numerical economy is not intended for representing the U.S. economy but used to explain economic intuitions implied by the model. Thus, we will use this set of parameters to illustrate the benchmark economy.

Figure 3.7 provides a graphical representation of the long-run equilibrium of the baseline economy. The upper panel shows LHS and RHS curves. At the intersection point, we can determine the long-run equilibrium employment in renewable resource extraction sector. Specifically, the equilibrium employment share in the baseline is about 1.0052 percent. The lower panel depicts the rate of return on capital, $\Phi[L_z]$, and the growth rate, $\beta(1 + (1 - \tau)\Phi[L_z] - \delta)$. Given the long-run equilibrium employment, we can get R^* and g^* as the values assigned at L_z^*.

Variable	Steady State Result	U.S.Data
Non-renewable resource sector employment (L_x)	0.0084	0.0084
Renewable resource sector employment (L_z)	0.0101	0.0045
Capital-Output Ratio (K/Y)	3.1310	3.2255
(After-tax) net user cost of capital($(1 - \tau)R - \delta$)	0.0406	0.0385
Long-Run Output per capita growth (g)	0.0220	0.0200

Table 3.5: Benchmark Result

Natural Resource Substitution and the Long-Run Economic Growth in the Benchmark Model

In this part, we examine the impact of changes in the substitution parameter on the long-run economic growth. As in the previous model, we examine growth effect via normalisation. We assume the BGP associated with $\rho = 0.0005$ is our baseline economy. The sensitivity analysis is illustrated in Figure 3.8.

Figure 3.8 shows the sensitivity to natural resource substitution variability within a range neighbouring the benchmark value 0.0005, from 0.0003 to 0.99. The top panel

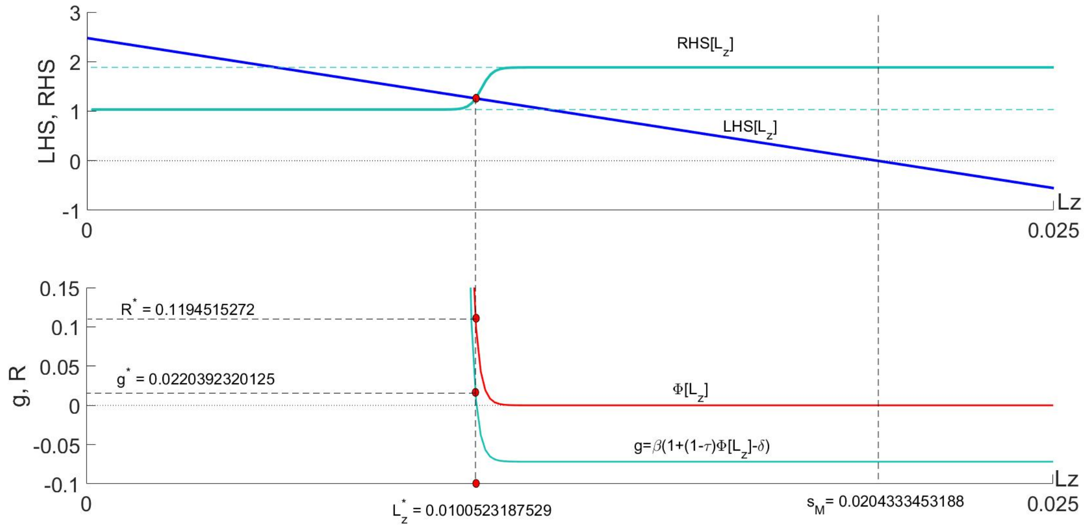

Figure 3.7: A specific BGP in Barro (1990) Model with Natural Resource Substitution

depicts the change in the share of labour devoted to renewable resource extraction sector, the middle panel depicts the change in the rate of return on capital and the bottom panel shows the change in the long-run growth rate of output per capita. It can be seen from the graph that as the substitution parameter increases the share of labour working in renewable resource extraction sector declines. In addition, the rate of return on capital decreases and thus the long-run growth rate decreases.

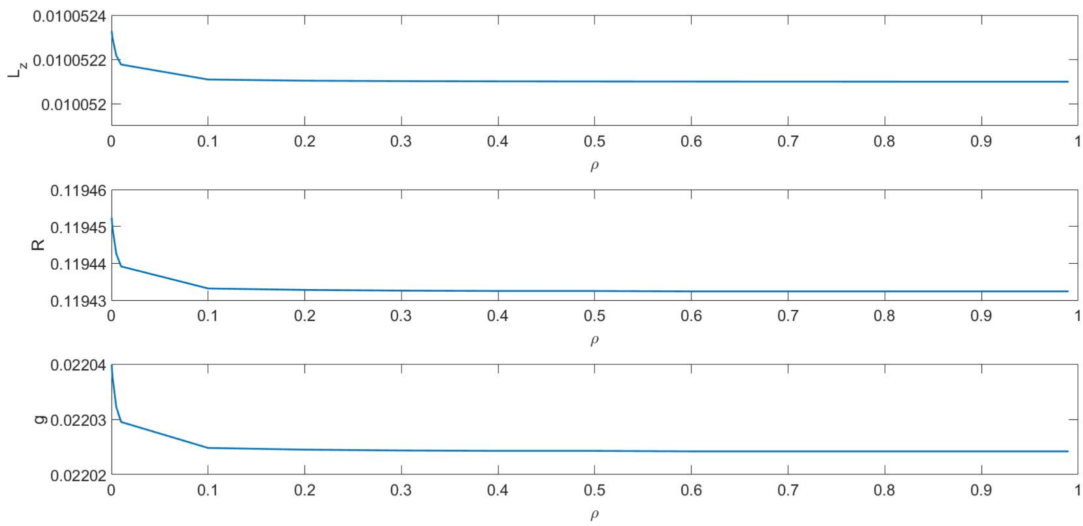

Figure 3.8: Growth effect under steady state normalisation in Barro (1990) Model with Natural Resource Substitution

Why does the substitution parameter weaken the long-run economic growth in the baseline economy? We can see that cut-off level $\overline{L}_z$ of the the benchmark economy is strictly greater than the equilibrium employment in renewable resource sector associated with $\rho = 0.0005$. Since $L_z^* < \overline{L}_z$, then increases in the state of natural resource substitution will weaken the percentage of employment in renewable resource sector.[15] This reduces the rate of return on capital and the the long-run growth rate as suggested by Proposition 3.2.

3.5 Discussions

Two points will be discussed here. To begin with, we emphasise that the labour allocation effect is necessary for the existence of the long-run growth effect. Even though labour allocation effect is an indirect effect when considering the impact of changes in the substitution parameter, the long-run growth effect will not appear without such an effect. Form the graphical analysis stated in subsection 3.4.3, if labour allocation is independent of the variations in ρ; consider the case that $L_{z,old}^* = \overline{L}_z$, the rate of return on capital will neither increase nor decrease. This result emphasises a vital role of the labour allocation effect.

In addition, policy implications based on our study seem to be practically beneficial. On the one hand, we are unsure if an increase in the ability to substitute renewable to non-renewable resources will enhance the long-run economic growth since it depends on the economic structure which reflects the percentage of employment in the renewable energy sector. In particular, the model suggests that if the share of worker in renewable energy sector is high enough, the improvement in such degree would guarantee the positive growth effect. On the other hand, one find that it is unlikely for the government to control the substitution parameter since this parameter is deep and it depends on the production technique of the economy. Improvement this parameter takes time, efforts, technological advancement and so on. These matters mostly rely on private decisions. The government should, instead, focus on how to stimulate the renewable resource employment to support the improvement in the degree of natural resource substitution.

3.6 Conclusion

There is by now a number of natural resource based economic growth models with natural resource substitution. What is lacking is a work determining a marginal effect of long-run economic growth when the degree of natural resource substitution increases. In this paper we have examined the analytical implications of such mechanism in two endogenous growth models: Romer (1986) and Barro (1990) models.

In order to analyse the effect of changes in the elasticity of substitution, isolation the impact from other arbitrary impacts is needed. To do so, we have introduced the concept

[15]In this benchmark economy, we have $\overline{L}_z = 0.0100523419953$ and $L_z^* = 0.0100523187529$.

of CES normalisation into the study. Apply this into the Romer (1986) model with natural resource substitution, we have shown that the degree of natural resource substitutability is growth-independent. This result is due to the fact that as such degree changes, the labour allocation across the three production sectors remains the same.

We, then, introduce an alternative model, á la Barro (1990) productive government spending endogenous growth model. In this model, we can show that labour allocation effect exists as the degree of natural resource substitution changes. The existence of labour allocation effect induces a change in the rate of return on capital so that growth effect appears. However, the direction is unclear depending on the initial equilibrium allocation in labour market and the state of natural resource substitution. In particular, improvement in natural resource substitutability will foster economic growth if the equilibrium employment in renewable resource extraction sector is sufficiently high, and the growth rate of non-renewable resource augmenting technology is strictly positive but not too large and the two types of natural inputs are gross substitute.

Chapter 4

Cross-Border Pollution: Growth and Structural Change Effects of Poor Countries

4.1 Introduction

It is widely accepted that climate change problem is one of the most pressing issues of our time. According to NASA (2019), as of August 2019, the average global temperature has increased by 0.85°C since 1880. The rapid increase in global temperature is alarming because it will affect every aspect of human life.

It is well-known that the main culprit of global warming is the stock of greenhouse gas emissions (chiefly CO_2) produced by fossil fuel based industrial activities and accumulated in the Earth's atmosphere (IPCC, 2014).[1] When fossil energies are extracted and utilised, the emitted greenhouse gases spread freely across the globe to integrate with the global climate system. This creates an unfair burden of climate change because most of the emissions are originated from developed and emerging countries who are also better equipped to deal with the impact of global warming than underdeveloped countries. According to WDI (2019), the OECD and BRICS countries account for 73 percent of global annual CO_2 emissions between 1960 and 2014 (see Figure 4.1). Moreover, recent studies reveal that countries that are most vulnerable to temperature increases are the poor ones, especially those in sub-Saharan Africa, South Asia, and the Pacific islands. The major emitters, on the other hand, do not seem to suffer any significant drawback from global climate changes (Althor et al., 2016; Bathiany et al., 2018; King and Harrington, 2018; Letta and Tol, 2019, among many others). The disproportionate burden of climate change effects on poor

[1] The report from IPCC (2014) reveals that 74 percent of such emissions come from industrial sectors around the world while the rest including agriculture, forestry, and other land use accounted for 24 percent in 2010.

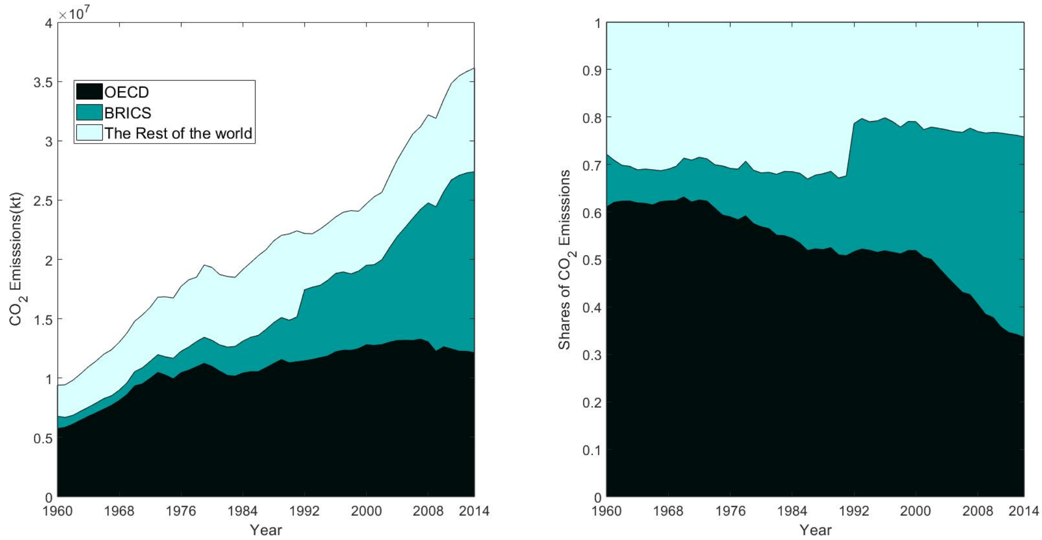

Figure 4.1: The share of CO2 emissions (kt) of BRICS and OECD members relative to global emission, 1960-2014.
Source: WDI (2019)

countries is an issue that warrants our attention.

In parallel with the studies on the differential effects of climate change across nations, there is now a large body of theoretical research in macroeconomics and development economics that aim at explaining the historical development of today's advanced economies and projecting the potential development trajectories of currently less-developed ones (Gollin et al., 2007; Yang and Zhu, 2013, among many others). A main research agenda of this literature is to reconcile two sets of empirical observations, known as the Kaldor facts and the Kuznets facts, within the neoclassical growth framework. The main objective is to ensure that this framework is consistent with the empirical observations in both aggregate and sectoral levels.[2] On the one hand, the Kaldor facts is a list of stylized facts regarding the generally stable pattern of major aggregate variables, such as the growth rate of per-capita output, capital-labour ratio, capital-output ratio, factor income shares, over a long period of time. On the other hand, the Kuznet facts summarise the main observations regarding sectoral transformation during economic development. Most notably, as economies grow, the employment share and valued-added share of non-agriculture increase but those of agriculture shrink.

Theoretical models of structural transformation are developed based on the now standard neoclassical growth model á la Ramsey (1928), Solow (1956), Koopmans (1963) and Cass (1965). The standard neoclassical growth model is a single-sector dynamic general

[2]See Herrendorf et al. (2014), for an intensive review of theoretical contributions and see van Neuss (2019) which also provides an integrated survey of the literature devoted to identifying the drivers of structural transformation. In addition, see Kaldor (1961) and Kuznets (1955) for both empirical regularities.

equilibrium model that can satisfactorily explain the Kaldor facts. While the assumption of a single final goods is convenient, it sidesteps the relationship between sectoral structure and aggregate growth. In particular, abstracting from multiple productive sectors implies that the Kuznets facts cannot be analysed. To overcome this limitation, studies in the structural transformation literature have extended the standard neoclassical framework to allow for multiple final goods and explore different mechanisms that can initiate structural change. Such extension broadens the ability of the theory as it can capture the Kaldor facts and the Kuznet facts at the same time. Examples of these studies include Kongsamut et al. (2001), Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008) and Alvarez-Cuadrado et al. (2017).

Theoretically, structural change mechanisms can be classified into two groups, namely demand side and supply side mechanisms. Studies that explore demand side mechanisms typically postulate that consumers have non-homothetic preferences over different types of goods. This captures the idea that consumers have a well-defined priority over different types of consumption. In particular, goods with higher priority (e.g., necessities like food consumption) will account for a large share of total expenditure when income level is low. Then, the expenditure share declines in relative terms as the economy develops and general income level rises. This variation in relative demand pattern, then, generates structural transformation. Studies that explore this mechanism include Echevarria (1997), Kongsamut et al. (2001) and Gollin et al. (2007). The supply side mechanism, on the other hand, uses sectoral differences in production technology to explain structural transformation. For example, Ngai and Pissarides (2007) focus on the bias in technological change across production sectors, while Acemoglu and Guerrieri (2008) focus on differences in capital intensity across sectors. These differences will induce changes in relative price which then lead to structural changes. More recently, Alvarez-Cuadrado et al. (2017) consider another supply-side factor, namely differences in the sectoral elasticity of substitution between capital and labour. They show that this type of differences can also induce a process of structural transformation via the imbalance in the marginal product of labour across sectors.

Building on these existing studies, in this chapter we explore a potential channel through which climate change can affect economic growth and the allocation of productive inputs across sectors. As mentioned before, Ngai and Pissarides (2007) suggest that biased technological change among production sectors could generate structural change. In particular, these authors analyse the conditions for structural transformation and aggregate balanced growth in a multi-sector model in which different sectors have the same production function but different growth rates in labour productivity. Biased technological change means that labour productivity tends to grow faster in one than in another sector, so supply could outgrow demand for the higher productive sector. This then leads to a rebalance of employment and value-added. A natural follow-up question is what causes the differences in productivity growth rate across productive sectors. One possible answer is climate change. In a recent study, Burke et al. (2015) provide evidence showing that agricultural productivity is more affected by global warming than in other sectors. This means that climate change could induce technological biasedness. As a consequence, it could affect the long-term development pattern in poor countries. These fundamentals lead us to address the question: how are the long-run growth and structural transformations affected by climate change?

Recent evidence on macroeconomic impacts of climate change suggest that such effects vary across countries and economic activities. To begin with, Burke et al. (2015) use data on economic production for 166 countries over the period 1960-2010 to estimate the country-specific output changes due to stochastic atmospheric changes. The study shows that countries located in tropical areas are more sensitive to climate changes than those located in temperate zones. The main finding of the study is that there is a hump-shaped relation between output growth and temperature. Specifically, if the initial level of temperature is around 13°C, then an increase in temperature has insignificant impact on the growth rate of per-capita output . However, this growth rate will significantly decline with a marginal increase in temperature if the initial temperature is higher than the threshold 13°C and the negative impact accelerates at higher temperatures. This non-linear and concave effect is affirmed by more recent studies (Althor et al., 2016; Pretis et al., 2018; IMF, 2017; Bathiany et al., 2018, for example). For example, IMF (2017) trace an impulse response of real per capita GDP to a 1°C increases in temperature of more than 180 economies during 1950-2015. They find that an increase in temperature lessens economic activity in countries with high average temperatures, while having the opposite effect in much colder countries. In terms of channels of effect, the IMF study also suggests that the adverse effect of climate change is biggest on agricultural output. Adverse effect on industrial output is also observed. The services sector is the only one that appears to be immune from bad weather.

The above discussion suggests at least two reasons why poor countries are more vulnerable to climate changes than rich countries. The first one is geographical location. Most of the poor countries are located in tropical areas which are most sensitive to temperature changes. In contrast, more affluent countries generally locate in temperate zones (See Figure 4.2). Based on the study of Burke et al. (2015), the average temperature in high income countries are very close to 13°C but for the poor the average weather is 25°C. Thus, with the same increase in temperature, the adverse effect of climate change will fall more heavily on the poor countries than on the rich countries. The second reason is that the poor countries rely more heavily on agriculture. The shares of agricultural employment and output tend to be higher in these countries than in developed countries. To illustrate

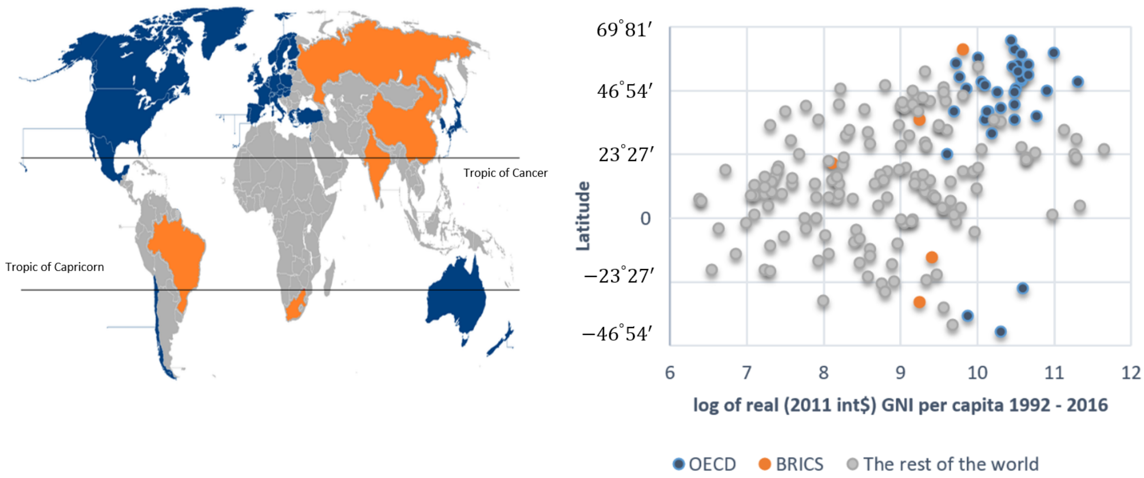

Figure 4.2: Income per capita and country location, 1992-2016.
Source: World Bank Data

Table 4.1: Employment Share 1997-2015 (World Bank)

Group	Agriculture	Non-Agriculture
OECD	0.06	0.94
Sub-Saharan Africa	0.57	0.43
South Asia	0.54	0.46
Pacific Islands and small states	0.37	0.63

this, the data from WDI (2019) reveal that the share of labour devoted to agricultural activities in OECD countries is approximately 6 percent but 57 percent in Sub-Saharan Africa between 1997 and 2015 (See Table 4.1). Also, in the same period, agricultural GDP in OECD countries is approximately 2 percent whereas 21 percent in Sub-Saharan Africa (See Table 4.2). As agricultural activity is mostly sensitive and being a core in tropical zone economies, climate change could generate the larger gap of agricultural productivity between the rich and the poor. This problem could further amplify the income difference between these countries (Gollin et al., 2002, 2007). In sum, the unequally distributional impact of climate change on rich and poor countries arises from relative geography and relative economic structure.

To analyse the effects of climate change on economic growth and the allocation of productive inputs across sectors, we develop a multi-sector neoclassical growth model along

Table 4.2: real GDP Share 1997-2015 (World Bank)

Group	Agriculture	Non-Agriculture
OECD	0.02	0.98
Sub-Saharan Africa	0.21	0.79
South Asia	0.21	0.79
Pacific Islands and small states	0.17	0.83

the line of Kongsamut et al. (2001), Ngai and Pissarides (2007), Acemoglu and Guerrieri (2008) and Alvarez-Cuadrado et al. (2017). In the model, there are two final-good sectors including agricultural sector and non-agricultural sector. Household preference is non-homothetic such that agricultural consumption is necessity. The interactions between the environment and macro-economy are specified as follows. Firstly, greenhouse gas emissions are exogenous in poor countries perspective. This assumption is reasonable as the emission from a single low-income country emission is negligible (WDI, 2019). Secondly, we follow Burke et al. (2015) by assuming that the temperature change affects productivity in agricultural and non-agricultural activities. It is important to note that climate-induced productivity changes are not necessarily identical across sectors. Within this framework, we characterise the market equilibrium and analyse the sensitivity of local growth and structural change when global temperature increases/decreases marginally. Comparative steady states analysis suggests that climate change always deteriorate the long-run economic growth. However, the effect on the long-run shares of sectoral employment and value-added are ambiguous.

We argue that analysing the problem using the model developed here is advantageous. To begin with, the general structure of our analysis is consistent to the literature concerning the climate-macroeconomic modelling according to Nordhaus (2008), among others, as the one-sector neoclassical growth model is the common ground of our and their frameworks. Secondly, structural change augmented-neoclassical growth model explicitly illustrates the stage of the economic development in a country. In particular, the shares of agricultural output and employment can be depicted so that the interactions between climate change and the stage of the development can be captured. Finally, as climate change effect varies across sectors, allowing multi-sectoral setting can highlight the role of sectoral heterogeneity corresponding to climate change.

To the best of our knowledge, Engstrom (2016) is the only work studying the interaction between climate change and macroeconomic impacts based on structural change augmented-neoclassical growth model. In that paper, the author develop a growth-climate model combining structural change originated by heterogeneity in the growth rates of TFP across sectors as in Ngai and Pissarides (2007). Also, the analysis follows Nordhaus (2008) by assuming that climate change only impacts on current output. The model is developed to set the optimal carbon tax. Also, numerical examples based on US and India data are provided to show the sensitivity of tax changes in various macro variables.

Our work is closely related to Engstrom (2016) but differs in some crucial aspects. Firstly, optimal policy setting is the main object of that paper but our work is the poor's growth and structural change effects of the global climate change problem. Secondly, Engstrom model uses supply-side mechanism to generate structural change; as in Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008), but we add demand-side mechanism to the analysis as well. As a consequence, the role of heterogeneity in income elasticities among various kinds of consumption goods will be emphasised here. We argue that our approach is more appropriate to study the poor countries as recent study suggests that the demand-side mechanism is the main source of structural change in the poor countries (Swiecki, 2017). Lastly, instead of applying Nordhaus (2008) measurement of climate damage, we follows Burke et al. (2015) by assuming that global warming will curb economic growth of the poor through deteriorating sectoral productivity growth rates.

This chapter will be organised as follows. In Section 4.2, we illustrate the structure of the model. Market equilibrium allocation is derived in Section 4.3. In Section 4.4, we illustrate comparative steady state analysis when climate change occurs. Section 4.5 makes concluding remarks.

4.2 The Model

We develop a closed economy in discrete time; indexed by $t = 0, 1, 2, \ldots$, to represent a hypothetical poor country with a potential mechanism through which climate change could affect it. The economy is comprised of two production sectors and populated by a representative infinitely-lived household whose preferences are derived from non-homothetic preferences towards two goods produced from the two sectors. Climate change is exogenously given and affects the hypothetical economy via altering productivity growths on both production technologies. Our economy is modeled along the line of structural transformation literature; inspired by the works by Kongsamut et al. (2001) and Ngai and Pissarides (2007). Furthermore, climate change-macro economy interaction is motivated by recent empirical investigations by Burke et al. (2015). The model description is as follows.

4.2.1 Production and Accumulation Technology

Consider the supply side of the economy with two productive sectors including agricultural sector (a) and non-agricultural sectors (n). Sector a employs workers to produce agricultural goods which are purely consumed. Sector n employs workers and hires physical capital to produce non-agricultural goods which are available for consumption or investment. These two goods are sold in competitive markets.

The general structures of input-endowments and technologies are as follows. In any period t, the economy is endowed with $K_t > 0$ units of physical capital and $N_t > 0$ units of labour. We assume that these two inputs are supplied inelastically in competitive markets. Given such endowments, the production technology of the sectors a is given by the following linear technology:

$$Y_{a,t} = A_{a,t}(l_{a,t}N_t). \tag{4.1}$$

In the sector n it is

$$Y_{n,t} = K_t^{\alpha_n}(A_{n,t}l_{n,t}N_t)^{1-\alpha_n}. \tag{4.2}$$

In the above expressions, $Y_{i,t} > 0$ and $l_{i,t} \in (0,1)$ represent outputs and employment share of sector i in period t, where $i \in \{a,n\}$, respectively. The parameter $\alpha_n \in (0,1)$ is the capital share of non-agricultural output. The variables $A_{a,t} > 0$ and $A_{n,t} > 0$ represent sector-specific labour-augmenting productivity terms. The sector-specific growth factors are allowed to be different. In regard to this matter, we assume that $A_{i,t}$ will grow exogenously by the time-variant rate $\tilde{\gamma}_{i,t}$. Specifically, we assume $A_{i,t} = A_{i,0}(1+\tilde{\gamma}_{i,t})^t$, where $A_{i,0}$ is the sector i's initial stock of technology. Time-varying exogenous process of the growth rate of $A_{i,t}$ reflects the evidence that changes in the average annual temperature will affect the growth rates of $A_{a,t}$ and $A_{n,t}$ in different degrees (Burke et al., 2015). We will discuss more about this later. Throughout, the non-agricultural good will be acting as the numerare. Then, its price is normalized to unity every period and all other prices and quantities are expressed in terms of the numerare good.

Let p_t, w_t, R_t and r_t be the unit price of agricultural good, wage rate, the rate of return on capital and the real interest rate in any period t, respectively. The representative firm in agricultural sector solves

$$\max_{\{l_{a,t}N_t\}} \Bigg\{ p_t Y_{a,t} - w_t(l_{a,t}N_t) \Bigg\}$$

and the representative firm in non-agricultural sector solves

$$\max_{\{l_{n,t}N_t, K_t\}} \Bigg\{ Y_{n,t} - w_t(l_{n,t}N_t) - R_t K_t \Bigg\}.$$

Under perfect competition each factor will be paid according its (value of) marginal product. Also, and no-arbitrage condition in labour market ensures that workers will be paid equally across sectors. These imply

$$w_t = p_t A_{a,t} = (1-\alpha_n)A_{n,t}\Big(\frac{K_t}{A_{n,t}l_{n,t}N_t}\Big)^{\alpha_n} \tag{4.3}$$

and

$$R_t \equiv r_t + \delta = \alpha_n\Big(\frac{K_t}{A_{n,t}l_{n,t}N_t}\Big)^{\alpha_n - 1}, \tag{4.4}$$

must hold in every period, where $\delta \in [0,1]$ is the capital depreciation rate.

We finish the supply-side structure by expressing the input-output market clearing conditions. To begin with, labour market clearing condition requires

$$l_{a,t} + l_{n,t} = 1. \tag{4.5}$$

Next, agricultural goods market clearing requires that

$$Y_{a,t} = N_t c_{a,t} \quad, \forall t \tag{4.6}$$

and non-agricultural market clearing requires that

$$Y_{n,t} = N_t c_{n,t} + K_{t+1} - (1-\delta)K_t \quad, \forall t \tag{4.7}$$

where $c_{a,t}$ and $c_{n,t}$ are agricultural consumption per capita and non-agricultural consumption per capita, respectively. As mentioned before, the agricultural goods market clearing condition (4.6) states that the aggregate output will only be used for the aggregate consumption $N_t c_{a,t}$ whereas market clearing condition in the non-agricultural goods market (4.7) states that the total amount of non-agricultural goods is allocated between the aggregate consumption $N_t c_{n,t}$ and gross investment $K_{t+1} - (1-\delta)K_t$.[3]

4.2.2 Preferences, Endowments, and Utility Maximisation

Let us move to the demand side of the economy. On this side, there is an infinitely long lived representative household, composed of N_t identical individuals in any period t. In each period, the representative agent derives his utility by consuming agricultural and non-agricultural goods. The individual preference over the two goods is defined along the line of macroeconomic development literature; see Gollin et al. (2004), Gollin et al. (2007) and Yang and Zhu (2013), for example. In particular, we assume that the individual will always prioritise his spendings. He begin spending for agricultural goods until a subsistence level is met. An intuitive explanation of this preference relation is that agricultural goods , i.e. fruit, vegetables, milk products, cereals, bread and fishery products, are necessity prerequisite for sustaining life. Once this level is met, the individual will allocate the remaining income to either consume over the two goods or to save in terms of capital accumulation. This preference relation can be represented by the following Stone-Geary function:

$$u(c_{a,t}, c_{n,t}) = \frac{\left[(c_{a,t} - \bar{c}_{a,t})^{\theta} c_{n,t}^{1-\theta}\right]^{1-\sigma}}{1-\sigma} \tag{4.8}$$

[3]In poor countries the financial markets are incomplete and, in turn, no any good can be transformed to an investment good.

where $\bar{c}_{a,t} \geq 0$ is subsistence level of consumption which is assumed to be time-varying, $\sigma > 0$ is the degree of relative risk aversion which is the reciprocal of the intertemporal elasticity of substitution (IES) and $\theta \in (0,1)$ represents a preference weight for agricultural goods. The assumption that $\bar{c}_{a,t}$ can change overtime implies that primarily necessary amount of agricultural goods needed could vary as the economy develops.[4] Here, we assume that $\bar{c}_{a,t}$ has a linear time trend, i.e. $\bar{c}_{a,t} = (1+\chi)^t \mu$ where $\chi \geq 0$ and $\mu \geq 0$ are the growth rate and the initial subsistence level of agricultural consumption, respectively.

At the beginning of time t, the household is endowed with the physical capital K_t and N_t units of labour input. They earn their income from supplying both endowments inelastically to input markets at the rates of returns w_t and r_t, respectively. The periodic-income will be then allocated to consumption $N_t(p_t c_{a,t} + c_{n,t})$ and savings $K_{t+1} - K_t$. The household's sequential budget constraint can be formally stated as

$$(1 + r_t)K_t + N_t w_t = N_t(p_t c_{a,t} + c_{n,t}) + K_{t+1}. \tag{4.9}$$

Given an initial capital$K_0 > 0$, the sequence of prices $\{p_{,t}, r_t, w_t\}_{t=0}^{\infty}$, and the sequence of primary necessity $\{\bar{c}_{a,t}\}_{t=0}^{\infty}$, the representative household solves

$$\max_{\{c_{a,t}, c_{n,t}, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t N_t u(c_{a,t}, c_{n,t}) \tag{4.10}$$

subject to (4.9) and the regularity constraints: $c_{n,t} \geq 0, c_{a,t} \geq \bar{c}_{a,t}$ and $K_{t+1} \geq 0$, for all t, where $\beta \in (0,1)$ is the discount factor.[5] The first-order conditions for utility maximisation are

$$\begin{aligned}
c_{a,t}: &\quad \left(\frac{\theta}{c_{a,t} - \bar{c}_{a,t}}\right)\left[(c_{a,t} - \bar{c}_{a,t})^{\theta} c_{n,t}^{1-\theta}\right]^{1-\sigma} - \lambda_t p_t = 0, \\
c_{n,t}: &\quad \left(\frac{1-\theta}{c_{n,t}}\right)\left[(c_{a,t} - \bar{c}_{a,t})^{\theta} c_{n,t}^{1-\theta}\right]^{1-\sigma} - \lambda_t = 0, \\
K_{t+1}: &\quad -\beta^t \lambda_t + \beta^{t+1}\lambda_{t+1}(1 + r_{t+1}) = 0,
\end{aligned}$$

where λ_t is the co-state variable associated to the state variable K_t. The first two conditions

[4]The subsistence level $\bar{c}_{a,t}$ is usually treated as a constant level; see Gollin et al. (2004), Gollin et al. (2007), Dekle and Vandenbroucke (2012) and Leukhina and Turnovsky (2016), among others. We subsume this restriction by allowing a more flexible trajectory for this value. The time-varying assumption is also used in previous studies as in Christiano (1989), Alvarez-Pelez and Diaz (2005) and Alonso-Carrera et al. (ming), among others. Potentially, he climate change problem could drive an increased minimum demand including expenditure on adaptation and mitigation (Stern, 2007). Intuitively, due to climate changes, as temperature rises, the tendency of diseases rises and more food is necessary to compensate the negative consequences of temperature rises. Thus, assuming time-varying subsistence level of consumption seems to be sensible.

[5]Given the utility function in (4.8), it is never optimal to have $c_{n,t} = 0$ or $c_{a,t} = \bar{c}_{a,t}$. Hence, we can focus on the interior solution of the household problem.

imply

$$\frac{c_{n,t}}{p_t c_{a,t}} = \frac{1-\theta}{\theta}\left(1 - \frac{\bar{c}_{a,t}}{c_{a,t}}\right), \tag{4.11}$$

known as an intratemporal tradeoff condition. This condition determines an equilibrium allocation of individual consumption expenditure across consumption goods.[6] Next, the second and the third conditions imply the Euler condition:

$$\beta(1 + r_{t+1}) = \left(\frac{c_{n,t+1}}{c_{n,t}}\right)^{\sigma}\left(\frac{p_{t+1}}{p_t}\right)^{\theta(1-\sigma)}. \tag{4.12}$$

which governs the intertemporal decision between current and future consumptions of non-agricultural goods. Optimising behaviour must also satisfy the following transversality condition (TVC, hereafter):

$$\lim_{t\to\infty} \beta^t \lambda_t K_{t+1} = 0 \tag{4.13}$$

which states that the rational household have no interest in holding valuable assets at the end of their life. Together with the TVC (4.13) and the household budget constraint (4.9), the first order conditions (4.11) and (4.12) characterise the optimising behaviour of the representative household.

4.2.3 Temperature and Productivity Growth

The climate change component is modelled consistently with the evidence from empirical studies focusing on poor countries as well as the literature concerning macroeconomics of climate change. According to recent empirical studies, it is observed that poor countries have contributed negligible emissions but suffered the greatest devastation from climate change (Dell et al., 2012; Burke et al., 2015; Althor et al., 2016; IMF, 2017, for example). The observed problem comes from the fact that the effects of GHG emissions can be felt beyond a country's border. Since the poor have contributed far less to climate change we assume that for a hypothetical poor economy the emission is exogenous. Next, we know that climate change problem can be approximated by the tendency of the annual average temperature observation (Shakun et al., 2012; Haustein et al., 2017). In particular, Shakun et al. (2012) have found the tightly co-varying atmospheric CO2 levels and surface temperature anomalies during the last 20,000 years. Let T_t be the period t average temperature

[6] Note that if $\bar{c}_{a,t} = 0$ for all t the consumption expenditure share in the agricultural good $\left(E_{a,t} \equiv \frac{p_t c_{a,t}}{p_t c_{a,t} + c_{n,t}}\right)$ will be constant and equal to θ. In another case, if $\bar{c}_{a,t} > 0$ for all t and growing at the same rate as $c_{a,t}$, then $E_{a,t}$ will be constant and equal to $\frac{\theta}{\theta + \left(\frac{1-\theta}{\theta}\right)\left(1 - \frac{\mu}{c_{a,0}}\right)}$. If $\bar{c}_{a,t} > 0$ for all t and growing at the rate which is less than that of $c_{a,t}$, the consumption expenditure share in the agricultural good is decreasing as the economy develops such that $E_{a,t} = \frac{\theta + (1-\theta)\frac{p_t \bar{c}_{a,t}}{c_{n,t}}}{1 + (1-\theta)\frac{p_t \bar{c}_{a,t}}{c_{n,t}}}$. This feature is as a result of non-homothetic preferences.

of the hypothetical poor country. Then, exogenous emissions implies that the sequence $\{T_t\}_{t=0}^{\infty}$ is also exogenously given.

In order to relate the climate change problem to the economy, we follow Burke et al. (2015) by assuming that climate change affect the economy via altering the sectoral total factor productivity growths. In particular, for $i \in \{a, n\}$, the growth rate of $A_{i,t}$ between time $t-1$ and time t is given by

$$\ln\Big(\frac{A_{i,t}}{A_{i,t-1}}\Big) \equiv \ln(1+\tilde{\gamma}_{i,t}) = \gamma_i + \Omega_i(T_t). \tag{4.14}$$

From the above expression, γ_i is a non-climate sensitive component labour-augmenting technological progress whereas $\Omega_i(T_t)$ is the growth-rate sensitivity to temperature.[7] In addition, as in their study, we introduces a non-linear specification:

$$\Omega_i(T_t) = \xi_{i,1}T_t + \xi_{i,2}(T_t)^2, i \in \{a, n\}. \tag{4.15}$$

Combined with the quadratic specification (4.15), sectoral productivity growth condition (4.14) describe a quadratic relationship between sectoral peroductivity growth and temperature. In particular, if $\xi_{a,2}$ and $\xi_{n,2}$ are strictly negative, then there exist optimal levels of temperature under which the growth factors of $A_{a,t}$ and $A_{n,t}$ are maximised. This feature is consistent with the empirical findings of Burke et al. (2015), which show a hump-shaped relationship between annual growth rate of GDP per capita and annual average temperature. Another remarkable point is that given an exogenous process of the average annual temperature $\{T_t\}_{t=0}^{\infty}$, a sequence of sectoral productivities $\{A_{a,t}, A_{a,t}\}_{t=0}^{\infty}$ is given as well.

The climate-change induced economic loss through sectoral productivity growth effect is a channel that we emphasise in this study. In mainstream studies of macroeconomic impacts of climate change, pioneered by Nordhaus's DICE Model (Nordhaus, 2008), the scholars intorduce a damage function which is increasing and convex in average temperature anomalies (relative to pre industrials) and it performs as a scale factor. This factor multiplies the aggregate production function in each period to decompose net output out of damages. An implicit implication of this measurement of climate change effect is that climate change appears to have temporary impacts on economic output, and thus temperature anomalies has only level effects (Dietz and Stern, 2015). It is argued if this scale quantification is under estimating the climate change effect in terms of production loss. Recent studies, e.g. Dell et al. (2012), Burke et al. (2015) and Letta and Tol (2019), have shown that the impact seems to be long lasting as they have found that climate change

[7] Some potential functional forms of $\Omega_i(T_t)$ have been specified and tested empirically. It is assumed to be linear and tested in Dell et al. (2012). After that, Moore and Diaz (2015) applies this empirical specification to extend DICE Nordhaus (2008) model to compare the social cost of carbons between their model and the original DICE. However, the sectoral heterogeneity is ignored in both studies. More recently, a study from Burke et al. (2015) introduce a non-linear specification with sectoral decomposition.

can potentially cause GDP per capita growth reduction. We follows this finding when developing the model to rationalise the effect on the hypothetical poor economy. Alternatively, some other channels; for example physical capital depreciations and poorer human health, have been discussed as some potential channels as well (IMF, 2017). However, in this analysis, we focus only on the sectoral productivity growth effect.

4.3 Dynamic General Equilibrium

In this section competitive equilibrium allocation will be defined. Then, dynamical system associated with the competitive equilibrium will be characterised. Finally, the balanced growth equilibrium will be illustrated.

4.3.1 Market Equilibrium

Definition 4.1. *A market equilibrium is a sequence of prices* $\{p_t, w_t, r_t\}_{t=0}^{\infty}$*, an allocation for the representative household* $\{c_{a,t}, c_{n,t}, K_{t+1}\}_{t=0}^{\infty}$*, an allocation for representative firms in both sectors* $\{l_{a,t}, l_{n,t}\}_{t=0}^{\infty}$ *such that:*

(i) the allocation $\{c_{a,t}, c_{n,t}, K_{t+1}\}_{t=0}^{\infty}$ *solves the household problem (4.10) subject to (4.9) given the sequence of prices* $\{p_t, w_t, r_t\}_{t=0}^{\infty}$*, the sequence of subsistence level of agricultural consumptions* $\{\bar{c}_{a,t}\}_{t=0}^{\infty}$ *and the initial stock of capital* $K_0 > 0$*,*

(ii) factor prices satisfy (4.3) and (4.4) given the sequence of sectoral productivities $\{A_{a,t}, A_{n,t}\}_{t=0}^{\infty}$*,*

(iii) markets clear, i.e. (4.5), (4.6) and (4.7) hold.

4.3.2 Intertemporal Equilibrium Characterisation

It will prove useful to characterise an intertemporal equilibrium using a modified dynamical system that contains a steady state corresponding to an (original) intertemporal equilibrium along which all variables grow at constant rates (possibly zero), if exists.[8] To begin with, we define the following de-trended variables:

$$\tilde{p}_t \equiv p_t\Big(\frac{A_{a,t}}{A_{n,t}}\Big), \quad \tilde{c}_{n,t} \equiv \frac{c_{n,t}}{A_{n,t}} \quad \text{and} \quad z_t \equiv \frac{K_t}{A_{n,t}N_t}.$$

Next, given such de-trended variables, we combine the sectoral zero-profit conditions (4.3)-(4.4), the market-clearing conditions (4.5)-(4.7), and the conditions (4.9), (4.11)-(4.13) which deliver the solution to the household's problem to obtain a set of three static conditions, two dynamic equations, one transversality condition:

$$\tilde{p}_t = (1-\alpha_n)\Big(\frac{z_t}{l_{n,t}}\Big)^{\alpha_n}, \tag{4.16}$$

[8] Formal definitions of stationary equilibria will be defined in the next subsection. See Definition 4.2.

$$\tilde{c}_{n,t} = \Big(\frac{1-\theta}{\theta}\Big)\tilde{p}_t\Big[1 - l_{n,t} - \frac{\overline{c}_{a,t}}{A_{a,t}}\Big], \tag{4.17}$$

$$\beta(1 + R_{t+1} - \delta) = \Big(\frac{\tilde{c}_{n,t+1}}{\tilde{c}_{n,t}}\Big)^{\sigma}(1 + \tilde{\gamma}_{n,t+1})^{\sigma}\Big(\frac{\tilde{p}_{t+1}}{\tilde{p}_t}\Big)^{\theta(1-\sigma)}\Big(\frac{1 + \tilde{\gamma}_{n,t+1}}{1 + \tilde{\gamma}_{a,t+1}}\Big)^{\theta(1-\sigma)}, \tag{4.18}$$

$$z_t^{\alpha_n} l_{n,t}^{1-\alpha_n} = \tilde{c}_{n,t} + (1+n)(1 + \tilde{\gamma}_{n,t+1})z_{t+1} - (1-\delta)z_t, \tag{4.19}$$

$$R_t = \alpha_n\Big[\frac{z_t}{l_{n,t}}\Big]^{\alpha_n - 1}, \tag{4.20}$$

and

$$\lim_{t \longrightarrow \infty} \beta^t \frac{\partial u(\cdot)}{\partial c_{n,t}} A_{n,t+1}N_{t+1}z_{t+1} = 0 \tag{4.21}$$

, given an exogenous process $\{A_{a,t}, A_{n,t}, \overline{c}_{a,t}\}_{t=0}^{\infty}$ and an initial condition $z_0 > 0$. Again, all proofs are relegated to several appendices at the end of the document.

Brief interpretations of these equilibrium conditions are in order. Firstly, similar to Alvarez-Cuadrado et al. (2017), condition (4.16) is known as the labour mobility condition. This condition ensures that the two sectors pay the same wage. Secondly, condition (4.17) is a version of the intratemporal tradeoff condition which ensures that the marginal utility per last dollar spent must be equal across all consumption goods. Thirdly, condition (4.18) is a restatement of the Euler condition (4.11). Fourthly, condition (4.19) is an alternative expression of the resource constraint which governs the evolution of the state variable z_t. Next, condition (4.20) is another expression of the rate of return on capital. Lastly, condition (4.21) is a version of the TVC (4.13).

In sum, we can formally state that an inter-temporal equilibrium is characterised by a sequence $\{l_{n,t}, R_t, \tilde{p}_t, \tilde{c}_{n,t}, z_{t+1}\}_{t=0}^{\infty}$ that satisfies (4.16)-(4.21), given an exogenous process $\{A_{a,t}, A_{n,t}, \overline{c}_{a,t}\}_{t=0}^{\infty}$ and an initial condition $z_0 > 0$.

Before moving onto the next steps of this study, it would be beneficial to characterise another crucial variable in structural transformation literature which is non-agricultural output share. The total output in period t, denoted by $Y_{a,t}$, is equal to the value of production $Y_t = p_{a,t}Y_{a,t} + Y_{n,t}$. We can apply (4.1)-(4.3) and (4.5) to the previous expression to obtain the total output function:

$$Y_t = A_{n,t}N_t\Big[\frac{z_t}{l_{n,t}}\Big]^{\alpha_n}(1 - \alpha_n + \alpha_n l_{n,t}). \tag{4.22}$$

Non-agricultural output share is defined as $s_{Yn,t} \equiv \frac{Y_{n,t}}{Y_t}$. Since $Y_{n,t} = \Big[\frac{z_t}{l_{n,t}}\Big]^{\alpha_n} l_{n,t}A_tN_t$, then

$$s_{Yn,t} = \frac{l_{n,t}}{1 - \alpha_n + \alpha_n l_{n,t}} \tag{4.23}$$

is an expression of non-agricultural output share represented as a function of $l_{n,t}$. Note that

$$\frac{ds_{Yn,t}}{dl_{n,t}} = \frac{1-\alpha_n}{[1-\alpha_n+\alpha_n l_{n,t}]^2} > 0. \tag{4.24}$$

This means that the non-agricultural output share is strictly increasing in the non-agricultural labour share.

4.3.3 Steady State Equilibria and Stationary Growth Paths

In this subsection, we will discuss about a special case of dynamic general equilibria, known as balanced growth paths (BGPs, hereafter). We start with featuring stationary state of climate change. Then, we define a formal definition of BGPs. Next, we characterise equilibrium allocation corresponding to this long-run equilibrium as well as the sufficient conditions under which the equilibrium allocations exists and is unique.

Zero Anomaly and Stable Sectoral Productivity Growths

To characterise a stationary equilibrium of a growth model with climate change phenomena, a precise definition of stationary situation of climate change is needed. In regard to this matter, we state that the climatic steady state corresponds to the state in which the temperature anomaly disappears. Combined with Paris Agreement assertion, there are scientific studies projecting some possible trajectories of average annual temperature anomalies; see Goodwin et al. (2018) and Nicholls et al. (2018), for example. The studies generally accept that no matter whether the Paris agreement will be achieved, the long-run temperature will reach a stable level as it is widely agreed that the main source of climate change is generated from burning fossil energy which is finite.[9]

We rationalise the climatic steady state as follows. Suppose that there is $\tau \geq 0$ such that

$$t \geq \tau \quad \Rightarrow \quad T_t = T^* > 0.$$

After defining the climatic steady state precisely, we can evaluate the stationary growth rates of the labour augmenting technologies in agricultural and non-agricultural sectors. Then, by using (4.14) and (4.15), we can state that in the long-run

$$\frac{A_{i,t+1}}{A_{i,t}} = 1 + \tilde{\gamma}_i^*; i \in \{a, n\}, \forall t \geq \tau \tag{4.25}$$

where $\tilde{\gamma}_i^*$ is a stationary net growth rate of the labour augmenting technologies in sector i.

[9]It does not mean that climate change is not as serious as we expect. We just clarify the physical constraint of this kind of natural resources. Without limiting the depletion of fossil fuels, the climate change problem would reach a very high degree that could cause an environmental catastrophe; see Hope et al. (2017, p.87).

Balanced Growth Path

We define a balanced growth path (BGP) under the assumption that temperature remains constant all the time.[10] Suppose that $T_t = T^* > 0$ for all t. Then, the constant sectoral growth rates can be derived, denoted by $\tilde{\gamma}_a^*$ and $\tilde{\gamma}_n^*$. Given these constant productivity growths, we can define a BGP as follows.

Definition 4.2. *A market equilibrium is said to be a BGP if it satisfies three additional conditions:*

(iv) the sectoral allocation of labour input is constant overtime, so that

$$l_{n,t} = l_n^* \in (0,1),$$

(v) the rate of return on capital is constant overtime, so that

$$r_t = r^* \; and \, R_t = R^* = r^* + \delta > 0,$$

(vi) growth rates of all variables are constant over time.

Some restrictions on parameters are required in order to establish the existence of a BGP. The following lemma highlight two such restrictions.

Lemma 4.1. *The following results hold:*

(i) a BGP exists only if $\bar{c}_{a,t} = \mu A_{a,t}$, for some $\mu > 0$.

(ii) The TVC (4.13) is satisfied in any BGP if and only if

$$\beta(1+n)\Big[(1+\tilde{\gamma}_n^*)^{(1-\theta)}(1+\tilde{\gamma}_a^*)^{\theta}\Big]^{1-\sigma} < 1. \tag{4.26}$$

Lemma 4.1 is standard in models with perpetual growth in per-capita variables and a minimum consumption requirement. Specifically, the statement (i) states that the subsistence level of consumption must be growing over time in order for a balanced growth equilibrium to remain in existence. In particular, it must grow at the same rate as that of $A_{a,t}$ to maintain the equality in (4.17) along the BGP. On the other hand, the statement (ii) provides a parameter restriction which ensures that the TVC (4.21) condition; and thus (4.13), is satisfied.

Proposition 4.1. *Suppose that conditions (i) and (ii) in Lemma 4.1 and $0 \leq \mu < 1$ hold. Then there exists a unique BGP such that*

$$R^* = \frac{1}{\beta}(1+\tilde{\gamma}_n^*)^{\sigma}\left(\frac{1+\tilde{\gamma}_n^*}{1+\tilde{\gamma}_a^*}\right)^{\theta(1-\sigma)} - (1-\delta) > 0, \tag{4.27}$$

[10] In practice, T_t may be interpreted as the average temperature within a certain time period, i.e., a calendar year. In this case, constant temperature means a stable pattern of average temperature across years.

$$\tilde{p}^* = (1-\alpha_n)\Big[\frac{R^*}{\alpha_n}\Big]^{\frac{\alpha_n}{\alpha_n - 1}} > 0, \tag{4.28}$$

$$l_n^* = \frac{\Big(\frac{1-\theta}{\theta}\Big)(1-\alpha_n)(1-\mu)}{\Big(\frac{1-\theta}{\theta}\Big)(1-\alpha_n) + 1 - \alpha_n \frac{\Gamma_1^*}{R^*}} \in (0,1), \tag{4.29}$$

$$z^* = \Big[\frac{R^*}{\alpha_n}\Big]^{\frac{1}{\alpha_n - 1}} l_n^* > 0 \tag{4.30}$$

$$\tilde{c}_n^* = \Big(\frac{1-\theta}{\theta}\Big)\tilde{p}^*\Big[1 - l_n^* - \mu\Big] > 0 \tag{4.31}$$

where $\Gamma_1^* \equiv \Big[(1+n)(1+\tilde{\gamma}_n^*) - (1-\delta)\Big] > 0.$

The above proposition states that if the temperature is stationary, the life-time utility converge and subsistence level of agricultural consumption is growing at a proper rate while its initial level is not too large, then there exist a unique BGP.

4.4 Comparative Steady States

In this section, we compare the long-run BGPs with and without climate change. In particular, we compare the two BGPs that differ only in their stationary temperatures. Regarding this issue, we emphasise four aspects including the long-run growth rate of output per capita, the rate of return on capital and structural transformations including employment share and value-added share.

4.4.1 General Picture

Before doing so, we need a primitive about the climate damages. As mentioned in literature, climate change induces heterogeneous effects on poor nations through reducing sectoral output growth rates (Burke et al., 2015; IMF, 2017). We follows Burke et al. (2015) by assuming that

$$\frac{\partial \tilde{\gamma}_a^*}{\partial T^*} < \frac{\partial \tilde{\gamma}_n^*}{\partial T^*} < 0. \tag{4.32}$$

The above inequalities states that the sectoral growth rates will decreases with a slightly increase in temperature change such that the magnitude is larger for agricultural sector.

After imposing the climate damage primitive, we analyse the two stationary states that differs only in their steady state temperatures. First, we consider the effect on the long-run growth rate of output per capita. Apply the Euler condition (4.18) with the stationary rate of return (4.24) and the restriction $\frac{c_{n,t+1}}{c_{n,t}} = 1 + g^*$, where g^* is the long-run growth rate of output per capita. Then, $g^* = \tilde{\gamma}_n^*$. This implies

$$\frac{\partial g^*}{\partial T^*} = \frac{\partial \tilde{\gamma}_n^*}{\partial T^*} < 0. \tag{4.33}$$

The result is consistent with structural transformation literature concerning agricultural and non-agricultural disaggregation as it appears that the long-run economic growth is equal to the growth rate of labour-augmenting technological factor in the sector that provides goods used for capital accumulation (Kongsamut et al., 2001). As non-agricultural productivity growth determines the long-run economic growth, the climate change effect plays a role in determining the long-run growth effect via altering the non-agricultural productivity growth factor.

Second, we draw an impact on the rate of return on capital. From (4.27), straightforward differentiation yields

$$\frac{\partial R^*}{\partial T^*} = \frac{1}{\beta}\frac{(1+\tilde{\gamma}_n^*)^{\sigma+\theta(1-\sigma)}}{(1+\tilde{\gamma}_a^*)^{\theta(1-\sigma)}}\left\{\frac{\sigma+\theta(1-\sigma)}{1+\tilde{\gamma}_n^*}\frac{\partial \tilde{\gamma}_n^*}{\partial T^*} + \frac{\theta(\sigma-1)}{1+\tilde{\gamma}_a^*}\frac{\partial \tilde{\gamma}_a^*}{\partial T^*}\right\}. \tag{4.34}$$

The sign of $\frac{\partial R^*}{\partial T^*}$ is ambiguous. In particular, it depends on the sign of the term in the bracket $\{\bullet\}$ on the RHS of (4.34). Nevertheless, one can observe that the parameter σ plays a crucial role in determining the sign of this expression:

$$\sigma \geq 1 \Rightarrow \{\bullet\} < 0 \Rightarrow \frac{\partial R^*}{\partial T^*} < 0,$$

$$\sigma < 1 \Rightarrow \{\bullet\} \lesseqqgtr 0 \Rightarrow \frac{\partial R^*}{\partial T^*} \lesseqqgtr 0.$$

It is obvious that climate change will always weaken the rate of return on capital when $\sigma \geq 1$. However, when $\sigma < 1$ the impact on the capital return becomes ambiguous. Particularly, climate change will weaken the rate of return on capital as long as the value of σ in not too low. Otherwise, it could stimulate the rate of return on capital if σ is sufficiently low. Evidently, meta-analysis from Havranek et al. (2015) suggest that the estimated EIS differs across countries (both rich and poor) but typically lying between 0 and 1. This implies that $\sigma \geq 1$ is very likely and climate change seems to harm the rate of return on capital in poor countries.

The effects of climate change on the long-run structural transformations are generally ambiguous. We begin by considering the impact on the sectoral employment share. As we have illustrated in Proposition 4.1, the long-run non-agricultural employment share is given by the condition (4.29). By differentiating this equation with respect to T^*, we get

$$\frac{\partial l_n^*}{\partial T^*} = \frac{\left(\frac{1-\theta}{\theta}\right)(1-\alpha_n)(1-\mu)}{\left[\left(\frac{1-\theta}{\theta}\right)(1-\alpha_n) + 1 - \alpha_n \frac{\Gamma_1^*}{R^*}\right]^2} \times \frac{\alpha_n}{(R^*)^2} \times \left[R^* \frac{\partial \Gamma_1^*}{\partial T^*} - \Gamma_1^* \frac{\partial R^*}{\partial T^*}\right]. \tag{4.35}$$

Intuitively, we argue that the ambiguity of the impact on the employment share arises due to the influence of the climate change on the rate of return on capital. To illustrate this, let's consider (4.35). There are two potential effects of climate change that could influence on the employment share. These two effects include *the direct non-agricultural productivity growth effect*; measured by $\frac{\partial \Gamma_1^*}{\partial T^*}$, and *the capital return effect*; measured by $\frac{\partial R^*}{\partial T^*}$. For the former effect, when the average annual temperature increases, it induces a decrease in the non-agricultural productivity growth. While keeping R^* unchanged, a decrease in $\tilde{\gamma}_n^*$ will induce a decrease in Γ_1^* and thus the non-agricultural labour share. In other words, without other influences the share of non-agricultural employment will be negatively affected by climate change. However, the ambiguity arises because the increasing in the average annual temperature also induces a change in the rate of return on capital through changes in both agricultural and non-agricultural productivity growth rates; see (4.34). This change will induce a change in the labour share indirectly via changes in R^* in ambiguous ways.

Finally, the impact on the non-agricultural value-added share is consistent with that of the labour share. Based on (4.23) and (4.24), it is obvious that

$$\frac{\partial s_{Yn}*}{\partial l_n^*} = \frac{1-\alpha_n}{[1-\alpha_n+\alpha_n l_n^*]^2} > 0.$$

Thus, the climate change affects the employment and value-added shares in the same direction.

In sum, what we have learnt from the general picture are as follows. Firstly, climate change matters for the long-run economic growth if it hurts the non-agricultural productivity growth rate. Climate change will affect the rate of return on capital via dampening the productivity growth rates in both sectors. Labour share will be affected via two channels including the direct non-agricultural productivity growth effect and the capital return effect. The ambiguity of the employment share is solely driven by the capital return effect. Finally, the impact on the value-added share is co-varied with that of the employment share.

4.4.2 A Special Case: The Role of Agricultural Productivity Growth Effect

Since poor countries rely heavily on agricultural activities, it would be useful if we emphasise the impacts of climate change through changes in the long-run growth rate of agricultural productivity. Regarding this matter, this subsection will be devoted to analyse the impacts on the long-run economic growth and structural transformation resulting from agricultural productivity growth detrimental.

As before, the analysis requires a primitive about climate change damages. In subsection 4.4.1, we conjecture that climate change harms productivity growth rate in both sectors. Now, we assume that

$$\frac{\partial \tilde{\gamma}_a^*}{\partial T^*} < \frac{\partial \tilde{\gamma}_n^*}{\partial T^*} = 0 \tag{4.36}$$

That is climate change will affect only the agricultural productivity growth and not that of another sector. Note that this analysis can be seen as a counter-factual experiment of the general picture that we have illustrated before.

Firstly, agricultural productivity growth plays no role in determining the long-run economic growth, i.e.

$$\frac{\partial g^*}{\partial T^*} = 0.$$

As mentioned before, along a balanced growth path, the long-run growth rate is determined by the productivity growth in the non-agricultural sector. When assuming that climate change will not affect non-agricultural production in poor countries, even if the negative effect on agricultural production remains, the long-run growth effect disappears.

Secondly, the impact on the rate of return on capital is unclear as before. From (4.34), the capital return effect turns out to be

$$\frac{\partial R^*}{\partial T^*} = \frac{1}{\beta} \frac{(1+\tilde{\gamma}_n^*)^{\sigma+\theta(1-\sigma)}}{(1+\tilde{\gamma}_a^*)^{\theta(1-\sigma)}} \left\{ \frac{\theta(\sigma-1)}{1+\tilde{\gamma}_a^*} \frac{\partial \tilde{\gamma}_a^*}{\partial T^*} \right\}.$$

As in the general picture, the relative risk aversion parameter σ plays a crucial role in determining the sign of the effect. In particular, climate change will decrease (increase) the rate of return on capital if $\sigma > (<) 1$. In addition, when $\sigma = 1$; which implies that the utility function is logarithmic, climate change will create no effect on the rate of return on capital.

Next, the effects on structural changes including labour share and value-added share are also ambiguous. For employment share, even if the direct non-agricultural productivity growth effect disappear, the capital return effect remains and turns out to be the only driver causing the reallocation in labour market. As a result, the climate change impact on the employment share remains but in ambiguous direction. For the value-added share, as in the general case, climate change still alters the share since the capital return effect is still functioning. In a specific situation when utility function is logarithmic, climate change will affect neither employment share nor value-added share. This happens because of two reasons. Firstly, in this experiment we mute the non-agricultural productivity growth detrimental. This turns out that $\frac{\partial l_n^*}{\partial \Gamma_1^*} = 0$, i.e. there is no direct non-agricultural productivity growth effect. Secondly, when the utility function is logarithmic, the capital return effect disappears. Thus, the effects on the employment share and the value-added share disappear.

In sum, this experiment suggests that agricultural productivity growth detrimental plays no role in determining the growth effect of climate change. Still, agricultural productivity growth detrimental causes changes in the stationary rate of return on capital and thus structural change. However, the directional effects are ambiguous.

4.5 An Extension

Having shown that climate change always weakens the long-run economic growth but may or may not induce de-industrialisation in a poor country, we explore the robustness of such predictions to the model with more generalised production technologies. We extend an otherwise baseline model by allowing the sectoral outputs to be produced under two CES production functions, using capital and labour. One may think that the generalisation of productions may affect some of the baseline results.

The model extension is straightforward. Firstly, we add another market clearing condition. Let $s_{i,t}$ be the share of capital input used in sector $i \in \{a, n\}$. In each period t,capital market clearing condition is

$$s_{a,t} + s_{n,t} = 1. \tag{4.37}$$

Secondly, we specify the new sectoral technologies. Sector i's output in period t, $Y_{i,t}$, is produced by a representative firm using physical capital devoted to the sector i, $s_{i,t}K_t$, and labour, $l_{i,t}N_t$, according to a CES production technology with substitution parameter $\psi_i \in (-\infty, 1]$,

$$Y_{i,t} = \left[\alpha_i \Big(s_{i,t}K_t\Big)^{\psi_i} + (1-\alpha_i)\Big(A_{i,t}l_{i,t}N_t\Big)^{\psi_i}\right]^{\frac{1}{\psi_i}}; \alpha_i \in (0,1), i \in \{a, n\} \tag{4.38}$$

where $A_{i,t}$ is defined as in the baseline model and α_i is the sector i's distributive share parameter. The specification (4.37) allows for two additional sector-specific features in the production technologies. First, the two CES production functions may have different substitution parameter values. This relaxation reflects that the elasticity of substitution between capital and labour can be different across sectors and, then, potentially affects structural change via *factor rebalancing effect* as suggested by (Alvarez-Cuadrado et al., 2017). Second, the two CES production functions may have different distributive parameter values. While capital input can be utilised in both sectors, the difference on the importance of capital input among sectors could generate *different capital intensities* between the two sector which could affect structural change as shown in Acemoglu and Guerrieri (2008). Combined with the fact that structural changes via *non-homothetic preference effect* (Kongsamut et al., 2001) and via *biased technological change effect* (Ngai and Pissarides, 2007) have been addressed in the baseline model, the extended model can be seen as a hybrid model of structural change allowing the four generators into one model.

Finally, we re-characterise the input price equations. The representative firm in agricultural sector solves

$$\max_{\{l_{a,t}N_t, s_{a,t}K_t\}} \left\{ p_t Y_{a,t} - w_t(l_{a,t}N_t) - R_t(s_{a,t}K_t) \right\}$$

and the representative firm in non-agricultural sector solves

$$\max_{\{l_{n,t}N_t, s_{n,t}K_t\}} \left\{ Y_{n,t} - w_t(l_{n,t}N_t) - R_t(s_{n,t}K_t) \right\}.$$

Under perfect competition and no-arbitrage conditions each factor will be paid according its (value of) marginal product and will be paid equally across sectors. This implies

$$w_t = p_t Y_{a,t}^{1-\psi_a}(1-\alpha_a)(A_{a,t}l_{a,t}N_t)^{\psi_a-1}A_{a,t} = Y_{n,t}^{1-\psi_n}(1-\alpha_n)(A_{n,t}l_{n,t}N_t)^{\psi_n-1}A_{n,t}, \quad (4.39)$$

$$R_t \equiv r_t + \delta = p_t Y_{a,t}^{1-\psi_a}\alpha_a(s_{a,t}K_t)^{\psi_a-1} = Y_{n,t}^{1-\psi_n}\alpha_n(s_{n,t}K_t)^{\psi_n-1}, \quad (4.40)$$

must hold in every period. It is notable that the baseline model can be seen as a special case of this extension by setting $\psi_a = \psi_n = \alpha_a = s_{a,t} = 0$.

While this extension does not alter consumer behaviour and climate component, these two parts of the economy remain unchanged. With the revised technologies (4.37), input prices (38)-(39) and the capital market clearing condition (4.38), we can characterise an inter-temporal equilibrium, balanced growth path and the climate change impacts as in the baseline model. The equilibrium can be formally stated by the following definition.

Definition 4.3. *A market equilibrium is a sequence of prices* $\{p_t, w_t, r_t\}_{t=0}^{\infty}$*, an allocation for the representative household* $\{c_{a,t}, c_{n,t}, K_{t+1}\}_{t=0}^{\infty}$*, an allocation for representative firms in both sectors* $\{s_{a,t}, s_{n,t}, l_{a,t}, l_{n,t}\}_{t=0}^{\infty}$ *such that:*

(i) the allocation $\{c_{a,t}, c_{n,t}, K_{t+1}\}_{t=0}^{\infty}$ *solves the household problem (4.10) subject to (4.9) given the sequence of prices* $\{p_t, w_t, r_t\}_{t=0}^{\infty}$*, the sequence of subsistence level of agricultural consumptions* $\{\bar{c}_{a,t}\}_{t=0}^{\infty}$ *and the initial stock of capital* $K_0 > 0$*,*

(ii) factor prices satisfy (4.39) and (4.40) given the sequence of sectoral productivities $\{A_{a,t}, A_{n,t}\}_{t=0}^{\infty}$*,*

(iii) markets clear, i.e. (4.5)-(4.7) and (4.37) hold.

4.5.1 Intertemporal Equilibrium Characterisation

Let us redefine the de-trended relative price as $\tilde{p}_t \equiv p_t \left(\frac{A_{a,t}}{A_{n,t}}\right)^{1-\alpha_a}$. Then, we apply a procedure similar to Alvarez-Cuadrado et al. (2017) to construct an equilibrium sequence $\{s_{n,t}, l_{n,t}, R_t, \tilde{p}_t, \tilde{c}_{n,t}, z_{t+1}\}_{t=0}^{\infty}$.[11] In particular, such a sequence can be characterised by using a system of four static conditions, two dynamic equations and one transversality condition:

$$\left(\frac{1-\alpha_a}{\alpha_a}\right)\left(\frac{\alpha_n}{1-\alpha_a}\right)\left(\frac{A_{a,t}}{A_{n,t}}\right)^{\psi_a}\left(\frac{1-s_{n,t}}{1-l_{n,t}}z_t\right)^{1-\psi_a} = \left(\frac{s_{n,t}z_t}{l_{n,t}}\right)^{1-\psi_n}, \tag{4.41}$$

$$\left(\frac{1-\alpha_a}{1-\alpha_n}\right)\tilde{p}_t\left(\frac{A_{a,t}}{A_{n,t}}\right)^{\alpha_a}\left[\alpha_a\left(\frac{A_{a,t}}{A_{n,t}}\right)^{-\psi_a}\left(\frac{1-s_{n,t}}{1-l_{n,t}}z_t\right)^{\psi_a}+(1-\alpha_a)\right]^{\frac{1-\psi_a}{\psi_a}} = \left(\frac{s_{n,t}z_t}{l_{n,t}}\right)^{1-\psi_n}\frac{R_t}{\alpha_n}, \tag{4.42}$$

$$\frac{\tilde{p}_t\left(\frac{A_{a,t}}{A_{n,t}}\right)^{\alpha_a-1}}{\left(\frac{\theta}{1-\theta}\right)}\left(\left(\frac{A_{a,t}}{A_{n,t}}\right)(1-l_{n,t})\left[\alpha_a\left(\frac{A_{a,t}}{A_{n,t}}\right)^{-\psi_a}\left(\frac{1-s_{n,t}}{1-l_{n,t}}z_t\right)^{\psi_a}+(1-\alpha_a)\right]^{\frac{1}{\psi_a}} - \frac{\bar{c}_{a,t}}{A_{n,t}}\right) = \tilde{c}_{n,t}, \tag{4.43}$$

$$\beta(1+R_{t+1}-\delta) = \left(\frac{\tilde{c}_{n,t+1}}{\tilde{c}_{n,t}}\right)^{\sigma}(1+\tilde{\gamma}_{n,t+1})^{\sigma}\left(\frac{\tilde{p}_{t+1}}{\tilde{p}_t}\right)^{\theta(1-\sigma)}\left(\frac{1+\tilde{\gamma}_{n,t+1}}{1+\tilde{\gamma}_{a,t+1}}\right)^{(1-\alpha_a)\theta(1-\sigma)}, \tag{4.44}$$

$$l_{n,t}\left(\frac{s_{n,t}z_t}{l_{n,t}}\right)\left(\frac{R_t}{\alpha_n}\right)^{\frac{1}{1-\psi_n}} = \tilde{c}_{n,t} + (1+n)(1+\tilde{\gamma}_{n,t+1})z_{t+1} - (1-\delta)z_t, \tag{4.45}$$

$$R_t = \alpha_n\left[\alpha_n + (1-\alpha_n)\left(\frac{s_{n,t}z_t}{l_{n,t}}\right)^{-\psi_n}\right]^{\frac{1-\psi_n}{\psi_n}} \tag{4.46}$$

and the TVC condition:

$$\lim_{t\longrightarrow\infty}\beta^t\frac{\partial u(\cdot)}{\partial c_{n,t}}A_{n,t+1}N_{t+1}z_{t+1} = 0 \tag{4.47}$$

[11] Alvarez-Cuadrado et al. (2017) construct a two-sector Solow growth model with structural transformation to show that difference in elasticity of subsitutions among sectors could cause a structural change during the transition dynamics. To do so, the authors develop two static conditions, called contract curve and labour mobility condition, to determine equilibrium allocations in input markets. Then, given the static equilibrium allocation, the model is closed by developing a transition equation governing the state variable of the economy. Since the consumption/saving decision is given, by definition of Solow model, the only dynamic problem in their model is the evolution of the capital stock of the economy. Nevertheless, we cannot decompose the system into two sub-systems as in their study. The simplified decomposition is impossible because when determining the equilibrium consumption plan (c_t, c_{t+1}); and thus (z_t, z_{t+1}), using Euler condition we need to know $(l_{n,t}, s_{n,t})$ as well as $(l_{n,t+1}, s_{n,t+1})$ at the same time. Instead, we have to solve the whole system simultaneously.

, given an exogenous process $\{A_{a,t}, A_{n,t}, \bar{c}_{a,t}\}_{t=0}^{\infty}$ and an initial condition $z_0 > 0$.[12]

According to Alvarez-Cuadrado et al. (2017), conditions (4.41) and (4.42) are called the contract curve (CC) and the labour mobility condition (LM), respectively. The CC represents the equilibrium pairs $(s_{n,t}, l_{n,t})$ where the marginal rates of technical substitution are equalised across the two sectors, given $A_{a,t}, A_{n,t}$ and z_t. Next, the LM collects input market allocations $(s_{n,t}, l_{n,t})$ where sectors pay the same wage, given $A_{a,t}, A_{n,t}$ and z_t. If we depict the CC and the LM in $s_{n,t}, l_{n,t}$ space, the two curve will necessarily cross at the equilibrium point.

The other conditions are standard. Conditions (4.43) is a version of the intratemporal tradeoff condition (4.11). Condition (4.44) is a restatement of the Euler condition (4.12). Condition (4.45) is an alternative expression of the resource constraint which governs the evolution of the state variable z_t. Condition (4.46) is another expression of the rate of return on capital. Condition (4.47) is a version of the TVC (4.13).

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4.5.2 Balanced Growth Path

Suppose that $T_t = T^* > 0$ for all t. Then, the constant sectoral growth rates can be derived, denoted by $\tilde{\gamma}_a^*$ and $\tilde{\gamma}_n^*$. Given these constant productivity growths, a BGP can be defined by Definition 4.2 with an additional condition:

$$s_{n,t} = s_n^* \in (0, 1).$$

, which ensure that the sectoral allocation of capital input is constant overtime.[13]

As in the baseline model, some restrictions on parameters are required in order to establish the existence of a BGP. The following two lemmas highlight three such restrictions.

Lemma 4.2. *A BGP exists only if* $\psi_a(\tilde{\gamma}_a^* - \tilde{\gamma}_n^*) = 0$.

Lemma 4.2 states that some restrictions on either the sector-specific productivity growth rates or the agricultural production function are necessary in order to ensure the existence of a BGP. More specifically, if $A_{a,t}$ and $A_{n,t}$ are assumed to grow at different rates in the long run, then it is necessary to have $\psi_a = 0$ which means the elasticity of substitution between capital and labour is one in the agricultural sector; i.e., the agri-

[12]See Appendix C.4.

[13]When discussing about the long-run behaviour of economic growth with structural transformation, most structural change models also feature another kind of long-run stationary equilibria which is known as asymptotic balanced growth paths (ABGPs, hereafter), e.g. Echevarria (1997), Kongsamut et al. (2001), Acemoglu and Guerrieri (2008) and Alonso-Carrera and Raurich (2018). One can find a formal definition of ABGPs in, e.g. Palivos et al. (1997). In our study, we will not focus on this kind of long-run equilibria. However, we devote an optional section in Appendix to Chapter 4 to characterise this kind of equilibrium paths.

cultural production function must be Cobb-Douglas.[14] Alternatively, if $\psi_a \neq 0$ then the two technological factors must have the same long-run growth rates, i.e., $\tilde{\gamma}_a^* = \tilde{\gamma}_n^*$.[15] This result holds even if the manufacturing production is Cobb-Douglas ($\psi_n = 0$) or the two production functions have the same elasticity of substitution, i.e., $\psi_a = \psi_n$.[16]

Lemma 4.3. *Suppose that $\psi_a = 0$. Then the following results hold:*

(i) a BGP exists only if $\bar{c}_{a,t} = \mu A_{a,t}^{1-\alpha_a} A_{n,t}^{\alpha_a}$, for some $\mu > 0$.

(ii) The TVC (4.13) is satisfied in any BGP if and only if

$$\beta(1+n)\Big[(1+\tilde{\gamma}_n^*)^{(1-\theta)+\theta\alpha_a}(1+\tilde{\gamma}_a^*)^{\theta(1-\alpha_a)}\Big]^{1-\sigma} < 1. \tag{4.48}$$

Economic intuitions of Lemma 4.3 is analogous to that of Lemma 4.1. Actually, this lemma is a more generalised version of Lemma 4.1. If we set $\alpha_a = 0$, then this lemma turns out to be Lemma 4.1.

We now provide a formal characterisation of a balanced growth path under the assumption that there is a finite $\tau \geq 0$ such that $\tilde{\gamma}_{i,t} = \tilde{\gamma}_i^* > 0$ for $i \in \{a, n\}$ and for all $t \geq \tau$. We need the following auxiliary notations:

$$\Lambda_1^* \equiv \Big[(\alpha_n)^{\frac{-1}{1-\psi_n}}(R^*)^{\frac{\psi_n}{1-\psi_n}} - 1\Big]^{\frac{-1}{\psi_n}}\Big(\frac{1-\alpha_n}{\alpha_n}\Big)^{\frac{1}{\psi_n}},$$

$$\Lambda_2^* \equiv \Big(\frac{\tilde{p}^*\alpha_a}{R^*}\Big)^{\frac{1}{1-\alpha_a}},$$

$$\Gamma_1^* \equiv \Big[(1+n)(1+\tilde{\gamma}_n^*) - (1-\delta)\Big],$$

$$\Gamma_2^* \equiv \Big[(\alpha_n)^{\frac{-1}{1-\psi_n}}(R^*)^{\frac{\psi_n}{1-\psi_n}} - 1\Big]^{-\frac{1}{\psi_n}}\Big[(1-\alpha_n)(R^*)^{\frac{\psi_n}{1-\psi_n}}(\alpha_n)^{-\frac{1}{1-\psi_n}}\Big]^{\frac{1}{\psi_n}},$$

where $\tilde{p}^*$ is the stationary value of $\tilde{p}_t$ along the BGP.

Proposition 4.2. *Suppose $\psi_a = 0$, $\bar{c}_{a,t} = \mu A_{a,t}^{1-\alpha_a} A_{n,t}^{\alpha_a}$ for some $\mu > 0$, and (4.48) is satisfied. Then, the following results hold.*

(i) In any BGP (provided that one exists), the value of R^* and $\tilde{p}^*$ are given by

$$R^* = \frac{1}{\beta}(1+\tilde{\gamma}_n^*)^{\sigma}\Big(\frac{1+\tilde{\gamma}_n^*}{1+\tilde{\gamma}_a^*}\Big)^{(1-\alpha_a)\theta(1-\sigma)} - (1-\delta), \tag{4.49}$$

$$\tilde{p}^* = \Big(\frac{1-\alpha_n}{1-\alpha_a}\Big)(\Lambda_1^*)^{(1-\alpha_a)(1-\psi_n)}\Big(\frac{\alpha_a}{1-\alpha_a}\cdot\frac{1-\alpha_n}{\alpha_n}\Big)^{-\alpha_a}\Big(\frac{R^*}{\alpha_n}\Big). \tag{4.50}$$

[14] Assuming Cobb-Douglas in agricultural sector is widely used in recent literature; see Alvarez-Cuadrado and Poschke (2011), Alonso-Carrera and Raurich (2015) and Alonso-Carrera and Raurich (2018) for example, concerning structural transformation.

[15] The equality $\tilde{\gamma}_a^* = \tilde{\gamma}_n^*$ can be justified by assuming that productivity growth in both sectors are driven by some general purpose technological improvements that benefit all the workers in the economy.

[16] Under the assumption that $\psi_a = 0$, the condition (4.43) turns out to be (C.5.4); See Appendix C.5.

(ii) Suppose the following conditions are satisfied:

$$(R^*)^{\psi_n} > \alpha_n \tag{4.51}$$

and

$$\Big[(\Lambda_2^*)^{\alpha_a} - \mu\Big](\Gamma_2^* - \Gamma_1^*\Lambda_1^*) > \Gamma_1^*\Lambda_2^*\mu. \tag{4.52}$$

Then, a unique BGP exists and the value of l_n^*, s_n^*, z^* *and* c_n^* *are determined by*

$$l_n^* = \frac{\Gamma_1^*\Lambda_2^* + \tilde{p}^*(\frac{1-\theta}{\theta})(\Lambda_2^*)^{\alpha_a} - \mu\tilde{p}^*(\frac{1-\theta}{\theta})}{\Gamma_1^*\Lambda_2^* + \tilde{p}^*(\frac{1-\theta}{\theta})(\Lambda_2^*)^{\alpha_a} + \Gamma_2^* - \Gamma_1^*\Lambda_1^*} \in (0,1), \tag{4.53}$$

$$s_n^* = \frac{\Lambda_1^* l_n^*}{\Lambda_1^* l_n^* + \Lambda_2^*(1-l_n^*)} \in (0,1), \tag{4.54}$$

$$z^* = \Lambda_1^* l_n^* + \Lambda_2^*(1-l_n^*) > 0, \tag{4.55}$$

$$\tilde{c}_n^* = \tilde{p}^*\Big(\frac{1-\theta}{\theta}\Big)\Big((1-l_n^*)(\Lambda_2^*)^{\alpha_a} - \mu\Big) > 0. \tag{4.56}$$

Proposition 4.2 states that if the temperature is stationary, agricultural production is Cobb-Douglas, subsistence level of agricultural consumption is growing at a proper rate and the life-time utility converge, then there exist a unique BGP only if parameter values are assigned properly so that the long-run labour share is feasible. Parameter restrictions (4.50) and (4.51) ensure that $\tilde{c}_n^*, \tilde{p}^* > 0$, $s_n^*, l_n^* \in (0,1)$ while $R^* > 0$ holds due to (4.48).

4.5.3 Climate Change Effects in the Extended Model

As in the baseline model, we assume that $\frac{\partial\tilde{\gamma}_a^*}{\partial T^*} < \frac{\partial\tilde{\gamma}_n^*}{\partial T^*} < 0$, i.e. climate change will adversely affect the sectoral productivity growth rates and that of the agricultural sector is higher vulnerable to climate change impacts. Given this climate damage primitive, we begin by illustrating the long-run growth effect. After that, we explore the capital return effect and then the long-run structural change effects.

Firstly, climate change also weakens the long-run growth rate of the extended model. To illustrate this, we can apply (4.44), (4.48) and the restriction $\frac{c_{n,t+1}}{c_{n,t}} = 1 + g^*$ to show that $g^* = \tilde{\gamma}_n^*$. As a consequence, we can conclude that climate change is growth-detrimental since $\frac{\partial g^*}{\partial T^*} = \frac{\partial\tilde{\gamma}_n^*}{\partial T^*} < 0$.

Secondly, the impact on the rate of return on capital is unclear as in the baseline model. From (4.48), the capital return effect turns out to be

$$\frac{\partial R^*}{\partial T^*} = \frac{1}{\beta}\frac{(1+\tilde{\gamma}_n^*)^{\hat{\sigma}}}{(1+\tilde{\gamma}_a^*)^{\hat{\sigma}-\sigma}}\left\{\frac{\hat{\sigma}}{1+\tilde{\gamma}_n^*}\frac{\partial\tilde{\gamma}_n^*}{\partial T^*} + \frac{(1-\alpha_a)\theta(\sigma-1)}{1+\tilde{\gamma}_a^*}\frac{\partial\tilde{\gamma}_a^*}{\partial T^*}\right\} \tag{4.57}$$

where $\hat{\sigma} \equiv \sigma + (1-\alpha_a)\theta(1-\sigma) \geq 0$. As in the general picture, the relative risk aversion parameter σ plays a crucial role in determining the sign of the effect:

$$\sigma \geq 1 \Rightarrow \{\bullet\} < 0 \Rightarrow \frac{\partial R^*}{\partial T^*} < 0,$$

$$\sigma < 1 \Rightarrow \{\bullet\} \lesseqgtr 0 \Rightarrow \frac{\partial R^*}{\partial T^*} \lesseqgtr 0.$$

Thirdly, the impact on employment share is ambiguous and such ambiguity arises from the impacts of the capital return effect. To illustrate this, when considering (4.46) we can show that

$$\begin{aligned}\frac{\partial l_n^*}{\partial T^*} = \frac{(\tilde{p}^*\hat{\theta}\mu - \Theta)}{(\Theta + \Gamma_2^* - \Gamma_1^*\Lambda_1^*)^2} \cdot \Big(\frac{\partial \Gamma_2^*}{\partial T^*} - \Gamma_1^*\frac{\partial \Lambda_1^*}{\partial T^*} - \Lambda_1^*\frac{\partial \Gamma_1^*}{\partial T^*}\Big) \\ + \frac{(\Gamma_2^* - \Gamma_1^*\Lambda_1^* + \tilde{p}^*\hat{\theta}\mu)}{(\Theta + \Gamma_2^* - \Gamma_1^*\Lambda_1^*)^2} \cdot \frac{\partial \Theta}{\partial T^*} \\ + \frac{(-1)(\Theta + \Gamma_2^* - \Gamma_1^*\Lambda_1^*)\hat{\theta}\mu}{(\Theta + \Gamma_2^* - \Gamma_1^*\Lambda_1^*)^2} \cdot \frac{\partial \tilde{p}^*}{\partial T^*}\end{aligned} \tag{4.58}$$

where $\Theta \equiv \Gamma_1^*\Lambda_2^* + \tilde{p}^*\hat{\theta}(\Lambda_2^*)^{\alpha_a} > 0$ and $\hat{\theta} \equiv \frac{1-\theta}{\theta} > 0$. According to the feasibility for $l_n^* \in (0,1)$, we can show that

$$(4.26) \text{ and } (4.27) \Rightarrow \underbrace{(\bullet)}_{\text{(-)}} \cdot \left(\frac{\partial \Gamma_2^*}{\partial T^*} - \Gamma_1^*\frac{\partial \Lambda_1^*}{\partial T^*} - \Lambda_1^*\frac{\partial \Gamma_1^*}{\partial T^*}\right) + \underbrace{(\bullet)}_{(+)} \cdot \frac{\partial \Theta}{\partial T^*} + \underbrace{(\bullet)}_{\text{(-)}} \cdot \frac{\partial \tilde{p}^*}{\partial T^*}, \tag{4.59}$$

The sign of $\frac{\partial l_n^*}{\partial T^*}$ is ambiguous depending on the parameter values which reflect the magnitudes and directions of

$$\left(\frac{\partial \Gamma_2^*}{\partial T^*} - \Gamma_1^*\frac{\partial \Lambda_1^*}{\partial T^*} - \Lambda_1^*\frac{\partial \Gamma_1^*}{\partial T^*}\right), \frac{\partial \Theta}{\partial T^*}, \frac{\partial \tilde{p}^*}{\partial T^*}.$$

In Appendix C.5, we show that the magnitudes and directions of these derivatives are inconclusive. As a consequence, the climate change impact on the non-agricultural employment share is ambiguous. To show that the ambiguity of the impact on the employment share arises due to the influence of the capital effect, let's consider (4.52). While keeping R^* unchanged (so that $\tilde{p}^*, \Lambda_1^*, \Lambda_2^*$ and Γ_2^* remain unchanged as well), a decrease in $\tilde{\gamma}_n^*$ will induce a decrease in Γ_1^* and thus the non-agricultural labour share because:

$$\frac{\partial l_n^*}{\partial \Gamma_1^*} = \frac{\Gamma_2^*\Lambda_2^* + \Lambda_1^*\tilde{p}^*(\frac{1-\theta}{\theta})[\Lambda_2^*)^{\alpha_a} - \mu] + \tilde{p}^*(\frac{1-\theta}{\theta})\mu\Lambda_2^*}{\Big[\Gamma_1^*\Lambda_2^* + \tilde{p}^*(\frac{1-\theta}{\theta})(\Lambda_2^*)^{\alpha_a} + \Gamma_2^* - \Gamma_1^*\Lambda_1^*\Big]^2} > 0. \tag{4.60}$$

This means that without other influences the share of non-agricultural employment will be negatively affected by climate change. However, the ambiguity arises because the increasing in the average annual temperature also induces a change in the rate of return on capital (in an inconclusive direction) through changes in both agricultural and non-agricultural productivity growth rates; see (4.56). This change will induce a change in the labour share indirectly via changes in $\tilde{p}^*, \Lambda_1^*, \Lambda_2^*$ and Γ_2^* in ambiguous ways.

Similar to the impact on the sectoral employment share, the climate change impact on the sectoral value-added share is also ambiguous. To illustrate this, we begin by characterising the total output. Based on (4.2) and the assumption that $\psi_a = 0$, we have

$$Y_{n,t} = A_{n,t}N_t\left[\alpha_n(s_{n,t}z_t)^{\psi_n} + (1-\alpha_n)(l_{n,t})^{\psi_n}\right]^{\frac{1}{\psi_n}}$$

and

$$p_tY_{a,t} = A_{n,t}N_t\tilde{p}_t(1-s_{n,t})^{\alpha_a}z_t^{\alpha_a}(1-l_{n,t})^{1-\alpha_a}.$$

The total output at time t, denoted by Y_t, is given by the sum of the gross value-added in both sectors, i.e. $Y_t = p_tY_{a,t} + Y_{a,t}$. Or equivalently,

$$Y_t = A_{n,t}N_t\left\{\left[\alpha_n(s_{n,t}z_t)^{\psi_n} + (1-\alpha_n)(l_{n,t})^{\psi_n}\right]^{\frac{1}{\psi_n}} + \tilde{p}_t(1-s_{n,t})^{\alpha_a}z_t^{\alpha_a}(1-l_{n,t})^{1-\alpha_a}\right\}. \quad (4.61)$$

Next, we define the the non-agricultural value-added share, denoted by $s_{Yn,t}$, such that $s_{Yn,t} \equiv \frac{Y_{n,t}}{Y_t}$. Along a BGP the share can be expressed as

$$s_{Yn}^* = \frac{\left[\alpha_n\left(\frac{s_n^*}{l_n^*}z^*\right)^{\psi_n} + (1-\alpha_n)\right]^{\frac{1}{\psi_n}}}{\left[\alpha_n\left(\frac{s_n^*}{l_n^*}z^*\right)^{\psi_n} + (1-\alpha_n)\right]^{\frac{1}{\psi_n}} + \tilde{p}^*\left(\frac{1-s_n^*}{1-l_n^*}z^*\right)^{\alpha_a}\left(\frac{1-l_n^*}{l_n^*}\right)}.$$

Since $\left(\frac{s_n^*}{l_n^*}z^*\right) = \Lambda_1^*$ and $\left(\frac{1-s_n^*}{1-l_n^*}z^*\right) = \Lambda_2^*$, then

$$s_{Yn}^* = \frac{\left[\alpha_n\left(\Lambda_1^*\right)^{\psi_n} + (1-\alpha_n)\right]^{\frac{1}{\psi_n}}}{\left[\alpha_n\left(\Lambda_1^*\right)^{\psi_n} + (1-\alpha_n)\right]^{\frac{1}{\psi_n}} + \tilde{p}^*\left(\Lambda_2^*\right)^{\alpha_a}\left(\frac{1-l_n^*}{l_n^*}\right)} \in (0,1). \quad (4.62)$$

In Appendix C.6, we show that the sign of $\frac{\partial s_{Yn}^*}{\partial T^*}$ is ambiguous due to the influence of the capital return effect. As we can see, the value of s_{Yn}^* depends on $\Lambda_1, \Lambda_2, \tilde{p}^*$ and l_n^*. Without the capital return effect, climate change will not induce changes in $\Lambda_1, \Lambda_2, \tilde{p}^*$ but weaken l_n^*. As a result, the non agricultural value-added share will decrease. However, if the capital return effect is activated, the net effect of such a share becomes ambiguous due to the the unclear magnitudes and directions of $\frac{\partial \Lambda_1}{\partial T^*}$, $\frac{\partial \Lambda_2}{\partial T^*}$ and $\frac{\partial \tilde{p}^*}{\partial T^*}$ which are solely driven by changes in the rate of return on capital. As a consequence, the long-run impact on the non agricultural value-added share is ambiguous.

In sum, it seems that climate change will lead to deteriorating the long-run economic growth. However, the climate change effects on the rate of return on capital as well as structural change are inconclusive. This confirm the robustness of our results.

4.6 Conclusion

This chapter is devoted to investigate the long-term consequences of cross-border pollution on poor countries in terms of growth and structural transformation. To investigate such long-term impacts, we develop a two-sector growth model along the line of structural transformation literature. In the long-run, climate change clearly reduces long-run growth of output per capita due to the impact on the growth rate of labour augmenting technological progress in non-agricultural sector. However, the impact on the structural changes are ambiguous but it depends on various model parameters. A policy implication is that when imposing Paris agreement it is helpful for a poor country, if achieved, by stimulating the long-run economic growth. However, the gain from structural change seems to be inconclusive. Due to this limitation, matching the model with empirical evidence to see the long-run structural change effect and economic growth as well as the counter part impacts during transitional dynamic effects would be interesting to go further in this research direction.

Chapter 5

Conclusion

This thesis consists of three original research studies. The studies in Chapter 2 and Chapter 3 cover interesting topics related to theoretical economic growth analysis and utilisation of natural resources whereas the impact of cross-border pollution on the long-run growth and structural changes in poor countries is addressed in Chapter 4.

In Chapter 2, we investigate for which partition of the CES class of production functional forms that allows endogenous growth to emerge in a neoclassical growth model with productive non-renewable resources. To do so, we examine the preservation of this property by extending Agnani et al. (2005) model in a certain way. In particular, we relax the Double Cobb-Douglas specification by replacing it with a more generalised CES functional form. The main finding is that the combination of the effective flow of non-renewable input and the effective labour under Cobb-Douglas basis is necessary for endogenous growth to emerge. It is also noteworthy that the analysis is consistent with the celebrated Uzawa's Steady State Growth Theorem(Uzawa, 1961) as it shows that the Cobb-Douglas combination between these two effective inputs implies the aggregate production function is equivalent to labour-augmenting technology.

In Chapter 3, we explain when the degree of the elasticity of substitution between renewable and non-renewable resources in final good production matters for the long-run economic growth. By extending the two endogenous growth models including Romer (1986) and Barro (1990) along the line of Golosov et al. (2014), we find that natural resource substitution will create the long-run growth effect if it induces a change in labour allocation across sectors. However, the directional impact is ambiguous. In particular, the substitutability will stimulate growth if the two natural resources are gross substitute while parameters are restricted in a proper way. What we find shed the light on both theoretical and empirical motivations. For the former, we highlight the role of labour reallocation effect that becomes necessary for the growth effect. For the latter, as we show that growth enhancing occurs when the initial share of renewable resource employment share is high. It seems that motivating people to get involved more and more on renewable resource-related activities is necessary to promote sustainable growth.

Chapter 4 is motivated by a crucial observation that poorer countries, which tend to have contributed less to climate change, seem to be more seriously affected by climate change. According to this evidence, we develop a climate change-augmented two-sector neoclassical model with structural transformations [along the line of Kongsamut et al. (2001); Ngai and Pissarides (2007); Acemoglu and Guerrieri (2008) and Alvarez-Cuadrado et al. (2017)] to illustrate the potential mechanisms that climate change could hurt the poor economy. Comparative steady states analysis suggests that climate change always deteriorate the long-run economic growth. Nevertheless, the impacts on the employment and value-added shares are unclear. Such ambiguities arise because of the influence of the capital return effect.

In sum, the economic system and the environment are closely related: the environment acts as a source that provides natural resources to the economy, and acts as a sink for emissions. As the true connection is complicated, it is unlikely to generate a general conclusion using only a single study. Small-scale analysis might be necessary even if it is questionable due to its generalisation.

Appendix A

Appendix to Chapter 2

A.1 Nested CES Production Functions

In this appendix, we will verify that Assumption 2.2 is satisfied by all the nested CES production functions considered in Subsections 2.2.2 and 2.3.2. We begin with the specification considered in Section 3.1, which is

$$F(K_t, Z_t) = \left[\alpha K_t^{\eta} + (1-\alpha) Z_t^{\eta}\right]^{\frac{1}{\eta}}, \qquad \text{with } \alpha \in (0,1) \text{ and } \eta < 1,$$

$$G(Q_t X_t, A_t N_t) \equiv \left[\varphi (Q_t X_t)^{\psi} + (1-\varphi)(A_t N_t)^{\psi}\right]^{\frac{1}{\psi}}, \qquad \text{with } \varphi \in (0,1) \text{ and } \psi < 1.$$

First, consider capital input. If $\eta \leq 0$, then

$$\lim_{K_t \to 0} F(K_t, G(Q_t X_t, A_t N_t)) = 0$$

regardless of the value of ψ. In other words, physical capital is essential for production when $\eta \leq 0$. If $\eta \in (0,1)$, then

$$\lim_{K_t \to 0} F_1(K_t, G(Q_t X_t, A_t N_t)) = \infty,$$

regardless of the value of ψ. Next, consider the inputs of $G(\cdot)$. When $\psi \leq 0$, we have

$$\lim_{X_t \to 0} G(Q_t X_t, A_t N_t) = \lim_{N_t \to 0} G(Q_t X_t, A_t N_t) = 0,$$

$$\lim_{X_t \to 0} G_1(Q_t X_t, A_t N_t) = \varphi^{\frac{1}{\psi}} Q_t \quad \text{and} \quad \lim_{N_t \to 0} G_2(Q_t X_t, A_t N_t) = (1-\varphi)^{\frac{1}{\psi}} A_t.$$

There are now two sub-cases to consider: If $\psi \leq 0$ and $\eta \leq 0$, then both natural resources and labour are essential for production. In particular, we can show that

$$\lim_{X_t \to 0} F(K_t, G(Q_t X_t, A_t N_t)) = \lim_{N_t \to 0} F(K_t, G(Q_t X_t, A_t N_t)) = 0.$$

If $\psi \leq 0$ and $\eta \in (0,1)$, then we can show that

$$\lim_{X_t \to 0} \frac{\partial Y_t}{\partial X_t} = (1-\alpha) \left\{ \alpha \lim_{X_t \to 0} \left[\frac{G(Q_t X_t, A_t N_t)}{K_t} \right]^{-\eta} + 1 - \alpha \right\}^{\frac{1}{\eta}-1} \cdot \lim_{X_t \to 0} G_1 (Q_t X_t, A_t N_t),$$

$$\lim_{N_t \to 0} \frac{\partial Y_t}{\partial N_t} = (1-\alpha) \left\{ \alpha \lim_{N_t \to 0} \left[\frac{G(Q_t X_t, A_t N_t)}{K_t} \right]^{-\eta} + 1 - \alpha \right\}^{\frac{1}{\eta}-1} \cdot \lim_{N_t \to 0} G_2 (Q_t X_t, A_t N_t).$$

Both of these limits diverge to infinity as

$$\lim_{X_t \to 0} \left[\frac{G(Q_t X_t, A_t N_t)}{K_t} \right]^{-\eta} = \lim_{N_t \to 0} \left[\frac{G(Q_t X_t, A_t N_t)}{K_t} \right]^{-\eta} = \infty.$$

If $\psi \in (0,1)$, then we have

$$\lim_{X_t \to 0} G(Q_t X_t, A_t N_t) = (1-\varphi)^{\frac{1}{\psi}} (A_t N_t) \qquad \text{and} \qquad \lim_{N_t \to 0} G(Q_t X_t, A_t N_t) = \varphi^{\frac{1}{\psi}} (Q_t X_t),$$

$$\lim_{X_t \to 0} G_1 (Q_t X_t, A_t N_t) = \lim_{N_t \to 0} G_2 (Q_t X_t, A_t N_t) = \infty.$$

Using these we can obtain

$$\lim_{X_t \to 0} \frac{\partial Y_t}{\partial X_t} = F_2 \left(K_t, (1-\varphi)^{\frac{1}{\psi}} A_t N_t \right) \left[\lim_{X_t \to 0} G_1 (Q_t X_t, A_t N_t) \right] = \infty,$$

$$\lim_{N_t \to 0} \frac{\partial Y_t}{\partial N_t} = F_2 \left(K_t, \varphi^{\frac{1}{\psi}} Q_t X_t \right) \left[\lim_{N_t \to 0} G_2 (Q_t X_t, A_t N_t) \right] = \infty.$$

Note that these results hold regardless of the value of η. **Q.E.D.**

A.2 Nested CES Production Functions(Con't)

Next, we turn to the production function in (2.34). There are now only two possible cases: If $\psi \leq 0$, then all three inputs are essential for production. If $\psi \in (0,1)$, then we can obtain

$$\lim_{N_t \to 0} \frac{\partial Y_t}{\partial N_t} = \varphi A_t \left\{ \varphi + (1-\varphi) \lim_{N_t \to 0} \left[\frac{A_t N_t}{K_t^{\alpha} (Q_t X_t)^{1-\alpha}} \right]^{-\psi} \right\}^{\frac{1}{\psi}-1} = \infty,$$

$$\lim_{K_t \to 0} \frac{\partial Y_t}{\partial K_t} = \alpha (1-\varphi) \left\{ \varphi \lim_{N_t \to 0} \left[\frac{K_t^{\alpha} (Q_t X_t)^{1-\alpha}}{A_t N_t} \right]^{-\psi} + 1 - \varphi \right\}^{\frac{1}{\psi}-1} \left[\lim_{N_t \to 0} \left(\frac{K_t}{Q_t X_t} \right)^{\alpha - 1} \right] = \infty,$$

$$
\begin{aligned}
\lim_{X_t \to 0} \frac{\partial Y_t}{\partial X_t} &= (1-\alpha)(1-\varphi) \left\{ \varphi \lim_{X_t \to 0} \left[\frac{K_t^{\alpha} (Q_t X_t)^{1-\alpha}}{A_t N_t} \right]^{-\psi} + 1 - \varphi \right\}^{\frac{1}{\psi}-1} \left[\lim_{X_t \to 0} \left(\frac{K_t}{Q_t X_t} \right)^{\alpha} \right] \\
&= \infty.
\end{aligned}
$$

Note that the production functions in (2.34) and (2.35) are essentially identical, except that $A_t N_t$ and $Q_t X_t$ have switched place. Thus, using the same line of argument we can show that (2.35) satisfies Assumption 2.2.

We now consider the production function in (2.36). The first thing to note is that labour input is essential for production regardless of the value of ψ. If $\psi \leq 0$, then both physical capital and natural resources are essential for production. What remains is to consider the marginal product of these inputs when $\psi \in (0,1)$. Straightforward differentiation gives

$$
\frac{\partial Y_t}{\partial K_t} = (1-\beta)\varphi (A_t N_t)^{\beta} \left[\varphi + (1-\varphi) \left(\frac{K_t}{Q_t X_t} \right)^{-\psi} \right]^{\frac{1}{\psi}-1} \left[\varphi K_t^{\psi} + (1-\varphi)(Q_t X_t)^{\psi} \right]^{-\frac{\beta}{\psi}},
$$

$$
\frac{\partial Y_t}{\partial X_t} = (1-\beta)(1-\varphi)(A_t N_t)^{\beta} \left[\varphi \left(\frac{Q_t X_t}{K_t} \right)^{-\psi} + (1-\varphi) \right]^{\frac{1}{\psi}-1} \left[\varphi K_t^{\psi} + (1-\varphi)(Q_t X_t)^{\psi} \right]^{-\frac{\beta}{\psi}}.
$$

Since

$$
\lim_{K_t \to 0} \left[\varphi + (1-\varphi) \left(\frac{K_t}{Q_t X_t} \right)^{-\psi} \right]^{\frac{1}{\psi}-1} = \lim_{X_t \to 0} \left[\varphi \left(\frac{Q_t X_t}{K_t} \right)^{-\psi} + (1-\varphi) \right]^{\frac{1}{\psi}-1} = \infty,
$$

it follows that

$$
\lim_{K_t \to 0} \frac{\partial Y_t}{\partial K_t} = (1-\beta)\varphi (1-\varphi)^{-\frac{\beta}{\psi}} \left(\frac{Q_t X_t}{A_t N_t} \right)^{-\beta} \lim_{K_t \to 0} \left[\varphi + (1-\varphi) \left(\frac{K_t}{Q_t X_t} \right)^{-\psi} \right]^{\frac{1}{\psi}-1} = \infty,
$$

$$
\lim_{X_t \to 0} \frac{\partial Y_t}{\partial X_t} = (1-\beta)\varphi^{-\frac{\beta}{\psi}} (1-\varphi) \left(\frac{K_t}{A_t N_t} \right)^{-\beta} \lim_{X_t \to 0} \left[\varphi \left(\frac{Q_t X_t}{K_t} \right)^{-\psi} + (1-\varphi) \right]^{\frac{1}{\psi}-1} = \infty.
$$

Since (2.36) and (2.37) are symmetric, the same line of argument can be used to show the desired properties for (2.37). **Q.E.D.**

A.3 Proof of Theorem 2.1

The proof is divided into a number of steps:

Step 1 This part of the proof uses the same line of argument as in Schlicht (2006) and Jones and Scrimgeour (2008). In any balanced growth equilibrium, Y_t grows at a constant rate $\hat{\gamma} \equiv \gamma^*(1+n)$ in every period, so that

$$Y_{t+1} = \hat{\gamma} Y_t, \forall t. \tag{A.3.1}$$

Rearranging terms and applying the CRTS property of $F(\cdot)$ gives

$$\begin{aligned} Y_t &= F\Big(\hat{\gamma}^{-1} K_{t+1}, \hat{\gamma}^{-1} G(Q_{t+1}X_{t+1}, A_{t+1}N_{t+1})\Big) \\ &= F\Big(K_t, \hat{\gamma}^{-1} G(Q_{t+1}X_{t+1}, A_{t+1}N_{t+1})\Big) \end{aligned}$$

The second line uses the condition that K_t and Y_t grow at the same rate in any balanced growth path competitive equilibrium. For any $K_t > 0$, $F(K_t, \cdot)$ is a strictly increasing function. Hence, the following equality must be satisfied in any balanced growth path competitive equilibrium,

$$G(Q_t X_t, A_t N_t) = \hat{\gamma}^{-1} G(Q_{t+1}X_{t+1}, A_{t+1}N_{t+1}). \tag{A.3.2}$$

In other words, $G(\cdot)$ grows at the same rate as Y_t and K_t along a balanced growth path. Note that (A.3.2) holds even if $G(\cdot)$ is not a Cobb-Douglas function.

Suppose now $G(\cdot)$ is given by

$$G(Q_t X_t, A_t N_t) = (Q_t X_t)^{1-\phi} (A_t N_t)^{\phi}, \quad \text{for some} \quad \phi \in (0,1).$$

Combining this with $A_{t+1} = (1+a)A_t$, $Q_{t+1} = (1+q)Q_t$, $X_{t+1} = (1-\tau^*)X_t$ and $N_{t+1} = (1+n)N_t$, we can rewrite (A.3.2) as

$$(Q_t X_t)^{1-\phi}(A_t N_t)^{\phi} = \hat{\gamma}^{-1}\Big[(1+a)(1+n)\Big]^{\phi}\Big[(1+q)(1-\tau^*)\Big]^{1-\phi}(Q_t X_t)^{1-\phi}(A_t N_t)^{\phi}.$$

If we ignore the trivial case in which $(Q_t X_t)^{1-\phi}(A_t N_t)^{\phi} = 0$, then (A.3.2) is valid if and only if

$$\gamma^* = (1+a)^{\phi}\left[\frac{(1+q)(1-\tau^*)}{1+n}\right]^{1-\phi}.$$

This is equation (2.22) in the theorem.

Step 2 In this step, we will prove that the ratio $\chi_t \equiv \frac{\hat{x}_t^{1-\phi}}{\hat{k}_t}$ is constant, say $\chi_t = \chi^*$, along any balanced growth path competitive equilibrium.

Using the fact that $r_{t+1} = r_t = r^*$ along a balanced growth path competitive equilibrium and the fact that the marginal product of capital $F_1(\cdot)$ is homogeneous of degree zero, then (2.11) implies

$$F_1\Big(1, \frac{G(\cdot)}{K_t}\Big) = F_1\Big(1, \chi_t\Big) = r^* + \delta > 0.$$

Since $F_1(1,\chi_t)$ is continuous and strictly decreasing in χ_t, then, by intermediate value theorem, there exists a unique $\chi^* > 0$ such that

$$F_1\Big(1,\chi^*\Big) = r^* + \delta \tag{A.3.3}$$

It follows that χ_t must be time-invariant in any balanced growth equilibrium. This is equation (2.23) in the theorem.

Step 3 Equation (2.24) can be derived by showing that $\frac{p_t X_t}{Y_t}$ is constant along a balanced growth path competitive equilibrium. By the linear homogeneity property of $F(\cdot)$ and $F_2(\cdot)$, we can write

$$F_2\Big(K_t, G(Q_t X_t, A_t N_t)\Big) = F_2(1,\chi^*),$$
$$F\Big(K_t, G(Q_t X_t, A_t N_t)\Big) = K_t F(1,\chi^*).$$

Using these and (2.12), we can get

$$\begin{aligned}
\frac{p_t X_t}{Y_t} &= \frac{Q_t X_t}{K_t} \cdot \frac{F_2(1,\chi^*) G_1(Q_t X_t, A_t N_t)}{F(1,\chi^*)} \\
&= \frac{F_2(1,\chi^*)}{F(1,\chi^*)} \cdot \frac{G(Q_t X_t, A_t N_t)}{K_t} \cdot \frac{Q_t X_t G_1(Q_t X_t, A_t N_t)}{G(Q_t X_t, A_t N_t)} \\
&= \frac{F_2(1,\chi^*)}{F(1,\chi^*)} \cdot \chi^* \cdot (1-\phi).
\end{aligned}$$

Hence, $\frac{p_t X_t}{Y_t}$ must be strictly positive and time-invariant. This in turn implies

$$\frac{p_{t+1}}{p_t} \cdot \frac{X_{t+1}}{X_t} = \frac{Y_{t+1}}{Y_t} \Rightarrow (1+r^*)(1-\tau^*) = \gamma^*(1+n).$$

This is equation (2.24) in the theorem.

Step 4 We now derive equation (2.25), which is based on the capital market clearing condition. In any competitive equilibrium, the market for physical capital clears when

$$K_{t+1} = N_t s_t = N_t \cdot \Big(\frac{w_t}{2+\theta} - p_t m_t\Big).$$

The second equality follows from (2.7). Substitution (2.12) and (2.13) into the above equation gives

$$K_{t+1} = F_2\Big(K_t, G(Q_t X_t, A_t N_t)\Big)\left[\frac{1}{2+\theta} A_t N_t G_2(Q_t X_t, A_t N_t) - N_t m_t Q_t G_1(Q_t X_t, A_t N_t)\right]. \tag{A.3.4}$$

As shown in Step 3, we can rewrite $F_2\Big(K_t, G(Q_t X_t, A_t N_t)\Big)$ as $F_2(1,\chi^*)$. In addition, the market clearing condition for natural resources implies that

$$N_t m_t = M_{t+1} = (1-\tau^*) \cdot \frac{M_t}{X_t} \cdot X_t = \Big(\frac{1-\tau^*}{\tau^*}\Big) X_t.$$

Substituting these into (A.3.4) gives

$$K_{t+1} = F_2(1,\chi^*) \Bigg[\frac{1}{2+\theta} A_t N_t G_2(Q_t X_t, A_t N_t) - \Big(\frac{1-\tau^*}{\tau^*}\Big) X_t Q_t G_1(Q_t X_t, A_t N_t) \Bigg].$$

Finally, using the Cobb-Douglas specification for $G(\cdot)$, we can simplify this to become

$$K_{t+1} = F_2(1,\chi^*) \Bigg[\frac{\phi}{2+\theta} - \Big(\frac{1-\tau^*}{\tau^*}\Big)(1-\phi) \Bigg] G(Q_t X_t, A_t N_t).$$

Dividing both sides by K_t and using the fact that $\frac{G(\cdot)}{K_t} = \chi^*$ along a balanced growth equilibrium once more gives (2.25):

$$\frac{K_{t+1}}{K_t} = \gamma^*(1+n) = \chi^* F_2(1,\chi^*) \Bigg[\frac{\phi}{2+\theta} - \Big(\frac{1-\tau^*}{\tau^*}\Big)(1-\phi) \Bigg].$$

Step 5 In this step, we are to show that the steady state value of extraction rate (if exists) is strictly greater than $\overline{\tau}$. Consider (2.25). We can see that the $\gamma^*(1+n)$ and $\chi^* F_2(1,\chi^*)$ are positive. This means that $\Big[\frac{\phi}{2+\theta} - \Big(\frac{1-\tau^*}{\tau^*}\Big)(1-\phi)\Big]$ must be positive as well. This is true as long as $\tau^* > \overline{\tau}$. This completes the proof of Theorem 2.1. **Q.E.D.**

A.4 Proof of Proposition 2.1

Using (2.22) and (2.24), we can get

$$\gamma^*(1+n) = (1+a)^{\phi}(1+q)^{1-\phi}(1-\tau^*)^{1-\phi}(1+n)^{\phi}, \tag{A.4.1}$$

$$r^* = (1+a)^{\phi}(1+q)^{1-\phi}(1-\tau^*)^{-\phi}(1+n)^{\phi} - 1 \equiv r(\tau^*). \tag{A.4.2}$$

Next, we differentiate $F(\cdot)$ with respect to K to get

$$F_1(1,\chi^*) = \alpha \Big[\alpha + (1-\alpha)(\chi^*)^{\eta}\Big]^{\frac{1-\eta}{\eta}}. \tag{A.4.3}$$

Since capital input is paid its marginal product, we can say that

$$r(\tau^*) + \delta = \alpha \Big[\alpha + (1-\alpha)(\chi^*)^{\eta}\Big]^{\frac{1-\eta}{\eta}} \tag{A.4.4}$$

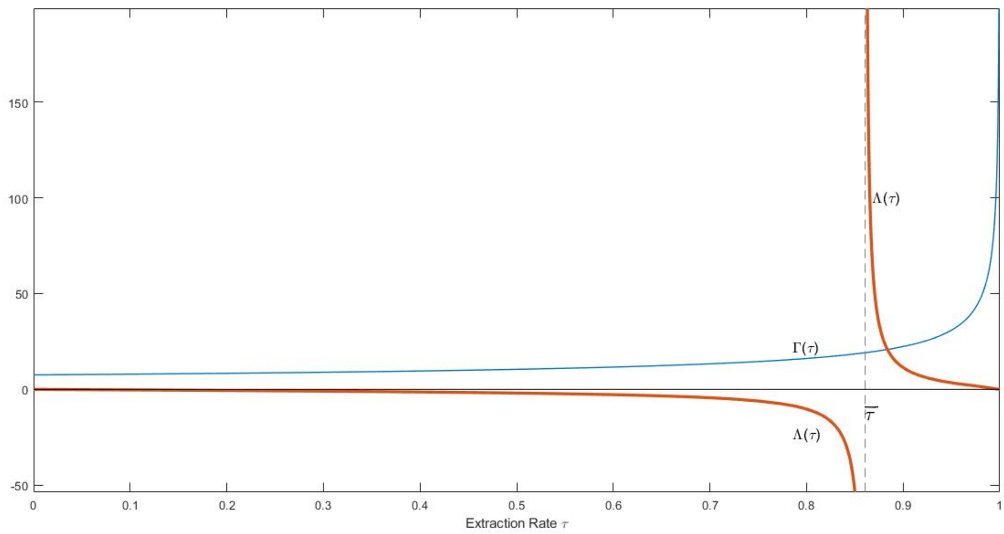

Figure A.1: *Proof of Proposition 1*

which also implies

$$(1-\alpha)(\chi^*)^\eta = \Big(\frac{r(\tau^*)+\delta}{\alpha}\Big)^{\frac{\eta}{1-\eta}} - \alpha. \tag{A.4.5}$$

Then, we differentiate $F(\cdot)$ with respect to Z to get

$$\chi^* F_2(1,\chi^*) = (1-\alpha)(\chi^*)^\eta \Big[\alpha + (1-\alpha)(\chi^*)^\eta\Big]^{\frac{1-\eta}{\eta}}. \tag{A.4.6}$$

Recall (2.25):

$$\gamma^*(1+n) = \chi^* F_2(1,\chi^*)\Big[\frac{\phi}{2+\theta} - \Big(\frac{1-\tau^*}{\tau^*}\Big)(1-\phi)\Big].$$

Apply (A.4.1)-(A.4.6) so that the above expression becomes

$$\frac{(2+\theta)(1+a)^\phi(1+q)^{1-\phi}(1-\tau^*)^{1-\phi}(1+n)^\phi}{\phi - \Big(\frac{1-\tau^*}{\tau^*}\Big)(2+\theta)(1-\phi)} = \Bigg[\Big(\frac{r(\tau^*)+\delta}{\alpha}\Big)^{\frac{\eta}{1-\eta}} - \alpha\Bigg]\frac{r(\tau^*)+\delta}{\alpha}. \tag{A.4.7}$$

This equation is a non-linear equation of τ. If a BGP competitive equilibrium exists, then there is $\tau^* \in (\overline{\tau}, 1)$ satisfying the above expression.

Define two auxiliary functions $\Lambda(\cdot)$ and $\Gamma(\cdot)$ according to

$$\Lambda(\tau) \equiv \frac{(2+\theta)(1+a)^\phi(1+q)^{1-\phi}(1-\tau)^{1-\phi}(1+n)^\phi}{\phi - \Big(\frac{1-\tau}{\tau}\Big)(2+\theta)(1-\phi)},$$

$$\Gamma(\tau) \equiv \Bigg[\Big(\frac{r(\tau)+\delta}{\alpha}\Big)^{\frac{\eta}{1-\eta}} - \alpha\Bigg]\frac{r(\tau)+\delta}{\alpha}.$$

To illustrate key properties of $\Lambda(\cdot)$, we adopt a proof that is so similar to that of Proposition 1 in Agnani et al. (2005, p.403). It is straightforward to show that $lim_{\tau\to\overline{\tau}^+}\Lambda(\tau) = \infty, lim_{\tau\to\overline{\tau}^-}\Lambda(\tau) = -\infty, lim_{\tau\to 0}\Lambda(\tau) = 0$ and $\Lambda(1) = 0$. In addition, we can show that

$$\Lambda'(\tau) = \frac{(2+\theta)(1+n)^{\phi}(1+a)^{\phi}(1+q)^{1-\phi}}{(1-\tau)^{\phi}}\Bigg[(1-\phi)\Big(\phi-\Big(\frac{1-\tau}{\tau}\Big)(2+\theta)(1-\phi)\Big)-\frac{\tau^{-2}(2+\theta)(1-\phi)}{1-\tau}\Bigg]$$

is strictly negative for all $\tau \in (\overline{\tau}, 1)$. Consider the function $\Gamma(\cdot)$. We can see that $\Gamma(0)$ is finite and $\Gamma(\tau)$ reaches to infinity as τ approaches one; $\Gamma(\tau)$ has a vertical asymptote at $\tau = 1$. In addition, by continuity of $r(\tau)$ over $(\overline{\tau}, 1)$, $\Gamma(\tau)$ will be continuous in that domain as well. Since $\Lambda(\tau)$ and $\Gamma(\tau)$ are both continuous over $(\overline{\tau}, 1)$, there will be at least one value $\tau^* \in (\overline{\tau}, 1)$ such that $\Lambda(\tau^*) = \Gamma(\tau^*)$.

If $\Gamma(\tau)$ is a strictly increasing function over $(\overline{\tau}, 1)$, the steady state solution is unique. Straightforward differentiation shows that

$$\Gamma'(\tau) = \Bigg[\frac{1}{1-\eta}\Big(\frac{r(\tau)+\delta}{\alpha}\Big)^{\frac{\eta}{1-\eta}}\frac{1}{\alpha} - 1\Bigg]r'(\tau).$$

Since $r'(\tau)$ is strictly increasing over $(0, 1)$, then $\Gamma(\tau)$ is strictly increasing over $(\overline{\tau}, 1)$ if

$$\frac{1}{1-\eta}\Big(\frac{r(\tau)+\delta}{\alpha}\Big)^{\frac{\eta}{1-\eta}}\frac{1}{\alpha} - 1 > 0, \forall\tau \in (\overline{\tau}, 1).$$

It suffices if

$$\frac{1}{1-\eta}\Big(\frac{r(\overline{\tau})+\delta}{\alpha}\Big)^{\frac{\eta}{1-\eta}}\frac{1}{\alpha} - 1 > 0.$$

Then, it is straightforward. This completes the proof of Proposition 1. **Q.E.D.**

A.5 Proof of Theorem 2.2

The proof is divided into a number of steps:

Step 1 We derive two additional conditions that are satisfied along a BGP competitive equilibrium. Recall (A.3.2):

$$G(Q_tX_t, A_tN_t) = \hat{\gamma}^{-1}G(Q_{t+1}X_{t+1}, A_{t+1}N_{t+1}).$$

This condition holds along a BGP competitive equilibrium. In addition,

$$Q_{t+1} = (1+q)Q_t, X_{t+1} = (1-\tau^*)X_t, A_{t+1} = (1+a)A_t \quad \text{and} \quad N_{t+1} = (1+n)N_t$$

also hold along this path. Then

$$G(Q_tX_t, A_tN_t) = \hat{\gamma}^{-1}G((1+q)(1-\tau^*)Q_tX_t, (1+a)(1+n)A_tN_t). \tag{A.5.1}$$

Since $G(\cdot)$ is homogeneous of degree one, then (A.5.1) can be expressed as

$$G(\hat{x}_t, 1) = \varsigma_2 G\Big(\frac{\varsigma_1}{\varsigma_2}\hat{x}_t, 1\Big). \tag{A.5.2}$$

where $\hat{x}_t \equiv \frac{Q_tX_t}{A_tN_t}$, $\varsigma_1 \equiv \frac{(1+q)(1-\tau^*)}{\hat{\gamma}}$ and $\varsigma_2 \equiv \frac{(1+a)(1+n)}{\hat{\gamma}}$. Define $g(\hat{x}_t) \equiv G(\hat{x}_t, 1)$. Then, the first additional condition that must hold along any BGP competitive equilibrium is

$$g(\hat{x}_t) = \varsigma_2 g\Big(\frac{\varsigma_1}{\varsigma_2}\hat{x}_t\Big). \tag{A.5.3}$$

Since $g(\cdot)$ is continuously differentiable and (A.5.3) holds for all $\hat{x} > 0$, we can derive another condition that must hold along any BGP competitive equilibrium:

$$\frac{\hat{x}_t g'(\hat{x}_t)}{g(\hat{x}_t)} = \frac{\frac{\varsigma_1}{\varsigma_2}\hat{x}_t g'(\frac{\varsigma_1}{\varsigma_2}\hat{x}_t)}{g(\frac{\varsigma_1}{\varsigma_2}\hat{x}_t)}. \tag{A.5.4}$$

Step 2 Claim that

$$\frac{d}{d\hat{x}_t}\left(\frac{\hat{x}_t g'(\hat{x}_t)}{g(\hat{x}_t)}\right) \left\{\begin{matrix} > \\ = \\ < \end{matrix}\right\} 0 \quad \text{if and only if} \quad \sigma_G(\hat{x}_t)\left\{\begin{matrix} > \\ = \\ < \end{matrix}\right\} 1. \tag{A.5.5}$$

To show this, we start with straightforward differentiation which yields

$$\frac{d}{d\hat{x}_t}\left(\frac{\hat{x}_t g'(\hat{x}_t)}{g(\hat{x}_t)}\right) = \frac{g'(\hat{x})}{g(\hat{x})} - \frac{\hat{x}[g'(\hat{x})]^2}{[g(\hat{x})]^2} + \frac{\hat{x}g''(\hat{x})}{g(\hat{x})}. \tag{A.5.6}$$

Next, using the expression in (2.19), $\sigma_G(\hat{x}) \gtreqless 1$ if and only if

$$\begin{aligned} \frac{g'(\hat{x})[g(\hat{x}) - \hat{x}g'(\hat{x})]}{g(\hat{x})} &\gtreqless -\hat{x}g''(\hat{x}) \\ \Leftrightarrow \frac{g'(\hat{x})}{g(\hat{x})}\left[1 - \frac{\hat{x}g'(\hat{x})}{g(\hat{x})}\right] &\gtreqless \frac{-\hat{x}g''(\hat{x})}{g(\hat{x})} \end{aligned}$$

$$\Leftrightarrow \frac{g'(\hat{x})}{g(\hat{x})} - \frac{\hat{x}[g'(\hat{x})]^2}{[g(\hat{x})]^2} + \frac{\hat{x}g''(\hat{x})}{g(\hat{x})} = \frac{d}{d\hat{x}}\left[\frac{\hat{x}g'(\hat{x})}{g(\hat{x})}\right] \gtreqless 0. \tag{A.5.7}$$

By combining (A.5.6) and (A.5.7), the intermediate result can be obtained. This result essentially says that if $\sigma_G(\hat{x})$ is not equal to one, then $\hat{x}g'(\hat{x})/g(\hat{x})$ must be either strictly increasing or strictly decreasing for all $\hat{x} > 0$.

Step 3 We will now apply (A.5.5) to (A.5.3) and (A.5.4). As mentioned before, if $\sigma_G(\cdot)$ differs from one, then $\hat{x}g'(\hat{x})/g(\hat{x})$ must be either strictly increasing or strictly decreasing for all $\hat{x} > 0$. Hence, the equality in (A.5.4) holds if and only if $\varsigma_1 = \varsigma_2$. Using this equality, we can write (A.5.3) as

$$g'(\hat{x}_t) = \varsigma_2 g'(\hat{x}_t),$$

which implies that $\varsigma_2 = 1$. In sum, when $\sigma_G(\cdot)$ differs from one, the existence of a BGP competitive equilibrium requires

$$\varsigma_1 = \varsigma_2 = 1. \tag{A.5.8}$$

Step 4 Claim that

$$\varsigma_1 = \varsigma_2 = 1 \quad \text{implies} \quad \hat{k}_{t+1} = \hat{k}_t = \hat{k}^* \quad \text{and} \quad \hat{x}_{t+1} = \hat{x}_t = \hat{x}^*$$

along a BGP competitive equilibrium when $\sigma_G(\hat{x}_t) \neq 1$. Due to CRTS of $F(\cdot)$ and $G(\cdot)$, we obtain

$$\frac{Y_t}{A_t N_t} = F(\hat{k}_t, g(\hat{x}_t)). \tag{A.5.9}$$

Since $\varsigma_2 = 1$, then the LHS of (A.5.9) is constant along a BGP competitive equilibrium and hence the RHS. Constancy of the RHS of (A.5.9) is possible when both $\hat{k}_t$ and $\hat{x}_t$ oppositely change in a proper way or both variables remain unchanged. Nevertheless, we claim that only the latter is applied. To illustrate this, we recall that the rate of interest must be constant along any BGP competitive equilibrium. Recall (2.11):

$$r^* = F_1\Big(K_t, G(Q_t X_t, A_t N_t)\Big) - \delta.$$

, provided that $r_t = r^*$ along any BGP competitive equilibrium. Linear homogeneity of $F(\cdot)$ implies that $F_1(\cdot)$ is homogeneous of degree zero. Using this property, the above condition can be expressed as

$$r^* = F_1\Big(\hat{k}_t, g(\hat{x}_t)\Big) - \delta. \tag{A.5.10}$$

The only way to keep the RHS of (A.5.10) constant is to keep $\hat{k}_t$ and $\hat{x}_t$ unchanged. We, then, reach the conclusion.

Step 5 Claim that

$$\varsigma_1 = \varsigma_2 = 1 \quad \text{implies} \quad \gamma^* = 1 + a, 1 - \tau^* = \frac{(1+a)(1+n)}{1+q} \quad \text{and} \quad r^* = q$$

along a BGP competitive equilibrium when $\sigma_G(\hat{x}_t) \neq 1$. The first two conditions are obvious, we are to illustrate the last equality. To do so, recall (2.12):

$$p_t = Q_t F_2\Big(K_t, G(Q_t X_t, A_t N_t)\Big) G_1(Q_t X_t, A_t N_t).$$

Since F_2 and G_1 are homogeneous of degree zero, the above condition can be expressed as

$$p_t = Q_t F_2(\hat{k}_t, g(\hat{x}_t)) G_1(\hat{x}_t, 1) \tag{A.5.11}$$

Having shown that $(\hat{k}_t, \hat{x}_t) = (\hat{k}^*, \hat{x}^*)$ along a BGP competitive equilibrium when $\sigma_G(\hat{x}_t) \neq 1$, we can apply this with Hotelling condition (2.5) to show that $r^* = q$.

Step 6 We derive (2.30) and (2.31) that can be used to jointly determine the steady state values $\hat{k}^*$ and $\hat{x}^*$. We can derive (2.30) using (A.5.10) with the facts that $(\hat{k}_t, \hat{x}_t) = (\hat{k}^*, \hat{x}^*)$ and that $r^* = q$ along a BGP competitive equilibrium when $\sigma_G(\hat{x}_t) \neq 1$. Next, we derive (2.31) as follows. As shown in the proof of Theorem 2.1, the capital market clearing condition can be expressed as

$$K_{t+1} = F_2\Big(K_t, G(Q_t X_t, A_t N_t)\Big)\left[\frac{1}{2+\theta} A_t N_t G_2(Q_t X_t, A_t N_t) - \Big(\frac{1-\tau^*}{\tau^*}\Big) X_t Q_t G_1(Q_t X_t, A_t N_t)\right].$$

Dividing both sides by $A_t N_t$ gives

$$(1+a)(1+n)\hat{k}_{t+1} = F_2\Big(\hat{k}_t, G(\hat{x}_t, 1)\Big)\left[\frac{G_2(\hat{x}_t, 1)}{2+\theta} - \Big(\frac{1-\tau^*}{\tau^*}\Big)\hat{x}_t G_1(\hat{x}_t, 1)\right].$$

Equation (2.31) can be obtained by setting $\hat{k}_{t+1} = \hat{k}_t = \hat{k}^*$ and $\hat{x}_t = \hat{x}^*$. This completes the proof of Theorem 2.2. **Q.E.D.**

A.6 Proof of Proposition 2.2

Suppose the $F(\cdot)$ and $G(\cdot)$ take the CES forms (2.26) and (2.20), respectively. Also, as before, define $\hat{k}_t \equiv \frac{K_t}{A_t N_t}$ and $\hat{x}_t \equiv \frac{Q_t X_t}{A_t N_t}$. Then, the partial derivatives of $F(\cdot)$ with respect to the first and the second arguments yield, respectively,

$$F_1(\hat{k}_t, G(\hat{x}_t, 1)) = \alpha\left[\alpha + (1-\alpha)\Big(\frac{G(\hat{x}_t, 1)}{\hat{k}_t}\Big)^{\eta}\right]^{\frac{1-\eta}{\eta}}, \tag{A.6.1}$$

$$F_2(\hat{k}_t, G(\hat{x}_t, 1)) = (1-\alpha)\left(\frac{G(\hat{x}_t, 1)}{\hat{k}_t}\right)^{\eta-1}\left[\alpha + (1-\alpha)\Big(\frac{G(\hat{x}_t, 1)}{\hat{k}_t}\Big)^{\eta}\right]^{\frac{1-\eta}{\eta}} \tag{A.6.2}$$

where $G(\hat{x}_t,1)=\Big[\varphi(\hat{x}_t)^{\psi}+(1-\varphi)\Big]^{\frac{1}{\psi}}$. Similarly, the partial derivatives of $G(\cdot)$ with respect to the first and the second arguments yield, respectively,

$$G_1(\hat{x}_t,1)=\frac{\varphi(\hat{x}_t)^{\psi-1}G(\hat{x}_t,1)}{\varphi(\hat{x}_t)^{\psi}+(1-\varphi)}, \tag{A.6.3}$$

$$G_2(\hat{x}_t,1)=\frac{(1-\varphi)G(\hat{x}_t,1)}{\varphi(\hat{x}_t)^{\psi}+(1-\varphi)}. \tag{A.6.4}$$

Next, we suppose that the economy is established along a BGP competitive equilibrium. In such a path, $\hat{k}_{t+1}=\hat{k}_t=\hat{x}^*$ and $\hat{x}_{t+1}=\hat{x}_t=\hat{x}^*$. We, then, characterise the BGP competitive equilibrium as follows. To begin with, we apply (2.30) to get

$$\alpha\left[\alpha+(1-\alpha)\Big(\frac{G(\hat{x}^*,1)}{\hat{k}^*}\Big)^{\eta}\right]^{\frac{1-\eta}{\eta}}=q+\delta \tag{A.6.5}$$

, which implies

$$(1-\alpha)\Big(\frac{G(\hat{x}^*,1)}{\hat{k}^*}\Big)^{\eta}=\left(\Big(\frac{q+\delta}{\alpha}\Big)^{\frac{\eta}{1-\eta}}-\alpha\right). \tag{A.6.6}$$

We, then, combine (A.6.2) and (A.6.6) to get

$$\frac{G(\hat{x}^*,1)}{\hat{k}^*}F_2(\hat{k}^*,G(\hat{x}^*,1))=\Big(\frac{q+\delta}{\alpha}\Big)\left(\Big(\frac{q+\delta}{\alpha}\Big)^{\frac{\eta}{1-\eta}}-\alpha\right)=(2+\theta)\Theta. \tag{A.6.7}$$

Next, we manipulate (2.31) by using (A.6.3), (A.6.4):

$$\begin{aligned}
(1+a)(1+n)\hat{k}^* &= F_2\Big(\hat{k}^*,G(\hat{x}^*,1)\Big)\left[\frac{G_2(\hat{x}^*,1)}{2+\theta}-\Big(\frac{1-\tau^*}{\tau^*}\Big)\hat{x}^*G_1(\hat{x}^*,1)\right]\\
&= F_2\Big(\hat{k}^*,G(\hat{x}^*,1)\Big)\left[\frac{(1-\varphi)G(\hat{x}^*,1)}{\varphi(\hat{x}^*)^{\psi}+(1-\varphi)}\frac{1}{2+\theta}-\Big(\frac{1-\tau^*}{\tau^*}\Big)\hat{x}^*\frac{\varphi(\hat{x}^*)^{\psi-1}G(\hat{x}^*,1)}{\varphi(\hat{x}^*)^{\psi}+(1-\varphi)}\right]\\
&= F_2\Big(\hat{k}^*,G(\hat{x}^*,1)\Big)\frac{G(\hat{x}^*,1)}{\varphi(\hat{x}^*)^{\psi}+(1-\varphi)}\left[\frac{1-\varphi}{2+\theta}-\Big(\frac{1-\tau^*}{\tau^*}\Big)\varphi(\hat{x}^*)^{\psi}\right]\\
&= F_2\Big(\hat{k}^*,G(\hat{x}^*,1)\Big)\frac{G(\hat{x}^*,1)}{\varphi(\hat{x}^*)^{\psi}+(1-\varphi)}\frac{1}{2+\theta}\left[(1-\varphi)-\Big(\frac{1-\tau^*}{\tau^*}\Big)\varphi(\hat{x}^*)^{\psi}(2+\theta)\right]
\end{aligned}$$

, which leads to

$$(1+a)(1+n)\Big(\varphi(\hat{x}^*)^{\psi}+(1-\varphi)\Big)=F_2\Big(\hat{k}^*,G(\hat{x}^*,1)\Big)\frac{G(\hat{x}^*,1)}{\hat{k}^*}\frac{1}{2+\theta}\left[(1-\varphi)-\Big(\frac{1-\tau^*}{\tau^*}\Big)\varphi(\hat{x}^*)^{\psi}(2+\theta)\right].$$

Apply (A.6.7), the above expression becomes

$$(1+a)(1+n)\Big(\varphi(\hat{x}^*)^\psi + (1-\varphi)\Big) = \Theta\left[(1-\varphi) - \Big(\frac{1-\tau^*}{\tau^*}\Big)\varphi(\hat{x}^*)^\psi(2+\theta)\right]. \quad \text{(A.6.8)}$$

Next, we solve the previous condition for $(\hat{x}^*)^\psi$:

$$(\hat{x}^*)^\psi = \frac{\frac{1-\varphi}{\varphi}\left[\frac{\Theta}{(1+a)(1+n)} - 1\right]}{1 + \frac{\Theta}{(1+a)(1+n)}\left(\frac{1-\tau^*}{\tau^*}\right)(2+\theta)}. \quad \text{(A.6.9)}$$

We can characterise the BGP competitive equilibrium $\hat{x}^*$ using (A.6.9). For the other variables, it is straightforward.

We finish the proof by finding under which conditions the BGP competitive equilibrium is feasible. To prove this, it suffices to prove under the additional conditions that ensure $\hat{x}^* > 0, \tau^* \in (0,1)$ and $\Theta > 0$. Firstly, to ensure that $\hat{x}^* > 0$, it is necessary that $\Theta > (1+a)(1+n)$; see (A.6.9). Secondly, in order to have $\tau^* \in (0,1)$, it is necessary that $1+q > (1+a)(1+n)$; see (2.29). Finally, in order to have $\Theta > 0$, it is necessary that $\delta + q > \alpha^{\frac{1}{\eta}}$; see the definition of Θ. These conditions can be combined into one by imposing that $min\{\Theta, 1+q\} > (1+a)(1+n)$. This completes the proof of Proposition 2.2. **Q.E.D.**

A.7 Proof of Theorem 2.3

We will consider each specifications in (2.34)-(2.37) separately.

Specification 1 We begin with the production function in (2.34). Under this specification, the-first order conditions for the representative firm's are given by

$$(1-\varphi)\alpha Y_t^{1-\psi} K_t^{\psi\alpha-1}(Q_t X_t)^{(1-\alpha)\psi} = r_t + \delta, \quad \text{(A.7.1)}$$

$$(1-\varphi)(1-\alpha) Y_t^{1-\psi} K_t^{\psi\alpha}(Q_t X_t)^{(1-\alpha)\psi-1} Q_t = p_t, \quad \text{(A.7.2)}$$

$$\varphi Y_t^{1-\psi}(A_t N_t)^{\psi-1} A_t = w_t. \quad \text{(A.7.3)}$$

In any balanced growth equilibrium, the capital-output ratio is constant over time, i.e.,

$$Y_t = \frac{1}{\kappa^*} K_t \quad \text{for some} \quad \kappa^* > 0.$$

Using this claim, (A.7.1)-(A.7.3) become, respectively,

$$(1-\varphi)\alpha(\kappa^*)^{\psi-1}\left(\frac{\hat{k}_t}{\hat{x}_t}\right)^{\psi(\alpha-1)} = r_t + \delta, \tag{A.7.4}$$

$$(1-\varphi)(1-\alpha)(\kappa^*)^{\psi-1}\left(\frac{\hat{k}_t}{\hat{x}_t}\right)^{1-\psi(1-\alpha)} Q_t = p_t, \tag{A.7.5}$$

$$\varphi(\kappa^*)^{\psi-1}\hat{k}_t^{1-\psi}A_t = w_t. \tag{A.7.6}$$

Combined with the condition that $r_t = r^*$, for some $r^* > -\delta$ along any BGP competitive equilibrium, (A.7.4) implies that $\hat{k}_t$ and $\hat{x}_t$ grow at the same rate along such a path, i.e.

$$\frac{\hat{k}_{t+1}}{\hat{k}_t} = \frac{\hat{x}_{t+1}}{\hat{x}_t}. \tag{A.7.7}$$

Using (A.7.7) and the Hotelling condition (2.5), we obtain the long-run real interest rate which is

$$r^* = q. \tag{A.7.8}$$

Next, we derive an equation governing the state variable K_t; by using the capital market clearing condition, saving function (2.6) and natural resource market clearing condition, as follows.

$$\begin{aligned} K_{t+1} &= N_t S_t \\ &= N_t\left[\frac{w_t}{2+\theta} - p_t m_t\right] \\ &= N_t\left[\frac{w_t}{2+\theta} - p_t \frac{M_{t+1}}{N_t}\right]. \end{aligned}$$

Then, apply $M_{t+1} = \left(\frac{1-\tau^*}{\tau^*}\right)X_t$ so that

$$K_{t+1} = N_t\left[\frac{w_t}{2+\theta} - \left(\frac{1-\tau^*}{\tau^*}\right)p_t\frac{X_t}{N_t}\right]. \tag{A.7.9}$$

This equation will be very useful throughout the proofs of all specifications.

Now, we state an equation of motion for the transformed variable $\hat{k}_t$. Substitute (A.7.5) and (A.7.6) into (A.7.9) so that

$$(1+a)(1+n)\frac{\hat{k}_{t+1}}{\hat{k}_t} = \left[\frac{\varphi(\kappa^*)^{\psi-1}\hat{k}_t^{-\psi}}{2+\theta} - \left(\frac{1-\tau^*}{\tau^*}\right)(1-\varphi)(1-\alpha)(\kappa^*)^{\psi-1}\left(\frac{\hat{k}_t}{\hat{x}_t}\right)^{\psi(\alpha-1)}\right]. \tag{A.7.10}$$

Since $\frac{\hat{k}_{t+1}}{\hat{k}_t}$ and $\left(\frac{\hat{k}_t}{\hat{x}_t}\right)$ are constant along any BGP competitive equilibrium, these imply that $\hat{k}_{t+1} = \hat{k}_t = \hat{k}^*$ and thus

$$\frac{\hat{k}_{t+1}}{\hat{k}_t} = \frac{\hat{x}_{t+1}}{\hat{x}_t} = 1. \tag{A.7.11}$$

This implies that

$$\gamma^* = 1+a \quad \text{and} \quad (1-\tau^*) = \frac{(1+a)(1+n)}{(1+q)}. \tag{A.7.12}$$

We finish the proof the the first specification by characterising a set of equations characterising a BGP competitive equilibrium (if exists). Using (2.34) and $Y_t\kappa_t = K_t$, we can show that

$$(\kappa_t)^{-1} = \left[\varphi\hat{k}_t^{-\psi} + (1-\varphi)\left(\frac{\hat{k}_t}{\hat{x}_t}\right)^{\psi(\alpha-1)}\right]^{\frac{1}{\psi}}. \tag{A.7.13}$$

We use (A.7.4), (A.7.10) and (A.7.13) to characterise a BGP competitive equilibrium (if exists). In particular, if such a path exists, then $(\kappa^*, \hat{k}^*, \hat{x}^*) \gg 0$ can be jointly determined by the following system:

$$(1-\varphi)\alpha(\kappa^*)^{\psi-1}\left(\frac{\hat{k}^*}{\hat{x}^*}\right)^{\psi(\alpha-1)} = r^* + \delta,$$

$$(1+a)(1+n) = \left[\frac{\varphi(\kappa^*)^{\psi-1}(\hat{k}^*)^{-\psi}}{2+\theta} - \left(\frac{1-\tau^*}{\tau^*}\right)(1-\varphi)(1-\alpha)(\kappa^*)^{\psi-1}\left(\frac{\hat{k}^*}{\hat{x}^*}\right)^{\psi(\alpha-1)}\right],$$

$$(\kappa^*)^{-1} = \left[\varphi(\hat{k}^*)^{-\psi} + (1-\varphi)\left(\frac{\hat{k}^*}{\hat{x}^*}\right)^{\psi(\alpha-1)}\right]^{\frac{1}{\psi}}.$$

Specification 2 Consider the production function in (2.35). Under this specification, the first-order conditions for the representative firm's are given by

$$(1-\varphi)\alpha Y_t^{1-\psi}K_t^{\psi\alpha-1}(A_tN_t)^{(1-\alpha)\psi} = r_t + \delta, \tag{A.7.14}$$

$$\varphi Y_t^{1-\psi}(Q_tX_t)^{\psi-1}Q_t = p_t, \tag{A.7.15}$$

$$(1-\varphi)(1-\alpha)Y_t^{1-\psi}K_t^{\alpha\psi}(A_tN_t)^{\psi(1-\alpha)-1}A_t = w_t. \tag{A.7.16}$$

In any balanced growth equilibrium, the capital-output ratio is constant over time, i.e.,

$$Y_t = \frac{1}{\kappa^*} K_t \quad \text{for some} \quad \kappa^* > 0.$$

Using this claim, (A.7.14)-(A.7.16) become, respectively,

$$(1-\varphi)\alpha(\kappa^*)^{\psi-1}\left(\hat{k}_t\right)^{\psi(\alpha-1)} = r_t + \delta, \tag{A.7.17}$$

$$\varphi(\kappa^*)^{\psi-1}\left(\frac{\hat{k}_t}{\hat{x}_t}\right)^{1-\psi} Q_t = p_t, \tag{A.7.18}$$

$$(1-\varphi)(1-\alpha)(\kappa^*)^{\psi-1}\hat{k}_t^{1-\psi(1-\alpha)} A_t = w_t. \tag{A.7.19}$$

Combined with the condition that $r_t = r^*$, for some $r^* > -\delta$ along any BGP competitive equilibrium, (A.7.17) implies that $\hat{k}_t$ is time-invariant along such a path, i.e.

$$\hat{k}_{t+1} = \hat{k}_t = \hat{k}^*. \tag{A.7.20}$$

This also implies that

$$\gamma^* = 1 + a. \tag{A.7.21}$$

Recall (A.7.9):

$$K_{t+1} = N_t \left[\frac{w_t}{2+\theta} - \Big(\frac{1-\tau^*}{\tau^*}\Big) p_t \frac{X_t}{N_t} \right].$$

Apply (A.7.18) and (A.7.19), we get

$$(1+a)(1+n)\frac{\hat{k}_{t+1}}{\hat{k}_t} = (\kappa^*)^{\psi-1}\left[\frac{(1-\varphi)(1-\alpha)\hat{k}_t^{-\psi(1-\alpha)}}{2+\theta} - \Big(\frac{1-\tau^*}{\tau^*}\Big)\varphi\left(\frac{\hat{k}_t}{\hat{x}_t}\right)^{-\psi} \right]. \tag{A.7.22}$$

Since $\hat{k}_t$ is constant along any BGP competitive equilibrium, this implies

$$\hat{x}_{t+1} = \hat{x}_t = \hat{x}^*. \tag{A.7.23}$$

This also implies that

$$1 - \tau^* = \frac{(1+a)(1+n)}{1+q}. \tag{A.7.24}$$

Also, we use the result that $\hat{k}_t / \hat{x}_t$ is constant along any BGP competitive equilibrium with Hotelling condition (2.5) and (A.7.18) to get

$$r^* = q. \tag{A.7.25}$$

We state a system of equations that can be used to characterise the steady state values κ^*, $\hat{k}^*$ and $\hat{x}^*$ as follows. To begin with, we formulate an equation determining capital to output ratio. In particular, by using (35) and the definition of κ_t:

$$\frac{Y_t}{K_t} = (\kappa_t)^{-1} = \left[\varphi\Big(\frac{\hat{x}_t}{\hat{k}_t}\Big)^{\psi} + (1-\varphi)(\hat{k}_t^{1-\alpha})^{\psi}\right]^{\frac{1}{\psi}}. \tag{A.7.26}$$

We use (A.7.17), (A.7.22) and (A.7.26) to characterise a BGP competitive equilibrium (if exists). In particular, if such a path exists, then $(\kappa^*, \hat{k}^*, \hat{x}^*) \gg 0$ can be jointly determined by the following system:

$$(1-\varphi)\alpha(\kappa^*)^{\psi-1}\Big(\hat{k}^*\Big)^{\psi(\alpha-1)} = r^* + \delta,$$

$$(1+a)(1+n) = (\kappa^*)^{\psi-1}\left[\frac{(1-\varphi)(1-\alpha)(\hat{k}^*)^{-\psi(1-\alpha)}}{2+\theta} - \Big(\frac{1-\tau^*}{\tau^*}\Big)\varphi\left(\frac{\hat{k}^*}{\hat{x}^*}\right)^{-\psi}\right],$$

$$(\kappa^*)^{-1} = \left[\varphi\Big(\frac{\hat{x}^*}{\hat{k}^*}\Big)^{\psi} + (1-\varphi)(\hat{k}^*)^{\psi(1-\alpha)}\right]^{\frac{1}{\psi}}.$$

Specification 3 Consider the production function in (2.36). Under this specification, the-first order conditions for the representative firm's are given by

$$(1-\beta)\varphi Y_t^{1-\frac{\psi}{1-\beta}}(A_tN_t)^{\frac{\beta\psi}{1-\beta}}K_t^{\psi-1} = r_t + \delta, \tag{A.7.27}$$

$$(1-\beta)(1-\varphi)Y_t^{1-\frac{\psi}{1-\beta}}(A_tN_t)^{\frac{\beta\psi}{1-\beta}}(Q_tX_t)^{\psi-1}Q_t = p_t, \tag{A.7.28}$$

$$\beta Y_t(A_tN_t)^{-1}A_t = w_t. \tag{A.7.29}$$

In any balanced growth equilibrium, the capital-output ratio is constant over time, i.e.,

$$Y_t = \frac{1}{\kappa^*}K_t \quad \text{for some} \quad \kappa^* > 0.$$

Using this claim, (A.7.27)-(A.7.29) become, respectively,

$$(1-\beta)\varphi(\kappa^*)^{\frac{\psi}{1-\beta}-1}(\hat{k}_t)^{-\frac{\beta\psi}{1-\beta}} = r_t + \delta, \tag{A.7.30}$$

$$(1-\beta)(1-\varphi)(\kappa^*)^{\frac{\psi}{1-\beta}-1}\hat{k}_t^{1-\frac{\psi}{1-\beta}}(\hat{x}_t)^{\psi-1}Q_t = p_t, \tag{A.7.31}$$

$$\beta(\kappa^*)^{-1}\hat{k}_tA_t = w_t. \tag{A.7.32}$$

Combined with the condition that $r_t = r^*$, for some $r^* > -\delta$ along any BGP competitive equilibrium, (A.7.17) implies that $\hat{k}_t$ is time-invariant along such a path, i.e.

$$\hat{k}_{t+1} = \hat{k}_t = \hat{k}^*. \tag{A.7.33}$$

This also implies that

$$\gamma^* = 1 + a. \tag{A.7.34}$$

Recall (A.7.9):

$$K_{t+1} = N_t \left[\frac{w_t}{2+\theta} - \Big(\frac{1-\tau^*}{\tau^*}\Big) p_t \frac{X_t}{N_t} \right].$$

Apply (A.7.31) and (A.7.32), we get

$$(1+a)(1+n)\hat{k}_{t+1} = \left[\frac{\beta(\kappa^*)^{-1}\hat{k}_t}{2+\theta} - \Big(\frac{1-\tau^*}{\tau^*}\Big)(1-\beta)(1-\varphi)(\kappa^*)^{\frac{\psi}{1-\beta}-1} \hat{k}_t^{1-\frac{\psi}{1-\beta}} (\hat{x}_t)^{\psi} \right]. \tag{A.7.35}$$

Since $\hat{k}_t$ is constant along any BGP competitive equilibrium, this implies

$$\hat{x}_{t+1} = \hat{x}_t = \hat{x}^*. \tag{A.7.36}$$

This also implies that

$$1 - \tau^* = \frac{(1+a)(1+n)}{1+q}. \tag{A.7.37}$$

Also, we use this results (A.7.33) with (A.7.36) with Hotelling condition (2.5) and (A.7.31) to get

$$r^* = q. \tag{A.7.38}$$

We state a system of equation that can be used to characterise the steady state values κ^*, $\hat{k}^*$ and $\hat{x}^*$ as follows. To begin with, we formulate an equation determining capital to output ratio. In particular, by using (2.36) and the definition of κ_t:

$$\frac{Y_t}{K_t} = (\kappa_t)^{-1} = \hat{k}_t^{-\beta} \left[\varphi + (1-\varphi)\Big(\frac{\hat{x}_t}{\hat{k}_t}\Big)^{\psi} \right]^{\frac{1-\beta}{\psi}}. \tag{A.7.39}$$

We use (A.7.30), (A.7.35) and (A.7.39) to characterise a BGP competitive equilibrium (if exists). In particular, if such a path exists, then $(\kappa^*, \hat{k}^*, \hat{x}^*) \gg 0$ can be jointly determined by the following system:

$$(1-\beta)\varphi(\kappa^*)^{\frac{\psi}{1-\beta}-1}(\hat{k}^*)^{-\frac{\beta\psi}{1-\beta}} = r^* + \delta,$$

$$(1+a)(1+n) = \left[\frac{\beta(\kappa^*)^{-1}}{2+\theta} - \Big(\frac{1-\tau^*}{\tau^*}\Big)(1-\beta)(1-\varphi)(\kappa^*)^{\frac{\psi}{1-\beta}-1}(\hat{k}^*)^{-\frac{\psi}{1-\beta}}(\hat{x}^*)^{\psi} \right],$$

$$(\kappa^*)^{-1} = (\hat{k}^*)^{-\beta}\left[\varphi + (1-\varphi)\Big(\frac{\hat{x}^*}{\hat{k}^*}\Big)^{\psi}\right]^{\frac{1-\beta}{\psi}}.$$

Specification 4 Consider the production function in (2.37). Under this specification, the-first order conditions for the representative firm's are given by

$$(1-v)\varphi Y_t^{1-\frac{\psi}{1-v}}(Q_tX_t)^{\frac{v\psi}{1-v}}K_t^{\psi-1} = r_t + \delta, \tag{A.7.40}$$

$$vY_t(Q_tX_t)^{-1}Q_t = p_t, \tag{A.7.41}$$

$$(1-v)(1-\varphi)(Q_tX_t)^{\frac{v\psi}{1-v}}Y_t^{1-\frac{\psi}{1-v}}(A_tN_t)^{\psi-1}A_t = w_t. \tag{A.7.42}$$

In any balanced growth equilibrium, the capital-output ratio is constant over time, i.e.,

$$Y_t = \frac{1}{\kappa^*}K_t \quad \text{for some} \quad \kappa^* > 0.$$

Using this claim, (A.7.40)-(A.7.42) become, respectively,

$$(1-v)\varphi(\kappa^*)^{\frac{\psi}{1-v}-1}\Big(\frac{\hat{x}_t}{\hat{k}_t}\Big)^{\frac{v\psi}{1-v}} = r_t + \delta, \tag{A.7.43}$$

$$v(\kappa^*)^{-1}\Big(\frac{\hat{k}_t}{\hat{x}_t}\Big)Q_t = p_t, \tag{A.7.44}$$

$$(1-v)(1-\varphi)(\kappa^*)^{\frac{\psi}{1-v}-1}\hat{k}_t^{1-\psi}\Big(\frac{\hat{x}_t}{\hat{k}_t}\Big)^{\frac{v\psi}{1-v}}A_t = w_t. \tag{A.7.45}$$

Combined with the condition that $r_t = r^*$, for some $r^* > -\delta$ along any BGP competitive equilibrium, (A.7.43) implies that $\hat{k}_t$ are growing at a common growth rate along such a path, i.e.

$$\frac{\hat{k}_{t+1}}{\hat{k}_t} = \frac{\hat{x}_{t+1}}{\hat{x}_t}. \tag{A.7.46}$$

Using this result with the Hotelling condition (2.5) and (A.7.44), we obtain

$$r^* = q. \tag{A.7.47}$$

Recall (A.7.9):

$$K_{t+1} = N_t\left[\frac{w_t}{2+\theta} - \Big(\frac{1-\tau^*}{\tau^*}\Big)p_t\frac{X_t}{N_t}\right].$$

Apply (A.7.44) and (A.7.45), we get

$$(1+a)(1+n)\frac{\hat{k}_{t+1}}{\hat{k}_t} = \left[\frac{(1-v)(1-\varphi)(\kappa^*)^{\frac{\psi}{1-v}-1}\hat{k}_t^{-\psi}\Big(\frac{\hat{x}_t}{\hat{k}_t}\Big)^{\frac{v\psi}{1-v}}}{2+\theta} - \Big(\frac{1-\tau^*}{\tau^*}\Big)v(\kappa^*)^{-1}\right]. \tag{A.7.48}$$

Since $\frac{\hat{k}_{t+1}}{\hat{k}_t}$ and $\Big(\frac{\hat{x}_t}{\hat{k}_t}\Big)$ are constant along any BGP competitive equilibrium, (A.7.48) implies

$$\hat{k}_{t+1} = \hat{k}_t = \hat{x}^*. \tag{A.7.49}$$

Combined with (A.7.46), (A.7.49) implies $\frac{\hat{k}_{t+1}}{\hat{k}_t} = \frac{\hat{x}_{t+1}}{\hat{x}_t} = 1$ which leads to

$$1-\tau^* = \frac{(1+a)(1+n)}{1+q} \tag{A.7.50}$$

and

$$\gamma^* = 1+a. \tag{A.7.51}$$

We state a system of equation that can be used to characterise the steady state values κ^*, $\hat{k}^*$ and $\hat{x}^*$ as follows. To begin with, we formulate an equation determining capital to output ratio. In particular, by using (2.36) and the definition of κ_t:

$$\frac{Y_t}{K_t} = (\kappa_t)^{-1} = \Big(\frac{\hat{x}_t}{\hat{k}_t}\Big)^v \left[\varphi + (1-\varphi)\Big(\frac{1}{\hat{k}_t}\Big)^\psi\right]^{\frac{1-v}{\psi}}. \tag{A.7.52}$$

We use (A.7.43), (A.7.48) and (A.7.52) to characterise a BGP competitive equilibrium (if exists). In particular, if such a path exists, then $(\kappa^*, \hat{k}^*, \hat{x}^*) \gg 0$ can be jointly determined by the following system:

$$(1-v)\varphi(\kappa^*)^{\frac{\psi}{1-v}-1}\Big(\frac{\hat{x}^*}{\hat{k}^*}\Big)^{\frac{v\psi}{1-v}} = r^* + \delta,$$

$$(1+a)(1+n) = \left[\frac{(1-v)(1-\varphi)(\kappa^*)^{\frac{\psi}{1-v}-1}(\hat{k}^*)^{-\psi}\Big(\frac{\hat{x}^*}{\hat{k}^*}\Big)^{\frac{v\psi}{1-v}}}{2+\theta} - \Big(\frac{1-\tau^*}{\tau^*}\Big)v(\kappa^*)^{-1}\right],$$

$$(\kappa^*)^{-1} = \Big(\frac{\hat{x}^*}{\hat{k}^*}\Big)^v \left[\varphi + (1-\varphi)\Big(\frac{1}{\hat{k}^*}\Big)^\psi\right]^{\frac{1-v}{\psi}}.$$

We finish the proof. **Q.E.D.**

Appendix B

Appendix to Chapter 3

B.1 Household Optimization

The Lagrangian is[1]

$$\max_{\{c_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[lnc_t - \mu_t \Big(c_t + K_{t+1} - (w_t + R_t K_t + \pi_t) - (1-\delta) K_t \Big) \right] \tag{B.1.1}$$

where μ_t is the (undiscounted) shadow value of (a unit of) K_{t+1}. The first order conditions are

$$\frac{\partial(\cdot)}{\partial c_t} = 0 \Leftrightarrow \mu_t = \frac{1}{c_t} \tag{B.1.2}$$

and

$$\frac{\partial(\cdot)}{\partial K_{t+1}} = 0 \Leftrightarrow \frac{\mu_{t+1}}{\mu_t} = \frac{1}{\beta} \frac{1}{(1 + R_{t+1} - \delta)}. \tag{B.1.3}$$

Combining these two conditions yields Euler condition:

$$\frac{c_{t+1}}{c_t} = \beta \Big(1 + R_{t+1} - \delta \Big). \tag{B.1.4}$$

In order to obtain TVC, we assume for the moment that the problem was finite with terminal date $T < \infty$. The Lagrangian would become

$$\max_{\{c_t, K_{t+1}\}_{t=0}^{T}} \sum_{t=0}^{T} \beta^t \left[lnc_t - \mu_t \Big(c_t + K_{t+1} - (w_t + R_t K_t + \pi_t) - (1-\delta) K_t \Big) \right]. \tag{B.1.5}$$

The objective function is strictly concave in $-K_{T+1}$. Then, the Kuhn-Tucker condition with respect to $-K_{T+1}$ will give

$$\frac{\partial(\cdot)}{\partial K_{T+1}} \geq 0; K_{T+1} \geq 0; \text{with complementary slackness} \tag{B.1.6}$$

[1]Since marginal utility at zero and time frame are infinite we can ignore non-negativity constraint for every choice variable.

or equivalently,

$$\beta^T \mu_T \geq 0; K_{T+1} \geq 0; \text{with } \beta^T \mu_T K_{T+1} = 0. \tag{B.1.7}$$

This means that either leaving nothing after the terminal period ($K_{T+1} = 0$) or otherwise leaving something only if the shadow value of (a unit of) K_{T+1} is zero, i.e. $\mu_T = 0$; note that β^T is finite, is needed. When $T \to \infty$, the terminal condition $\beta^T \mu_T K_{T+1} = 0$ will be replaced by the transversality condition

$$\lim_{T\to\infty} \beta^T \mu_T K_{T+1} = 0. \tag{B.1.8}$$

That is the (discounted) shadow value of (a unit of) K_{T+1} converge to zero. Equivalently, we can write

$$\lim_{T\to\infty} \beta^T u'[c_T] K_{T+1} = 0. \tag{B.1.9}$$

where $u'[\cdot]$ is the marginal utility in any period.

Note that if we iterate Equations (B.1.2) and (B.1.3) we obtain

$$\mu_0 = \frac{1}{c_0};$$

$$\mu_1 = \frac{1}{\beta}\frac{1}{1+R_1-\delta}\mu_0;$$

$$\mu_2 = \frac{1}{\beta}\frac{1}{1+R_1-\delta}\frac{1}{\beta}\frac{1}{1+R_2-\delta}\mu_0;$$

$$\vdots$$

$$\vdots$$

$$\mu_T = \frac{1}{\beta^T}\frac{1}{\prod_{v=1}^T(1+R_v-\delta)}\mu_0.$$

Thus, Equation (B.1.8) becomes

$$\lim_{T\to\infty} \frac{1}{\prod_{v=1}^T(1+R_v-\delta)}\mu_0 K_{T+1} = 0. \tag{B.1.10}$$

Since $\lambda_0 = \frac{1}{c_0} > 0$, then the transversality condition can be expressed as

$$\lim_{T\to\infty} \frac{K_{T+1}}{\prod_{v=1}^T(1+R_v-\delta)} = 0. \tag{B.1.11}$$

which is stated in the main text. **Q.E.D.**

B.2 Non-Renewable Resource Extraction Firm Optimization

By Lagrangian method, the firm solves

$$\max_{\{X_t, M_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} q_t \left[p_{x,t} X_t - w_t \frac{X_t}{M_t} + \lambda_t (M_t - X_t M_{t+1}) \right]$$

Two first order conditions are:

$$\frac{\partial(\bullet)}{\partial X_t} = 0 \Longleftrightarrow q_t \left[p_{x,t} - w_t \frac{1}{M_t} - \lambda_t \right] = 0 \tag{B.2.1}$$

$$\frac{\partial(\bullet)}{\partial M_{t+1}} = 0 \Longleftrightarrow -q_t \lambda_t + q_{t+1} \left[\frac{w_{t+1}}{M_{t+1}} \frac{X_{t+1}}{M_{t+1}} + \lambda_{t+1} \right] = 0 \tag{B.2.2}$$

and the transversality condition is:

$$\lim_{T \to \infty} q_T \lambda_T M_{T+1} = 0 \tag{B.2.3}$$

Combine Equations (B.2.1) and (B.2.2) to yield

$$\begin{aligned} p_{x,t} - \frac{w_t}{M_t} &= \frac{q_{t+1}}{q_t} \left[\frac{w_{t+1}}{M_{t+1}} \frac{X_{t+1}}{M_{t+1}} + p_{x,t+1} - \frac{w_{t+1}}{M_{t+1}} \right] \\ &= \frac{q_{t+1}}{q_t} \left[p_{x,t+1} + \left(\frac{w_{t+1}}{M_{t+1}} \frac{X_{t+1}}{M_{t+1}} - \frac{w_{t+1}}{M_{t+1}} \right) \right] \\ &= \frac{q_{t+1}}{q_t} \left[p_{x,t+1} - \frac{w_{t+1}}{M_{t+1}} \left(1 - \frac{X_{t+1}}{M_{t+1}} \right) \right] \end{aligned} \tag{B.2.4}$$

By using the definition of q_t, we get the Hotelling condition which is

$$p_{x,t} - \frac{w_t}{M_t} = \frac{1}{1 + r_{t+1}} \left[p_{x,t+1} - \frac{w_{t+1}}{M_{t+1}} \left(1 - \frac{X_{t+1}}{M_{t+1}} \right) \right] \tag{B.2.5}$$

, which is equivalent to Equation (3.22) in the text.

Finally, in order to get Equation (3.23), we know from Equation (B.2.1) that $p_{x,t} - \frac{w_t}{M_t} = \lambda_t > 0$. Use this with the fact that $q_t > 0$, we can get the result. **Q.E.D.**

B.3 Proof of Lemma 3.1

The proof is divided in nine steps as follows.

Step 1. *We claim that Equation (3.34) holds along a BGP. We also claim that this condition is the first-no arbitrage condition in labour market.*

By the free mobility of workers across different sectors, the wage rates must be equalised across sectors in equilibrium. Thus, there is no difference between the wage rates between the final-good sector and renewable resource extraction sector. Using Equations (3.13), (3.15) and (3.17), we obtain *the first no-arbitrage condition*:

$$\alpha_2 \frac{Y_t}{L_{Y,t}} = \frac{(1-\alpha_1-\alpha_2)Y_t}{\left(\left(\frac{\varpi}{1-\varpi}\right)\left(\frac{A_tX_t}{Z_t}\right)^{\rho}+1\right)Z_t}. \tag{B.3.1}$$

We can insert the RHSs of Equations (3.16) and (3.18) into the above equation to express the first no-arbitrage condition in terms of $L_{Y,t}, L_{Y,t}, L_{Y,t}$ and B_t. That is

$$\left(\left(\frac{\varpi}{1-\varpi}\right)\left(\frac{B_tL_{x,t}}{L_{z,t}}\right)^{\rho}+1\right) = \frac{1-\alpha_1-\alpha_2}{\alpha_2}\cdot\frac{L_{Y,t}}{L_{z,t}}. \tag{B.3.2}$$

Along a BGP, this condition is

$$\left(\left(\frac{\varpi}{1-\varpi}\right)\left(\frac{B^*L_x^*}{L_z^*}\right)^{\rho}+1\right) = \frac{1-\alpha_1-\alpha_2}{\alpha_2}\cdot\frac{L_Y^*}{L_z^*} \tag{B.3.3}$$

which is Equation (3.34) in the main text.

Step 2. *We claim that B_t is constant along any BGP.*

From Equation (B.3.2), as $L_{Y,t}, L_{Y,t}$ and $L_{Y,t}$ are all constant along any BGP, the term B_t must be constant as well.

Step 3. *We claim that Equation (3.28) holds along any BGP.*

By definition, $B_t = A_tM_t$. Along a BGP, we have $B_{t+1} = B_t \Leftrightarrow A_{t+1}M_{t+1} = A_tM_t$. Moreover, by Equations (3.18) and (3.19), we can show that $M_{t+1} = (1-L_{x,t})M_t$. Thus,

$$\frac{A_{t+1}}{A_t}\frac{M_{t+1}}{M_t} = (1+g_A)(1-L_{x,t}) = 1.$$

This implies

$$L_x^* = \frac{g_A}{1+g_A} \tag{B.3.4}$$

along the BGP. Moreover, $\frac{g_A}{1+g_A} \in (0,1)$ as $g_A > 0$, by assumption. Thus, equation (3.28) is verified, i.e., the size constancy of B_t along a BGP implies that the stationary value of labour share employed in the non-renewable resource extraction sector L_x^* is solely determined by the rate g_A such that $L_x^* = \frac{g_A}{1+g_A}$.

Step 4. *We claim that Equation (3.29) holds along any BGP.*

By using labour market clearing condition, it is straightforward to show that

$$L_Y^* = 1 - L_x^* - L_z^* \tag{B.3.5}$$

This condition is Equation (3.29) in the main text.

Step 5. *We claim that along a BGP Euler condition can be expressed by Equation (3.30).*

Recall Equations (3.3). Then, use the facts that $\frac{c_{t+1}}{c_t} - 1 = g^*$ and $R_{t+1} = R_t = R^*$

along any BGP to obtain

$$1+g^* = \beta(1+R^*-\delta). \tag{B.3.6}$$

This is Equation (3.30) in the main text.

Step 6. *We claim that Equation (3.31) holds along any BGP.*

By using Equations (3.7), (3.16) and (3.18), it is straightforward to show that, along a BGP,

$$\Omega^* = \Omega[B^* L_x^*, L_z^*] = D\Big(\varpi(B^* L_x^*)^\rho + (1-\varpi)(L_z^*)^\rho\Big)^{\frac{1}{\rho}} \tag{B.3.7}$$

This condition is Equation (3.31) in the main text.

Step 7. *We claim that Equation (3.32) holds along any BGP.*

By using Equations (3.7), (3.12) and (3.27), it is straightforward to show that, along a BGP,

$$R^* = \alpha_1 (L_Y^*)^{\alpha_2} (\Omega^*)^{1-\alpha_1-\alpha_2} \tag{B.3.8}$$

This condition is Equation (3.32) in the main text.

Step 8. *We claim that Θ_t is constant along any BGP.*

Apply (3.12)-(3.15) so that

$$Y_t\left\{\alpha_1+\alpha_2+(1-\alpha_1-\alpha_2)\left(\frac{1}{1+(\frac{1-\omega}{\omega})(\frac{Z_t}{A_tX_t})^\rho}+\frac{1}{(\frac{\omega}{1-\omega})(\frac{A_tX_t}{Z_t})^\rho+1}\right)\right\} = R_tK_t+w_t+p_{x,t}X_t-w_tL_{x,t}.$$

The second term on the LHS of the above expression is one. Thus, along a BGP:

$$Y_t = R_tK_t + w_t + p_{x,t}X_t - w_tL_{x,t} = R_tK_t + w_t + \Theta_t w_t L_{x,t} - w_t L_{x,t}.$$

Along a BGP, we have $R_t = R^*, L_{x,t} = L_x^*$ and

$$\frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}}{K_t} = \frac{w_{t+1}}{w_t} = 1+g^*.$$

As a consequence, Θ_t is constant along any BGP. **Q.E.D.**

Step 9. *We claim that along a BGP Hotelling condition can be expressed by Equation (3.33). In addition, we claim that this condition can be seen as the second no-arbitrage condition in labour market.*

Recall Hotelling condition:

$$p_{x,t} - \frac{w_t}{M_t} = \frac{1}{1+R_{t+1}-\delta}\left[p_{x,t+1} - \frac{w_{t+1}}{M_{t+1}} + \frac{X_{t+1}w_{t+1}}{(M_{t+1})^2}\right]. \tag{B.3.9}$$

This implies

$$\frac{w_t}{M_t}\Big(\frac{p_{x,t}M_t}{w_t}-1\Big) = \frac{1}{1+R_{t+1}-\delta}\cdot\frac{w_{t+1}}{M_{t+1}}\Big(\frac{p_{x,t+1}M_{t+1}}{w_{t+1}} - (1-L_{x,t+1})\Big). \tag{B.3.10}$$

Along any BGP, $\frac{p_{x,t}M_t}{w_t} = \frac{p_{x,t}X_t}{w_t L_{x,t}}$ must be constant. Also, along any BGP, we have $\frac{w_{t+1}}{w_t} = 1 + g^*$. Since $\Theta_t = \frac{p_{x,t}X_t}{w_t L_{x,t}}$, then along any BGP we have

$$\Big(\Theta^* - 1\Big) = \frac{1}{1 + R^* - \delta} \cdot \frac{1 + g^*}{1 - L_x^*}\Big(\Theta^* - 1 + L_x^*)\Big). \tag{B.3.11}$$

As no-arbitrage condition holds in equilibrium, workers in final-good sector and non-renewable resource extraction sector will be paid an equal wage. As long as no-arbitrage condition holds, i.e. as long as Equation (3.9) holds, Hotelling condition can be seen as *the second no-arbitrage condition.*

Step 10. *We claim that Equation (3.35) holds along any BGP.*

By definition, $\Theta_t = \frac{p_{x,t}X_t}{L_{x,t}w_t}$. Thus, using the factor demand conditions (3.13) and (3.14), we get

$$\Theta_t = \frac{(1 - \alpha_1 - \alpha_2)Y_t}{\left(1 + \left(\frac{1-\varpi}{\varpi}\right)\left(\frac{Z_t}{A_t X_t}\right)^{\rho}\right)} \cdot \frac{1}{X_t} \cdot X_t \cdot \frac{1}{L_{x,t}} \cdot \frac{L_{Y,t}}{\alpha_2 Y_t}. \tag{B.3.12}$$

Use extraction technologies, the above condition becomes

$$\Theta_t = \frac{(1 - \alpha_1 - \alpha_2)}{\left(1 + \left(\frac{1-\varpi}{\varpi}\right)\left(\frac{L_{z,t}}{B_t L_{x,t}}\right)^{\rho}\right)} \cdot \frac{1}{L_{x,t}} \cdot \frac{L_{Y,t}}{\alpha_2}. \tag{B.3.13}$$

This equation is equivalent to Equation (3.35) when evaluated along a BGP, i.e.

$$\Theta^* = \frac{(1 - \alpha_1 - \alpha_2)}{\left(1 + \left(\frac{1-\varpi}{\varpi}\right)\left(\frac{L_z^*}{B^* L_x^*}\right)^{\rho}\right)} \cdot \frac{1}{L_x^*} \cdot \frac{L_Y^*}{\alpha_2}. \tag{B.3.14}$$

Q.E.D.

B.4 Proof of Proposition 3.1

To begin with, we solve the Hotelling condition (3.33) for Θ^* to obtain

$$\Theta^* = 1 + \frac{(1 + g^*)L_x^*}{(1 + R^* - \delta)(1 - L_x^*) - (1 + g^*)}.$$

Use Euler condition to eliminate g^* so that the previous condition becomes

$$\Theta^* = 1 + \frac{\beta L_x^*}{(1 - L_x^*) - \beta}. \tag{B.4.1}$$

Using (3.28), we can simplify (B.4.1) to become

$$\Theta^* = \frac{1-\beta}{1-\beta(1+g_A)}. \tag{B.4.2}$$

Next, rewrite (3.35) as

$$\Theta^* = \left(\frac{1-\alpha_1-\alpha_2}{\alpha_2} \frac{L_Y^*}{L_x^*} \right) \frac{\frac{\varpi}{1-\varpi} \left(\frac{B^* L_x^*}{L_z^*} \right)^\rho}{1 + \frac{\varpi}{1-\varpi} \left(\frac{B^* L_x^*}{L_z^*} \right)^\rho}.$$

Using (3.34), we can simplify this to become

$$\begin{aligned}
\Theta^* &= \frac{L_z^*}{L_x^*} \left(\frac{1-\alpha_1-\alpha_2}{\alpha_2} \frac{L_Y^*}{L_z^*} - 1 \right) \\
&= \frac{L_z^*}{L_x^*} \left[\frac{1-\alpha_1-\alpha_2}{\alpha_2} \left(\frac{1-L_x^*-L_z^*}{L_z^*} \right) - 1 \right] \\
&= \frac{L_z^*}{L_x^*} \left[\frac{1-\alpha_1-\alpha_2}{\alpha_2} \frac{1}{1+g_A} \frac{1}{L_z^*} - \frac{1-\alpha_1}{\alpha_2} \right] \\
&= \frac{1-\alpha_1-\alpha_2}{\alpha_2} \frac{1}{g_A} - \frac{1-\alpha_1}{\alpha_2} \frac{1+g_A}{g_A} L_z^*.
\end{aligned}$$

Combining this and (B.4.2) gives

$$1-\alpha_1-\alpha_2-(1-\alpha_1)(1+g_A)L_z^* = \frac{\alpha_2(1-\beta)g_A}{1-\beta(1+g_A)}$$

$$\Rightarrow L_z^* = \frac{1}{(1-\alpha_1)(1+g_A)} \left\{ 1-\alpha_1-\alpha_2 \left[1 + \frac{(1-\beta)g_A}{1-\beta(1+g_A)} \right] \right\}. \tag{B.4.3}$$

The condition

$$1 > \frac{\alpha_2}{1-\alpha_1} \left[1 + \frac{(1-\beta)g_A}{1-\beta(1+g_A)} \right] > 0 \tag{B.4.4}$$

ensures that $L_z^* \in \left(0, \frac{1}{1+g_A}\right)$.

In order to gain some economic intuitions, we rearrange (B.4.4) as follows. Firstly, we consider the case that $1-\beta(1+g_A) > 0$; i.e. $g_A < \frac{1-\beta}{\beta}$. This implies that the second inequality holds automatically. Not only that, the first inequality is equivalent to

$$g_A < \frac{1-\beta}{\beta} \cdot \frac{1}{1 + \frac{1-\beta}{\beta} \frac{\alpha_2}{1-\alpha_1-\alpha_2}}. \tag{B.4.5}$$

If this condition holds, then $g_A < \frac{1-\beta}{\beta}$ holds automatically. Thus, it suffices to impose (B.4.5) to guarantee the feasibility. Secondly, we consider the case that $1-\beta(1+g_A) > 0$;

i.e. $g_A > \frac{1-\beta}{\beta}$. In this case, the second inequality of (B.4.4) implies that

$$0 > \frac{1-\beta}{2\beta-1} > g_A$$

, which is infeasible. As a consequence, we eliminate this one. Finally, the case that $1-\beta(1+g_A)=0$; i.e. $g_A = \frac{1-\beta}{\beta}$ will be eliminated as it implies that the right-hand side of (B.4.3) will be undefined.**Q.E.D.**

B.5 Proof of Lemma 3.2

We combine Equations (3.28), (3.29), (3.31) and (3.32) to obtain

$$R^* = \alpha_1\Big(\frac{1}{1+g_A} - L_z^*\Big)^{\alpha_2}(DL_z^*)^{1-\alpha_1-\alpha_2}\Big(\varpi\Big(\frac{B^*L_x^*}{L_z^*}\Big)^{\rho} + (1-\varpi)\Big)^{\frac{1-\alpha_1-\alpha_2}{\rho}}. \tag{B.5.1}$$

After rearranging, the condition (3.34) becomes

$$\frac{B^*L_x^*}{L_z^*} = \Big(\frac{1-\varpi}{\varpi}\Big)^{\frac{1}{\rho}}\Xi[L_z^*]^{\frac{1}{\rho}}. \tag{B.5.2}$$

where $\Xi[L_z^*] \equiv \Big(\Big(\frac{1-\alpha_1-\alpha_2}{\alpha_2}\Big)\Big(\frac{1}{1+g_A}\Big)\frac{1}{L_z^*} - \Big(\frac{1-\alpha_1}{\alpha_2}\Big)\Big)$.

Use the RHS of (B.5.2), (B.5.1) becomes

$$\begin{aligned} R^* &= \alpha_1\Big(\frac{1}{1+g_A} - L_z^*\Big)^{\alpha_2}(DL_z^*)^{1-\alpha_1-\alpha_2}\Big(\varpi\Big(\Big(\frac{1-\varpi}{\varpi}\Big)^{\frac{1}{\rho}}\Xi[L_z^*]^{\frac{1}{\rho}}\Big)^{\rho} + (1-\varpi)\Big)^{\frac{1-\alpha_1-\alpha_2}{\rho}} \\ &= \alpha_1\Big(\frac{1}{1+g_A} - L_z^*\Big)^{\alpha_2}(DL_z^*)^{1-\alpha_1-\alpha_2}\Big((1-\varpi)\Xi[L_z^*] + (1-\varpi)\Big)^{\frac{1-\alpha_1-\alpha_2}{\rho}} \\ &= \alpha_1\Big(\frac{1}{1+g_A} - L_z^*\Big)^{\alpha_2}(DL_z^*)^{1-\alpha_1-\alpha_2}(1-\varpi)^{\frac{1-\alpha_1-\alpha_2}{\rho}}\Big(\Xi[L_z^*]+1\Big)^{\frac{1-\alpha_1-\alpha_2}{\rho}} \end{aligned}$$

From now, it is straightforward. **Q.E.D.**

B.6 Derivation of condition (3.40)

Using Equation (3.39), it is fairly straightforward to show that

$$\frac{dR^*}{d\rho}\begin{cases} >0 & \text{iff } 0 < (1-\varpi)\Big(\frac{1-\alpha_1-\alpha_2}{\alpha_2}\frac{1}{1+g_A}\frac{1}{L_z^*} - \frac{1-\alpha_1-\alpha_2}{\alpha_2}\Big) < 1 \\ =0 & \text{iff } \quad (1-\varpi)\Big(\frac{1-\alpha_1-\alpha_2}{\alpha_2}\frac{1}{1+g_A}\frac{1}{L_z^*} - \frac{1-\alpha_1-\alpha_2}{\alpha_2}\Big) = 1 \\ <0 & \text{iff } \quad (1-\varpi)\Big(\frac{1-\alpha_1-\alpha_2}{\alpha_2}\frac{1}{1+g_A}\frac{1}{L_z^*} - \frac{1-\alpha_1-\alpha_2}{\alpha_2}\Big) > 1 \end{cases}. \tag{B.6.1}$$

Or equivalently,

$$\frac{dR^*}{d\rho}\begin{cases} >0 & \text{iff } L_z^* \in \left(\frac{(1-\varpi)(1-\alpha_1-\alpha_2)}{\alpha_2+(1-\varpi)(1-\alpha_1-\alpha_2)}\frac{1}{1+g_A}, \frac{1}{1+g_A}\right) \\ =0 & \text{iff } L_z^* = \frac{(1-\varpi)(1-\alpha_1-\alpha_2)}{\alpha_2+(1-\varpi)(1-\alpha_1-\alpha_2)}\frac{1}{1+g_A} \\ <0 & \text{iff } L_z^* \in \left(0, \frac{(1-\varpi)(1-\alpha_1-\alpha_2)}{\alpha_2+(1-\varpi)(1-\alpha_1-\alpha_2)}\frac{1}{1+g_A}\right) \end{cases}. \tag{B.6.2}$$

This means that ρ is a growth-enhancing parameter if the share of labour employed in the renewable resource sector is sufficiently high. Otherwise, this parameter becomes either growth-neutral or even growth-reducing. **Q.E.D.**

B.7 Proof of Lemma 3.3

The proof contains nine steps as follows.

Step 1: Show that $B_{t+1} = B_t = B^* \geq 0$ along a BGP.

We use the same argument stated in Step 2 in Proof of Lemma 3.1.

Step 2: Show that condition (3.53) along a BGP.

We use the same argument stated in Step 3 in Proof of Lemma 3.1.

Step 3: Show that condition (3.54) along a BGP.

It is obvious by labour market clearing condition (3.24) and the definition of a BGP.

Step 4: Show that condition (3.55) holds along a BGP.

It is obvious by Euler condition (3.46) and the definition of a BGP.

Step 5: Show that condition (3.56) holds along a BGP.

It is obvious by the CES aggregate condition (3.7), the extraction technologies (3.16) and (3.18) and the definition of a BGP.

Step 6: Show that condition (3.57) holds along a BGP.

By using Equations (3.50) and (3.52), we get

$$Y_t = K_t \tau^{\frac{1-\alpha_1}{\alpha_1}} L_{Y,t}^{\frac{\alpha_2}{\alpha_1}} \Omega_t^{\frac{1-\alpha_1-\alpha_2}{\alpha_1}} \tag{B.7.1}$$

which implies

$$\frac{Y_t}{K_t} = \tau^{\frac{1-\alpha_1}{\alpha_1}} L_{Y,t}^{\frac{\alpha_2}{\alpha_1}} \Omega_t^{\frac{1-\alpha_1-\alpha_2}{\alpha_1}}. \tag{B.7.2}$$

Next, combine Equation (B.7.2) and the final-good firm's demand for capital (3.12) to eliminate $\frac{Y_t}{K_t}$ again. Accordingly,

$$R_t = \alpha_1 \tau^{\frac{1-\alpha_1}{\alpha_1}} L_{Y,t}^{\frac{\alpha_2}{\alpha_1}} \Omega_t^{\frac{1-\alpha_1-\alpha_2}{\alpha_1}}. \tag{B.7.3}$$

This condition holds in competitive equilibrium. Thus, along a BGP we have

$$R^* = \alpha_1 \tau^{\frac{1-\alpha_1}{\alpha_1}} (L_Y^*)^{\frac{\alpha_2}{\alpha_1}} (\Omega^*)^{\frac{1-\alpha_1-\alpha_2}{\alpha_1}}. \tag{B.7.4}$$

Then, we get the result.

Step 7: Show that condition (3.58) holds along a BGP.

See Steps 8 and 9 in Proof of Lemma 3.1.

Step 8: Show that condition (3.59) holds along a BGP.

Apply Equations (3.13), (3.15)-(3.18) along a BGP. Then

$$\left(\left(\frac{\varpi}{1-\varpi}\right)\left(\frac{B^* L_x^*}{L_z^*}\right)^{\rho} + 1\right) = \frac{1-\alpha_1-\alpha_2}{\alpha_2} \cdot \frac{L_Y^*}{L_z^*} \tag{B.7.5}$$

which is the equation (3.59).

Step 9: Show that condition (3.60) holds along a BGP.

See Step 10 in Proof of Lemma 3.1. **Q.E.D.**

B.8 Proof of Lemma 3.4

Having derived Equations (3.53) - (3.60) stated in Lemma 3.3, we now can get Equation (3.61) using these conditions.

By using conditions (3.53), (3.54), (3.58) and (3.60), we can get

$$\frac{\left(\frac{1-\alpha_1-\alpha_2}{\alpha_2}\right)\left(\frac{1-(1+g_A)L_z^*}{g_A}\right)}{\left(1+\left(\frac{1-\varpi}{\varpi}\right)\left(\frac{L_z^*}{B^*}\right)^{\rho}\left(\frac{1+g_A}{g_A}\right)^{\rho}\right)} = 1 + \frac{(1+g^*)g_A}{(1+R^*-\delta)-(1+g_A)(1+g^*)}. \tag{B.8.1}$$

Next, we rearrange condition (3.59) to obtain

$$B^* = \left(\left(\frac{1-\varpi}{\varpi}\right)\left(\frac{1-\alpha_1-\alpha_2}{\alpha_2}\right)\left(\frac{1}{1+g_A}\right)\frac{1}{L_z^*} - \left(\frac{1-\varpi}{\varpi}\right)\left(\frac{1-\alpha_1}{\alpha_2}\right)\right)^{\frac{1}{\rho}}\left(\frac{1+g_A}{g_A}\right)L_z^*. \tag{B.8.2}$$

As the same as in the previous model, $B^* > 0$ for all ρ if and only if $L_z^* \in S$. Thus, this restriction is necessary.

Next, we combine Equations (3.53), (3.55), (3.56) and (3.57) to obtain

$$1 + g^* = \beta\Big(1 + (1-\tau)R^* - \delta\Big). \tag{B.8.3}$$

and

$$R^* = \alpha_1 \tau^{\frac{1-\alpha_1}{\alpha_1}} \Big(\frac{1}{1+g_A} - L_z^*\Big)^{\frac{\alpha_2}{\alpha_1}} D^{\frac{1-\alpha_1-\alpha_2}{\alpha_1}} \Big(\varpi(\frac{g_A}{1+g_A})^\rho (B^*)^\rho + (1-\varpi)(L_z^*)^\rho\Big)^{\frac{1}{\rho}\frac{1-\alpha_1-\alpha_2}{\alpha_1}}. \tag{B.8.4}$$

If Equations (B.8.2)-(B.8.4) hold, then the equation (B.8.1) is a single non-linear equation with one unknowns - namely L_z^*. This equation can be used to determine a BGP.

Define $V[L_z^*] = \Bigg(\Big(\frac{1-\varpi}{\varpi}\Big)\Big(\frac{1-\alpha_1-\alpha_2}{\alpha_2}\Big)\Big(\frac{1}{1+g_A}\Big)\frac{1}{L_z^*} - \Big(\frac{1-\varpi}{\varpi}\Big)\Big(\frac{1-\alpha_1}{\alpha_2}\Big)\Bigg)$. Then $B^* = V[L_z^*]^{\frac{1}{\rho}}\Big(\frac{1+g_A}{g_A}\Big)L_z^*$. The LHS of Equation (B.8.1) becomes

$$\frac{\Big(\frac{1-\alpha_1-\alpha_2}{\alpha_2}\Big)\Big(\frac{1-(1+g_A)L_z^*}{g_A}\Big)}{\Bigg(1+\Big(\frac{1-\varpi}{\varpi}\Big)\Big(\frac{L_z^*}{V[L_z^*]^{\frac{1}{\rho}}\Big(\frac{1+g_A}{g_A}\Big)L_z^*}\Big)^\rho\Big(\frac{1+g_A}{g_A}\Big)^\rho\Bigg)} = \frac{(1-\alpha_1-\alpha_2)-(1-\alpha_1)(1+g_A)L_z^*}{\alpha_2 g_A}. \tag{B.8.5}$$

Note that this term is independent of ρ. For the RHS of (B.8.1), we can apply Euler condition so that

$$\begin{aligned} 1+\frac{(1+g^*)g_A}{(1+R^*-\delta)-(1+g_A)(1+g^*)} &= 1+\frac{\beta\Big(1+(1-\tau)R^*-\delta\Big)g_A}{(1+R^*-\delta)-(1+g_A)\beta\Big(1+(1-\tau)R^*-\delta\Big)} \\ &= 1+\frac{g_A}{\frac{1+R^*-\delta}{\beta\Big(1+(1-\tau)R^*-\delta\Big)}-(1+g_A)}. \end{aligned} \tag{B.8.6}$$

By using Equations (B.8.5) and (B.8.6), Equation (B.8.1) becomes

$$\frac{(1-\alpha_1-\alpha_2)-(1-\alpha_1)(1+g_A)L_z^*}{\alpha_2 g_A} = 1+\frac{g_A}{\frac{1+R^*-\delta}{\beta\Big(1+(1-\tau)R^*-\delta\Big)}-(1+g_A)}. \tag{B.8.7}$$

Similarly, using $V[L_z^*]$, the RHS of Equation (B.8.4) becomes

$$\alpha_1 \tau^{\frac{1-\alpha_1}{\alpha_1}} D^{\frac{1-\alpha_1-\alpha_2}{\alpha_1}} \Big(\frac{(1-\varpi)(1-\alpha_1-\alpha_2)}{\alpha_2}\Big)^{\frac{1-\alpha_1-\alpha_2}{\rho\alpha_1}} \Big(\frac{1}{1+g_A} - L_z^*\Big)^{\frac{1-\alpha_1-\alpha_2+\rho\alpha_2}{\rho\alpha_1}} (L_z^*)^{\frac{1-\alpha_1-\alpha_2}{\alpha_1}(1-\frac{1}{\rho})}. \tag{B.8.8}$$

Hence, an alternative expression of R^* is

$$R^* = \alpha_1 \tau^{\frac{1-\alpha_1}{\alpha_1}} D^{\frac{1-\alpha_1-\alpha_2}{\alpha_1}} \Big(\frac{(1-\varpi)(1-\alpha_1-\alpha_2)}{\alpha_2}\Big)^{\frac{1-\alpha_1-\alpha_2}{\rho\alpha_1}} \Big(\frac{1}{1+g_A} - L_z^*\Big)^{\frac{1-\alpha_1-\alpha_2+\rho\alpha_2}{\rho\alpha_1}} (L_z^*)^{\frac{1-\alpha_1-\alpha_2}{\alpha_1}(1-\frac{1}{\rho})}. \tag{B.8.9}$$

We define the RHS of the above expression as $\Phi[L_z^*]$. Note that $\Phi[L_z^*]$ can be seen as an alternative expression of the long-run equilibrium rate of returns on capital.

To sum up, a BGP exists if there exists $L_z^* \in S$ that solves Equation (B.8.7) subject to

Equation (B.8.9). **Q.E.D.**

B.9 Proof of Lemma 3.5

This proof consists of three steps. In the first and second steps, we verify some properties of the terms on the LHS and RHS of the equation (3.65). Given the properties have been given from the previous steps, the sufficient conditions ensuring the existence and uniqueness of a BGP will be shown in the last step.

Step 1 : Properties of the LHS of Equation(3.65)

Consider the LHS term of Equation(3.68). Define

$$LHS[L_z] = \frac{(1-\alpha_1-\alpha_2)-(1-\alpha_1)(1+g_A)L_z}{\alpha_2 g_A} \tag{B.9.1}$$

as a function of L_z. Having shown that $L_z^* \in S$, it suffices to focus on $L_z \in [0, s_M]$. Obviously, $LHS[L_z]$ is a linear function. In addition, according to the parameter restrictions $\alpha_1, \alpha_2, \alpha_1+\alpha_2 \in (0,1)$ and $g_A > 0$, this function is linear with negative slope. The line crosses the vertical axis at the point $\left(0, \frac{1-\alpha_1-\alpha_2}{\alpha_2 g_A}\right)$ while it crosses the horizontal axis at the point $(s_M, 0)$.

Step 2 : Properties of the RHS of Equation(3.65)

Define $\xi \equiv \alpha_1 \tau^{\frac{1-\alpha_1}{\alpha_1}} D^{\frac{1-\alpha_1-\alpha_2}{\alpha_1}} \left(\frac{(1-\varpi)(1-\alpha_1-\alpha_2)}{\alpha_2}\right)^{\frac{1-\alpha_1-\alpha_2}{\rho\alpha_1}} > 0$. Then from $\Phi[L_z]$ we can see that the rate of returns on capital can be expressed as a function of L_z, i.e.,

$$\Phi[L_z] = \xi\Big(\frac{1}{1+g_A} - L_z\Big)^{\frac{1-\alpha_1-\alpha_2+\rho\alpha_2}{\rho\alpha_1}} (L_z)^{\frac{1-\alpha_1-\alpha_2}{\alpha_1}(1-\frac{1}{\rho})}. \tag{B.9.2}$$

Clearly, the rate of returns on capital is unbounded above but bounded below by zero for all $L_z \in [0, s_M]$.

After having restricted the domain on the closed interval $[0, s_M]$, some properties of $\Phi[L_z]$ can be verified. First of all, we specify the properties at boundaries. If $L_z = 0$ we have

$$\Phi[0] \begin{cases} = \infty & \text{if } \rho \in (0,1) \\ = 0 & \text{if } \rho \in (-\infty, 0) \end{cases} \tag{B.9.3}$$

Also, when $L_z = s_M$ we have $\Phi[s_M] > 0$ and finite. The interior properties is investigated by differentiating $\Phi[L_z]$ which yields

$$\Phi'[L_z] = \xi\Phi[L_z]\Bigg((L_z)^{-1}\frac{1-\alpha_1-\alpha_2}{\alpha_1}\Big(1-\frac{1}{\rho}\Big) - \frac{1-\alpha_1-\alpha_2+\rho\alpha_2}{\rho\alpha_1}\Big(\frac{1}{1+g_A} - L_z\Big)^{-1}\Bigg). \tag{B.9.4}$$

If $\rho \in (0,1)$, then $\Phi'[L_z] < 0$ for all $L_z \in (0, s_M)$. Combining with the two boundaries conditions, we can say that if $\rho \in (0,1)$, then $\Phi[L_z]$ is strictly decreasing on $[0, s_M]$. On the other hand, if $\rho \in (-\infty, 0)$, (B.11.4) implies $\Phi'[L_z] \gtrless 0$ if and only if

$$\frac{1-\alpha_1-\alpha_2}{\alpha_1}\left(1-\rho^{-1}\right)L_z^{-1} \gtrless \frac{1-\alpha_1-\alpha_2+\rho\alpha_2}{\rho\alpha_1}\left(\frac{1}{1+g_A}-L_z\right)^{-1}$$

$$\Leftrightarrow (1-\alpha_1-\alpha_2)\left(1-\rho^{-1}\right)\left(\frac{1}{1+g_A}-L_z\right) \gtrless \left[(1-\alpha_1-\alpha_2)\rho^{-1}+\alpha_2\right]L_z$$

$$\Leftrightarrow \left(1-\rho^{-1}\right)\frac{(1-\alpha_1-\alpha_2)}{(1-\alpha_1)(1+g_A)} \gtrless L_z.$$

Note that

$$\left(1-\frac{1}{\rho}\right)\frac{(1-\alpha_1-\alpha_2)}{(1-\alpha_1)(1+g_A)} = \left(1-\frac{1}{\rho}\right)s_M > s_M$$

for $\rho < 0$. Hence, for any $L_z \in [0, s_M]$, we have $\Phi'[L_z] > 0$.

Now, let us consider crucial properties of the function $RHS[\cdot]$. After rearranging and using the fact that $\Phi[L_z] = R$, $RHS[\cdot]$ can be expressed as

$$RHS[R] = \frac{(1-\delta)(1-\beta)+R[1-\beta(1-\tau)]}{(1-\delta)[1-\beta(1+g_A)]+R[1-\beta(1+g_A)(1-\tau)]}.$$

We are only interested in non-negative values of $RHS[\cdot]$ over the range $[0,\infty)$. Note that the numerator is always strictly positive for all $R \geq 0$. For the denominator, there are 3 possible cases:

Case 1:

$$0 < g_A < \frac{1}{\beta} - 1 < \frac{1}{\beta(1-\tau)} - 1.$$

In this case, $RHS[R] > 0$ for all $R \geq 0$ and we have

$$RHS[0] = \frac{1-\beta}{1-\beta(1+g_A)} > 0,$$

$$\lim_{R\to\infty} RHS[R] = \frac{1-\beta(1-\tau)}{1-\beta(1+g_A)(1-\tau)} > 0,$$

$$RHS'[R] = -\frac{\tau\beta g_A(1-\delta)}{\{(1+R^*-\delta)-\beta(1+g_A)[1+(1-\tau)R^*-\delta]\}^2} < 0.$$

Case 2:

$$\frac{1}{\beta} - 1 < g_A < \frac{1}{\beta(1-\tau)} - 1.$$

Then $RHS(R) > 0$ if and only if

$$R > \frac{(1-\delta)[\beta(1+g_A)-1]}{1-\beta(1+g_A)(1-\tau)}.$$

Case 3:

$$\frac{1}{\beta} - 1 < \frac{1}{\beta\,(1-\tau)} - 1 < g_A.$$

In this case, $RHS\,(R) < 0$ for all $R \geq 0$ so we should rule out this case.

We consider that Case 2 will create a lot of (unnecessary) complications which are largely technical in nature. Thus, we choose to avoid these by focusing on Case 1. Thus, in Lemma 5 we begin with the primitive that $\beta < \beta(1+g_A) < 1$ holds.

Step 3 : The Existence and Uniqueness of a BGP

Thus, if $\rho \in (0,1)$, then $\Phi\,(L_z)$ is a strictly decreasing function with $\Phi\,(0) = +\infty$ and $\Phi\,(s_M) > 0$. Hence, $RHS = \Theta\,[\Phi\,(L_z)]$ is strictly increasing over the range $(0, s_M)$ with

$$\Theta\,[\Phi\,(0)] = \Theta\,(\infty) = \frac{1-\beta\,(1-\tau)}{1-\beta\,(1+g_A)\,(1-\tau)} > 0,$$

$$\Theta\,[\Phi\,(s_M)] > 0.$$

Thus, a unique solution L_z^* exists if and only if

$$\frac{1-\alpha_1-\alpha_2}{\alpha_2}\frac{1}{g_A} > \frac{1-\beta\,(1-\tau)}{1-\beta\,(1+g_A)\,(1-\tau)}$$

$$\Leftrightarrow \frac{\left(\frac{1-\alpha_1-\alpha_2}{\alpha_2}\right)\left[\beta^{-1}\,(1-\tau)^{-1} - 1\right]}{\beta^{-1}\,(1-\tau)^{-1} - 1 + \left(\frac{1-\alpha_1-\alpha_2}{\alpha_2}\right)} > g_A.$$

Next, consider the case that $\rho < 0$. Combining with the results shown in Steps 1 and 2, we now know that if $g_A < \beta^{-1} - 1$ and $\rho < 0$, then $RHS\,[\Phi\,(L_z)]$ is a strictly decreasing function over the range $[0, s_M]$ with

$$RHS\,[\Phi\,(0)] = RHS\,(0) = \frac{1-\beta}{1-\beta\,(1+g_A)} > 0 \qquad \text{and} \qquad RHS\,[\Phi\,(s_M)] > 0.$$

Hence, (3.61) has at least one solution if

$$\frac{1-\alpha_1-\alpha_2}{\alpha_2}\frac{1}{g_A} > \frac{1-\beta}{1-\beta\,(1+g_A)}$$

$$\Leftrightarrow \frac{\left(\frac{1-\alpha_1-\alpha_2}{\alpha_2}\right)\left(\beta^{-1} - 1\right)}{\beta^{-1} - 1 + \left(\frac{1-\alpha_1-\alpha_2}{\alpha_2}\right)} > g_A. \tag{B.9.5}$$

Q.E.D.

Appendix C

Appendix to Chapter 4

C.1 Characterisation of the Dynamical System (4.16) - (4.21)

The intertemporal equilibrium is governed by a system of six conditions governing five variables including $l_{n,t}, z_{t+1}, R_t, \tilde{p}_t$ and $\tilde{c}_{n,t}$, provided that $z_0 > 0$ is given. These conditions are (i) labour mobility condition, (ii) intratemporal tradeoff condition, (iii) the Euler condition, (iv) the resource constraint, (v) the rate of return on capital and (vi) the transversality condition.

To get the labour mobility condition, we apply the definitions of z_t and $\tilde{p}_t$ to (4.3) so that

$$\tilde{p}_t = (1-\alpha_n)\Big(\frac{z_t}{l_{n,t}}\Big)^{\alpha_n}. \tag{C.1.1}$$

To get the intratemporal tradeoff condition, we manipulate (4.11) by using (4.6) and the definitions of $\tilde{c}_{n,t}$ and $\tilde{p}_t$ so that

$$\tilde{c}_{n,t} = \Big(\frac{1-\theta}{\theta}\Big)\tilde{p}_t\Big[1 - l_{n,t} - \frac{\overline{c}_{a,t}}{A_{a,t}}\Big]. \tag{C.1.2}$$

For the Euler condition, it is straightforward by applying the definitions of $\tilde{p}_t$ and $\tilde{c}_{n,t}$ to (4.12) so that

$$\beta(1+R_{t+1}-\delta) = \Big(\frac{\tilde{c}_{n,t+1}}{\tilde{c}_{n,t}}\Big)^{\sigma}(1+\tilde{\gamma}_{n,t+1})^{\sigma}\Big(\frac{\tilde{p}_{t+1}}{\tilde{p}_t}\Big)^{\theta(1-\sigma)}\Big(\frac{1+\tilde{\gamma}_{n,t+1}}{1+\tilde{\gamma}_{a,t+1}}\Big)^{\theta(1-\sigma)}. \tag{C.1.3}$$

For the resource constraint, we manipulate (4.7) so that

$$\frac{Y_{n,t}}{A_{n,t}N_t} = \frac{c_{n,t}}{A_{n,t}} + (1+n)(1+\tilde{\gamma}_{n,t+1})\frac{K_{t+1}}{A_{n,t+1}N_{t+1}} - (1-\delta)\frac{K_t}{A_{n,t}N_t}.$$

Use (4.2) and apply the the definitions of z_t and $\tilde{c}_{n,t}$ to the above expression so that

$$z_t^{\alpha_n} l_{n,t}^{1-\alpha_n} = \tilde{c}_{n,t} + (1+n)(1+\tilde{\gamma}_{n,t+1})z_{t+1} - (1-\delta)z_t. \tag{C.1.4}$$

For the rate of return on capital, we revise (4.4) by using the definition of z_t so that

$$R_t = \alpha_n \Big[\frac{z_t}{l_{n,t}}\Big]^{\alpha_n - 1}. \tag{C.1.5}$$

For the transversality condition, it is straightforward. **Q.E.D.**

C.2 Proof of Lemma 4.1

Along a BGP, the rate of return R_t must be constant. This implies that z_t and $l_{n,t}$ must grow at the same rate along such a path; see (4.20). Taking into account that $l_{n,t}$ is bounded by $(0, 1)$, the common growth rate of them must be zero. Using this fact with (4.16) and (4.19), we can also show that both $\tilde{c}_{n,t}$ and $\tilde{p}_t$ must be constant along a BGP as well. Since $l_{n,t}$, $\tilde{c}_{n,t}$ and $\tilde{p}_t$ are constant along a BGP, condition (4.17) implies that $\bar{c}_{a,t}$ and $A_{a,t}$ must grow at the same rate along such a path. Then, we finish the proof of (i) in the lemma.

For the proof of (ii) in Lemma 4.2, we observe that

$$\frac{\partial U}{\partial c_n} = (1-\theta)(c_a - \bar{c}_a)^{\theta - \theta\sigma} c_n^{-\sigma-\theta+\sigma\theta} = (1-\theta)(c_a - \bar{c}_a)^{\theta(1-\sigma)} c_n^{(1-\sigma)(1-\theta)-1}.$$

Thus, the growth factor of the above marginal utility is

$$(1+\tilde{\gamma}_a^*)^{\theta(1-\sigma)t}(1+\tilde{\gamma}_n^*)^{(1-\sigma)(1-\theta)t}(1+\tilde{\gamma}_n^*)^{-t}.$$

Then, we can use this growth factor with (4.21) to get the inequality (4.26). **Q.E.D.**

C.3 Proof of Proposition 4.1

A BGP is a time-invariant path associated with the system (4.16)-(4.20) and Lemma 4.1. To begin with, we solve for R^*. By using the Euler condition (4.18), we obtain

$$R^* = \frac{1}{\beta}(1+\tilde{\gamma}_n^*)^{\sigma}\Big(\frac{1+\tilde{\gamma}_n^*}{1+\tilde{\gamma}_a^*}\Big)^{\theta(1-\sigma)} - (1-\delta). \tag{C.3.1}$$

Secondly, we solve for $\tilde{p}^*$. By using (4.16) and (4.20), we obtain

$$\tilde{p}^* = (1-\alpha_n)\Big[\frac{R^*}{\alpha_n}\Big]^{\frac{\alpha_n}{\alpha_n - 1}}. \tag{C.3.2}$$

Thirdly, we solve z^* and $\tilde{c}_n^*$ in terms of l_n^*. By using (4.20), we obtain

$$z^* = \Big[\frac{R^*}{\alpha_n}\Big]^{\frac{1}{\alpha_n - 1}} l_n^*. \tag{C.3.3}$$

By using 4.17) and Lemma 4.1 (i), we obtain

$$\tilde{c}_n^* = \Big(\frac{1-\theta}{\theta}\Big)\tilde{p}^*\Big[1 - l_n^* - \mu\Big]. \tag{C.3.4}$$

Fourthly, we solve for l_n^*. Apply (C.3.2), (C.3.3) and (C.3.4) to (4.19) so that

$$\Big[\frac{R^*}{\alpha_n}\Big]^{\frac{\alpha_n}{\alpha_n-1}} l_n^* = \Big(\frac{1-\theta}{\theta}\Big)(1-\alpha_n)\Big[\frac{R^*}{\alpha_n}\Big]^{\frac{\alpha_n}{\alpha_n-1}}\Big[1 - l_n^* - \mu\Big] + \Gamma_1^*\Big[\frac{R^*}{\alpha_n}\Big]^{\frac{1}{\alpha_n-1}} l_n^*.$$

We, then, solve the above expression for l_n^* so that

$$l_n^* = \frac{\Big(\frac{1-\theta}{\theta}\Big)(1-\alpha_n)(1-\mu)}{\Big(\frac{1-\theta}{\theta}\Big)(1-\alpha_n) + 1 - \alpha_n\frac{\Gamma_1^*}{R^*}}. \tag{C.3.5}$$

Finally, we want to ensure feasibility. The inequality (4.26) guarantees that $R^* > \Gamma_1^* > 0$, this implies that R^* and $\tilde{p}^* > 0$. It suffices to guarantee the feasibility if we can guarantee that both $\tilde{c}_n^* > 0$ and $l_n^* \in (0,1)$ hold. To begin with, we can see that (4.26) ensures that $l_n^* < 1$ for any $\mu \geq 0$ because this condition guarantees that $\Big(1 - \alpha_n\frac{\Gamma_1^*}{R^*}\Big) > 0$. Next, from (C.3.4) we can show that

$$\tilde{c}_n^* > 0 \Leftrightarrow 1 - l_n^* > \mu.$$

Combined with (C.8.5), we have

$$\tilde{c}_n^* > 0 \Leftrightarrow \frac{1 - \alpha_n\frac{\Gamma_1^*}{R^*} + \mu(1-\alpha_n)\Big(\frac{1-\theta}{\theta}\Big)}{1 - \alpha_n\frac{\Gamma_1^*}{R^*} + (1-\alpha_n)\Big(\frac{1-\theta}{\theta}\Big)} > \mu.$$

Condition (4.26) ensures that $1 - \alpha_n\frac{\Gamma_1^*}{R^*} > 0$. Thus,

$$\tilde{c}_n^* > 0 \Leftrightarrow 1 > \mu. \tag{C.3.6}$$

Restricting $1 > \mu$ not only ensures $\tilde{c}_n^* > 0$, but also $l_n^* > 0$; see (C.3.5). Then, we reach the conclusion.

In sum, we can say that if Lemma 4.1 and $\mu \in (0,1)$ hold, then there exists a unique BGP:

$$(l_n^*, R^*, \tilde{c}_n^*, z^*, \tilde{p}^*) \in (0,1) \times \mathbb{R}_{++}^4$$

where these values are given by (C.3.1) - (C.3.5). **Q.E.D.**

C.4 Characterisation of the Dynamical System (4.41)-(4.47)

The contract curve: As we have mentioned in the main text, the contract curve refers to the equilibrium pairs $(s_{n,t}, L_{n,t})$ where the marginal rates of technical substitution are equalised across the two sectors, given $A_{a,t}, A_{n,t}$ and z_t. We know that the equilibrium rate of technical of substitution is the relative input price $\frac{w_t}{r_t+\delta}$. Accordingly, combining (4.39) and (4.40) yields:

$$\frac{w_t}{r_t+\delta} = \Big(\frac{1-\alpha_a}{1-\alpha_n}\Big)\frac{\alpha_n}{\alpha_a}\left[\frac{l_{a,t}}{s_{a,t}\Big(\frac{K_t}{A_{n,t}N_t}\Big)}\right]^{\psi_a-1}\Big(\frac{A_{a,t}}{A_{n,t}}\Big)^{\psi_a} = \left[\frac{l_{n,t}}{s_{n,t}\Big(\frac{K_t}{A_{n,t}N_t}\Big)}\right]^{\psi_n-1}.$$

Imposing the two input market clearing conditions (4.5) and (4.37) and then using the definition of z_t to the above expression yields the contract curve:

$$CC\Big(l_{n,t}, s_{n,t}; A_{a,t}, A_{n,t}, z_t\Big) \equiv \Big(\frac{1-\alpha_a}{1-\alpha_n}\Big)\frac{\alpha_n}{\alpha_a}\left[\frac{1-l_{n,t}}{(1-s_{n,t})z_t}\right]^{\psi_a-1}\Big(\frac{A_{a,t}}{A_{n,t}}\Big)^{\psi_a} = \left[\frac{l_{n,t}}{s_{n,t}z_t}\right]^{\psi_n-1} \tag{C.4.1}$$

, which is (4.41) in the main text.

The labour mobility condition: As we have mentioned in the main text, the labour mobility condition refers to the equilibrium pairs $(s_{n,t}, l_{n,t})$ where workers in both sectors will be paid at the same rate, given $A_{a,t}, A_{n,t}$ and z_t. Thus, by manipulating (4.39) using (4.38) and some algebras:

$$w_t = p_t(1-\alpha_a)\frac{Y_{a,t}^{1-\psi_a}}{(A_{a,t}l_{a,t}N_t)^{1-\psi_a}}A_{a,t} = \frac{Y_{n,t}^{1-\psi_n}}{(A_{n,t}l_{n,t}N_t)^{1-\psi_n}}(1-\alpha_n)A_{n,t}$$

$$\Leftrightarrow p_t(1-\alpha_a)\frac{Y_{a,t}^{1-\psi_a}}{(A_{a,t}l_{a,t}N_t)^{1-\psi_a}}A_{a,t} = \frac{Y_{n,t}^{1-\psi_n}}{(A_{n,t}l_{n,t}N_t)^{1-\psi_n}}(1-\alpha_n)A_{n,t}$$

$$\Leftrightarrow p_t\frac{A_{a,t}}{A_{n,t}}\Big(\frac{1-\alpha_a}{1-\alpha_n}\Big)\frac{\left[\alpha_a\Big(s_{a,t}K_t\Big)^{\psi_a} + (1-\alpha_a)\Big(A_{a,t}l_{a,t}N_t\Big)^{\psi_a}\right]^{\frac{1-\psi_a}{\psi_a}}}{(A_{a,t}l_{a,t}N_t)^{1-\psi_a}} = \frac{Y_{n,t}^{1-\psi_n}}{(A_{n,t}l_{n,t}N_t)^{1-\psi_n}}$$

$$\Leftrightarrow p_t\frac{A_{a,t}}{A_{n,t}}\Big(\frac{1-\alpha_a}{1-\alpha_n}\Big)\left[\alpha_a\Big(\frac{s_{a,t}}{l_{a,t}}\frac{K_t}{A_{n,t}N_t}\Big)^{\psi_a}\Big(\frac{A_{a,t}}{A_{n,t}}\Big)^{-\psi_a} + (1-\alpha_a)\right]^{\frac{1-\psi_a}{\psi_a}} = \frac{Y_{n,t}^{1-\psi_n}}{(A_{n,t}l_{n,t}N_t)^{1-\psi_n}}$$

$$\Leftrightarrow \frac{\left[\alpha_a\Big(\frac{s_{a,t}}{l_{a,t}}\frac{K_t}{A_{n,t}N_t}\Big)^{\psi_a}\Big(\frac{A_{a,t}}{A_{n,t}}\Big)^{-\psi_a} + (1-\alpha_a)\right]^{\frac{1-\psi_a}{\psi_a}}}{\left[p_t\frac{A_{a,t}}{A_{n,t}}\Big(\frac{1-\alpha_a}{1-\alpha_n}\Big)\right]^{-1}} = \frac{\left[\alpha_n\Big(s_{n,t}K_t\Big)^{\psi_n} + (1-\alpha_n)\Big(A_{n,t}l_{n,t}N_t\Big)^{\psi_n}\right]^{\frac{1-\psi_n}{\psi_n}}}{(A_{n,t}l_{n,t}N_t)^{1-\psi_n}}$$

$$\Leftrightarrow \frac{\left[\alpha_a\left(\frac{s_{a,t}}{l_{a,t}}\frac{K_t}{A_{n,t}N_t}\right)^{\psi_a}\left(\frac{A_{a,t}}{A_{n,t}}\right)^{-\psi_a}+(1-\alpha_a)\right]^{\frac{1-\psi_a}{\psi_a}}}{\left[p_t\frac{A_{a,t}}{A_{n,t}}\left(\frac{1-\alpha_a}{1-\alpha_n}\right)\right]^{-1}}=\left[\alpha_n\left(\frac{s_{n,t}}{l_{n,t}}\frac{K_t}{A_{n,t}N_t}\right)^{\psi_n}+(1-\alpha_n)\right]^{\frac{1-\psi_n}{\psi_n}}$$

which eventually leads to

$$\frac{\left[\alpha_a\left(\frac{s_{a,t}}{l_{a,t}}\frac{K_t}{A_{n,t}N_t}\right)^{\psi_a}\left(\frac{A_{a,t}}{A_{n,t}}\right)^{-\psi_a}+(1-\alpha_a)\right]^{\frac{1-\psi_a}{\psi_a}}}{\left[p_t\left(\frac{A_{a,t}}{A_{n,t}}\right)^{1-\alpha_a+\alpha_a}\left(\frac{1-\alpha_a}{1-\alpha_n}\right)\right]^{-1}}=\left[\alpha_n\left(\frac{s_{n,t}}{l_{n,t}}\frac{K_t}{A_{n,t}N_t}\right)^{\psi_n}+(1-\alpha_n)\right]^{\frac{1-\psi_n}{\psi_n}}.$$

Applying the definitions of $\tilde{p}_t$ and z_t and input market clearing conditions stated in (4.5) and (4.37) to the above expression yields

$$\frac{\left[\alpha_a\left(\frac{1-s_{n,t}}{1-l_{n,t}}z_t\right)^{\psi_a}\left(\frac{A_{a,t}}{A_{n,t}}\right)^{-\psi_a}+(1-\alpha_a)\right]^{\frac{1-\psi_a}{\psi_a}}}{\left[\tilde{p}_t\left(\frac{A_{a,t}}{A_{n,t}}\right)^{\alpha_a}\left(\frac{1-\alpha_a}{1-\alpha_n}\right)\right]^{-1}}=\left[\alpha_n\left(\frac{s_{n,t}}{l_{n,t}}z_t\right)^{\psi_n}+(1-\alpha_n)\right]^{\frac{1-\psi_n}{\psi_n}}. \quad \text{(C.4.2)}$$

From (4.38) and (4.40), the rate of return on capital is

$$\begin{aligned}
R_t &= \alpha_n\frac{Y_{n,t}^{1-\psi_n}}{(s_{n,t}K_t)^{1-\psi_n}} \\
&= \alpha_n\frac{\left[\alpha_n\Big(s_{n,t}K_t\Big)^{\psi_n}+(1-\alpha_n)\Big(A_{n,t}l_{n,t}N_t\Big)^{\psi_n}\right]^{\frac{1-\psi_n}{\psi_n}}}{(s_{n,t}K_t)^{1-\psi_n}} \\
&= \alpha_n\left[\alpha_n+(1-\alpha_n)\left(\frac{s_{n,t}K_t}{A_{n,t}l_{n,t}N_t}\right)^{-\psi_n}\right]^{\frac{1-\psi_n}{\psi_n}}.
\end{aligned}$$

Apply the definition of z_t to the above expression. Then,

$$R_t=\alpha_n\left[\alpha_n+(1-\alpha_n)\left(\frac{s_{n,t}}{l_{n,t}}z_t\right)^{-\psi_n}\right]^{\frac{1-\psi_n}{\psi_n}}. \quad \text{(C.4.3)}$$

We then combine (C.4.2) and (C.4.3) to get $LM\Big(l_{n,t},s_{n,t};A_{a,t},A_{n,t},z_t\Big)\equiv$:

$$\left(\frac{1-\alpha_a}{1-\alpha_n}\right)\tilde{p}_t\left(\frac{A_{a,t}}{A_{n,t}}\right)^{\alpha_a}\left[\alpha_a\left(\frac{A_{a,t}}{A_{n,t}}\right)^{-\psi_a}\left(\frac{1-s_{n,t}}{1-l_{n,t}}z_t\right)^{\psi_a}+(1-\alpha_a)\right]^{\frac{1-\psi_a}{\psi_a}}=\left(\frac{s_{n,t}z_t}{l_{n,t}}\right)^{1-\psi_n}\frac{R_t}{\alpha_n} \quad \text{(C.4.4)}$$

, which is (4.42) in the main text.

The intratemporal tradeoff condition: We revise the intratemporal tradeoff condition (4.11) as follows. To begin with, we manipulate (4.11) using the definitions of $\tilde{p}_t$ and $\tilde{c}_{n,t}$ to get

$$\tilde{p}_t\Big(\frac{A_{a,t}}{A_{n,t}}\Big)^{\alpha_a-1}\Big(\frac{c_{a,t}}{A_{n,t}}-\frac{\overline{c}_{a,t}}{A_{n,t}}\Big)=\frac{\theta}{1-\theta}\tilde{c}_{n,t}.$$

Apply (4.6) and (4.38) to the above expression:

$$\tilde{p}_t\Big(\frac{A_{a,t}}{A_{n,t}}\Big)^{\alpha_a-1}\left(\frac{\Big[\alpha_a\Big(s_{a,t}K_t\Big)^{\psi_a}+(1-\alpha_a)\Big(A_{a,t}l_{a,t}N_t\Big)^{\psi_a}\Big]^{\frac{1}{\psi_a}}}{N_tA_{n,t}}-\frac{\overline{c}_{a,t}}{A_{n,t}}\right)=\frac{\theta}{1-\theta}\tilde{c}_{n,t}$$

$$\Leftrightarrow \tilde{p}_t\Big(\frac{A_{a,t}}{A_{n,t}}\Big)^{\alpha_a-1}\left(l_{a,t}\Big(\frac{A_{a,t}}{A_{n,t}}\Big)\Big[\alpha_a\Big(\frac{s_{a,t}}{l_{a,t}}\frac{K_t}{A_{n,t}N_t}\Big)^{\psi_a}\Big(\frac{A_{a,t}}{A_{n,t}}\Big)^{-\psi_a}+(1-\alpha_a)\Big]^{\frac{1}{\psi_a}}-\frac{\overline{c}_{a,t}}{A_{n,t}}\right)=\frac{\theta}{1-\theta}\tilde{c}_{n,t}.$$

Apply (4.5), (4.37) and the definition of z_t, we get the intratemporal tradeoff condition

$$\tilde{p}_t\Big(\frac{A_{a,t}}{A_{n,t}}\Big)^{\alpha_a-1}\left((1-l_{n,t})\Big(\frac{A_{a,t}}{A_{n,t}}\Big)\left[\alpha_a\Big(\frac{1-s_{n,t}}{1-l_{n,t}}z_t\Big)^{\psi_a}\Big(\frac{A_{a,t}}{A_{n,t}}\Big)^{-\psi_a}+(1-\alpha_a)\right]^{\frac{1}{\psi_a}}-\frac{\overline{c}_{a,t}}{A_{n,t}}\right)=\Big(\frac{\theta}{1-\theta}\Big)\tilde{c}_{n,t} \tag{C.4.5}$$

, which is (4.43).

The Euler condition: We revise the Euler condition (4.12) by using the definitions of $\tilde{p}_t$ and $\tilde{c}_{n,t}$ and $R_t = r_t + \delta$. Then, it is straightforward to get (4.44).

The resource constraint: We revise the economy wide resource constraint as follows. Premultiply both sides of (4.7) by $\frac{1}{A_{n,t}N_t}$. Then,

$$\begin{aligned}\frac{Y_{n,t}}{A_{n,t}N_t}&=\frac{N_tc_{n,t}}{A_{n,t}N_t}+\frac{K_{t+1}}{A_{n,t}N_t}-(1-\delta)\frac{K_t}{A_{n,t}N_t}\\&=\frac{c_{n,t}}{A_{n,t}}+\frac{K_{t+1}}{1}\frac{1}{A_{n,t}N_t}-(1-\delta)\frac{K_t}{A_{n,t}N_t}\\&=\frac{c_{n,t}}{A_{n,t}}+\frac{K_{t+1}}{A_{n,t+1}N_{t+1}}\frac{A_{n,t+1}N_{t+1}}{A_{n,t}N_t}-(1-\delta)\frac{K_t}{A_{n,t}N_t}.\end{aligned}$$

This leads to

$$\frac{Y_{n,t}}{A_{n,t}N_t}=\frac{c_{n,t}}{A_{n,t}}+(1+n)(1+\tilde{\gamma}_{n,t+1})\frac{K_{t+1}}{A_{n,t+1}N_{t+1}}-(1-\delta)\frac{K_t}{A_{n,t}N_t}. \tag{C.4.6}$$

By using (4.38), we can show that

$$\frac{Y_{n,t}}{A_{n,t}N_t}=l_{n,t}\left[\alpha_n\Big(\frac{s_{n,t}}{l_{n,t}}\frac{K_t}{A_{n,t}N_t}\Big)^{\psi_n}+(1-\alpha_n)\right]^{\frac{1}{\psi_n}}. \tag{C.4.7}$$

By combining (C.4.6) and (C.4.7), we can get (4.45) by using the definitions of $\tilde{c}_{n,t}$ and z_t.

The rate of return on capital: We have verified already; see (C.4.3).
The TVC: Since $\lambda_t = \frac{\partial u(\cdot)}{\partial c_{n,t}}$ and by asset market clearing $a_{t+1} = \frac{K_{t+1}}{N_{t+1}}$, then

$$\lim_{t\to\infty} \beta^t \frac{\partial u(\cdot)}{\partial c_{n,t}} A_{n,t+1} N_{t+1} \frac{K_{t+1}}{A_{n,t+1}N_{t+1}} = 0. \tag{C.4.8}$$

By using the definitions of z_t, we get (4.46). We finish the proof. **Q.E.D.**

C.5 Proof of Lemma 4.2

Based on Definition 4.2, the constancy of R_t along a BGP requires that $l_{t,n}$ and $s_{t,n}z_t$ must grow at a common rate; see (4.46). Taking into account that $l_{t,n}, s_{t,n} \in (0,1)$, these two variables cannot grow (or decay) along a BGP and thus z_t must be time-invariant along a BGP as well. Formally, along a BGP,

$$l_{n,t+1} = l_{n,t} = l_n^* \in (0,1), s_{n,t+1} = s_{n,t} = s_n^* \in (0,1) \text{ and } z_{t+1} = z_t = z^* > 0. \tag{C.5.1}$$

Since (C.5.1) must hold along a BGP, the conditions (4.41) and (4.42) implies that either the growth rates of labour augmenting technologies in both sectors are equal (unbiased technological progress) or the elasticity of substitution between capital and labour is unity along a BGP. In our context, the former requirement seems to be too restricted as climate change would affect the growth rates of $A_{a,t}$ and $A_{n,t}$ in different degrees. In the long-run, it is very unlikely that the two growth rate will be identical. In stead, we assume the latter.

If we assume that $\psi_a \to 0$ which implies $Y_{a,t} = (s_{a,t}K_t)^{\alpha_a}(A_{a,t}l_{a,t}N_t)^{1-\alpha_a}$. Then, the conditions (4.41)-(4.43) becomes, respectively,

$$\Big(\frac{1-\alpha_a}{\alpha_a}\Big)\Big(\frac{\alpha_n}{1-\alpha_a}\Big)\Big(\frac{1-s_{n,t}}{1-l_{n,t}}z_t\Big) = \Big(\frac{s_{n,t}z_t}{l_{n,t}}\Big)^{1-\psi_n}, \tag{C.5.2}$$

$$\Big(\frac{1-\alpha_a}{1-\alpha_n}\Big)\tilde{p}_t\Big(\frac{1-s_{n,t}}{1-l_{n,t}}z_t\Big)^{\alpha_a} = \Big(\frac{s_{n,t}z_t}{l_{n,t}}\Big)^{1-\psi_n}\frac{R_t}{\alpha_n}, \tag{C.5.3}$$

$$\tilde{p}_t\left((1-l_{n,t})\Big(\frac{1-s_{n,t}}{1-l_{n,t}}z_t\Big)^{\alpha_a} - \frac{\bar{c}_{a,t}}{A_{a,t}^{1-\alpha_a}A_{n,t}^{\alpha_a}}\right) = \Big(\frac{\theta}{1-\theta}\Big)\tilde{c}_{n,t}. \tag{C.5.4}$$

Q.E.D.

C.6 Proof of Lemma 4.3

We omit the proof as this lemma is a more generalised version of Lemma 4.1. **Q.E.D.**

C.7 Proof of Proposition 4.2

The proof is divided into a number of steps:

Step 1 Evaluate R^*. According to (4.44), the rate of return on capital along a BGP is

$$R^* = \frac{\kappa}{\beta} - (1-\delta) \tag{C.7.1}$$

where $\kappa \equiv (1+\tilde{\gamma}_n^*)^{\sigma}\left(\frac{1+\tilde{\gamma}_n^*}{1+\tilde{\gamma}_a^*}\right)^{(1-\alpha_a)\theta(1-\sigma)}$. This is the condition (4.49) stated in the main text.

Step 2 Ensure $R^* > 0$. The inequality (4.48) implies that

$$(1+n)(1+\tilde{\gamma}_n^*) < \frac{\kappa}{\beta}. \tag{C.7.2}$$

From (C.7.1) and (C.7.2), we have

$$R^* > \Gamma_1^* > 0. \tag{C.7.3}$$

Note that $R^* > \Gamma_1^*$ holds due to (4.48) while $\Gamma_1^* > 0$ holds because of the parameter values of n, γ_n^* and δ.

Step 3 Evaluate $\tilde{p}^*$. From (C.4.3) we get

$$\left(\frac{R^*}{\alpha_n}\right) = \left[\alpha_n + (1-\alpha_n)\left(\frac{s_n^*}{l_n^*}z^*\right)^{-\psi_n}\right]^{\frac{1-\psi_n}{\psi_n}} \tag{C.7.4}$$

and

$$\left(\frac{s_n^*}{l_n^*}z^*\right) = \left(\frac{1-\alpha_n}{\alpha_n}\right)^{\frac{1}{\psi_n}}\left[(\alpha_n)^{\frac{-1}{1-\psi_n}}(R^*)^{\frac{\psi}{1-\psi_n}} - 1\right]^{\frac{-1}{\psi_n}} = \Lambda_1^*. \tag{C.7.5}$$

Apply (C.7.5) to (C.5.2) so that

$$\left(\frac{1-s_n^*}{1-l_n^*}z^*\right) = (\Lambda_1^*)^{1-\psi_n}\left(\frac{\alpha_a}{1-\alpha_a}\cdot\frac{1-\alpha_n}{\alpha_n}\right). \tag{C.7.6}$$

Then, we can apply (C.7.5) and (C.7.6) to (the stationary expression of) the condition (C.5.3) to get

$$\tilde{p}^* = \left(\frac{1-\alpha_n}{1-\alpha_a}\right)(\Lambda_1^*)^{(1-\alpha_a)(1-\psi_n)}\left(\frac{\alpha_a}{1-\alpha_a}\cdot\frac{1-\alpha_n}{\alpha_n}\right)^{-\alpha_a}\left(\frac{R^*}{\alpha_n}\right), \tag{C.7.7}$$

, which is the condition (4.50) in the main text.

Step 4 Ensure $\tilde{p}^* > 0$. From (C.7.7), $\tilde{p}^* > 0$ for all ψ_n when $\Lambda_1^* > 0$. In particular, this equation implies that $\Lambda_1^* > 0$ if and only if

$$(R^*)^{\psi_n} > \alpha_n. \tag{C.7.8}$$

In this study we assume that this inequality holds.

Step 5 Show that if $(R^*, \tilde{p}^*, \Lambda_1^*) \gg 0$, then there exists a unique $\Lambda_2^* > 0$. After rearranging (C.7.7), we obtain

$$\Lambda_2^* \equiv \Big(\frac{\tilde{p}^*\alpha_a}{R^*}\Big)^{\frac{1}{1-\alpha_a}} = \Big(\frac{\alpha_a}{1-\alpha_a} \cdot \frac{1-\alpha_n}{\alpha_n}\Big)(\Lambda_1^*)^{(1-\psi_n)} \tag{C.7.9}$$

Then it is straightforward. Note that (C.7.6) and (C.7.9) imply

$$\Lambda_2^* \equiv \Big(\frac{\tilde{p}^*\alpha_a}{R^*}\Big)^{\frac{1}{1-\alpha_a}} = \Big(\frac{\alpha_a}{1-\alpha_a} \cdot \frac{1-\alpha_n}{\alpha_n}\Big)(\Lambda_1^*)^{(1-\psi_n)} = \Big(\frac{1-s_n^*}{1-l_n^*} z^*\Big). \tag{C.7.10}$$

Step 6 Evaluate $(s_n^*, z^*, \tilde{c}_n^*)$ in terms of l_n^*. For s_n^*, we can use (C.7.7) and (C.7.10) so that

$$s_n^* = \frac{\Lambda_1^* l_n^*}{\Lambda_1^* l_n^* + \Lambda_2^*(1-l_n^*)}. \tag{C.7.11}$$

For z^*, we can use (C.7.5) and (C.7.11) so that

$$z^* = \Lambda_1^* l_n^* + \Lambda_2^*(1-l_n^*). \tag{C.7.12}$$

For $\tilde{c}_n^*$, we can use (C.5.4), (C.7.10) and Part (i) in Lemma 4.3 so that

$$\tilde{c}_n^* = \Big(\frac{1-\theta}{\theta}\Big)\tilde{p}^*\Bigg[(1-l_n^*)(\Lambda_2^*)^{\alpha_a} - \mu\Bigg]. \tag{C.7.13}$$

These are (4.54), (4.55) and (4.56), respectively.

Step 7 Evaluate l_n^*. We use (C.7.13) to eliminate $\tilde{c}_n^*$ from (4.45) so that

$$l_n^*(\Lambda_1^*)\Bigg(\frac{R^*}{\alpha_n}\Bigg)^{\frac{1}{1-\psi_n}} = \Big(\frac{1-\theta}{\theta}\Big)\tilde{p}^*\Bigg[(1-l_n^*)(\Lambda_2^*)^{\alpha_a} - \mu\Bigg] + \Gamma_1^* z^*.$$

Apply (C.7.4), (C.7.5) and (C.7.12) to the above expression so that

$$l_n^*(\Lambda_1^*)\Big[\alpha_n + (1-\alpha_n)(\Lambda_1^*)^{-\psi_n}\Big]^{\frac{1}{\psi_n}} = \Big(\frac{1-\theta}{\theta}\Big)\tilde{p}^*\Big[(1-l_n^*)(\Lambda_2^*)^{\alpha_a} - \mu\Big] + \Gamma_1^*(\Lambda_1^* - \Lambda_2^*)l_n^* + \Gamma_1^*\Lambda_2^*.$$

We solve the above expression for l_n^* so that

$$l_n^* = \frac{(\frac{1-\theta}{\theta})\tilde{p}^*(\Lambda_2^*)^{\alpha_a} + \Gamma_1^*\Lambda_2^* - \mu(\frac{1-\theta}{\theta})\tilde{p}^*}{(\frac{1-\theta}{\theta})\tilde{p}^*(\Lambda_2^*)^{\alpha_a} + \Gamma_1^*\Lambda_2^* + (\Lambda_1^*)\Big[\alpha_n + (1-\alpha_n)(\Lambda_1^*)^{-\psi_n}\Big]^{\frac{1}{\psi_n}} - \Gamma_1^*\Lambda_1^*}. \tag{C.7.14}$$

It would be useful to show; by using (C.7.5), that

$$\begin{aligned}
(\Lambda_1^*)\Big[\alpha_n + (1-\alpha_n)(\Lambda_1^*)^{-\psi_n}\Big]^{\frac{1}{\psi_n}} &= \Big[\alpha_n(\Lambda_1^*)^{\psi_n} + (1-\alpha_n)\Big]^{\frac{1}{\psi_n}} \\
&= \Big[\alpha_n\Big(\frac{1-\alpha_n}{\alpha_n}\Big)^{\frac{\psi_n}{\psi_n}}(\Upsilon^*)^{\frac{-\psi_n}{\psi_n}} + (1-\alpha_n)\Big]^{\frac{1}{\psi_n}} \\
&= \Big[\frac{(1-\alpha_n)}{\Upsilon^*} + (1-\alpha_n)\frac{\Upsilon^*}{\Upsilon^*}\Big]^{\frac{1}{\psi_n}} \\
&= (\Upsilon^*)^{\frac{-1}{\psi_n}}\Big[(1-\alpha_n)(\Upsilon^*-1)\Big]^{\frac{1}{\psi_n}} \equiv \Gamma_2^*
\end{aligned}$$

where $\Upsilon^* \equiv (\alpha_n)^{-\frac{1}{1-\psi_n}}(R^*)^{\frac{\psi_n}{1-\psi_n}} - 1$. In sum, the stationary labour share devoted to non-agricultural sector corresponding to a BGP is

$$l_n^* = \frac{(\frac{1-\theta}{\theta})\tilde{p}^*(\Lambda_2^*)^{\alpha_a} + \Gamma_1^*\Lambda_2^* - \mu(\frac{1-\theta}{\theta})\tilde{p}^*}{(\frac{1-\theta}{\theta})\tilde{p}^*(\Lambda_2^*)^{\alpha_a} + \Gamma_1^*\Lambda_2^* - (\Gamma_1^*\Lambda_1^* - \Gamma_2^*)} \tag{C.7.15}$$

which is (4.53) in the main text.

Step 8 Ensure feasibility. We finish the proof by showing parameter restrictions ensuring the feasibility of the solution. Regarding this matter, we want to ensure that $(l_n^*, s_n^*, \tilde{c}_n^*, z^*) \in (0,1)^2 \times \mathbb{R}_{++}^2$. However, since (C.7.8) holds; so $\Lambda_1^*, \Lambda_2^* > 0$, it suffices to guarantee the feasibility by ensuring that $l_n^* \in (0,1)$ and $\tilde{c}_n^* > 0$.

To begin with, we claim that

$$(R^*)^{\psi_n} > \alpha_n \Rightarrow l_n^* < 1. \tag{C.7.16}$$

To illustrate this, we begin by showing an alternative expression of Γ_2^* such that

$$\Gamma_2^* = \Lambda_1^*\left(\frac{R^*}{\alpha_n}\right)^{\frac{1}{1-\psi_n}}.$$

This implies that

$$\Gamma_2^* - \Lambda_1^*\Gamma_1^* = \Lambda_1^*\left[\left(\frac{R^*}{\alpha_n}\right)^{\frac{1}{1-\psi_n}} - \Gamma_1^*\right]. \tag{C.7.17}$$

Under the assumption that $(R^*)^{\psi_n} > \alpha_n$, we have $\left(\frac{R^*}{\alpha_n}\right)^{\frac{1}{1-\psi_n}} > R^*$. Since $R^* > \Gamma_1^*$; see (C.7.3), then (C.7.17) is strictly positive and thus $l_n^* < 1$.

Next, we claim that

$$\tilde{c}_n^* > 0 \Rightarrow l_n^* > 0. \tag{C.7.18}$$

From (C.7.13), we can see that $\tilde{c}_n^* > 0$ if and only if

$$(1 - l_n^*)(\Lambda_2^*)^{\alpha_a} - \mu > 0.$$

By using (C.7.15), we can show that

$$1 - l_n^* = \frac{\Gamma_2^* - \Gamma_1^*\Lambda_1^* + \mu(\frac{1-\theta}{\theta})\tilde{p}^*}{(\frac{1-\theta}{\theta})\tilde{p}^*(\Lambda_2^*)^{\alpha_a} + \Gamma_1^*\Lambda_2^* + \Gamma_2^* - \Gamma_1^*\Lambda_1^*},$$

$$(1 - l_n^*)(\Lambda_2^*)^{\alpha_a} = \frac{\Gamma_2^*(\Lambda_2^*)^{\alpha_a} - \Gamma_1^*\Lambda_1^*(\Lambda_2^*)^{\alpha_a} + \mu(\frac{1-\theta}{\theta})\tilde{p}^*(\Lambda_2^*)^{\alpha_a}}{(\frac{1-\theta}{\theta})\tilde{p}^*(\Lambda_2^*)^{\alpha_a} + \Gamma_1^*\Lambda_2^* + \Gamma_2^* - \Gamma_1^*\Lambda_1^*}$$

and

$$(1 - l_n^*)(\Lambda_2^*)^{\alpha_a} - \mu = \frac{\Big[(\Lambda_2^*)^{\alpha_a} - \mu\Big]\Big[\Gamma_2^* - \Lambda_1^*\Gamma_1^*\Big] - \Lambda_2^*\Gamma_1^*\mu}{(\frac{1-\theta}{\theta})\tilde{p}^*(\Lambda_2^*)^{\alpha_a} + \Gamma_1^*\Lambda_2^* + \Gamma_2^* - \Gamma_1^*\Lambda_1^*}.$$

This means

$$\tilde{c}_n^* > 0 \Longleftrightarrow \Big[(\Lambda_2^*)^{\alpha_a} - \mu\Big]\Big[\Gamma_2^* - \Lambda_1^*\Gamma_1^*\Big] > \Lambda_2^*\Gamma_1^*\mu > 0 \tag{C.7.19}$$

Since $\Big[\Gamma_2^* - \Lambda_1^*\Gamma_1^*\Big] > 0$, the above inequalities will hold *only if* $\Big[(\Lambda_2^*)^{\alpha_a} - \mu\Big] > 0$ which turns out that $l_n^* > 0$ is necessarily true.

In sum, assuming Lemma 4.3 holds, inequalities (C.7.8) and (C.7.19) ensure that there exists a unique BGP. **Q.E.D.**

C.8 Further Characterisations of Employment Share Effect

From now, we go further by investigating the expressions of (4.58). This exposition will give us some more accurate conclusions about the climate impact on the long-run employment share.

The sign of $\frac{\partial \tilde{p}^*}{\partial T^*}$: From Proposition 4.2, the detrended relative price can be expressed as

$$\tilde{p}^* = (\hat{\alpha})^{1-\alpha_a}(\Lambda_1^*)^{(1-\alpha_a)(1-\psi_n)}\frac{R^*}{\alpha_a}.$$

The feasibility condition (4.51); which implies $\Lambda_1^* > 0$, ensures that $\tilde{p}^* > 0$. This parameter restriction implies that the long-run rate of return on capital (4.49) must be bounded below by a certain value which is strictly positive but less than one when the two inputs are gross substitute ($\psi_n > 0$) and bounded above by a certain value which is strictly greater than one when the two inputs are gross compliment ($\psi_n < 0$):

$$\tilde{p}^* > 0 \Rightarrow R^* \in \begin{cases} (\alpha_n^{\frac{1}{\psi_n}}, +\infty) & ; \psi_n > 0 \\ (0, \alpha_n^{-\frac{1}{|\psi_n|}}) & ; \psi_n < 0 \end{cases} \tag{C.8.1}$$

where $\hat{\alpha} \equiv \frac{\alpha_a}{1-\alpha_a} \cdot \frac{1-\alpha_n}{\alpha_n}$. Note that when $\psi_n \to 0$, (4.51) is always satisfied. Define $\Upsilon^* \equiv (\alpha_n)^{\frac{-1}{1-\psi_n}} (R^*)^{\frac{\psi_n}{1-\psi_n}} - 1 > 0$; due to (4.51). Then differentiating $\tilde{p}^*$ with respect to T^* yields

$$\frac{\partial \tilde{p}^*}{\partial T^*} = \frac{(\hat{\alpha})^{1-\alpha_a}}{\alpha_a} \cdot \Big(\frac{1-\alpha_n}{\alpha_n}\Big)^{\frac{(1-\alpha_a)(1-\psi_n)}{\psi_n}} \cdot (\Upsilon^*)^{-\frac{(1-\alpha_a)(1-\psi_n)}{\psi_n}} \cdot \Big[\alpha_a(\Upsilon^* + 1) - 1\Big] \frac{\partial R^*}{\partial T^*} \tag{C.8.2}$$

Clearly, the sign of $\frac{\partial \tilde{p}^*}{\partial T^*}$ is as the same as that of

$$\Big[\alpha_a(\Upsilon^* + 1) - 1\Big] \frac{\partial R^*}{\partial T^*} = \Big[\alpha_a \alpha_n^{-\frac{1}{1-\psi_n}} (R^*)^{\frac{\psi_n}{1-\psi_n}} - 1\Big] \frac{\partial R^*}{\partial T^*}.$$

Depending on the signs of $\frac{\partial R^*}{\partial T^*}$ and ψ_n, we can state that

$$\text{case i } \frac{\partial R^*}{\partial T^*} < 0 \text{ and } \psi_n < 0 : \frac{\partial \tilde{p}^*}{\partial T^*} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \Leftrightarrow R^* \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \alpha_a^{\frac{1+|\psi_n|}{|\psi_n|}} \Big(\frac{1}{\alpha_n}\Big)^{\frac{1}{|\psi_n|}}, \tag{C.8.3}$$

$$\text{case ii } \frac{\partial R^*}{\partial T^*} < 0 \text{ and } \psi_n \to 0 : \qquad \frac{\partial \tilde{p}^*}{\partial T^*} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \Leftrightarrow \alpha_a \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \alpha_n, \tag{C.8.4}$$

$$\text{case iii } \frac{\partial R^*}{\partial T^*} < 0 \text{ and } \psi_n > 0 : \frac{\partial \tilde{p}^*}{\partial T^*} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \Leftrightarrow R^* \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \alpha_n^{\frac{1}{\psi_n}} \Big(\frac{1}{\alpha_a}\Big)^{\frac{1-\psi_n}{\psi_n}} \tag{C.8.5}$$

$$\text{case iv } \frac{\partial R^*}{\partial T^*} = 0 : \qquad \frac{\partial \tilde{p}^*}{\partial T^*} = 0, \forall \psi_n \in (-\infty, 1), \tag{C.8.6}$$

$$\text{case v } \frac{\partial R^*}{\partial T^*} > 0 \text{ and } \psi_n < 0 : \frac{\partial \tilde{p}^*}{\partial T^*} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \Leftrightarrow R^* \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} \alpha_a^{\frac{1+|\psi_n|}{|\psi_n|}} \Big(\frac{1}{\alpha_n}\Big)^{\frac{1}{|\psi_n|}}, \tag{C.8.7}$$

$$\text{case vi } \frac{\partial R^*}{\partial T^*} > 0 \text{ and } \psi_n \to 0 : \qquad \frac{\partial \tilde{p}^*}{\partial T^*} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \Leftrightarrow \alpha_a \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \alpha_n, \tag{C.8.8}$$

$$\text{case vii } \frac{\partial R^*}{\partial T^*} > 0 \text{ and } \psi_n > 0 : \frac{\partial \tilde{p}^*}{\partial T^*} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \Leftrightarrow R^* \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \alpha_n^{\frac{1}{\psi_n}} \Big(\frac{1}{\alpha_a}\Big)^{\frac{1-\psi_n}{\psi_n}}. \tag{C.8.9}$$

In sum, the sign of $\frac{\partial \tilde{p}^*}{\partial T^*}$ is inconclusive in general. In order to explore the climate change impact on the detrend price we need to know the impact on the rate of return on capital and the degree of substituability between capital and labour in the non-agricultural sector as well as the stationary value of the rate of return on capital.

The sign of $\frac{\partial \Gamma_1^*}{\partial T^*}$: By definition of Γ_1^*, it is straightforward to show that

$$\frac{\partial \Gamma_1^*}{\partial T^*} = (1+n)\frac{\partial \tilde{\gamma}_n^*}{\partial T^*} < 0. \tag{C.8.10}$$

The signs of $\frac{\partial \Gamma_2^*}{\partial T^*}, \frac{\partial \Lambda_1^*}{\partial T^*}, \frac{\partial \Lambda_2^*}{\partial T^*}$: By definition of Γ_2^*, Λ_1^* and Λ_2^*, it is straightforward to show that

$$\frac{\partial \Gamma_2^*}{\partial T^*} = -\frac{\Gamma_2^*}{1-\psi_n}\frac{1}{\Upsilon^*}\frac{1}{R^*}\frac{\partial R^*}{\partial T^*}, \tag{C.8.11}$$

$$\frac{\partial \Lambda_1^*}{\partial T^*} = -\frac{\Lambda_1^*}{1-\psi_n}\frac{\Upsilon^*+1}{\Upsilon^*}\frac{1}{R^*}\frac{\partial R^*}{\partial T^*}, \tag{C.8.12}$$

$$\frac{\partial \Lambda_2^*}{\partial T^*} = -(\Lambda_1^*)^{1-\psi_n}\hat{\alpha}\frac{\Upsilon^*+1}{\Upsilon^*}\frac{1}{R^*}\frac{\partial R^*}{\partial T^*}. \tag{C.8.13}$$

Clearly, the signs of these are as the same as that of $\Big(-\frac{\partial R^*}{\partial T^*}\Big)$.

The sign of $\Big(\frac{\partial \Gamma_2^*}{\partial T^*} - \Gamma_1^*\frac{\partial \Lambda_1^*}{\partial T^*} - \Lambda_1^*\frac{\partial \Gamma_1^*}{\partial T^*}\Big)$: After some mathematical manipulations, we can show that

$$\begin{aligned} &\frac{\partial \Gamma_2^*}{\partial T^*} - \Gamma_1^*\frac{\partial \Lambda_1^*}{\partial T^*} - \Lambda_1^*\frac{\partial \Gamma_1^*}{\partial T^*} = \\ &\frac{\Gamma_1^*}{R^*}\frac{\Big\{\Gamma_1^*(\Upsilon+1) - \Big[\alpha_n + (1-\alpha_n)(\Lambda_1^*)^{-\psi_n}\Big]^{\frac{1}{\psi_n}}\Big\}\frac{\partial R^*}{\partial T^*}}{(1-\psi_n)\Upsilon^*} - \Lambda_1^*(1+n)\frac{\partial \tilde{\gamma}_n^*}{\partial T^*}. \end{aligned} \tag{C.8.14}$$

In general, the sign of $\frac{\partial \Gamma_2^*}{\partial T^*} - \Gamma_1^*\frac{\partial \Lambda_1^*}{\partial T^*} - \Lambda_1^*\frac{\partial \Gamma_1^*}{\partial T^*}$ is ambiguous. In some cases, if the sign of the term $\{\bullet\}\frac{\partial R^*}{\partial T^*}$ on the right-hand side of the above expression is positive, we can conclude that $\frac{\partial \Gamma_2^*}{\partial T^*} - \Gamma_1^*\frac{\partial \Lambda_1^*}{\partial T^*} - \Lambda_1^*\frac{\partial \Gamma_1^*}{\partial T^*} > 0$.

Another interesting point from this expression is that

$$\{\bullet\} \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} 0 \Leftrightarrow (1+\tilde{\gamma}_n^*) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \frac{1}{1+n}\Big[1+R^*-\delta\Big]. \tag{C.8.15}$$

This will be very useful when analysing the structural change effect. We will discuss about this later.

The sign of $\frac{\partial\Theta}{\partial T^*}$: Straightforward differentiating Θ with respect to T^* yields

$$\frac{\partial\Theta}{\partial T^*} = \Gamma_1^*\frac{\partial\Lambda_2^*}{\partial T^*} + \Lambda_2^*\frac{\partial\Gamma_1^*}{\partial T^*} + \hat{\theta}\tilde{p}^*\alpha_a(\Lambda_2^*)^{\alpha_a-1}\frac{\partial\Lambda_2^*}{\partial T^*} + \hat{\theta}(\Lambda_2^*)^{\alpha_a}\frac{\partial\tilde{p}^*}{\partial T^*}. \tag{C.8.16}$$

After some mathematical manipulations, we can show that

$$\frac{\partial\Theta}{\partial T^*} = \Lambda_2^*(1+n)\frac{\partial\tilde{\gamma}_n^*}{\partial T^*} + \frac{\left[\frac{\partial R^*}{\partial T^*}\right]}{\Upsilon^*}\left\{\begin{array}{c} \hat{\theta}(\Lambda_2^*)^{\alpha_a}\frac{\hat{\alpha}^{1-\alpha_a}}{\alpha_a}\left(\frac{1}{\Upsilon^*}\cdot\frac{1-\alpha_n}{\alpha_n}\right)^{\frac{(1-\alpha_a)(1-\psi_n)}{\psi_n}} \\ \times[\alpha_a(\Upsilon^*+1)-1] \\ - \\ [\Gamma_1^*+\hat{\theta}\tilde{p}^*\alpha_a(\Lambda_2^*)^{\alpha_a-1}](\Lambda_1^*)^{1-\psi_n}\hat{\alpha}\left(\frac{\Upsilon^*+1}{R^*}\right) \end{array}\right\}. \tag{C.8.17}$$

Clearly, the sign of the above expression is unclear since the term $\left[\frac{\partial R^*}{\partial T^*}\right]\{\bullet\}$ on the righ-hand side can be positive, zero or negative. However, if $\left[\frac{\partial R^*}{\partial T^*}\right]\{\bullet\} \leq 0$, we can conclude that $\frac{\partial\Theta}{\partial T^*} < 0$.

In sum, the impact on employment share is inconclusive as it depends on the parameter values and a certain set of feasibility restrictions. In some cases, we can say that

$$\sigma \geq 1, \psi_n < 0, R^* > \alpha_a^{\frac{1+|\psi_n|}{|\psi_n|}}\left(\frac{1}{\alpha_n}\right)^{\frac{1}{|\psi_n|}} \Rightarrow \frac{\partial R^*}{\partial T^*} < 0, \frac{\partial\tilde{p}^*}{\partial T^*} > 0.$$

Combined with

$$(1+\tilde{\gamma}_n^*) < \frac{1}{1+n}\Big[1+R^*-\delta\Big] \Rightarrow \Big(\frac{\partial\Gamma_2^*}{\partial T^*} - \Gamma_1^*\frac{\partial\Lambda_1^*}{\partial T^*} - \Lambda_1^*\frac{\partial\Gamma_1^*}{\partial T^*}\Big) > 0,$$

$$\left[\frac{\partial R^*}{\partial T^*}\right]\{\bullet\}\text{on the RHS of (C.8.17)} \leq 0 \Rightarrow \frac{\partial\Theta}{\partial T^*} < 0$$

, then we can say $\frac{\partial l_n^*}{\partial T^*} < 0$ if the restrictions (4.51) and (4.52) holds. **Q.E.D.**

C.9 Further Characterisations of Value-Added Share Effect

Define $y_{n,t} \equiv \frac{Y_{n,t}}{A_{n,t}N_t}$ as the non-agricultural output per effective unit. Then,

$$y_{n,t} = \Big[\alpha_n(s_{n,t}z_t)^{\psi_n} + (1-\alpha_n)(l_{n,t})^{\psi_n}\Big]^{\frac{1}{\psi_n}} \tag{C.9.1}$$

Along a BGP, $y_{n,t} = y_n^*$ is constant and

$$\frac{\partial y_n^*}{\partial T^*} = \Big[\frac{y_n^*}{l_n^*}\Big]^{1-\psi_n}\cdot\Big\{\alpha_n\Big(\frac{s_n^*z^*}{l_n^*}\Big)^{\psi_n-1}\Big(s_n^*\frac{\partial z^*}{\partial T^*} + z^*\frac{\partial s_n^*}{\partial T^*}\Big) + (1-\alpha_n)\frac{\partial l_n^*}{\partial T^*}\Big\}. \tag{C.9.2}$$

Similarly, define $p_t y_{a,t} \equiv \frac{p_t Y_{a,t}}{A_{n,t} N_t}$ as the value of agricultural output per effective unit. Then,

$$p_t y_{a,t} = \tilde{p}_t [(1 - s_{n,t}) z_t]^{\alpha_a} (1 - l_{n,t})^{1-\alpha_a} \tag{C.9.3}$$

Along a BGP, $p_t y_{a,t} = (p_t y_a)^*$ is constant and

$$\frac{\partial (p_t y_a)^*}{\partial T^*} = \frac{\partial \tilde{p}^*}{\partial T^*} + \tilde{p}^* (1 - s_n^*)^{\alpha_a} (1 - l_n^*)^{1-\alpha_a} (z^*)^{\alpha_a} \left\{ \frac{\alpha_a}{z^*} \frac{\partial z^*}{\partial T^*} - \frac{1-\alpha_a}{1 - l_n^*} \frac{\partial l_n^*}{\partial T^*} - \frac{\alpha_a}{1 - s_n^*} \frac{\partial s_n^*}{\partial T^*} \right\}. \tag{C.9.4}$$

Clearly, the signs of both and are ambiguous depending on the signs of $\frac{\partial s_n^*}{\partial T^*}$ and $\frac{\partial z^*}{\partial T^*}$.

The sign of $\frac{\partial s_n^*}{\partial T^*}$: Using (4.54), we obtain

$$\frac{\partial s_n^*}{\partial T^*} = \frac{1}{(\Lambda_1^* l_n^* + \Lambda_2^* (1 - l_n^*))^2} \cdot \begin{Bmatrix} \Lambda_2^* (1 - l_n^*) \Big[\Lambda_1^* \frac{\partial l_n^*}{\partial T^*} + l_n^* \frac{\partial \Lambda_1^*}{\partial T^*} \Big] \\ + \\ \Lambda_1^* l_n^* \Big[\Lambda_2^* \frac{\partial l_n^*}{\partial T^*} - (1 - l_n^*) \frac{\partial \Lambda_2^*}{\partial T^*} \Big] \end{Bmatrix} \tag{C.9.5}$$

which is ambiguous.

The sign of $\frac{\partial z^*}{\partial T^*}$: Using (4.55), we obtain

$$\frac{\partial z^*}{\partial T^*} = (\Lambda_1^* - \Lambda_2^*) \frac{\partial l_n^*}{\partial T^*} + \Big(\frac{\partial \Lambda_1^*}{\partial T^*} - \frac{\partial \Lambda_2^*}{\partial T^*} \Big) + \frac{\partial \Lambda_2^*}{\partial T^*} \tag{C.9.6}$$

which is ambiguous. Note that

$$(\Lambda_1^* - \Lambda_2^*) = -\frac{1}{1 - \alpha_a} \Big[\alpha_a (\Upsilon^* + 1) - 1 \Big].$$

It turns out that

$$\text{case i } \psi_n < 0: \quad (\Lambda_1^* - \Lambda_2^*) \gtreqless 0 \Leftrightarrow R^* \gtreqless \alpha_a^{\frac{1+|\psi_n|}{|\psi_n|}} \alpha_n^{-\frac{1}{|\psi_n|}}, \tag{C.9.7}$$

$$\text{case ii } \psi_n \to 0: \quad (\Lambda_1^* - \Lambda_2^*) \gtreqless 0 \Leftrightarrow \alpha_a \lesseqgtr \alpha_n, \tag{C.9.8}$$

$$\text{case iii } \psi_n > 0: \quad (\Lambda_1^* - \Lambda_2^*) \gtreqless 0 \Leftrightarrow R^* \lesseqgtr \alpha_n^{\frac{1}{\psi_n}} \alpha_a^{-\frac{1-\psi_n}{\psi_n}}. \tag{C.9.9}$$

For that of $\Big(\frac{\partial \Lambda_1^*}{\partial T^*} - \frac{\partial \Lambda_2^*}{\partial T^*} \Big)$, we can show that

$$\Big(\frac{\partial \Lambda_1^*}{\partial T^*} - \frac{\partial \Lambda_2^*}{\partial T^*} \Big) = \Big[(\Lambda_1^*)^{1-\psi_n} \hat{\alpha} - \frac{\Lambda_1^*}{1 - \psi_n} \Big] \frac{\Upsilon^* + 1}{\Upsilon^*} \frac{1}{R^*} \frac{\partial R^*}{\partial T^*}.$$

Clearly, the sign of the above expression depends on that of $\Big[(\Lambda_1^*)^{1-\psi_n} \hat{\alpha} - \frac{\Lambda_1^*}{1-\psi_n} \Big] \frac{\partial R^*}{\partial T^*}$ and we can show that

$$\text{case i}\frac{\partial R^*}{\partial T^*} < 0\text{and}\psi_n < 0: \Big(\frac{\partial \Lambda_1^*}{\partial T^*} - \frac{\partial \Lambda_2^*}{\partial T^*}\Big) \gtreqless 0 \Leftrightarrow R^* \gtreqless \alpha_n^{-\frac{1}{|\psi_n|}} \left(1 + \Big(\frac{1-\alpha_a}{\alpha_a}\Big)\frac{1}{1+\mid \psi_n \mid}\right)^{-\frac{1+|\psi_n|}{|\psi_n|}}, \tag{C.9.10}$$

$$\text{case ii}\,\frac{\partial R^*}{\partial T^*} < 0\,\text{and}\,\psi_n \to 0: \ \Big(\frac{\partial \Lambda_1^*}{\partial T^*} - \frac{\partial \Lambda_2^*}{\partial T^*}\Big) \gtreqless 0 \ \Leftrightarrow \ \alpha_a \lesseqgtr \alpha_n, \tag{C.9.11}$$

$$\text{case iii}\frac{\partial R^*}{\partial T^*} < 0\text{and}\psi_n > 0: \Big(\frac{\partial \Lambda_1^*}{\partial T^*} - \frac{\partial \Lambda_2^*}{\partial T^*}\Big) \gtreqless 0 \Leftrightarrow R^* \lesseqgtr \alpha_n^{\frac{1}{\psi_n}} \left(1 + \frac{1}{1-\psi_n}\Big(\frac{1-\alpha_a}{\alpha_a}\Big)\right)^{\frac{1-\psi_n}{\psi_n}}, \tag{C.9.12}$$

$$\text{case iv}\ \frac{\partial R^*}{\partial T^*} = 0: \ \Big(\frac{\partial \Lambda_1^*}{\partial T^*} - \frac{\partial \Lambda_2^*}{\partial T^*}\Big) = 0 \ \ \forall \psi_n \in (-\infty, 1), \tag{C.9.13}$$

$$\text{case v}\frac{\partial R^*}{\partial T^*} > 0\text{and}\psi_n < 0: \Big(\frac{\partial \Lambda_1^*}{\partial T^*} - \frac{\partial \Lambda_2^*}{\partial T^*}\Big) \gtreqless 0 \Leftrightarrow R^* \lesseqgtr \alpha_n^{-\frac{1}{|\psi_n|}} \left(1 + \Big(\frac{1-\alpha_a}{\alpha_a}\Big)\frac{1}{1+\mid \psi_n \mid}\right)^{-\frac{1+|\psi_n|}{|\psi_n|}}, \tag{C.9.14}$$

$$\text{case vi}\ \frac{\partial R^*}{\partial T^*} > 0 \text{ and } \psi_n \to 0: \ \Big(\frac{\partial \Lambda_1^*}{\partial T^*} - \frac{\partial \Lambda_2^*}{\partial T^*}\Big) \gtreqless 0 \ \Leftrightarrow \ \alpha_a \gtreqless \alpha_n, \qquad , \tag{C.9.15}$$

$$\text{case vii}\frac{\partial R^*}{\partial T^*} > 0\text{and}\psi_n > 0: \Big(\frac{\partial \Lambda_1^*}{\partial T^*} - \frac{\partial \Lambda_2^*}{\partial T^*}\Big) \gtreqless 0 \Leftrightarrow R^* \gtreqless \alpha_n^{\frac{1}{\psi_n}} \left(1 + \frac{1}{1-\psi_n}\Big(\frac{1-\alpha_a}{\alpha_a}\Big)\right)^{\frac{1-\psi_n}{\psi_n}}. \tag{C.9.16}$$

Value-Added Share: We define the aggregate output in any period t, denoted by Y_t, as

$$Y_t = p_t Y_{a,t} + Y_{n,t}. \tag{C.9.17}$$

Let $s_{Yn,t}$ be the share of non-agricultural output: $s_{Yn,t} = \frac{Y_{n,t}}{Y_t}$. Then, we can show that

$$s_{Yn,t} = \frac{y_{n,t}}{p_t y_{a,t} + y_{n,t}}. \tag{C.9.18}$$

To evaluate the climate impact on the value-added share, we differentiate the above expression with respect to T^* so that

$$\frac{\partial s^*_{Yn}}{\partial T^*} = \frac{1}{(p_t y_{a,t} + y_{n,t})^2}\Big\{(p_t y_{a,t} + y_{n,t})\frac{\partial y^*_n}{\partial T^*} - y^*_n \frac{\partial (p_t y_{a,t})^*}{\partial T^*}\Big\}. \tag{C.9.19}$$

Clearly, in general, the impact is ambiguous. **Q.E.D.**

C.10 Optional: Characterisation of an Asymptotic Balanced Growth Path(ABGP)

Definition C.1. *A market equilibrium is said to be an asymptotically balanced growth path (ABGP) if such equilibrium path implies that the rate on return on capital converges to a positive value and for any variable X_t, $\lim_{t\to\infty}\left(\frac{X_{t+1}}{X_t} - 1\right) = g_X$ exists and is finite, where g_X is a constant growth rate of the variable X_t.*

By definition, BGP can be seen as an ABGP. To distinguish between the two equilibria, we will only consider ABGP as a path along which the rate of return on capital and the growth rates of all endogenous variables converge but will never be constant in finite periods. As a result, the constancy in limit of R_t requires

$$\lim_{t\to\infty} l_{n,t} = l^{**}_n \in (0,1), \lim_{t\to\infty} s_{n,t} = s^{**}_n \in (0,1), \lim_{t\to\infty} z_t = z^{**} > 0. \tag{C.10.1}$$

If we consider (4.41) and (4.42), eventual convergences of these three variables require either eventually unbiased technological progress or unitary elasticity of substitution between capital and labour in agricultural sector. By logical consistency, we keep assuming $\psi_a = 0$.

Under the assumption $\psi_a = 0$, the intratemporal tradeoff condition is given by (C.2.4), as illustrated in the proof C.5. Condition (C.5.3) still holds in ABGP, this condition implies that

$$\lim_{t\to\infty} \tilde{p}_t = \tilde{p}^{**} > 0. \tag{C.10.2}$$

To ensure the existence of an ABGP, it is required that

$$\lim_{t\to\infty} \tilde{c}_{n,t} = \tilde{c}^{**}_n > 0. \tag{C.10.3}$$

Focusing on (C.5.4), this means that the ratio $\frac{\bar{c}_{a,t}}{A^{1-\alpha_a}_{a,t} A^{\alpha_a}_{n,t}}$ must disappear in limit in order to establish an ABGP. This leads to the following assumption.

Assumption C.1. *There exists $\overline{\tau} > 0$ such that*

$$\frac{\bar{c}_{a,t+1}}{\bar{c}_{a,t}} < \frac{A^{1-\alpha_a}_{a,t+1} A^{\alpha_a}_{n,t+1}}{A^{1-\alpha_a}_{a,t} A^{\alpha_a}_{n,t}}$$

for all $t \geq \overline{\tau}$.

Strictly speaking, this assumption requires that the growth factor $A_{a,t}^{1-\alpha_a} A_{n,t}^{\alpha_a}$ will eventually dominate the subsistence level of agricultural consumption $\bar{c}_{a,t}$ in the long-run. An immediate implication of this assumption is that the intratemporal tradeoff condition in limit is given by

$$\tilde{p}_t \left((1 - l_{n,t}) \Big(\frac{1 - s_{n,t}}{1 - l_{n,t}} z_t \Big)^{\alpha_a} \right) = \Big(\frac{\theta}{1-\theta} \Big) \tilde{c}_{n,t}. \tag{C.10.4}$$

Note that if $\bar{c}_{a,t}$ is constant as usual assumed in literature, Assumption C.1 will be satisfied automatically.

An ABGP exists under certain conditions and the following proposition verify such a path. The following theorem provide an ABGP characterisation.

Theorem C.1. *Let $\psi_a = 0$ and **Assumption C.1** hold. Suppose that there is a finite $\tau > 0$ such that $\tilde{\gamma}_{i,t} = \tilde{\gamma}_i^{**}$ for $i \in \{a, n\}, t \geq \tau$. Then, there exists a unique (non-trivial) ABGP along which the rate of return on capital converges to*

$$R^{**} = \frac{\kappa}{\beta} - (1 - \delta) \tag{C.10.5}$$

where $\kappa \equiv (1 + \tilde{\gamma}_n^*)^\sigma \Big(\frac{1+\tilde{\gamma}_n^*}{1+\tilde{\gamma}_a^{**}} \Big)^{(1-\alpha_a)\theta(1-\sigma)}$. Suppose that the parameter restrictions

$$R^{**} > 0 \quad and \quad \Upsilon^{**} \equiv (\alpha_n)^{\frac{-1}{1-\psi_n}} (R^{**})^{\frac{\psi_n}{1-\psi_n}} - 1 > 0 \tag{C.10.6}$$

hold. Then, the unique (non-trivial) ABGP is associated with the fixed point $(s_n^{**}, l_n^{**}, z^{**}, \tilde{p}^{**}, \tilde{c}_n^{**}) \in (0,1)^2 \times \mathbb{R}_{++}^3$ such that

$$\tilde{p}^{**} = \Big(\frac{1-\alpha_n}{1-\alpha_a} \Big) (\Lambda_1^{**})^{(1-\alpha_a)(1-\psi_n)} \Big(\frac{\alpha_a}{1-\alpha_a} \cdot \frac{1-\alpha_n}{\alpha_n} \Big)^{-\alpha_a} \Big(\frac{R^{**}}{\alpha_n} \Big), \tag{C.10.7}$$

$$l_n^{**} = \frac{\Gamma_1^{**}\Lambda_2^{**} + \tilde{p}^{**}(\frac{1-\theta}{\theta})(\Lambda_2^{**})^{\alpha_a}}{\Gamma_1^{**}\Lambda_2^{**} + \tilde{p}^{**}(\frac{1-\theta}{\theta})(\Lambda_2^{**})^{\alpha_a} + \Gamma_2^{**} - \Gamma_1^{**}\Lambda_1^{**}}, \tag{C.10.8}$$

$$s_n^{**} = \frac{\Lambda_1^{**} l_n^{**}}{\Lambda_1^{**} l_n^{**} + \Lambda_2^{**}(1 - l_n^*)}, \tag{C.10.9}$$

$$z^{**} = \Lambda_1^{**} l_n^{**} + \Lambda_2^{**}(1 - l_n^{**}), \tag{C.10.10}$$

$$\tilde{c}_n^{**} = \tilde{p}^{**} \Big(\frac{1-\theta}{\theta} \Big) \left((1 - l_n^{**}) (\Lambda_1^{**})^{\alpha_a(1-\psi_n)} \Big(\frac{\alpha_a}{1-\alpha_a} \cdot \frac{1-\alpha_n}{\alpha_n} \Big)^{\alpha_a} \right), \tag{C.10.11}$$

if the inequality $\Gamma_1^{**}\Lambda_1^{**} - \Gamma_2^{**} < 0$ holds, where $\Lambda_1^{**} \equiv (\Upsilon^{**})^{\frac{-1}{\psi_n}} \Big(\frac{1-\alpha_n}{\alpha_n} \Big)^{\frac{1}{\psi_n}}$, $\Lambda_2^{**} \equiv \Big(\frac{\tilde{p}^{**}\alpha_a}{R^{**}} \Big)^{\frac{1}{1-\alpha_a}}$, $\Gamma_1^{**} \equiv \Big[(1+n)(1+\tilde{\gamma}_n^*) - (1-\delta) \Big]$ and $\Gamma_2^{**} \equiv (\Upsilon^{**})^{-\frac{1}{\psi_n}} \Big[(1-\alpha_n)(R^{**})^{\frac{\psi_n}{1-\psi_n}} (\alpha_n)^{-\frac{1}{1-\psi_n}} \Big]^{\frac{1}{\psi_n}}$.

Proof: The proof is analogous to that of **Proposition 4.2** except that we use the intratemporal trade off condition (C.10.4) instead of (C.5.4). As a result, we can see that the (non-trivial) fixed point in this proposition is a special case of the one obtained in Proposition 4.2. Clearly, $R^{**} = R^* > 0$. In addition, the fixed point here is the point $(z^*, l_n^*, s_n^*, \tilde{p}^*, \tilde{c}_n^*)$ while setting $\mu = 0$. From now, it is straightforward. **Q.E.D.**

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